Signaling Effects of Monetary Policy

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Abstract

We develop a DSGE model in which the policy rate signals to price setters the central bank's view about macroeconomic developments. The model is estimated with likelihood methods on a U.S. data set that includes the *Survey of Professional Forecasters* as a measure of price setters' inflation expectations. The estimated model with signaling effects delivers large and persistent real effects of monetary disturbances, even though the average duration of price contracts is fairly short. While the signaling effects do not substantially alter the transmission of technology shocks, they bring about deflationary pressures in the aftermath of positive demand shocks. In the 1970s, the Federal Reserve's disinflation policy, which was characterized by *gradual* increases in the policy rate, was counterproductive because it ended up signaling inflationary shocks.

Keywords: Bayesian estimation; higher-order beliefs; endogenous signals; price puzzle; persistent real effects of nominal shocks; marginal likelihood.

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1 Introduction

An important feature of economic systems is that information is dispersed across market participants and policymakers. Dispersed information implies that publicly observable policy actions transfer information to market participants. An important example is the monetary policy rate, which conveys information about the central bank's view on macroeconomic developments. Such an information transfer may strongly influence the transmission of monetary impulses and the central bank's ability to stabilize the economy. Consider the case in which a central bank expects that an exogenous disturbance will raise inflation in the next few quarters. On the one hand, as predicted by standard macroeconomic models, tightening monetary policy has the effect of mitigating the inflationary effects of the shock. On the other hand, raising the policy rate might also cause higher inflation if this action *signals* to unaware market participants that an inflationary shock is about to hit the economy. While the first type of monetary transmission has been intensively investigated in the economic literature, the signaling effects of monetary policy have received far less attention.

This paper develops a dynamic stochastic general equilibrium (DSGE) model to study the empirical relevance of the signaling effects of monetary policy and their implications for the propagation of policy and non-policy disturbances. In the model, price-setting firms face nominal rigidities and dispersed information. Firms observe their own specific technology conveying noisy private information about aggregate technology shocks that influence the future dynamics of firms' nominal marginal costs. Furthermore, price setters observe a noisy private signal about demand shocks and they observe the policy rate set by the central bank according to a Taylor-type reaction function. The policy signal provides public information about the central bank's view on current inflation and the output gap to firms. We call this model the *dispersed information model* (DIM).

The DIM features two channels of monetary transmission. The first channel emerges because the central bank can affect the real interest rate because of both nominal rigidities, as in standard New Keynesian models, and dispersed information. Changes in the real interest rate induce households to adjust their consumption. The second channel arises because the policy rate signals non-redundant information to price setters and hence influences their beliefs about macroeconomic developments. We label this second channel *the signaling channel* of monetary transmission. The signaling effects of monetary policy on the propagation of shocks critically rely on how price setters interpret the change in the policy rate. Raising the policy rate can be interpreted by price-setting firms in two ways. First, a monetary tightening might imply that the central bank is responding to a contractionary monetary shock, leading the central bank to deviate from its monetary rule. Second, a higher interest rate may also be interpreted as the response of the central bank to inflationary non-policy shocks, which, in the model, are an adverse aggregate technology shock or a positive demand shock. If the first interpretation prevails among price setters, tightening (easing) monetary policy curbs (raises) firms' inflation expectations and hence inflation. If the second interpretation prevails, raising (cutting) the policy rate induces firms to expect higher (lower) inflation, and hence, inflation tends to increase (decrease).

The model is estimated through likelihood methods on a U.S. data set that includes the *Survey of Professional Forecasters* (SPF) as a measure of price setters' inflation expectations. The data range includes the 1970s, which were characterized by one of the most notorious episodes of heightened inflation and inflation expectations in recent U.S. economic history. The estimated model features a fairly sluggish response of inflation to monetary disturbances and, at the same time, a very small amount of nominal rigidities. In fact, the likelihood seems to favor dispersed information to explain the sluggish effects of monetary impulses. Furthermore, the signaling channel is found to have non-negligible effects on the transmission of shocks. In the estimated model, raising the policy rate signals to firms that the central bank is likely to be responding to either a positive demand shock or a contractionary monetary shock. Firms, however, do not critically change their expectations about the aggregate technology shock from observing the policy rate as they hold fairly precise private information about this type of shock. A number of important implications follow. First, the signaling channel magnifies the real effects of monetary shocks. Second, inflation expectations respond positively to monetary shocks. These two features are present because price setters interpret − to some extent − a rise in the policy rate as the central bank's response to a positive demand shock that pushes up inflation expectations. Third, the inflationary effects of the signaling channel are negligible after an aggregate technology shock. When the Federal Reserve raises the policy rate to counter a negative technology shock, firms are induced to believe that the central bank may be reacting to either a positive demand shock or a contractionary monetary shock. These two effects on firms' inflation expectations are conflicting and turn out to cancel each other out, leading to tiny signaling effects of monetary policy in the aftermath of a technology shock. Fourth, the signaling effects of monetary policy bring about deflationary pressures in the aftermath of a positive demand shock. When the Federal Reserve raises the interest rate in response to a positive demand shock, firms attach some probability that a contractionary monetary shock might have occurred. Expecting a contractionary monetary shock tends to lower inflation expectations.

Signaling effects of monetary policy are found to be sizable in the 1970s and provide an explanation for the Federal Reserve's failure to lower inflation in that decade. Figure 1 shows that inflation soared while the Federal Reserve tightened monetary policy from 1972 through 1974 and from 1977 through the appointment of Paul Volcker as Federal Reserve Chairman in August of 1979. We introduce *Bayesian counterfactuals* to assess the signaling effects associated

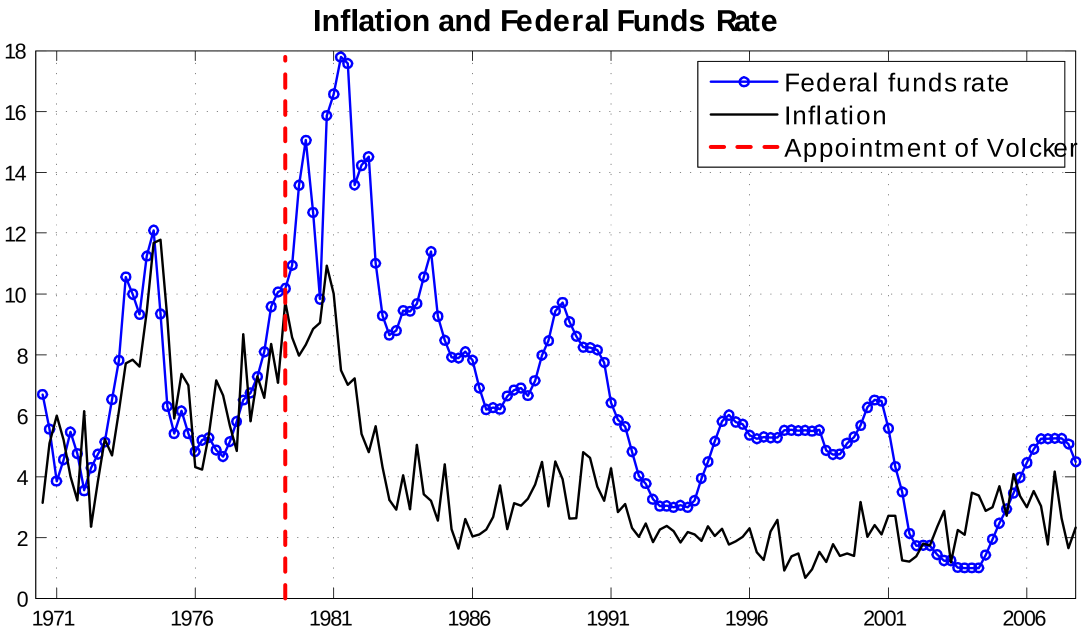

Figure 1: The inflation rate is obtained from the GDP deflator (*GDPDEF*) computed by the U.S. Bureau of Economic Analysis (BEA). The federal funds rate is the average of daily figures of the effective federal funds rate (*FEDFUNDS*) reported by the Federal Reserve Economic Data (FRED). The vertical axis measures units of percentage points of annualized rates.

with these two disinflation policies. On the one hand, we find that the *timid* increases of the federal funds rate substantially contributed to raising inflation expectations by signaling expansionary monetary shocks to price setters. On the other hand, the increases of the policy rate were *too gradual* to offset the rise in inflation expectations owing to the signaling effects of the monetary tightening itself. In relation to this second finding, we find that a more aggressive approach toward inflation stabilization would have offset these signaling effects and would have prevented inflation from rising to two digits in the 1970s. The model also predicts that this hawkish policy course would have led to a contraction of the quarter-to-quarter real gross domestic product (GDP) growth of up to 1.2 percentage points in annualized terms.

Furthermore, we find that the signaling effects of monetary policy also account for the *sluggish* adjustment of inflation during the robust monetary contraction conducted after the appointment of Paul Volcker as Federal Reserve Chairman in August 1979 through the second quarter of 1981. In that period, the increases in the policy rate were interpreted by price setters as evidence that the monetary authority was responding to inflationary demand shocks. Moreover, we find that the signaling effects of monetary policy had positively contributed to the good macroeconomic performance of the U.S. economy from the 1980s through 2003.

It is important to emphasize that the signaling effects of monetary policy are not the only possible channel through which the model could explain why both the rate of inflation and the federal funds rate swiftly rose in the 1970s and why price dynamics appear to respond sluggishly to the robust monetary tightening conducted by the Federal Reserve under Chairman Paul Volcker. For instance, the model could have explained these dynamics through a sequence of adverse aggregate technology shocks, which are associated with tiny signaling effects because firms have fairly accurate private information about these shocks.

Furthermore, the DIM is found to fit the data better than a model in which price setters have perfect information (i.e., the perfect information model, or PIM). This finding validates the use of the DIM to study the signaling effects of monetary policy, since the PIM is a prototypical New Keynesian model that has been extensively used by scholars for conducting quantitative analysis about monetary policy (e.g., Rotemberg and Woodford 1997; Clarida, Gali, and Gertler 2000; Lubik and Schorfheide 2004; and Coibion and Gorodnichenko 2011*a*). Quite interestingly, if the two models were estimated using a narrower data set that does not include the SPF, we could not have concluded that the DIM fits the data better than the PIM. This finding suggests that the advantage of the DIM stems from its ability to fit the observed inflation expectations. The fact that the DIM fits the SPF better than a perfect information model is not obvious. In fact, Del Negro and Eusepi (2011) find that the imperfect information model by Erceg and Levin (2003) is outperformed by a standard perfect information model in fitting the SPF.

This paper also makes a methodological contribution by providing an algorithm to solve DSGE models in which agents find it optimal to *forecast the forecasts of other agents*. The solution routine proposed in the paper turns out to be sufficiently fast and reliable to allow likelihood-based estimation. The proposed algorithm belongs to the general solution methods developed by Nimark (2011). The proposed algorithm improves upon the one used in Nimark (2008) as it does not require solving a system of nonlinear equations. Furthermore, the paper shows how to quantify information flows conveyed by the policy signal to private agents in the model.

The idea that the monetary authority sends public signals to an economy in which agents have dispersed information was pioneered by Morris and Shin (2003a , 2003b). While technical hurdles have prevented empiricists to conduct a structural investigation of the signaling effects of monetary policy so far, the theoretical literature has been flourishing quickly. The space in this section is regrettably too small to do justice to all these theoretical contributions. Angeletos, Hellwig, and Pavan (2006) study the signaling effects of policy decisions in a coordination game. Walsh (2010) shows that the (perceived or actual) signaling effects of monetary policy alter the central bank's decisions, resulting in a bias (i.e., an opacity bias) that distorts the central bank's optimal response to shocks. Unlike this paper, Walsh's study is based on a model that does not feature dispersed information. Baeriswyl and Cornand (2010) study optimal monetary policy in a DSGE model in which the central bank can use its policy instrument to disclose information about its assessment of the fundamentals. Price setters face two sources of information limitation: sticky information à la Mankiw and Reis (2002) and dispersed information of a type that is very similar to that of this paper. That contribution is mostly theoretical, whereas this paper carries out a full-fledged likelihood estimation of a model in which monetary policy has signaling effects. Hachem and Wu (2012) develop a model in which firms update their heterogeneous inflation expectations through social dynamics to study the effects of central bank communication.

The model studied in this paper is built on Nimark (2008). A particularly useful feature of Nimark's model is that the supply side of the model economy can be analytically worked out and characterized by an equation that nests the standard New Keynesian Phillips curve. The model studied in this paper shares this feature. Nonetheless, in Nimark (2008) the signaling channel does not arise because assumptions on the Taylor-rule specification imply that the policy rate conveys only redundant information to price setters.

This paper is also related to a quickly growing empirical literature that uses the SPF to study the response of public expectations to monetary policy decisions. Del Negro and Eusepi (2011) perform an econometric evaluation of the extent to which the inflation expectations generated by DSGE models are in line with the observed inflation expectations. There are two main differences between that paper and this one. First, in our settings, price setters have heterogeneous and dispersed higher-order expectations as they observe private signals. Second, this paper fits the model to a data set that includes the 1970s, whereas Del Negro and Eusepi (2011) use a data set starting from the early 1980s. Coibion and Gorodnichenko (2012*b*) find that the Federal Reserve raises the policy rate more gradually if the private sector's inflation expectations are lower than the Federal Reserve's forecasts of inflation. This empirical evidence can be rationalized in a model in which monetary policy has signaling effects and the central bank acts strategically to stabilize public inflation expectations. Coibion and Gorodnichenko (2012*a*) use the SPF to document robust evidence in favor of models with informational rigidities.

This paper also belongs to a quite thin literature that carries out likelihood-based analyses on models with dispersed information. Nimark (2012) estimates an island model built on Lorenzoni (2009) and augmented with man-bites-dog signals, which are signals that are more likely to be observed when unusual events occur. Maćkowiak, Moench, and Wiederholt (2009) use a dynamic factor model to estimate impulse responses of sectorial price indexes to aggregate shocks and to sector-specific shocks for a number of models, including a rational inattention model. Melosi (Forthcoming) conducts an econometric analysis of a stylized DSGE model with dispersed information à la Woodford (2002).

Bianchi and Melosi (2012) develop a DSGE model that features waves of agents' pessimism about how aggressively the central bank will react to future changes in inflation to study the welfare implications of monetary policy communication. Gorodnichenko (2008) introduces a model in which firms make state-dependent decisions on both pricing and acquisition of information and shows that this model delivers delayed response of inflation to monetary shocks. Trabandt (2007) analyzes the empirical properties of a state-of-the-art sticky-information DSGE model à la Mankiw and Reis (2002) and compares them with those of a state-of-the-art DSGE model with sticky prices à la Calvo.

Finally, Hetzel (1998) and Romer and Romer (2013) analyze historical documentation, including minutes of the Federal Open Market Committee (FOMC) meetings and Federal Reserve Chairmen' speeches, showing that the Federal Reserve adopted a *gradualistic approach* to disinflate the U.S. economy in the 1970s.

The paper is organized as follows. Section 2 describes the dispersed information model, in which monetary policy has signaling effects, as well as a model in which firms have complete information. The latter model will be used as a benchmark to evaluate the empirical performance of the dispersed information model. In Section 3, we perform some numerical experiments to show the macroeconomic propagation of monetary disturbances through the signaling channel. Section 4 deals with the empirical analysis of the paper. In Section 5, we conclude.

2 Models

Section 2.1 introduces the model with dispersed information and signaling effects of monetary policy. In Section 2.2, we present the time protocol of the model. Section 2.3 presents the problem of households. Section 2.4 presents firms' price-setting problem. In Section 2.5, the central bank's behavior and government's behavior are modeled. Section 2.6 deals with the log-linearization and the solution of the dispersed information model. Finally, Section 2.7 presents the perfect information model, which will turn out to be useful for evaluating the empirical significance of the dispersed information model.

2.1 The Dispersed Information Model (DIM)

The economy is populated by a continuum $(0,1)$ of households, a continuum $(0,1)$ of monopolistically competitive firms, a central bank (or monetary authority), and a government (or fiscal authority). A Calvo lottery establishes which firms are allowed to reoptimize their prices in any given period t (Calvo 1983). Households consume the goods produced by firms, demand government bonds, pay taxes to or receive transfers from the fiscal authority, and supply labor to the firms in a perfectly competitive labor market. Firms sell differentiated goods to households. The fiscal authority has to finance maturing government bonds. The fiscal authority can issue new government bonds and can either collect lump-sum taxes from households or pay transfers to households. The central bank sets the nominal interest rate at which the government's bonds pay out their return.

Aggregate and idiosyncratic shocks hit the model economy. The aggregate shocks are a technology shock, a monetary policy shock, and a demand shock. All of these shocks are orthogonal to each other at all leads and lags. Idiosyncratic shocks include a firm-specific technology shock and the outcome of the Calvo lottery for price optimization.

2.2 The Time Protocol

Any period t is divided into three stages. All actions that are taken in any given stage are simultaneous. At stage 0, shocks are realized and the central bank sets the interest rate for the current period t. At stage 1, firms update their information set by observing (i) their idiosyncratic technology, (ii) a private signal about the demand shocks, and (iii) the interest rate set by the central bank. Given these observations, firms set their prices at stage 1. At stage 2, households learn about the realization of all the shocks in the economy and therefore become perfectly informed. Households then decide their consumption, C_t; their demand for (one-period) nominal government bonds, B_t; and their labor supply, N_t. At this stage, firms hire labor and produce so as to deliver the demanded quantity at the price they have set at stage 1. The fiscal authority issues bonds and collects taxes from households or pays transfers to households. The markets for goods, labor, and bonds clear.

2.3 Households

Households have perfect information, and hence, we can use the representative household to solve their problem at stage 2 of every period t:

$$\max_{C_{t+s},B_{t+s},N_{t+s}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} g_{t+s} \left[\ln C_{t+s} - \chi_n N_{t+s}\right],$$

where β is the deterministic discount factor and g_t denotes a preference shifter that scales up or down the period utility function. The logarithm of the preference shifter follows an autoregressive (AR) process: $\ln g_t = \rho_g \ln g_{t-1} + \sigma_g \varepsilon_{g,t}$ with Gaussian shocks $\varepsilon_{g,t} \backsim \mathcal{N}(0,1)$. We refer to g_t as demand conditions and to the innovation $\varepsilon_{g,t}$ as demand shock. Disutility from labor linearly enters the period utility function. Note that χ_n is a parameter that affects the marginal disutility of labor.

The flow budget constraint of the representative household in period t reads as

$$P_t C_t + B_t = W_t N_t + R_{t-1} B_{t-1} + \Pi_t - T_t, \tag{1}$$

where P_t is the price level of the composite good consumed by households and W_t is the (competitive) nominal wage rate, R_t stands for the nominal (gross) interest rate, Π_t is the (equally shared) dividends paid out by the firms, and T_t stands for the lump-sum transfers/taxes. Composite consumption in period t is given by the Dixit-Stiglitz aggregator $C_t = \left(\int_0^1 C_{j,t}^{\frac{\nu-1}{\nu}} dj\right)^{\frac{\nu}{\nu-1}}$, where $C_{j,t}$ is consumption of the good produced by firm j in period t and ν is the elasticity of substitution between consumption goods.

At stage 2 of every period t, the representative household chooses its consumption of the good produced by firm j, labor supply, and bond holdings subject to the sequence of the flow budget constraints and a no-Ponzi-scheme condition. The representative household takes as given the nominal interest rate, the nominal wage rate, nominal aggregate profits, nominal lump-sum transfers/taxes, and the prices of all consumption goods. It can be shown that the demand for the good produced by firm j is:

$$C_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\nu} C_t, \tag{2}$$

where the price level of the composite good is defined as $P_t = \left(\int (P_{j,t})^{1-\nu} di\right)^{\frac{1}{1-\nu}}$.

2.4 Firms' Price-Setting Problem

Firms are endowed with a linear technology $Y_{j,t} = A_{j,t} N_{j,t}$, where $Y_{j,t}$ is the output produced by the firm j at time t, $N_{j,t}$ is the amount of labor employed by firm j at time t, and $A_{j,t}$ is the firm-specific level of technology that can be decomposed into a level of aggregate technology (A_t) and a white-noise firm-specific component ($\varepsilon_{j,t}^a$). More specifically,

$$\ln A_{j,t} = \ln A_t + \widetilde{\sigma}_a \varepsilon_{j,t}^a, \tag{3}$$

where $\varepsilon_{j,t}^a \overset{iid}{\backsim} \mathcal{N}(0,1)$ and $A_t = \gamma^t a_t$ with deterministic trend $\gamma > 1$ and a_t is the detrended level of aggregate technology that evolves according to the AR process $\ln a_t = \rho_a \ln a_{t-1} + \sigma_a \varepsilon_{a,t}$ with Gaussian shocks $\varepsilon_{a,t} \overset{iid}{\backsim} \mathcal{N}(0,1)$.

Following Calvo (1983), we assume that a fraction θ of firms are not allowed to reoptimize their prices at stage 1 of any period. Those firms that are not allowed to reoptimize are assumed to index their prices to the steady-state inflation rate. Furthermore, we assume that firms have limited knowledge about the history of shocks that have hit the economy. More specifically, it is assumed that firms' information set includes the history of firm-specific technology $\ln A_{j,t}$ and the history of a private signal $g_{j,t}$ on the demand conditions g_t, which evolves according to the following process:

$$\ln g_{j,t} = \ln g_t + \widetilde{\sigma}_g \varepsilon_{j,t}^g, \tag{4}$$

where $\varepsilon_{j,t}^g \overset{iid}{\backsim} \mathcal{N}(0,1)$. This signal is meant to capture the fact that arguably firms are used to carrying out market analyses to gather information about demand conditions g_t before setting their price. Moreover, firms observe the history of the nominal interest rate R_t set by the central bank, and the history of the price set by the firm. To sum up, the information set $\mathcal{I}_{j,t}$ of firm j at time t is given by

$$\mathcal{I}_{j,t} \equiv \{\ln A_{j,\tau}, \ln g_{j,\tau}, R_\tau, P_{j,\tau} : \tau \leq t\}\,, \tag{5}$$

Firms receive the signals in $\mathcal{I}_{j,t}$ at stage 1 when they are called to set their price. Firms are assumed to know the model transition equations and their structural parameters. Furthermore, note that observing the history of their own price $\{P_{j,\tau} : \tau \leq t\}$ conveys only redundant information to firms because their price is either adjusted to the steady-state inflation rate, which is known by firms, or a function of the history of the signals that have been already observed in the past. Thus, this signal does not play any role in the formation of firms' expectations and will be called the redundant signal. Henceforth, when we refer to signals, we mean only the non-redundant signals (namely, $\ln A_{j,t}$, $\ln g_{j,t}$, and R_t). Finally, we assume that firms have received an infinitely long sequence of signals at any time t. This assumption substantially simplifies the task of solving the model by ensuring that the Kalman gain matrix is time invariant and is the same across firms.

Let us denote the (gross) steady-state inflation rate as π_*, the nominal marginal costs for firm j as $MC_{j,t} = W_t/A_{j,t}$, the time t value of one unit of the composite consumption good in period $t+s$ to the representative household as $\Xi_{t|t+s}$, and the expectation operator conditional on firm j's information set $\mathcal{I}_{j,t}$ as $\mathbb{E}_{j,t}$. At stage 1 of every period t, an arbitrary firm j that is allowed to reoptimize its price $P_{j,t}$ solves

$$\max_{P_{j,t}} \quad \mathbb{E}_{j,t}\left[\sum_{s=0}^{\infty} (\beta\theta)^s\, \Xi_{t|t+s}\left(\pi_*^s P_{j,t} - MC_{j,t+s}\right) Y_{j,t+s}\right],$$

subject to $Y_{j,t} = C_{j,t}$ (i.e., firms commit themselves to satisfying any demanded quantity that will arise at stage 2), to the firm j's specific demand in equation (2), and to the linear production function. When solving the price-setting problem at stage 1, firms have to form expectations about the evolution of their nominal marginal costs, which will be realized in the next stage of the period (i.e., stage 2), using their information set $\mathcal{I}_{j,t}$. At stage 2, firms produce and deliver the quantity the representative household demands for their specific goods at the prices they set in the previous stage 1. It is important to recall that at stage 2, firms do not receive any further information or any additional signals to what they have already observed at stage 1.

Since firms find it optimal to set their prices in response to changes in their *nominal* marginal costs, they raise their prices, *ceteris paribus*, when they expect the price level to increase, that is, when they expect that the *other* price setters, on average, are raising their prices. Such a coordination motive in price setting and the availability of private information make it optimal for price setters to *forecast the forecasts* of other price setters (Townsend 1983a, 1983b). This feature of the price-setting problem raises technical challenges, which will be tackled in Section 2.6, when we solve the dispersed information model.

2.5 The Monetary and Fiscal Authorities

There is a monetary authority and a fiscal authority. The monetary authority sets the nominal interest rate according to the reaction function:

$$R_t = (r_* \pi_*) \left(\frac{\pi_t}{\pi_*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\phi_y} \eta_{r,t}, \tag{6}$$

where r_* is the steady-state real interest rate, π_t is the (gross) inflation rate, and Y_t^* is potential output, which is the output level that would be realized if prices were perfectly flexible (i.e., $\theta = 0$) and firms were perfectly informed. Note that $\eta_{r,t}$ is a random variable that affects the nominal interest rate in period t and is driven by the following process: $\ln \eta_{r,t} = \rho_r \ln \eta_{r,t-1} + \sigma_r \varepsilon_{r,t}$, with Gaussian shocks $\varepsilon_{r,t} \overset{iid}{\backsim} \mathcal{N}(0,1)$.[1] We will refer to the exogenous variable $\eta_{r,t}$ as the central bank's *deviation from the monetary rule* and to the innovation $\varepsilon_{r,t}$ as a monetary policy shock. The deviation from the monetary rule $\eta_{r,t}$ is intended to capture any exogenous deviation from the monetary policy, including the *central bank's errors* in estimating the current rate of inflation and the current output gap or the *central bank's willingness to smooth the dynamics of the policy rate R_t*.

The flow budget constraint of the fiscal authority in period t reads as $R_{t-1}B_{t-1} - B_t = T_t$. The fiscal authority has to finance maturing government bonds. The fiscal authority can collect lump-sum taxes or issue new government bonds. Since there is neither capital accumulation nor government consumption, the resource constraint implies $Y_t = C_t$.

2.6 Log-linearization and Model Solution

First, we solve the firms' and households' problems, described in Sections 2.3 and 2.4, and obtain the consumption Euler equation and the price-setting equation. Second, we detrend the non-stationary variables before log-linearizing the model equations around their value at the nonstochastic steady-state equilibrium. Let us define the detrended real output as $y_t \equiv Y_t/\gamma^t$. We denote the log-deviation of an arbitrary (stationary) variable x_t from its steady-state value as $\widehat{x}_t$. As in Nimark (2008), we obtain the imperfect-common-knowledge Phillips curve that

[1] Endowing the firms with a private signal about the deviations from the monetary rule would be an alternative specification of the information set $\mathcal{I}_{j,t}$. However, the noise variance of this additional private signal turns out not to be identified in our estimation. Therefore, we decide to not adopt that specification of the information set.

reads as[2]

$$\hat{\pi}_t = (1-\theta)(1-\beta\theta)\sum_{k=1}^{\infty}(1-\theta)^{k-1}\widehat{mc}_{t|t}^{(k)} + \beta\theta\sum_{k=1}^{\infty}(1-\theta)^{k-1}\widehat{\pi}_{t+1|t}^{(k)}, \tag{7}$$

where $\widehat{\pi}_{t+1|t}^{(k)}$ denotes the average k-th order expectations about the next period's inflation rate, $\widehat{\pi}_{t+1}$, that is, $\widehat{\pi}_{t+1|t}^{(k)} \equiv \underbrace{\int \mathbb{E}_{j,t} \ldots \int \mathbb{E}_{j,t}}_{k} \widehat{\pi}_{t+1} dj...dj$, any integer $k > 1$. In addition, $\widehat{mc}_{t|t}^{(k)}$ denotes the average k-th order expectations about the real aggregate marginal costs $\widehat{mc}_t \equiv \int \widehat{mc}_{j,t} dj$, which evolve according to the equation $\widehat{mc}_{t|t}^{(k)} = \hat{y}_{t|t}^{(k)} - \widehat{a}_{t|t}^{(k-1)}$, any integer $k > 1$. Average higher-order expectations (HOE) enter the specification of the Phillips curve because price setters find it optimal to *forecast the forecasts* of other price setters, as pointed out in Section 2.4.

The log-linearized Euler equation is standard and reads as follows:

$$\widehat{g}_t - \widehat{y}_t = \mathbb{E}_t\widehat{g}_{t+1} - \mathbb{E}_t\widehat{y}_{t+1} - \mathbb{E}_t\hat{\pi}_{t+1} + \hat{R}_t, \tag{8}$$

where $\mathbb{E}_t(\cdot)$ denotes the expectation operator conditional on the complete information set, which includes the history of the three aggregate shocks. The central bank's reaction function (6) can be written as the following:

$$\hat{R}_t = \phi_\pi \hat{\pi}_t + \phi_y (\hat{y}_t - \widehat{y}_t^*) + \widehat{\eta}_{r,t}. \tag{9}$$

The demand conditions evolve according to $\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \sigma_g \varepsilon_{g,t}$. The process for aggregate technology becomes $\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \sigma_a \varepsilon_{a,t}$. The process leading the central bank's deviation from the monetary rule becomes $\widehat{\eta}_{r,t} = \rho_r \widehat{\eta}_{r,t-1} + \sigma_r \varepsilon_{r,t}$. We detrend and then log-linearize the signal equation concerning the level of aggregate technology (3) and obtain

$$\widehat{a}_{j,t} = \widehat{a}_t + \widetilde{\sigma}_a \varepsilon_{j,t}^a. \tag{10}$$

The signal equation concerning the demand conditions (4) is written as

$$\widehat{g}_{j,t} = \widehat{g}_t + \widetilde{\sigma}_g \varepsilon_{j,t}^g. \tag{11}$$

The signal about monetary policy is given by equation (9).

When firms solve their price-setting problem, they have to form expectations about the dynamics of their nominal marginal costs $MC_{j,t}$ by using their information set $\mathcal{I}_{j,t}$. To this end, firms solve a signal extraction problem using the log-linearized model equations, which are listed earlier, and the signal equations (9), (10), and (11).[3]

[2] See Appendix A for a detailed derivation of the Philips curve.

[3] In the context of the log-linear model, observing R_t, as indicated in the information set (5), or $\widehat{R}_t$, as

A detailed description of how we solve the model is provided in Appendix B. The proposed solution algorithm improves upon the one used in Nimark (2008) as our approach does not require solving a system of nonlinear equations.[4] When the model is solved, the law of motion of the endogenous variables $\mathbf{s}_t \equiv \left[\widehat{y}_t, \widehat{\pi}_t, \widehat{R}_t\right]'$ reads as follows:

$$\mathbf{s}_t = \mathbf{v}_0 X_{t|t}^{(0:k)}, \tag{12}$$

where $X_{t|t}^{(0:k)} \equiv \left[\widehat{a}_{t|t}^{(s)}, \widehat{\eta}_{r,t|t}^{(s)}, \widehat{g}_{t|t}^{(s)} : 0 \leq s \leq k\right]'$ is the vector of the average HOE about the exogenous state variables (i.e., $\widehat{a}_t, \widehat{\eta}$, and $\widehat{g}_t$). The average s-th order expectations about the level of aggregate technology, $\widehat{a}_{t|t}^{(s)}$, are defined as the integral of firms' expectations about the average $(s-1)$-th order expectations across firms. In symbols, $\widehat{a}_{t|t}^{(s)} = \int \mathbb{E}_{j,t}\left(\widehat{a}_{t|t}^{(s-1)}\right) dj$ for $1 \leq s \leq k$, where conventionally $\widehat{a}_{t|t}^{(0)} = \widehat{a}_t$. Analogously, the average HOE about the central bank's deviation from the monetary rule and demand conditions are given by $\widehat{\eta}_{r,t|t}^{(s)} = \int_{r,t|t}^{(s-1)} \widehat{\eta} dj$ for $1 \leq s \leq k$ and $\widehat{g}_{t|t}^{(s)} = \int \widehat{g}_{t|t}^{(s-1)} dj$ for $1 \leq s \leq k$, respectively. Note that we truncate the infinite hierarchy of average higher-order expectations, considering only orders smaller than or equal to the positive integer k. Henceforth, we set $k = 20$. The vector of average HOE is assumed to follow a Vector AutoRegressive (VAR) model of order one[5]

$$X_{t|t}^{(0:k)} = \mathbf{M} X_{t-1|t-1}^{(0:k)} + \mathbf{N}\boldsymbol{\varepsilon}_t. \tag{13}$$

The parameter set of the log-linearized dispersed information model is given by the vector

$$\Theta_{DIM} = \left[\theta, \phi_\pi, \phi_y, \beta, \rho_a, \rho_g, \rho_r, \sigma_a, \widetilde{\sigma}_a, \sigma_g, \widetilde{\sigma}_g, \sigma_r, \gamma\right]'.$$

---

specified in the signal equation (9), does not change the price-setting decisions in that firms are assumed to know the model and, hence, the nonstochastic steady-state, including the nominal interest rate arising at the deterministic steady-state $\pi_* r_*$.

[4] Nimark (2009) introduces a method to improve the efficiency of these types of solution methods for dispersed information models in which agents (e.g., firms) use lagged endogenous variables to form their beliefs. An alternative solution algorithm based on rewriting the equilibrium dynamics partly as a moving-average process and setting the lag with which the state is revealed to be a very large number is analyzed by Hellwig (2002) and Hellwig and Vankateswaran (2009).

[5] As is standard in the literature (e.g., Woodford 2002), we focus on equilibria where the higher-order expectations about the exogenous state variables follow a VAR model of order one. To solve the model we also assume common knowledge of rationality. See Nimark (2008, Assumption 1, p. 373) for a formal formulation of the assumption of common knowledge of rationality. Rondina and Walker (2012) study a new class of rational expectations equilibria in dynamic economies with dispersed information and signal extraction from endogenous variables.

2.7 The Perfect Information Model (PIM)

If the noise variance of the private exogenous signals ($\widetilde{\sigma}_a$ and $\widetilde{\sigma}_g$) is equal to zero, higher-order uncertainty would fade away (i.e., $X_{t|t}^{(k)} = X_t$, for any integer k) and the linearized model would boil down to a prototypical (perfect information) three-equation New Keynesian DSGE model (e.g., Rotemberg and Woodford 1997; Lubik and Schorfheide 2004; and Rabanal and Rubio-Ramírez 2005). More specifically, the imperfect-common-knowledge Phillips curve (7) would become $\hat{\pi}_t = \kappa_{pc}\widehat{mc}_t + \beta \mathbb{E}_t \widehat{\pi}_{t+1}$, where $\kappa_{pc} \equiv (1-\theta)(1-\theta\beta)/\theta$ and the real marginal costs $\widehat{mc}_t = \hat{y}_t - \widehat{a}_t$. The Euler equation and the Taylor rule would be the same as in the dispersed information model. In the perfect information model, the monetary shock propagates by affecting the intertemporal allocation of consumption. The real effects of money solely emerge as a result of price stickiness as opposed to the sluggish adjustments of firms' expectations in the dispersed information model. We call this prototypical New Keynesian DSGE model the perfect information model (PIM). The parameter set of the log-linearized PIM is given by the vector

$$\Theta_{PIM} = \left[\theta, \phi_\pi, \phi_y, \beta, \rho_a, \rho_g, \rho_r, \sigma_a, \sigma_g, \sigma_r, \gamma\right]'.$$

3 The Signaling Channel of Monetary Transmission

A salient feature of the dispersed information model is that the policy rate R_t transfers information about the output gap and inflation to price setters. We call this transfer of information the *signaling channel of monetary transmission*. Price setters use the policy rate as a signal that helps them to track non-policy shocks (namely, technology shocks $\varepsilon_{a,t}$ and demand shocks $\varepsilon_{g,t}$) and, at the same time, to infer shocks to central bank's exogenous deviations from the monetary rule (i.e., monetary policy shocks $\varepsilon_{r,t}$).

This section is organized as follows. In Section 3.1, we define a set of measures to quantify the amount of information conveyed by the signals observed by firms. Measuring information flows will simplify the task of interpreting the macroeconomic implications of the signaling channel later on. In Section 3.2, we introduce a set of tools that will help us characterize the role of the signaling channel in the transmission of monetary disturbances. In Section 3.3, we present and analyze three numerical examples that shed light on how the signaling channel influences the transmission of monetary impulses to inflation and inflation expectations.

3.1 Measuring Information Flows from the Signaling Channel

Following a standard practice in information theory (Cover and Thomas 1991), we use an entropy-based measure to assess how much information is provided by the signals firms observe in every period. The entropy measures the uncertainty about a random variable. For instance, the entropy associated with the level of aggregate technology $\widehat{a}_t$, which is normally distributed with (unconditional) covariance matrix $var(\widehat{a}_t)$, is defined as $H(\widehat{a}_t) \equiv 0.5\log_2[2\pi e \cdot var(\widehat{a}_t)]$.

We quantify the information flow conveyed by the signals as *the reduction of uncertainty* (i.e., entropy) at time t due to observing the signals in the information set $\mathcal{I}_{j,t}$.[6] For instance, the information flow about aggregate technology conveyed by the signals in the information set $\mathcal{I}_{j,t}$ can be computed as $\mathcal{H}(\widehat{a}_t;\mathcal{I}_{j,t}) = H(\widehat{a}_t) - H(\widehat{a}_t|\mathcal{I}_{j,t})$, where the conditional entropy $H(\widehat{a}_t|\mathcal{I}_{j,t}) \equiv 0.5\log_2[2\pi e \cdot var(\widehat{a}_t|\mathcal{I}_{j,t})]$ and $var(\widehat{a}_t|\mathcal{I}_{j,t})$ denotes the variance of aggregate technology conditional on firms having observed the signals in their information set $\mathcal{I}_{j,t}$.[7] Note that having assumed that firms have received an infinitely long sequence of signals at any time t implies that the conditional covariance matrix $var(\widehat{a}_t|\mathcal{I}_{j,t})$ is time invariant and is the same across firms at any time. Hence, information flows do not vary over time or across firms and we can omit indexing the information flow $\mathcal{H}$ with j and t.

Analogously, define the entropy conditional on firms having observed *only* their private signals as $H(\widehat{a}_t|\mathcal{I}_{j,t}\diagup R^t) = 0.5\log_2[2\pi e \cdot var(\widehat{a}_t|\mathcal{I}_{j,t}\diagup R^t)]$, where $var(\widehat{a}_t|\mathcal{I}_{j,t}\diagup R^t)$ denotes the conditional variance of aggregate technology conditional on the history of private signals. We can hence measure the information flow that firms receive about aggregate technology from observing *solely the private signals* as $\mathcal{H}(\widehat{a}_t;\mathcal{I}_{j,t}\diagup R^t) \equiv H(\widehat{a}_t) - H(\widehat{a}_t|\mathcal{I}_{j,t}\diagup R^t)$. Let us define the entropy of aggregate technology conditional on firms having observed only the history of the policy signal as $H\left(\widehat{a}_t|\widehat{R}^t\right) \equiv 0.5\log_2\left[2\pi e \cdot var\left(\widehat{a}_t|\widehat{R}^t\right)\right]$, where $var\left(\widehat{a}_t|\widehat{R}^t\right)$ denotes the variance of aggregate technology conditional on firms having observed only the history of the policy signal R^t. We measure the information flow about aggregate technology only conveyed by the policy signal $\widehat{R}_t$ as $\mathcal{H}\left(\widehat{a}_t;\widehat{R}^t\right) \equiv H(\widehat{a}_t) - H\left(\widehat{a}_t|\widehat{R}^t\right)$.

We compute the fraction of private information about the aggregate technology $\widehat{a}_t$ as the ratio of the private information flow to the information flow from all the signals in the information set $\mathcal{I}_{j,t}$; that is, $\vartheta_a \equiv \mathcal{H}(\widehat{a}_t;\mathcal{I}_{j,t}\diagup R^t)/\mathcal{H}(\widehat{a}_t;\mathcal{I}_{j,t})$. It should be noted that $\vartheta_a \in [0,1]$. If ϑ_a is close to zero, then most of the information about aggregate technology stems from the policy signal. On the contrary, if ϑ_a is close to unity, then most of the information about aggregate technology stems from the private signal $\widehat{a}_{j,t}$.[8] Analogously, we can define the fraction of private information about the demand conditions $\widehat{g}_t$ as $\vartheta_g \equiv \mathcal{H}(\widehat{g}_t;\mathcal{I}_{j,t}\diagup R^t)/\mathcal{H}(\widehat{g}_t;\mathcal{I}_{j,t})$.

Another useful statistic for assessing the macroeconomic effects of the signaling channel is the fraction of information about the non-policy exogenous state variables (i.e., the level of aggregate technology $\widehat{a}_t$ and the demand conditions $\widehat{g}_t$) conveyed by the policy signal. For

[6] This approach is extensively followed by the literature of rational inattention pioneered by Sims (2003) and followed by Maćkowiak and Wiederholt (2009 and 2010), Paciello and Wiederholt (Forthcoming), and Matejka (2011).

[7] The units of the measure $\mathcal{H}(\widehat{a}_t)$ are *bits* of information. The conditional variance can be pinned down by applying the Kalman-filter recursion, as shown in Appendix C.

[8] Note that the other private signal (i.e., $\widehat{g}_{j,t}$) does not convey any information about the level of aggregate technology because of the assumed orthogonality of structural shocks at all leads and lags.

instance, the fraction of information about the level of aggregate technology is computed as follows:

$$\Phi_a \equiv \frac{\mathcal{H}\left(\widehat{a}_t;\widehat{R}^t\right)}{\mathcal{H}\left(\widehat{a}_t;\widehat{R}^t\right)+\mathcal{H}\left(\widehat{\eta}_{r,t};\widehat{R}^t\right)+\mathcal{H}\left(\widehat{g}_t;\widehat{R}^t\right)}. \tag{14}$$

The numerator quantifies the information flow about the level of aggregate technology $\widehat{a}_t$ conveyed by the public signal. The denominator quantifies the information flow about the three exogenous state variables (i.e., $\widehat{a}_t$, $\widehat{\eta}_{r,t}$, and $\widehat{g}_t$) conveyed by the policy signal. This ratio Φ_a assumes values between zero and one.

In summary, the ratio ϑ_a measures the accuracy of the private signal $\widehat{a}_{j,t}$ about the level of aggregate technology $\widehat{a}_t$ *relative to that of the policy signal.* The ratio Φ_a evaluates the accuracy of the public signal *relative to the three exogenous state variables* (i.e., $\widehat{a}_t$, $\widehat{\eta}_{r,t}$, and $\widehat{g}_t$). Analogously, we can define the fraction of information about the deviation from the monetary rule conveyed by the policy signal Φ_m and the fraction of information about the demand conditions conveyed by the policy signal Φ_g. Note that $\Phi_a + \Phi_m + \Phi_g = 1$.

Let us consider the following example. When the ratio ϑ_a is close to one, firms mostly rely on private information to learn about the aggregate technology. When ϑ_a is close to zero and Φ_a is sufficiently larger than zero, the quality of private information is rather poor relative to that of public information, and hence, firms mostly rely on the policy signal to learn about the level of aggregate technology. In this case, as we shall show in the next section, firms tend to interpret any changes in the policy rate as a response of the central bank to aggregate technology shocks, and so the inflationary effects of the signaling channel will be important. By the same token, the inflationary effects of the signaling channel depend on the relative precision of private information regarding the demand conditions relative to public information, as captured by the ratios ϑ_g and Φ_g.

3.2 A Useful Decomposition

In this section, we analyze the signaling effects of monetary policy on *the propagation of structural shocks* (i.e., technology shocks, monetary policy shocks, and demand shocks) in the dispersed information model. It is illustrative to use equation (12) to decompose the effects of the average HOE about the three exogenous state variables (i.e., $\widehat{a}_t$, $\widehat{\eta}_{r,t}$, and $\widehat{g}_t$) on inflation; doing so leads us to the following:

$$\frac{\partial \pi_{t+h}}{\partial \varepsilon_{i,t}} = \mathbf{v}_a \cdot \frac{\partial X^a_{t+h}}{\partial \varepsilon_{i,t}} + \mathbf{v}_m \cdot \frac{\partial X^m_{t+h}}{\partial \varepsilon_{i,t}} + \mathbf{v}_g \cdot \frac{\partial X^g_{t+h}}{\partial \varepsilon_{i,t}}, \tag{15}$$

where $i \in \{a, r, g\}$ is the subscript that determines the shock of interest (i.e., technology, monetary, or demand shock) and the row vectors $\mathbf{v}_a$, $\mathbf{v}_m$, and $\mathbf{v}_g$ are subvectors of the second row of the matrix $\mathbf{v}_0$ in equation (12). Additionally, X^a_{t+h} is the column vector of (average) h-step-ahead HOE about the level of aggregate technology $\widehat{a}_t$; that is, $X^a_{t+h} \equiv \left[\widehat{a}^{(s)}_{t+h|t} : 0 \leq s \leq k\right]$.[9] Note that the vector X^a_{t+h} also includes the average zero-order expectations $a^{(0)}_{t+h|t}$, which are conventionally equal to the actual aggregate technology; that is, $\widehat{a}^{(0)}_{t+h|t} = \widehat{a}_{t+h}$, for any t and h. Analogously, $X^m_{t+h} \equiv \left[\widehat{\eta}^{(s)}_{r,t+h|t} : 0 \leq s \leq k\right]$ and $X^g_{t+h} \equiv \left[\widehat{g}^{(s)}_{t+h|t} : 0 \leq s \leq k\right]$ are the column vectors of (average) h-step-ahead HOE about the deviation from the monetary rule and the demand conditions, respectively.

It is important to emphasize that the signaling channel would be *inactive* if firms never observed the policy rate in the dispersed information model; that is, $R^t \notin \mathcal{I}_{j,t}$. Note that if the signaling channel were inactive, the impulse response vectors $\frac{\partial X^a_{t+h}}{\partial \varepsilon_{r,t}}$ and $\frac{\partial X^g_{t+h}}{\partial \varepsilon_{r,t}}$ would be equal to zero vectors at any time t and horizon h. This means that the average higher-order expectations about aggregate technology and demand conditions would not respond to monetary shocks because the policy rate is not observed and the private signals (i.e., $\widehat{a}_{j,t}$ and $\widehat{g}_{j,t}$) are orthogonal to monetary shocks. Furthermore, average HOE about central bank's deviations from the monetary rule X^m_{t+h} would not respond to non-policy shocks (i.e., $\varepsilon_{a,t}$ and $\varepsilon_{g,t}$) at any time t and horizon h if the signaling channel were inactive. In summary, when the signaling channel is inactive, the following zero restrictions hold: $\frac{\partial X^j_{t+h}}{\partial \varepsilon_{i,t}} = \mathbf{0}_{(k+1)\times 1}$ for all $i, j \in \{a, r, g\}^2$ and $i \neq j$. However, if the signaling channel is *active*, these zero restrictions do not necessarily apply because the policy signal is endogenous and conveys information about *all the aggregate shocks* that hit the economy. In other words, the policy signal is a *source of confusion* about the nature of shocks that have hit the economy. For instance, an increase in the policy rate can be interpreted by firms as the central bank's response to a contractionary monetary shock, an adverse technology shock, or a positive demand shock.

The decomposition in equation (15) provides us with a powerful tool to interpret the signaling effects of monetary policy on the propagation of structural shocks. To shed light on the signaling effects of monetary policy on inflation expectations, we work out a decomposition for the average inflation expectations of the form (15), using the fact that the law of motion for the average first-order expectations about the endogenous variables $\mathbf{s}_t$ reads as $\mathbf{s}^{(1)}_{t+h|t} = \mathbf{v}_0 \mathbf{M}^h X^{(1:k+1)}_{t|t}$.[10]

3.3 Some Numerical Examples

Endowed with these tools, we conduct a simple numerical experiment, in which we set the Calvo parameter $\theta = 0.65$, the autoregressive parameters $\rho_a = 0.85$ and $\rho_r = 0.65$, and the

[9] Conventionally, the average zero-order expectation about a random variable is equal to the variable itself; that is, $\widehat{a}^{(0)}_{t+h|t} = \widehat{a}_{t+h}$, for any h.

[10] For a formal proof of this result, see Appendix D, Proposition 2.

variances $\sigma_a = 0.7$ and $\sigma_r = 0.5$. The central bank's response to inflation is determined by $\phi_\pi = 1.5$. For the sake of simplicity, we shut down the demand shocks (i.e., $\sigma_g = 0$) and the policy response to output gap (i.e., $\phi_y = 0$). Let us first consider the propagation of a contractionary monetary policy shock when the signal noise $\widetilde{\sigma}_a$ is set to be equal to the standard deviation of the aggregate technology shock, σ_a (i.e., the signal-to-noise ratio $\sigma_a/\widetilde{\sigma}_a = 1$). In this numerical case, firms receive 92 percent of their overall information about the aggregate technology from the private signal (i.e., $\vartheta_a = 0.92$), suggesting that firms mostly rely on private information to learn about the non-policy shock. Firms will use the policy signal to mainly learn about monetary policy shocks. The bottom graphs of Figure 2 report the response of the average higher-order expectations about aggregate technology $\partial X^a_{t+h}/\partial \varepsilon_{r,t}$ (bottom left graph) and about the deviation from the monetary rule $\partial X^m_{t+h}/\partial \varepsilon_{r,t}$ (bottom right graph) from the first order up to the third order for h periods after the monetary shock.[11] Note that the average HOE about the deviation from the monetary rule are very close to the true value $\widehat{\eta}_{r,t+h}$, which is denoted by the red circles, showing that firms can rather easily figure out that the observed rise in the policy rate is due to a contractionary monetary policy shock.

The top left graph of Figure 2 shows that the response of inflation to the contractionary monetary shock is negative. The vertical bars in the top left graph are related to the decomposition (15) and isolate the effects of the change in the average HOE about aggregate technology on inflation h periods after the shock, $\mathbf{v}'_a \partial X^a_{t+h}/\partial \varepsilon_{r,t}$, from those about the deviation from the monetary rule, $\mathbf{v}'_m \cdot \partial X^m_{t+h}/\partial \varepsilon_{r,t}$. If the signaling channel were inactive (i.e., firms never observe the policy rate), the gray bars would have zero length. It can be observed that the inflationary effects of HOE about aggregate technology (i.e., the gray vertical bars) are positive. This happens because the observed rise in the policy rate misleads firms, inducing them to believe $-$ to some extent $-$ that the central bank has raised the policy rate in response to a negative technology shock, as confirmed by the response of the average HOE about aggregate technology in the bottom left plot of Figure 2. However, note that these effects are very small quantitatively because firms acquire most of the information about aggregate technology from their private signals ($\vartheta_a = 0.92$).

Let us now turn our attention to the case in which (i) firms have imprecise private information (i.e., the signal-to-noise ratio $\sigma_a/\widetilde{\sigma}_a = 0.05$) and (ii) the central bank's estimates about inflation and output gap are more accurate than those in the previous example (i.e., $\sigma_r = 0.1$). The following two features emerge. First, now firms acquire less private information about aggregate technology compared with the previous example, as $\vartheta_a = 0.02 < 0.92$. Such a tiny ratio ϑ_a implies that firms have to almost entirely rely on the policy signal to learn about aggregate technology. Second, most of the information that firms receive from the policy signal is now about aggregate technology, since $\Phi_a = 0.84$. That the signaling channel conveys a lot

[11]Note that because we shut down the demand shock, $\partial X^g_{t+h}/\partial \varepsilon_{r,t} = \mathbf{0}_{(k+1)\times 1}$.

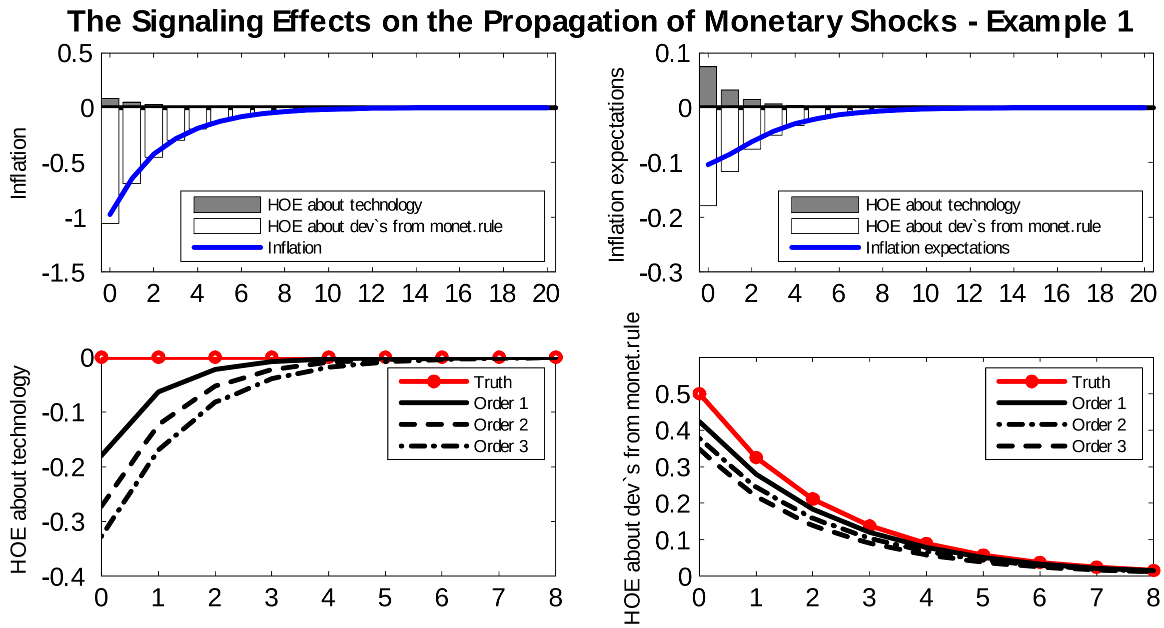

Figure 2: Impulse response functions to a one-standard deviation contractionary monetary shock: the case of $\sigma_a/\widetilde{\sigma}_a = 1$ and $\sigma_r = 0.5$. HOE means higher-order expectations. *Top plots*: Response of inflation (left) and four-quarter ahead inflation expectations (right). The gray (white) bars denote the effects of average higher-order expectations about the deviation from the monetary rule $\widehat{\eta}_{r,t}$ (about the level of aggregate technology $\widehat{a}_t$) on inflation or inflation expectations. *Bottom plots*: Response of average expectations about level of aggregate technology $\widehat{a}_t$ (left) and about the central bank's deviation from the monetary rule $\widehat{\eta}_{r,t}$ (right). The red line with circles marks the true value of the level of aggregate technology $\widehat{a}_t$ (left) or the deviation from the monetary rule $\widehat{\eta}_{r,t}$ (right). The horizontal axis in all graphs measures the number of quarters after the shock. The vertical axis in the top graphs represents units of percentage points of annualized rates.

of information about the non-policy shocks is confirmed by the bottom graphs of Figure 3. The response of the average HOE about the deviation from the monetary rule are quite far from the truth (i.e., the red line with circles), suggesting that these average HOE respond weakly to the observed rise in the policy rate due to the monetary shock. Conversely, the average HOE about aggregate technology respond more strongly to the monetary shocks, suggesting that firms mainly interpret the monetary tightening as the response of the central bank to an inflationary technology shock.

This is exactly the situation in which the inflationary effects of the signaling channel are *very strong*. The top graphs of Figure 3 show the effects of the signaling channel on inflation and inflation expectations as captured by the gray vertical bars. Signaling effects are so strong that inflation and inflation expectations actually rise in response to a contractionary monetary shock. Positive responses of the inflation rate to monetary shocks have been empirically documented in the VAR literature (Sims 1992) and dubbed by Eichenbaum (1992) as a *price puzzle*. Having inflation respond positively to monetary shocks is not puzzling in this model with the signaling effects of monetary policy, as rises (cuts) in the policy rate signal to price setters the occurrence

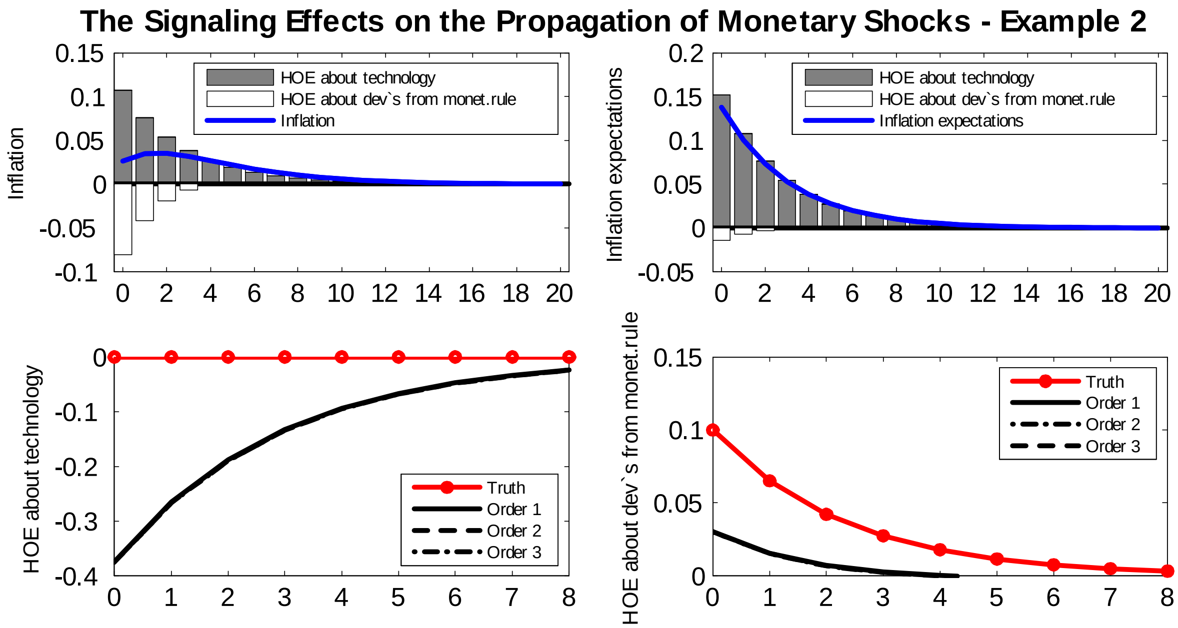

Figure 3: Impulse response functions to a one-standard deviation contractionary monetary shock: the case of $\sigma_a/\widetilde{\sigma}_a = 0.05$ and $\sigma_r = 0.1$. HOE means higher-order expectations.

of inflationary (deflationary) non-policy shocks.

We now focus on the case in which firms' private information is still quite imprecise (i.e., $\sigma_a/\widetilde{\sigma}_a = 0.05$), but the variance of the monetary shock is $\sigma_r = 0.5$ and hence bigger than that of the previous example. As in the previous case, firms do not rely on the private signal to learn about aggregate technology because this signal provides less information than the policy signal ($\vartheta_a = 0.17$). Nevertheless, unlike the previous case but similar to the first case, the policy signal is relatively less informative about aggregate technology than about the deviation from the monetary rule ($\Phi_a = 0.07$), since the central bank is less precise in estimating current inflation and output gap. Figure 4 shows that in this numerical example, the signaling channel has a quite *weak* inflationary effect and the price puzzle disappears, similar to the first numerical example depicted in Figure 2.

To sum up, we note that the inflationary consequences of the signaling channel are strong when the following two conditions hold jointly: (i) firms' private information is noisier than public information, which is conveyed by the policy rate, and (ii) the policy rate is more informative about non-policy shocks than about monetary policy shocks.

4 Empirical Analysis

This section contains the quantitative analysis of the model. Section 4.1 presents the data set and the state-space model for the econometrician. In Section 4.2, we discuss the prior

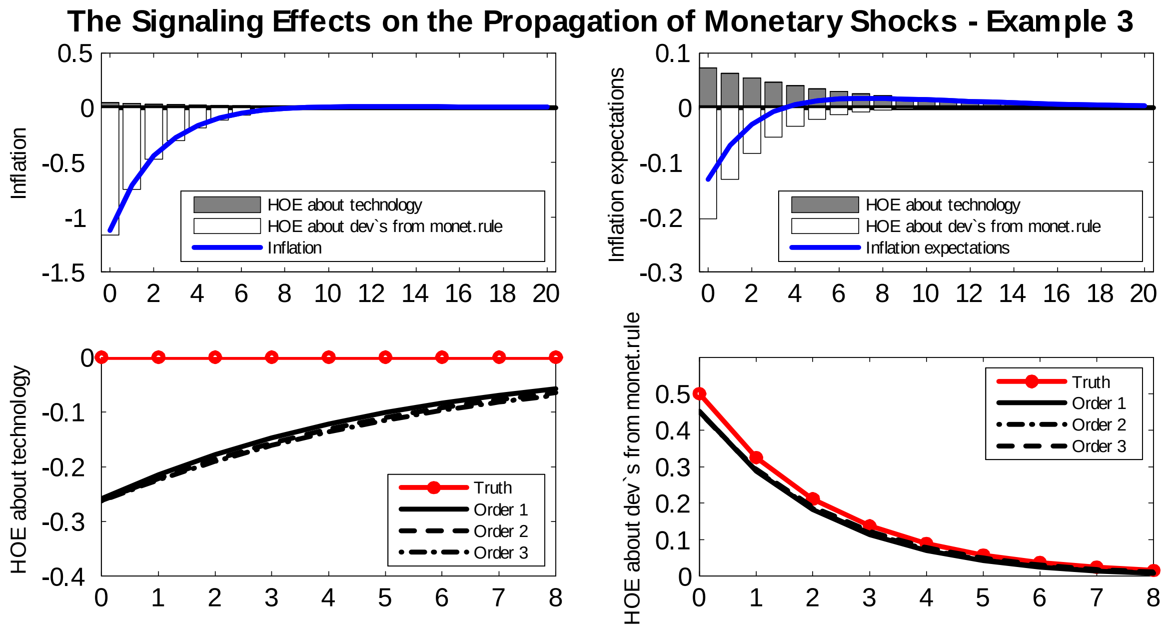

Figure 4: Impulse response functions to a one-standard deviation contractionary monetary shock: the case of $\sigma_a/\widetilde{\sigma}_a = 0.05$ and $\sigma_r = 0.5$. HOE means higher-order expectations.

and posterior distribution for the model parameters. In Section 4.3, we study the propagation of unanticipated monetary disturbances with particular emphasis on the functioning of the signaling channel. In Section 4.4, we deal with how the signaling channel affects the propagation of non-policy disturbances, such as the technology shock and the demand shock. In Section 4.5, we evaluate the signaling effects of monetary policy on the observed dynamics of GDP, the rate of inflation, and inflation expectations. In Section 4.6, we formally assess the empirical performance of the DIM relative to that of the PIM introduced in Section 2.7.

4.1 The State-Space Model for the Econometrician

The data set includes five observable variables: the U.S. real GDP growth rate, U.S. inflation rate (from the GDP deflator), the federal funds rate, one-quarter-ahead inflation expectations, and four-quarters-ahead inflation expectations. The data are quarterly and range from 1970:Q3 through 2007:Q4. Data on inflation expectations are obtained from the *Survey of Professional Forecasters* (SPF). The measurement equations are:

$$\ln\left(\frac{GDP_t}{POP_t^{\geq 16}}\right) - \ln\left(\frac{GDP_{t-1}}{POP_{t-1}^{\geq 16}}\right) = \ln\gamma + \hat{y}_t - \hat{y}_{t-1},$$

$$\ln\left(\frac{PGDP_t}{PGDP_{t-1}}\right) = \ln\pi_* + \hat{\pi}_t,$$

$$FEDRATE_t = \ln R_* + \hat{R}_t,$$

$$\ln\left(\frac{PGDP3_t}{PGDP2_t}\right) = \ln\pi_* + \hat{\pi}^{(1)}_{t+1|t} + \sigma_{m_1}\varepsilon_t^{m_1},$$

$$\ln\left(\frac{PGDP6_t}{PGDP5_t}\right) = \ln\pi_* + \hat{\pi}^{(1)}_{t+4|t} + \sigma_{m_2}\varepsilon_t^{m_2},$$

where GDP_t is the real gross domestic product computed by the U.S. Bureau of Economic Analysis (BEA) (haver analytics' mnemonic: *GDPC96*); $POP_t^{\geq 16}$ is the civilian non-institutional population aged 16 years old and over as computed by the U.S. Bureau of Labor Statistics (BLS) (haver analytics' mnemonic: *LNS10000000*); $PGDP_t$ is the GDP deflator computed by the BEA (haver analytics' mnemonic: *GDPDEF*); $FEDRATE$ is the average of daily figures of the effective federal funds rate (haver mnemonic: *FEDFUNDS*) reported by the Federal Reserve Economic Data (FRED) database managed by the Federal Reserve Bank of St. Louis; and $PGDP2_t$, $PGDP3_t$, $PGDP5_t$, and $PGDP6_t$ are the SPF's mnemonics for the median forecasts about the current, one-quarter-ahead, three-quarters-ahead, and four-quarters-ahead GDP price indexes, respectively. We relate these statistics with the first moment of the distribution of firms' expectations implied by the model. To avoid stochastic singularity, we introduce two independently and identically distributed (i.i.d.) Gaussian measurement errors $\varepsilon_t^{m_1}$ and $\varepsilon_t^{m_2}$.

4.2 Bayesian Estimation

At the deterministic steady state, the discount factor β depends on the linear trend of real output γ and the steady-state real interest rate R_*/π_*. Hence, we fix the value for this parameter so that the steady-state nominal interest rate R_*, the inflation rate π_*, and the growth rate γ match their respective sample means. The prior and posterior statistics for the model parameters are reported in Table 1. The prior distribution for the Calvo parameter θ is chosen so as to put probability mass to values that are consistent with studies on price setting at micro levels (Bils and Klenow 2004; Klenow and Kryvtsov 2008; Nakamura and Steinsson 2008; Klenow and Malin 2010). The priors for the autoregressive parameters ρ_a, ρ_r, and ρ_g are broad enough to accommodate a wide range of persistence degrees for the three exogenous processes. The prior for the monetary policy parameters is set to be fairly tight because the inflation parameter ϕ_π seems to be only weakly identified in the perfect information model.[12] The volatility of the monetary policy shock (σ_r) and that of the demand shock (σ_g) are informally taken according to the rule proposed by Del Negro and Schorfheide (2008) that the overall variance of GDP

[12]The posterior statistics for the DIM parameters reported in Table 1 and all the results in the subsequent sections would be substantially unchanged if one estimates the models using a broader prior for the policy parameters ϕ_π and ϕ_y.

	DIM - Posterior			PIM - Posterior			Prior		
Name	Mean	5%	95%	Mean	5%	95%	Type	Mean	Std.
θ	0.2613	0.2450	0.2801	0.5796	0.5468	0.6114	$\mathcal{B}$	0.50	0.30
ϕ_π	1.0629	1.0451	1.0820	1.3234	1.2324	1.4200	$\mathcal{G}$	1.50	0.10
ϕ_y	0.3416	0.3212	0.3607	0.4356	0.1918	0.6560	$\mathcal{G}$	0.25	0.10
ρ_r	0.8613	0.8520	0.8713	0.4690	0.4163	0.5224	$\mathcal{B}$	0.50	0.20
ρ_a	0.9932	0.9911	0.9963	0.9751	0.9667	0.9832	$\mathcal{B}$	0.50	0.20
ρ_g	0.8505	0.8408	0.8597	0.8192	0.7949	0.8435	$\mathcal{B}$	0.50	0.20
$100\sigma_a$	0.7569	0.6440	0.8516	0.9961	0.8973	1.0957	$\mathcal{IG}$	0.80	1.50
$100\widetilde{\sigma}_a$	1.6048	1.3517	1.8332	–	–	–	$\mathcal{U}$	50.00	28.87
$100\sigma_g$	2.7843	2.6976	2.8610	0.8169	0.6908	0.9421	$\mathcal{IG}$	0.80	1.50
$100\widetilde{\sigma}_g$	34.277	30.789	38.068	–	–	–	$\mathcal{U}$	50.00	28.87
$100\sigma_r$	0.6372	0.6267	0.6429	0.6832	0.5717	0.7947	$\mathcal{IG}$	0.80	1.50
$100\sigma_{m_1}$	0.1291	0.1145	0.1452	0.1753	0.1585	0.1923	$\mathcal{IG}$	0.10	0.08
$100\sigma_{m_2}$	0.1222	0.1087	0.1381	0.1727	0.1565	0.1892	$\mathcal{IG}$	0.10	0.08
$100\ln\gamma$	0.4889	0.3786	0.5927	0.3302	0.3030	0.3556	$\mathcal{N}$	0.62	0.10
$100\ln\pi_*$	0.8327	0.7181	0.9514	0.7374	0.6124	0.8655	$\mathcal{N}$	0.65	0.10

Table 1: Prior and Posterior Statistics for the parameters of the dispersed information model (DIM) and the perfect information model (PIM)

growth rate, inflation, and policy rate is roughly close to that observed in the presample, which ranges from 1960:Q1 through 1970:Q2. The variance of the idiosyncratic technology ($\widetilde{\sigma}_a$) and that of the private signal concerning the demand conditions ($\widetilde{\sigma}_g$) are crucial for the macroeconomic implications of the signaling channel, as shown in Section 3.3. To avoid conjecturing *a priori* that the signaling channel strongly influences firms' beliefs about non-policy shocks, we set a loose uniform prior over these parameters. It can be shown that the signaling channel implied by the prior has quantitatively weak inflationary effects.[13] The implied 95 percent prior credible set for the fraction of public information about the three exogenous state variables (i.e., Φ_a, Φ_m, and Φ_g) spans the spectrum of admissible values $(0,1)$. Finally, the prior mean for the measurement errors (i.e., σ_{m1}, σ_{m2}) is set so as to match the variance of inflation expectations reported in the *Livingston Survey*.

We combine a prior distribution for the parameter set of the two models (i.e., the DIM and the PIM) with their likelihood function and conduct Bayesian inference. As explained in Fernández-Villaverde and Rubio-Ramírez (2004) and An and Schorfheide (2007), a closed-form expression for the posterior distribution is not available, but we can approximate the moments of the posterior distributions via the Metropolis-Hastings algorithm. We obtain 250,000 posterior draws for the dispersed information model and 1,000,000 draws for the perfect information model. The posterior statistics for the parameters of the perfect information model are fairly standard. As far as the DIM is concerned, the posterior mean for the Calvo parameter θ

[13] See Appendix E.

implies very flexible price contracts, whose implied duration is roughly four months. This finding suggests that the likelihood favors sources of persistence that are unrelated to sticky prices. Similar to Melosi (Forthcoming), the DIM relies on the sluggish adjustment of the higher-order expectations to fit the high serial correlation of the macro data.

The posterior mean for the inflation coefficient of the Taylor rule (ϕ_π) is substantially smaller than its prior mean. The posterior mean for the variance of the monetary shock (σ_r) is slightly smaller than what is conjectured in the prior. As discussed in Section 3, a small variance of monetary shocks makes the policy signal $\widehat{R}_t$ more informative about non-policy shocks (i.e., demand and aggregate technology shocks) and, hence, tends to strengthen the inflationary effects of the signaling channel.

The posterior mean for the variance of the firm-specific technology shock $\widetilde{\sigma}_a$ implies that the posterior mean of the signal-to-noise ratio $\sigma_a/\widetilde{\sigma}_a$ is 0.47. The posterior mean for the signal-to-noise ratio $\sigma_g/\widetilde{\sigma}_g$ is extremely small, suggesting that firms' private information is less accurate about demand shocks than about aggregate technology shocks. These estimates suggest that, *ceteris paribus*, firms will rely more on the policy signal to learn about demand shocks than to learn about aggregate technology shocks. This intuition is confirmed by looking at the statistics introduced in Section 3.1. The posterior distribution implies that the policy signal is mainly informative about aggregate technology, as roughly ($\Phi_a =$)80 percent of the public information flow is about aggregate technology. Nevertheless, firms almost entirely learn about the aggregate technology from their private signal (i.e., the level of their idiosyncratic technology): the 95-percent posterior credible set for the fraction of private information about the aggregate technology ϑ_a ranges from 0.9972 to 0.9983. These numbers suggest that the accuracy of private information about the level of aggregate technology is much higher than that of public information. Conversely, firms have to rely on the policy signal $\widehat{R}_t$ to learn about the demand conditions $\widehat{g}_t$, since the private signal conveys only $\vartheta_g = 0.10$ of the overall information firms gathered about this exogenous state variable. It is important to notice that the remaining ($1 - \Phi_a =$) 20 percent of information conveyed by the policy signal is equally split between information about the deviation from the monetary rule $\widehat{\eta}_{r,t}$ and demand conditions $\widehat{g}_t$, as the posterior means for Φ_m and Φ_g are roughly equal to 10 percent. As we shall see, this feature will make it hard for firms to tell whether the observed changes in the policy rate are due to monetary shocks or demand shocks.

To sum up, we find that firms receive most of their information about aggregate technology from their private signal but have to rely on the policy signal to learn about monetary shocks and demand shocks. The policy rate is found to be equally informative about these two types of shocks. In such an environment, the inflationary effects of the signaling channel are expected to be important, since firms will tend to interpret changes in the policy rate as the central bank's responses to demand shocks. We will analyze the implication of this feature in the next

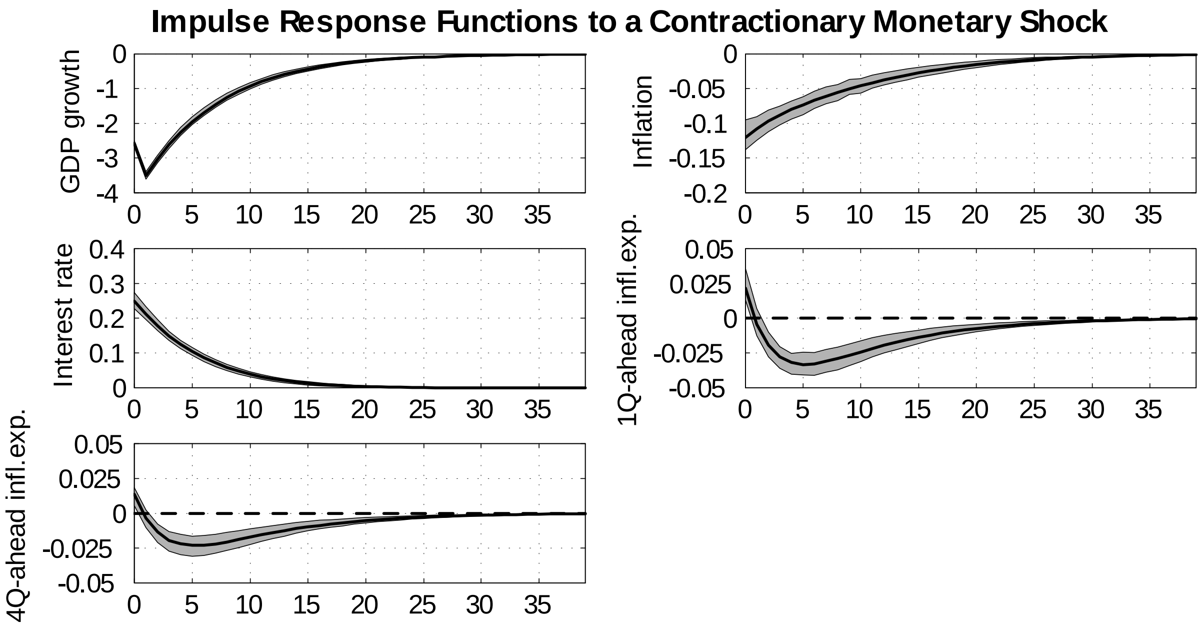

Figure 5: Impulse response function of the observable variables to a monetary shock that raises the interest rate by 25 bps. The solid line denotes posterior means computed for every 500 posterior draws. The horizontal axis in all graphs measures the number of quarters after the shock. The vertical axis in all graphs reports units of percentage points of annualized rates.

two sections.

4.3 Propagation of Monetary Shocks

Figure 5 shows the impulse response functions (and their 95 percent posterior credible sets in gray) of the real GDP growth rate, the inflation rate, the federal funds rate, one-quarter-ahead inflation expectations, and four-quarters-ahead inflation expectations to a monetary shock that raises the interest rate by 25 basis points. Two features of these impulse response functions have to be emphasized. First, inflation expectations respond positively to a monetary policy shock. Second, inflation and inflation expectations seem to react fairly sluggishly, even though the estimated average duration of the price contract is only four months.

To shed light on the origins of such a high price rigidity, we plot the vertical bars in the top left graphs of Figure 6 that capture the decomposition (15) and measure the inflationary effects of the change in the average higher-order expectations (HOE) about aggregate technology $\mathbf{v}_a' \partial X_{t+h}^a / \partial \varepsilon_{r,t}$ (the gray bars), the deviation from the monetary rule $\mathbf{v}_m' \cdot \partial X_{t+h}^m / \partial \varepsilon_{r,t}$ (the white bars), and demand conditions $\mathbf{v}_g' \cdot \partial X_{t+h}^g / \partial \varepsilon_{r,t}$ (the black bars) for h periods after the monetary shock. The sum of the three vertical bars in the top graphs delivers the response of inflation and inflation expectations (i.e., the solid lines in the upper graphs). We observe a large effect of the average HOE about the demand conditions (i.e., the black bars), which can be interpreted as a

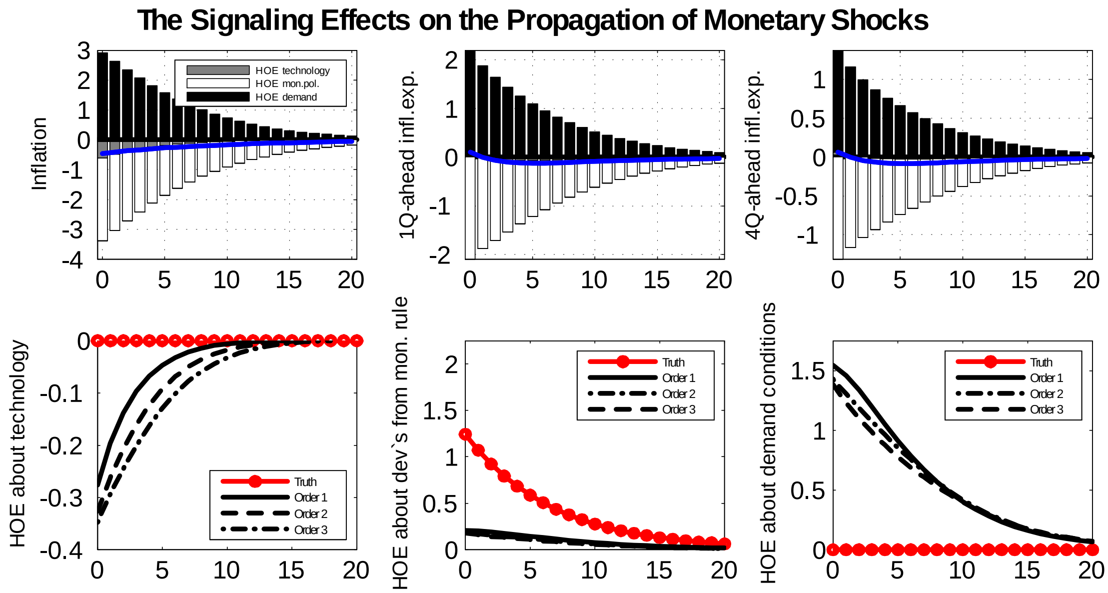

Figure 6: Impulse response functions of inflation, inflation expectations, and average higher-order expectations (HOE) to a monetary shock that raises the interest rate by 25 bps. Parameter values are set equal to the posterior mean. *Top graphs:* The solid blue line is the response of inflation (left graph), one-quarter-ahead inflation expectations (middle graph), and four-quarters-ahead inflation expectations (right graph) and its decomposition to the effects of average HOE about the level of aggregate technology $\widehat{a}_t$, the deviation from the monetary rule $\widehat{\eta}_{r,t}$, and the demand conditions $\widehat{g}_t$ (i.e., the vertical bars). *Bottom graphs:* Response of the true level of aggregate technology (left graph), the deviation from the monetary rule (middle graph), and the demand conditions (right graph) and the associated average HOE up to the third order. The vertical axis in the top graphs reports units of percentage points of annualized rates. The vertical axis in the bottom graphs reports the percent deviations from the value of the corresponding variable at the deterministic steady-state equilibrium.

situation in which price setters believe that the federal funds rate has been raised in response to a positive demand shock (see the lower right graph). Such an interpretation gives rise to inflationary pressures (captured by the black bars), which dampen the deflationary consequences (mainly captured by the white bars) associated with the contractionary monetary shock.

The three bottom graphs of Figure 6 report the response of the average HOE about the three exogenous state variables. The average HOE about aggregate technology fall right after the shock and then go back to zero rather quickly. Average HOE about the demand conditions rise substantially and remain misaligned with the truth for quite a long time (see the lower right graph). The average HOE about the deviation from the monetary rule are quite far from the truth and remain so for a long period of time (see the lower middle graph). These responses of the average HOE about the three exogenous state variables confirm that firms mostly interpret the rise in the interest rate as the central bank's response to an inflationary demand shock. This triggers persistently high black bars in the top graphs, delivering a fairly high degree of price rigidity.

To understand why firms interpret the rise in the policy rate as the central bank's response

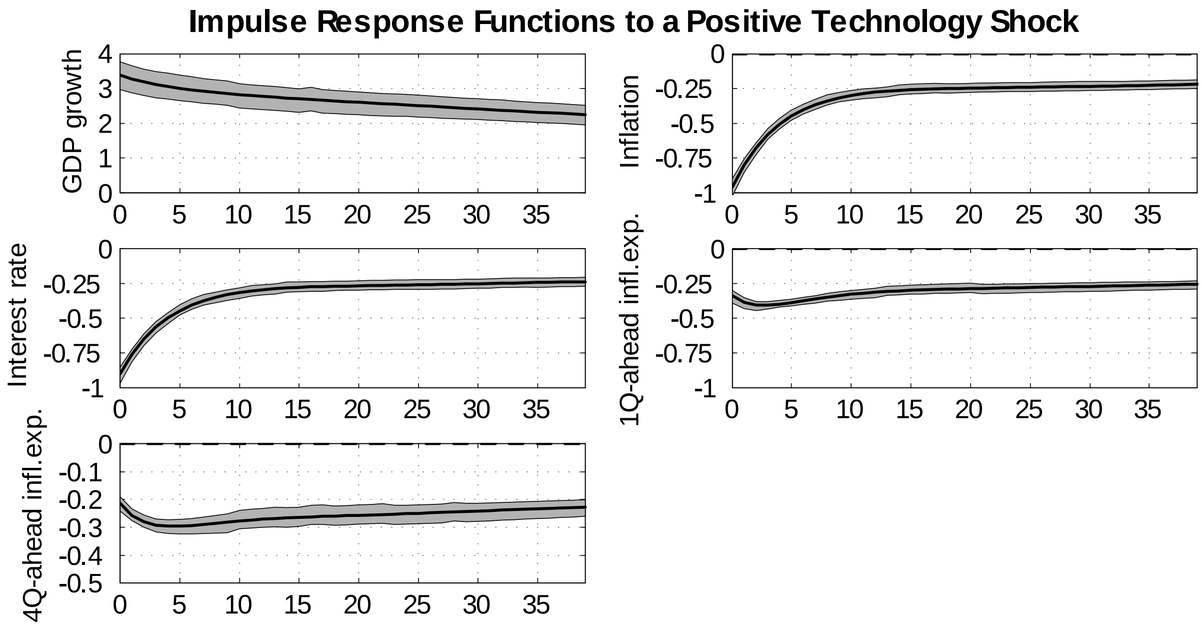

Figure 7: Impulse response function of the observable variables to a one-standard-deviation positive technology shock. The solid line denotes posterior means computed for every 500 posterior draws. The horizontal axis in all graphs measures the number of quarters after the shock. The vertical axis in all graphs reports units of percentage points of annualized rates.

to a demand shock, recall from Section 4.2 that firms receive poor private information about the demand conditions ($\vartheta_g = 0.10$) and hence have to rely on the policy signal. The policy signal is equally informative about the demand conditions and the deviation from the monetary rule with the ratio Φ_g and Φ_m being roughly equal.

The fact that firms interpret a change in the policy rate as a signal that the central bank is responding to a demand shock also affects the response of inflation expectations to monetary shocks. The upper middle and right graphs show that inflation expectations react weakly to monetary shocks. They initially respond positively because of very strong inflationary effects of the HOE about the demand conditions. They turn negative later as the inflationary effects of the average HOE about the demand conditions and the aggregate technology decay faster than those associated with the average HOE about the deviation from the monetary rule.

Finally, note that the effects of the HOE about aggregate technology (i.e., the gray bars in the upper left graph) are deflationary – albeit quantitatively fairly small – as the gray vertical bars in the top left graph lie in negative territory. This is a general-equilibrium result that is due to the sharp contraction of the average HOE about real marginal costs in response to the monetary shock. Since this is a general-equilibrium effect, it is hard to find one exact cause. However, the substantial price rigidity due to the presence of the signaling channel plays some role in exacerbating the negative elasticity of the average HOE about real marginal costs to

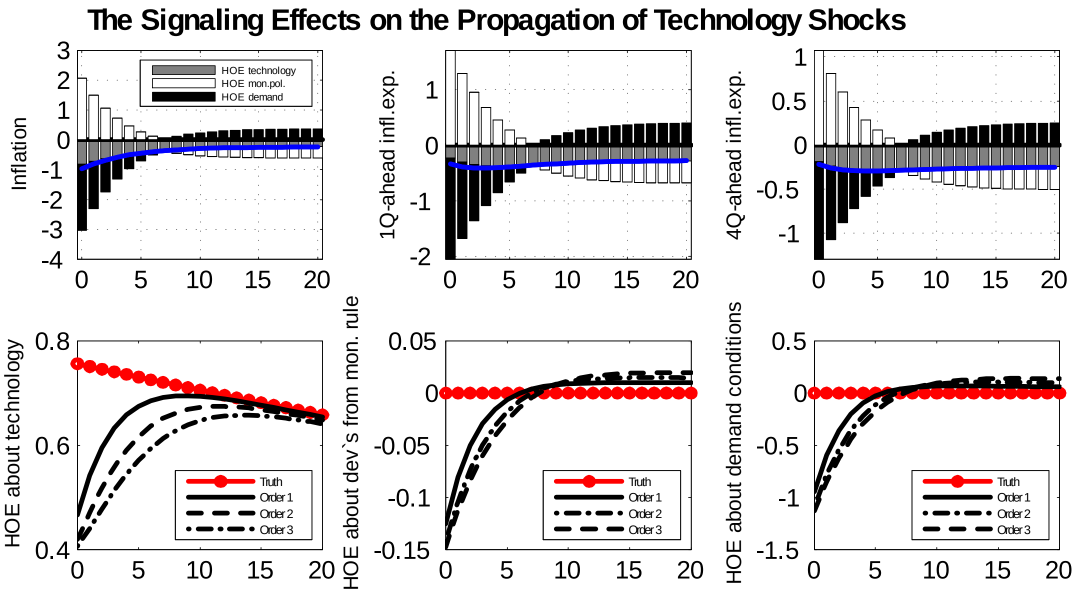

Figure 8: Impulse response functions of inflation, inflation expectations, and average higher-order expectations (HOE) to a positive aggregate technology shock. Parameter values are set equal to the posterior mean. *Top graphs:* The solid blue line is the response of inflation (left graph), one-quarter-ahead inflation expectations (middle graph), and four-quarters-ahead inflation expectations (right graph) and its decomposition to the effects of average HOE about the level of aggregate technology $\widehat{a}_t$, the deviation from the monetary rule $\widehat{\eta}_{r,t}$, and the demand conditions $\widehat{g}_t$ (i.e., the vertical bars). *Bottom graphs:* Response of the true level of aggregate technology (left graph), the deviation from the monetary rule (middle graph), and the demand conditions (right graph) and the associated average HOE up to the third order. The vertical axis in the top graphs reports units of percentage points of annualized rates. The vertical axis in the bottom graphs reports the percent deviations from the value of the corresponding variable at the deterministic steady-state equilibrium.

monetary shocks (i.e., $\frac{\partial \widehat{y}_{t|t}^{(l)}}{\partial \widehat{a}_{t|t}^{(0:k)}} \frac{\partial \widehat{a}_{t|t}^{(0:k)}}{\partial \varepsilon_{r,t}} < 0$, for any order $0 \leq l \leq k$).

In summary, the signaling channel turns out to enhance price rigidity and, consequently, the real effects of monetary disturbances. Furthermore, the signaling effects of monetary policy cause inflation expectations to respond positively to monetary shocks. The reason for these two results is that in the aftermath of a monetary shock firms tend to attach a non-negligible probability that the central bank has adjusted the policy rate to react to a demand shock.

4.4 Propagation of Non-policy Shocks

Figure 7 shows the response of the real GDP growth rate, the inflation rate, the federal funds interest rate, one-quarter-ahead inflation expectations, and four-quarters-ahead inflation expectations to a one-standard-deviation positive aggregate technology shock. Since the aggregate technology shock is almost unit root, the response of variables exhibits a high degree of persistence. In the aftermath of a positive aggregate technology shock, the GDP growth rate becomes positive, while both inflation and inflation expectations fall.

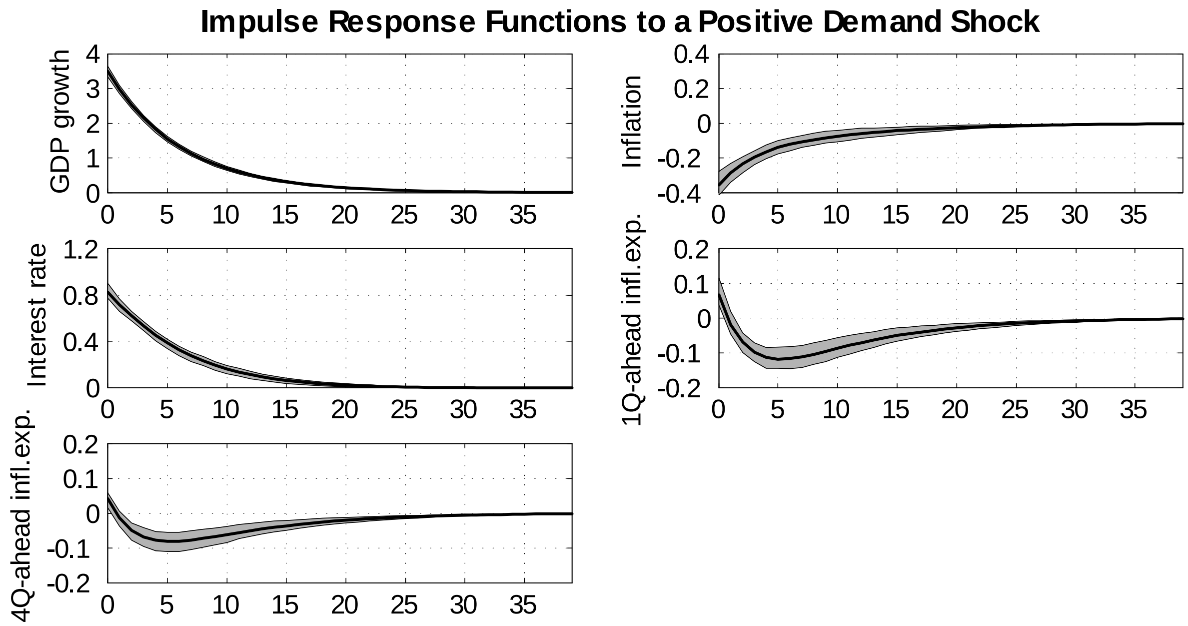

Figure 9: Impulse response function of the observable variables to a one-standard-deviation positive demand shock. The solid line denotes posterior means computed for every 500 posterior draws. The horizontal axis in all graphs measures the number of quarters after the shock. The vertical axis in all graphs reports units of percentage points of annualized rates.

Figure 8 graphs the decomposition of the response of inflation and inflation expectations to a positive aggregate technology shock (upper graphs) and the responses of average HOE about the three exogenous state variables $\widehat{a}_t$, $\widehat{\eta}_{r,t}$, and $\widehat{g}_t$ (lower graphs). A drop in the policy rate owing to a positive technology shock induces firms to believe that the central bank is responding to either an expansionary monetary shock or a negative demand shock. If firms are persuaded that an expansionary monetary shock has occurred, then the confusion generated by the signaling channel would limit the response of inflation to the technology shock. This effect is captured by the white bars lying in positive territory in the top graphs. However, if monetary easing leads firms to believe that a negative demand shock has hit the economy, the opposite effect on inflation would prevail. Firms' inflation expectations would further decrease, and inflation would go further down. This effect is captured by the black bars lying in negative territory in the top graphs.

One feature of Figure 8 deserves to be emphasized. Most importantly, the top left graph shows that the response of average expectations about the deviation from the monetary rule (i.e., the white bars) and that about the demand conditions (i.e., the black bars) contribute to the adjustment of inflation by similar amounts in every quarter after the technology shock. Thus, there are two conflicting signaling effects of monetary policy on inflation and inflation expectations in the aftermath of a technology shock. These two effects turn out to cancel each other out. This result squares with the previous finding that the signaling channel provides

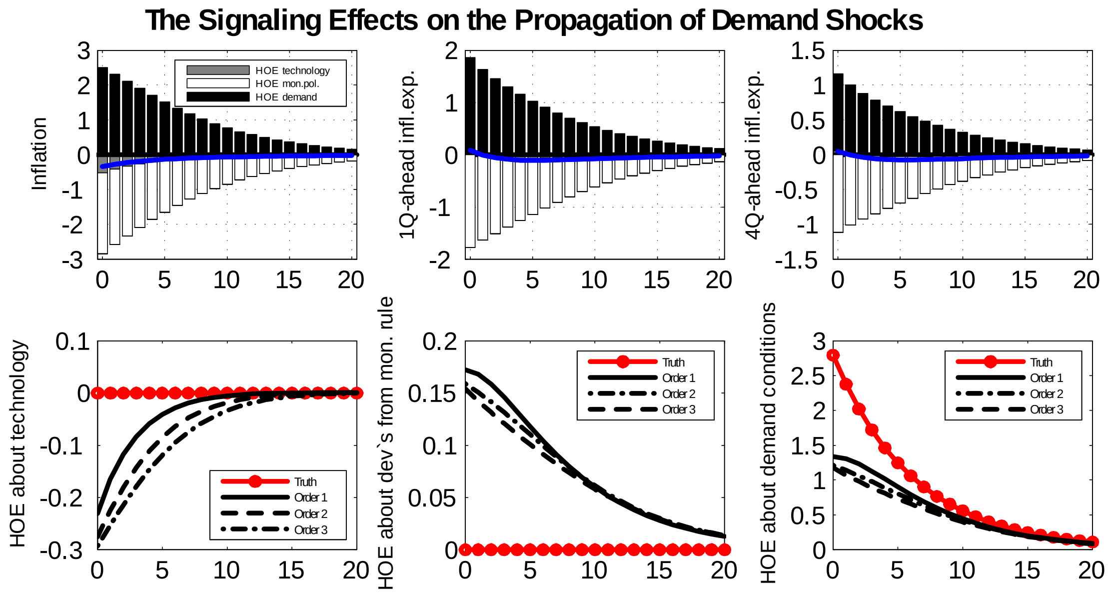

Figure 10: Impulse response functions of inflation, inflation expectations, and average higher-order expectations (HOE) to a positive demand shock. Parameter values are set equal to the posterior mean. *Top graphs:* The solid blue line is the response of inflation (left graph), one-quarter-ahead inflation expectations (middle graph), and four-quarters-ahead inflation expectations (right graph) and its decomposition to the effects of average HOE about the level of aggregate technology $\widehat{a}_t$, the deviation from the monetary rule $\widehat{\eta}_{r,t}$, and the demand conditions $\widehat{g}_t$ (i.e., the vertical bars). *Bottom graphs:* Response of the true level of aggregate technology (left graph), the deviation from the monetary rule (middle graph), and the demand conditions (right graph) and the associated average HOE up to the third order. The vertical axis in the top graphs reports units of percentage points of annualized rates. The vertical axis in the bottom graphs reports the percent deviations from the value of the corresponding variable at the deterministic steady-state equilibrium.

firms with virtually the same amount of information about the deviation from the monetary rule (Φ_m) and the demand conditions (Φ_g), as noticed in Section 4.2. Importantly, this result also implies that the signaling effects of monetary policy on price dynamics are *negligible* in the aftermath of an aggregate technology shock.[14]

The propagation of a one-standard-deviation positive demand shock is described in Figure 9. It is important to emphasize that inflation responds *negatively* to demand shocks, while the GDP growth rate responds positively. Inflation expectations initially respond positively, but their response turns negative a few quarters after the shock. Note that the central bank raises its policy rate in the aftermath of a positive demand shock.

As reported in Figure 10, the signaling channel critically affects the propagation of demand shocks, causing inflation to respond negatively to these shocks. There are two signaling effects

[14]It should also be noted that the middle and right lower graphs of Figure 8 show that six quarters after the shock, firms change their minds about the deviation from the monetary rule and the demand conditions. While firms initially expect the response of these state variables to be negative, later they expect it to become positive. This leads to a reversion of the effects of the HOE about these state variables on inflation and inflation expectations in the top graphs. Signaling effects of monetary policy are still negligible at this longer horizon.

of monetary policy in the aftermath of a demand shock. First, the signaling effects may induce firms to believe that a contractionary monetary shock has prompted the central bank to raise the policy rate (see the white bars). Second, the signaling effects may also induce firms to believe that a negative technology shock might be the reason behind the observed rise in the federal funds rate (see the gray bars). Both effects push inflation down,[15] countering the rise in inflation due to the occurrence of a positive demand shock (see the black bars lying in positive territory in the upper graphs of Figure 10). Note also that while the second effect (captured by the gray bars in the top graphs) has quantitatively a fairly small impact on inflation and inflation expectations, the first effect (captured by the white bars) appears to substantially contribute to pushing inflation and inflation expectations down. This result is explained by the fact that private information about demand shocks is fairly inaccurate, whereas price setters receive quite precise information about aggregate technology shocks. Therefore, price setters mostly rely on the information conveyed by the policy rate to learn about demand shocks.

In summary, the signaling effects of monetary policy on the rate of inflation and inflation expectations are found to be negligible in the aftermath of a technology shock. Furthermore, the signaling effects of monetary policy are deflationary (inflationary) in the aftermath of a positive (negative) demand shock because the resulting increase (cut) in the policy rate *signals* $-$to some extent$-$ to price setters that a contractionary (expansionary) monetary shock has occurred.

4.5 Bayesian Evaluation of the Signaling Channel

In this section, we use the DIM to empirically assess the signaling effects of monetary policy on GDP, inflation, and inflation expectations. To this end, we run a *Bayesian counterfactual experiment* using an algorithm that can be described as follows. In Step 1, for every posterior draw of the DIM parameters, we obtain the model's predicted series for the three structural shocks (aggregate technology shock $\varepsilon_{a,t}$, monetary shock $\varepsilon_{r,t}$, and the demand shock, $\varepsilon_{g,t}$) using the two-sided Kalman filter. In Step 2, these filtered series of shocks are used to simulate real output, the rate of inflation, one-quarter-ahead inflation expectations, and four-quarters-ahead inflation expectations from the following two models: (i) the DIM and (ii) the DIM in which monetary policy has no signaling effects. The latter model is obtained from the DIM by assuming that the history of the policy rate does not belong to firms' information set (i.e., $R^t \notin \mathcal{I}_{j,t}$, for all periods t and firms j). As discussed in Section 3.2, this assumption implies that the signaling channel is inactive, and hence, firms form their expectations by using only their private information (i.e., the history of the signals $\widehat{a}_{j,t}$ and $\widehat{g}_{j,t}$). In Step 3, we compute

[15] The deflationary effects associated with expecting an adverse technology shocks are due to general-equilibrium effects, which are discussed in Section 4.3.

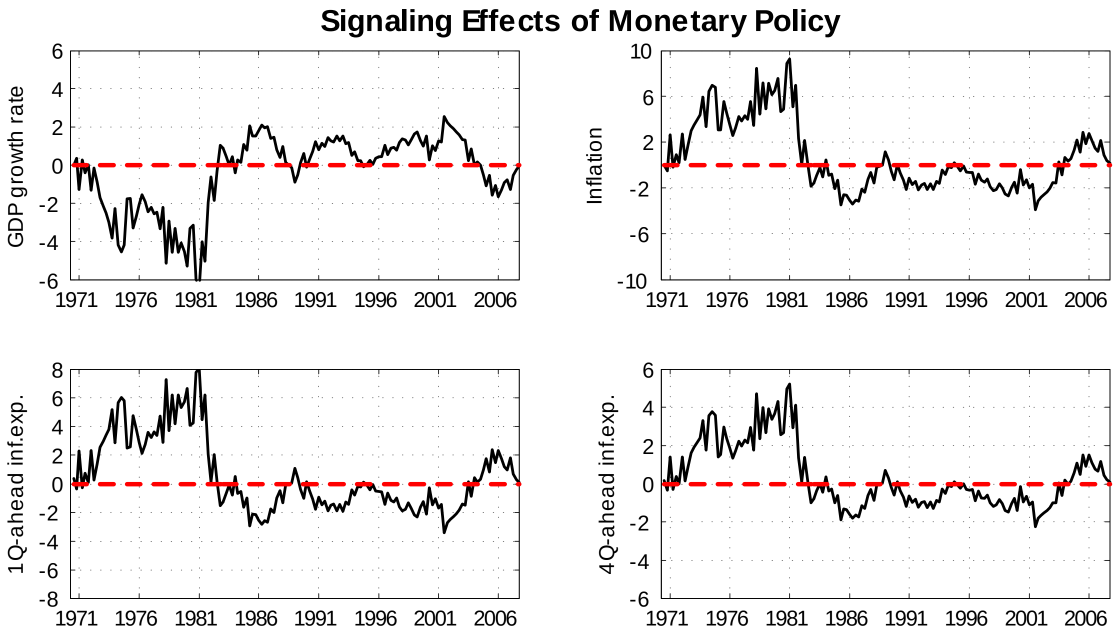

Figure 11: Signaling effects of monetary policy on gross domestic product (GDP), inflation, and inflation expectations. The solid black line is obtained by subtracting the across-posterior-draws mean of the series obtained by simulating the dispersed information model (DIM) without the signaling channel from that of the series obtained by simulating the DIM in which monetary policy has signaling effects. The two-sided filtered shocks of the estimated DIM are used to simulate the models. The vertical axis in all graphs measures units of percentage points of annualized rates.

the mean of the simulated series across posterior draws for the two models.

The solid line in Figure 11 marks the signaling effects of monetary policy on GDP (upper left graph), inflation (upper right graph), one-quarter-ahead inflation expectations (lower left graph), and four-quarters-ahead inflation expectations (lower right graph), taken by subtracting the mean of the simulated series of the DIM without signaling effects from the corresponding mean of the simulated series of the DIM with signaling effects. Let us focus first on the signaling effects of monetary policy on the rate of inflation (i.e., the upper right graph) and inflation expectations (i.e., the lower graphs). The solid line of these graphs lies in positive territory throughout the 1970s, suggesting that the signaling effects of monetary policy contributed to raising inflation and inflation expectations in that decade. In particular, the signaling effects on inflation were very strong when the Federal Reserve raised the policy rate to disinflate the economy during the 1970s and early 1980s. As shown in Figure 1, the first attempt at disinflating the U.S. economy in the 1970s was carried out by the Federal Reserve by *gradually* raising the federal funds rate from 1972 through 1974.[16] The second attempt is captured by the *gradual* increases of the policy rate from 1977 through the second quarter of the 1979, when

[16] See Hetzel (1998) and Rotemberg (2013) for an analysis of the historical reasons that led the Federal Reserve to adopt a gradualistic approach to disinflate the U.S. economy under Chairman Arthur Burns.

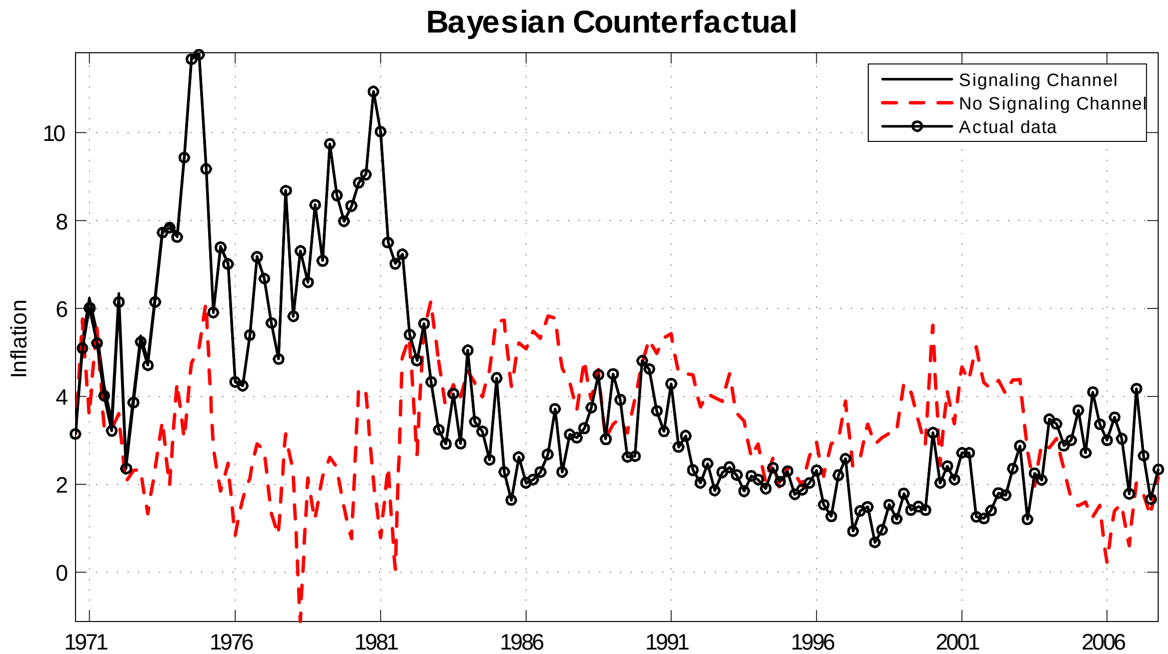

Figure 12: Comparison of the inflation rate simulated from the estimated dispersed information model (DIM) (solid black line) with those simulated from the DIM in which the signaling channel is shut down (red dashed line). Model simulation is based on two-sided filtered shocks from the estimated DIM. The black line with circles denotes actual data. The vertical axis in all graphs measures units of percentage points of annualized rates.

gradualism was abandoned coincidentally with the appointment of Paul Volcker as Federal Reserve Chairman. The third attempt was the robust monetary tightening carried out by the Federal Reserve under Chairman Paul Volcker from the third quarter of 1979 through 1982.

Figure 12 reports the simulated series of inflation from the DIM with signaling effects and the counterfactual DIM in which the signaling channel is shut down. It can be observed that if the signaling channel were inactive (i.e., if the imperfectly informed firms did not observe the policy rate), inflation would have been substantially lower throughout the 1970s. Figure 12 illustrates that the *first* and *second attempts* at disinflating the U.S. economy in the 1970s failed because of the strong signaling effects of monetary policy.

It is important to emphasize that the signaling effects associated with these two attempts at reducing inflation were inflationary because the 1970s were mainly characterized by adverse demand conditions (i.e., the filtered series for $\widehat{g}_t < 0$). Recall from Section 4.4 that the signaling effects of monetary policy are inflationary in the aftermath of a negative demand shock because the policy rate *signals* –to some extent– to price setters that an expansionary monetary shock has occurred.[17] As a result, firms started expecting higher inflation, which, as we shall show,

[17]The white vertical bars lying in negative territory in the upper left graph of Figure 10 provide a visual perception of the *deflationary* consequences of the signaling channel in the aftermath of a *positive* demand shock.

the monetary tightening was too timid to subdue.[18]

Figure 1 shows that the period from the third quarter of 1979 through 1982 (when the *third attempt* at disinflating the U.S. economy beginning in the 1970s was actually carried out) was characterized by a robust monetary tightening and by an inflation rate that adjusted very sluggishly. The upper right graph of Figure 11 shows that the signaling effects of this aggressive monetary contraction played a role in making the adjustment of inflation *sluggish* in the late 1970s and early 1980s. The signaling effects of the Volcker's monetary contraction initially contributed to raise the rate of inflation because the associated sequence of contractionary monetary shocks partially induced firms to believe that the central bank was raising the policy rate to respond to an inflationary demand shock. As shown in Section 4.3, the signaling effects associated with contractionary monetary shocks are captured by the vertical black bars in the upper left graph of Figure 6. In summary, the sequence of contractionary monetary shocks caused the signaling channel to delay the adjustment of the inflation rate in the late 1970s and early 1980s because the resulting monetary contractions ended up signaling inflationary demand shocks to price setters.

It is worthwhile to point out that the signaling effects of monetary policy are not the only possible channel through which the model could explain why both inflation and the federal funds rate rapidly rose in the 1970s and why price dynamics appear to respond sluggishly to the robust monetary tightening conducted by the Federal Reserve under Volcker. For instance, these patterns could be explained through a sequence of adverse aggregate technology shocks. Recall from Section 4.4 that these shocks would have given rise to *tiny signaling effects* of monetary policy because firms hold fairly precise private information about aggregate technology and hence do not rely on the policy signal to learn about these shocks. If the likelihood estimation had favored this story, then we would have found that the signaling effects of monetary policy played a small role in explaining the two-digit inflation rate of the 1970s and the sluggish adjustment of inflation to the disinflation policies of the late 1970s and the early 1980s.

Would a more hawkish monetary policy have subdued inflation throughout the 1970s by offsetting the signaling effects of monetary policy on the price dynamics? We find that the answer to this question is positive. We reach this conclusion by considering a counterfactual

[18]Despite the rising federal funds rate from 1972:Q1 through 1979:Q2, U.S. monetary policy was more expansionary than what its Taylor-type reaction function would predict (i.e., the smoothed estimates for $\widehat{\eta}_{r,t} < 0$). Recall from Section 4.3 that the signaling effects of monetary policy are *deflationary* in the aftermath of an *expansionary* monetary shock. See the black vertical bars in the upper left graph of Figure 6 for the case of a contractionary monetary shock. Nonetheless, judging from the upper right graph of Figure 11, we note that these deflationary pressures are dominated by the inflationary consequences of the signaling effects associated with the adverse demand shocks. Furthemore, the (filtered) level of aggregate technology $\widehat{a}_t$ was negative in the 1970s. While this fact certainly contributed to keeping the inflation rate high during that decade, it is unlikely to have triggered significant signaling effects on inflation and inflation expectations. The reason is that the signaling effects do not significantly alter the transmission of aggregate technology shocks, as shown in Section 4.4.

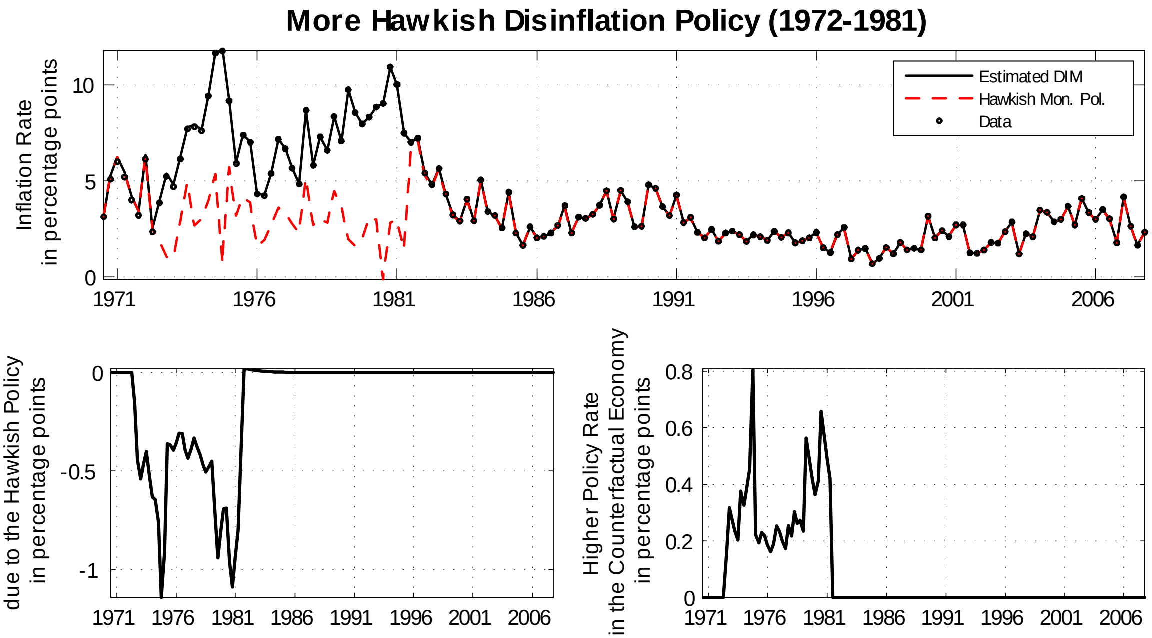

Figure 13: The upper graph reports the evolution of inflation simulated (i) from the DIM using the filtered series of shocks (Estimated DIM) and (ii) from the counterfactual DIM in which the central bank is assumed to be more hawkish (i.e., the historical sequence of $\widehat{\eta}_{r,t}$ is set to be higher than the filtered one) from 1972 through 1981 (Hawkish Mon. Pol.), as well as the evolution of the actual inflation (data). The lower left graph depicts the impact of the hawkish policy on the annualized real GDP growth in percentage points over the sample period. The lower right graph reports the difference between the filtered $\eta_{r,t}$ and its more hawkish counterpart. All simulated series are median across posterior draws for the DIM parameters.

DIM in which the Federal Reserve sets its policy rate more hawkishly from 1972 through 1981 than what the estimated DIM predicts. This is obtained by simulating the DIM using larger deviations ($\widehat{\eta}_{r,t}$) of the policy rate from the monetary rule than the ones we filtered using the estimated DIM.[19] The upper graph of Figure 13 shows that this *hawkish* monetary policy would have prevented the *Great Inflation* of the 1970s from arising. The lower right graph shows how much higher the Federal Reserve should have set the policy rate (above the level required by the rule) to offset the signaling effects of monetary policy on inflation and hence to subdue the run-up in inflation during the 1970s. Note that the lower left graph of Figure 13 suggests that this more hawkish policy would have contracted the quarter-to-quarter real GDP growth rate

[19] An alternative counterfactual experiment would have been to set a higher value for the policy parameter of inflation stabilization ϕ_π. This more systematic disinflation policy effectively limits the inflationary implications of the signaling channel. The reason is as follows. While firms still fear that a change in the policy rate may correspond to an inflationary non-policy shock, they are also confident that the central bank will be able to shelter the economy from the inflationary consequences of such a shock. It can be shown that rasing the systematic response of the policy rate to inflation also works to subdue inflation throughout the 1970s. However, this approach is more controversial than the one considered in the main text because changing the policy parameter requires assuming that agents suddenly learn and fully believe that the Federal Reserve has become more hawkish.

by up to 1.2 percentage points in annualized terms.

In summary, these results suggest that the signaling effects associated with the first two disinflation attempts of the 1970s contributed to raising inflation because the Federal Reserve tightened monetary policy *too gradually*. On the one hand, the timid increases of the federal funds rate substantially contributed to raising inflation expectations by signaling expansionary monetary shocks to price setters. On the other hand, the increases of the policy rate were *too gradual* (i.e., $\widehat{\eta}_{r,t}$ was too low) to offset the rise in inflation expectations owing to the signaling effects of the monetary tightening itself. Therefore, the signaling effects associated with the Federal Reserve's *gradualism* of the 1970s paved the way to one of the most notorious period of high inflation in the postwar U.S. history. While Volcker's tightening was more aggressive, it was also not enough to fully offset the signaling effects, which, in fact ended up delaying the adjustment of the inflation rate.[20]

From 1981 through the end of the 1980s disinflation was achieved because of the hawkish policy conducted by the Federal Reserve (i.e., $\widehat{\eta}_{r,t} > 0$) and the positive aggregate technology conditions (i.e., $\widehat{a}_t > 0$), as well as positive demand conditions (i.e., $\widehat{g}_t > 0$),[21]. These muted conditions ended the *Great Inflation* that had characterized the previous decade. Positive demand conditions also led to a *deflationary* contribution of the signaling channel, as shown in Figure 11, because the resulting response of the policy rate strengthened firms' beliefs that contractionary monetary shocks had occurred. See the white vertical bars lying in negative territory in the upper left graph of Figure 10.

Furthermore, the upper left graph of Figure 11 shows that the signaling effects of monetary policy contributed to the slowdown in economic activity observed in the 1970s. This is mostly due to a number of negative demand shocks that occurred in the 1970s. Furthermore, the signaling effects of monetary policy seem to have contributed to exacerbating the recession at the beginning of the 1980s with a contribution to growth that reached -6 percentage points in 1981:Q1 in annualized terms. Signaling effects of monetary policy have been mostly stimulative from the disinflation period through 2003. The stimulus due to these signaling effects reached up to 50 basis points in some quarters.

4.6 Empirical Fit of the DIM

In this section, we evaluate the ability of the DIM to fit the data relative to the PIM. Since the PIM is a prototypical New Keynesian DSGE model that has been extensively used for monetary policy analysis (e.g., Rotemberg and Woodford 1997; Clarida, Gali, and Gertler 2000; Lubik and Schorfheide 2004; and Coibion and Gorodnichenko 2011*a*), the goal of this exercise is to

[20] Investigating *why* the Federal Reserve did not adopt a more hawkish disinflation policy is clearly beyond the scope of this paper. A potential reason is provided by Chari, Christiano and Eichenbaum (1998).

[21] Recall from Section 4.4 that positive demand shocks are disinflationary as shown in Figure 9.

validate the DIM as a viable modeling framework with which study U.S. monetary policy.

In Bayesian econometrics, non-nested model comparison is based on computing the posterior probability of the two candidate models. The marginal likelihood is the appropriate density for updating researcher's prior probabilities over a set of models. Furthermore, Fernández-Villaverde and Rubio-Ramírez (2004) show that the marginal likelihood allows the researcher to select the best model to approximate the true probability distribution of the data-generating process under the Kullback-Leibler distance. Since the marginal likelihood penalizes for the number of model parameters (An and Schorfheide 2007), it can be applied to gauge the relative fit of models that feature different numbers of parameters, including the DIM and the PIM.

Denote the data set used for estimation and presented in Section 4.1 as Y. The marginal likelihood associated with the DIM is defined as $P\left(Y|\mathcal{M}_{DIM}\right) = \int \mathcal{L}\left(Y|\Theta_{DIM}\right) p\left(\Theta_{DIM}\right) d\Theta_{DIM}$, where $\mathcal{L}\left(Y|\Theta_{DIM}\right)$ denotes the likelihood function derived from the model and $p\left(\Theta_{DIM}\right)$ is the prior density, whose moments are reported in Table 1.

A Bayesian test of the null hypothesis that the DIM is at odds with the data can be performed by comparing the marginal likelihood associated with the DIM ($\mathcal{M}_{DIM}$) and the PIM ($\mathcal{M}_{PIM}$). Under a 0-1 loss function, the test rejects the null if the DIM has a larger posterior probability than the PIM (Schorfheide 2000). The posterior probability of model $\mathcal{M}_s$, where $s \in \{DIM, PIM\}$, is given by:

$$\pi_{T,\mathcal{M}_s} = \frac{\pi_{0,\mathcal{M}_s} \cdot P\left(Y|\mathcal{M}_s\right)}{\sum_{s\in\{DIM,PIM\}} \pi_{0,\mathcal{M}_s} \cdot P\left(Y|\mathcal{M}_s\right)}, \tag{16}$$

where $\pi_{0,\mathcal{M}_s}$ stands for the prior probability of the model $\mathcal{M}_s$. Also, note that $P\left(Y|\mathcal{M}_s\right)$ is the marginal likelihood associated with the model $\mathcal{M}_s$. If the prior probabilities are equal across models (i.e., $\pi_{0,\mathcal{M}_{DIM}} = \pi_{0,\mathcal{M}_{PIM}} = 0.50$), then the model with the highest marginal likelihood is the one that attains the largest posterior probability.

The column labeled *Full Data Set* in Table 2 shows that the DIM attains the largest marginal likelihood, and hence, adopting equal prior probabilities across models leads us to reject the null. Note that the posterior probability in favor of the DIM is larger than that in favor of the PIM unless the prior probability in favor of the former (i.e., $\pi_{0,\mathcal{M}_{DIM}}$) is as small as $9.75E-8$. Such a low prior probability suggests that only if one has extremely strong *a priori* information against the DIM, one can favor the PIM over the DIM.

This finding validates the use of the DIM to study the signaling effects of monetary policy, since the PIM is a prototypical New Keynesian model that has been extensively used by scholars to study the effects of monetary policy (e.g., Rotemberg and Woodford 1997; Clarida, Gali, and Gertler 2000; and Lubik and Schorfheide 2004).

We check if there is any specific observable variable that the DIM fits particularly better than the PIM. To this end, we estimate the two models using a narrower data set that does

Log-Marginal Likelihood			
Full Data Set		Excluding SPF	
DIM	PIM	DIM	PIM
-212.4445	-228.5888	-306.4532	-304.87466

Table 2: The table reports the log-marginal likelihood for the dispersed information model (DIM) and the perfect information model (PIM) based on the full data set described in Section 4.1 (Full Data Set) and on a narrower data set that does not include the Survey of Professional Forecasters (Excluding SPF). We use Geweke's harmonic mean estimator (Geweke 1999) to estimate the marginal likelihood for the two competing models.

not include the SPF as a measure of inflation expectations.[22] The log-marginal likelihood for the two competing models estimated using the narrower data set is reported in Table 2 under the column labeled *Excluding SPF*. It can be observed that now the two models fit the data similarly well, with the PIM having actually a little edge over the DIM. Therefore, the observed inflation expectations seem to play an important role in selecting the DIM as the best-fitting model. If we had estimated the two models using the data set that does not include the SPF, we could not have concluded that the DIM fits the data better than the PIM. This suggests that the DIM is better than the PIM at fitting the observed inflation expectations.

That an imperfect information model, such as the DIM, fits the SPF better than a perfect information model, such as the PIM, is by no means obvious. In fact, Del Negro and Eusepi (2011) find that the imperfect information model by Erceg and Levin (2003) is outperformed by a perfect information model in fitting the SPF.

5 Concluding Remarks

This paper studies a DSGE model in which information is dispersed across price setters and in which the interest rate that is set by the central bank has signaling effects. In this model, monetary impulses propagate through two channels: (i) the channel based on the central bank's ability to affect the real interest rate due to price stickiness and dispersed information and (ii) the signaling channel. The latter arises because changing the policy rate conveys information about the central bank's assessment of inflation and the output gap to price setters. This paper fits the model to a data set that includes the *Survey of Professional Forecasters* (SPF) as a measure of price setters' inflation expectations. We perform a formal econometric evaluation of the model with signaling effects of monetary policy. While the likelihood selects a very short average duration for price contracts, the signaling channel causes the real effects of monetary

[22]The posterior statistics of this estimation based on a narrower data set are reported in Appendix F.

shocks to be very sizable and persistent. Furthermore, this channel generates deflationary pressure in the aftermath of positive demand shocks, while it does not substantially alter the transmission of technology shocks.

An important lesson we learn from this paper is that signaling effects of monetary policy were sizable in the 1970s and provide an explanation for the Federal Reserve's failure to lower inflation in that decade. The gradual disinflation policies carried out by the Federal Reserve ended up *signaling* higher inflation to firms, leading the central bank to lose control over inflation and inflation expectations. We show that a more aggressive monetary tightening would have offset these signaling effects of monetary policy on price dynamics, allowing the Federal Reserve to prevent the inflation rate from rising to two-digit levels in the 1970s. Furthermore, we argue that signaling effects substantially contributed to explaining the *sluggish* response of inflation to the robust monetary contraction carried out by the Federal Reserve under Chairman Paul Volcker in the first years of his tenure. From 1983 on, the signaling effects of monetary policy seem to have contributed to keeping inflation low and to stimulating economic activity.

We make a number of assumptions to keep the model sufficiently tractable to allow for likelihood analysis. First, households are perfectly informed. Relaxing this assumption could affect the inflationary consequences of the signaling channel. For instance, if a monetary shock is partly perceived as a negative productivity shock, there will be at least two additional effects: (i) households will lower their expectations about the next period's real rate of interest and hence increase consumption and (ii) households will expect to be less productive in the future and therefore consume less. Note that a change in consumption will affect marginal costs in the model, and hence, depending on which effect dominates, the signaling effects on inflation could be either stronger or weaker than what we obtain in the current setup. Second, another convenient shortcut is the assumption that firms use the linearized version of the true model to solve their signal extraction problem. This assumption allows us to use the convenient characterization of the law of motion of the average expectations provided by the Kalman filter. Third, we assume no capital accumulation. Studying a dispersed information model with capital accumulation would be a fascinating extension of the current setup, since this is a standard feature of state-of-the-art DSGE models (e.g., Christiano, Eichenbaum, and Evans 2005 and Smets and Wouters 2007).

In the dispersed information model, the central bank's sole avenue of communications with firms is by setting the policy rate. While this assumption is not too strong for a study that mainly focuses on the signaling effects of U.S. monetary policy in the 1970s and in the 1980s, this hypothesis has to be relaxed to extend the analysis to the most recent period, in which central bank's announcements play a central role in monetary policy making.

Our analysis also provides support for the signaling effects of monetary policy as a potential explanation for the run-up of inflation in the 1970s. A formal econometric evaluation of whether this theory is more empirically plausible than the alternative theories[23] is beyond the scope of this paper whose main focus is to construct a modeling framework in which signaling effects can be empirically evaluated and to provide an explanation for why the U.S. disinflation policies were unsuccessful throughout the 1970s.

Changes in the Federal Reserve's attitude toward inflation stabilization have been documented by Davig and Leeper (2007), Justiniano and Primiceri (2008), Fernández-Villaverde, Guerrón-Quintana and Rubio-Ramírez (2010) and Bianchi (Forthcoming). One can show that the signaling effects on inflation are lower if the central bank reacts more aggressively to inflation deviations from the target. Future research should empirically evaluate whether structural changes in central bank's behaviors have affected the signaling effects of monetary policy on the macroeconomy. However, estimating a dispersed information model featuring parameter instability in the form of either time-varying parameters or Markov-switching parameters is computationally unmanageable at this stage.

[23]For instance, some of the most popular theories for why inflation rose in the 1970s are (*i*) the bad luck view (e.g., Cogley and Sargent 2005; Sims and Zha 2006; Primiceri 2005; and Liu, Waggoner, and Zha 2011), (*ii*) the lack of commitment view (e.g., Chari, Christiano, and Eichenbaum 1998 and Christiano and Gust 2000), (*iii*) the policy mistakes view (e.g., Sargent 2001; Clarida, Gali, and Gertler 2000; Lubik and Schorfheide 2004; Primiceri 2006; and Coibion and Gorodnichenko 2011*a*), and (*iv*) fiscal and monetary interactions view (e.g., Sargent, Williams, and Zha 2006; Bianchi and Ilut 2012; and Bianchi and Melosi, forthcoming).

References

Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan. 2006. "Signaling in a Global Game: Coordination and Policy Traps." *Journal of Political Economy*, 114(3): 452–484.

An, Sungbae, and Frank Schorfheide. 2007. "Bayesian Analysis of DSGE Models." *Econometric Reviews*, 26(2-4): 113–172.

Baeriswyl, Romain, and Camille Cornand. 2010. "The signaling role of policy actions." *Journal of Monetary Economics*, 57(6): 682–695.

Bianchi, Francesco. Forthcoming. "Regime Switches, Agents' Beliefs, and Post-World War II U.S. Macroeconomic Dynamics." *Review of Economic Studies.*

Bianchi, Francesco, and Cosmin Ilut. 2012. "Monetary/Fiscal Policy Mix and Agents' Beliefs." *Duke University. Mimeo.*

Bianchi, Francesco, and Leonardo Melosi. 2012. "Constrained Discretion and Central Bank Transparency." Mimeo.

Bianchi, Francesco, and Leonardo Melosi. Forthcoming. "Dormant Shocks and Fiscal Virtue." In *NBER Macroeconomics Annual 2013, Volume 28. NBER Chapters*, 319–378. National Bureau of Economic Research, Inc.

Bils, Mark, and Pete J. Klenow. 2004. "Some Evidence on the Importance of Sticky Prices." *Journal of Political Economy*, 112(5): 947–985.

Blanchard, Olivier Jean, and Charles M Kahn. 1980. "The Solution of Linear Difference Models under Rational Expectations." *Econometrica*, 48(5): 1305–11.

Calvo, Guillermo A. 1983. "Staggered Prices in a Utility Maximizing Framework." *Journal of Monetary Economics*, 12(3): 383–398.

Chari, V. V., Lawrence J. Christiano, and Martin Eichenbaum. 1998. "Expectation Traps and Discretion." *Journal of Economic Theory*, 81(2): 462–492.

Christiano, Lawrence J., and Christopher Gust. 2000. "The expectations trap hypothesis." *Economic Perspectives*, 21–39.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy*, 113(1): 1–45.

Clarida, Richard, Jordi Gali, and Mark Gertler. 2000. "Monetary Policy Rules And Macroeconomic Stability: Evidence And Some Theory." *The Quarterly Journal of Economics*, 115(1): 147–180.

Cogley, Timothy, and Thomas J. Sargent. 2005. "Drift and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S." *Review of Economic Dynamics*, 8(2): 262–302.

Coibion, Olivier, and Yuriy Gorodnichenko. 2011. "Monetary Policy, Trend Inflation, and the Great Moderation: An Alternative Interpretation." *American Economic Review*, 101(1): 341–70.

Coibion, Olivier, and Yuriy Gorodnichenko. 2012*a*. "What Can Survey Forecasts Tell Us about Information Rigidities?" *Journal of Political Economy*, 120(1): 116 – 159.

Coibion, Olivier, and Yuriy Gorodnichenko. 2012*b*. "Why Are Target Interest Rate Changes So Persistent?" *American Economic Journal: Macroeconomics*, 4(4): 126–62.

Cover, Thomas M., and Joy A. Thomas. 1991. *Elements of Information Theory.* New York:Wiley.

Davig, Troy, and Eric M. Leeper. 2007. "Generalizing the Taylor Principle." *American Economic Review*, 97(3): 607–635.

Del Negro, Marco, and Frank Schorfheide. 2008. "Forming Priors for DSGE Models (and How It Affects the Assessment of Nominal Rigidities)." *Journal of Monetary Economics*, 55(7): 1191–1208.

Del Negro, Marco, and Stefano Eusepi. 2011. "Fitting observed inflation expectations." *Journal of Economic Dynamics and Control*, 35(12): 2105–2131.

Eichenbaum, Martin. 1992. "Comment on Interpreting the macroeconomic time series facts: the effects of monetary policy." *European Economic Review*, 36(5): 1001–1011.

Erceg, Christopher J., and Andrew T. Levin. 2003. "Imperfect Credibility and Inflation Persistence." *Journal of Monetary Economics*, 50(4): 915–944.

Fernández-Villaverde, Jesús, and Juan F. Rubio-Ramírez. 2004. "Comparing Dynamic Equilibrium Models to Data: A Bayesian Approach." *Journal of Econometrics*, 123(1): 153–187.

Fernández-Villaverde, Jesús, Pablo Guerrón-Quintana, and Juan F. Rubio-Ramírez. 2010. "Fortune or Virtue: Time-Variant Volatilities Versus Parameter Drifting in U.S. Data." National Bureau of Economic Research, Inc NBER Working Papers 15928.

Geweke, John. 1999. "Using Simulation Methods for Bayesian Econometric Models: Inference, Development, and Communications." *Econometric Reviews*, 18(1): 1–127.

Gorodnichenko, Yuriy. 2008. "Endogenous information, menu costs and inflation persistence." National Bureau of Economic Research, Inc NBER Working Papers 14184.

Hachem, Kinda, and Jing Cynthia Wu. 2012. "Inflation Announcements and Social Dynamics." Chicago Booth mimeo.

Hellwig, C., and V. Vankateswaran. 2009. "Setting the Right Price for the Wrong Reasons." *Journal of Monetary Economics*, 56(S): S57–S77.

Hellwig, Christian. 2002. "Public Announcements, Adjustment Delays and the Business Cycle." Mimeo.

Hetzel, Rober L. 1998. "Arthur Burns and Inflation." FRB of Richmond Economic Quarterly.

Justiniano, Alejandro, and Giorgio E. Primiceri. 2008. "The Time-Varying Volatility of Macroeconomic Fluctuations." *American Economic Review*, 98(3): 604–41.

Klenow, Peter J., and Benjamin A. Malin. 2010. "Microeconomic Evidence on Price-Setting." In *Handbook of Monetary Economics*. Vol. 3A, , ed. Benjamin Friedman and Michael Woodford. New York:North Holland.

Klenow, Peter J., and Oleksiy Kryvtsov. 2008. "State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?" *Quarterly Journal of Economics*, 123(3): 863–904.

Liu, Zheng, Daniel F. Waggoner, and Tao Zha. 2011. "Sources of macroeconomic fluctuations: A regime-switching DSGE approach." *Quantitative Economics*, 2(2): 251–301.

Lorenzoni, Guido. 2009. "A Theory of Demand Shocks." *American Economic Review*, 99(5): 2050–2084.

Lubik, Thomas A., and Frank Schorfheide. 2004. "Testing for Indeterminacy: An Application to U.S. Monetary Policy." *American Economic Review*, 94(1): 190–217.

Maćkowiak, Bartosz, and Mirko Wiederholt. 2009. "Optimal Sticky Prices under Rational Inattention." *American Economic Review*, 99(3): 769–803.

Maćkowiak, Bartosz, and Mirko Wiederholt. 2010. "Business Cycle Dynamics under Rational Inattention." CEPR Discussion Paper No. 7691.

Maćkowiak, Bartosz, Emanuel Moench, and Mirko Wiederholt. 2009. "Sectoral Price Data and Models of Price Settings." *Journal of Monetary Economics*, 56(S): 78–99.

Mankiw, Gregory N., and Ricardo Reis. 2002. "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve." *Quarterly Journal of Economics*, 117(4): 1295–1328.

Matejka, Filip. 2011. "Rationally Inattentive Seller: Sales and Discrete Pricing." CERGE-EI Working Papers wp408.

Melosi, Leonardo. Forthcoming. "Estimating Models with Dispersed Information." *American Economic Journal: Macroeconomics.*

Morris, Stephen, and Hyun S. Shin. 2003*a*. "Global Games: Theory and Applications." In *Advances in Economics and Econometrics, the Eighth World Congress.* , ed. M. Dewatripont, L. Hansen and S. Turnovsky. Cambridge:Cambridge University Press.

Morris, Stephen, and Hyun S. Shin. 2003*b*. "Social Value of Public Information." *American Economic Review*, 92(5): 1521–1534.

Nakamura, Emi, and Jon Steinsson. 2008. "Five Facts about Prices: A Reevaluation of Menu Cost Models." *Quarterly Journal of Economics*, 123(4): 1415–1464.

Nimark, Kristoffer. 2008. "Dynamic Pricing and Imperfect Common Knowledge." *Journal of Monetary Economics*, 55(8): 365–382.

Nimark, Kristoffer. 2009. "A Low Dimensional Kalman Filter for Systems with Lagged Observables." Mimeo.

Nimark, Kristoffer. 2011. "Dynamic Higher Order Expectations." Mimeo.

Nimark, Kristoffer. 2012. "Man-Bites-Dog Business Cycles." CREI mimeo.

Paciello, Luigi, and Mirko Wiederholt. Forthcoming. "Exogenous Information, Endogenous Information and Optimal Monetary Policy." *Review of Economic Studies.*

Primiceri, Giorgio E. 2005. "Time Varying Structural Vector Autoregressions and Monetary Policy." *Review of Economic Studies*, 72(3): 821–852.

Primiceri, Giorgio E. 2006. "Why Inflation Rose and Fell: Policymakers' Beliefs and US Postwar Stabilization Policy." *The Quarterly Journal of Economics*, 121: 867–901.

Rabanal, Pau, and Juan F. Rubio-Ramírez. 2005. "Comparing New Keynesian models of the business cycle: A Bayesian approach." *Journal of Monetary Economics*, 52: 1151–1166.

Romer, Christina D., and David H. Romer. 2013. "The Most Dangerous Idea in Federal Reserve History: Monetary Policy Doesn't Matter." *American Economic Review*, 103(3): 55–60.

Rondina, Giacomo, and Todd B. Walker. 2012. "Information Equilibria in Dynamic Economies with Dispersed Information." Mimeo.

Rotemberg, Julio. 2013. "Penitence After Accusation of Error: 100 Years of Monetary Policy at the U.S. Federal Reserve." NBER Mimeo.

Rotemberg, Julio, and Michael Woodford. 1997. "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy." In *NBER Macroeconomics Annual 1997, Volume 12. NBER Chapters*, 297–361. National Bureau of Economic Research, Inc.

Sargent, Thomas J. 2001. *The Conquest of American Inflation.* Princeton NJ:Princeton University Press.

Sargent, Thomas, Noah Williams, and Tao Zha. 2006. "Shocks and Government Beliefs: The Rise and Fall of American Inflation." *American Economic Review*, 96(4): 1193–1224.

Schorfheide, Frank. 2000. "Loss Function-Based Evaluation of DSGE Models." *Journal of Applied Econometrics*, 15(6): 645–670.

Sims, Christopher A. 1992. "Interpreting the macroeconomic time series facts: the effects of monetary policy." *European Economic Review*, 36(5): 975–1000.

Sims, Christopher A. 2002. "Solving Linear Rational Expectations Models." *Computational Economics*, 20(1-2): 1–20.

Sims, Christopher A. 2003. "Implications of Rational Inattention." *Journal of Monetary Economics*, 50(3): 665–690.

Sims, Christopher A., and Tao Zha. 2006. "Were There Regime Switches in U.S. Monetary Policy?" *American Economic Review*, 96(1): 54–81.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97(3): 586–606.

Townsend, Robert M. 1983*a*. "Equilibrium Theory with Learning and Disparate Information." In *Individual Forecasting and Aggregate Outcomes.* , ed. R. Frydman and Edmund S. Phelps, 169–197. Cambridge:Cambridge University Press.

Townsend, Robert M. 1983*b*. "Forecasting the Forecasts of Others." *Journal of Political Economy*, 91(4): 546–588.

Trabandt, Mathias. 2007. "Sticky Information vs. Sticky Prices: A Horse Race in a DSGE Framework." Sveriges Riksbank (Central Bank of Sweden) Working Paper Series 209.

Walsh, Carl. 2010. "Transparency, the Opacity Bias, and Optimal Flexible Inflation Targeting." mimeo.

Woodford, Michael. 2002. "Imperfect Common Knowledge and the Effects of Monetary Policy." In *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps.* , ed. Philippe Aghion, Roman Frydman, Joseph Stiglitz and Michael Woodford, 25–58. Princeton:Princeton University Press.

Appendices

The Appendices are organized as follows. In Appendix A, we derive of the imperfect-common-knowledge Phillips curve (7). Appendix B details an algorithm to solve the dispersed information model. In Appendix C, we characterize the transition equations for the average higher-order expectations about the exogenous state variables $-$ that is, equation (13). In Appendix D, we characterize the laws of motion for the three endogenous state variables (i.e., inflation $\hat{\pi}_t$, real output $\hat{y}_t$ and the interest rate $\hat{R}_t$). In Appendix E, we discuss the impulse response function of inflation and inflation expectations implied the prior for the model parameters. This is to verify that the main findings of the paper do not stem from our prior choice, but are actually driven by the data. Appendix F reports the posterior statistics for the parameters of the dispersed information model and the perfect information model estimated to a narrower data set that does not include the SPF as a measure of firms' inflation expectations.

A The Imperfect-Common-Knowledge Phillips Curve

The log-linear approximation of the labor supply can be shown to be given by $\widehat{c}_t = \widehat{w}_t$. Recalling that the resource constraint implies that $\hat{y}_t = \hat{c}_t$, we can then rewrite the labor supply as follows:

$$\hat{y}_t = \hat{w}_t. \tag{17}$$

Log-linearizing the equation for the real marginal costs yields

$$\widehat{mc}_{j,t} = \hat{w}_t - \widehat{a}_t - \varepsilon^a_{j,t}.$$

Recall that $(\ln A_{j,t} - \ln\gamma \cdot t) \in \mathcal{I}_{j,t}$ and write

$$\mathbb{E}_{j,t}\widehat{mc}_{j,t} = \mathbb{E}_{j,t}\hat{w}_{j,t}\underbrace{-\widehat{a}_t - \varepsilon^a_{j,t}}_{\ln A_{j,t} - \ln\gamma\cdot t},$$

where $\mathbb{E}_{j,t}$ is expectations conditioned on firm j's information set at time t ($\mathcal{I}_{j,t}$) defined in (5). Using equation (17) for replacing $\hat{w}_t$ yields

$$\mathbb{E}_{j,t}\widehat{mc}_{j,t} = \mathbb{E}_{j,t}\hat{y}_t - \widehat{a}_t - \varepsilon^a_{j,t}.$$

By integrating across firms, we obtain the average expectations on marginal costs:

$$\widehat{mc}^{(1)}_{t|t} = \hat{y}^{(1)}_{t|t} - \widehat{a}_t.$$

The linearized price index can be written as

$$\int \hat{p}^*_{j,t} dj = \frac{\theta}{1-\theta}\hat{\pi}_t.$$

Recall that we defined $\hat{p}^*_{j,t} = \ln P^*_{j,t} - \ln P_t$ and $\hat{\pi}_t = \ln P_t - \ln P_{t-1} - \ln \pi_*$. After some algebraic manipulation, we write

$$\ln P_t = \theta \left(\ln P_{t-1} + \ln \pi_*\right) + (1-\theta) \int \left(\ln P^*_{j,t}\right) dj. \tag{18}$$

The price-setting problem leads to the following first-order conditions:

$$\mathbb{E}\left[\sum_{s=0}^{\infty} (\beta\theta)^s \frac{\Xi_{j,t+s}}{P_{t+s}} \left[(1-\nu)\pi_*^s + \nu \frac{MC_{j,t+s}}{P^*_{j,t}}\right] Y_{j,t+s} | \mathcal{I}_{j,t}\right] = 0.$$

We define the stationary variables:

$$\begin{aligned} y_t &= \frac{Y_t}{\gamma^t},\ c_t = \frac{C_t}{\gamma^t};\ p^*_{j,t} = \frac{P^*_{j,t}}{P_t},\ y_{j,t} = \frac{Y_{j,t}}{\gamma^t}, \\ w_t &= \frac{W_t}{\gamma^t P_t},\ a_t = \frac{A_t}{\gamma^t}, R_t = \frac{R_t}{R_*},\ mc_{j,t} = \frac{MC_{j,t}}{P_t}, \\ \xi_{j,t} &= \gamma^t \Xi_{j,t}. \end{aligned}$$

And then we write

$$\mathbb{E}\left\{\xi_{j,t}\left[1-\nu+\nu\frac{mc_{j,t}}{p^*_{j,t}}\right] y_{j,t} + \sum_{s=1}^{\infty} (\beta\theta)^s \xi_{j,t+s} \left[(1-\nu)\pi_*^s + \nu\frac{mc_{j,t+s}}{p^*_{j,t}}\left(\Pi_{\tau=1}^s \pi_{t+\tau}\right)\right] y_{j,t+s} | \mathcal{I}_{j,t}\right\} = 0. \tag{19}$$

First realize that the square brackets are equal to zero at the steady state, and hence, we do not care about the terms outside them. We can write

$$\mathbb{E}\left[\left[1-\nu+\nu mc_{j,*} e^{\widehat{mc}_{j,t}-\widehat{p}^*_{j,t}}\right] + \sum_{s=1}^{\infty} (\beta\theta)^s \left[(1-\nu)\pi_*^s + \nu mc_{j,*} e^{\widehat{mc}_{j,t+s}-\widehat{p}^*_{j,t}+\sum_{\tau=1}^s \hat{\pi}_{t+\tau}}\right] | \mathcal{I}_{j,t}\right] = 0.$$

Taking the derivatives yields

$$\mathbb{E}\left[\widehat{mc}_{j,t} - \widehat{p}^*_{j,t} + \sum_{s=1}^{\infty} (\beta\theta)^s \left[\left(\widehat{mc}_{j,t+s} - \widehat{p}^*_{j,t} + \sum_{\tau=1}^{s} \hat{\pi}_{t+\tau}\right)\right] | \mathcal{I}_{j,t}\right] = 0.$$

We can take the term $\widehat{p}_{j,t}$ out of the sum operator in the second term and gather the common term to obtain

$$\mathbb{E}\left[\widehat{mc}_{j,t} - \frac{1}{1-\beta\theta}\widehat{p}^*_{j,t} + \sum_{s=1}^{\infty}(\beta\theta)^s\left(\widehat{mc}_{j,t+s} + \sum_{\tau=1}^{s}\hat{\pi}_{t+\tau}\right)|\mathcal{I}_{j,t}\right] = 0.$$

Recall that $\widehat{p}^*_{j,t} \equiv \ln P^*_{j,t} - \ln P_t$ and cannot be taken out of the expectation operator. We write

$$\ln P^*_{j,t} = (1-\beta\theta)\,\mathbb{E}\left[\widehat{mc}_{j,t} + \frac{1}{1-\beta\theta}\ln P_t + \sum_{s=1}^{\infty}(\beta\theta)^s\left(\widehat{mc}_{j,t+s} + \sum_{\tau=1}^{s}\hat{\pi}_{t+\tau}\right)|\mathcal{I}_{j,t}\right]. \tag{20}$$

Rolling this equation one step ahead yields

$$\ln P^*_{j,t+1} = (1-\beta\theta)\,\mathbb{E}\left[\widehat{mc}_{j,t+1} + \frac{1}{1-\beta\theta}\ln P_{t+1} + \sum_{s=1}^{\infty}(\beta\theta)^s\left(\widehat{mc}_{j,t+s+1} + \sum_{\tau=1}^{s}\hat{\pi}_{t+\tau+1}\right)|\mathcal{I}_{j,t+1}\right].$$

Taking firm j's conditional expectation at time t on both sides and applying the law of iterated expectations, we obtain the following:

$$\mathbb{E}\left(\ln P^*_{j,t+1}|\mathcal{I}_{j,t}\right) = (1-\beta\theta)\,\mathbb{E}\left[\widehat{mc}_{j,t+1} + \frac{1}{1-\beta\theta}\ln P_{t+1} + \sum_{s=1}^{\infty}(\beta\theta)^s\left(\widehat{mc}_{j,t+s+1} + \sum_{\tau=1}^{s}\hat{\pi}_{t+\tau+1}\right)|\mathcal{I}_{j,t}\right].$$

We can take $\widehat{mc}_{j,t+1}$ inside the sum operator and write

$$\mathbb{E}\left(\ln P^*_{j,t+1}|\mathcal{I}_{j,t}\right) = (1-\beta\theta)\,\mathbb{E}\left[\frac{1}{1-\beta\theta}\ln P_{t+1} + \frac{1}{\beta\theta}\sum_{s=1}^{\infty}(\beta\theta)^s\,\widehat{mc}_{j,t+s} + \sum_{s=1}^{\infty}(\beta\theta)^s\sum_{\tau=1}^{s}\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right].$$

Therefore,

$$\sum_{s=1}^{\infty}(\beta\theta)^s\,\mathbb{E}\left[\widehat{mc}_{j,t+s}|\mathcal{I}_{j,t}\right] = \frac{\beta\theta}{1-\beta\theta}\left[\mathbb{E}\left(\ln P^*_{j,t+1}|\mathcal{I}_{j,t}\right) - \mathbb{E}\left(\ln P_{t+1}|\mathcal{I}_{j,t}\right)\right] - \beta\theta\sum_{s=1}^{\infty}(\beta\theta)^s\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right]. \tag{21}$$

Hence, the equation (20) can be rewritten as:

$$\begin{aligned}
\ln P^*_{j,t} &= (1-\beta\theta)\left\{\mathbb{E}\left[\widehat{mc}_{j,t}|\mathcal{I}_{j,t}\right] + \frac{1}{1-\beta\theta}\mathbb{E}\left[\ln P_t|\mathcal{I}_{j,t}\right] + \sum_{s=1}^{\infty}(\beta\theta)^s\,\mathbb{E}\left[\widehat{mc}_{j,t+s}|\mathcal{I}_{j,t}\right]\right\} \\
&\quad + (1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^s\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right].
\end{aligned}$$

By substituting the result in equation (21), we obtain

$$
\begin{aligned}
\ln P_{j,t}^{*} &= (1-\beta\theta)\left[\mathbb{E}\left[\widehat{mc}_{j,t}|\mathcal{I}_{j,t}\right]+\frac{1}{1-\beta\theta}\mathbb{E}\left[\ln P_{t}|\mathcal{I}_{j,t}\right]\right] \\
&\quad +\beta\theta\left[\mathbb{E}\left(\ln P_{j,t+1}^{*}|\mathcal{I}_{j,t}\right)-\mathbb{E}\left(\ln P_{t+1}|\mathcal{I}_{j,t}\right)\right]-(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right] \\
&\quad +(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right].
\end{aligned}
$$

We consider the last term and write

$$
\begin{aligned}
(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right] &= (1-\beta\theta)\,\beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]+(1-\beta\theta)\sum_{s=2}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right] \\
&= (1-\beta\theta)\,\beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]+ \\
&\quad +(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\left(\sum_{\tau=1}^{s}\left[\left(\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right]\right)\right]+\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]\right).
\end{aligned}
$$

It then follows that

$$
\begin{aligned}
(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right] &= (1-\beta\theta)\,\beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right] \\
&\quad +(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right] \\
&\quad +(1-\beta\theta)\left(\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\right)\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right].
\end{aligned}
$$

Because $\left(\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\right)=\frac{(\beta\theta)^{2}}{1-\beta\theta}$, then after simplifying, we can write that

$$
\begin{aligned}
(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right] &= \beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right] \\
&\quad +(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right].
\end{aligned}
$$

We substitute this result into the original equation to get the following expression:

$$
\begin{aligned}
\ln P_{j,t}^{*} &= (1-\beta\theta)\,\mathbb{E}\left[\widehat{mc}_{j,t}|\mathcal{I}_{j,t}\right]+\mathbb{E}\left[\ln P_{t}|\mathcal{I}_{j,t}\right] \\
&\quad +\beta\theta\left[\mathbb{E}\left(\ln P_{j,t+1}^{*}|\mathcal{I}_{j,t}\right)+\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]-\mathbb{E}\left(\ln P_{t+1}|\mathcal{I}_{j,t}\right)\right].
\end{aligned}
\tag{22}
$$

Note that by definition $\hat{\pi}_{t+1} \equiv \ln P_{t+1} - \ln P_t - \ln \pi_*$. Hence, we can write

$$\begin{aligned} \ln P^*_{j,t} &= (1-\beta\theta) \cdot \mathbb{E}\left[\widehat{mc}_{j,t} | \mathcal{I}_{j,t}\right] + (1-\beta\theta)\, \mathbb{E}\left[\ln P_t | \mathcal{I}_{j,t}\right] \\ &\quad + \beta\theta \cdot \mathbb{E}\left(\ln P^*_{j,t+1} | \mathcal{I}_{j,t}\right) - \beta\theta \ln \pi_*. \end{aligned} \tag{23}$$

We denote firm j's average k-th order expectation about an arbitrary variable $\hat{x}_t$ as

$$\mathbb{E}^{(k)}\left(\hat{x}_t | \mathcal{I}_{j,t}\right) \equiv \int \mathbb{E}\left(\int \mathbb{E}\left(\dots\left(\int \mathbb{E}\left(\hat{x}_t | \mathcal{I}_{j,t}\right) dj\right)\dots | \mathcal{I}_{j,t}\right) dj | \mathcal{I}_{j,t}\right) dj,$$

where expectations and integration across firms are taken k times.

Let us denote the average reset price as $\ln P^*_t = \int \ln P^*_{j,t} dj$. Note that we can rewrite equation (18) as follows

$$\ln P_t = \theta\left(\ln P_{t-1} + \ln \pi_*\right) + (1-\theta) \ln P^*_t. \tag{24}$$

Furthermore, we can integrate equation (23) across firms to obtain an equation for the average reset price:

$$\begin{aligned} \ln P^*_t &= (1-\beta\theta) \cdot \widehat{mc}^{(1)}_{t|t} + (1-\beta\theta) \ln P^{(1)}_{t|t} \\ &\quad + \beta\theta \ln P^{*(1)}_{t+1|t} - \beta\theta \ln \pi_*, \end{aligned} \tag{25}$$

where $x^{(1)}_{t|t}$ denotes the average first-order expectations about an arbitrary variable x_t of the model (e.g., the real marginal costs).

Let us plug equation (25) into equation (24) as follows:

$$\begin{aligned} \ln P_t &= \theta \ln P_{t-1} + \left(\theta - (1-\theta)\beta\theta\right) \ln \pi_* \\ &\quad + (1-\theta)\left[(1-\beta\theta) \cdot \widehat{mc}^{(1)}_{t|t} + (1-\beta\theta) \ln P^{(1)}_{t|t} + \beta\theta \ln P^{*(1)}_{t+1|t}\right]. \end{aligned} \tag{26}$$

From the fact that $\ln P_t = \widehat{\pi}_t + \ln P_{t-1} + \ln \pi_*$ and from the price index (18) we get the following:[24]

$$\ln P^*_{t+1} = \frac{\hat{\pi}_{t+1}}{1-\theta} + \ln P_t + \ln \pi_*.$$

Furthermore, the following fact is easy to establish:

$$\ln P_{t+1} = \widehat{\pi}_{t+1} + \ln P_t + \ln \pi_*.$$

Applying these three results to equation (26) yields

$$\begin{aligned} \widehat{\pi}_t + \ln P_{t-1} + \ln \pi_* \quad = \quad & \theta \ln P_{t-1} + (\theta - (1-\theta)\,\beta\theta) \ln \pi_* \\ & + (1-\theta) \left[(1-\beta\theta) \cdot \widehat{mc}^{(1)}_{t|t} + (1-\beta\theta) \ln P^{(1)}_{t|t} + \beta\theta \left(\frac{\hat{\pi}^{(1)}_{t+1|t}}{1-\theta} + \ln P^{(1)}_{t|t} + \ln \pi_* \right) \right]. \end{aligned} \tag{27}$$

And after simplifying, we get the following:

$$\widehat{\pi}_t = (1-\theta)\,(1-\beta\theta) \cdot \widehat{mc}^{(1)}_{t|t} + (1-\theta)\,\widehat{\pi}^{(1)}_{t|t} + \beta\theta \left(\hat{\pi}^{(1)}_{t+1|t} \right). \tag{28}$$

By repeatedly taking firm j's expectation and average the resulting equation across firms, we get

$$\widehat{\pi}^{(k)}_{t|t} = (1-\theta)\,(1-\beta\theta) \cdot \widehat{mc}^{(k+1)}_{t|t} + (1-\theta)\,\widehat{\pi}^{(k+1)}_{t|t} + \beta\theta \left(\hat{\pi}^{(k+1)}_{t+1|t} \right).$$

Repeatedly substituting these equations for $k \geq 1$ back to equation (28) yields the imperfect-common-knowledge Phillips curve:

$$\hat{\pi}_t = (1-\theta)\,(1-\beta\theta) \sum_{k=1}^{\infty} (1-\theta)^{k-1}\,\widehat{mc}^{(k)}_{t|t} + \beta\theta \sum_{k=1}^{\infty} (1-\theta)^{k-1}\,\widehat{\pi}^{(k)}_{t+1|t}.$$

B Solving the Dispersed Information Model

We solve the model assuming common knowledge of rationality (Nimark 2008) and focusing on equilibria where the higher-order expectations about the exogenous state variables; that is,

[24] This last result comes from observing that

$$\ln P_t = \theta\,(\ln P_{t-1} + \ln \pi_*) + (1-\theta) \ln P^*_t.$$

By using the fact that $\ln P_t = \widehat{\pi}_t + \ln P_{t-1} + \ln \pi_*$:

$$\widehat{\pi}_t + \ln P_{t-1} + \ln \pi_* = \theta\,(\ln P_{t-1} + \ln \pi_*) + (1-\theta) \ln P^*_t.$$

Rolling one period forward, we get

$$\widehat{\pi}_{t+1} = (\theta - 1)\,(\ln P_t + \ln \pi_*) + (1-\theta) \ln P^*_{t+1}.$$

And finally, by rearranging, we get the result in the text.

$X_{t|t}^{(0:k)} \equiv \left[\widehat{a}_{t|t}^{(s)}, \widehat{\eta}_{r,t|t}^{(s)}, \widehat{g}_{t|t}^{(s)} : 0 \le s \le k\right]'$, follow the VAR(1) process in equation (13). Note that we truncate the order of the average expectations at $k < \infty$. Furthermore, we guess the matrix $\mathbf{v}_0$ that determines the dynamics of the endogenous variables $\mathbf{s}_t \equiv \left[\widehat{y}_t, \widehat{\pi}_t, \widehat{R}_t\right]$ in equation (12). As shown in Appendix D, the structural equations of the model can be written in the following linear form:

$$\Gamma_0 \mathbf{s}_t = \Gamma_1 \mathbb{E}_t \mathbf{s}_{t+1} + \Gamma_2 X_{t|t}^{(0:k)}, \tag{29}$$

where $\mathbb{E}_t$ denotes the expectation operator conditional on a complete information set (i.e., an information set that includes the history of all structural shocks).

For a given parameter set Θ_{DIM}, take the following steps:

Step 0 Set $i = 1$ and guess the matrices $\mathbf{M}^{(i)}$, $\mathbf{N}^{(i)}$, and $\mathbf{v}_0^{(i)}$.

Step 1 Set $\mathbf{M} = \mathbf{M}^{(i)}$ and $\mathbf{N} = \mathbf{N}^{(i)}$ and solve the model given by equation (13) and equation (29) through a standard linear rational expectations model solver (e.g., Blanchard and Kahn 1980; Sims 2002). The solver delivers the matrix $\mathbf{v}_0^{(i+1)}$, such that $\mathbf{s}_t = \mathbf{v}_0^{(i+1)} X_{t|t}^{(0:k)}$. As we will show in Appendix D, the matrices Γ_0, Γ_1, and Γ_2 in equation (29) are functions of the model parameter Θ_{DIM} as well as the guessed matrices $\mathbf{M}^{(i)}$ and $\mathbf{v}_0^{(i)}$.

Step 2 Given the law of motion (13) for $X_{t|t}^{(0:k)}$, in which we set $\mathbf{M} = \mathbf{M}^{(i)}$ and $\mathbf{N} = \mathbf{N}^{(i)}$, equation (10) for the signal concerning the aggregate technology, equation (11) for the signal concerning the demand conditions, and the equation

$$\widehat{R}_t = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \mathbf{v}_0^{(i+1)} X_{t|t}^{(0:k)},$$

for the endogenous policy signal $\widehat{R}_t \in \mathbf{s}_t$ solve the firms' signal extraction problem through the Kalman filter and determine the matrices $\mathbf{M}^{(i+1)}$ and $\mathbf{N}^{(i+1)}$. Appendix C provides a detailed explanation of how we characterize these matrices.

Step 3 If $\left\|\mathbf{M}^{(i)} - \mathbf{M}^{(i+1)}\right\| < \varepsilon_m$, $\left\|\mathbf{N}^{(i)} - \mathbf{N}^{(i+1)}\right\| < \varepsilon_n$, and $\left\|\mathbf{v}_0^{(i)} - \mathbf{v}_0^{(i+1)}\right\| < \varepsilon_v$ for any $\varepsilon_m > 0$, $\varepsilon_n > 0$, and $\varepsilon_v > 0$ and small, STOP or else set i=i+1 and go to STEP 1.

Given equation (13) and equation $\mathbf{s}_t = \mathbf{v}_0^{(i)} X_{t|t}^{(0:k)}$ obtained in step 1, the law of motion of the model variables is as follows:

$$\begin{bmatrix} X_{t|t}^{(0:k)} \\ \mathbf{s}_t \end{bmatrix} = \begin{bmatrix} \mathbf{M}^{(i+1)} & \mathbf{0} \\ \mathbf{v}_0^{(i+1)} \mathbf{M}^{(i+1)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} X_{t-1|t-1}^{(0:k)} \\ \mathbf{s}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{N}^{(i+1)} \\ \mathbf{v}_0^{(i+1)} \mathbf{N}^{(i+1)} \end{bmatrix} \boldsymbol{\varepsilon}_t. \tag{30}$$

C Transition Equation of High-Order Expectations

In this section, we show how to derive the law of motion of the average higher-order expectations of the exogenous state variables (i.e., $\widehat{a}_t$, $\widehat{\eta}_{r,t}$, $\widehat{g}_t$,) for given parameter values and the matrix of coefficients $\mathbf{v}_0$. We focus on equilibria where the HOE evolve according to

$$X_{t|t}^{(0:k)} = \mathbf{M}X_{t-1|t-1}^{(0:k)} + \mathbf{N}\boldsymbol{\varepsilon}_t, \tag{31}$$

where $\boldsymbol{\varepsilon}_t \equiv \begin{bmatrix} \varepsilon_{a,t} & \eta_{r,t} & \varepsilon_{g,t} \end{bmatrix}'$. Denote $\mathbf{X}_t \equiv X_{t|t}^{(0:k)}$, for notational convenience. Firms' reduced-form state-space model can be concisely cast as follows:

$$\mathbf{X}_t = \mathbf{M}\mathbf{X}_{t-1} + \mathbf{N}\boldsymbol{\varepsilon}_t, \tag{32}$$

$$\mathbf{Z}_t = \mathbf{D}\mathbf{X}_t + \mathbf{Q}e_{j,t}, \tag{33}$$

where

$$\mathbf{D} = \begin{bmatrix} \mathbf{d}_1' & \mathbf{d}_2' & \left(\mathbf{1}_3^T \mathbf{v}_0\right)' \end{bmatrix}',$$

with $\mathbf{d}_1' = \left[1, \mathbf{0}_{1\times 3(k+1)-1}\right]$, $\mathbf{d}_2' = [\mathbf{0}_{1\times 2}, 1, \mathbf{0}_{1\times 3k}]$, $\mathbf{1}_3^T = [0, 0, 1]$, and $e_{j,t} = \left[\varepsilon_{j,t}^a, \varepsilon_{j,t}^g\right]'$ and

$$\mathbf{Q} = \begin{bmatrix} \widetilde{\sigma}_a & 0 \\ 0 & \widetilde{\sigma}_g \\ 0 & 0 \end{bmatrix}.$$

Solving the firms' signal extraction problem requires applying the Kalman filter. The Kalman equation pins down firm j's first-order expectations about the model's state variables $\mathbf{X}_{t|t}(j)$ and the associated conditional covariance matrix $\mathbf{P}_{t|t}$:

$$\begin{aligned} \mathbf{X}_{t|t}(j) &= \mathbf{X}_{t|t-1}(j) + \mathbf{P}_{t|t-1}\mathbf{D}'\mathbf{F}_{t|t-1}^{-1}\left[\mathbf{Z}_t - \mathbf{Z}_{t|t-1}(j)\right], & (34) \\ \mathbf{P}_{t|t} &= \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{D}'\mathbf{F}_{t|t-1}^{-1}\mathbf{D}\mathbf{P}_{t|t-1}', & (35) \end{aligned}$$

where

$$\mathbf{P}_{t|t-1} = \mathbf{M}\mathbf{P}_{t-1|t-1}\mathbf{M}' + \mathbf{N}\mathbf{N}', \tag{36}$$

and the matrix $\mathbf{F}_{t|t-1} \equiv E\left[\mathbf{Z}_t\mathbf{Z}_t'|\mathbf{Z}^{t-1}\right]$, which can be shown to be

$$\mathbf{F}_{t|t-1} = \mathbf{D}\mathbf{P}_{t|t-1}\mathbf{D}' + \mathbf{Q}\mathbf{Q}'. \tag{37}$$

Therefore, combining equation (35) with equation (36) yields

$$\mathbf{P}_{t+1|t} = \mathbf{M}\left[\mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{D}'\mathbf{F}_{t|t-1}^{-1}\mathbf{D}\mathbf{P}_{t|t-1}'\right]\mathbf{M}' + \mathbf{N}\mathbf{N}'. \tag{38}$$

Denote the Kalman-gain matrix as $\mathbf{K}_t \equiv \mathbf{P}_{t|t-1}\mathbf{D}'\mathbf{F}_{t|t-1}^{-1}$. Write the law of motion of firm j's first-order beliefs about $\mathbf{X}_t$ as

$$\mathbf{X}_{t|t}(j) = \mathbf{X}_{t|t-1}(j) + \mathbf{K}_t\left[\mathbf{D}\mathbf{X}_t + \mathbf{Q}e_{j,t} - \mathbf{D}\mathbf{X}_{t|t-1}(j)\right],$$

where we have combined equations (34) and (33). By recalling that $\mathbf{X}_{t|t-1}(j) = \mathbf{M}\mathbf{X}_{t-1|t-1}(j)$, we obtain

$$\mathbf{X}_{t|t}(j) = (\mathbf{M} - \mathbf{KDM})\,\mathbf{X}_{t-1|t-1}(j) + \mathbf{K}\left[\mathbf{DM}\cdot\mathbf{X}_{t-1} + \mathbf{DN}\cdot\boldsymbol{\varepsilon}_t + \mathbf{Q}e_{j,t}\right]. \tag{39}$$

The vector $\mathbf{X}_{t|t}(j)$ contains firm j's first-order expectations about the model's state variables. Integrating across firms yields the law of motion of the average expectation about $\mathbf{X}_{t|t}^{(1)}$:

$$\mathbf{X}_{t|t}^{(1)} = (\mathbf{M} - \mathbf{KDM})\,\mathbf{X}_{t-1|t-1}^{(1)} + \mathbf{K}\left[\mathbf{DM}\cdot\mathbf{X}_{t-1} + \mathbf{DN}\cdot\boldsymbol{\varepsilon}_t\right].$$

Note that $X_{t|t}^{(0:\infty)} = \left[X_t, X_{t|t}^{(1:\infty)}\right]'$ and that

$$X_t = \underbrace{\begin{bmatrix} \rho_a & 0 & 0 & \mathbf{0} \\ 0 & \rho_r & 0 & \mathbf{0} \\ 0 & 0 & \rho_g & \mathbf{0} \end{bmatrix}}_{\mathbf{R}_1} X_{t-1|t-1}^{(0:k)} + \underbrace{\begin{bmatrix} \sigma_a & 0 & 0 \\ 0 & \sigma_r & 0 \\ 0 & 0 & \sigma_g \end{bmatrix}}_{\mathbf{R}_2} \cdot \boldsymbol{\varepsilon}_t.$$

So by using the assumption of common knowledge in rationality, we can fully characterize the matrices $\mathbf{M}$ and $\mathbf{N}$:

$$\mathbf{M} = \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3k} \\ \mathbf{0}_{3k\times3} & (\mathbf{M} - \mathbf{KDM})\,|_{(1:3k,1:3k)} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{K}\,(\mathbf{DM})\,|_{(1:3k,1:3(k+1))} \end{bmatrix}, \tag{40}$$

$$\mathbf{N} = \begin{bmatrix} \mathbf{R}_2 \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{KDN}|_{(1:3k,1:3)} \end{bmatrix}, \tag{41}$$

where $\cdot|_{(n_1:n_2,m_1:m_2)}$ denotes the submatrix obtained by taking the elements lying between the n_1-th row and the n_2-th row and between the m_1-th column and the m_2-th column. Note that $\mathbf{K}$ in equation (40) and equation (41) denotes the steady-state Kalman gain matrix, which is obtained by iterating the equations (36) and (38) until convergence.

D The Laws of Motion for the Endogenous State Variables

In this section we introduce some useful results and characterize the law of motion (29) for the endogenous state variables, which are inflation $\hat{\pi}_t$, real output $\hat{y}_t$, and the (nominal) interest rate $\hat{R}_t$.

D.1 Preliminaries

The *assumption of common knowledge in rationality* ensures that agents use the actual law of motion of higher-order expectations to forecast the dynamics of the higher-order expectations. The following propositions turn out to be useful for what follows:

Proposition 1 $\mathbf{s}_{t|t}^{(s)} = \mathbf{v}_0 X_{t|t}^{(s:k+s)}$ *for any* $0 \leq s \leq k$.

Proof. We conjectured that $\mathbf{s}_t = \mathbf{v}_0 X_{t|t}^{(0:k)}$. Then common knowledge in rationality implies that $\mathbf{s}_{t|t}^{(s)} = \mathbf{v}_0 X_{t|t}^{(s:k+s)}$. ■

Since we truncate beliefs after the k-th order, we define the matrix $\mathbf{T}^{(s)}$ as follows:

$$\mathbf{T}^{(s)} \equiv \begin{bmatrix} \mathbf{0}_{3(k-s+1)\times 3s} & \mathbf{I}_{3(k-s+1)} \\ \mathbf{0}_{3s\times 3s} & \mathbf{0}_{3s\times(k+1-s)3} \end{bmatrix},$$

and we approximate the law of motion for $\mathbf{s}_{t|t}^{(s)}$ as $\mathbf{s}_{t|t}^{(s)} = \mathbf{v}_0 \mathbf{T}^{(s)} X_{t|t}^{(0:k)}$ for any $0 \leq s \leq k$.

Proposition 2 *The following holds true:* $\mathbf{s}_{t+h|t}^{(s)} = \mathbf{v}_0 \mathbf{M}^h X_{t|t}^{(s:k+1)}$ *for any* $0 \leq s \leq k$.

Proof. We conjectured that $\mathbf{s}_{t+h} = \mathbf{v}_0 X_{t+h|h+1}^{(0:k)}$. Given equation (13), it follows that $\mathbf{s}_{t+h} = \mathbf{v}_0 \left(\mathbf{M}^h X_{t|t}^{(0:k)} + \mathbf{N}\boldsymbol{\varepsilon}_{t+1} \right)$. Common knowledge in rationality implies that repeatedly taking firms' expectations and then averaging across firms leads to an expression for the law of motion of the average higher-order expectations: $\mathbf{s}_{t+h|t}^{(s)} = \mathbf{v}_0 \mathbf{M}^h X_{t|t}^{(s:k+1)}$ for any s. ■

Since we truncate beliefs after the k-th order, we can approximate the law of motion for the average higher-order expectations as $\mathbf{s}_{t+h|t}^{(s)} = \mathbf{v}_0 \mathbf{M}^h \mathbf{T}^{(s)} X_{t|t}^{(0:k)}$ for any $0 \leq s \leq k$.

D.2 The Laws of Motion of the Endogenous State Variables

The laws of motion of the three endogenous state variables, which are inflation $\hat{\pi}_t$, real output $\hat{y}_t$, and the (nominal) interest rate $\hat{R}_t$, are given by the Euler equation (8), the Phillips curve (7), and the Taylor Rule (9). We want to write this system of linear equations as

$$\Gamma_0 \mathbf{s}_t = \Gamma_1 \mathbb{E}_t \mathbf{s}_{t+1} + \Gamma_2 X_{t|t}^{(0:k)}, \tag{42}$$

where $\mathbf{s}_t \equiv \left[\hat{\pi}_t, \hat{y}_t, \hat{R}_t\right]'$. It is obvious how to write equations (8) and (9) in the form (42). However, how to write the Phillips curve (7) in the form (42) requires a bit of work. First, note that given Propositions 1−2 and the equation $\widehat{mc}_{t|t}^{(k)} = \hat{y}_{t|t}^{(k)} - \widehat{a}_{t|t}^{(k-1)}$, the imperfect-common-knowledge Phillips curve (7) can be rewritten as follows:

$$
\begin{aligned}
\mathbf{a}_0 X_{t|t}^{(0:k)} &= (1-\theta)(1-\beta\theta)\sum_{s=0}^{k-1}(1-\theta)^s \mathbf{1}_2^T \left[\mathbf{v}_0 \mathbf{T}^{(s+1)} X_{t|t}^{(0:k)}\right] + \\
&-(1-\theta)(1-\beta\theta)\sum_{s=0}^{k-1}(1-\theta)^s \left[\boldsymbol{\gamma}_a^{(s)\prime} X_{t|t}^{(0:k)}\right] \\
&+\beta\theta\sum_{s=0}^{k-1}(1-\theta)^s \mathbf{1}_1^T \left[\mathbf{v}_0 \mathbf{M}\mathbf{T}^{(s+1)} X_{t|t}^{(0:k)}\right],
\end{aligned}
$$

where $\mathbf{1}_1^T = [1,0,0]$, $\mathbf{1}_2^T = [0,1,0]$, and $\boldsymbol{\gamma}_a^{(s)} = \left[\mathbf{0}_{1\times 3s}, (1,0,0), \mathbf{0}_{1\times 3(k-s)}\right]'$. The following restrictions upon vectors of coefficients $\mathbf{a}_0$ and $\mathbf{a}_1$ can be derived from the rewritten Phillips curve:

$$
\widehat{\pi}_t = \left[(1-\theta)(1-\beta\theta)\left[\boldsymbol{\nu}\mathbf{m}_1 - \left(\sum_{s=0}^{k-1}(1-\theta)^s \boldsymbol{\gamma}_a^{(s)\prime}\right)\right] + \beta\theta\boldsymbol{\nu}\mathbf{m}_2\right] X_{t|t}^{(0:k)}, \tag{43}
$$

where we define:

$$
\begin{aligned}
\mathbf{m}_1 &\equiv \begin{bmatrix} \mathbf{1}_2^T \mathbf{v}_0 \mathbf{T}^{(1)} \\ (1-\theta)\left[\mathbf{1}_2^T \mathbf{v}_0 \mathbf{T}^{(2)}\right] \\ \vdots \\ (1-\theta)^k \left[\mathbf{1}_2^T \mathbf{v}_0 \mathbf{T}^{(k)}\right] \end{bmatrix}, \ \mathbf{m}_2 \equiv \begin{bmatrix} \mathbf{1}_1^T \mathbf{v}_0 \mathbf{M}\mathbf{T}^{(1)} \\ (1-\theta)\left[\mathbf{1}_1^T \mathbf{v}_0 \mathbf{M}\mathbf{T}^{(2)}\right] \\ \vdots \\ (1-\theta)^k \left[\mathbf{1}_1^T \mathbf{v}_0 \mathbf{M}\mathbf{T}^{(k)}\right] \end{bmatrix}, \\
\boldsymbol{\nu} &\equiv \mathbf{1}_{1\times k}.
\end{aligned}
$$

E Signaling Effects Implied by the Prior

The aim of this section is to assess the extent to which the likelihood favors the idea that signaling effects associated with monetary policymaking are important. In other words, we want to rule out the possibility that this finding totally follows from the choice of the prior. Figure 14 reports the response of inflation (top left graph), inflation expectations (top middle and right graphs), and the average HOE about the three exogenous state variables (bottom graphs) to a monetary shock that raises the policy rate by 25 basis points when the DIM parameters are set to equal the *prior means* reported in Table 1. The vertical bars in the upper graphs are related to the decomposition (15) and isolate the inflationary effects of the change in

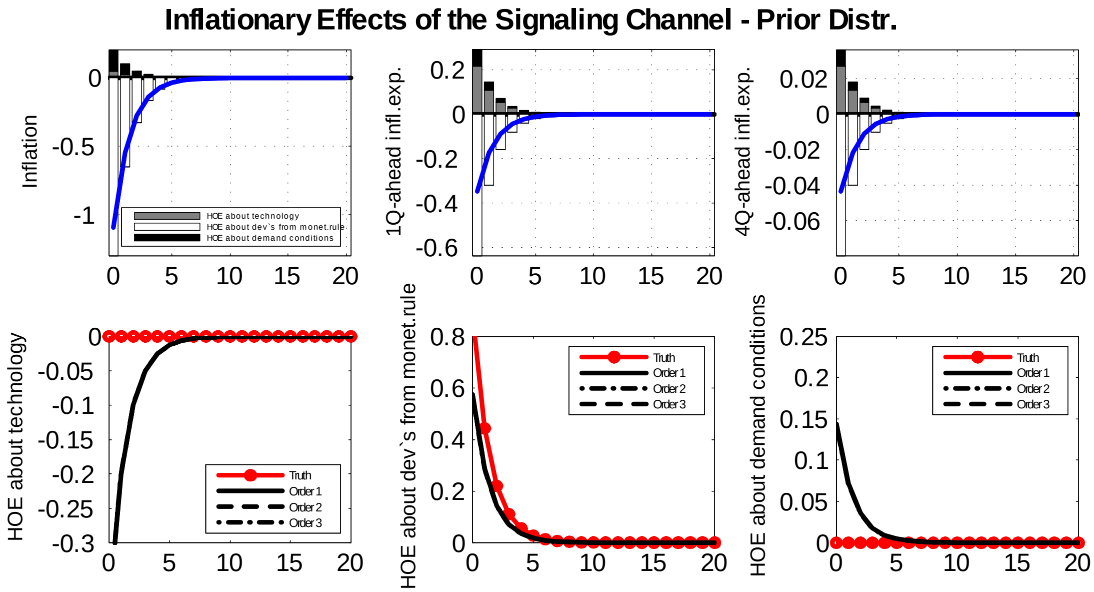

Figure 14: Impulse response functions of inflation, inflation expectations, and average higher-order expectations (HOE) to a monetary shock that raises the interest rate by 25 bps. Parameter values are set equal to the prior mean. *Top graphs:* The solid blue line is the response of inflation (left graph), one-quarter-ahead inflation expectations (middle graph), and four-quarters-ahead inflation expectations (right graph) and its decomposition to the effects of average HOE about the level of aggregate technology $\widehat{a}_t$, the deviation from the monetary rule $\widehat{\eta}_{r,t}$, and the demand conditions $\widehat{g}_t$ (i.e., the vertical bars). *Bottom graphs:* Response of the true level of aggregate technology (left graph), the deviation from the monetary rule (middle graph), and the demand conditions (right graph) and the associated average HOE up to the third order. The vertical axis in the top graphs reports units of percentage points of annualized rates. The vertical axis in the bottom graphs reports the percent deviations from the value of the corresponding variable at the deterministic steady state equilibrium.

the average higher-order expectations (HOE) about technology $\mathbf{v}_a' \partial X_{t+h}^a / \partial \varepsilon_{r,t}$ (the gray bars), those about the deviation from the monetary rule $\mathbf{v}_m' \cdot \partial X_{t+h}^m / \partial \varepsilon_{r,t}$ (the white bars), and those about the demand conditions $\mathbf{v}_g' \cdot \partial X_{t+h}^g / \partial \varepsilon_{r,t}$ (the black bars) for h periods after the monetary shock.

Note that the sum of the gray vertical bars and the black vertical bars captures the effect of the signaling channel on inflation and inflation expectations. In Figure 14, these effects appear to be fairly small, especially for the response of inflation that is almost entirely explained by the shock and the average HOE about the deviation from the monetary rule on inflation (i.e., the white bars). More importantly, comparing Figure 14 with Figure 6, which is based on using the posterior mean to calibrate the DIM parameters, reveals that the Bayesian updating boosts the inflationary effects of the average HOE about the demand conditions (i.e., the black bars). Recall that high black bars imply that firms mostly interpret a rise in the policy rate as the central bank's response to a positive demand shock. As shown before, this result gives rise to important inflationary effects of the signaling channel in the estimated model. Such signaling effects of monetary policy dampen the response of inflation to monetary shocks and hence boost the real effects of monetary disturbances. The comparison of Figure 14 with Figure 6 reveals that high and persistent real effects of monetary shocks are not originated by the prior and actually appear to be mainly driven by the likelihood through the Bayesian updating.

F DIM and PIM Estimated with a Narrower Data Set

	DIM - Posterior			PIM - Posterior			Prior		
Name	Mean	5%	95%	Mean	5%	95%	Type	Mean	Std.
θ	0.3387	0.2615	0.4508	0.5297	0.4687	0.5907	$\mathcal{B}$	0.50	0.30
ϕ_π	1.0289	1.0001	1.1261	1.6471	1.4820	1.7939	$\mathcal{G}$	1.50	0.10
ϕ_y	0.1951	0.1199	0.2648	0.2710	0.0973	0.4377	$\mathcal{G}$	0.25	0.10
ρ_r	0.6643	0.6211	0.7149	0.4765	0.4111	0.5446	$\mathcal{B}$	0.50	0.20
ρ_a	0.9138	0.8491	0.9680	0.9927	0.9868	0.9991	$\mathcal{B}$	0.50	0.20
ρ_g	0.7291	0.6814	0.7737	0.9116	0.8697	0.9538	$\mathcal{B}$	0.50	0.20
$100\sigma_a$	0.3652	0.2169	0.6590	0.9808	0.8738	1.0837	$\mathcal{IG}$	0.80	1.50
$100\widetilde{\sigma}_a$	0.2035	0.0010	1.4803	–	–	–	$\mathcal{U}$	50.00	28.87
$100\sigma_g$	1.5582	1.4281	1.7191	1.4953	0.9122	2.0525	$\mathcal{IG}$	0.80	1.50
$100\widetilde{\sigma}_g$	35.3479	34.8243	35.8393	–	–	–	$\mathcal{U}$	50.00	28.87
$100\sigma_r$	0.4797	0.4372	0.5464	0.6858	0.5884	0.7841	$\mathcal{IG}$	0.80	1.50
$100\ln\gamma$	0.5087	0.3906	0.6214	0.4098	0.3308	0.4880	$\mathcal{N}$	0.62	0.10
$100\ln\pi_*$	0.7131	0.5994	0.8868	0.7847	0.6511	0.9127	$\mathcal{N}$	0.65	0.10

Table 3: Posterior statistics for the parameters of the dispersed information model (DIM) and the perfect information model (PIM) using a data set that does not include inflation expectations from the Survey of Professional Forecasters

Table 3 reports the posterior statistics for the DIM and the PIM parameters when the two models are estimated using a narrower data set that does not include the SPF as a measure of price setters' inflation expectations. For the estimation, the same prior as the one detailed in Table 1 is adopted.