

Networks of Military Alliances, Wars, and International Trade

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Abstract

We investigate the role of networks of alliances in preventing (multilateral) interstate wars. We first show that, in the absence of international trade, no network of alliances is peaceful and stable. We then show that international trade induces peaceful and stable networks: trade increases the density of alliances so that countries are less vulnerable to attack and also reduces countries' incentives to attack an ally. We present historical data on wars and trade, noting that the dramatic drop in interstate wars since 1950, and accompanying densification and stabilization of alliances, are consistent with the model but not other prominent theories.

Keywords: Alliances, Conflict, War, Networks, International Trade, Treaties

JEL Classification Codes: D74, D85, F10

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1 Introduction

Wars are caused by undefended wealth. *Ernest Hemingway (repeated by Douglas MacArthur in lobbying to fortify the Philippines in the 1930's¹)*

The enormous costs of war make it imperative to understand the conditions under which wars are likely to occur, and the ways in which they can be prevented. Although much is known about bilateral conflicts, there is no formal theory of how networks of multilateral international relationships foster and deter interstate wars. In this paper we introduce a model of networks of military alliances and analyze its predictions, and demonstrate the important role of international trade in understanding the network structure of alliances and in enabling peace.

In terms of background, the history of the networks of international alliances is rich and nuanced. Arranging multiple alliances to ensure world peace found perhaps its most famous proponent in Otto von Bismarck and his belief that the European states could be allied in ways that would maintain a peaceful balance of power.² The alliances that emerged were briefly stable following the unification and expansion of Germany that took place up through the early 1870s, but were ultimately unable to prevent World War I. Indeed, many world conflicts involve multiple countries allied together in defensive and offensive groups, from the shifting alliances of the Peloponnesian and Corinthian wars of ancient Greece to the Axis and Allies of World War II, and so studying the fabric of alliances is necessary for understanding international (in)stability. Based on the “Correlates of War” data set, between 1823 and 2003, 40 percent of wars with more than 1000 casualties involved more than two countries, and indeed some of the most destructive (e.g., the World Wars, Korean War, Vietnam,...) involved multilateral conflicts.³ Most importantly, this is really a network problem. As we detail in Section 2.4, multilateral wars never involved cliques (fully allied coalitions of more than two countries) against cliques. Out of the 95 wars between 1823 and 2003 that qualify as having at least one side with three or more countries, *none* of them involved a clique versus a clique. Thus, a network approach of understanding alliances, rather than a coalitional one (in which countries are partitioned into allied groups) is warranted. As we also show, a network approach meshes well with patterns of international trade, which are far from coalitional and instead involve rich network patterns.

The historical background on the networks of alliances between the early 1800s and the

¹See the biography by Bob Considine, source for Chapter 1: Deseret News, Feb 24, 1942.

²E.g., see Taylor (1969).

³This is based on the COW data for which there is data regarding initiators of the war, which we then couple with other data for our analysis. This does not even include the Napoleonic wars, as the data begin afterwards. Also, there are some wars that might be thought of as civil, but that involve substantial interstate conflict: e.g., the Second Congo War, the Russian Revolution, etc.

present basically breaks into two periods, with the break occurring after World War II⁴. This can be seen in Figures 6 through 11. The early period (pre-1950) involved relatively sparse, very dynamic and unstable networks, and many wars. The time series of these early networks exhibits rapid shifts, with very different alliances existing decade to decade. The later period (post-1950) involves increasingly dense, highly stable networks, and relatively very few wars. The networks stabilize and become substantially denser and with alliances that are separated by continent and ideology - there are large cliques, corresponding to large geographical areas, which are bridged by a few larger states. As a preview of the analysis in Section 5.1, between 1816 and 1950 a country had on average 2.525 alliances, while from 1951 to 2003 this grows by a factor of more than four to 10.523. In terms of turnover: between 1816 to 1950, for an alliance that is present in one year, there is only a 0.695 probability that it will still be present five years later. In contrast, during the period of 1950 to 2000 the frequency increases to 0.949.

To gain insights into networks of alliances and the incidence of wars, we model the incentives of countries to attack each other, to form alliances, and to trade with each other. We first present a base concept of networks that are stable against wars from a purely military point of view, when trade is ignored. A group of countries can attack some other country if all members of the attacking coalition share a mutual ally. The idea is that alliances represent the necessary means for coordinating military action. A country that is attacked can be defended by its allies. A country is *vulnerable* if there is some aggressor country and a coalition of its allies whose collective military strength outweighs that of the country and its remaining allies who are not in the attacking coalition (adjusted by a parameter that captures technological considerations that may give an advantage to offensive or defensive forces).⁵

In addition to not having any vulnerable countries, endogenizing the networks is essential to understanding stability. Thus, we define a concept of *war-stable* networks that accounts for the incentives of countries to form and drop alliances. We build upon the concept of pairwise stability of Jackson and Wolinsky (1996), adapting it to the current setting. In particular, a network of alliances is *war-stable* if three conditions are met:

- first, no country is vulnerable to a successful attack by others,
- second, no two countries that are not allied could form an alliance that would allow them, together with allies, to successfully attack another country, and

⁴We use alliance data reported by the Alliance Treaty Obligation and Provisions Project atop.rice.edu, including alliances marked as containing at least one of a defensive, offensive, or consultation provision.

⁵ We also explore other definitions based on other rules of which connections are needed between countries in order to attack and/or defend, and show that the results hold for those alternative definitions.

- third, each existing alliance serves a purpose - any country that deletes any of its alliances would end up being vulnerable.

This concept embodies the simple principles that countries prefer to win a war and not to lose one, and that alliances are costly and so should serve some purpose in order to be maintained.

It turns out that there are no war-stable networks, even with this definition that imposes minimal requirements. The tension is understood as follows. Requiring that countries not be vulnerable to attack and having every alliance serve some purpose leads networks to be relatively sparse - with each country having a few alliances but a network that is not overly dense. However, this can make a country susceptible to some of its allies joining forces and defeating it. Essentially, the pressure to economize on alliances conflicts with stability against the formation of new alliances, which leads to instability and would suggest chaotic dynamics.

This instability provides insights into the constantly shifting structures and recurring wars that occurred throughout the nineteenth and first half of the twentieth centuries.⁶ Wars, however, have greatly subsided in parallel with the huge increase of trade (partly coincidental with the introduction of containerized shipping in the 1960s): between 1820 and 1959 each pair of countries averaged .00056 wars per year, while from 1960 to 2000 the average was .00005 wars per year, less than *one tenth* as much. We see this pattern quite clearly in Figure 1.⁷ These changes also follow the advent of nuclear weapons, which impacted the technology of war. Indeed, we show how nuclear weapons can lead to some changes in stability, but does not generate peace on its own. Indeed, in order to capture the actual patterns that have emerged one must add other considerations - such as trade considerations - since the base model shows that networks of alliances would not be stable with nuclear weapons but without trade.⁸

Thus, the second part of our analysis is to enrich the base model to include international trade. Indeed, there has been a rapid increase in global trade since World War II (partly coincident with the growth of container shipping among other stimuli). The empirical relationship between war and trade is an active area of research, with strong suggestions (e.g.,

⁶There was a relatively quiet period prior to World War I that was prosperous and during which trade increased and there was some temporary stability. However, stability was partly due to the relative asymmetries in the strengths of Germany and Austria-Hungary compared to France and Russia (and the fact that Germany had already gained much territory from those countries); but this subsided as France and Russia regained the relative strength that they had lost during the nineteenth century. Interlocking trade was not yet sufficient to prevent the ‘Great War’ from occurring, and the alliance structure proved far from stable.

⁷Even if one measures this per country rather than per pair of potential combatants, the decrease has been more than threefold, as discussed in Section 5.1.2.

⁸The cold war was accompanied by a (temporary) change to a form of bilateralism, that we come back to in Section 5.1.4. Again, to understand the accompanying peace and patterns of alliances, international trade is instrumental.

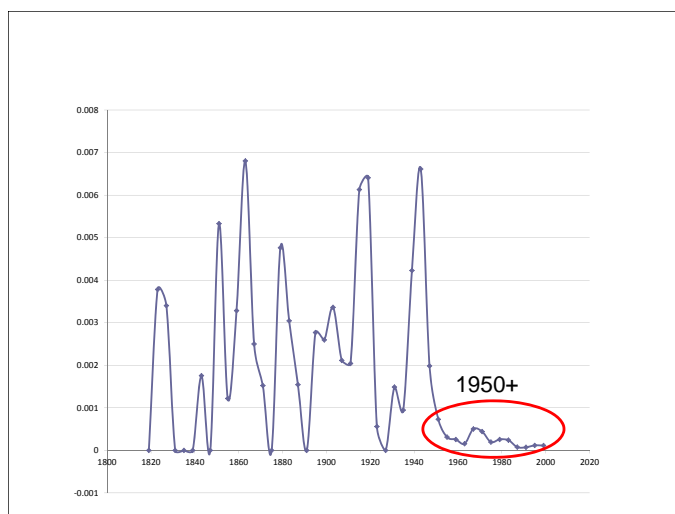


Figure 1: Wars per pair of countries by year, 1820-2000. (Participant level observations from COW MIDB 4.01 dataset, number of entries with hostility level 5 divided by number of pairs of countries in COW State System Membership)

Martin, Mayer, and Thoenig (2008)) that network concerns may be important. So, we introduce a concept of a network of alliances being *war and trade stable*, which allows countries to form alliances for either economic or military considerations. In this richer model, an alliance allows countries to trade with each other and to coordinate military activities, and so can be formed for either reason. This restores existence of networks of alliances that are stable against the addition or deletion of alliances. Trade provides two helpful incentives: first it provides economic motivations to maintain alliances, and the resulting denser network of alliances then has a deterrent effect; and second, it can reduce the incentives of a country to attack another since trade will be disrupted. This reduces the potential set of conflicts and, together with the denser networks, allows for a rich family of stable networks that can exhibit structures similar to networks we see currently.

We provide some results on the existence and structure of war and trade stable networks of alliances, showing that structures similar to those observed over the past few decades are economically stable under apparently reasonable parameters. It is important to note that another dramatic change during the post-war period was the introduction of nuclear weapons, which changes the technology of war and is generally thought to have greatly increased the defensive advantage to those with such weapons.⁹ Our model suggests that although world-wide adoption of nuclear weapons could stabilize things in the absence of

⁹Another change has been in the number of democracies. The endogeneity of such changes makes it difficult to factor in, but even accounting for democratization, trade still seems to be an important factor, as discussed by Oneal and Russett (1999).

trade, it would result in an empty network of alliances as the stable network. To explain the much denser and more stable networks in the modern age along with the paucity of war in a world where nuclear weapons are limited to a small percentage of countries, our model points to the enormous growth in trade as a big part of the answer. We close the paper with some discussion of this potential role that the growth in trade has played in reducing wars over the past half century, and how this relates to the advent of the nuclear age.

Before proceeding, let us say a few words about how this paper contributes to the study of war. The literature on war provides many rationales for why wars occur. Our analysis here fits firmly into what has become a “rationalist” tradition based on cost and benefit analyses by rational actors, with roots seen in writings such as Hobbes (1651) *Leviathan*, and has become the foundation for much of the recent international relations literature.¹⁰

To our knowledge, there are no previous models of conflict that game-theoretically model networks of alliances between multiple agents/countries based on costs and benefits of wars.¹¹ There are previous models of coalitions in conflict settings (e.g., see Bloch (2012) for a survey). Here, network structures add several things to the picture. Our model is very much in a similar rationalist perspective of the literature that examines group conflict (e.g., Esteban and Ray (1999, 2001); Esteban and Sakovicz (2003)), but enriching it to admit network structures of alliances and of international trade. This allows us to admit patterns that are consistent with the networks of alliances that are actually observed, which are far from being partitions (e.g., the U.S. is currently allied with both Israel and Saudi Arabia, Pakistan and India, just to mention a couple of many prominent examples). More importantly, our Theorem 3 provides a first model in which such non-partitional such structures are stable and provide insight into peace. Moreover, as we already mentioned above, the observed patterns of wars and of alliances are not partitional, and so this provides an important advance in moving the models towards matching observed patterns of wars, trade and alliances. Our model thus serves as a foundation upon which one can eventually build more elaborate analyses of multilateral interstate alliances, trade, and wars. It is also important to emphasize that the network of international trade is complex and can in fact be stable (and prevent conflict) precisely because it cuts *across* coalitions. This is in contrast to coalitional models that generally predict only the grand coalition can be stable or that very exact balances are possible (e.g., see Bloch, Sanchez-Pages, and Soubeyran (2006)). Again, this is something illustrated in our Theorem 3, and which does not exist in the previous literature. Finally, our model illuminates the relationships between international trade, stable network structures,

¹⁰Background can be found in Fearon (1995) and Jackson and Morelli (2011).

¹¹There is a literature that adapts the balance theory of Heider (1946) to examine network patterns of enmity (e.g., Hiller (2012); Reitzke and Roberson (2013); Koenig et al. (2014)). The ideas in those works build upon notions of the form that “the enemy of my enemy is my friend,” and are quite different from the sort of cost-benefit analysis underlying the military and trading alliances considered here.

and peace, something not appearing in the previous literature - as the previous literature that involves international trade and conflict generally revolves around bilateral reasoning or focuses on instability and armament (e.g., Garfinkel, Skaperdas, and Syropoulos (2014)) and does not address the questions that we address here.

The complex relationship between trade and conflict is the subject of a growing empirical literature (e.g., Barbieri (1996); Mansfield and Bronson (1997); Martin, Mayer, and Thoenig (2008); Glick and Taylor (2010); Hegre, Oneal, and Russett (2010)). The literature not only has to face challenges of endogeneity and causation, but also of substantial heterogeneity in relationships, as well as geography, and the level of conflict. The various correlations between conflict and trade are complex and quite difficult to interpret, and a model such as ours that combines military and economic incentives, and others that may follow, can provide some structure with which to interpret some of the empirical observations, as we discuss in the concluding remarks.

2 The Basic Model

2.1 Countries and Networks

There is a set $N = \{1, \dots, n\}$ of countries.

Countries are linked through *alliances*, represented by a network of alliances $g \subset N^2$ with the interpretation that if $ij \in g$ countries i and j are allies.¹² Alliances represent channels through which countries *can* coordinate military actions, either offensively or defensively. The presence of alliances does not require countries to come to each other's aid, as that will have to be incentive compatible, as embodied in our definitions below. The operative part of the assumption is that countries either need to have an alliance or add one in order to coordinate their military activities. (For more discussion on alliances, see Section 4.1.)

$N_i(g) = \{j : ij \in g\}$ are the allies of i .

For a given alliance $ij \notin g$, let $g + ij$ denote the network obtained by adding the alliance ij to g . Similarly, given an alliance $ij \in g$, let $g - ij$ denote the network obtained by deleting the alliance ij from g . In a slight abuse of notation, let $g - i$ denote the network obtained by deleting all alliances of the form ik , $k \in N$, from g ; that is, removing i from the network.

Let

$$\mathcal{C}(g) = \{C \subset N \mid ij \in g \text{ for all } i, j \in C\}$$

denote the set of all cliques in a network g : that is the set of all groups of countries such

¹²Here we represent a network by the list of unordered pairs ij that it comprises. So, for instance, the network $g = \{12, 23, 45\}$ on a set of countries $N = \{1, 2, 3, 4, 5\}$ represents situations where country 2 is allied to both 1 and 3, and 4 is allied with 5, and where no other alliances are present.

that every pair of countries in the group are allied.

2.2 Military Strengths and Wars

Each country $i \in N$ is endowed with a military strength $M_i \in \mathbb{R}$.¹³

For any group of countries $C \subset N$, let $M(C) = \sum_{i \in C} M_i$ be their collective military strength.

If there is a war between sets of countries C and C' , with C being the aggressor, then C “wins” if

$$M(C) > \gamma M(C').$$

The parameter $\gamma > 0$ is the defensive (if $\gamma > 1$) or offensive (if $\gamma < 1$) advantage in the war.

This modeling of a war outcome based on relative strengths is reminiscent of the approach of Niou and Ordeshook (1991a,b). One could instead work with contest success functions (e.g., as in Skaperdas (1996); Jackson and Morelli (2009)), which would provide for random chances of winning. In our model it would not add anything since we are not focused on arming, and so all that would matter is whether the expected benefits computed via a probability of winning exceed a threshold of potential costs/losses, and so the decisions would still be either to attack or not based on relative strengths and costs and benefits, exactly as already in our model, simply with a different functional form.

2.3 Vulnerable Countries and Networks

We say that a country i is *vulnerable* at a network g if there exists j and $C \subset N_j(g) \cup \{j\}$ such that $j \in C$, $i \notin C$ and

$$M(C) > \gamma M(i \cup (N_i(g) \cap C^c)),$$

where C^c is the complement of C . In this case, we say that country j is a *potential aggressor* at a network g .¹⁴

Thus, no country is vulnerable at a network g if for any coalition C of a potential aggressor j and some its allies, and any target country $i \notin C$, the aggressors cannot successfully attack the country: $M(C) \leq \gamma M(C')$ where $C' = i \cup (N_i(g) \cap C^c)$.

The incentives of countries to attack or defend are embodied in the definitions below. The above definitions just define the technology of war.

¹³Although it would be interesting to endogenize the strengths, for our purposes in this paper we take these as given. For a bilateral model of endogenous military strengths see Jackson and Morelli (2009).

¹⁴A country can be both vulnerable and a potential aggressor at some networks.

If some country is vulnerable, then a group that can defeat the country and its remaining allies has an incentive to attack and defeat the country. This presumes that the benefits from defeating a country outweigh costs of war. If a country that is not vulnerable were to be attacked then it and its allies would be successful in holding off the attackers. Implicit in the definition is that if the country and its allies could be successful in fending off an attack, then they would do so. For now, we simply assume that winning a war (even successfully aiding an ally in defense) is desired and losing a war is not. When we explicitly model trade and economics below, we will be more explicit about gains and losses.

2.4 Alternative Definitions of Vulnerability

In the above definition, in order for a group of countries to attack they must coordinate via some country j , and then the target country i is defended by its neighbors.

We refer to this as ‘NN-vulnerability’ since the attacking and defending countries can each receive aid from their allies or ‘neighbors’ (hence the ‘N’), without any additional restrictions. Of course, we can also consider other definitions. For example, if we think of alliances as a channel of communication between countries, then it could be in some circumstances that greater coordination is needed. For instance, to initiate an attack, it might be that all countries need to be in communication with all of the others in the coalition: i.e., they must form a clique. Moreover, there could be some asymmetries in military operations. For example, it might be in some circumstances that attacking coalitions need to be cliques, while a country can be defended by all neighbors without requiring that the defending coalition be a clique. This would capture the fact that more coordination is needed when attacking, while defense might only require each neighbor to lend aid to the attacked country. We refer to this case as ‘CN-vulnerability’ (attacking coalitions as ‘C’liques, defending coalitions as ‘N’ighbors).

This is mostly an empirical question, and so let us examine the data on this issue. Indeed, the fraction of links that are present is higher among attacking coalitions than defending ones. When considering wars between 1823 and 2003 (from the Franco-Prussian War through the Invasion of Iraq, based on what is available from the Correlates of War Inter-State War database [“COW”] intersected with the ATOP alliance data), 61 percent of the links in attacking coalitions were present while 33 percent of the links in defending coalitions were present (out of 95 wars). In terms of actual clique counts, Figure 2 shows the fraction of wars between 1823 and 2003 that fall into various categories in terms of whether the attacking/defending coalition was a ‘C’ (clique) or ‘N’ (non-clique - generally a country and some of its neighbors, not forming a full clique, or having fewer than three members).

Figure 2 shows that the wars predominately involve NN, so fall under our current definition, while a few fall in the categories CN (offense=Clique, defense=Non-clique) and NC,

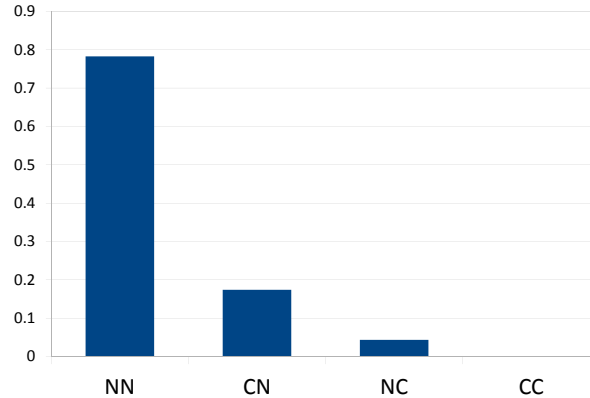


Figure 2: Categorization of wars by type (COW data set from 1823 to 2003 such that at least one side had at least three countries, so some possibility of a clique). C indicates that the coalition was a clique (three or more countries with all possible alliances present), while N indicates that the coalition was missing some alliances or involved fewer than three countries. CN indicates that the “offense” (side initiating the conflict according to the COW data classification) was a clique and the “defense” (other side) was not a clique. NN indicates that neither side was a clique, and so forth.

but none in the category CC. Thus, we focus on the NN-vulnerability definition, but we also provide results for the CN and NC cases, and simply comment on the CC case. In general, we will take the ‘NN’ prefix as understood and simply refer to ‘vulnerability’.

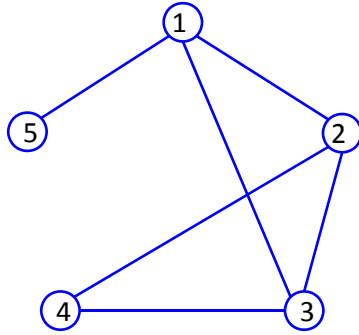
As we mentioned in the introduction, this diagram also shows that multilateral wars exhibit ‘network’ patterns and do not fall along the lines of (even subsets) of some partition of countries into coalitions. This reiterates the motivation for a network-based approach, that becomes even more paramount when we come to the connections with international trade, which exhibits rich network patterns.

2.5 Illustrations of Vulnerability

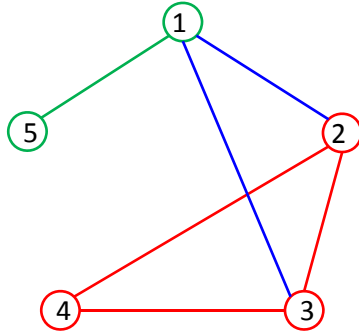
Before moving on to the main definitions and analysis, we present a simple observation and some illustrations of networks and vulnerability.

For an illustration of our definition of vulnerability, consider Figure 3.

In this network, country 1 is vulnerable if $\gamma(M_1 + M_5) < M_2 + M_3 + M_4$. Countries 2,3, and 4 form a clique and hence can attack country 1 under either requirement of C or N for attacking coalitions, and country 1 has country 5 to defend it under either definition C or N. Country 5 cannot join countries 2, 3, and 4 in attacking country 1 since it is not allied with any of them.



(a) A Network of Alliances



(b) 2 and its allies 3 and 4 attack 1 who is defended by 5

Figure 3: 1 is vulnerable if $\gamma(M_1 + M_5) < M_2 + M_3 + M_4$.

Let $\overline{M} = \max_i M_i$ and $\underline{M} = \min_i M_i$.

OBSERVATION 1. *If $M(N \setminus \{i\}) > \gamma \underline{M}$ for $i = \operatorname{argmin} M_i$, then some country is vulnerable*

in the complete network.¹⁵ If $\overline{M} > \gamma \underline{M}$, then any network which has no vulnerable countries is nonempty and incomplete.

This simple observation points out that in most settings of interest, requiring that no country be vulnerable implies that networks will be “intermediate.”

2.6 Incentives and War-Stable Networks

We now introduce the concept of *war-stability* that accounts for countries’ incentives to conquer other countries and to add or delete alliances. At this point we do not include trade, focusing on the world in which alliances serve only military purposes, and the motivations for going to war are to gain land or resources from other countries.

The motivation for attacking another country comes from the economic spoils - which historically have included land, natural resources, slaves, and access to markets and other economic resources. Netted from this are expected damages and other costs of war. The expected net gain from winning a war is then represented as $E_{ik}(g, C)$, which are the total economic gains that accrue to country k if country i is conquered by a coalition C with $k \in C$ when the network is g and i is defended by the coalition $C' = \{i\} \cup N_i(g) \cap C^c$.¹⁶ For example these include natural resources or other potential spoils of war.¹⁷

Finally, there are costs to maintaining alliances. The cost of country i having an alliance with country j is some $c_{ij}(g) > 0$. This could include costs of opening diplomatic, military and communication channels, coordinating military operations or intelligence, or other related costs. We generally take costs of alliances to be small relative to the potential spoils of winning a war, as otherwise the analysis is degenerate.¹⁸ The costs are also sufficiently small that any country i is willing to pay $c_{ij}(g + ij)$ to add an alliance with j , provided that the addition of the alliance would change i from being vulnerable to not.

Define a network g to be *war-stable* if three conditions are met:

- S1** no country is vulnerable at g ;
- S2** no two countries both benefit by adding an alliance to g ; and
- S3** no country has an incentive to delete any of its alliances.

¹⁵I.e., the network containing all possible alliances

¹⁶We allow for the dependence upon the network g , since once we allow for trade, the economic spoils available will be a function of the network.

¹⁷For important discussions of the spoils of inter-state wars, see Caselli, Morelli, and Rohner (2012); Garfinkel, Skaperdas, and Syropoulos (2014).

¹⁸In particular, costs are small enough so that if there is some g and $jk \notin g$ such that j is a potential aggressor at $g + ij$, but not at g , with i being vulnerable to being conquered by j , then $c_{jk}(g + jk) + \sum_{s \in N_g(j)} [c_{js}(g + jk) - c_{js}(g)] \leq E_{ij}(g + jk, C)$. Thus, j is always willing to add an alliance to some k that would be pivotal in winning a war.

Given that no country is vulnerable at g , the only way in which two countries j and k could have an incentive to add a link to g would be that some other country i must become vulnerable at $g + jk$, and both j and k would have to be part of the winning coalition. The change in payoffs to j (with an analogous expression for k) would be at least $E_{ij}(g + jk, C) - c_{jk}(g + jk) - \sum_{s \in N_g(j)} [c_{js}(g + jk) - c_{js}(g)]$, with j being part of a coalition C that includes k and conquers some i . By assumption, this is positive. Thus, **[S2]** is equivalent to saying that no country is vulnerable at $g + jk$, $\forall jk \notin g$. Similarly, given that links are costly, a country not having an incentive to delete any of its alliances implies that it must be that by deleting any alliance a country becomes vulnerable at the new network. Therefore **[S3]** is equivalent to saying that both j and k are vulnerable at $g - jk$, $\forall jk \in g$

So, g is *war-stable* if three conditions are met:

- S1** no country is vulnerable at g ;
- S2** $\forall jk \notin g$, no country is vulnerable at $g + jk$,
- S3** $\forall jk \in g$, both j and k are vulnerable at $g - jk$.

That is, g is war-stable if no pair of countries can destabilize the network by adding an alliance and making some other country vulnerable and there are no superfluous alliances.

This definition is similar to that of pairwise stability of Jackson and Wolinsky (1996) in that we consider changes in the network one alliance at a time, and both additions or deletions - requiring two countries to benefit to form an alliance, but only one country to benefit to break an alliance. One can enrich the definition in various directions, by allowing groups of countries to add alliances, countries to delete multiple alliances, payments for forming links, forward-looking countries, and so forth. Given that there is already a large literature on possible variations on definitions of network formation, we focus on this base definition here.¹⁹

As a reminder, note that for the definition of ‘war stability’, we use ‘NN-vulnerability’. When another notion of vulnerability is used then we explicitly note this in the name (e.g., ‘CN-war-stability’ refers to ‘CN-vulnerability’).

3 Nonexistence of War-Stable Networks

For the case of 2 countries, it is direct to check that the only possible stable network is the empty network and it is war-stable if and only if $\gamma \geq \frac{\bar{M}}{\underline{M}}$. Thus, we consider the more interesting case with $n \geq 3$.

¹⁹See Jackson (2008); Bloch and Jackson (2006) for overviews of alternative network formation definitions.

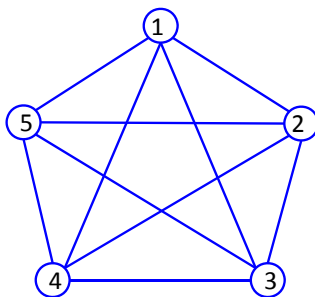


Figure 4: A network that is *not* war-stable under any of the NN, CN, NC, CC definitions for any value of γ .

Before presenting the results on lack of existence of war-stable networks, let us illustrate some of the main insights.

We start with a very simple network that is not war-stable, pictured in Figure 4. First, in order for no country to be vulnerable it is clear from Observation 1 that it would have to be that $M(N \setminus \{i\}) \leq \gamma M_i$ for all i . Next, note that if $M(N \setminus \{i\}) \leq \gamma M_i$ for all i , then any country i can delete any of its alliances and still not be vulnerable, violating the condition for war stability.

The argument is slightly different, but the same conclusion applies to less connected networks, as in the network pictured in Figure 5. Begin with the ring network in which each country has two links. Let us examine the NN case (variations on the argument apply in other cases).²⁰ In order for 1 not to be vulnerable under the addition of the link 53, it must be that $\gamma M_1 \geq M_2 + M_3 + M_4 + M_5$. However, this implies that 1 is not vulnerable in the original network if it deletes an alliance regardless of the attacking coalition, and so this contradicts war-stability.

The following theorem shows that there are no war-stable networks except empty networks in extreme cases, regardless of country's strengths. This particular theorem applies under NN-vulnerability.

Label countries in order of their strength: $M_1 \geq M_2 \geq \dots \geq M_n$.

THEOREM 1. *Let $n \geq 3$. There are no nonempty war-stable networks. The empty network is war-stable if and only if $\gamma \geq \frac{M_1 + M_2}{M_n}$.*

²⁰For the cases of CN and CC, the argument is more involved (see Claims 1 and 2 for details).

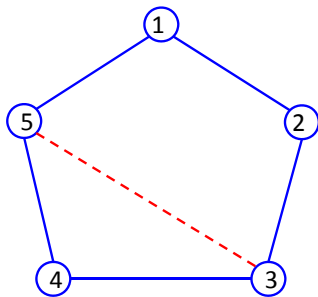


Figure 5: A network that is *not* war-stable for any value of γ .

Theorem 1 shows that war-stable networks only exist in the extreme case in which the defensive parameter is so high that the weakest country can withstand an attack by the two strongest countries in the world, in which case the empty network is stable. Outside of that case, there are no war-stable networks. The intuition behind the proof of Theorem 1 is similar to that of the examples: outside of the extreme case, requiring that a country not be vulnerable, nor vulnerable to the addition of any alliances, implies that a country has extraneous alliances.

The nonexistence of war-stable networks extends to other definitions of vulnerability as we now verify.

THEOREM 2. *Let $n \geq 3$.*

- **NC-vulnerability:** *There are no nonempty NC-war-stable networks. The empty network is NC-war-stable if and only if $\gamma \geq \frac{M_1+M_2}{M_n}$.*
- **CN-vulnerability:** *Under the uniform strength case of $M_i = \bar{M} \forall i$, if $1 \leq \gamma < 2$ then there are no CN-war-stable networks. If $\gamma \geq 2$, then the unique CN-war-stable network is the empty network. If $\gamma < 1$, then for large enough n , there exist nonempty CN-war-stable networks.*

Even though the arguments for any particular network's instability are straightforward, showing that there do not exist any nonempty war-stable networks under these variety of definitions requires covering all possible configurations, and so is quite involved. Thus, the full proof of the theorems, including the case of CN-vulnerability, uses a combinatorial

pigeonhole argument, showing that certain sorts of contradictions arise in all nonempty graphs. The case of CN-vulnerability turns out to be quite intricate, and the proof is limited to the case of equal strengths.²¹

4 War and Trade Stable Networks

As we have seen, other than in some exceptional cases, pure military considerations do not lead to stable networks, since countries only maintain alliances if they serve a purpose, which leads to sparse networks, but then sparse networks are destabilized by the addition of new alliances. As we now show, accounting for economic incentives associated with gains from trade can restore stability.

We are now more explicit about the payoffs that accrue to countries as a function of the network and in the event of a war.

A country i gets a payoff or utility from the network g given by $u_i(g)$. This represents the economic benefits from the trade that occurs in the network g .

4.1 Alliances

A link now represents a potential trading relationship and potential to coordinate military activities. The important assumption is that if two countries trade (significantly) with each other, then they can come to each other's aid in the event of a military conflict.

The two assumptions that we are using are thus: (i) having an ‘alliance’ involves some costs, however tiny, which must be offset by some benefits either via trade or war, and (ii) without having any relationship, countries are not able to coordinate either in attacking or in defending.

Clearly, (ii) is a simple assumption that ‘alliances’ have some meaning, otherwise there is really nothing to model. We do allow the formation of new alliances in cases in which that would benefit the countries, as part of the definition. Alliances can be fairly inexpensive, but still serve a purpose of making clear who could defend whom in various situations. One

²¹ For CN-war-stability, the restriction to $\gamma \geq 1$ is important. If the offense has a substantial advantage and $\gamma < 1$, for the case of CN-vulnerability there exist war-stable networks. At first blush it might be surprising that a world where attackers have an advantage over defenders leads to more stability, but it can be understood as follows. An offensive advantage provides incentives for countries to maintain alliances, as without alliances countries easily become vulnerable. This allows one to build up networks of alliances that are denser. The key to then getting CN-war-stability is to have each country be involved in several separate cliques, so that no attacking clique is large enough to overcome the country and its other allies (see Section 7). The case of CC-war stability is particularly challenging. We can show nonexistence when $\gamma < 1$, and conjecture that it also holds for $\gamma > 4/3$, but can find some CC-war-stable networks for $1 \leq \gamma \leq 4/3$. Given that it is not a case of empirical interest, we leave it aside.

could imagine a more complicated model in which there is some incomplete information, and in which making public a non-binding alliance is useful.²²

Let us also comment on our presumption that substantial trade allows for potential military coordination *regardless of whether there is then an explicit military alliance or not*. This captures the idea that both the interests and channels of communication are then generally present. For example, this was exactly what happened in the U.S. aid to Kuwait in the Persian Gulf War. Moreover, even though China and the U.S. do not have explicit trading relationships, it would be difficult to imagine the U.S. not reacting if there was an unprovoked attack by some other country on China, which severely disrupted trade with the U.S.

This also fits with the empirical evidence. Mansfield and Bronson (1997) examine correlations between alliances, trade, and participation in preferential trading agreements over the period of 1960 to 1990. They find that alliances (and participation in a preferential trading agreement) lead to increased bilateral trade, with the effect being considerably larger when the pair of countries have both an alliance and mutual participation in a preferential trading agreement. Interestingly, this relationship differs in the recent 1960-1990 period compared to pre-World War II. Long and Leeds (2006), looking at pre-World War II Europe, finds that trade between allies is only statistically larger than trade between non-allied countries when economic provisions are explicitly mentioned in the alliance. This fact is consistent with our analysis in that opportunities for trade were substantially limited in pre-World War II Europe, and so the economic trade incentives emerge to a much greater extent in the 1950s and thereafter when costs of trade begin to plummet and incomes increase and trade grows significantly. Regardless of the relationships between *explicit* alliances and trade, the open lines of communication are what is essential for our theory.

4.2 Vulnerability and Stability with Trade

We now introduce a concept of vulnerability based on the incentives of countries to attack others when explicitly accounting for the benefits and costs associated with conquering a country.

We say that a country i is *vulnerable despite trade* in a network g to a country j and coalition $C \subset N_j(g) \cup \{j\}$ if $j \in C$, $i \notin C$ and

- $M(C) > \gamma M(i \cup (N_i(g) \cap C^c))$ (i.e., C could conquer i), and
- $u_k(g - i) + E_{ik}(g, C) \geq u_k(g) \forall k \in C$ with some strict inequality: every $k \in C$ would benefit from conquering i , factoring in economic gains of conquering and gains or losses

²²Such a rationale for formal alliances can be found in, e.g., Morrow (2000).

in subsequent payoffs from the network.²³

The second item is new to this definition of vulnerability and incorporates two aspects of economic incentives of countries to attack each other:

Recall that the $E_{ik}(g, C)$ represents the potential net benefits that k enjoys from conquering i as part of the coalition C in a network g . If a country is poor in natural resources, and much of its economy is built upon nontransferable or difficult to extract human capital, it would tend to have a lower E_{ik} and would be less attractive.

The $u_k(g - i)$ accounts for the fact that as i is conquered then the network of trade will adjust. If k is a trading partner of i , then k could lose via the elimination of i , with $u_k(g - i) < u_k(g)$.²⁴ Note that this effect works both ways: it might also be that a country k benefits from the elimination of some country i , for instance if it improves k 's position in the resulting trade network.

With this framework, we now define a stability notion corresponding to war stability but adding the economic considerations.

Our definition of war and trade stability now incorporates two incentives for adding or deleting alliances. First, countries might add or maintain an alliance because of its military value in either preventing a war or in instigating one, just as with war stability. This is similar to what we considered before, except that countries now consider the economic spoils and consequences of war in deciding whether to take part in an attack. Second, countries add or maintain alliances for the economic benefits in terms of trade.

Let us now consider the incentives for countries to add an alliance and attack another country.

Starting from a network g , some alliance $jk \notin g$ is *war-beneficial* if there exists some $C \in N_j(g + jk) \cup \{j\}$ with $j \in C$, $k \in C$ and $i \notin C$ such that i is vulnerable despite trade to C at $g + jk$ and

- $u_j(g + jk - i) + E_{ij}(g, C) \geq u_j(g)$, so, j would benefit from forming the link and attacking, and
- $u_k(g + jk - i) + E_{ik}(g, C) \geq u_k(g)$, similarly for k , with one of these inequalities holding strictly.

We say that a network g is *war and trade stable* if three conditions are met:

ES1 no country is vulnerable despite trade at g ;

²³It is not essential whether the strict inequality is required for all countries or just some, or must include j , as generically in the E function ensures there will not be equality for any countries.

²⁴As Glick and Taylor (2010) documents, the economic loss resulting from trade disruption during wars can be of the same order as more traditional estimates of losses resulting from interstate conflict. This does not even account for the potential loss of trade if a partner is lost altogether.

ES2 $\forall jk \notin g$: if $u_j(g + jk) > u_i(g)$ then $u_k(g + jk) < u_k(g)$, and also jk is not war-beneficial

ES3 $\forall jk \in g$ either $u_j(g - jk) \leq u_j(g)$ or j is vulnerable despite trade at $g - jk$, and similarly for k .

So, a network of alliances is war and trade stable if no country is vulnerable despite trade, if no two countries can add an alliance and both profit either through economic or war means, and either economic or war considerations prevent any country from severing any of its alliances.

REMARK 1. *If $u_i(\cdot)$ is constant for all i , then war and trade stability reduces to war stability.*

We say that a network g is *strongly war and trade stable* if it is *war and trade stable* for any (nonnegative) specification of the E_{ij} 's.

Again, the default definition will be relative to a country and its neighbors attacking or defending, but the same extensions to cliques hold as in the earlier sections (so, there are CN, NC, and CC variations on the definitions). The default definition refers to the NN case.

We remark that we have not explicitly mentioned the incentives of a country that loses a war. Clearly, there are costs to losing, and what they are exactly does not matter in the model. Being explicit about the losing countries' payoffs is not necessary, since our stability notions only need to check whether countries would benefit by attacking, benefit by adding an alliance and then attacking, or could safely remove an alliance; and the payoff to the losers is irrelevant in these calculations.

4.3 Results on, and Examples of, War and Trade Stable Networks

Let us examine the set of war and trade stable networks. We begin by identifying a condition that is sufficient for war and trade stability.

PROPOSITION 1. *Suppose that g is pairwise stable with respect to u . If no country is vulnerable despite trade at g or $g + jk$ for any $jk \notin g$, then g is war and trade stable. Moreover, if no country is vulnerable at g or $g + jk$ for any $jk \notin g$, then g is strongly war and trade stable.*

The proof of the proposition is straightforward and thus omitted (and also extends to the CN, NC, CC definitions).

There are many examples of networks that are war and trade stable but not war stable. The following theorem outlines a whole class of war and trade stable networks, showing that economic considerations restore general existence results.

For what remains, which are constructive results, we specialize to the case of symmetric countries (so the $u_i(\cdot)$, $E_{ij}(\cdot)$, and M_i 's are independent of i and j), but it will be clear that similar results extend to the asymmetric case with messier statements of conditions.

We also consider a canonical case in which

$$u_i(g) = f(d_i(g)) - c \cdot d_i(g),$$

where $d_i(g)$ is the degree of i and f is concave, nondecreasing, and such that there exists some $d \leq n-1$ such that $f(d) < c \cdot d$. This is a simple model of gains from trade and costs of having trading relationships, abstracting from heterogeneity in goods and trading partners and inter-dependencies in trading relationships beyond diminishing returns - but illustrates our main point and it should be clear that similar results hold for richer models. Let d^* maximize $f(d) - c \cdot d$ among nonnegative integers.

In addition, in this model and given the symmetry, let $E_{ij}(g, C) = \frac{E(d_i(g))}{|C|}$, so that each country's economic spoils from a war depend only on that country's degree, and then are divided equally among the attacking countries.

THEOREM 3. *Consider the symmetric model with $d^* \geq 2$.*

- *Any d^* -regular network (i.e., such that each country has d^* alliances) for which no two countries have more than $k < d^* - 1$ allies in common is strongly war and trade stable network if $\gamma \geq \frac{d^*+1}{d^*-k-1}$.*
- *If $E(d^*) \leq 2[f(d^*) - f(d^* - 1) - c]$, then any d^* -regular network (in any configuration, including combinations of cliques) is war and trade stable network if $\gamma \geq \frac{d^*+1}{d^*-1}$.*

Theorem 3 provides two existence results that each work from a different idea. The first part is based on the motivation that trade provides for countries to maintain relationships for trade purposes, and the fact that this results in networks in which no country is vulnerable, and no country would be vulnerable even with the addition of new alliances. This first result is independent of the E_{ij} 's and the relative costs of war, but does require some specific structures (for example, simply forming cliques where each country has d^* allies will not work, as then all of a country's partners can attack the country and win). The second result applies for more specific gains from war (E_{ij} s), but for a wider set of networks. It works off of the fact that with sufficient gains from trade, the potential spoils of a war are outweighed by the lost trade value - and so countries are never attacked by one of their own trading partners. In that case, each country then has enough alliances to protect itself against attacks from outside, and then a wide range of networks becomes stable. This allows for more cliquish structures to be stable, which are more consistent with the emerging networks

that we observe in the world today. Thus, we see two different ways in which trade stabilizes the world.

Variations on the result hold for other definitions. For the case of CN-war-and-trade stability, if $d^* \geq 4$ then any network in which all countries have d^* alliances, the largest clique is of size at most $\lceil \frac{d^*}{2} \rceil$, and any two cliques intersect in at most one country is CN-strongly war and trade stable for any $\gamma \geq 1$.²⁵

5 Concluding Remarks

In closing, we briefly discuss some of what is known regarding trade and wars and then comment on some issues for further research.

5.1 Empirics of Trade and Wars

5.1.1 Trends in Military Alliance Networks

Marked differences occur between the military alliances we see in the ATOP data over time.²⁶ There are two major changes that we see in the period before and after the Second World War. These changes are also easy to see in the Figures in Section 7.

The first major change is that there is a great deal of turnover in alliances, which constantly shift in the period from 1816 to 1950. In particular, let us do a simple calculation: how frequently do alliances disappear? Specifically, consider an alliance that is present in year t , and calculate the frequency with which it is also present in year $t + 5$. Doing this for each year from 1816 to 1950, we find the frequency to be 0.695. When doing this for each year from 1950 to 2003 the frequency becomes 0.949. Thus, there is an almost one-third chance that any given alliance disappears in the next five years in the pre-WWII period, and then only a five-percent chance that any given alliance at any given time will disappear within the next five years in the post-WWII period.

²⁵ A particularly interesting class of CN-strongly war and trade stable networks is one that is built up from a set of cliques, called quilts. A network is said to be a *quilt* if all nodes have at least two links and the network can be written as a union of cliques, each of size at least 3, and such that any two cliques share at most one node in common. (This definition of quilts differs slightly from that of social quilts introduced by Jackson, Rodriguez-Barraquer, and Tan (2012) in that larger cycles are permitted here.) In particular, if $d^* \geq 4$ then any quilt in which all countries have d^* alliances and the largest clique is of size at most $\lceil \frac{d^*}{2} \rceil$ is strongly war and trade stable for any $\gamma \geq 1$. Quilts are of interest as we see their underpinnings in, for example, the network in Figure 11, consists of a number of cliques that have some small overlap. With $d^* = 3$, the same proposition holds with $\gamma \geq 3/2$ and with $d^* = 2$ it then moves to $\gamma \geq 2$.

²⁶ The number of countries in the data set grows over time, and so everything we do adjusts on a per country basis, as otherwise the trends are even magnified further. The number of states in 1816 was 23, in 1950 it was 75, and by 2003 it reached 192.

The second major change is that the network of alliances greatly densifies. Between 1816 and 1950 a country had on average 2.525 alliances (standard deviation 3.809). If one drops the WWII decade of the 1940s during which most countries were allied in one of two blocks, then this number drops down even further to 1.722 between 1816 to 1940 (standard deviation of 1.366). During the period of 1951 to 2003 this grows by a factor of more than four to 10.523 (standard deviation of 1.918). Thus, there are substantially more alliances per country in the post war than the pre-war period.

To summarize, countries have just a couple of alliances on average and those alliances rapidly turned over in the pre-WWII period; while in contrast countries form on average more than ten alliances and do not turn them over in the post-WWII period.

5.1.2 Trends of Wars and Conflicts

Another trend that is quite evident is that the number of wars per country has decreased dramatically post World War II, and that this decrease comes even though the number of countries has increased - so that there are many more pairs of countries that could be going to war. For example, the average number of wars per pair of countries per year from 1820 to 1959 was .00056 while from 1960 to 2000 it was .00005, less than a tenth of what it was in the previous period. We saw this in Figure 1.

This finding is robust to when the cut takes place: from 1850 to 1949 it was .00059 while from 1950 to 2000 it was .00006, from 1850 to 1969 it was .00053 while from 1970 to 2000 it was .00005. If one looks at wars per country instead of per pair of countries, then from 1820 to 1959 it was .012 while from 1960 to 2000 it was .004. One could also include all Militarized Interstate Disputes (MID2-5) instead of just wars (MID5s - involving at least 1000 deaths). In that case, from 1820 to 1959 there are .006 MID5s per pair of countries while from 1960 to 2000 there were .003. Thus, the decrease in wars is quite robust to the way in which this is measured.²⁷

5.1.3 Nuclear Weapons

An obvious trend that occurs post WWII is that nuclear weapons were invented during the war and greatly enhanced in both power and delivery methods through the following decades, leading to dramatic changes in the technology of war. Although rarely used, their existence changes the technology and potentially the opportunities for stability.²⁸ This change in

²⁷It is also interesting to note that with the exceptions of the Korean and Vietnam wars, which had major cold-war considerations, (as well as the anomalous Falklands war), the 24 other MID 5's since 1950 generally involved lesser-developed countries as the major protagonist on at least one (and often both) sides of the dispute. Moreover, major trading partners at the time do not appear on opposite sides of the dispute.

²⁸There is a large literature on the cold war and a contentious debate on the potential stabilizing or destabilizing impact of nuclear technology (e.g., see Schelling (1966); Mueller (1988); Geller (1990).

technology can be captured within our model, as follows. One obvious change is that it increases the military strengths of countries possessing the technology. Another is that it can increase the defensive advantage γ dramatically, as a country that is attacked can inflict severe damage on its attackers.²⁹ Finally, it can also (potentially) lead to more devastation, possibly increasing the costs of war and decreasing the value of the spoils of war. All of these changes in parameters lead to the potential for increased stability under Theorem 1, but interestingly only for the stability of the *empty* network. Moreover, that would only be true if all countries possessed nuclear weapons. Thus, it is important to emphasize that these changes, taken together or separately, would not account for the actual patterns that we see post WWII, which involve a combination of increased stability and peace, but also increased density. Therefore, one still needs trade to explain why we see denser networks that are stable and why many non-nuclear countries also live in relative peace; and the combination of nuclear technology together with the increased benefits from international trade leads to this richer picture of the trends we observe.

5.1.4 Multipolar vs Bipolar Systems and Collective Security

A notable change in alliances during the cold-war period was from a ‘multipolar’ to a ‘bipolar’ structure, somethings which has been extensively discussed in the cold-war literature (e.g., see Bloch (2012) for references). Although this lasted for part of the post-war period, and was characterized by a stalemate between the eastern and western blocks, such a system of two competing cliques of alliances is only war stable if there are sufficient trade benefits between members of a clique, as shown in Theorem 3. Moreover, it is more of a historical observation than a theory, and it does not account at all for the continued peace that has ensued over the last several decades.

Another institutional observation regarding the post-WWII calm is that institutions have allowed for coordination of countries onto a peaceful “collective security” equilibrium where any country disrupting international peace is punished by all other countries, so that war against one is war against all. However, as shown by Niou and Ordeshook (1991b), this equilibrium is in some sense “weak”: it relies heavily upon the assurance that a country tempted to join an attacking coalition will refuse and that all countries will follow through on their punishment commitments, so that far-sighted expectations of off-equilibrium behavior

²⁹This is a bit tricky, as it could also introduce assymetries, as it would increase γ for countries with nuclear weapons, but not for those without. So, one could extend the model to have different γ s for different countries. Put together with changes in military strength, nuclear arms would entail a large defensive advantage if a defending country has nuclear weapons, and a large offensive advantage if an attacking coalition has such weapons and a defending coalition does not. This could amplify gains from trade in parts of the developed world to lead to greater stability there, and could also explain why remaining recent wars tend to involve at least one country without nuclear capabilities. This raises the question of the endogeneity of arms, which is another interesting issue (e.g., see Baliga and Sjöström (2004); Jackson and Morelli (2009)).

are correct. Given that various small conflicts since WWII did not precipitate a global response, such doubts of some countries' commitment to follow through on punishments seems reasonable.

Finally, one might think of the rise of international institutions as allowing larger groups of countries to simultaneously add alliances, rather than the pairwise addition modeled above. Note, however, that altering our definition of stability to admit coalitions of countries adding alliances only decreases the set of potentially war-stable networks, once again indicating that trade needs to be incorporated into a model of alliances in order to account for the dramatic drop in conflict and simultaneous increase in alliances (strongly correlated with trade).

5.1.5 Trade

International trade has had two major periods of growth over the last two centuries, one in the latter part of the nineteenth century and beginning of the twentieth, disrupted by the first world war, and then picking up again after the second world war, recovering to its 1914 levels through the 1960s and then continuing to grow at an increasing rate thereafter. In particular, Estevadeordal, Frantz, and Taylor (2003) finds that trade per capita grew by more than 1/3 in each decade from 1881 to 1913, while it grew only 3 percent per decade from 1913 to 1937. Table 1, from Krugman (1995),³⁰ provides a view of this dynamic.³¹

Table 1: World merchandise exports as percent of GDP: Krugman (1995)

Year	1850	1880	1913	1950	1973	1985	1993	2012
Percent	5.1	9.8	11.9	7.1	11.7	14.5	17.1	25.3

The trade has been further bolstered or accompanied by the advent of container shipping as well as increases in world per capita income. Hummels (2007) looks at the interaction between transportation costs and international trade, while Bernhofen, El-Sahli, and Kneller (2013) and Rua (2012) investigate the rise of containerization and its spread through international shipping. The relative correlations between income and trade and transportation costs and trade have been open to some debate. Baier and Bergstrand (2001), looking at trade between OECD countries from the late 1950s through the late 1980s, argues that decreasing transportation costs explains 8 percent of the growth in trade, with the lion's-share of the increase (67 percent) correlating with increased incomes. Regardless of the source,

³⁰The figure for 2012 is directly from the World Bank indicator (<http://data.worldbank.org/topic/private-sector?display=graph>, December 11, 2013), from which Krugman (1995) quotes the other numbers.

³¹Dean and Sebastia-Barriel (2004) provide an overview of changes in the level of world trade in relation to world output over the course of the 20th century, while Estevadeordal, Frantz, and Taylor (2003) looks at the period 1870 to 1939.

trade has increased dramatically over time, and especially post World War II, where it has increased by almost a factor of four.

5.1.6 Relations Between Trade and Wars

Putting these two trends together, we see that the decrease in wars is mirrored by an increase in trade. The percentage of trade varies mainly between 5 and 12 percent from 1850 to 1959 and between 12 and 25 percent from 1960 onwards.

These numbers cannot be taken as evidence for the theory. There are many confounding variables in the relationship between trade and wars, so although there was an unprecedented growth in trade post World War II, coincident with an unprecedented drop in the frequency of wars, there was also a cold-war and many technological changes (the advent of nuclear weapons, as discussed above), as well as an increase in income and wealth levels worldwide, and a growth in the number of democracies, among other changes (many of which are endogenous to peace), which make it difficult to test the theory directly. Moreover, one could also hypothesize that the absence of war led to the increase in trade instead of the reverse. Thus, although we do see a strong correlation that is in line with what our theory would suggest, there are many confounders which make causation impossible to infer, and there may be multiple forces at work. Nonetheless, the theory provides insight into the data and can help develop further theories and pointed predictions to test with data.

There are many papers that have investigated the empirical relationship between conflict and trade at a more dyadic level, and as one might expect causation and the specifics of the relationships are difficult to disentangle. Indeed, Barbieri (1996) – investigating the period 1870 to 1938 in Europe and including conflicts that fall substantially short of war – finds that although low to moderate levels of economic interdependence may be accompanied by a decrease in military conflicts; high levels of economic interdependence can be accompanied by increased incidence of conflicts. This inversion is nuanced, as Martin, Mayer, and Thoenig (2008) – looking at trade and militarized disputes over the period 1950-2000 – find that an increase in bilateral trade between two countries correlates with a decreased likelihood of these countries entering military dispute with each other, while an increase in one of the country’s multilateral trade (i.e. an overall increase in a country’s trade share without an increase in the bilateral trade between the two countries) leads to an increased likelihood of war between the pair. The definition of dispute is broader than that of war and could include posturing for bargaining purposes, or simply the increase in contact that accompanies trade leading to an increase in minor incidents. Oneal and Russett (1999) provide evidence that with a careful examination of proximity and trade, trade significantly reduces conflict, although again such results are correlations and not proof of causation.³²

³²See also Hegre, Oneal, and Russett (2010), who provide some evidence for the aspect of our model that

In summary, although the broad data are consistent with the theory, establishing a causal relationship would require finding appropriate exogenous variation that could be used to test the theory, or to enrich the theory with a detailed modeling of the specific gains from trade and war technology. A study of this kind does not yet exist, and is an important one for future research for which our model provides explicit hypotheses.

5.2 Other Issues for Further Research

We have provided a first model through which to analyze networks of military alliances and the interactions of those with international trade. Starting with a purely militaristic model of networks of alliances, we've found that stable networks fail to exist. We then include economic considerations, and show that with sufficient benefits from international trade, stability is restored.

There are two obvious ways in which to enrich the model. First, one can enrich the modeling of trade. There are many ways to introduce heterogeneity, for instance along the lines of Dixit and Stiglitz (1977); or else, capturing the complexity of trade dynamics as discussed in Gowa and Mansfield (2004), Long and Leeds (2006), and Mansfield and Bronson (1997). Second, and relatedly, is a question of geography. Both trade and war have strong relationships with geography (see, e.g., Eaton and Kortum (2002), as well as Caselli, Morelli, and Rohner (2012), who find that from 1945 to 1987 eighty six percent of significant international wars were between neighboring states). Geography constrains both the opportunities and benefits from trade and war, and so it has ambiguous effects on stability. Nonetheless, it plays an important role in explaining realized networks of trade and alliances and deserves further attention.

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6 Appendix: Proofs

Proof of Theorems 1 and 2:

For any of the definitions of vulnerability (NN, CN, NC, CC), the conditions on stability can be recast as requirements on the γ parameter. Let $\mathcal{C}^*(g)$ denote the feasible attacking coalitions under the corresponding definition - either some j and a subset of its neighbors, or a clique. The first condition that no country be vulnerable, [S1], translates as:

$$\gamma \geq \max_{C \in \mathcal{C}^*(g)} \left(\max_{i \in C^c} \frac{M(C)}{M(i \cup (N_i(g) \cap C^c))} \right). \quad [\text{S1}] \quad (1)$$

The second condition that no additional link leads to [S2] translates as:

$$\gamma \geq \max_{jk \notin g} \left(\max_{C \in \mathcal{C}^*(g+jk)} \left(\max_{i \in C^c} \frac{M(C)}{M(i \cup (N_i(g+jk) \cap C^c))} \right) \right). \quad (2)$$

Note that given (1), we need only check (2) with respect to C such that $j \in C$ and $k \in C$. Thus, we can change the denominator in (2) to be $M(i \cup (N_i(g) \cap C^c))$. Therefore, stability implies that

$$\gamma \geq \max_{jk \notin g} \left(\max_{C \in \mathcal{C}^*(g+jk)} \left(\max_{i \in C^c} \frac{M(C)}{M(i \cup (N_i(g) \cap C^c))} \right) \right). \quad [\text{S2}] \quad (3)$$

The third condition [S3] translates as (providing g is nonempty):

$$\gamma < \min_{ij \in g} \left(\min \left\{ \max_{C \in \mathcal{C}^*(g-ij)} \frac{M(C)}{M(i \cup (N_i(g-ij) \cap C^c))}, \max_{C \in \mathcal{C}^*(g-ij)} \frac{M(C)}{M(j \cup (N_j(g-ij) \cap C^c))} \right\} \right). \quad [\text{S3}] \quad (4)$$

Label countries in order of decreasing strength, so that $M_i \geq M_{i+1}$.

First, note that the empty network is stable (under any of the vulnerability definitions), if and only if $(M_1 + M_2)/M_n \leq \gamma$. This follows since in that case [S1] is clearly satisfied, and also [S3] is vacuously satisfied since there are no links to delete, and since $(M_1 + M_2)/M_n \leq \gamma$ corresponds to the cases where [S2] is satisfied. Thus, to prove the theorems, it is enough to show that there are no nonempty war-stable networks for NN or NC; and for CN when $\gamma \geq 1$.

We begin with a claim that applies regardless of the vulnerability definition (NN, CN, NC, CC)

CLAIM 1. *There does not exist a non-empty war stable network with maximum degree less than 2.*

Proof of Claim 1: Consider a network with a maximum degree of 1. If $\gamma < 1$, then the network must violate [S1], since a (strongest) country in any linked pair can defeat the other country. So, consider the case in which $\gamma \geq 1$. Let n be the weakest country. Let i either be the ally of n , or else some other country if n has no allies. It follows that $\gamma \geq 2$, as otherwise i , together with some country k different from i and n that is either an existing ally of i 's or by forming a new link ik , could defeat n , which would violate [S1] or [S2] respectively. However, $\gamma \geq 2$ implies that the network cannot be war stable. This is seen as follows. Consider the strongest country i that has positive degree. Either i can sever its link violating [S3], or else (given that $\gamma \geq 2$ and i is the strongest among those having connections and cliques are at most pairs) it must be that there is some country k that has no ties that could defeat i if i severed its link to its ally j . However, then by adding ik they would defeat j (since j is no stronger than i and i would be defeated by k when k is all alone) violating [S3]. ■

Before treating the CN case, we complete the proof for the cases of NN and NC. Consider a country of maximum degree in a nonempty network, say i , which then has alliance to some k . In order for [S3] to be satisfied, it must be that i is vulnerable in $g - ik$. Thus, there is some j and $C \subset N_j(g - ik) \cup j$ with $M(C) > \gamma M(C')$ where C' is the maximum strength permissible coalition (depending on the NN or NC case) out of $\{i\} \cup N_i(g - ik) \cap C^c$ that can defend i . Given [S1], it must be that i was not vulnerable at g , and so it must be that $k \notin C$ and in particular that $jk \notin g$. However, if the link jk is added (so that the network $g + jk$ is formed), then $C \cup \{k\}$ can defeat i , since then C' is then also the maximum strength

permissible coalition (depending on the NN or NC case) out of $\{i\} \cup N_i(g) \cap C^c$ that can defend i , and $M(C \cup \{k\}) > M(C) > \gamma M(C')$. This violates [S2], which is a contradiction. This establishes that any network that is NN or NC-war-stable must be empty.

We now specialize to equal strengths for the remainder of the proof which covers the case of CN.

CLAIM 2. *There does not exist a non-empty CN-war stable network with maximum degree less than 3.*

Proof of Claim 2: Given Claim 1, consider a network g with maximum degree two.

First, consider the case in which $\gamma \geq 2$. Given that the biggest clique is of size 3 and $\gamma \geq 2$, then a country i with degree 2 could sever one of its links and not be CN-vulnerable (its remaining ally cannot be part of any clique of size more than 2), and any clique of size 3 could not defeat i and its remaining ally. Thus there is no country with degree 2 if the maximum degree is 2, which is a contradiction.

So, consider the case in which $\gamma < 2$, and consider a country i and links $ij \in g$ and $ik \in g$. It cannot be that $jk \in g$ as otherwise jk can defeat i , violating [S1]. Similarly, if $jk \notin g$ then by adding that link jk would defeat i violating [S2]. So, again we reach a contradiction.

Thus, it must be that the maximum degree is at least three. ■

CLAIM 3. *Consider i of maximum degree and some $ij \in g$. There exists $C \in \mathcal{C}(g - ij)$ such that*

$$\gamma < \frac{M(C)}{M(i \cup (N_i(g - ij) \cap C^c))} \quad (5)$$

and every such C satisfies $C \cap N_i(g - ij) \neq \emptyset$ and $i \notin C$ and $j \notin C$.

Proof of Claim 3:

We know from [S3] there exists $C \in \mathcal{C}(g - ij)$ such that

$$\gamma < \frac{M(C)}{M(i \cup (N_i(g - ij) \cap C^c))}.$$

Suppose that some such C has $C \cap N_i(g - ij) = \emptyset$. This implies that $|C| > \gamma d_i$, and since $\gamma \geq 1$ and d_i is maximal, this implies that $|C| = d_i + 1$. However, this is a contradiction since then all but one member of C can defeat a remaining member (who necessarily has degree d_i and thus only has connections to other members of C). This follows since $d_i > \gamma$ given that $|C| = d_i + 1 > \gamma d_i$ and $d_i \geq 1$.

Thus, any $C \in \mathcal{C}(g - ij)$ satisfying (8) must satisfy $C \cap N_i(g - ij) \neq \emptyset$. The fact that $i \notin C$ is by definition, and that $j \notin C$ is that otherwise we would violate [S1] (as C would defeat i in the network g with ij present). ■

CLAIM 4. Consider i of maximum degree and some $ij \in g$. Consider any $C \in \mathcal{C}(g - ij)$ such that

$$\gamma < \frac{M(C)}{M(i \cup (N_i(g - ij) \cap C^c))}.$$

It follows that

$$C \subset N_i(g)$$

and that³³

$$|C| = \lceil \frac{\gamma d_i}{1 + \gamma} \rceil \quad \text{and} \quad |C| > \frac{\gamma d_i}{1 + \gamma}.$$

Moreover, for any $C \in \mathcal{C}(g)$ with $i \notin C$,

$$|(C \cap N_i(g))| \leq \lceil \frac{\gamma d_i}{1 + \gamma} \rceil.$$

Proof of Claim 4:

Let $x = |C \cap N_i(g)|$, and let $y = |C \cap N_i(g)^c|$ be the number of members of C who are not connected to i . Then $\gamma < \frac{M(C)}{M(i \cup (N_i(g - ij) \cap C^c))}$ implies that

$$x + y > \gamma(1 + d_i - 1 - x) = \gamma(d_i - x). \quad (6)$$

Let $k \in C \cap N_i(g)$ (by Claim 3). [S1] implies that the remaining members of C cannot defeat k and so:

$$x + y - 1 \leq \gamma(d_k + 1 - (x + y - 1)).$$

The fact that $d_k \leq d_i$ (i is of maximal degree) and the two above inequalities imply that

$$\gamma(d_i - x) - 1 < \gamma(d_i + 2 - x - y),$$

or $\gamma(y - 2) < 1$. Given that $\gamma \geq 1$ and y is an integer, $\gamma(y - 2) < 1$ implies that $y \leq 2$. Now, let us argue that $y = 0$. Suppose to the contrary that $y = 2$ (a similar argument will show that $y \neq 1$). Let k' and k'' be the countries in $C \cap N_i(g)^c$. Consider the network $g + ik'$, and the clique of $C' = (C \setminus \{k''\}) \cup \{i\}$, and note that $|C'| = |C|$. By (6) with $y = 2$ and $x = |C| - 2$, we know that $|C| > \gamma(d_i - (|C| - 2))$. But then, since i has maximal degree and $|C'| = |C|$, it follows that $|C'| > \gamma(d_{k''} - |C'| + 2)$. However, this contradicts [S2], since then i and k' can form a link and the resulting clique C' defeats k'' . (To prove $y \neq 1$, take k'' to be any country in C not equal to k' .)

Next, using (6) and $y = 0$ it then follows that

$$x(1 + \gamma) > \gamma d_i$$

³³Note that this implies that $\frac{\gamma d_i}{1 + \gamma}$ cannot be an integer.

or

$$x > \frac{\gamma d_i}{1 + \gamma}.$$

Given that $\gamma \geq 1$ and x is an integer, this implies that

$$x = |C| \geq \lceil \frac{\gamma d_i}{1 + \gamma} \rceil. \quad (7)$$

To see the last part of the claim, let $z = |(C \cap N_i(g)) \setminus \{i\}|$. By [S1] (with C not defeating i):

$$z \leq |C| \leq \gamma(d_i + 1 - z)$$

and so

$$z \leq \frac{\gamma(d_i + 1)}{1 + \gamma},$$

which, given that z is an integer, implies that

$$z = |(C \cap N_i(g)) \setminus \{i\}| \leq \lceil \frac{\gamma d_i}{1 + \gamma} \rceil.$$

as claimed.

The second part of the claim then follows from the last part of the claim and (7). ■

CLAIM 5. *Consider i of maximal degree and $ij \in g$. It must be that*

$$N_i(g) \setminus \{j\} \neq N_j(g) \setminus \{i\}.$$

Proof of Claim 5:

Consider i of maximum degree and some $ij \in g$. By Claim 3, there exists $C \in \mathcal{C}(g - ij)$ such that

$$\gamma < \frac{M(C)}{M(i \cup (N_i(g - ij) \cap C^c))} \quad (8)$$

and every such C satisfies $C \cap N_i(g - ij) \neq \emptyset$ and $i \notin C$ and $j \notin C$.

By Claim 4, $C \subset N_i(g - ij)$. If $N_i(g) \setminus \{j\} = N_j(g) \setminus \{i\}$, then $C \cup \{j\}$ is also a clique. But then $|C \cup \{j\}| > |C|$, and we violate the last part of Claim 4. ■

CLAIM 6. *There are no nonempty CN-war-stable networks (when $\gamma \geq 1$).*

Proof of Claim 6:

Let i be of maximum degree. In order to satisfy [S3], it must be that for each $j \in N_i(g)$ there exists $C_j \in \mathcal{C}(g)$ such that

$$\gamma < \frac{M(C_j)}{M(i \cup (N_i(g - ij) \cap C_j^c))}.$$

Moreover, it follows from Claim 3 that each C_j can be taken to lie in $C(g - ij)$. From Claim 4, each such C_j is such that $C_j \subset N_i(g)$ and $|C_j| = \lceil \frac{\gamma d_i}{1+\gamma} \rceil > \frac{d_i}{2}$.

Moreover, by Claim 4 it must be that for each $j' \in N_i(g)$, $\cup_{C_j \ni j'} C_j \neq N_i(g)$. This follows since i is of maximum degree and otherwise this would imply that $N_{j'}(g) \setminus \{i\} = N_i \setminus \{j'\}$, contradicting Claim 5.

Finding such sets C_j for each j in $N_i(g)$ thus becomes the following combinatorics problem: create subsets $\{C_1, C_2, \dots, C_S\}$ of a set $M = \{1, 2, \dots, d_i\}$ of d_i elements (the neighbors of i) such that:

1. $\forall C_s, |C_s| = x > \frac{d_i}{2}$,
2. $\forall j \in M, \exists C_s$ such that $j \notin C_s$,
3. $\forall j \in M, \cup_{C_s \ni j} C_s \neq M$, and
4. $\nexists D \subset M$ such that $\forall \{k, j\} \subset D, \exists C_s$ such that $\{k, j\} \subset C_s$ and $|D| > x$.

4 follows from Claim 4 as otherwise D would be a clique of size larger than $x = \lceil \frac{\gamma d_i}{1+\gamma} \rceil$.

We now show that such a collection of subsets is not possible. To do this, we start with just the set C_1 and see what implications hold as we consider each additional C_s , ultimately reaching a contradiction. For reference, we introduce the three new series of sets: $\{W_s\}_{s=1}^S$, $\{Y_s\}_{s=1}^S$, and $\{Z_s\}_{s=1}^S$. W_s are the set of elements of M which have been in at least one of the sets C_1, \dots, C_s (i.e. $W_s = \cup_{i=1}^s C_s$). Y_s are the set of elements of M which have been in all of the sets C_1, \dots, C_s (i.e. $Y_s = \cap_{i=1}^s C_s$). Z_s are the set of elements of M which have been in none of C_1, \dots, C_s (i.e. $Z_s = M \setminus W_s$).

Let us now complete the proof. Note that, if a set of subsets $\{C_1, \dots, C_S\}$ satisfying 1-4 existed, then $Y_S = \emptyset$ follows from point 2 since every element of M has some C_s that doesn't contain it. Note also that with each additional C_s , W_s (weakly) grows larger while Y_s and Z_s (weakly) grow smaller.

To complete the proof we show that

$$|Y_{s-1} \setminus Y_s| \leq |Z_{s-1} \setminus Z_s|$$

and that

$$|Y_1| > |Z_1|.$$

Together these imply that $Y_S \neq \emptyset$, which is then a contradiction.

We start with $Y_1 = W_1 = C_1$. Thus, $|Y_1| = |W_1| = x > d_i - x = |Z_1|$ since $x > \frac{d_i}{2}$. So, let us show that $|Y_{s-1} \setminus Y_s| \leq |Z_{s-1} \setminus Z_s|$. At each subsequent addition of a C_s , either $C_s \cap Y_{s-1} = Y_{s-1}$ or $C_s \cap Y_{s-1} \subsetneq Y_{s-1}$. In the first case, the result follows directly since then by definition $Y_{s+1} = Y_s$ and $0 \leq |Z_{s-1} \setminus Z_s|$. So consider the second case. In the second

case, we show that $|Y_{s-1}| - |Y_{s-1} \cap C_s| \leq |Z_{s-1} \setminus Z_s|$. Let $A = Y_{s-1} \setminus Y_s$ be the set of j such that $j \in \bigcap_{i=1}^{s-1} C_i$ but $j \notin C_s$. We show that $|C_s \cap Z_{s-1}| \geq |A|$ - that is, C_s contains at least as many elements which aren't in any $C_{s'}$, $s' < s$ as there are elements of which are in every $C_{s'}$, $s' < s$ but not in C_s (this establishes our result since $|C_s \cap Z_{s-1}| = |Z_{s-1} \setminus Z_s|$). To see this, suppose it weren't true. That is, suppose $|C_s \cap Z_{s-1}| < |A|$. Then, we would have set $D = (C_s \setminus Z_{s-1}) \cup A$ of size at least $x + 1$ that would contradict 4. To see that the size is at least $x + 1$, note that C_s has by assumption x members; by excluding C_s 's intersection with Z_{s-1} , we are excluding at most $|A| - 1$ members of C_s , and adding in the $|A|$ elements of A . To see that D satisfies the conditions of 4, note that any pair of elements k, j both of elements will satisfy $\{k, j\} \in C_1$. Likewise, any pair of elements k, j both in $C_s \setminus Z_{s-1}$ will satisfy $\{k, j\} \in C_s$. Finally, any pair of elements k, j with $k \in A$, $j \in (C_s \setminus Z_{s-1}) \setminus A$ will satisfy $\{k, j\} \in C_{s'}$ for some $s' < s$ since k is in all such $C_{s'}$ and since $j \in C_s \setminus Z_{s-1} \subset W_{s-1}$, j is in at least one such $C_{s'}$. So, we have found a set of size at least $x + 1$ satisfying the restrictions of point 4. The contradiction establishes the impossibility of satisfying the combinatorics problem and thus the claim. ■

We have thus shown that there exist no nonempty CN-war stable networks when $\gamma \geq 1$. The final part of the theorem, that there are no CN-war stable networks if $\gamma < 2$, then follows from the fact that the empty network fails to satisfy [S2] if $\gamma < 2$ (but satisfies it if $\gamma \geq 2$), as already established. ■

Proof of Theorem 3: We apply Proposition 1. It is clear that any network that is d^* regular is pairwise stable. Thus, we need only show that no country is vulnerable despite trade and that this remains true with the addition of any link. For the first part of the proposition we also need to show that this is true regardless of $\delta E(\cdot)$ for at least some d^* -regular networks. For the second part of the proposition, we need to show this is true under the given assumption on $E(\cdot)$, but for any d^* regular network.

First, note that no country i is vulnerable to any coalition C that does not include any of its neighbors (even if this comes from the addition of a link not involving any neighbors), since under either part of the theorem $\gamma \geq \frac{d^*+1}{d^*-1} > \frac{d^*+2}{d^*+1}$. Thus, we need only verify vulnerability to a coalition that involves at least one neighbor, and might possibly involve the addition of a link.

So, consider a country i and a coalition C that involves at least one of its neighbors. Under the first part of the theorem, the maximum strength of the coalition (involving adding a link) would be $d^* + 2$ (if the center is not one of i 's neighbors) and then the defending coalition would involve at least $d^* - k$ members, or else the center is one of i 's neighbors in which case the strength is at most $d^* + 1$ and the defense involves at least $d^* - k - 1$ members. Given that $\gamma \geq \frac{d^*+1}{d^*-k-1}$, it follows that $\gamma \geq \frac{d^*+2}{d^*-k}$, and so i is not vulnerable in either case.

Under the second part of the theorem, if any of those neighbors of i in C still has only d^* links, then since $\frac{E(d^*)}{2} \leq f(d^*) - f(d^* - 1) - c$, and the attacking coalition must involve at least two countries (given γ and i at a minimum defending itself), then the country will not be willing to follow through with the attack of i since it will lose a link. Thus, all of i 's neighbors in the coalition C must be gaining a link. This implies that the coalition involves at most two of i 's neighbors, but then since $\gamma \geq \frac{d^*+1}{d^*-1} \geq \frac{d^*+2}{d^*}$, the attacking coalition cannot defeat i and its remaining neighbors, regardless of whether it involves one or two of i 's neighbors. ■

7 Appendix: Snapshots of Networks of Alliances: 1815 to 2000



Figure 6: Network of Alliances, 1815, red for multilateral alliance, grey for bilateral alliance, green for both

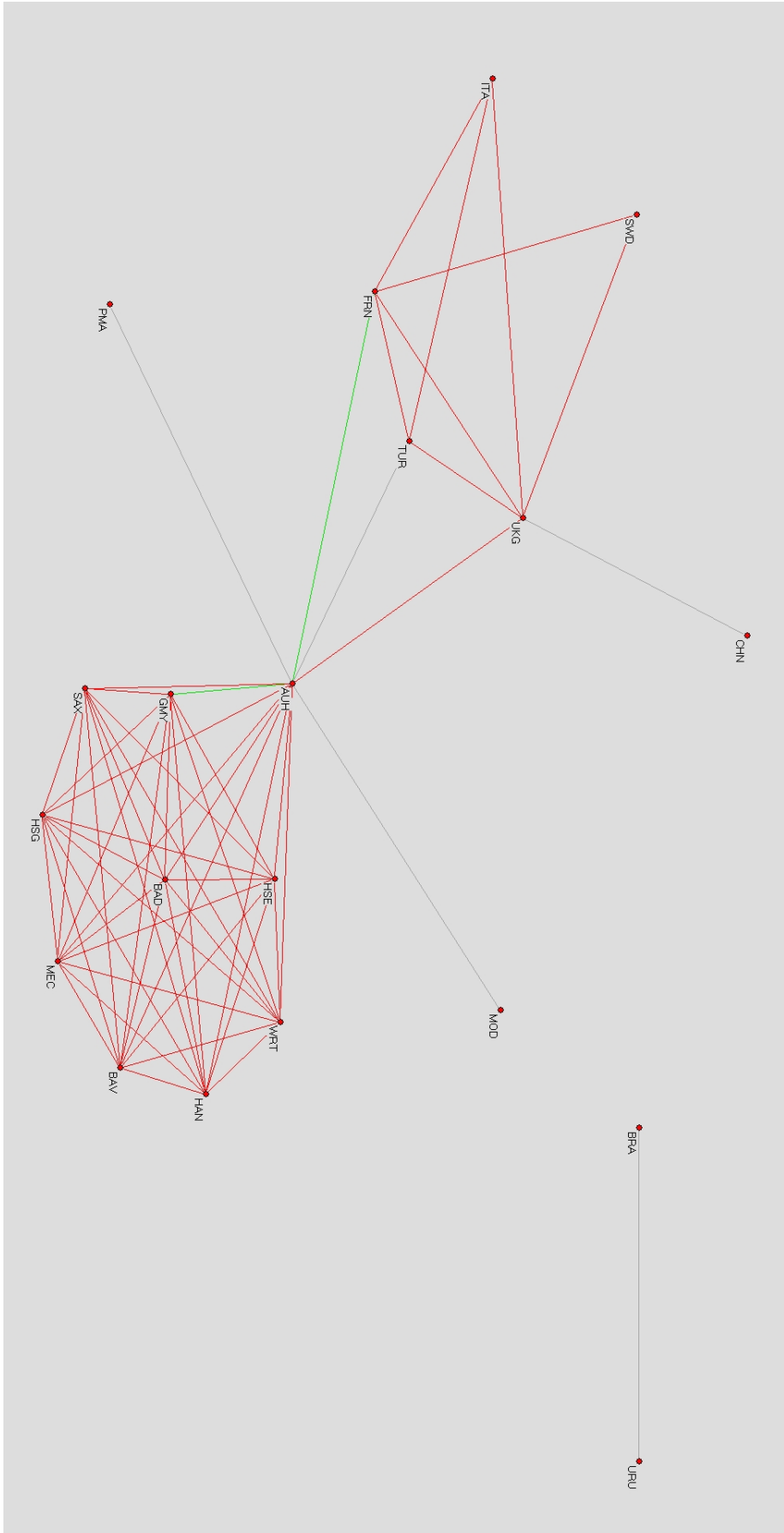


Figure 7: Network of Alliances, 1855, red for multilateral alliance, grey for bilateral alliance, green for both

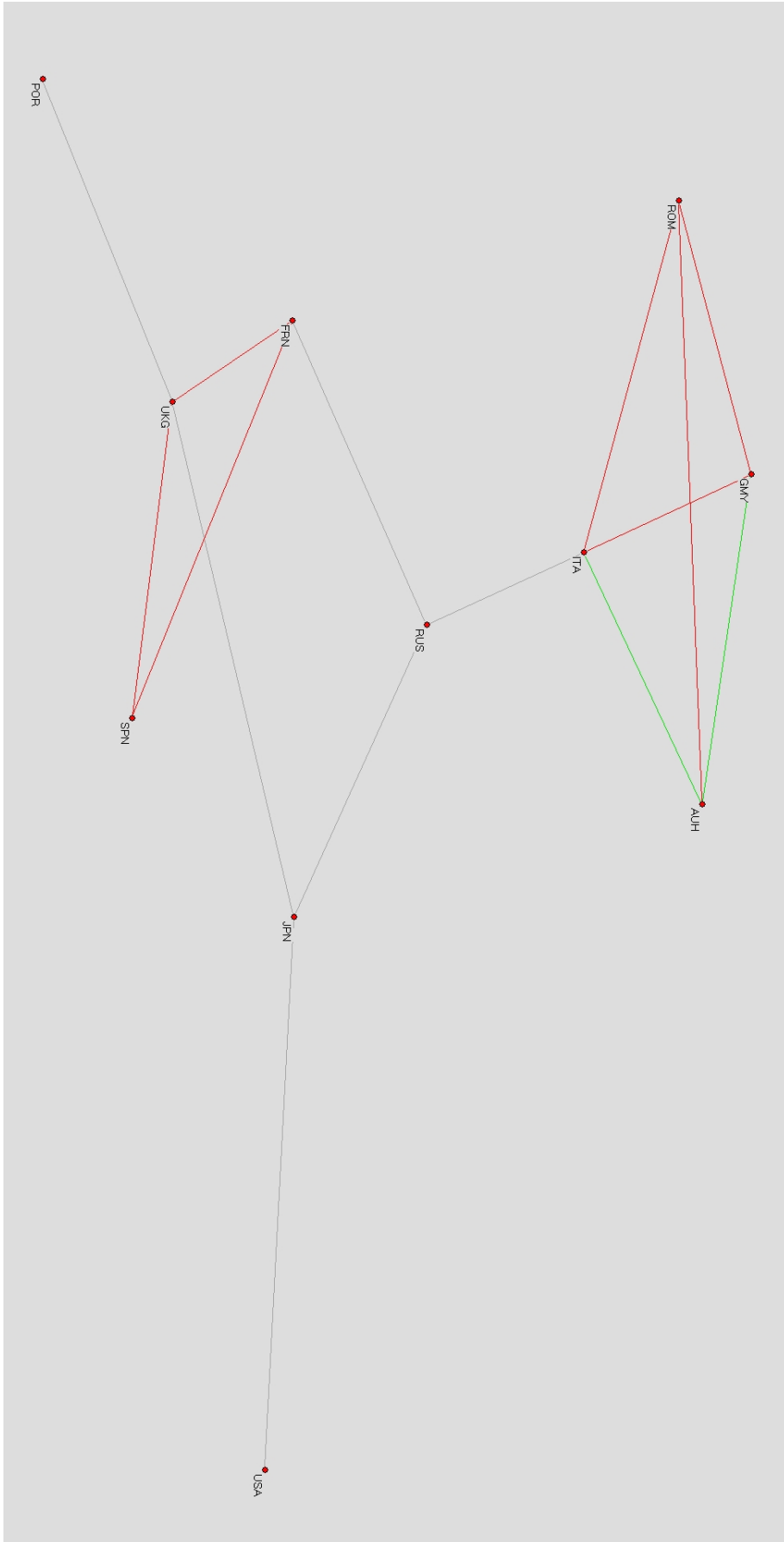


Figure 8: Network of Alliances, 1910, red for multilateral alliance, grey for bilateral alliance, green for both

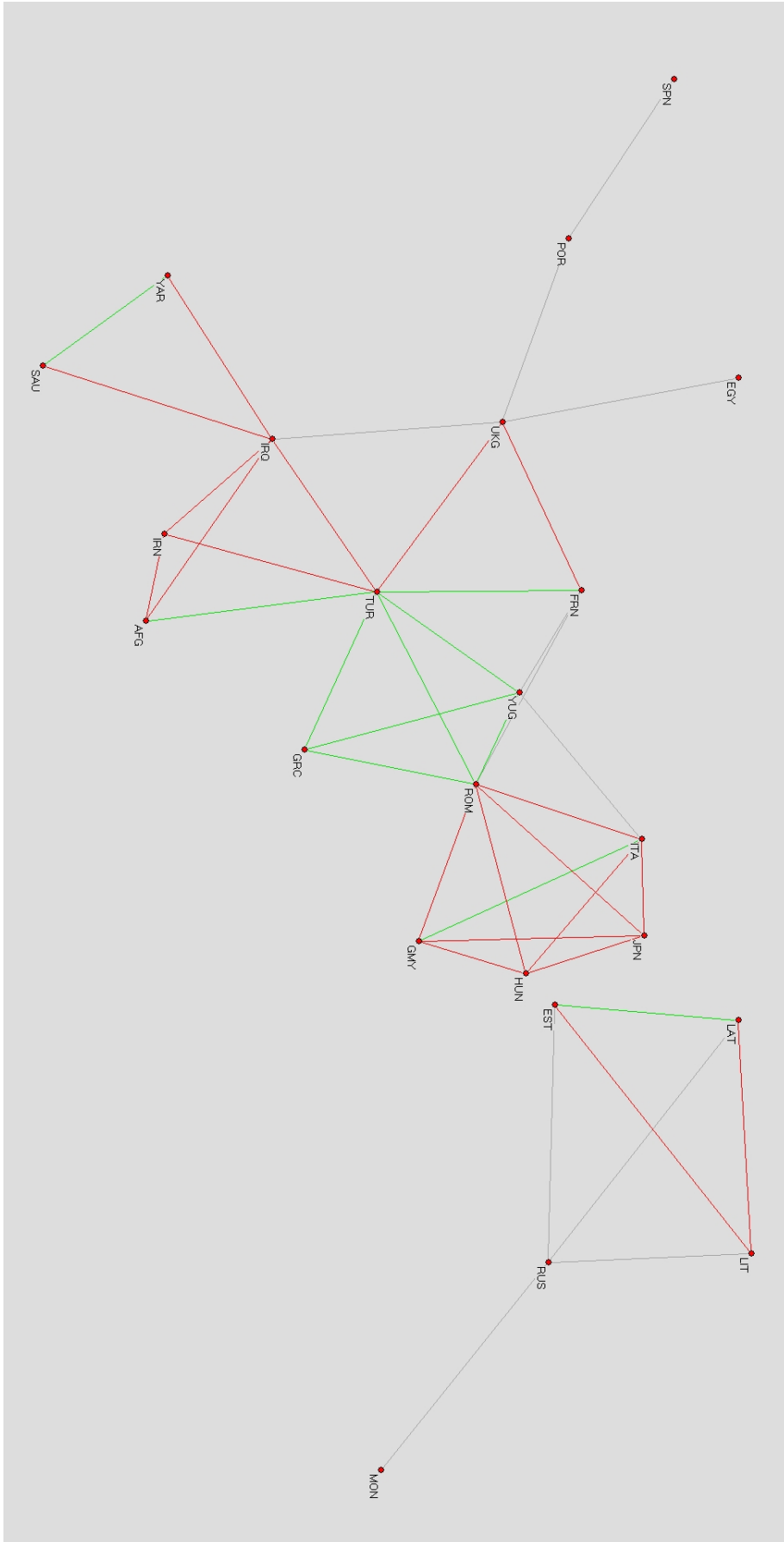


Figure 9: Network of Alliances, 1940, red for multilateral alliance, grey for bilateral alliance, green for both

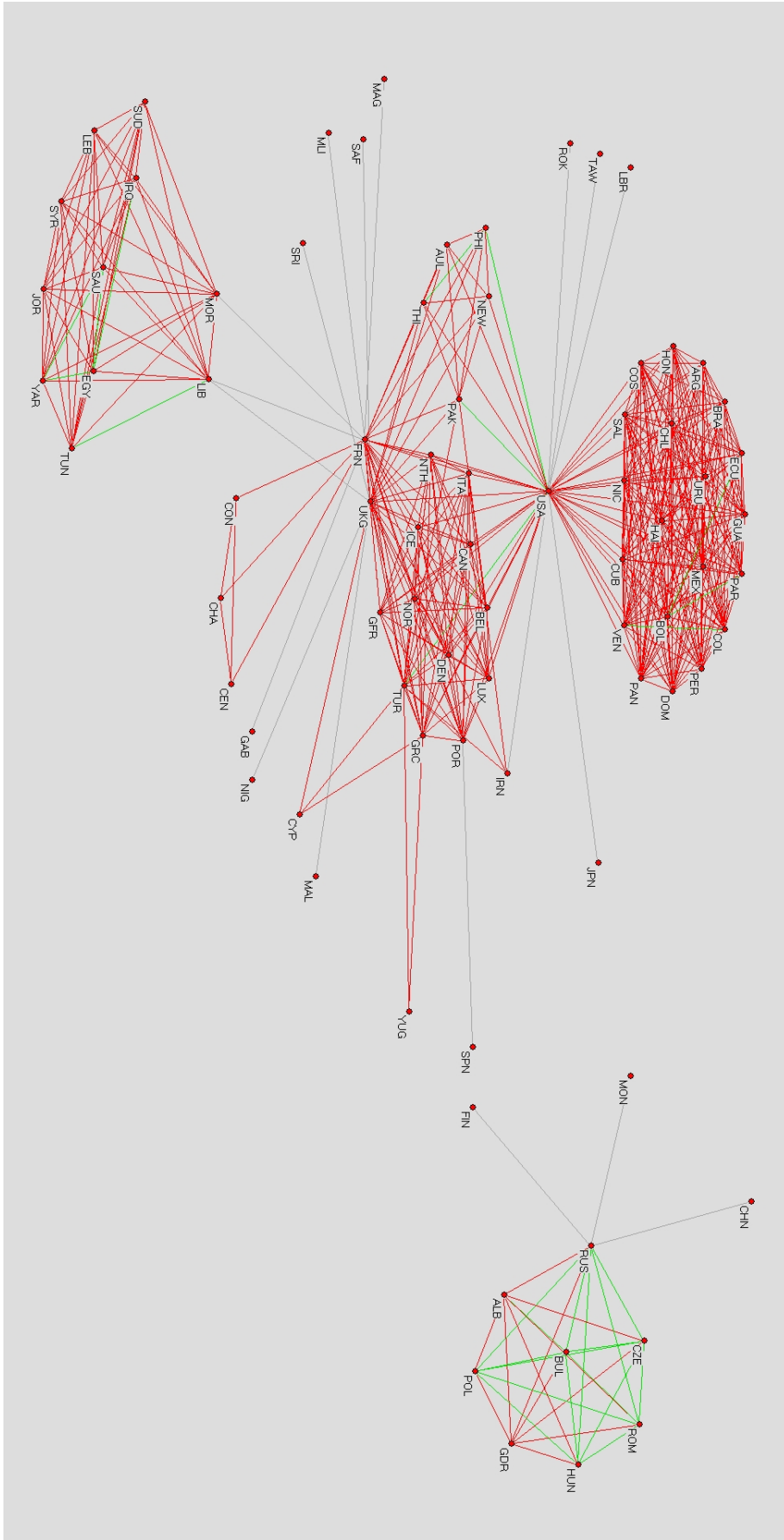


Figure 10: Network of Alliances, 1960, red for multilateral alliance, grey for bilateral alliance, green for both

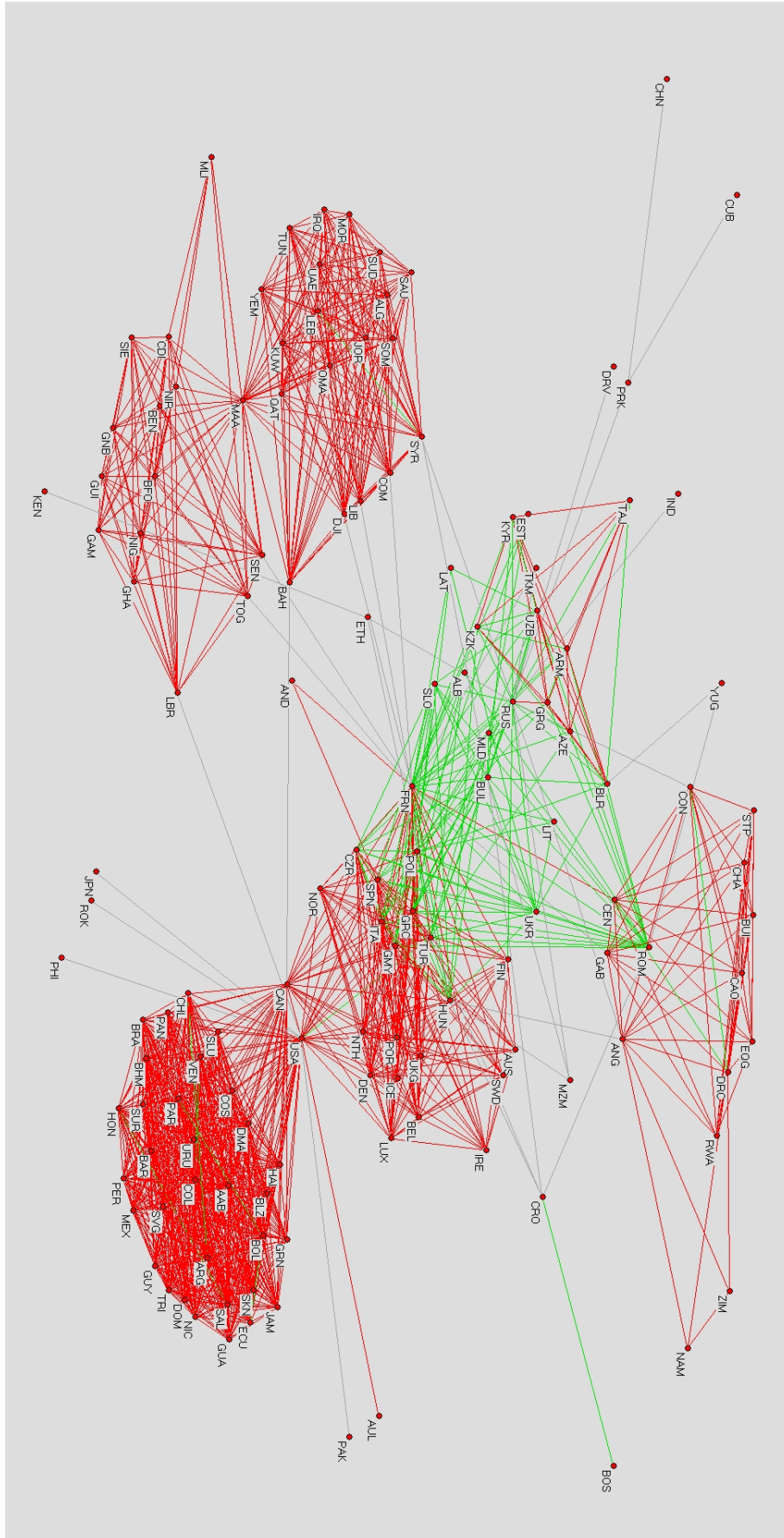


Figure 11: Network of Alliances, 2000, red for multilateral alliance, grey for bilateral alliance, green for both

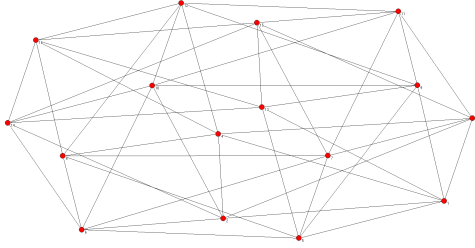


Figure 12: Stable Alliance Network

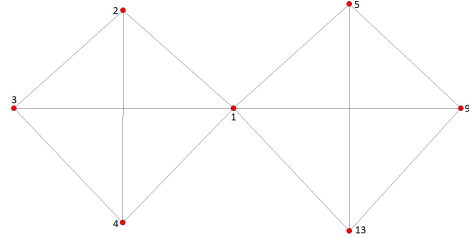


Figure 13: Close-up of Country 1 and Its Allies

For Online Publication - Supplementary Material: Stability Under CN-Vulnerability

To illustrate the point from Footnote 21, a war-stable configuration when $\gamma < 1$ under CN-vulnerability is pictured in Figure 12.

Figure 13 is a subgraph of Figure 12 focusing on country 1, its neighbors, and the cliques it is involved in (note that each other country in 13 has 3 other neighbors in the larger network, with each pair of countries one from the 1-2-3-4 clique and the other from the 1-5-9-13 clique sharing one neighbor in common). The network is isomorphic, so that country 1 is perfectly representative of what is faced by all countries. It can be easily verified that this network has no vulnerable countries if $\gamma \geq \frac{3}{4}$. For instance if country 1 were attacked by 2,3,4, it would be defended by 5,9,13 and so there would be 3 attackers and four defenders (counting 1) - and the defenders would win if $\gamma \geq \frac{3}{4}$. Any other cliques outside of 1's neighbors would face 7 defenders if they attacked 1, and so would also lose. It is easy to check that this network is stable against the addition of new links, since adding a new link does not increase the size of any clique, it just adds a new pair and pairs cannot win when attacking any country. Thus, [S2] is easily checked. So, it remains to check [S3]. If country 1 drops one of its links, e.g. with country 2, it will be vulnerable if $\gamma < 1$ (country 1 could be attacked by countries 5, 9, and 13 and only defended by 3 and 4). Since similar arguments can be made for every other country (so that [S3] is satisfied for the network as a whole), if $\gamma \in [\frac{3}{4}, 1)$, the network is stable.

Similar examples can be constructed for even lower γ s, by having countries be part of more separate cliques. For instance having each country be part of 3 separate cliques of size 4 (adding one more clique to the right hand side of Figure 12 and for every country) would lead to a stable network for $\gamma \in [\frac{3}{7}, \frac{1}{2})$. By varying the sizes of the cliques and the number of cliques that each country is in involved in, even arbitrarily small γ s nonempty stable networks can be found for large enough n .

PROPOSITION 2. *Consider any $\gamma < 1$ and the uniform strength case in which $M_i = \bar{M} \forall i$. There exists a large enough n such that there is a nonempty war-stable network under CN-*

vulnerability in which every country has at least one alliance.

Proposition 2 provides an interesting contrast to Theorem 2. With an offensive advantage, stable networks exist under CN-vulnerability as the offensive advantage provides incentives for countries to maintain relationships as a deterrent that they might sever in conditions where there is a defensive advantage.

Proof of Proposition 2: We prove by constructing a network such that every country i is a member of 2 cliques, each of size 4, and with i being the only country in both cliques. Further, we let i have a additional neighbors, none of whom are connected to each other or to other countries in the cliques i is in. The required number of countries will be $32a$.

Start with a network with 16 countries, each involved in 2 cliques of size 4. To construct it, take four copies of k_4 (the complete network on 4 vertices). In each copy of k_4 , label the countries 1 through 4. Then, connect all countries labelled 1 to each other, all countries labelled 2 to each other, and so on.

To construct the final network g , take this network on 16 countries, relabel the countries “a” through “p” (the 16th letter) arbitrarily. Then, create $2a$ copies of the network on 16 countries, numbering each copy 1 through $2a$. Label each vertex by (number of the position in the first network, letter of the position in the second network). There will be $32a$ labels, from (1,a) through (32a,p). Now connect each country to all other countries which agree on letter but differ on parity of the number (e.g. connect (1,a) with (3,a), (5,a), and so on; in essence, each subset of countries of the same letter forms a a -regular network with no cliques larger than 2).

In this final network, there are only cliques of size 4 (in the copies of the starting network) and of size 2. Further, adding any link to the network can create a new clique of size at most 3, involving no more than 2 of any given country’s neighbors. Since each country can already be threatened by a clique of 3 of its neighbors, we may safely ignore what restrictions on γ the requirement that no country be vulnerable in $g + ij$. For no country i to be vulnerable in g , we need $\gamma \geq \frac{3}{4+a}$ (i can be attacked by 3 of its neighbors connected in one of the size 4 cliques i is in, and i is defended by its neighbors in the other size 4 clique it is in, as well as its a other neighbors). To prevent i from wanting to drop a link, we need $\gamma < \frac{3}{3+a}$. Combining, we need $\gamma \in [\frac{3}{4+a}, \frac{3}{3+a})$. For $a = 0$, this interval is $[\frac{3}{4}, 1)$. Rearranging the interval, we need $\frac{1}{\gamma} \in (1 + \frac{a}{3}, \frac{4}{3} + \frac{a}{3}]$, so that for any $\gamma < 1$, we can take the largest a satisfying $a < \frac{3}{\gamma} - 3$, satisfying the lower bound, and then the upper bound will necessarily be satisfied. ■

7.1 Heterogeneous Military Strengths under CN-vulnerability

Although we have not been able to find any examples of nonempty war-stable networks under CN-vulnerability when $\gamma \geq 1$ and conjecture that the result extends generally, it appears

quite difficult to prove. The complexity of the proof of Theorem 2 shows the logic is involved. Parts of that proof are combinatorial in nature, and not directly extendable to asymmetries.

Nonetheless, we can show that the CN-result in Theorem 2 is robust in the sense that it holds for any (unequal) military strengths in an open neighborhood around equal ones. The question of whether the open neighborhood can be expanded to the full set remains open.

Let us say that γ has the *no-tie* property relative to n if there does not exist any positive integers m_1 and m_2 for which $m_1 + m_2 \leq n$ and $m_1 = \gamma m_2$. This is clearly a generic property, as it holds directly for all irrational levels of γ , as well as many rational levels - and a full measure set of values.

THEOREM 4. *Let $n \geq 3$ and consider γ that satisfies the no-tie property relative to n and some base military strength \bar{M} . There exists an open set of military strengths \mathcal{M} with $\bar{M} \in \mathcal{M}$ for which the following holds. If $1 \leq \gamma < 2$ and $(M_1, \dots, M_n) \in \mathcal{M}^n$, then there are no CN-war-stable networks. If $\gamma \geq 2$ and $(M_1, \dots, M_n) \in \mathcal{M}^n$, then the unique CN-war-stable network is the empty network.*

Theorem 4 shows that Theorem 2 is robust to some heterogeneity in military strengths.

Proof of Theorem 4: The proof follows easily from the proof of Theorem 2 simply noting that if γ admits no ties, then there is an open set of military strengths for $M(C) > \gamma M(C')$ for any coalitions when all countries have strengths within \mathcal{M} if and only if the same is true when all have strength \bar{M} . ■

7.2 CN-War and Trade Stability

In illustrating Footnote 25, an example illustrating the existence of CN-war-and-trade stability appears in Figure 12 in which $d^* = 6$, and which is in fact strongly CN-war and trade stable for all $\gamma \geq \frac{3}{4}$.

The quilts take advantage of the fact that no clique is large enough to outweigh any country and its remaining allies, and yet the alliances are held in place by trading concerns so that no alliances are deleted. Thus, the quilts leverage the economic incentives to maintain links, thereby overcoming the sparsity issues that can prevent stability.

Another aspect that trade incentives bring is that they reduce incentives for members of a clique to follow through with attacks. For example, there are networks that are war and trade stable, but for which there is a $ij \notin g$ such that some country is vulnerable despite trade at $g + ij$. For instance, let $d^* = 3$ and the number of countries be 6. Let g be formed of two triangles, with each country in one triangle connected to exactly one country in the other triangle. In particular, consider network $g = \{12, 13, 23, 45, 46, 56, 14, 25, 36\}$, as pictured in Figure 14. Let $\gamma < \frac{3}{2}$. Then, adding another link between the two triangles would make

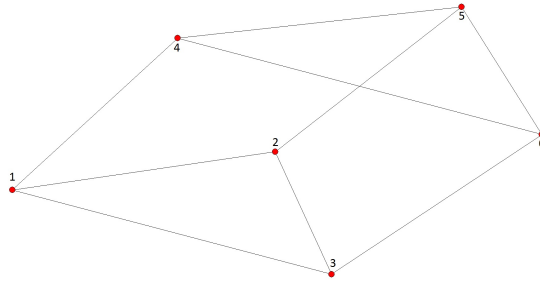


Figure 14: An alliance network

some country vulnerable despite trade at $g + ij$ (with an attacking coalition consisting of two countries from one triangle and one country from the other attacking the third country from the first triangle). In particular, if 15 is added, then 125 can defeat 3 who only has ally 6 remaining. However, there are several possible deterrents to this attack. One is that δ is low enough so that the gains from conquering 3 are not worth it for 2 who then only has 2 trading partners in the resulting network. Similarly it might be that E_3 is small. Another possibility is that 5 might not wish to have four allies instead of three, and that the resulting gains from conquering country 3 are not worth the cost of an additional ally.