Sovereign Default, Inequality, and Progressive Taxation

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Abstract

A sovereign’s willingness to repay its foreign debt depends on the cost of raising taxes. The allocation of this tax burden across households is a key factor in this decision. To study the interaction between the incentive to default and the distributional cost of taxes, I extend the canonical Eaton-Gersovitz-Arellano model to include heterogeneous agents, progressive taxation, and elastic labor supply. When the progressivity of the tax schedule is exogenous, progressivity and the incentive to default are inversely related. Less progressive taxes, and hence higher after-tax inequality, encourage default since the cost of raising tax revenue from a larger mass of low-income households outweighs the cost of default in the form of lost insurance opportunities. When tax progressivity is endogenous and chosen optimally, the government internalizes the influence of progressivity on default risk and the cost of borrowing. As such, committing to a more progressive tax system emerges as an effective policy tool to reduce sovereign credit spreads in highly indebted countries.

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1 Introduction

During the recent Eurozone sovereign debt turmoil, the burden that fiscal austerity placed on low-income households was a widespread concern. For example, in response to the May 2010 credit spreads spike, Greece adopted a series of rather regressive austerity measures (Matsaganis and Leventi, 2013).\textsuperscript{1} The events that followed - in particular, the continued rise in spreads (Figure 1), and the political success of the radical left coalition’s platform to cancel both public debt and the austerity plan at the May 2012 legislative election - reflect the importance of these fiscal measures’ welfare costs on Greece’s eventual default decisions.\textsuperscript{2}

These events suggest that the distribution of debt repayment costs, in the form of domestic fiscal policies, affects a country’s willingness to repay. As such, how should a country optimally distribute taxes to repay foreign sovereign debt? To answer this question, I analyze a model of sovereign default with heterogeneous agents, progressive taxes, and elastic labor. With exogenous tax progressivity, I first establish that progressivity and the incentive to default are inversely related. Then, I show that this finding has implications for how the government should design fiscal policies: committing to a more progressive tax system is an effective policy tool to reduce sovereign credit spreads in highly indebted countries. The results suggest that redistributive concerns should be included in debt sustainability assessment.

The point of departure is the workhorse model of sovereign default with a representative agent, as described by Eaton and Gersovitz (1981) and Arellano (2008). In this setup, foreign sovereign debt is typically rationalized as an insurance tool against domestic aggregate shocks. Default is determined by an intertemporal insurance tradeoff. In adverse times, a government can increase current consumption by defaulting; households consume what would otherwise have been paid to foreign creditors. On the other hand, defaulting also makes future consumption more volatile, since the sovereign is typically excluded from global financial markets for a time. When the contemporaneous gains outweigh future costs, the country chooses to default. To study the distributional

\textsuperscript{1}Matsaganis and Leventi (2013) argue that the poor shouldered a larger share of the 2010 fiscal consolidation than the richer, relative to their income, as documented in Annex A. This is because a large part of the austerity was implemented through increases in VAT and cuts in pension benefits.

\textsuperscript{2}SYRIZA, the Coalition of Radical Left, finished second in May 2012, but appeared unable to form a coalition to actually govern.
consequences involved in these tradeoffs, this paper extends the literature in three dimensions: heterogenous agents, elastic labor supply, and some limitation to the government’s capacity to redistribute in the form of progressive taxes on labor income.

The key economic force for distributional concerns to interact with default decisions is how ratios of marginal utilities across agents vary across aggregate shocks. Under CRRA preferences, when these ratios are constant the economy aggregates to a representative agent; debt and default decisions are only driven by aggregate insurance motives and thus are independent of after-tax inequality. This condition would hold if households could trade a complete set of Arrow securities across themselves, or if the government could use a sufficiently nonlinear tax system (lump sum taxes indexed by households). On the other hand, interactions between concerns for redistribution and default decisions are magnified when (i) markets for private insurance are absent (hand-to-mouth households) and (ii) the government uses a linear tax. For most of the analysis, the paper imposes these two assumptions; this makes the setup tractable and allows for deriving a quantitative bound of how large these interactions can be.

Given that distributional concerns matter, Section 2 analyzes how tax progressivity alters incentives to default. I begin with the case where the progressivity of the tax schedule is exogenous.
and compare countries with different levels of progressivity. Progressivity alters both after-tax inequality and efficiency.

The effect of inequality on incentives to default is ambiguous as both the contemporaneous gains and the future costs of default increase with inequality. First, when a country does not repay its existing debt, the amount of revenues raised by the government can decrease. This tax decrease is valued more by low-income households, so the larger the mass of low-income households, the larger the contemporaneous gain of default. On the other hand, future costs of default also increase in inequality: because of financial autarky, the government cannot use foreign debt to smooth taxes. More volatile taxes are also more welfare-costly for low-income households, so the larger the mass of low-income households, the larger the future costs of default. Typically, I find that the first force dominates, and more progressive economies have less incentives to default. The intuition for this asymmetry comes from an aggregation property that holds in this setting: more unequal economies can be described by a representative-agent, whose implied risk aversion endogenously increases with after-tax inequality, in particular for low levels of aggregate consumption. The main finding is explained by the fact that countries tend to default in adverse times when aggregate consumption is indeed low, resulting in gains of defaulting increasing sharply with inequality.

Tax progressivity does not only change inequality but also adds an efficiency consideration, as incentives to work are distorted. A more progressive economy features a smaller output and consequently decreases the maximum amount of debt it can sustain. This force, which could in principle reverse the result, appears dominant only for very high levels of tax progressivity.

Finally, to quantify how much tax progressivity affects default sets, I calibrate the model, including pre-tax and after-tax income distribution, to Argentina. With hand-to-mouth households and a linear tax, I find that average debt-over-GDP ratios are as much as three times smaller than in an economy populated with a representative agent, reflecting the fact that a country with higher after-tax inequality has larger default incentives.

In Section 3, I report two pieces of evidence to support my results. First, I document a negative relationship between foreign debt and inequality. To illustrate this, I show how external public debt to GDP ratios vary across countries with different levels of inequality and tax progressivity. Second,
I report that the empirical results of Aizenman and Jinjarak (2012), who show that spreads are positively correlated with inequality, controlling for per capita income and debt over GDP ratios, are in line with the model’s predictions.

Finally, Section 4 turns to the optimal policy for tax progressivity in presence of default risk. Inequality, by changing incentives to default, alters prices at which the government can issue debt in equilibrium. As a consequence, when the government is able to commit to future progressivity, it internalizes the influence of progressivity on default risk and the cost of borrowing, beyond the usual efficiency-redistribution tradeoff. To analyze this new forward-looking role of progressivity, I assume a timing protocol where tax progressivity is chosen one period in advance and non-state-contingently; this setup could reflect tax inertia. The main result is that committing to a more progressive tax system - which decreases future inequality, and hence lowers future incentives to default - emerges as an effective policy tool to reduce sovereign credit spreads in highly indebted countries. This force is isolated by comparing optimal progressivity across settings that differ in the government’s ability to commit. Taken together, the findings suggest that redistributive concerns are an important dimension for debt sustainability assessment.

1.1 Literature review

This paper is related to two lines of research: (i) sovereign default, and (ii) optimal fiscal policy. The modeling of sovereign default used in this paper is based on the two canonical models of Eaton and Gersovitz (1981) and Arellano (2008). As in Cuadra, Sanchez, and Sapriza (2010), the setup allows for elastic labor and a distortionary tax. With respect to the literature on optimal fiscal policy, as in Werning (2007) and Bhandari, Evans, Golosov, and Sargent (2013), this paper studies optimal redistribution across aggregate shocks, in particular when fiscal tools are limited to linear taxes. The two papers mentioned above develop a richer setup for private insurance, with complete and incomplete markets, respectively; this paper incorporates external sovereign debt and no commitment on taxes and debt repayments. In addition, the paper relates to the literature that quantitatively evaluates optimal progressivity of labor taxes in models with heterogeneous agents.

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3 A large literature has emerged to enrich the model of Arellano (2008) and bring it closer to the data; among others, Mendoza and Yue (2012) endogenize part of the output cost of autarky with a production firm.
Relative to the efficiency vs. redistribution tradeoff that is the main focus of Conesa, Kitao, and Krueger (2009) and Heathcote, Storesletten, and Violante (2014), the presence of default risk adds an extra motive for tax progressivity. In particular, it is shown how commitment on future tax progressivity gives a government the possibility to manipulate interest rates at which it can issue bonds. This channel is a particular example of how commitment on future taxes can be used to manipulate current prices, a well-known mechanism in the literature of optimal fiscal policy since Lucas and Stokey (1983).

Two papers explore more directly the distributional costs of sovereign debt. Mengus (2014) provides a rationale for external sovereign borrowing using limits to fiscal redistribution and internal default. In particular, he shows that in absence of these features, autarky is too valuable to make external borrowing sustainable. In this paper, the rationale for borrowing is standard, and the focus is on how the tradeoffs which determine default are sensitive to costs of inequality resulting from the government’s limited ability to redistribute. Finally, D’Erasmo and Mendoza (2014) propose a tractable incomplete markets model with labor income risk to explore how the distribution of domestic public debt across households changes the government’s incentives to default. On the other hand, this paper ignores domestic public debt and internal default to focus on the distributional effects of taxes and external default.

2 Tax progressivity and incentives to default: a positive analysis

This section describes how incentives to default change with tax progressivity, when progressivity is taken as exogenous. The main finding is that more progressive economies have a weaker incentive to default, and therefore feature smaller default sets. In that setup, tax progressivity alters both after-tax inequality (redistribution) and labor supply (efficiency). To disentangle which forces drive

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4 Both papers study richer tax functions in setups where tax progressivity is constant across time. Heathcote, Storesletten, and Violante (2014) also discuss human capital accumulation.

5 In addition, in some preliminary work, Dovis, Golosov, and Shourideh (2014) characterize the long-term optimal lending-borrowing contract between a country and foreign investors, where there is limited enforcement on the government’s side. As in this paper, the domestic economy features some motives for redistribution that interact with the government’s incentives to work away from the contract.

6 Impatience of the domestic economy and costs of default (financial autarky and productivity loss).

7 To make the problem tractable, the set of policies chosen optimally by the government are restricted to the default decision; taxes, and the policy for debt when no default, are assumed exogenous.
the result, I discuss each channel by analyzing first inelastic, then elastic labor.

2.1 Environment

Time is discrete and indexed by \( t = 0, 1, 2, \ldots \). The economy is populated with three types of agents: a continuum of households, a government, and foreign lenders. Aggregate uncertainty is driven by aggregate productivity shocks \( a_t \). Households value consumption and leisure, and discount the future at rate \( \beta \). Each household \( i \) has idiosyncratic labor productivity \( \epsilon_i \). The distribution of idiosyncratic productivities is fixed over time, with mean \( \bar{\epsilon} \) and standard deviation \( \sigma_\epsilon \). Every period, households choose labor supply, given its own productivity, the aggregate TFP shock that multiplies their idiosyncratic productivity, and the tax schedule \( T(. \) ). Finally, households are assumed hand-to-mouth; they consume their after-tax income.\(^8\) A Utilitarian government finances constant government spending \( g \), using two fiscal tools: an income tax, described by \( T(.) \); and foreign non-contingent debt. Notice that in this set-up, the role of debt is twofold: first, smooth taxes to smooth consumption against the aggregate shock; and second, frontload or backload consumption, depending on the discrepancy between the households’ discount factor and the rate of return on the debt. However, the government cannot commit to repay its debt. Default triggers temporary exclusion from financial markets, together with additional costs of financial autarky. Finally, risk-neutral international lenders buy the debt issued by the government and have access to a risk-free bond at a price \( q^\star \). The price \( q \) of the sovereign debt is determined by a no-arbitrage condition in equilibrium.

A parsimonious tax function The tax function \( T(y; \tau_1, \tau_2) \), given a pre-tax income \( y \), is assumed a function of two parameters, \( \tau_1 \) and \( \tau_2 \), such that

\[
\frac{\partial}{\partial \tau_1} \left( \frac{\partial T(y; \tau_1, \tau_2)}{\partial y} \right) \geq 0, \quad \text{and} \quad \frac{\partial}{\partial \tau_2} \left( \frac{T(y; \tau_1, \tau_2)}{y} \right) \leq 0
\]

\(^8\)The modeling choice of hand-to-mouth consumers, although not fully satisfactory, makes the problem significantly more tractable. To understand the technical challenges and the economic implications of forward-looking households, an extension is presented in Annex.
A larger $\tau_1$ implies higher marginal tax rates, hence a steeper tax function, while a larger $\tau_2$ results in larger average tax rates. Most of the paper focuses on a linear tax function:

$$T(y; \tau_1, \tau_2) = (1 - \tau_1)y - \tau_2$$

(1)

Roughly speaking, $\tau_1$ captures progressivity, while $\tau_2$ describes the level of taxes. The most progressive tax system is $\tau_1 = 1$, which generates full redistribution; on the other hand, $\tau_1 = 0$ describes the least progressive environment, where the government relies on lump-sum taxes only to finance its spending. Importantly, $\tau_1$ is understood as a fixed characteristic of the economy in this section, while the level of the tax function $\tau_2$ is chosen optimally by the government every period to balance its budget.

**Households problem** Let $T'(.)$ be the derivative of the tax function with respect to its first argument. The households maximization problem (2) imposes the first-order constraints (3) to hold in equilibrium.

$$\max_{c_i, n_i} u(c_i, n_i) \text{ s.t. } c_i = a\epsilon_in_i - T(a\epsilon_in_i; \tau_1, \tau_2) \forall i$$

(2)

$$u_{n,i} = -u_{c,i}a\epsilon_i (1 - T'(a\epsilon_in_i; \tau_1, \tau_2)) \forall i$$

(3)

**Government problem** A recursive equilibrium is defined, in which the government does not have commitment and the government, foreign creditors, and households act sequentially. Assume a period where the country is not excluded from international financial markets. First the government observes the TFP shock $a$ and, given some debt $B$, chooses whether to repay or default. Let $V$ be the value function of the government at the beginning of the period:

$$V(B, a) = \max\{V^{nd}(B, a), V^d(a)\}$$
The value of non-default, $V^{nd}(B, a)$ is determined as follows:

$$V^{nd}(B, a) = \max_{B', \tau_2, \{c_i, n_i\}} \int u(c_i, n_i)di + \beta \mathbb{E}_{\alpha'} V(B', a')$$

s.t. $g + B = \int T(\alpha ; n_i; \tau_1, \tau_2)di + q(B', a)B'$

$$c_i = a\epsilon_i n_i - T(\alpha ; n_i; \tau_1, \tau_2) \forall i$$

$$u_{n,i} = -u_{c,i} \alpha_i (1 - T'(\alpha ; n_i; \tau_1, \tau_2)) \forall i$$

The Utilitarian government takes the price schedule of the debt $q(B', a)$ as given. If no default, it chooses taxes $\tau_2$ and debt $B'$ in order to maximize the households’ utilities, such that its budget constraint holds and households behave optimally. The value of default $V^d(a)$ is determined as:

$$V^d(a) = \max_{\tau_2, \{c_i, n_i\}} \int u(c_i, n_i)di + \beta \mathbb{E}_{\alpha'} \left\{ (1 - \pi) V^d(a') + \pi V(0, a') \right\}$$

s.t. $g = \int T(\alpha ; n_i; \tau_1, \tau_2)di$

$$c_i = \alpha \epsilon_i n_i - T(\alpha ; n_i; \tau_1, \tau_2) \forall i$$

$$u_{n,i} = -u_{c,i} \alpha_i (1 - T'(\alpha ; n_i; \tau_1, \tau_2)) \forall i$$

The additional cost of autarky is captured by a function $\chi(a) \leq a$. Let $S = \{B, a, \eta\}$ the aggregate state for the economy, where $\eta$ is an indicator function equal to 1 if the government has access to financial markets at the beginning of the period. Let $d(S)$ be the default policy function.

**Definition 1** A Markov perfect equilibrium is a set of policy functions for the government $\{d(S), B'(S), \tau_2(S)\}$, the households’ policy functions $\{c_i(S), n_i(S)\}_i$, and a price function $q(B', a)$ such that:

1. Given prices and households’ policies, the governments’ policy functions satisfy its optimization problem; given prices and government policies, the households behave optimally;
2. The price function reflects the government’s default probabilities:

\[ q(B', a) = q^* \left[ 1 - \int_{a'} d(B', a')dF(a'|a) \right] \]

2.2 Inequality

This subsection focuses on how incentives to default change with inequality. To shutdown concerns for efficiency, labor is assumed inelastic, such that the only effect of an increase in tax progressivity is to reduce after-tax inequality. First a necessary condition for inequality to alter debt and default policies is provided. Second inequality is shown to have ambiguous effects on incentives to default. Finally I explain why typically more unequal economies typically have a stronger incentive to default on their debt.

**Proposition 1** Let \( C \equiv a\bar{\epsilon} - \int T(\epsilon_i; \tau_1, \tau_2)di \). The economy admits aggregation with \( U \) the utility function of the representative agent defined as,

\[ U(C|a, \{\epsilon_i\}, \tau_1) \equiv \int u(c_i)di \]

Proposition 1 states that the economy admits a particular form of aggregation, in the sense that the problem can be solved in two steps: first, one can characterize debt policies solving for aggregate consumption \( C \), given the utility function \( U \); then, the tax function commands the allocation of \( C \) across households. The proof relies on the fact that there is a one-to-one mapping between aggregate consumption \( C \) and the level of the tax scheme \( \tau_2 \), given its slope \( \tau_1 \). The government policies for aggregate consumption \( C^{nd}(.) \) and \( C^{d}(.) \) are defined as:

\[ C^{nd}(B, a) = a\bar{\epsilon} + q(B', a)B'(B, a) - B - g \]
\[ C^{d}(a) = \chi(a)\bar{\epsilon} - g \]

The intuitive result appears useful to describe the main forces of the model. To understand how inequality alters debt and default policies, it is sufficient to analyze how \( U \) change with after-tax
inequality, that is, with pre-tax inequality \( \epsilon \) and redistribution \( \tau_1 \).

**Proposition 2** Assume CRRA preferences. If \( \exists \{\omega_{i,j}\} \) s.t. \( u_c(c_i(C,a)) = \omega_{i,j}u_c(c_j(C,a)) \forall(C,a) \), then optimal borrowing and default policies are independent of \( \tau_1, \sigma_\epsilon \).

Proposition 2 states that, when ratios of marginal utilities are constant across aggregate shocks, debt and default policies are independent of inequality; as a corollary, when such condition holds, \( U = u \).

Why are the sensitivity of spreads in marginal utilities to aggregate shocks the crucial statistic for the interaction between inequality and debt and default decisions? In a representative-agent model debt and default policies are driven by insurance motives against aggregate shocks. In a multi-agent model non-constant ratios of marginal utilities reflect the fact that households are affected differently across shocks. Therefore, fiscal policy is not only driven by insurance motives across aggregate shocks, but also by redistribution concerns: reallocating resources across households.

Proposition 2 is central to understand the implications of the modeling assumptions. Constant ratios of marginal utilities occur in two setups: (1) when households can trade together a complete set of Arrow securities in zero-net supply; or (2) when the government’s fiscal tools can redistribute resources costlessly across households. In these two cases, the economy aggregates to a representative agent, with \( U = u \). On the other hand, restrictions to private insurance and limitations to fiscal redistribution are the key forces for inequality to alter incentives to default.

Note that the degree of nonlinearity required for ratios of marginal utilities to be constant is model dependent. For instance, when labor is inelastic and utility is CRRA, a log linear tax function \( \log c = (1 - \tau_1) \log y - \tau_2 \) is sufficiently nonlinear for this condition to hold. With such a tax function, used by Heathcote, Storesletten, and Violante (2014) among others, \( \tau_1 = 1 \) ensures equal consumption across households, while \( \tau_1 = 0 \) implies a proportional tax scheme. With CRRA utility, no markets for private insurance, and a log linear tax function, a government chooses identical debt and default policies, whether the distribution of income is wide or degenerate, and whether the tax scheme is proportional or fully redistributive.

**Linear tax** In the rest of the section the analysis focuses on the linear tax described in (1). Together with the absence of markets for private insurance, this setup magnifies the effects of
inequality on default. This is because it maximizes the sensitivity of spreads in marginal utilities to the level of the tax scheme, optimally chosen given the productivity shock and the level of outstanding debt. With the linear tax function, the aggregate utility function $U$ can be explicated as

$$U(C, a) = \int \left[ C - a(1 - \epsilon_i)(1 - \tau_1) \right]^{1-\rho} di$$

**Remark 1** When $\tau_1$ is fixed, the relevant statistic of inequality for debt policies is the distribution of $(1 - \tau_1)\epsilon_i$, keeping $\bar{\epsilon}$ constant.

With CRRA utility $u(c) = c^{1-\rho}/(1-\rho)$, one can think of a more unequal economy as more risk-adverse. In particular, for a given $a$, define $\tilde{\rho}(C)$ such that

$$-\frac{\tilde{\rho}(C)}{C} = -\int \frac{u_{cc}(c_i)}{u_c(c_i)} di = -\int \frac{\rho}{c_i} di$$

The variable $\tilde{\rho}$ describes the risk-aversion of the representative-agent of an economy indexed by $(\sigma, \tau_1)$. As shown in Figure 2, $\tilde{\rho}$ is increasing in the after-tax income distribution spread (lower $\tau_1$, or larger $\sigma$). This is specifically true for low values of aggregate consumption $C$, that is, for high levels of taxes $\tau_2$ given an aggregate shock $a$. This insight is key to understand how inequality affects incentives to default. To see that, let $\bar{B}(.)$ be the default threshold function, defined as the minimum level of debt at which an economy defaults for each realization of the shock $a$. At the default threshold $\bar{B}(a)$ the country is indifferent between default and non-default:

$$U(C^d(a)) - U(C^{nd}(\bar{B}(a), a)) = \mathbb{E}_{a'|a} \left| V^{nd}(B'(\bar{B}(a), a), a') - V^d(a') \right|_+$$

The left-hand side of (4) captures the gains of default, while the right-hand side describes the costs. As in Arellano (2008), default is determined by an intertemporal insurance trade-off. When a country defaults, aggregate consumption is larger within the period of default: $C^d \geq C^{nd}$ since the right-hand side of the equation is positive. This result is well-known in the literature of sovereign default.

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Assuming $\pi = 0$ for clarity.
default: a government defaults only if it cannot roll over its foreign debt \((q(B', a)B' - B \geq 0)\).

Through the lens of this model, taxes decrease when a government defaults, as existing debt is not repaid; consumption increases. However, after default, the government is excluded from financial markets. Losing access to financial markets is costly, as the country cannot use foreign debt to insure against aggregate shocks. Budgets have to be balanced every period, hence consumption becomes more volatile.\(^{10}\)

Inequality has ambiguous effects on default incentives, since both gains and costs of default increase in after-tax inequality. First, the contemporaneous gains of default increase with inequality. Keeping policies constant, the increase in aggregate consumption \(C^d - C^{nd}\) is more valued in a more unequal economy. Indeed, default triggers an immediate decrease in taxes \(\tau_2\), which is more valued by low-income households. Thus, the larger the mass of low-income households, the larger the welfare gain of default, assuming a Utilitarian planner. However, the future costs of default also increase with inequality. After default, the country loses access to insurance against aggregate shocks.

\(^{10}\)In addition, in autarky, the government faces a productivity loss when productivity is high enough, which lowers the mean of future consumption.
shocks. The welfare costs of business cycle fluctuations are increasing in inequality.\textsuperscript{11}

Recalling the loose equivalence between risk-aversion and inequality, these two forces are easily mapped into the representative-agent economy with utility $U$. First, a more concave household values more an additional unit of consumption today, as its marginal utility is higher. However, a more risk-adverse household values more future insurance against aggregate shocks. To summarize, the value of having access to financial markets is larger for more unequal economies, but the cost of keeping access to financial markets is also larger for more unequal economies: keeping access to financial markets implies repaying some of the existing debt, thus raising taxes, which is more welfare-costly for low-income households.

Typically though, I find that more unequal economies have larger default sets, as shown in Subsection 2.5: the left-hand side of (4) increases faster with after-tax inequality than its right-hand side. This result holds because countries typically default in adverse times, where aggregate consumption is low. This is exactly when the aggregate utility function is particularly non-homothetic (Figure 2): a more unequal economy is as if more concave, in particular for low levels of consumption. Therefore, the welfare gains of consuming more resources today (default) are very sensitive to the level of inequality in the economy. On the other hand, continuation values capture presented discounted value of future consumption streams, higher in average than consumption at the moment of default. Why is aggregate consumption low at the moment of default? In equilibrium, trade balances are countercyclical, reflecting the fact that the government optimally raises taxes to pay back some of its large level of public debt, before eventually defaulting if keeping receiving a sequence of adverse shocks. The fact that governments typically repay debt, thus raise taxes, in adverse times push consumption in the case of no default even lower, exacerbating the effect of inequality on gains of default. Note that the empirical pattern of trade balances has been widely documented in the literature (see Arellano (2008) or Cuadra, Sanchez, and Sapriza (2010) among others): in adverse times emerging economies do experience positive trade balances, or negative

\textsuperscript{11}In addition, the productivity loss triggered by autarky in the case of high realizations of the shock is also more costly for more unequal economies. Indeed, when the government must finance some positive government spending, the productivity losses $\chi$ decrease the tax base for $\tau_1$. Therefore the government must increase the level of the tax scheme $\tau_2$, which is more costly for low-income households. See the end of the subsection for a more detailed discussion about the distributional effects of $\chi$. 

14
Finally, one should note how the modeling choice for $\chi$ affect the main result. When the additional costs of autarky do not alter spreads in marginal utilities, then they are irrelevant to assess how default sets change with inequality. With inelastic labor, setting government spending equal to zero would insure that ratios of marginal utilities are independent of the additional costs of autarky, hence without distributional effects.\textsuperscript{12} When, as standard in the literature, losses triggered by autarky only affect total output or aggregate productivity, a positive level of government spending generates larger spreads in marginal utilities, a pattern that could, in principle, flip the result. This appears not to be the case, in part because a country typically defaults in bad times, where costs of default are nil.\textsuperscript{13}

**A digression on the nature of aggregate shocks** In this setup, aggregate uncertainty is driven by productivity shocks. When $a$ falls, debt and government spending to be financed are relatively larger, as the revenues raised through the proportional tax $\tau_1$ decrease. If no default, the government increases the level of the tax scheme, which may magnify spreads in marginal utilities. However, it is important to notice that aggregate shocks in themselves have distributional effects, beyond the fiscal choices. One could have assumed stochastic government spending $\{g\}$ rather than stochastic productivity. In such setting, described in Appendix C, aggregate shocks do not intrinsically change spreads: distributional concerns are entirely driven by the government’s fiscal choices $\tau_2$. Then, Proposition 2 would also hold for linear taxes and CARA utility function. This is because, again, spreads in marginal utilities would be constant across aggregate shocks.

### 2.3 Labor supply

The previous part described why more progressive economies default on larger levels of debt, when concerns for efficiency are absent. In that setup, the main finding was that less progressive taxes, hence higher after-tax inequality, were associated with a stronger incentive to default. This subsection allows for elastic labor to restore concerns for efficiency. Then progressivity has a direct

\textsuperscript{12}Another way to achieve this result would be to assume that government spending are also rescaled by $\chi$ in autarky: $g^d = (\chi(a)/a)g$.

\textsuperscript{13}In the calibration, $\chi(a) = a$ when $a$ is low enough.
effect on incentives to work, and the strict equivalence between $\tau_1$ and $\sigma_\epsilon$ does not hold. Following insights from Proposition 2, it is possible to use a particular tax function, together with a particular utility function, to keep spreads in marginal utilities across households constant across aggregate shocks.

Proposition 3 Assume an economy with a log linear tax function indexed by $\tau$, a process $\{a\}$ and

$$u(c, n) = \log c - v(n)$$

Then, $\forall \tilde{\tau}_1, \exists \{\tilde{a}\}$ s.t. borrowing and default policies are identical. The rescaled process for government spending is defined as follows: $\tilde{a} = \omega a$, where $\omega = \check{\epsilon}(\tilde{\tau}_1)/\check{\epsilon}(\tau_1)$, where $\check{\epsilon}(\tau_1)$ describes aggregate efficient labor in the economy indexed by progressivity $\tau_1$.

Aggregate efficient labor is defined as $\check{\epsilon}(\tau_1) \equiv \int \epsilon n(\epsilon, tau_1) = \check{n}^*(\tau_1)$, where $n^*(\tau_1)$ solves the first-order condition of the household: $(1 - \tau_1) = n^* v'(n^*)$.\textsuperscript{14}

Corollary 2 Borrowing and default policies are identical to policies of an endowment economy with stochastic TFP $\{an^*(\tau_1)\}$.

Concerns for redistribution are absent as spreads in marginal utilities are constant across shocks: debt and default policies are independent of inequality $\sigma_\epsilon$. However, the level of progressivity $\tau_1$ does alter sovereign borrowing, through labor decisions; this set-up isolates the concerns for efficiency. When labor is elastic, a more progressive economy features a smaller output, and the economy can sustain smaller levels of debt.\textsuperscript{15} As a consequence, more progressive economies can sustain lower levels of debt, as far as efficiency is concerned. However this opposite force appears quantitatively small, as shown in Subsection 2.5.

\textsuperscript{14}A similar result is reported in Appendix C for government spending shocks rather than TFP shocks.

\textsuperscript{15}It is also true that the variance of the aggregate shock is relatively smaller. This could in principle increase incentives to default, as the need for insurance is smaller. However, this variance effect appears always dominated by the level effect.
2.4 Calibration

In this subsection, the model is calibrated to Argentina prior to its default in December 2001 to follow the tradition in the literature of sovereign default. In the next subsection I compare debt and default policies for different levels of tax progressivity $\tau_1$ to quantify the different forces of the model. The model is calibrated using a linear tax function. Together with the absence of private markets for households’ savings, this assumption maximizes changes in spreads in marginal utilities across aggregate shocks. Therefore, the numerical results should be read as an upper bound of the effects of tax progressivity on debt and default policies.

A standard utility function is used:

$$u(c, n) = \frac{c^{1-\rho}}{1-\rho} - \frac{n^{1+\varphi}}{1+\varphi}$$

where risk-aversion $\rho$ is set to 2 and Frisch elasticity $\varphi^{-1}$ to 2.22. The risk-free interest rate $1/q^*-1$ is set to 1.7%, the average yield of the five-year US treasury bond from 1983 to 2001. The stochastic process for aggregate productivity is a log AR(1)

$$\log a_t = \rho_a \log a_{t-1} + \sigma_a \epsilon_t$$

with persistence and variance of the normally distributed innovation calibrated such that the model matches persistence and standard deviation of GDP from 1983 to 2001, as reported by Arellano (2008). The functional form for the output cost of autarky is also borrowed to Arellano (2008):

$$\chi(a) = \min (a, \bar{a})$$

The discount factor $\beta$, the probability of reentering financial markets after default $\pi$, and the output costs of autarky $\bar{a}$, are jointly calibrated to match the following moments of the Argentinean economy: an annual default probability of 3%, an average debt service-to-GDP ratio of 5.5%, and a standard deviation of the trade balance of 1.75%, as reported by Arellano (2008) using Argentinian data from 1993 to 2001.
Table 1: Calibration.

<table>
<thead>
<tr>
<th>Calibration</th>
<th>Values</th>
<th>Target statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free interest rate</td>
<td>(1/q^* - 1 = 1.7%)</td>
<td>US 5-year bond quarterly yield</td>
</tr>
<tr>
<td>Risk-aversion (\rho)</td>
<td>2</td>
<td>Arellano (2008)</td>
</tr>
<tr>
<td>Frisch elasticity (\varphi^{-1})</td>
<td>2.22</td>
<td>Cuadra, Sanchez, and Sapriza (2010)</td>
</tr>
<tr>
<td>Stochastic process (a)</td>
<td>(\rho_a = 0.945, \sigma_a = 0.028)</td>
<td>Argentina’s GDP</td>
</tr>
<tr>
<td>Discount factor (\beta)</td>
<td>0.956</td>
<td>Default probability (annual)</td>
</tr>
<tr>
<td>Probability of reentry (\pi)</td>
<td>0.13</td>
<td>Debt service to GDP</td>
</tr>
<tr>
<td>Output costs (\bar{a})</td>
<td>0.94</td>
<td>Trade balance volatility</td>
</tr>
<tr>
<td>Productivity ({\epsilon})</td>
<td>{0.115; 0.213; 0.295; 0.394; 0.479; 0.589; 0.738; 0.943; 1.476; 4.757}</td>
<td>Pre-tax income distribution</td>
</tr>
<tr>
<td>Income tax (\tau_1)</td>
<td>0.30</td>
<td>After-tax Gini coefficient</td>
</tr>
</tbody>
</table>

Heterogeneity is calibrated as follows. The economy is populated with ten households, with an arbitrary distribution for productivity \(\{\epsilon\}\) chosen to match the average pre-tax income distribution per decile between 1993 and 1998, using SEDLAC data.\(^{16}\) Therefore, the distribution of pre-tax income generates an average pre-tax Gini coefficient consistent with the numbers reported by Gasparini and Cruces (2009).\(^{17}\) Finally, \(\tau_1\), the slope of the tax function, is calibrated to match the average after-tax Gini coefficient on that period. Table 1 summarizes the parameters. Tables 2 and 3 report targeted moments together with other business cycle properties of the model and their data counterpart, using empirical work of Arellano (2008) and Mendoza and Yue (2012).

### 2.5 The quantitative effects of tax progressivity

This last subsection reports the quantitative effects of changes in tax progressivity \(\tau_1\). Graph 3 plots price schedules and default sets, for the benchmark economy \((\tau_1 = 0.3)\), a less progressive economy \((\tau_1 = 0.15)\) and a more progressive economy \((\tau_1 = 0.45)\). In the benchmark case, fiscal policy reduces Gini coefficient from 0.46 to 0.38; the less progressive economy features after-tax

\(^{16}\)After 1998, the survey is extended to include more territories, making it non-comparable to previous years.

\(^{17}\)The model is calibrated to total income, as the after-tax Gini coefficient on labor income is not known. In SEDLAC the distribution of pre-tax labor income looks similar to pre-tax total income.
Table 2: Business cycle properties: aggregates

<table>
<thead>
<tr>
<th></th>
<th>Simulation Targeted</th>
<th>Data</th>
<th>Simulation Non-targeted</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default probability</td>
<td>2.75%</td>
<td>3%</td>
<td>1.86%</td>
<td>0.76%</td>
</tr>
<tr>
<td>$E(\text{DS/GDP})$</td>
<td>5.7%</td>
<td>5.5%</td>
<td>0.78%</td>
<td>1.51%</td>
</tr>
<tr>
<td>$\sigma(\text{TB/GDP})$</td>
<td>0.7%</td>
<td>1.75%</td>
<td>$\rho(\text{GDP, Spreads})$</td>
<td>-0.46</td>
</tr>
<tr>
<td>$\sigma(\text{GDP})$</td>
<td>4.7%</td>
<td>4.7%</td>
<td>$\rho(\text{TB/GDP, GDP})$</td>
<td>-0.18</td>
</tr>
<tr>
<td>$E(\text{G/GDP})$</td>
<td>15.3%</td>
<td>16%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Default probability is annual. All other moments are quarterly. Business cycle properties of the model are computed from a 10000-quarter simulation, when the country is not in autarky. The targeted statistics are compared to empirical observations reported by Arellano (2008), using quarterly data from 1983 to 2001 for GDP, from 1980 to 2001 for debt service, and from 1993 to 2001 for other moments. The other moments are compared to empirical observations reported by Mendoza and Yue (2012), using quarterly data from 1994 to 2001. The government spending to GDP ratio is 16% in 1999 (World Bank).

Table 3: Business cycle properties: pre-tax income distribution and Gini coefficients

<table>
<thead>
<tr>
<th>Decile</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target: pre-tax income distribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>0.015</td>
<td>0.027</td>
<td>0.038</td>
<td>0.050</td>
<td>0.061</td>
<td>0.075</td>
<td>0.094</td>
<td>0.120</td>
<td>0.168</td>
<td>0.353</td>
</tr>
<tr>
<td>Data (total)</td>
<td>0.015</td>
<td>0.027</td>
<td>0.037</td>
<td>0.048</td>
<td>0.069</td>
<td>0.074</td>
<td>0.093</td>
<td>0.120</td>
<td>0.168</td>
<td>0.357</td>
</tr>
<tr>
<td>Data (labor)</td>
<td>0.016</td>
<td>0.029</td>
<td>0.040</td>
<td>0.050</td>
<td>0.062</td>
<td>0.076</td>
<td>0.093</td>
<td>0.119</td>
<td>0.167</td>
<td>0.349</td>
</tr>
</tbody>
</table>

Gini coefficient  

<table>
<thead>
<tr>
<th>Pre-tax</th>
<th>After-tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>0.46</td>
</tr>
<tr>
<td>Data</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Note: $\{\epsilon\}$ and $\tau_1$ are calibrated to match the average pre-tax income distribution per decile in Argentina between 1993 and 1998 (source: SEDLAC) and the average after-tax Gini coefficient between 1993 and 1998 (source: Gasparini and Cruces, 2009), respectively.
Table 4: Average and maximum debt over GDP in the simulation

<table>
<thead>
<tr>
<th></th>
<th>E(B/GDP)</th>
<th>max((B/GDP))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less progressive</td>
<td>(\tau_1 = 0.15)</td>
<td>2.02%</td>
</tr>
<tr>
<td>Benchmark</td>
<td>(\tau_1 = 0.30)</td>
<td>5.67%</td>
</tr>
<tr>
<td>More progressive</td>
<td>(\tau_1 = 0.45)</td>
<td>7.70%</td>
</tr>
<tr>
<td>Representative Agent</td>
<td>(\tau_1 = 0.30, \sigma_\epsilon = 0)</td>
<td>15.31%</td>
</tr>
</tbody>
</table>

Gini coefficients equal to the pre-tax coefficients;\(^{18}\) in the more progressive economy, the after-tax Gini coefficient is equal to 0.31. The left panel describes the price schedule \(q\) of an economy as a function of debt issued, \(B'\), when productivity is at its mean level. For each level of debt \(B'\), the more progressive economy faces higher prices \(q(B',a)\) than the benchmark case, illustrating the fact that such economy has smaller incentives to default on its sovereign debt. As a consequence default sets are also ordered with progressivity. The right panel plots debt thresholds for each level of aggregate productivity, in the three economies; when debt is larger than the threshold, the country defaults. The more progressive economy has less incentives to default, thus the debt at which it starts defaulting is higher for each realization of the aggregate productivities. Therefore default sets shrink with tax progressivity. Over a simulation, time series for debt feature a similar pattern, as reported in Table 4: a more progressive economy sustains larger debt over GDP ratios, both in terms of average and maximum levels of debt issued along the simulation. The order of magnitude is large: an economy where the fiscal policy reduces after-tax Gini coefficients from 0.38 (baseline) to 0.31 (the more progressive economy) sustain maximum debt-over-GDP ratios that are about twice larger.

\(^{18}\)Note that government spending is positive; if \(\tau_1 = 0\), government spending is entirely financed with lump-sum taxes, which increases inequality. This is why a positive \(\tau_1\) leads to equal pre-tax and after-tax Gini coefficients.
Figure 3: Default sets and price schedules.

Note: The baseline case is calibrated to Argentina ($\tau_1 = 0.3$, pre-tax and after-tax Gini coefficients are equal to 0.46 and 0.38, respectively). The blue line depicts a less progressive economy ($\tau_1 = 0.15$, pre-tax and after-tax Gini coefficients are both equal to 0.45). The red line depicts a more progressive economy ($\tau_1 = 0.45$, pre-tax and after-tax Gini coefficients are equal to 0.47 and 0.31, respectively).
Figure 4: Default sets, inelastic and elastic labor.

Note: The top panels plot default sets for inelastic (left) and elastic (right) labor, for low levels of tax progressivity. The bottom panels depict the case for inelastic (left) and elastic (right) labor for much higher levels of tax progressivity.

Finally, to quantify concerns for redistribution vs. concerns for efficiency, Graph 4 shows default sets for varied levels of tax progressivity, for two economies: with inelastic labor ($\varphi^{-1} = 0$) and with elastic labor, as described in the calibration. The top panels depict default sets for relatively low levels of tax progressivity; despite efficiency concerns, default sets decrease with tax progressivity even when labor is elastic. The bottom panels plot default sets for very large levels of tax progressivity. When labor is inelastic, default sets keep decreasing but the gain from reducing after-tax inequality becomes smaller. When labor is elastic, default sets eventually increase with tax progressivity - the red line is below the green one. The loss from depressing output dominates the gain from reducing after-tax inequality. Note that this happens for very high levels of tax
progressivity \( (\tau_1 = 0.95) \), despite a relatively large Frisch elasticity.\(^{19}\)

As a conclusion, Section 2 explained why more progressive economies have less incentives to default. As a consequence, more progressive, or less (after-tax) unequal economies, face lower spreads for a given level of debt over GDP ratio, and can sustain larger levels of external debt. The very large quantitative effect of inequality and tax progressivity should be read as an upper bound, though, due to the joint assumption of hand-to-mouth households and linear taxes. To assess how large this effect actually is, one needs to take a stand on, (1) how incomplete domestic markets for private insurance are, in particular for the left part of the distribution, and (2) how severe the limits to redistribution faced by a government are. These two features are likely to be country-specific.

### 3 Evidence

This section reports two pieces of evidence connecting cross-country data on \((i)\) sovereign debt and \((ii)\) default probabilities, to measures of inequality and tax progressivity. First, I document a positive relationship between debt and tax progressivity, and a negative relationship between debt and after-tax inequality. Second, I argue that default probabilities are negatively correlated with tax progressivity, and positively correlated with inequality. These findings are in line with the model’s predictions, as discussed in Section 2.

As a first empirical check, I show that more progressive economies are associated with larger levels of debt over GDP ratios, consistent with the result that more progressive economies have smaller default sets. In Figure 6, tax progressivity is measured as the percentage decrease in Gini coefficients, from pre-tax (market) to post-tax (disposable) income distribution. A more negative number indicates a more redistributive tax system. The data is taken from Luebker (2011), who computes this measure for 22 countries for the most recent available year (typically in the 2000s), based on Luxembourg Income Study (LIS) Database. External government debt over GDP ratios are taken from Quarterly External Debt Statistics of the World Bank. The exercise is as follows.

\(^{19}\)Concerns for efficiency are small because of the wealth effect in this setup. Calibrating the model to GHH preferences could magnify the distortionary effect of progressivity on labor supply.
For a given year, log of GDP-per-capita and GDP-per-capita growth, are regressed against public debt:

\[
Debt-over-GDP_i = \beta_0 + \beta_1 \log GDP-per-capita_i + \beta_2 GDP-per-capital-growth_i + \epsilon_i
\]

Then, the residuals \(\{\epsilon_i\}\) are plotted against the measure for progressivity for each country.\(^{20}\) This exercise is repeated for three years: 2008, 2010, and 2012.\(^{21}\) A negative relationship holds: more progressive countries tend to sustain larger levels of external debt over GDP. As a robustness check, Annex F presents a similar calculation with total external debt rather than government debt only. A similar pattern holds (see Figures 10 and 11). In Figure 12, in Annex, inequality is measured as the ratio of top to bottom decile average after-tax income share, as documented in the Poverty and Inequality Database of the World Bank. A larger number indicates a more unequal economy. For each country, I compute the average measure for available data between 2006 and 2012, excluding countries that report inequality measures on consumption rather than income. Again, controlling for GDP-per-capita, in log and in growth, less unequal countries tend to sustain larger levels of public external debt over GDP.

It should be noted that those scatter plots control only for GDP-per-capita and GDP growth. Obviously, many other factors could result in heterogeneity in debt over GDP ratios. As a second piece of evidence, I argue that default probabilities are positively correlated with inequality, after conditioning on GDP per capita and debt-to-GDP ratios. This empirical result is established by Aizenman and Jinjarak (2012), using 50 countries in 2007, 2009 and 2011. Controlling for several variables, they find that an increase in the after-tax Gini coefficient reduces “fiscal space” - a statistic defined as the inverse of public debt over realized tax collection - which in turn has a positive effect on CDS, the proxy used for default probabilities. According to their estimation, the magnitude of this effect is large: an increase in the Gini coefficient of inequality by 1 (in a scale of 0-100), is associated in 2011 with lower tax collections and higher sovereign spreads (+45 basis points).

\(^{20}\)To get a sense of magnitudes, Figure 5 shows scatter plots of raw data for external government debt over GDP ratios and CDS against tax progressivity for a given year (2008). Annex F also lists countries used for each graphs.

\(^{21}\)I have only one measure of tax progressivity per country. To the extent that, as it is the case for Gini coefficients, differences across tax progressivity are larger across countries than across the business cycle for a given country, this exercise is still informative.
Figure 5: External debt over GDP, CDS, and tax redistribution across countries in 2008

Figure 6: Debt over GDP and tax redistribution across countries

I complement these findings by extending the analysis to tax progressivity. Figure 7 plots CDS against tax progressivity, when controlling for external government debt to GDP ratios, log of GDP per capita and GDP growth. The 5-year CDS are taken from Bloomberg (the annual value is obtained as the average of the end of the period quarter data), for years 2008, 2010, and 2012; control variables are also measured for these three years. Again, less progressive economies are associated with higher probabilities of default, given a level of GDP-per-capita and debt-over-GDP ratio. However, one should point out the limited number of observations, due to the fact that measures of tax progressivity are scarce, in particular for developing economies.

4 Optimal progressivity

The previous section established that more progressive economies typically face larger incentives to default, when progressivity is taken as exogenous. This section turns to the question of the optimal policy for progressivity. The literature generally characterizes optimal progressivity through the lens of the efficiency-redistribution trade-off. However, there are additional trade-offs for determining optimal progressivity when a government is unable to commit to repay debt, as is the case for many emerging economies. In particular, changes in future progressivity alter future inequality, and therefore future incentives to default. This translates into changes in contemporaneous prices at which the government can issue debt. Thus, the ability to commit to future progressivity opens a new channel: tax progressivity can be used to manipulate credit spreads. To study this new forward-looking role of progressivity, I start with the simplest optimal policy setup that activates the main channel: one period commitment to a non state-contingent tax progressivity. The importance of the credit spread manipulation channel is quantified by comparing outcomes to a case where we relax commitment. The last part of this section elaborates on the role of contingency of tax progressivity.

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22 See Werning (2007), or, being loose on the difference between redistribution and insurance, Heathcote, Storesletten, and Violante (2014) or Conesa, Kitao, and Krueger (2009) in a Bewley set-up, or Farhi and Werning (2013) and Golosov, Troshkin, and Tsyvinski (2014), in a Mirleesian set-up, among many others.
Note: CDS (5yr, $US), residuals. Source: Bloomberg. Progressivity: percentage change between pre-tax (disposable) and after-tax (market) Gini coefficient. Source: Luebker (2011), based on Luxembourg Income Study (LIS) Database, mostly 2000s. All countries with available data are reported, except for Argentina (a similar exercise including Argentina is presented Figure 13, in Annex). The plots control for GDP per capita, in logs, GDP per capita growth, and external government debt-over-GDP ratios.
4.1 Optimal policy problem

The main setup describes a government that chooses and can commit to a fixed tax progression for one period. This timing assumption resembles the one adopted in Farhi (2010). Setting progression one period in advance and non-state-contingently may be justified by tax inertia: a government needs one period to change the progression of the tax code. Therefore, progression in period \( t \) is chosen in period \( t - 1 \) (de facto one-period commitment), and cannot depend on the realization of the shock in \( t \) (non-state-contingent). On the other hand, the setup maintains the assumption that the government cannot commit to repay its foreign debt, hence tax level and debt repayments are decided every period, state-contingently, and without commitment. The timing and measurability restrictions for progression are summarized below.

**Assumption 1** *Tax progression is chosen one period in advance, non-state-contingently.*

Given the timing assumptions of this optimal policy problem, the state space is extended by one variable. A government that had access to financial markets last period enters with \((B, a, \tau_1)\), where \( B \) describes the government’s outstanding debt, \( a \) the realization of the shock, and \( \tau_1 \) the pre-determined progression of the tax. Within a period, the government chooses: (i) future level of debt \( B' \); (ii) the level of the tax \( \tau_2 \), which depends on the policy for debt \( B' \) and the pre-determined tax progression \( \tau_1 \), and (iii) the level of progression next period \( \tau'_1 \). The value function of a government that chooses not to default can be written as follows:

\[
V^{nd}(B, a, \tau_1) = \max_{B', \tau_2, c_i, n_i, \tau'_1} \int u(c_i, n_i) di + \beta \mathbb{E}_a V(B', a', \tau'_1) \\
\text{s.t. } g + B = \int T(a \epsilon_i n_i; \tau_1, \tau_2) di + q(B', \tau'_1, a)B' \\
c_i = a \epsilon_i n_i - T(a \epsilon_i n_i; \tau_1, \tau_2) \forall i \\
u_{n,i} = -u_{c,i} a \epsilon_i (1 - T'(a \epsilon_i n_i; \tau_1, \tau_2)) \forall i
\]
Similarly, the value function after defaulting is described by:

\[
V^d(a, \tau_1) = \max_{\tau_2, \bar{c}, n_i, \bar{\tau}_1'} \int u(c_i, n_i) di + \beta \mathbb{E}_{a'} \left[ (1 - \pi) V^d(a', \tau_1') + \pi V(0, a', \tau_1') \right]
\]

s.t. \[
g = \int T(\chi(a)\epsilon_i n_i; \tau_1, \tau_2) di
\]

\[
c_i = \chi(a)\epsilon_i n_i - T(\chi(a)\epsilon_i n_i; \tau_1, \tau_2) \forall i
\]

\[
u_{n,i} = -u_{c,i} \chi(a) \epsilon_i \left( 1 - T'(\chi(a)\epsilon_i n_i; \tau_1, \tau_2) \right) \forall i
\]

where \( V(B, a, \tau_1) \equiv \max \{ V^{nd}(B, a, \tau_1), V^d(a, \tau_1) \} \).

In the problem described above, the important variable, \( \bar{\tau}_1' \), appears twice: in the value function next period, and in the price function for sovereign borrowing today. This reflects the two main forces that optimally determine progressivity. First, progressivity affects value functions next period, through the usual efficiency-redistribution trade-off: it affects the size of total output by discouraging labor supply, and, it affects spreads in consumption, by redistributing income away from high-income to low-income households. Secondly, progressivity also appears in prices \( q(B', \tau_1', a) \), because of default risk. This reflects the new credit spread manipulation channel. Recall that a government that commits to a larger progressivity will have less incentives to default next period, as more equal economies feature smaller default sets. Therefore, probabilities of default will be smaller, and contemporaneous interest rates are lower: \( \partial q(B', \tau_1', a) / \partial \tau_1' \geq 0 \). Hence, when the government can commit to next-period progressivity of the tax system, it optimally chooses a larger progressivity than the one determined by the efficiency-redistribution trade-off. Commitment is key: if a country could reset progressivity in the beginning of period \( t + 1 \) - that is, after issuing debt at a lower interest rate - it would choose a smaller progressivity. To isolate this second channel, I compare the optimal policy in this setup to a modified problem, where progressivity is chosen non-state-contingently but without commitment.

**Assumption 2** Tax progressivity is chosen every period, before the realization of the shock.

This set-up is very close to Assumption 1, with one key difference: tax progressivity is chosen in the beginning of each period, rather than one period in advance; the country enters a period with some
debt $B$, and before the realization of the shock $a$, chooses $\tau_1$. Because the government cannot commit on tax progressivity, it cannot decrease prices by promising a larger progressivity. The credit spread manipulation channel is absent, and tax progressivity is chosen taken into account solely the expected revenues optimally raised in the period, across shocks. Therefore, contrasting optimal policy in Assumption 1 to the policy in Assumption 2 allows for isolating the price manipulation channel while keeping the same measurability restrictions.

Figure 8 shows the optimal choice for $\tau_1$ in both set-ups. The optimal policy for progressivity without commitment can be understood as follows. The larger the debt in $t$, the larger the revenues raised by the government both in $t$ and $t+1$, because of tax smoothing. Progressivity is increasing in the level of revenues optimally raised - in expectation, due to the measurability restriction: the more revenues the government needs to raise, the higher both the slope and the level of the tax function. Note that, if labor was inelastic, the optimal progressivity without commitment would be $\tau_1 = 1$. To isolate the price manipulation channel, we compare the policy for progressivity with and without commitment. When the economy enters period $t$ with a low level of debt, the probability of default in period $t+1$ is zero, even when $\tau_1$ is chosen without commitment. Hence, the price manipulation channel is mute, and optimal policies with and without commitment are identical. On the other hand, when the probability of default in $t+1$ is positive under the non-commitment policy, a government that can commit to its tax progressivity next period chooses a larger tax progressivity. By doing so, it decreases default sets next period, and therefore can buy debt at a better price today. The larger the probability of default when no commitment on progressivity, the larger this effect: this is the credit spread manipulation in action.

### 4.2 Taxes contingent to default

The optimal policy problem described in the previous subsection relied on two assumptions: commitment and non-state-contingency. While commitment is necessary for credit spread manipulation, the non-measurability restriction was mainly for tractability and justified by tax inertia. However, an implicit restriction is that the government cannot commit to tax progressivity conditional on

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23 Due to this minor change in timing of policies, Bellman equations of this problem involve a cumbersome notation relegated in Annex.
Figure 8: Tax progressivity to manipulate credit spreads

**Note:** Optimal policy for progressivity in $t + 1$ given debt in $t$, when the country can (red) and cannot (blue) commit to the progressivity of the tax scheme next period.

default policies. Giving the government access to a richer commitment device for progressivity will strengthen the price manipulation channel.

**Assumption 3** Tax progressivity is chosen one period in advance for each repayment decision: default and non-default.

The country now chooses a vector $\mathbf{\tau}_1 = (\tau_{1n}, \tau_{1d})$ instead of a single number $\tau'_1$. As compared to the previous Assumption 1, this setup gives a country more space to manipulate incentives to default next period. In particular, the government has an additional incentive to commit to an ex-post welfare-costly level of progressivity when defaults occur, in order to shrink default sets, and get even better prices. We highlight the differences in tax progressivity upon default or non-default, a simple two-period model with inelastic labor. When progressivity is not contingent to default policies (Assumption 1), it is always set to its maximum level: with linear taxes, the efficiency-redistribution trade-off implies $\tau_1 = 1$, as there are no concerns for efficiency. This is optimal.

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24 The complete description of this problem is presented in Annex.
both without and with commitment. Since the government cannot commit to a progressivity larger than $\tau_1 = 1$, the credit spread manipulation channel is mute. On the other hand, if a government can commit to policies conditional on default decisions, then the optimal policy appears to be $(\tau_1^{nd} = 1, \tau_1^{d} = 0)$. By increasing spreads in marginal utilities in case of default, the government makes default next period more welfare-costly: default sets shrink, and the government can borrow in $t = 0$ at a better price.25

This setup is presented as a way to highlight the potential ways the government can manipulate prices: making default welfare-costly through ex-post suboptimal levels of progressivity. In effect, the government is exploiting its ability to make tax progressivity contingent on its default decisions as a substitute for its inability to commit to debt repayments. This makes the inherent asymmetry in the governments’ ability to commit more tenuous in this current setting, Assumption 3, than in Assumption 1. However, to provide a complete answer to whether this asymmetry is sustainable or not, one would need a deeper theory of the sources of commitment.

5 Conclusion

In this paper, I explored how the distributional cost of taxes interacts with a country’s foreign borrowing and default policies. I first established that keeping the progressivity of the tax code fixed, more unequal economies tend to default at lower levels of debt. Since inequality affects incentives to default, I then studied how a government should use tax progressivity to influence future inequality. More particularly, I show that ability to commit to future progressivity, gives the government a tool to manipulate sovereign credit spreads. This price manipulation channel mandates more progressive tax schemes when default risk is higher. This finding relies on a minimal assumption for commitment - the government fixes the progressivity of its tax system one period ahead; more importantly, it sheds a new light on the suboptimality of regressive fiscal austerity plans adopted, for instance, in Greece or Portugal, in order to decrease credit spreads.

This research is work in progress and could be pursued in many directions. First, the envi-

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25Note that allowing for default- and state-contingent policy $\tau^*_1 = (\tau_1^{nd}(a'), \tau_1^{d}(a'))$ would keep the result unchanged.
ronment can be extended to allow the households to save. This is important for two reasons; it will allow to get a quantitatively better description of household inequality and hence, more discipline on the distributional costs of taxes and their effect on debt and default decisions. But on a theoretical level, forward-looking households impose further restrictions on the government’s debt, default and progressivity choices. Other avenues could be incorporating richer theories of costs and constraints of changing progressivity, from tax evasion to political economy frictions.

References


A  The austerity plan in Greece

Figure 9 reports the distribution of the 2010 fiscal austerity plan per income decile; the austerity plan fell more on low-income households, proportionally to their income. The distribution of the 2010 fiscal austerity plan per 2009 income decile (middle panel) is computed of Matsaganis and Leventi (2013) using EUROMOD data (Figure 3 of their paper). I compare this distribution to the distribution of income per decile in 2009 (left panel), and plot the difference between these two distributions in the right panel. A positive number for a decile indicate that the share of the fiscal plan supported by this decile is larger than its income share. As a robustness check, results are very similar when using the 2010 income distribution. The distribution of the fiscal plan for years 2011 and 2012 is not known, but it seems reasonable to expect a similar regressive pattern. The top-to-bottom quintile after-tax income raised from 9.77 in 2010 to 10.91 in 2011 and 13.94 in 2012, before decreasing to 12.7 in 2013 (source: Eurostat, SILC).

B  Proofs of Section 2

Proof of Proposition 1 Let $C \equiv \int c_i \, di$. The progressivity of the tax function $\tau_1$ is given, and the distribution $\{\epsilon\}$ is fixed. Assuming that $\partial \int T(a\epsilon_i; \tau_1, \tau_2) \, di / \partial \tau_2 > 0$, there exists a function $\hat{\tau}_2(C, a)$ defined such that: $C = \int a\epsilon_i - \int T(a\epsilon_i; \tau_1, \hat{\tau}_2(C, a)) \, di$. Hence, one can define the functions $c_i(C, a)$ and $U(C, a)$ such that:

$$c_i(C, a) \equiv a\epsilon_i - T(a\epsilon_i; \tau_1, \hat{\tau}_2(C, a)) \forall i$$

$$U(C, a) \equiv \int u(c_i(C, a)) \, di = \int u(a\epsilon_i - T(a\epsilon_i, \tau_1, \hat{\tau}_2(C, a))) \, di$$
Figure 9: The regressive nature of the Greek fiscal austerity plan in 2010.

Note: The left panel reports the share of total disposable household income per decile, in 2009. Source: Eurostat. The middle panel reports the distribution of the 2010 fiscal savings by 2009 decile. Source: Matsaganis and Leventi (2013), based on EUROMOD version F4.32. The right panel plots the difference between the two lines.
The value of non-default can be written as:

\[ V^{nd}(B, a) = \max_{B', C} U(C, a) + \beta \mathbb{E}_{a'} V(B', a') \quad \text{s.t.} \quad C + g + B = a\bar{\epsilon} + q(B', a)B'. \]

Similarly, the value of default is:

\[ V^{d}(a) = \max_C U(C, a) + \beta \mathbb{E}_{a'} \left\{ (1 - \pi)V^{d}(a') + \pi V^{nd}(0, a') \right\} \quad \text{s.t.} \quad C + g = \chi(a)\bar{\epsilon}. \]

**Proof of Proposition 2** Under the condition stated in the proposition,

\[ C = \int c_i(C, a) di = c_j(C, a) \int u_c^{-1}(\omega_{i,j}) di \ \forall j \]

As a consequence, there exists a positive real \( \bar{\omega}_i \) s.t. \( c_i = \omega_i C \), leading to the result when \( u \) is homothetic: \( U(C) \propto u(C) \).

Note that the generic condition for Proposition 2 to hold is that \( \exists \{\bar{\omega}_i\} \) such that, \( \forall C, a: \)

\[ \frac{\partial u(c_i(C, a))}{\partial C} = u_c(C)\bar{\omega}_i \quad (5) \]

This assumption imposes a joint restriction on utility and tax functions. With CRRA utility function, the condition of Proposition 2 holds when the tax function is log linear. Indeed,

\[ C = \int c_i di = \int (1 - \tau_2)(a\epsilon_i)^{1-\tau_i} di \]

Replacing \( \tau_2 \) by its implicit function \( \hat{\tau}_2(C, a) \):

\[ c_i(C, a) = C(a\epsilon_i)^{1-\tau_i} \left( \int (a\epsilon_i)^{1-\tau_i} di \right)^{-1} \]

\[ u(c_i(C, a)) = C^{1-\rho} \left( (a\epsilon_i)^{1-\tau_i} \left( \int (a\epsilon_i)^{1-\tau_i} di \right)^{-1} \right)^{1-\rho} / (1 - \rho) \]
And therefore, ratios of marginal utilities with respect to $C$ are independent of $C$ and $a$:

$$\frac{u_C(c_i(C,a))}{u_C(c_j(C,a))} = \frac{C^{-\rho} \left( (ae_i)^{1-\tau_1} (\int (ae_i)^{1-\tau_1} di)^{-1} \right)^{1-\rho}}{C^{-\rho} \left( (ae_j)^{1-\tau_1} (\int (ae_i)^{1-\tau_1} di)^{-1} \right)^{1-\rho}} = \left( \frac{\epsilon_i}{\epsilon_j} \right)^{(1-\rho)(1-\tau_1)}$$

However, one does not need to require CRRA utility function. For instance, when aggregate uncertainty comes from government spending shock rather than productivity (the setup is described below), this condition also holds with linear taxes and CARA preferences.

Finally, note that the generic condition 5 of Proposition 2 is satisfied in two benchmark cases: absence of limits to redistribution or access to complete private insurance. The proof for the case where the government does not face limits to redistribution is straightforward and does not require CRRA utility functions. With or without elastic labor, the government would use lump sum taxes indexed by $i$ to reach full redistribution, $c_i = C \forall (C,a,i)$, without distorting labor supply: $(u_c(C)) = -ae_i v'(n_i)$. Then, $u(c_i(C)) = u(C) \forall i$ and Condition 5 trivially holds with $\tilde{\omega}_i = 1 \forall i$. The proof for complete markets is similar: when households can trade a complete set of securities in zero net supplies, their spreads in marginal utilities are equalized across shocks. Together with the CRRA assumption, 5 holds - leading to the result. The complete definition of the economy with savings is presented below.

**Proof of Proposition 3** First, note that with separable preferences and log-utility, labor is independent of $\tau_2$: $n^*$ solves the first-order condition $(1 - \tau_1) = n_i v'(n_i) \forall i \forall \tau_2$. Then, spreads in marginal utilities are constant across shocks.

**C Government spending shocks**

In the paper, the source of aggregate uncertainty is aggregate productivity shocks. Instead, one could have assumed government spending shock, in the tradition of Lucas and Stokey (1983). As explained in the end of Section 2.2, such a setup is cleaner to isolate the distributional effects of
fiscal policy. The government’s problem would read as follows:

$$V^{nd}(B, g) = \max_{B', \tau_2, \epsilon_i, n_i} \int u(c_i, n_i) di + \beta E g V(B', g')$$

s.t. \( g + B = q(B', g)B' + \int T(\epsilon_i n_i; \tau_1, \tau_2) di \)

\( c_i = \epsilon_i n_i - T(\epsilon_i n_i; \tau_1, \tau_2) \forall i \)

\( u_{n,i} = -u_{c,i} \epsilon_i (1 - T'(\epsilon_i n_i; \tau_1, \tau_2)) \forall i \)

Then, assuming inelastic labor supply and fixed progressivity \( \tau_1 \), the aggregate utility function \( U \) as defined in Proposition 1, is independent of the realization of the aggregate shock; government spending, when non-valued by the households, have no distributional effects (do not alter ratios of marginal utilities across agents). In addition, spreads in marginal utilities would be constant across shocks in two setups: not only with CRRA utility and loglinear taxes, but also with CARA utility and linear taxes. Indeed, in the second case, when \( u(c) = -\alpha^{-1} e^{-\alpha c} \), with \( \alpha \in \mathbb{R}^+ \),

$$\frac{u_{c,i}}{u_{c,j}} = \frac{e^{-\alpha(1-\tau_1)\epsilon_i + \alpha \tau_2}}{e^{-\alpha(1-\tau_1)\epsilon_j + \alpha \tau_2}} = \frac{e^{-\alpha(1-\tau_1)\epsilon_i}}{e^{-\alpha(1-\tau_1)\epsilon_j}} \forall i, j \forall g$$

On the other hand, when the source of uncertainty is aggregate productivity, then productivity shocks do change spreads in marginal utilities in that second setup, even though fiscal policies, through \( \tau_2 \), do not.

**D Markov Perfect Equilibrium with private savings**

In this section, I define the Markov Perfect Equilibrium in an economy populated with households who can borrow and save. The environment regarding the private market is as follows: households can issue a complete set of securities that are traded domestically across themselves in zero net supply. To keep the problem simple, households are assumed not to default on their obligation, and the government is assumed to issue foreign debt only and to use a linear tax with \( \tau_1 \) fixed. Also, labor is inelastic.
Let $S$ be the aggregate state (defined later) including $a$. Assume $\epsilon_i$ is constant. Households take prices of Arrow securities $q^d$ and the government’s policies $\tau_2(S)$ taken as given. Then,

$$
v^i(b, S) = \max_{c, \{b'(S')\}} u(c) + \beta \mathbb{E}_{S'|S} \left[ v^i(b'(S'), S') \right] \\
\text{s.t. } c + \sum_{S'} q^d(S', S)b'(S') = (1 - \tau_1)a\epsilon_i + b - \tau_2(S) \text{ and } S' = \Phi(S)
$$

where $\Phi(.)$ is the function used by the households to forecast the aggregate state next period, as a function of other households’ and future government’s policies. The households’ policy functions, consumption $C^i(b, S)$ and savings $\{B^i(S'; b, S)\}$, must therefore satisfy two conditions $\forall i$:

$$
C^i(b, S) = (1 - \tau_1)a\epsilon_i + b - \tau_2(S) - \sum_{S'} q^d(S', S)B^i(S'; b, S)
$$

$$
q^d(S'|S) = \beta \pi(S'|S)u_c(C^i(B^i(S'; b, S), S'))/u_c(C^i(b, S))
$$

From the second condition, that holds $\forall i$, one can deduce that $\{\exists \omega_{i,j}\}$ s.t. $u_c(c_i) = u_c(c_j) \forall i, j$, and therefore, $\exists \tilde{\omega}_i$ s.t. $c_i = \omega_i C$, when $u$ is CRRA.

The government’s maximization problem is similar to the case with hand-to-mouth households. The state space is augmented to the distribution of wealth across households $\{b_i\}$: $S = (B, a, \{b_i\}, \eta)$. Given the policy function of the households and the future policy functions for default, debt, and taxes, of future governments $D(S)$, $B^g(S)$ and $\tau(S)$, a government that chooses not to default maximizes:

$$
V^{nd}(S) = \max_{\tau_2, B'} \int u(C^i(b, S))di + \beta \mathbb{E}_{S'|S} \left[ V(S') \right] \\
B + g = \tau_1a\bar{\epsilon} + \tau_2 + q(B', \{B^i(S'; b, S)\}|B, a)B' \text{ and } S' = \Phi_g(S)
$$

where $\Phi_g(.)$ is the function used by the government to forecast the aggregate state next period, as a function of its choice for $B'$ and $\tau_2$ and their effect on households’ and future government’s policies, and under the constraint that households behave optimally and markets for domestic savings clear. $V^d$ is defined similarly, and $V = \max\{V^d, V^{nd}\}$ pins down the optimal default policy.
for the government.

In a Markov Perfect Equilibrium, the following conditions must hold: given future government’s policies, households’ policies, and prices, the current government’s policies must be consistent with its optimization problem; given current and future governments’ policies and domestic prices, the households must behave optimally; domestic prices must clear the market for domestic savings; foreign prices must reflect probability of default as pinned down by the government’s policies, and \( \Phi \) and \( \Phi_g \) must be consistent with the government’s and households’ optimal policies.

E  Computational details

The aggregate productivity shocks are discretized on 17 points using a quadrature method as in Tauchen and Hussey (1991). The value function for non-default is approximated using projection methods (linear splines) on \( \omega = -(B + g) \) using 60 points, for each realization of the aggregate shock. A fixed grid (with space 1e-4) is used to find the optimal choice for \( B' \) when non-defaulting. Finally, following Hatchondo, Martinez, and Sapriza (2010), the fixed point for the value function is computed as the limit of the finite problem. The choice of a fixed grid to find the optimal policy for \( B' \) is explained by the fact that the value function admits more than one local maximum. This is because \( a \) is discretized, so the value function \( q(B', a) \) is a step function. Another way to proceed would be to fit splines in the price function to mimic a continuous process for \( a \) and use a Golden search maximization routine. This numerical method, that appeared much less stable, was abandoned in the short run, but could be a promising way to significantly increase both the speed and the precision of these types of models.
Figure 10: Total external debt over GDP and tax redistribution across countries

**Note:** Total external debt over GDP, residuals. Source: World Bank Quarterly External Debt Statistics. Progressivity: percentage change between pre-tax (disposable) and after-tax (market) Gini coefficient. Source: Luebker (2011), based on Luxembourg Income Study (LIS) Database, mostly 2000s. All countries with available data are reported.
Figure 11: Total external debt over GDP and tax redistribution across countries, excluding Ireland

Figure 12: Debt over GDP and inequality across countries

Note: External public debt over GDP, residuals. Source: World Bank Quarterly External Debt Statistics, 2013Q4. Inequality: Top to bottom average income decile. Source: World Bank, Poverty and Inequality Database, average available data 2006-2012. All countries with available data are reported, but it should be noted that Greece defaulted in 2012.
Figure 13: Default probabilities and progressivity, including Argentina

Note: CDS, residuals, including Argentina. Source: Bloomberg. Progressivity: percentage change between pre-tax (disposable) and after-tax (market) Gini coefficient. Source: Luebker (2011), based on Luxembourg Income Study (LIS) Database, mostly 2000s. Argentina was excluded from Figure 7 to show that the negative relationship between progressivity and CDS is not only driven by this outlier.
\section*{F \hspace{1em} Data}

\section*{G \hspace{1em} Optimal policy problem, Assumption 2}

Let $V_{\text{nd}}^-$ be the value function of a government that had access to financial markets last period, just before the realization of the shock $a$. Without commitment, the tax progressivity is not part of the state. The value function is described as:

$$V_{\text{nd}}^-(B, a) = \max_{\tau_1} \int_a \max \left( \tilde{V}_{\text{nd}}^d(B, a, \tau_1), \tilde{V}_d^d(a, \tau_1) \right) dF(a|a-)$$

where $\tilde{V}_{\text{nd}}$ and $\tilde{V}_d$ are defined as:

$$\tilde{V}_{\text{nd}}^d(B, a, \tau_1) = \max_{B', \tau_2, c, n_i} \int u(c_i, n_i) di + \beta V_{\text{nd}}^d(B', a)$$

s.t. $c_i = a_\epsilon n_i - T(a_\epsilon n_i; \tau_1, \tau_2) \forall i$

$$g + B = q(B', a)B' + \int T(a_\epsilon n_i; \tau_1, \tau_2)$$

$$u_{n, i} = -u_{c, i} a_\epsilon (1 - T'(a_\epsilon n_i; \tau_1, \tau_2)) \forall i$$

and

$$\tilde{V}_d^d(a, \tau_1) = \max_{\tau_2, c, n_i} \int u(c_i, n_i) di + \beta \left[ \pi V_{\text{nd}}^d(0, a) + (1 - \pi) V_d^d(a) \right]$$

s.t. $c_i = \chi(a)\epsilon_i n_i - T(\chi(a)\epsilon_i n_i; \tau_1, \tau_2) \forall i$

$$g = \int T(\chi(a)\epsilon_i n_i; \tau_1, \tau_2)$$

$$u_{n, i} = -u_{c, i} \chi(a)\epsilon_i (1 - T'(\chi(a)\epsilon_i n_i; \tau_1, \tau_2)) \forall i$$

The last value function, $V_d^d(g)$, is defined as follows:

$$V_d^d(a) = \max_{\tau'_1} \int_a \tilde{V}_d^d(a', \tau'_1) dF(a'|a)$$