Recognition for Sale

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Abstract

I examine the consequences of letting players compete for bargaining power in a multilateral bargaining game. In each period, the right to propose an offer is sold to the highest bidder, and all players pay their bids. If players vote according to any rule in which no player has veto power, then the first proposer captures the entire surplus. If a full consensus is needed for an offer to be accepted, then the first proposer shares the surplus with at most one other player, and as the period length between offers vanishes, one player may capture virtually the entire surplus. In settings with a stochastic or an endogenous surplus, players are unwilling to efficiently delay agreement or invest in the surplus.

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1 Introduction

A group of players have a surplus to divide. An individual may influence the negotiating process by deciding which proposals are put on the table. This is *agenda-setting power*. But how should agenda-setting power be allocated?

This paper studies the implications of selling agenda-setting power to the player most willing to pay for it. I consider a sequential bargaining model in the spirit of Rubinstein [14] and Baron and Ferejohn [2] but in which recognition is neither alternating-offer nor stochastic but is instead sold through an auction in each period. In each period, two or more players bid for recognition in an all-pay auction, and the winner is selected to be the *proposer*. That player proposes a division of the surplus and her proposal passes should it be accepted by a winning coalition (as defined by the voting rule); otherwise, the game continues to the next period.

The main finding is that in equilibrium, the sale of bargaining power fosters severe inequality: the first proposer captures the *entire surplus* for every voting rule in which no individual player has veto power. This degenerate bargaining outcome is independent of heterogeneity in patience and asymmetries that players may have in their bidding capabilities. When players are asymmetric, only the strongest player (appropriately defined by the analysis) expects a strictly positive surplus, and all other players dissipate their rents in competing for recognition. Thus, selling bargaining power discourages players from sharing resources with others, and fosters “winner-take-all” behavior.

Let me offer a snapshot of the intuition. From the study of all-pay auctions in Siegel [15], we know that not more than a single player anticipates having a positive surplus after the race for a prize. But the prize that I model—recognition—is unconventional: it is a position in an extensive-form game, and thus, has no inherent value. Instead, its value is the strategic behavior that it induces. I derive the value of recognition (and the cost of not being recognized) using the logic of dynamic bargaining: a proposer forms a minimal winning coalition with the cheapest coalition partners offering each of these players her discounted continuation value. Combining these forces implies the result: prior to bidding, at least \( n - 1 \) players dissipate so much surplus in the competition for recognition that their expected payoff prior to the auction corresponds to their payoff from not being recognized at all, which is at most their discounted expected payoff prior to tomorrow’s auction. This bound implies that at least \( n - 1 \) players have a continuation payoff of 0. Since the voting rule satisfies the no veto property, the first proposer can assure the passage of a proposal in which she offers \( \epsilon > 0 \) to \( (n - 2) \) of these players, and captures the remaining \( 1 - (n - 2)\epsilon \) for \( \epsilon > 0 \). Therefore, in equilibrium, she captures the entire surplus.

This result bears several implications. The competition for recognition is intense, and the auctioneer extracts a large surplus when selling bargaining power: at least \( n - 1 \) players dissipate all
of their surplus from the race for recognition. Because bargaining ends immediately, behavior is the
same with periodic auctions as it would be with a single auction for permanent recognition rights.
The outcome is not only inequitable but also (utilitarian) inefficient: if an individual’s benefits from
her share of the surplus are concave, an interior split leads to higher joint surplus. Heterogeneity in
bidding capabilities plays a critical role in deriving the payoff and identity of the strongest player.

The no veto property plays a fundamental role in the result described above but introducing
veto power has only a slight effect on inequality. In a symmetric environment, the first proposer
still captures the entire surplus as all players dissipate their surplus in competing for power in the
future. With heterogeneity in bidding costs, the proposer shares the surplus with at most one other
player, namely the player who finds it least costly to bid. Indeed, that player extracts virtually the
entire surplus as the period length converges to 0, even if she is not the proposer.

Selling power harms efficiency in a broader context in which players can influence the size of the
surplus. If surplus varies stochastically, when bargaining power is sold through an all-pay auction,
players are no longer willing to wait for the surplus to grow, even for an unanimity rule. This result
offers a sharp contrast to Merlo and Wilson [12] who find that groups efficiently delay agreement
when recognition is stochastic. Similarly, if the surplus emerges from investments made by the
players, no player is willing to invest in this public good if at the negotiation stage, bargaining
power is sold through an auction.

I see two ways in which results from this stylized model may be useful. First, proposers are
indeed often recognized by a chair, CEO, or leader of an organization. Ideally, the leader may
screen proposers and select those that serve the group’s interests, but in practice, he may respond
to bribes or lobbying efforts. Second, because power is an unusual intermediate good whose value is
derived from its strategic impact, it is a priori unclear as to what happens when bargaining power
is sold to the highest bidder. The crisp logic that emerges here elucidates a general negative effect
of selling bargaining power. To the extent that social conventions prohibit or stigmatize the sale of
power as “corruption,” these results offer a theoretical rationale for these restrictions.

Related Literature: The most closely related paper is Yildirim Yildirim [16], which develops a
framework for multilateral bargaining in which players exert costly effort that influences recognition
by a “contest success function.” He studies how players with the same cost of effort compete if they
have different time preferences, and analyzes which voting rules induce the greatest rent dissipation.2

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1A separate literature—Crawford [5], Evans [8], and Pérez-Castrillo and Weinstein [13]—had studied the implications of competition for coalitional bargaining games.
2Also related, Yildirim [17] studies under an unanimity voting role as to how transitory recognition—in which players compete for recognition in each period (as in this paper)—compares to persistent recognition—in which there is a single lobbying stage that determines recognition probabilities throughout the bargaining game.
My approach complements his work. I model the race for recognition through all-pay and first-price auctions because I am interested in exploring the implications of selling power to the highest bidder, and not only in lobbying activities. By contrast, the contest success function is better suited to understand lobbying and perhaps does not facilitate as transparent an understanding of auctioning off bargaining power. The difference in approach leads to substantially different results; moreover, the tractability of an auction environment permits a unified treatment of a broad class of coalitional structures in transferable and non-transferable utility environments, and in settings with stochastic and endogenous surplus. Closer to his results, I consider a setting in Section 4.1 in which I restrict the amount of bargaining power that can be sold, and in that section, I derive several comparative statics that parallel those of his work.

A small but growing literature has investigated different models of how players may influence their bargaining power. Board and Zweibel [4] analyze a bilateral bargaining framework in which players are budget-constrained, and recognition is awarded by a first-price auction. McKelvey and Reizman [11], Eguia and Shepsle [7], Bassi [3], and Diermeier, Prato, and Vlaicu [6] develop frameworks in which players influence procedures that determine bargaining power.3

I use the rich insights derived in the study of all-pay auctions. I apply the payoff characterization from Siegel [15] to establish that only one player can hope to gain from an all-pay auction, and I prove the analogue for first-price auctions. This paper illustrates that this logic applies even if the value of prizes emerges endogenously from future strategic behavior.

2 Contests for Power

A group of players $N = \{1, \ldots, n\}$ bargains over the division of a dollar, i.e., choosing a policy in $\mathcal{X} \equiv \{x \in [0, 1]^n : \sum_{i \in N} x_i = 1\}$. Bargaining takes places at discrete times in $\mathcal{T} \equiv \{t \in \mathbb{N} : t \leq \widehat{t}\}$ in which $\widehat{t} \leq \infty$ is the deadline for bargaining. In each period $t$, players bid for bargaining power, and the proposer for time $t$, $p^t$, is determined. The proposer proposes a division $x^t$ in $\mathcal{X}$. Players vote to accept or reject the proposal sequentially.4

**Scores:** Each player $i$ chooses a score $s_i \geq 0$ and incurs cost $c_i(s_i)$, which is continuous in $s_i$. Player $i$ may have a head-start $\bar{s}_i \geq 0$ where for every $s \leq \bar{s}_i$, $c_i(s) = 0$, and in the domain $[\bar{s}_i, \infty]$.

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3This paper also connects to my prior work with B. Douglas Bernheim and Xiaoqen Fan [1] in which we study the implications of information about bargaining power being revealed in prior periods. We show that predictability can also concentrate power in the hands of the first proposer, but the logic does not apply to the setting here since no player can be ruled out from being the next proposer, even in equilibrium.

4Sequential voting avoids coordination failures at the voting stage, and is equivalent to assuming that players vote as if pivotal.
\(c_i(\cdot)\) is strictly increasing. The auctioneer chooses a player among those with the highest score to be the proposer, and can use any procedure to break ties.

**Voting:** Let \(\mathcal{C} \subset 2^N \setminus \emptyset\) be a set of winning coalitions. For every period \(t\), a proposal is accepted in period \(t\) if and only if there is a coalition of players in \(\mathcal{C}\) that individually votes to accept the proposal. The set of winning coalitions is monotone: if \(C \in \mathcal{C}\) and \(C \subset C'\), then \(C' \in \mathcal{C}\). A voting rule satisfies no veto power if \(\mathcal{C}\) includes every coalition that has at least \(n-1\) players, and satisfies veto power otherwise. A unanimous voting rule is \(\mathcal{C} = \{N\}\).

**Payoffs:** If policy \(x\) is accepted in round \(t\), player \(i\)'s payoff after choosing scores \((s_i^0, \ldots, s_i^t)\) is \(\delta_i u_i(x_i) - \sum_{s=0}^{t} \delta_i c_i(s_i^s)\), where for each \(i\), \(u_i\) is strictly increasing, continuous, weakly concave, and satisfies \(u_i(0) = 0\). The “maximum willingness to pay” is bounded: for each player \(i\), \(\lim_{s_i \to \infty} c_i(s_i) > u_i(1)\). Players vary in patience, but no player is perfectly patient. If no policy is ever accepted, then each player obtains 0 from negotiations but incurs all the costs from bidding. Players are one-shot symmetric if \(c_i(\cdot) = c_j(\cdot)\) and \(u_i(\cdot) = u_j(\cdot)\) for each pair \(i\) and \(j\). Contrastingly, players are ordered if whenever \(i < j\), then \(c_j(s) \leq u_j(1)\) implies that \(c_i(s) < u_i(1)\) for every \(s \in [0, \infty)\).\(^5\) Players are uniformly ordered if whenever \(i < j\), player \(i\) has a (weakly) greater head-start \((s_i \geq s_j)\) and is more willing to compete for every feasible prize: for every \(\hat{x} \in [0, 1]\), \(s > s_i\), \(u_i(\hat{x}) - c_i(s) > u_j(\hat{x}) - c_j(s)\).

Utility is transferable if \(u_i(\cdot)\) is the identity function.

**Solution Concept:** I denote by \(h^t\) the history of scores chosen, proposers selected, proposals made, and voting decisions in periods \(0, \ldots, t-1\), and by \(\mathcal{H}^t\) the set of all such feasible histories. The set of all feasible histories is \(\mathcal{H} \equiv \cup_{t \in T} \mathcal{H}^t\). A bidding strategy is \(\sigma_i^x : \mathcal{H} \to \Delta[0, \infty)\). Let \(\mathcal{H}_i\) denote all interim histories in which player \(i\) is the proposer; \(\sigma_i^p : \mathcal{H}_i \to \Delta\mathcal{X}\) is her mixture over proposals. Finally, \(\sigma_i^x : \mathcal{H} \times [0, \infty]^n \times \mathcal{X} \to \Delta\{\text{Yes, No}\}\) is her probability of accepting the proposal on the table. In the finite-horizon setting, I study subgame perfect equilibria (henceforth SPE). In the infinite-horizon, I study stationary subgame perfect equilibria (henceforth SSPE), in which behavior is independent of time and history.\(^6\)

### 2.1 A Key Lemma from All-Pay Auctions

An important simplification comes from Siegel [15]'s payoff characterization. Suppose \(n\) players participate in an all-pay auction. The value of winning for player \(i\) is \(\bar{v}_i \geq 0\) and that of losing is

\(^5\)Being ordered is a generic property insofar as whenever it is the case that for \(i\) and \(j\), and \(s\), \(c_i(s) = u_i(1)\) and \(c_j(s) = u_j(1)\), then perturbations to \((c_i(\cdot), c_j(\cdot), u_i(1), u_j(1))\) result in the players being ordered.

\(^6\)In an SSPE, player \(i\)'s strategy reduces to \((\sigma_i^x, \sigma_i^p)\) where \(\sigma_i^x \in \Delta[0, \infty)\), \(\sigma_i^p \in \Delta\mathcal{X}\), and \(\sigma_i^x : \mathcal{X} \to \Delta\{\text{Yes, No}\}\).
\(v_i < \bar{v}_i\). Each player \(i\) chooses a score \(s_i\) at cost \(c_i(s_i)\). The auctioneer selects a winner from among those who choose the highest scores. I use the following results from Siegel [15].

**Lemma 1.** The all-pay auction has a Nash equilibrium. In every equilibrium, there exists a set of players \(J\) such that \(|J| \geq n - 1\), and for each \(j \in J\), player \(j\)’s expected equilibrium payoff is \(v_j\).

### 2.2 No Veto Power

**Theorem 1.** If the voting rule satisfies no veto power, the proposer selected at \(t = 0\) captures the entire surplus in every SPE of the finite horizon, and in every SSPE of the infinite horizon.

**Proof.** **Step 0: The General Reduced Game.** Consider the all-pay auction in which \(\bar{v}_i = u_i(1)\), \(v_i = 0\), and the costs of bidding are those described in Section 2. Denote this game as \(G^R\). By Lemma 1, \(G^R\) has a Nash equilibrium; denote one Nash equilibrium by \((\sigma_i^R, \ldots, \sigma_n^R)\) and let \(w_i\) be player \(i\)’s ex ante expected equilibrium payoff. By Lemma 1, \(w_i > 0\) for at most one player \(i\).

**Step 1: Existence of Equilibrium.** The following strategy profile is an equilibrium:

1. for each history \(h \in H_i\), player \(i\) chooses \(\sigma_i(h) = \sigma_i^R\),
2. for each history \(h_i \in H_i\), player \(i\) makes the proposal \(x\) that sets \(x_i = 1\) with probability 1,
3. for each history, at the voting stage, a player \(i\) votes to accept any proposal that offers her a payoff no less than \(\delta_i w_i\).

Players have no incentives to deviate, either when proposing or voting. At the bidding stage, the game reduces to the all-pay auction \(G^R\), and the bidding strategies are an equilibrium of that game.

**Step 2: Unique SPE Outcome of Finite Horizon.** The proposer in the final terminal period captures the entire surplus, and so the auction at \(\bar{t}\) reduces to \(G^R\). By Lemma 1, at least \(n - 1\) players have a continuation payoff of 0 at the beginning of period \(\bar{t}\). Because the voting rule satisfies no veto power, the proposer in \(\bar{t} - 1\) can guarantee passage of a proposal that promises \(\epsilon\) to \(n - 2\) players, and in equilibrium, captures the entire surplus. Thus, the auction at \(\bar{t} - 1\) also reduces to \(G^R\), and the result follows from induction.

**Step 3: Unique SSPE Outcome of Infinite Horizon.** In an SSPE, there exists a vector \((W_1, \ldots, W_n)\) such that at the beginning of period \(t\), player \(i\)’s value is \(W_i\). Lemma 3 in the Appendix establishes that every equilibrium proposal offered with strictly positive probability is accepted with probability 1. Once a proposer is recognized, she forms a minimal winning coalition

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\(^7\)The existence result is his Corollary 1, the bound on the payoff for \(n - 1\) players is his Zero Lemma.
by offering to \( j \in C \setminus \{ i \} \) player \( j \)'s discounted continuation value. Therefore, player \( i \) as proposer has an expected payoff of \( u_i \left( 1 - \min_{C \setminus \{ i \}} \sum_{j \in C \setminus \{ i \}} u_j^{-1}(\delta_j W_j) \right) \); denote this term as \( V_i \), which I show in Lemma 3 is well-defined. When player \( i \) is not the proposer, she obtains \( \delta_i W_i \) whenever she is included in a minimal winning coalition. Therefore, her expected payoff conditional on losing at the bidding stage is

\[
V_i = \sum_{j \in C \setminus \{ i \}} Pr(j \text{ is the proposer} \mid i \text{ is not the proposer}) \sum_{x \in \mathcal{X} \setminus \{ i \}} \sigma_j(x) (\delta_j W_j).
\]  

(1)

At the bidding stage, behavior reduces to that of an all-pay auction in which player \( i \) obtains \( V_i \) if she wins the auction and obtains the expected payoff of \( V_i \) if she loses the auction. Lemma 1 implies that for at least \( n - 1 \) players, in equilibrium, \( W_i = V_i \). Combining this equation with (1) implies that for at least \( n - 1 \) players, \( W_i = 0 \). Since the voting rule satisfies no veto power, for each player \( i \), there exists a coalition \( C \in \mathcal{C} \) such that \( \sum_{j \in C \setminus \{ i \}} u_j^{-1}(\delta_j W_j) = 0 \). Therefore, \( V_i = u_i(1) \), which implies that the first proposer captures the entire surplus.

Because the first proposer captures the entire surplus, players compete strongly for recognition. If players are one-shot symmetric, players dissipate all surplus in this race: each has an expected equilibrium payoff of 0 and expenditure of \( u_i(1) \). If players are ordered, then all players among 2, \ldots, \( n \) have an expected equilibrium payoff of 0, and player 1’s expected equilibrium payoff is \( u_1(1) - c_1(c_2^{-1}(u_2(1))) \). Thus, only the player who finds it least costly to compete for the entire prize has a strictly positive payoff. Patience has no impact on bargaining shares, payoffs, or bidding / lobbying behavior; relative concavity of a player’s utility function affects only the identity and payoff of the strongest player, and bidding behavior. Negotiations and payoffs are invariant across voting rules that satisfy no veto power.

### 2.3 Unanimity Rule and Veto Power

What can be said for other voting rules? My first result describes symmetric players.

**Theorem 2.** Suppose that players are one-shot symmetric. Then for every voting rule, the proposer selected at \( t = 0 \) captures the entire surplus in every SPE of the finite horizon and in every SSPE of the infinite horizon.

Symmetric players dissipate all surplus in competing for power, and so a proposer can obtain unanimous consent to her taking the entire surplus. This result fails when players are heterogeneous:

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8Since the generic conditions of Siegel [15] are satisfied, his Theorem 1 implies the above expected payoffs.
when a player has veto power, and has a strictly positive expected equilibrium payoff, the first proposer cannot capture the entire surplus. Indeed, slight advantages can let a player capture the entire surplus in the continuous-time limit, even if she is not the first proposer. Let the period length between offers be $\Delta$ and player $i$’s discount rate be $\rho_i$.

**Theorem 3.** Suppose that the voting rule is unanimity, utility is transferable, and players are uniformly ordered. For every $\Delta > 0$, in every SSPE, all players $2, \ldots, n$ have ex ante expected payoffs of 0, and only player 1 has a strictly positive ex ante expected payoff. As $\Delta \rightarrow 0$, the ex ante expected payoff of player 1 converges to 1 in every SSPE.

*Proof.* Let $W^\Delta_i$ summarize player $i$’s continuation value at the beginning of the period, $V^\Delta_i$ summarize how much she obtains when she is the proposer, and $W^\Delta_i$ summarize her expected payoff conditional on losing the auction when the period length is $\Delta$. Since $C = \{N\}$, $V^\Delta_i = 1 - \sum_{j \in N \setminus \{i\}} e^{-\rho_j \Delta} W^\Delta_j$ and $W^\Delta_i = e^{-\rho_i \Delta} W^\Delta_i$. **Lemma 1** implies that for at least $n - 1$ players, $W^\Delta_i = V^\Delta_i$, and therefore $W^\Delta_i = 0$. I argue that $W^\Delta_i > 0$ by eliminating other possibilities.

Suppose that $W^\Delta_i = 0$ for all $i$. Then $V^\Delta_i = 1$ and $V^\Delta_i = 0$ for each player $i$. But then players $2, \ldots, n$ never choose a score beyond $c_2^1(1)$, in which case, player 1 can attain a payoff of $1 - c_1(c_2^1(1)) > 0$, because players are uniformly ordered, and thereby contradicting $W^\Delta_i = 0$.

Now suppose that $W^\Delta_i = 0$ but $W^\Delta_j > 0$ for some $j \neq 1$. It then follows that for all $i \neq j$, $V^\Delta_i = 1 - e^{-\rho_j \Delta} W^\Delta_j$, and $V^\Delta_j = 0$, and $W^\Delta_j = e^{-\rho_j \Delta} W^\Delta_j$, which implies that $V^\Delta_i - V^\Delta_j = V^\Delta_j - V^\Delta_j$. Then no player chooses a score beyond $c_2^1(1 - e^{-\rho_j \Delta} W^\Delta_j)$. Therefore, for every $\epsilon > 0$, $W^\Delta_i \geq 1 - e^{-\rho_i \Delta} W^\Delta_j - c_1(c_2^1(1 - e^{-\rho_j \Delta} W^\Delta_j)) - \epsilon$: player 1 can always bid slightly more than $c_2^1(1 - e^{-\rho_j \Delta} W^\Delta_j)$ with probability 1 and ensure that she is the proposer. Because $e^{-\rho_j \Delta} W^\Delta_j < 1$, $c_2^1(1 - e^{-\rho_j \Delta} W^\Delta_j) > \theta_2 \geq \theta_1$. Since players are uniformly ordered, $c_1(c_2^1(1 - e^{-\rho_j \Delta} W^\Delta_j)) < 1 - e^{-\rho_j \Delta} W^\Delta_j$. Therefore, for sufficiently small $\epsilon$, $1 - e^{-\rho_j \Delta} W^\Delta_j - c_1(c_2^1(1 - e^{-\rho_j \Delta} W^\Delta_j)) - \epsilon > 0$, contradicting $W^\Delta_i = 0$.

Thus, we have established that $W^\Delta_i > 0$ and $W^\Delta_i = 0$ for all $i \geq 2$, which implies that $V^\Delta_i = 1$, $V^\Delta_i = e^{-\rho_i \Delta} W^\Delta_i$, $V^\Delta_i = 1 - e^{-\rho_i \Delta} W^\Delta_i$, and $V^\Delta_i = 0$ for all $i \geq 2$. The reach of each player is the score at which she is indifferent between winning for sure and losing, i.e., $V^\Delta_i - c_i(r^\Delta_i) = V^\Delta_i$. Since $V^\Delta_i - V^\Delta_i$ is constant across players, and players are uniformly ordered, the contest is a “generic contest” in the terminology of Siegel [15]. By his Theorem 1, $W^\Delta_1 = V^\Delta_1 - c_1(r^\Delta_1) = 1 - c_1(r^\Delta_1)$, and $1 - e^{-\rho_1 \Delta} W^\Delta_1 - c_2(r^\Delta_2) = V^\Delta_2 - c_2(r^\Delta_2) = V^\Delta_2 = 0$. Combining the equations implies

$$1 - e^{-\rho_1 \Delta} = c_2(r^\Delta_2) - e^{-\rho_1 \Delta} c_1(r^\Delta_1).$$

(2)

As $\Delta \rightarrow 0$, the LHS converges to 0, and the RHS converges to $\lim_{\Delta \rightarrow 0}[c_2(r^\Delta_2) - c_1(r^\Delta_1)]$. Equality holds only if $c_2(r^\Delta_2) \rightarrow 0$, which implies that $c_1(r^\Delta_2) \rightarrow 0$. Therefore, $\lim_{\Delta \rightarrow 0} W^\Delta_1 = 1$. □
Heterogeneity generates a *discouragement effect*: player 1 has a persistent advantage in future negotiations, whereas other players dissipate their surplus in the future. Thus, when player 1 is the proposer, she captures the entire surplus, and when she is not the proposer, she votes any proposal that offers less than her discounted continuation payoff. A disadvantaged player is deterred from competing in the continuous-time limit because even if she wins, she has to share so much with player 1 that there is little value from winning. In the limit, player 1 captures the entire prize at virtually no cost.\(^9\)

Now I discuss the outcome that emerges when utility may not be transferable, players may not be uniformly ordered, and some but not all players have veto power. The general insight is that the first proposer may share surplus with one other player but with no more.

**Theorem 4.** Suppose that the voting rule satisfies *veto power*. Then in every SPE of the finite horizon, and in every SSPE of the infinite horizon, at least \(n-2\) players obtain a payoff of 0.

I relegate the proof to the Appendix. The example below shows that this bound is tight.

**Example 1.** Suppose there are three players, each of whom has transferable utility, and a discount factor of \(\frac{1}{2}\). Suppose that the voting rule is unanimity. The environment is uniformly ordered: \(\tilde{\xi}_i = 0\) for every \(i\), and \(c_1(s) = \frac{\tilde{\xi}}{3}\), \(c_2(s) = \frac{\tilde{\xi}}{2}\), and \(c_3(s) = s\). By (2), the reach of player 2 solves \(\frac{1}{2} = \frac{v_2}{2} - \frac{1}{2}\left(\frac{v_2}{3}\right)\), which implies that \(v_2 = \frac{3}{2}\). Therefore, \(W_1 = 1 - \left(\frac{1}{3}\right)\left(\frac{3}{2}\right) = \frac{1}{2}\). So if player 2 or 3 is the first proposer, that player shares \(\frac{1}{4}\) of the surplus with player 1 and keeps the remainder to herself.

In summary, the first proposer captures the entire surplus either if players are symmetric or there is no veto power, and otherwise, at most two players share the surplus.

### 3 An Application to Stochastic and Endogenous Surplus

**Stochastic Surplus:** When the surplus varies stochastically, players may wish to delay negotiations until it is larger.\(^10\) Merlo and Wilson [12] show that a unanimity rule generates efficient delays in negotiations when recognition is stochastic. By contrast, I find that when recognition is for sale, players immediately agree *regardless of the initial state.*

Suppose that utility is transferable and that the size of the surplus in period \(t\), \(\pi^t\) is drawn i.i.d. according to cdf \(F\) with support \([\underline{\pi}, \bar{\pi}]\) in which \(\bar{\pi} > 0\). At the beginning of period \(t\), players first observe the size of the surplus \(\pi^t\). The remainder of the game follows that of Section 2, except that

\(^9\)The logic is similar to but distinct from that of multi-stage contests, e.g. Klump and Polborn [9] and Konrad and Kovenock [10], in which once a player is disadvantaged, she gains little from winning a stage of the contest because her future payoffs are dissipated through future competition.

\(^10\)I thank Huseyin Yildirim for suggesting that I study this setting.
the proposer selects proposals in $X(\pi^t) \equiv \{x \in [0,1]^n : \sum_{i \in N} x_i = \pi^t\}$. In a SSPE, players condition their choices in period $t$ also on the payoff relevant state, $\pi^t$, in period $t$.\footnote{If players were compelled to compete prior to observing the size of the surplus, the negative conclusion for efficiency emerges immediately from Theorem 2.}

**Theorem 5.** Suppose that players are one-shot symmetric. Then for every voting rule, there exists an SSPE in which bargaining ends in period 0, and the first proposer captures the surplus $\pi^0$. Moreover, this is the unique SSPE outcome when the voting rule is unanimity.

Because players have nothing to gain from the future, they are unwilling to wait to let the pie grow. This collective impatience emerges even though players may be heterogeneous in their patience, and all, if not some, players may be very patient. I prove Theorem 5 in the Appendix by adapting the argument of Theorem 2 to permit a stochastic surplus.

**Endogenous Surplus:** Suppose that players can influence the size of the surplus by investing in it: at $t = 0$, players simultaneously choose effort $e_i \geq 0$ leading to a total surplus of $\pi(e_1, \ldots, e_n) > 0$. The cost of effort for player $i$ is $K_i(e_i)$, which is strictly increasing in $e_i$ and satisfies $K_i(0) = 0$. After the surplus is created, players compete for recognition and negotiate as in Section 2.

**Theorem 6.** Suppose that players are one-shot symmetric. Then, for every voting rule, the unique SSPE outcome involves every player choosing $e_i = 0$, and the proposer capturing the entire surplus $\pi(0, \ldots, 0)$ at $t = 0$.

I omit the proof because it follows from Theorem 2: regardless of investments, the first proposer captures the entire surplus. Since every player dissipates her payoffs in competing to be that proposer, no player wishes to invest in surplus.

## 4 Other Extensions

### 4.1 Restricting the Sale of Bargaining Power

I vary the fraction of bargaining power that the leader can sell. For $\lambda$ in $[0,1]$, the winner of the auction is the proposer with probability $\lambda + \frac{1-\lambda}{n}$ and each loser is the proposer with probability $\frac{1-\lambda}{n}$. The model reduces to Baron and Ferejohn [2] at $\lambda = 0$ and to that of Section 2 at $\lambda = 1$. Players are one-shot symmetric, $\delta_i = \delta$ for each player $i$, utility is transferable, and I study symmetric SSPE in which proposers randomize uniformly across all possible minimal winning coalitions. Let $W$ be the
expected value from this round for a player, $\bar{V}^\lambda$ be a player’s payoff when she wins the auction, and $\bar{V}^\lambda$ be her payoff when she loses the auction. By calculation,

$$\bar{V}^\lambda \equiv \left( \lambda + \frac{1 - \lambda}{n} \right) \left( 1 - (q - 1)\delta W \right) + (n - 1) \left( \frac{1 - \lambda}{n} \right) \left( \frac{q - 1}{n - 1} \delta W \right),$$

$$V^\lambda \equiv \left( \frac{1 - \lambda}{n} \right) \left( 1 - (q - 1)\delta W \right) + \left( \lambda + (n - 1) \left( \frac{1 - \lambda}{n} \right) \right) \left( \frac{q - 1}{n - 1} \delta W \right).$$

Because players are symmetric, they must have the same payoffs from the all-pay auction and dissipate all of their surplus. Therefore, substituting $W = \bar{V}^\lambda$ implies that

$$V^\lambda = \frac{(1 - \lambda)(n - 1)}{n((n - 1) - \lambda \delta(q - 1))}, \quad \bar{V}^\lambda = \frac{\lambda((n - 1)^2 - \delta n(q - 1) + (n - 1)}{n((n - 1) - \lambda \delta(q - 1))}$$

The equilibrium value of the prize, $\bar{V}^\lambda - V^\lambda$, is strictly increasing in $\lambda$. I use these terms to obtain the following comparative statics.

**Theorem 7.** For every $\lambda < 1$, the proposer does not capture the entire surplus. As $\lambda$ increases, the proposer captures a larger share of the surplus and distributes less to each member of her coalition. Increases in $\lambda$ increase expected expenditures.

Theorem 7 is the main comparative static of this paper: increasing the sale of bargaining power monotonically increases the first proposer’s share and decreases the share of every other player. Restricting the sale of bargaining power always improves equity and efficiency.

The above terms also permit other comparative statics. Consider a sequence of voting games in which $\frac{q - 1}{n}$ is a constant $\tau$. The value of winning the auction, $\bar{V}^\lambda - V^\lambda$, can be re-written as $\frac{\lambda(1 - \delta \tau)}{1 - \lambda \delta \tau}$. Notice that this is constant in $n$, and therefore, the competition for bargaining power is unaffected by the population size for a fixed proportional rule.\footnote{Also, the amount received by any individual loser of the auction, $\delta V^\lambda$, converges to 0 but the amount offered by the proposer to a winning coalition, $n \rightarrow \infty$, $n \tau \delta V^\lambda$ converges to a strictly positive constant in $(0, 1)$, thereby bounding the first proposer’s share away from 1 for all population sizes.} Increasing $\delta$ and $\tau$ increases how much the proposer offers each coalition partner and decreases how much she keeps for herself; therefore, these variables attenuate the race for recognition. These comparative statics parallel those of Yildirim [16], and strengthen his message on how aspects of group size, patience, and consensus affect competing for power (since our frameworks are distinct).

### 4.2 First Price Auctions

I focus throughout the paper on all-pay auctions because the stigma and illegality of selling bargaining power may make it difficult for individuals to commit to contingent payments after the
auction, and thus, all payments may be upfront. If players could commit to contingent payments, a first-price auction environment is a more appropriate model. The analysis of such a setting is different, but similar results emerge. I first establish the analogue of Lemma 1.

**Lemma 2.** The first-price auction has a Nash equilibrium. In every equilibrium, there exists a set of players \( J \) such that \( |J| \geq n - 1 \), and for each \( j \in J \), player \( j \)'s expected equilibrium payoff is \( v_j \). Moreover, if the auction is generic, player 1 has an expected payoff of \( \overline{v}_1 - c_1(r_2) > v_1 \) and each other player \( j \) has an expected payoff of \( v_j \) in every weakly undominated Nash equilibrium.

Only minor modifications to the argument of Theorem 1 are needed to establish the following:

**Theorem 8.** Suppose the voting rule satisfies no veto power, and only the player who chooses the highest score pays her bid. Then the proposer selected at \( t = 0 \) captures the entire surplus in every SPE of the finite horizon, and in every SSPE of the infinite horizon.

Pinning down rent dissipation requires further assumptions. Suppose that utility is transferable. Let \( r_i \) be player \( i \)'s reach when the payoff from losing is 0 and that from winning is 1, and suppose that players are ordered so that \( r_1 \geq r_2 \geq \ldots \geq r_n \). When \( r_1 = r_2 \), then all players have an expected payoff of 0 in equilibrium so the race for recognition dissipates all surplus. When \( r_1 > r_2 > r_3 \), there exists multiple equilibrium outcomes, although in each of these equilibria, player 1 is the first proposer and captures the entire surplus. For every \( s \in (r_2, r_1] \), there exists an equilibrium in which all players bid \( s \) with probability 1, and the leader selects player 1 to be the proposer. However, players 2, \ldots, \( n \) are playing weakly dominated scores at the bidding stage. Ruling out such behavior pins player 1’s payoffs as \( 1 - c_1(r_2) \), exactly as in the all-pay auction.

With a unanimity voting rule, the analogue of Theorem 3 emerges, assuming the same conditions on costs as in Section 2.3: the first proposer shares surplus with only player 1 if she does so with anyone. Eliminating weakly dominated scores at the bidding stage leads to player 1 capturing virtually the entire surplus regardless of the identity of the first proposer as \( \Delta \to 0 \). Therefore, the conclusions of selling bargaining power extend to a first-price auction.

**Appendix: Omitted Proofs**

**Lemma 3.** Consider an SSPE.

1. For each player \( i \), \( \overline{V}_i = u_i \left( 1 - \min_{C \in \mathcal{C}} \sum_{j \in C \setminus \{i\}} u_j^{-1}(\delta_j W_j) \right) \) is well-defined.
2. Every proposal offered with strictly positive probability is accepted with probability 1.

**Proof.** The arguments below adapt proofs of Theorem 4 of [1].
Proof of Part 1: In equilibrium, let \( \pi \) be the undiscounted average of policies selected on the equilibrium path. Because \( u_j \) is concave for each \( j \) and \( \delta_j < 1 \), and bidding is only potentially costly, Jensen’s Inequality implies that \( u_j(\pi_j) > \delta_j W_j \), and therefore for every \( C \subset \mathcal{N} \), \( \sum_{j \in C \setminus \{i\}} u_j^{-1}(\delta_j W_j) \leq \sum_{j \in \mathcal{N}} u_j^{-1}(\delta_j W_j) < \sum_{j \in \mathcal{N}} \pi_j \leq 1 \). Therefore, \( 1 - \min_C \sum_{j \in C \setminus \{i\}} u_j^{-1}(\delta_j W_j) \in (0, 1] \) and so \( V_i \) is well-defined.

Proof of Part 2: Suppose there is an equilibrium proposal offered with strictly positive probability, \( x' \), by player \( i \) that is rejected with strictly positive probability. Select some coalition \( C \) such that \( i \in C \) and for every other coalition \( C' \in C \) such that \( i \in C' \), \( \sum_{j \in C \setminus \{i\}} u_j^{-1}(\delta_j W_j) \leq \sum_{j \in C \setminus \{i\}} u_j^{-1}(\delta_j W_j) \). In other words, \( C \) is a minimal winning coalition for player \( i \). Define a proposal \( x^\epsilon \) for small \( \epsilon \geq 0 \) in which \( x_j^\epsilon = \delta_j W_j + \epsilon \) for every \( j \in C \setminus \{i\} \), \( x_j^\epsilon = 0 \) for every \( j \notin C \), and the proposer \( i \) keeps the remainder. In equilibrium, the proposal \( x^\epsilon \) is accepted by all members of \( C \) with probability 1 if \( \epsilon > 0 \), and therefore is implemented. Observe that the proof of Part 1 implies that \( u_j^{-1}(\delta_i W_i) \) is strictly less than \( 1 - \sum_{j \in C \setminus \{i\}} u_j^{-1}(\delta_j W_j) \), and therefore, for sufficiently small \( \epsilon > 0 \), \( x_j^\epsilon \) is strictly greater than \( u_j^{-1}(\delta_i W_i) \). Thus, conditional on \( x^\epsilon \) being rejected, player \( i \) is discretely better off deviating to \( x^\epsilon \) for sufficiently small \( \epsilon > 0 \), and conditional on \( x^\epsilon \) being accepted, player \( i \) is no better off than when she offers \( x^0 \). Since proposal \( x^\epsilon \) is rejected with strictly positive probability, player \( i \) has a strictly profitable deviation for sufficiently small \( \epsilon > 0 \).

Proof of Theorem 2 on p. 6. Steps 0-2 follow the same steps as in Theorem 2, except that in the Nash equilibrium of the reduced game, \( \mathcal{G}^R \), all \( n \) players have an expected payoff of 0. To complete Step 3, let \( u(\cdot) \) and \( c(\cdot) \) represent the common utility and cost functions. By Lemma 1, \( W_i = 0 \) for at least \( n - 1 \) players. I argue that \( W_i = 0 \) for every player \( i \). Suppose otherwise that \( W_i > 0 \) for some player \( i \).

First, let player \( i \) have individual veto power. Therefore, \( V_i = u(1) \) and \( V_j = \delta_i W_i \), and in equilibrium, player \( i \) wins the contest with strictly positive probability. So player \( i \) always chooses a score weakly lower than \( r_i \) that solves \( c(s) = u(1) - \delta_i W_i \). Consider player \( j \neq i \): it follows that \( V_j = u(1 - u^{-1}(\delta_i W_i)) \), and \( V_j = W_j = 0 \). The highest score that player \( j \) is willing to choose is \( r_j \) that solves \( c(s) = u(1 - u^{-1}(\delta_i W_i)) \).

The analysis proceeds by considering two exhaustive cases.

Case 1: Suppose \( u \) is linear. There is no loss of generality in treating \( u(\cdot) \) as the identity function. Then \( \overline{V}_j - \underline{V}_j = \overline{V}_i - \underline{V}_i \) for all \( j \neq i \). Corollary 3 of Siegel [15] implies that \( W_i = W_j = 0 \), yielding a contradiction.

Case 2: Suppose \( u \) is concave and \( u \) is not linear. Therefore, for all \( x \in (0, 1) \), \( u(x) > xu(1) \). Substituting \( x = u^{-1}(\delta_i W_i) \) and \( x = 1 - u^{-1}(\delta_i W_i) \) generates the following inequalities:

\[
\delta_i W_i > u^{-1}(\delta_i W_i) u(1), \text{ and } u(1 - u^{-1}(\delta_i W_i)) > (1 - u^{-1}(\delta_i W_i)) u(1).
\]

Adding these inequalities and re-arranging yields that \( \overline{V}_j - \underline{V}_j = u(1 - u^{-1}(\delta_i W_i)) > u(1) - \delta_i W_i = \overline{V}_i - \underline{V}_i \). Since \( c \) is increasing, it follows that \( r_j > r_i \). Consider a deviation whereby player \( j \) bids slightly more than \( r_i \); now player \( j \) wins whenever the profile of others’ bids are such that player \( i \) would have won in equilibrium, and when she does so, her payoffs are arbitrarily close to \( \overline{V}_j - c(r_i) > 0 \). Since her equilibrium payoff \( W_j = 0 \), this is a strictly profitable deviation, yielding a contradiction.
Suppose that $i$ does not have veto power: there exists a coalition $C$ in $\mathcal{C}$ such that $i \notin C$. By monotonicity, $\mathcal{N}\setminus\{i\} \in C$, which implies that every $j \neq i$ can extract the entire surplus and exclude player $i$ from her coalition. Thus, $\overline{V}_j = u(1)$ and $\overline{V}_i = 0$. But then $\overline{V}_j - \overline{V}_j = \overline{V}_i - \overline{V}_j = u(1)$ for every $j \neq i$. Thus, by Corollary 3 of Siegel [15], $W_i = W_j = 0$ for all $i, j$, leading to a contradiction.

**Proof of Theorem 4 on p. 8.** Suppose that the voting rule fails the no veto property. Then there exists a player such that the coalition of all other players is not a winning coalition. I prove that the first proposer does not share surplus with more than a single other player in both the finite and infinite horizon.

**Finite Horizon:** The proposer in the final terminal period must capture the entire surplus, and so the auction at time $\bar{t}$ reduces to $\mathcal{G}^R$. By Lemma 1, at least $n - 1$ players have a continuation payoff of 0 at the beginning of period $\bar{t}$ (before bidding) regardless of the equilibrium behavior at $\bar{t}$. Let $w^\pi_i$ be the expected payoff of player $i$ at the beginning of $\bar{t}$, which is 0 for $n - 1$ players. Then in equilibrium, if the proposer in period $\bar{t} - 1$ is player $j$, she obtains $u_j(1 - \sum_{i \neq j} u_i^{-1}(\delta_i w^\pi_i))$, sharing surplus only with a veto player who has a strictly positive continuation payoff. Therefore, the auction at $\bar{t} - 1$ reduces to an all-pay auction with a prize structure of $\overline{\pi}_j = u_j(1 - \sum_{i \neq j} u_i^{-1}(\delta_i w^\pi_i))$ and $\overline{\pi}_j = \delta_j w^\pi_j 1_{\mathcal{N}\setminus\{j\} \in C}$ for each player $j$. Lemma 1 applies to this auction, and by induction, the result follows for the first period.

**Infinite Horizon:** In an SSPE, the argument of Theorem 1 establishes that for at least $n - 1$ players, $W_i = 0$. Therefore, the first proposer shares $\delta_j W_j 1_{\mathcal{N}\setminus\{j\} \in C}$ with each player $j$, which can be positive for only a single player $j$.

**Proof of Theorem 5 on p. 9.** Adapting the notation to this environment, let a history $h^t$ encode all past choices (as before) and the current period’s surplus, $\pi^t$, and similarly for interim histories $h_i$. I denote for a generic history $h$ the surplus in the current period by $\pi(h)$. I associate the SSPE with a family of general reduced games, construct an equilibrium, and then prove that it is the unique outcome.

**Step 0: A Family of Reduced Games.** Consider the all-pay auction in which $\overline{\pi}_i = \pi$, $\overline{\pi}_i = 0$, and the costs of bidding are those described in Section 2. Denote this game as $\mathcal{G}^R_\pi$, and a Nash equilibrium of it by $(\sigma_1^\pi, \ldots, \sigma_n^\pi)$. By Corollary 3 of Siegel [15], each player’s expected equilibrium payoff is 0.

**Step 1: Existence of Equilibrium.** Consider the following equilibrium:

1. for each history $h \in \mathcal{H}$, player $i$ chooses $\sigma_i^\pi(h) = \pi(h)$,
2. for each history $h_i \in \mathcal{H}_i$, player $i$ makes the proposal $x$ that sets $x_i = \pi(h_i)$ with probability 1,
3. for each history, at the voting stage, a player $i$ votes to accept any proposal.

**Step 2: Unique SSPE Outcome with an Unanimity Rule.** Suppose there were another equilibrium. Let $W_i^\pi$ summarize player $i$’s value at the beginning of the round when the surplus is $\pi$, $\overline{V}_i^\pi$ be her value from being the proposer, and $\overline{V}_i^\pi$ be her value when someone else is the proposer. Let $W_i = E[W_i^\pi]$ be the expected payoff at the beginning of the round, prior to the realization of the surplus.

Observe that there cannot be any equilibrium with perpetual disagreement. First, consider surplus $\pi$ such that no proposal is accepted in equilibrium when the surplus is $\pi$. In this case, $\overline{V}_i^\pi = \overline{V}_i^\pi = 0$, and
\( W_i^\pi = \delta_i W_i \). Second, consider surplus \( \pi \) when there is agreement. Since the voting rule is unanimity, it follows that if there is agreement at surplus \( \pi \), then \( \bar{V}_i^\pi = \pi - \sum_{j \neq i} \delta_j W_j, \) \( \bar{V}_i^\pi = \delta_i W_i \), which implies that for every player \( i \), \( \bar{V}_i^\pi - \bar{V}_i^\pi = \pi - \sum_{j} \delta_j W_j \). Since that term is independent of \( i \), behavior at the bidding stage reduces to a symmetric all-pay auction, which implies at the bidding stage that for every player \( i \), \( W_i^\pi = \bar{V}_i^\pi = \delta_i W_i \). Integrating across \( \pi \), we obtain that \( W_i = \delta_i W_i \), and hence, for every player \( i \), \( W_i = 0 \). □

Proof of Lemma 2 on p. 11. I first construct a Nash equilibrium: suppose that each player \( i > 1 \) bids her reach \( r_i \), and player 1 bids \( r_2 \). If \( r_1 > r_2 \), the auctioneer breaks ties in favor of player 1, and if \( r_1 = r_2 \), the auctioneer can use any tie-breaking rule. No player has an incentive to deviate from this profile. Now consider any other equilibrium. Let \( \sigma_i \) denote the equilibrium (mixed) strategy of player \( i \). Let \( s_i^l \equiv \inf[\text{Supp}[\sigma_i]] \) and \( s_i^h \equiv \sup[\text{Supp}[\sigma_i]] \), and let \( F_i(s) \) be the cdf of \( \sigma_i \).

Claim 1. Suppose that in equilibrium, each of a set of players \( J \) have an atom at score \( x \) in which \( |J| > 1 \). Each player \( j \) in \( J \) other than the lowest indexed player in \( J \) anticipates a payoff of \( v_j \).

Proof. Let player \( i \) be the lowest indexed player who choose an atom at score \( x \). Suppose towards a contradiction that \( j \in J \backslash \{i\} \) anticipates a payoff from choosing score \( x \) that strictly exceeds \( v_j \). By definition of \( r_j \), \( x \) must be strictly less than \( r_j \). By the ordering on labels, \( r_i \geq r_j \), and therefore, \( x < r_j \). Furthermore, there must be a strictly positive probability with which each of players \( i \) and \( j \) win at score \( x \), and so, \( \prod_{k \in N \backslash \{i, j\}} F_k(x) > 0 \). Since players in \( J \backslash \{i\} \) choose \( x \) with an atom, there exists \( \gamma > 0 \) such that player \( i \) can increase her probability of winning the prize by \( \gamma \) by bidding \( x + \epsilon \) for every \( \epsilon > 0 \). Therefore, in equilibrium, player \( i \) cannot choose \( x \) with strictly positive probability. □

Now consider any pair of players \( i \) and \( j > i \), and let \( s_{ij}^l = \min\{s_i^l, s_j^l\} \). If both players \( i \) and \( j \) choose score \( s_{ij}^l \) with strictly positive probability, then Claim 1 implies that player \( j \) expects \( v_j \). If only one of the players chooses score \( s_{ij}^l \) with strictly positive probability, then the other player is choosing a higher score with probability 1, which implies that once more, one of the players expects her payoff from losing. Finally, if neither player chooses \( s_{ij}^l \) with strictly positive probability, then by definition of \( s_{ij}^l \), one of players \( i \) and \( j \) is choosing scores arbitrarily close to \( s_{ij}^l \). Suppose that this is player \( i' \in \{i, j\} \), and let the other player in \( \{i, j\} \) be denoted by \( j' \). Since \( F_{j'}(s_{ij}^l) = 0 \), and \( F_{j'} \) is right-continuous, for every \( \epsilon > 0 \), player \( i' \) has best-responses that win with probability no more than \( \epsilon \). Therefore, player \( i' \) must have zero expected payoffs. Thus, for each pair of players, at least one’s equilibrium payoff coincides with her payoff from losing the auction, which establishes the first part of Lemma 2.

Now consider a generic auction, and a weakly undominated Nash equilibrium. In such an equilibrium, every player \( j > 1 \) must be choosing a score less than \( r_j \) with probability 1. Therefore, a lower bound on player 1’s equilibrium payoff is \( \pi_1 - c_1(r_2) \). If player 1 were to have a strictly higher equilibrium payoff, then \( s_i^l < r_2 \). Player 2 would then have a strictly profitable deviation to choosing a score in the interval \( (s_i^l, r_2) \cap (r_3, r_2) \) since all players are choosing scores that are below that interval (so player 2 would win for sure) and all scores are below player 2’s reach. □
References


