Cooperation, R&D Spillovers and Antitrust Policy*

Ángel L. López† Xavier Vives‡

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Preliminary Version

Abstract

We consider a model of process (cost-reducing) R&D investments with spillovers in Cournot oligopoly, and in which R&D cooperation cannot be disentangled from cooperation in the product market because of cross-shareholdings or because cooperation in R&D extends to cooperation in the product market. We characterize how R&D and output behave in response to a change in the degree of cooperation. We derive the threshold values of spillover above which some cooperation in both dimensions is optimal for welfare and consumers, and examine the optimal degree of toughness of the antitrust policy. If the objective is to maximize total surplus then there is scope for cooperation in both dimensions when spillovers are sufficiently large (and the scope is larger the more firms there are in the market), but if the objective is to maximize consumer surplus, then the scope for cooperation is greatly reduced. Furthermore, entry need not optimally induce more cooperation under the consumer surplus standard. Finally, our results show that the socially optimal degree of cooperation increases with the number of firms, the elasticity of demand and innovation function, and the intensity of spillover effects.

JEL classification numbers: D43, L13, O32

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†López: Departament d'Economia Aplicada, Universitat Autònoma de Barcelona, and Public-Private Sector Research Center, IESE Business School. E-mail address: angelluis.lopez@uab.cat

‡Vives: Public-Private Sector Research Center, IESE Business School.

1 Introduction

Any agreement on price or output that harms consumers is typically forbidden by antitrust laws. By contrast, R&D cooperation is widely allowed (and even encouraged) by public authorities.[1] R&D cooperation in the presence of spillovers is generally seen as welfare enhancing in part due to the results of the theoretical IO literature. d'Aspremont and Jacquemin (1988) is the seminal paper that provides a theoretical basis for the favorable attitude of antitrust authorities towards cooperative R&D; they show that when spillovers are high enough, R&D cooperation (with subsequent competition at the output stage) leads to more output, innovation, and welfare. The reason is that in the presence of spillovers, firms (partially) benefit from the innovation efforts of their rivals. As a result, if spillovers are high, firms may free ride and lower their investment in innovation. However cooperative R&D allows them to internalize externalities, and thereby, to preserve their incentives to invest in R&D.

d'Aspremont and Jacquemin's paper and some previous works[2] inspired a vast literature[3] that has thrown light on many aspects of R&D cooperation in oligopolistic markets. The main objective of this literature is to examine underprovision of R&D and the welfare effects of moving from a non-cooperative to a (full) cooperative regime. In an oligopoly model of process (cost-reducing) R&D investments with spillovers and quantity competition we aim to answer the following questions: What degree of cooperation is optimal in the presence of R&D investment and spillovers? How does it depend on structural parameters (such as demand and cost conditions, technological opportunity of industry and level of spillovers)? How does it depend on the objective of the competition authority (be it maximize total surplus or consumer surplus)? In our model, cooperation among firms may come from the presence of cross-ownership or because R&D cooperation extends to cooperation in the product market. Intermediate degrees of cooperation may arise as a result of toughness of competition policy in terms of limiting cross-shareholdings or increasing the degree of activism destined for persecuting collusion in the product market. Intermediate degrees of cooperation may be socially optimal as they help to

[1]For a comparison of US policy in cooperative R&D with the policy of the European Union and Japan see Martin (1996). For Europe see also Jacquemin and Soete (1994).

[2]Brander and Spencer (1984), Spence (1984) and Katz (1986) are pioneering works in the analysis of multiple stage strategic investments with spillovers.

[3]Among others, Suzumura (1992) extends the analysis to multiple firms and general demand and cost functions in Cournot competition. Ziss (1994) does the same but also considering product differentiation and price competition. Kamien et al. (1992) analyze the effects of R&D cartelization and research joint ventures. Leahy and Neary (1997) present a general analysis of the effect of strategic behavior and cooperative R&D in the presence of price and output competition. They also study optimal public policy towards R&D in the form of subsidies. For a survey see Gilbert (2006), Suetens (2004a), and de Bondt (1996).

internalize externalities and therefore to ameliorate the problem of under-investment in R&D, which may be important. In particular, using a panel of U.S. firms from 1981 to 2001, Bloom, Schankerman, and Van Reenen (2013) find that technology spillover effects are much larger than product market spillovers, and that the socially optimal level of R&D is between two and three times as high as the level of observed R&D.

There is growing interest among competition authorities in assessing the competitive effects of financial interest, in part due to the rapid growth of private-equity investment observed in recent years, in which private-equity firms often hold partial ownership interests in competing firms (Wilkinson and White, 2007), and because of some notorious cases (such as Ryanair's acquisition of Aer Lingus's stock) that have triggered discussion in Europe on the potential anticompetitive effects of partial ownership. Indeed, minority shareholdings are widespread in many industries, in particular they have become more important in telecommunications and high technology industries (Salop and O'Brien, 2000). There are generally two types of minority shareholding: (i) Financial interest, which refers to the right of the acquiring firm to receive a share of the profits of the target firm; (ii) Corporate control, which refers to the right of the acquiring firm to control or influence the target firm's decisions (in prices, output, product selection and other competition variables). We focus on the first type, also called *silent financial interest* or *passive structural links.*

In Canada and the United States minority shareholding is subject of scrutiny under merger control rules. In particular, in the U.S. minority shareholding are often examined under Section 7 of the Clayton Act and the Hart-Scott-Rodino Act.[4] Although no threshold is clearly established, acquisitions of less than 25%, and at least of 15%, have been found to violate Section 7 (Salop and O'Brien [2000], Miller, Raven and Went, [2012]). The European Commission, however, does not have competence under its merger control rules to examine passive investments, although it recognizes that *"Acquisitions of non-controlling minority shareholdings may in some cases lead to anticompetitive effects"* (EC, June 2013), and currently proposes[5] to extend the scope of the Merger Regulation to be able to intervene in some potential problematic cases such as those involving minority shareholding among competitors or in a vertical relationship.[6] Yet

[4]Section 7 prohibits acquisitions (of any part) of a company's stock that "may" substantially lessen competition by either (a) enabling the acquiring firm to raise prices or decrease output by controlling or influencing the target firm; or (b) altering the incentives of the acquiring firm to compete with the target firm.

[5]Commission consults on possible improvements to EU merger control to deal with anticompetitive effects arising from minority shareholding (Commission Staff Working Document "Towards more effective EU merger control", Brussels, 25.6.2013 SWD(2013) 239 final).

[6]Currently, the EC can only consider the competitive effects of (pre-existing) minority shareholdings in the

there are European countries, such as Austria, Germany and the UK, in which national merger control rules give competition authorities the competence to examine minority shareholdings.[7]

Anticompetitive effects of cross-ownership were first derived in Reynolds and Snapp (1986), and Bresnahan and Salop (1986). They consider the Cournot model and show that the presence of financial interests in the industry may result in less output and higher prices (even if interests are relatively small). The reason is that the competitive decision of one firm that has financial interests in its competitors' profit will take into account that by reducing output (or increasing the price) it will increase the competitors' profit and therefore its financial profit. While the anticompetitive effects tend to be weaker than those of a merger, minority shareholdings do not offer significant efficiencies as those that may arise with mergers (like rationalization or avoiding cost duplication). The common view is therefore that passive investments tend to lessen competition. Contrary to this view, we show that minority shareholdings may increase total surplus, and even consumer surplus, in industries where investment in R&D is important and spillovers are sufficiently high.

An attractive feature of considering partial cross-ownership is that it also encompasses the case in which R&D cooperation cannot be disentangled from cooperation in the product market. When shareholdings are symmetric across firms, maximizing the profit (with financial interests) of a given firm is equivalent to maximizing its own profit (i.e., with no financial interests) and a fraction λ of the profit of each of the remaining firms, where the size of the parameter λ will depend on the size of the shareholdings. In most of the literature on R&D, cooperation does not necessarily lead to coordination in the product market. Nevertheless, it is an old suspicion that R&D cooperation may facilitate coordination in the product market (see e.g. Pfeffer and Nowak [1976], Grossman and Shapiro [1986], Jacquemin [1988], Brodley [1990], Geroski [1992], and Jacquemin and Soete [1994]).[8] If this suspicion turns out to be true, then optimal public policy must balance a trade-off between market power and efficiency. Indeed there is growing evidence that R&D cooperation facilitates product market cooperation, such as empirical results

context of a notified merger (in which the merging firms have stakes on a third firm). EC (2013) highlights that there are many instances in which the merger is allowed on the basis of remedies that entail a divestiture of the (pre-existing) minority shareholding.

[7]E.g., the minority shareholding of Ryanair in Aer Lingus and the BskyB's acquisition of 17.9% of ITV would result according to the UK CC in a substantial lessening of competition (resp. on routes between Great Britain and Ireland and in the UK TV market). In August 2013, UK CC ordered Ryanair to sell its 29.8% stake in Aer Lingus down to 5%.

[8]Martin (1995), van Wegberg (1995), Greenlee and Cassiman (1999), Cabral (2000), Lambertini et al. (2002), and Miyagiwa (2009) analyze various channels through which cooperative R&D may facilitate coordination in the product market.

(Goeree and Helland [2010], Duso, Röller and Seldeslachts [2010]), experimental results (Suetens, 2008), antitrust cases[9], and the theory of ancillary restraints[10] and multimarket contact.[11] Our analysis therefore extends the traditional framework in two directions: no separation between coordination in R&D and output (be it because of cross-ownership or because cooperation in R&D extends to cooperation in the product market), and the presence of intermediate degrees of cooperation as a result of toughness of competition policy.[12]

Apart from limiting cross-ownership, one could think of λ as a function of an antitrust policy that limits the degree of collusion. Indeed, Besanko and Spulber (1989) show that when collusive behavior is unobservable and production costs are private information, the optimal antitrust authority may induce firms to collude up to some intermediate degree. The reason is that in the presence of asymmetric information, the antitrust authority does not know *a priori* if price is high because of price fixing or because of marginal costs are high. As monitoring is expensive and resources are limited, antitrust policy is committed to sue with a probability that depends on the price observed in the industry. If firms must pay a fine when collusion is detected and there is an upper limit on the level of that fine, then they may find it optimal to set a lower markup in order to reduce the risk of prosecution, which results in imperfect collusion.[13,14] We explore the model in terms of λ rather than in terms of shareholdings. In our framework, λ can take any value between 0 (no collusion in quantity and R&D) and 1 (full collusion in quantity

[9] There are many examples of cases in which R&D collaboration lead to anticompetitive abuses. Goeree and Helland (2010) gather a number of recent cases in the petroleum industry, the computer industry, the semiconductor memory market and in telecommunications.

[10] For example, when the research joint venture stipulates downstream market division for any patents that may result from the venture, or when there are collateral agreements that impose cross-licensing of old patents or a per-unit output royalty for using new patents since this chills the incentives for firms to increase output (see Grossman and Shapiro, [1986], and Brodley [1990]).

[11] Firms can sustain collusion more easily when they interact in multiple markets as compared to the situation in which they interact in one market only (Bernheim and Whinston [1990]; see also Spagnolo [1999]). Vonortas (2000) study whether large diversified firms use research joint ventures (RJVs) to create links to competitors in order to facilitate collusion across many markets. Using an extensive database of RJVs registered with the U.S. Department of Justice and the Federal Trade Commission from the mid-1980s to the mid-1990s, he finds evidence that the scope for collusion in the product market is larger in the presence of multi-R&D project and multimarket contact. Also, Snyder and Vonortas (2005) show that multiproject contact can facilitate explicit collusion. The reason is that multiproject contact allows firms to reduce the heterogeneity in private information, which in turn makes collusive agreements more efficient (see also Matsushima [2001]).

[12] While we focus on R&D investments, examining a framework in which cooperation in one dimension may facilitate cooperation in another dimension is relevant insofar as this situation may arise in other interesting applications. A clear case is joint marketing alliances (patent pools). In a recent paper, Rey and Tirole (2013) study tacit collusion in sales (or licensing), and its implications for users and society. In particular, they examine how independent marketing and joint marketing alliances may facilitate the sustainability of tacit collusion.

[13] In a different context, Besanko and Spulber (1990), consumers do not observe cartel costs and therefore have imperfect information about the existence of a price-fixing agreement. In this case it is also found that the cartel's pricing decision is constrained as the cartel's equilibrium price decreases with the level of the penalties. See also Souam (2000), Harrington (2004, 2005), and Schinkel and Tuinstra (2006).

[14] Also, Friedman, Jehiel and Thisse (1995) show that an intermediate antitrust policy maximizes social welfare when firms can collude in prices and must choose their product specification.

and R&D); intermediate values of λ represent imperfect collusion resulting from the toughness of antitrust policy. Besides, the analysis of imperfect collusion is also relevant once we recognize that firms face a number of problems that may make it impossible for them to achieve full collusion, thereby forcing them to settle for a lesser degree of cooperation.[15]

Another positive aspect of introducing the parameter λ is that it can be considered as a reduced form of repeated games with limited collusive behavior because of the (necessary) low discount factor. Since the extent of collusion is monotone in the discount factor, we can associate a high discount factor with a high λ. On the other hand, in a Cournot model with conjectural derivative, the parameter λ will be the constant elasticity of conjectural variation, which can be used to estimate the degree of cooperation in the industry (see e.g. Martin, 2001).[16] Finally, parametrizing the continuum of outcomes with varying competitive toughness is also interesting for dealing with the equilibrium indeterminacy that comes from the choice of the equilibrium concept itself (e.g., from Cournot to price competition in static games).[17]

We will examine static and two-stage competition, however we are primarily interested in the static model. The reason is that R&D investment effort of each firm is frequently not (perfectly) observable, moreover R&D programs are typically long-term investment plans and it is difficult to write a contract on future actions. Cabral (2000), Dasgupta and Stiglitz (1980), Hartwick (1984), Spence (1984), Levin and Reiss (1988), Leahy and Neary (1997), Vives (2008), and Ziss (1994), among others, analyze simultaneous models (in the presence or absence of R&D spillovers), but assuming that R&D cooperation does not necessarily lead to output cooperation.

The plan of the paper is as follows. We begin by presenting the general model in Section 2. We examine how output and R&D in equilibrium behave in response to a change in the degree of cooperation, λ. We show that R&D increases with the degree of cooperation when the level of spillover is higher than the cost pass-through coefficient multiplied by the perceived or internalized effect on each firm's marginal cost of a unit increase in R&D by all firms over the number of firms. We also show that output increases with the degree of cooperation when

[15]Imperfect information (or moral hazard) and incomplete information (or adverse selection) may limit the ability of firms to enforce the joint-profit-maximizing outcome. Also, leniency programs affect the degree of collusion that is attainable in the industry.

[16]In a recent paper, Michel (2013) estimates the degree of profit internalization after ownership changes in differentiated product industries. He allows each firm's objective function to potentially depend on other firm's profits by inserting the parameter λ_{ij}, which is the degree to which brand i takes into account brand j's profits when setting its optimal price.

[17]d'Aspremont and Dos Santos (2009) provide the canonical representation of oligopolistic competition that is game-theoretically founded and nets a continuum of theories of oligopolies with varying competitive toughness. See also d'Aspremont and Dos Santos (2010).

spillovers are sufficiently high that the effect on each firm's marginal cost of a unit increase in R&D by all firms is larger than a given measure of the slope of the optimal locus of output and R&D (so the positive effect of cooperation on R&D investment dominates its negative effect on output level). Next, we show that whenever an increase in λ decreases R&D expenditures, it also decreases output. However, when raising λ stimulates investment in R&D, then output may or may not increase depending on the size of the spillovers. These findings allow us to identify three regions that determine the response of R&D and output to different degrees of cooperation. Although in two of them it is not immediately clear whether a higher degree of cooperation will increase each firm's profit, we show that at equilibrium and independently of the intensity of the R&D spillover effects, each firm's profit increases with the degree of cooperation. This result has a clear policy implication: as firms have incentives to acquire (minority) shareholdings in the industry, there is scope for policy intervention so that the optimal degree of cooperation is achieved. In terms of spillovers, we find that at equilibrium the curvature of the innovation function is key to the response of output and R&D to the intensity of spillover effects.

In Section 3 we present three model specifications (the d'Aspremont-Jacquemin and the Kamien-Muller-Zang model specifications, and a constant elasticity model) as special cases of our framework, and conduct a comparative statics analysis for each of them. Section 4 examines the socially optimal degree of cooperation. We characterize the threshold values of spillover below which no cooperation is optimal for consumers and total surplus, and show that the latter is higher than the former. We also characterize the threshold value above which a positive λ will be socially optimal. We then turn to apply these results to our three model specifications. We first obtain the threshold values for each of them. We then show that under the d'Aspremont-Jacquemin and the Kamien-Muller-Zang model specifications we cannot exclude the possibility that inducing full cooperation (i.e. setting $\lambda = 1$) maximizes social welfare, but this will never be the case in the constant elasticity model that we consider. Numerical simulations illustrate and confirm these results, and allow us to examine the optimal degree of toughness of the antitrust policy. If the objective is to maximize total surplus then there is scope for cooperation in both dimensions when spillovers are sufficiently large (and the scope is larger the more firms there are in the market). Regarding consumer surplus, cooperation should be generally forbidden unless, and depending on the functional forms of demand, cost and investment, the number of firms or spillover effects are large enough. Finally, taken together our results indicate that the optimal degree of cooperation in terms of total surplus increases with the number of firms, the elasticity of demand and innovation function, and the intensity of spillover effects. Section 5 extends our model to consider two stages: in the first stage firms choose and commit to their R&D investments, and in the second stage they compete in output. We show that results are generally robust to this extension with the plus that the degree of toughness of the antitrust policy is moderated in the two-stage model when spillovers are high. Section 6 concludes.

2 Framework and equilibrium

We consider an industry composed of $n \geq 2$ identical firms, in which each firm $i = 1..n$ chooses simultaneously the R&D intensity (x_i) and quantity (q_i). Firms produce an homogeneous good with smooth inverse demand function $f(Q)$, with $Q = \sum_i q_i$. We assume the following

A.1. $f(Q)$ is twice continuously differentiable with $f'(Q) < 0$ for all $Q \geq 0$ such that $f(Q) > 0$, and

$$\delta(Q) \equiv \frac{Qf''(Q)}{f'(Q)}$$

is constant.

The elasticity of the slope of the inverse demand function is $-\delta$, and so it is equivalent to the relative degree of convexity or curvature of the inverse demand function. δ is also related with the marginal consumer surplus when output increases, that is, with $ms = -f'(Q)Q$. Letting ϵ_{ms} be the elasticity of the inverse marginal consumer surplus function (so $\epsilon_{ms} = ms/(ms'Q)$), Weyl and Fabinger (2013) argues that ϵ_{ms} measures the curvature of the logarithm of demand. Under A.1. we can write $1/\epsilon_{ms} = 1 + \delta$; when demand is log-concave $1 + \delta > 0$ and when demand is log-convex $1 + \delta < 0$. Furthermore, if demand is concave (convex) then $\delta > (<)0$. Assumption A.1. is always satisfied by inverse demand functions which are linear or constantly elastic. In particular, the family of inverse demand functions where $\delta(Q)$ is constant can be represented as

$$f(Q) = \begin{cases} a - bQ^{\delta+1} & \text{if} \quad \delta \neq -1 \\ a - b\log Q & \text{if} \quad \delta = -1 \end{cases},$$

where a is a non-negative constant and $b > 0$ ($b < 0$) if $\delta \geq -1$ ($\delta < -1$). We make the following two additional assumptions.

A.2. The marginal production cost or innovation function $c(\cdot)$ is independent of output and decreasing in own R&D and that of its rival in the following manner: $c(x_i + \beta \sum_{j \neq i} x_j)$ with $c' < 0$, $c'' \geq 0$, and $0 \leq \beta \leq 1$ ($i \neq j$).

A.3. The cost of investment is given by the function $\Gamma(x_i)$ with $\Gamma(0) = 0$, $\Gamma' > 0$ and $\Gamma" \geq 0$.

The parameter β represents the spillover level of the R&D activity. The presence of β is motivated by the fact that the outcome of R&D investment by one firm typically spills over and benefits other firms. Certainly, the intensity of spillover levels is quite heterogeneous across industries, which could be due to a negative relationship between spillover and patent protection levels: industries with low patent protection tend to have higher spillover levels (e.g., those that are low-tech) than industries with high patent protection (Griliches, 1990). In our model, as in most of the literature on R&D, we allow R&D outcomes to be imperfectly appropriable to a degree that can take values between 0 and 1. The particular case in which $\beta = 1$ represents the case in which firms form a Research Joint Venture (RJV) by which they fully share R&D outcomes among them and avoid duplication of R&D efforts. We may distinguish between six different cases: i) $\lambda = 0$ and $\beta \in [0, 1)$, firms compete and R&D outcomes may be imperfectly appropriable; ii) $\lambda = 0$ and $\beta = 1$, firms compete but they form an RJV and share R&D outcomes; iii) $\lambda = 1$ and $\beta \in [0, 1)$, firms form a cartel but not an RJV; iv) $\lambda = \beta = 1$, firms form a cartelized RJV; v) $\lambda \in (0, 1)$ and $\beta \in [0, 1)$, there is some degree of cooperation among firms (because of minority shareholdings or limited antitrust policy) and R&D outcomes may be imperfectly appropriable; vi) $\lambda \in (0, 1)$ and $\beta = 1$, as in the previous case there is some degree of cooperation among firms but now they form an RJV and share R&D outcomes.

Each firm's profit is given by

$$\pi_i = f(Q)q_i - c(x_i + \beta \sum_{j \neq i} x_j)q_i - \Gamma(x_i).$$

Let ω_{ji} be the jth firm's ownership interest in the ith firm. Total profit of firm i (including financial interests) is:

$$\Pi_i = \left(1 - \sum_{j \neq i}^{n} \omega_{ji}\right) \pi_i + \sum_{j \neq i}^{n} \omega_{ij} \pi_j.$$

Consider the symmetric case: $\omega_{ij} = \omega_{ji} = \omega$, then $\Pi_i = (1 - (n-1)\omega)\, \pi_i + \omega \sum_{j \neq i}^{n} \pi_j$. Maximizing Π_i is equivalent to maximizing,

$$\phi_i = \pi_i + \lambda \sum_{j \neq i} \pi_j, \tag{1}$$

where $\lambda \equiv [\omega/(1 - (n-1)\omega)] \in [0, 1]$. Two remarks are in order. First, in a symmetric equilibrium with identical firms, $\pi_i = \pi_j = \pi^*$ for $i \neq j = 1, .., n$, we have that $\Pi_i^* = (1 - (n - 1)\omega)\pi^* + \omega(n-1)\pi^* = \pi^*$. Second, the upper bound of cross-ownership is $\omega = 1/n$, in which case $\lambda = 1$, and n identical firms will maximize total joint profit. E.g., with 10 firms in the market, it suffices that each of them has 10% of each rival's stock to yield the monopoly level of output in equilibrium. In particular, the more firms there are in the market, the lower degree of partial cross-ownership is needed to achieve the monopoly level of output.

We explore the model in terms of λ, so the objective function is (1). Write $\Lambda = 1 + \lambda(n-1)$, $B = 1 + \beta(n-1)$ and $\tau = 1 + \lambda(n-1)\beta$. A symmetric interior equilibrium (Q^*, x^*) must solve the first-order necessary conditions:

$$f(Q^*)\left(1 - \varepsilon(Q^*)\frac{\Lambda}{n}\right) = c(Bx^*) \tag{2}$$

$$-c'(Bx^*)\frac{Q^*\tau}{n} = \Gamma'(x^*), \tag{3}$$

where $\varepsilon(Q^*) \equiv -f'(Q)(Q/f(Q))$ is the inverse of the elasticity of demand. Let $\alpha_z^i(\cdot)$, $\rho_z^i(\cdot)$ and $\varphi_{xq}(\cdot)$ be defined by $\alpha_z^i(\cdot) = (\partial^2/\partial z_i^2)\phi_i$, $\rho_z^i(\cdot) = (\partial^2/\partial z_i \partial z_j)\phi_i$, $\varphi_{\lambda z}^i(\cdot) = (\partial^2/\partial\lambda\partial z_i)\phi_i$ (with $z = q, x$) and $\varphi_{xq}^i(\cdot) = (\partial^2/\partial x_i \partial q_i)\phi_i$ $(i \neq j; i, j = 1, 2, ..., n)$. We assume that the following stability conditions hold: $\Delta_q \equiv \alpha_q + (n-1)\rho_q < 0$ and $\Delta_x \equiv \alpha_x + (n-1)\rho_x < 0$, and that

$$\Delta(Q^*, x^*) = \Delta_q \Delta_x - \left(\varphi_{xq}\right)^2 \tau B > 0. \tag{4}$$

Together imply that (2) and (3) have unique solution. If $\Delta(Q^*, x^*) > 0$, then we say that the equilibrium is regular, the meaning of which will become clear in the comparative statics analysis below. In particular, we assume that there is a unique regular symmetric interior equilibrium (Q^*, x^*) − the characterization of such an equilibrium will be the focus of the paper.

Comparative statics with respect to the degree of cooperation. We are particularly interested in how output and R&D in equilibrium behave in response to a change in λ. If we totally differentiate the two first-order necessary conditions, then after some manipulations we get

$$\frac{\partial q^*}{\partial \lambda} = \frac{1}{\Delta}[\varphi_{\lambda x}\varphi_{xq}B - \varphi_{\lambda q}\Delta_x] \tag{5}$$

$$\frac{\partial x^*}{\partial \lambda} = \frac{1}{\Delta}[\varphi_{\lambda q}\varphi_{xq}\tau - \varphi_{\lambda x}\Delta_q]. \tag{6}$$

The sign of these derivatives is not immediately clear. To see this, notice that for a given

x, λ has a negative effect on output: $\varphi^i_{\lambda q} = f'(Q)\sum_{j\neq i} q_j < 0$. This is the well-known effect of reducing output so as to increase price. Conversely, for a given q, λ has a positive effect on investment: $\varphi^i_{\lambda x} = -\beta\sum_{j\neq i} c'(x_j + \beta\sum_{k\neq j} x_k)q_j > 0$. This is the internalizing externalities effect, which, as it is clear from the previous equation, depends directly on the size of the spillovers. The final sign of the impact of λ on the equilibrium values of output and R&D per firm will depend on which of the two previous effects dominates. (Notice that $\varphi^i_{xq} > 0$.) When β is small, the positive effect on investment is small, so the negative effect on output dominates. Then, q^* decreases with λ, and as a result firms also invest less when λ increases since the benefit that firms obtain from investing in R&D decreases proportionally with output. Let us call the region in which this case occurs (i.e. when $\partial q^*/\partial\lambda < 0$ and $\partial x^*/\partial\lambda < 0$) as RI. Suppose now that β is sufficiently high, so that the positive effect on R&D reduces significantly the unit cost of production and this in turn stimulates output. Two effects are present in this case. On the one hand, the first mentioned effect still exists: firms want to reduce output so as to increase price in equilibrium. On the other hand, now firms have incentives to produce more as they are more efficient. If the first effect dominates, then $\partial q^*/\partial\lambda < 0$ and $\partial x^*/\partial\lambda > 0$ (we name this region RII). Conversely, if the second effect dominates, then $\partial q^*/\partial\lambda > 0$ and $\partial x^*/\partial\lambda > 0$ (region $RIII$). Which of these two cases arises in equilibrium will depend on the strength of the spillovers. In particular, from equations (5) and (6) we may derive threshold values (in terms of β) above which RII and $RIII$ exist.

Using equation (6) one obtains

$$sign\left\{\frac{\partial x^*}{\partial\lambda}\right\} = sign\left\{\beta\left(\Lambda(1+\delta)+n\right)-\tau\right\}. \tag{7}$$

Interestingly, the above condition can be rewritten in terms of the cost pass-through coefficient: $P'(c) = f'(nq^*)n(dq^*/dc)$, which is the rate at which the price changes with marginal cost (see e.g. Cowan [2012], and Weyl and Fabinger [2013]). By differentiating the first-order necessary condition $\partial\phi_i/\partial q_i = 0$, we obtain

$$\begin{aligned} P'(c) &= \frac{n}{\Lambda(1+\delta)+n} \\ &= \frac{n}{\Lambda/\epsilon_{ms}+n}. \end{aligned}$$

(The cost pass-through coefficient can be written in terms of the curvature of the inverse demand function and in terms of the elasticity of the inverse marginal consumer surplus.) Note that the stability condition $\Delta_q < 0$ holds if $\Lambda(1+\delta)+n > 0$, so $P'(c) > 0$. Furthermore, the pass-through increases with the number of firms when demand is log-concave ($\delta > -1$) and decreases with the term $\Lambda/\epsilon_{ms} = \Lambda(1+\delta)$. We thus have

LEMMA 1 *At equilibrium,*

$$sign\left\{\frac{\partial x^*}{\partial \lambda}\right\} = sign\left\{\beta P'(c)^{-1}n - \tau\right\},$$

where $\beta P'(c)^{-1}n - \tau = (n+1+\delta\Lambda)\beta - 1$.

We find that investment in R&D increases with the degree of cooperation when the size of spillovers is higher than the cost pass-through coefficient multiplied by the perceived or internalized effect on each firm's marginal cost of a unit increase in R&D by all firms (τ) over n, i.e., when $\beta > \tau P'(c)/n$. Thus, we have that $\partial x^*/\partial \lambda > 0$ if and only if $\beta > 1/(1+n+\Lambda\delta)$: allowing a higher degree of cooperation has a harmful effect on the equilibrium R&D expenditures, $\partial x^*/\partial \lambda$, when spillover effects are low.

Equation (5) can be rewritten as follows:

$$\frac{\partial q^*}{\partial \lambda} = -\frac{1}{f'(Q^*)\left(\Lambda(1+\delta)+n\right)}[\varphi_{\lambda q} + \varphi_{xq} B\frac{\partial x^*}{\partial \lambda}]. \tag{8}$$

The impact of a higher degree of cooperation on output at equilibrium depends directly on the marginal profit with respect to output, $\varphi_{\lambda q}$, and indirectly through its effect on the R&D received by each firm at equilibrium. Since $\Delta_q < 0$ requires that $\Lambda(1+\delta)+n > 0$, $\varphi_{\lambda q}$ is negative and φ_{xq} is positive, from equation (8) we have that if $\partial x^*/\partial \lambda \leq 0$, then $\partial q^*/\partial \lambda < 0$ (*RI*). This confirms that an increase in R&D investment is necessary (but not sufficient) for output to rise. In particular, if spillovers are sufficiently high, then $\partial x^*/\partial \lambda > 0$, and the sign of $\partial q^*/\partial \lambda$ can be negative (*RII*) or positive (*RIII*).

Let

$$H(\beta) = \left(\varphi_{\lambda q}/\varphi_{\lambda x}\right)\frac{\Delta_x}{\varphi_{xq}}.$$

We can establish the following Lemma:

LEMMA 2 *At equilibrium,*

$$sign\left\{\frac{\partial q^*}{\partial \lambda}\right\} = sign\left\{B - H(\beta)\right\}. \tag{9}$$

By totally differentiating the first-order condition with respect to R&D we get that its slope when there is no cooperation ($\lambda = 0$) is $dq/dx = -\Delta_x/\varphi_{xq}$. Since $\Delta_x < 0$ and $\varphi_{xq} > 0$, we have that $dq/dx > 0$: the locus of (q, x) combinations satisfying the first-order condition with respect to R&D is upward sloping. For $\lambda = 0$, we thus have $dq/dx = -(\varphi_{\lambda x}/\varphi_{\lambda q})H(\beta)$. The expression $\varphi_{\lambda x}$ and $\varphi_{\lambda q}$ is respectively the variation of the marginal profit with respect to R&D and output resulting from a higher degree of cooperation. As we discuss below, a higher degree of cooperation pushes firms to reduce output ($\varphi_{\lambda q} < 0$), thereby increasing revenues, but also to raise R&D ($\varphi_{\lambda x} > 0$), thereby decreasing marginal costs and increasing investment costs. Therefore, the ratio $\varphi_{\lambda q}/\varphi_{\lambda x}$ measures the firm's relative gain from some additional degree of cooperation, and $H(\beta) = -\left(\varphi_{\lambda q}/\varphi_{\lambda x}\right) dq/dx$ is then the slope of the optimal locus (q, x) weighted by this gain. Suppose that $\Gamma''(x^*)$ is strictly positive, then $H(\beta)$ can be written as

$$H(\beta) = \frac{1}{\eta(Q^*, x^*)}\left(1 + \frac{\chi(Bx^*)}{y(x^*)}\right),$$

where $\chi(Bx^*) = -c''(Bx^*)Bx^*/c'(Bx^*) \geq 0$ captures the elasticity of the slope of the innovation function, $y(x^*) = \Gamma''(x^*)x^*/\Gamma'(x^*) \geq 0$ captures the elasticity of the slope of the investment cost function, and $\eta(Q^*, x^*) = -\beta c'(Bx^*)^2/(f'(Q^*)\Gamma''(x^*)) > 0$ measures the relative effectiveness of R&D[18] weighted by β. Therefore, the ratio of the curvature of the innovation function to the curvature of the investment cost function, χ/y, plays an important role in capturing the slope of locus of (q, x) combinations satisfying the first-order condition with respect to R&D. In particular, H can be expressed as the sum of the inverse of relative effectiveness of R&D and the curvature ratio weighted by the inverse of relative effectiveness. B captures the effect on each firm's marginal cost of a unit increase in R&D by all firms. When $B > H(\beta)$ the positive effect of cooperation on R&D investments dominates its negative effect on output, so that a "little" more of cooperation raises output at equilibrium. Ideally, we would like to consider the class of games for which there exists a unique positive β, denoted by β', above which $\partial q^*/\partial\lambda > 0$. Suppose that $\beta \in \mathbb{R}^+$, since $H(\beta) \in \mathbb{R}^+$ we have that $H(\beta) : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$, therefore $\beta' \in \mathbb{R}^+$ is positive and unique when

[18] As defined by Leahy and Neary (1997, Section V., p. 654).

A.4. $H(\beta)$ has slope less than $n-1$.

Under A.4. $B = H(\beta)$ has a unique positive solution ($H(0) = \infty$). Whilst assumption A.4 does not guarantee that β' is lower than one, so $RIII$ may potentially fail to exist, it seems not to be restrictive given the model specifications that are typically used in the literature. For example, as we will see in next Section, in d'Aspremont and Jacquemin (1988), and Kamien, Muller and Zang (1992) with the innovation function considered below, H is strictly decreasing in β. Hence in both cases we can derive a unique positive threshold value β' above which $\partial q^*/\partial\lambda > 0$.

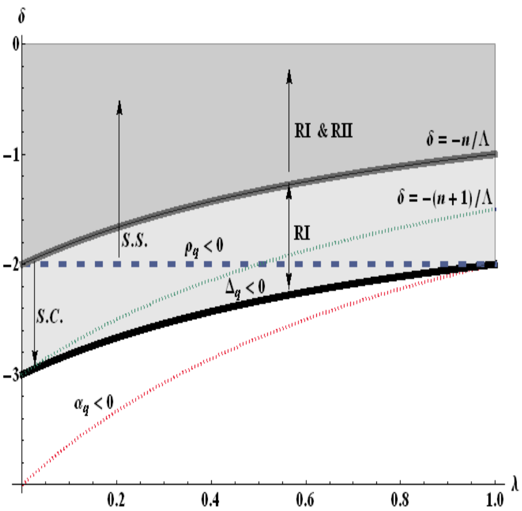

Fig. 1a. $n = 2$.

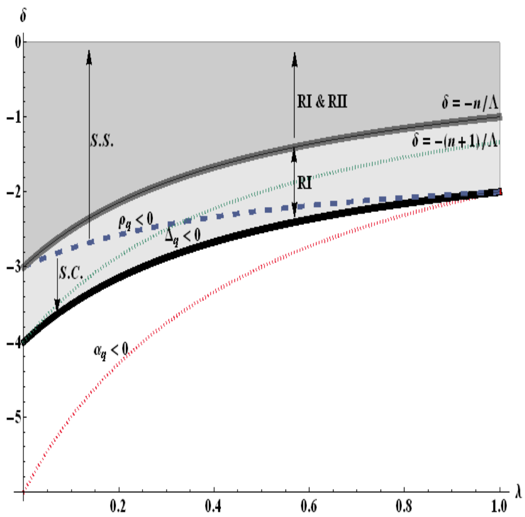

Fig. 1b. $n = 3$.

LEMMA 3 *(i) If $-(n+\Lambda)/\Lambda \leq \delta \leq -n/\Lambda$, then only region RI exists; (ii) If $\delta > -n/\Lambda$, then region RI and RII exist, region $RIII$ moreover exists ($\beta' < 1$) if $n - H(1) > 0$ holds.*

Proof. See Appendix A. ■

If $\delta < -n(1+\lambda)/\Lambda$, so that $\rho_q > 0$ holds, then condition $\delta < -n/\Lambda$ also holds: when quantities are strategic complements (S.C.) only RI exists. When δ is such that $-n(1+\lambda)/\Lambda < \delta < -n/\Lambda$, quantities are strategic substitutes but again only RI exists; however, when $\delta > -n/\Lambda$ quantities are strategic substitutes (S.S.) and at least RI and RII exist (see Figure 1a,b[19]). Therefore, as a corollary, we have that RII can only exist when quantities are strategic substitutes. Furthermore, RI and RII exist when demand is concave ($\delta > 0$). Using the above Lemmas we can establish:

[19] When $\delta > -(n+1)/\Lambda$, there exists a positive threshold of spillover above which $\partial x^*/\partial\lambda > 0$ since $1/(1+n+\Lambda\delta) > 0$, however such a threshold is above one unless $\delta > -n/\Lambda$.

PROPOSITION 1 *Under assumptions A.1.-A.4., when* $\delta \leq -n/\Lambda$ *only region RI exists, and quantities are strategic complements (respectively strategic substitutes) if* $\delta < (>) - n(1+\lambda)/\Lambda$*; when* $\delta > -n/\Lambda$*, quantities are strategic substitutes, and: (i) if* $\beta \leq 1/(1+n+\Lambda\delta)$*, then* $\frac{\partial x^*}{\partial \lambda} \leq 0$ *and* $\frac{\partial q^*}{\partial \lambda} < 0$ *(RI); (ii) if* $1/(1+n+\Lambda\delta) < \beta \leq \beta'$*, then* $\frac{\partial q^*}{\partial \lambda} \leq 0$ *and* $\frac{\partial x^*}{\partial \lambda} > 0$ *(RII); (iii) if* $\beta > \beta'$*, then* $\frac{\partial q^*}{\partial \lambda} > 0$ *and* $\frac{\partial x^*}{\partial \lambda} > 0$ *(RIII), where* β' *is the unique positive solution to the equation* $B - H(\beta) = 0$*.*

Finally, we are interested in analyzing the impact of λ on each firm's profit. As mentioned above, in a symmetric equilibrium with identical firms: $\Pi_i^* = \pi^*$. In Appendix A we show that

$$sign\{\pi^{*\prime}(\lambda)\} = sign\left\{-c'(Bx^*)\beta(\partial x^*/\partial\lambda) + f'(Q^*)(\partial q^*/\partial\lambda)\right\}. \tag{10}$$

Using (10) and noting that in RII we have that $\partial x^*/\partial\lambda > 0$ and $\partial q^*/\partial\lambda < 0$, it is clear that in this region: $\pi^{*\prime}(\lambda) > 0$. On the contrary, the sign of the impact of λ on π^* is less clear in RI (since there $\partial x^*/\partial\lambda < 0$ and $\partial q^*/\partial\lambda < 0$) and $RIII$ (since there $\partial x^*/\partial\lambda > 0$ and $\partial q^*/\partial\lambda > 0$). Nonetheless, the following proposition states that

PROPOSITION 2 *At the symmetric equilibrium, profit per firm,* π_i*, is increasing in* λ*.*

Proof. See Appendix A. ■

Therefore, according to Proposition 2, the positive effect on price dominates the negative effect on R&D in RI, and the negative effect on price is dominated by the positive effect on R&D in $RIII$, so that in both regions the profit rises with the degree of cooperation. Therefore, firms have incentives to acquire (minority) shareholdings in the industry independently of the intensity of the R&D spillover effects.[20] Before proceeding with the welfare analysis, we first examine the impact of β on equilibrium values.

Comparative statics with respect to the spillover effects. By totally differentiating the two first-order conditions with respect to β, we get

$$\frac{\partial q^*}{\partial \beta} = \frac{1}{\Delta}[\varphi_{\beta x}\varphi_{xq}B - \varphi_{\beta q}\left(\alpha_x + \rho_x(n-1)\right)]$$

$$\frac{\partial x^*}{\partial \beta} = \frac{1}{\Delta}[\varphi_{\beta q}\varphi_{xq}\tau - \varphi_{\beta x}\left(\alpha_q + \rho_q(n-1)\right)].$$

[20] Karle, Klein and Stahl (2011) analyze in a differentiated product market with two firms the incentives of an investor to acquire a controlling or non-controlling stake in a competitor.

Since $\varphi_{xq} > 0$ and $\varphi_{\beta q} > 0$, the sign of the impact of β on output and R&D in equilibrium depends on the sign of $\varphi_{\beta x}$. It can be shown that

$$\varphi_{\beta x} = -c'(Bx^*)\frac{(n-1)q^*}{B}[B\Lambda - \chi(Bx^*)].$$

Noting that the elasticity of the slope of the innovation function is non-negative, we have that $\varphi_{\beta x}$ is positive (respectively negative) when the curvature of the innovation function is sufficiently low (high). Therefore, we can establish the following proposition.

PROPOSITION 3 *When the curvature of the innovation function, χ, is sufficiently low, then $\partial q^*/\partial\beta > 0$ and $\partial x^*/\partial\beta > 0$. If the curvature of the innovation function is sufficiently high, then the sign of the impact of β on q^* and x^* is ambiguous.*

As a corollary, when marginal cost is linear ($\chi = 0$), as e.g. in d'Aspremont and Jacquemin (1988), increasing the size of spillover effects raises the equilibrium values of output and R&D. However, in the next section we will see that under the model specification used in Kamien, Muller and Zang (1992), $\chi > 0$ and, as a result, for low values of λ we may have that $\partial x^*/\partial\beta < 0$.

3 Model specification examples

This section gives a brief description of different model specifications, presents market outcomes and performs comparative statics. We will consider the well-known R&D model specifications with linear demand of d'Aspremont-Jacquemin (AJ) and Kamien-Muller-Zang (KMZ), and a constant elasticity model (CE) similar to the Dasgupta and Stiglitz's (1980) model but with spillover effects.

As shown in Amir (2000) the AJ and the KMZ model specifications are not equivalent for large spillover values (the critical value depends on the innovation function or unit cost of production function and on the number of firms). The difference between the two models lies on the unit cost of production function and the autonomous R&D expenditures. Under the KMZ specification, *the effective R&D investment* for each firm is the sum of its own expenditure x_i and a fixed fraction (β) of the sum of the expenditures of the rest of firms, i.e., $X_i = x_i + \beta\sum_{j\neq i} x_j$. Instead, under the AJ specification, *the effective cost reduction* for each firm is X_i, so $c(\cdot)$ is a linear function. Thus, in AJ decision variables are unit-cost reductions, whereas in KMZ

	AJ	KMZ	CE
Demand	$f = a - bQ$	$f = a - bQ$	$f = \sigma Q^{-\varepsilon}$
	$\delta = 0;\ a, b > 0$	$\delta = 0;\ a, b > 0$	$\delta = -(1+\varepsilon);\ a = 0, b = -\sigma < 0$
$c(\cdot)$	$\bar{c} - x_i - \beta \sum_{j \neq i} x_j$	$\bar{c} - [(2/\gamma)(x_i + \beta \sum_{j \neq i} x_j)]^{1/2}$	$\kappa(x_i + \beta \sum_{j \neq i} x_j)^{-\alpha}$
$\Gamma(x)$	$(\gamma/2)x^2$	x	x

Table 1: Model Specifications.

	AJ	KMZ	CE
S.O.C	$\gamma b > 1/2$	$\gamma b > \tau/(2\tilde{\lambda})$	$n > \frac{\Lambda(1+\varepsilon)}{2}$ *and* $\frac{\varepsilon(1+\alpha)}{\alpha} > \frac{n(n-\varepsilon\Lambda)}{\tilde{\lambda}(2n+\Lambda\delta)}$
Regularity Condition	$\gamma b > \tau B/(\Lambda + n)$	$\gamma b > \tau/(\Lambda + n)$	$(1+\alpha)/\alpha > 1/\varepsilon$

, with $\tilde{\lambda} \equiv 1 + \lambda(n-1)\beta^2$.

Table 2: Second-order conditions and regularity condition. (See Appendix B.)

decision variables are the autonomous R&D expenditures.[21] In particular, in KMZ the unit cost of firm i is $\bar{c} - h(x_i + \beta \sum_{j \neq i} x_j)$, where for given $x_i \geq 0$ $(i = 1, ..., n)$ the effective cost reductions to firm i, $h(\cdot)$, is a twice differentiable and concave function with $h(0) = 0$, $h(\cdot) \leq \bar{c}$, and $(\partial/\partial x_i)h(\cdot) > 0$. As in Amir (2000), to allow for a direct comparison between AJ and KMZ, we consider a particular case of the KMZ model specification: $h = [(2/\gamma)(x_i + \beta \sum_{j \neq i} x_j)]^{1/2}$ with $\gamma > 0$. The CE model considers constant elasticity demand and costs with $\alpha, \kappa > 0$ (see Table 1), and where α is the unit cost of production (or innovation function) elasticity with respect to the investment in R&D (in the absence of spillover effects). Finally, $\Gamma(x)$ is quadratic in AJ but linear in KMZ and CE. Table 1 summarizes these model specifications, and Table 2 contains sufficient second-order conditions and the regularity condition for each model specification.

Comparative statics. Table 3 collects equilibrium values of output and R&D that one obtains by solving equations (2) and (3). In Appendix B we shall show that in the constant elasticity model, $\Delta > 0$ if and only if $\varepsilon - \alpha(1-\varepsilon) > 0$. We next examine comparative statics with respect to demand and cost parameters, spillover effects and the degree of cooperation.

Demand and cost parameters. Clearly, in AJ and KMZ the R&D expenditure and output per firm increase with the size of the market (a). Conversely, both x^* and q^* decrease with the level of inefficiency of the technology employed, $\bar{c}$, the slope of inverse demand, b, and the slope of the marginal R&D costs, γ. As in the previous cases, in the constant elasticity model

[21] Furthermore, while in AJ the joint returns to scale (in R&D expenditure and number of firms) are decreasing, constant or increasing when spillover effects are less than, equal to, or greater than $1/(n+1)$, in KMZ the joint returns to scale are always nonincreasing (Proposition 4.1 in Amir [2000]).

	AJ	KMZ	CE
q^*	$\frac{\gamma(a-\bar{c})}{b\gamma(\Lambda+n)-B\tau}$	$\frac{\gamma(a-\bar{c})}{b\gamma(\Lambda+n)-\tau}$	$\frac{1}{\alpha\kappa\tau}\left[\sigma\left(\tau\alpha/n\right)^{\varepsilon}\kappa^{\varepsilon-1}\left(1-\varepsilon\Lambda/n\right)\right]^{(1+\alpha)/[\varepsilon-\alpha(1-\varepsilon)]}$
x^*	$\frac{\tau(a-\bar{c})}{b\gamma(\Lambda+n)-B\tau}$	$\frac{\gamma\tau^2(a-\bar{c})^2}{2B(b\gamma(\Lambda+n)-\tau)^2}$	$\frac{1}{B}\left[\sigma\left(\tau\alpha/n\right)^{\varepsilon}\kappa^{\varepsilon-1}\left(1-\varepsilon\Lambda/n\right)\right]^{1/[\varepsilon-\alpha(1-\varepsilon)]}$
H	$\frac{b\gamma}{\beta}$	$\frac{b\gamma B}{\beta}$	$\frac{B}{\beta}\left(\frac{\alpha+1}{\alpha}\right)\frac{\varepsilon}{n-\varepsilon\Lambda}\tau$

Table 3: Equilibrium Values.

R&D expenditure and output per firm increase with the size of the market, σ. In addition, the costlier is the technology employed, κ, the lower is total output, Q^*. However, x^* decreases (respectively, increases) with κ if demand is elastic (inelastic). The last two results hold for any value of λ and β.[22]

Spillover effects. With the AJ model specification, as mentioned in the previous section, for all λ we have that $\partial q^*/\partial\beta > 0$ and $\partial x^*/\partial\beta > 0$ because $\chi = 0$, and hence $\varphi_{\beta x} > 0$. Thus, spillovers do not reduce the incentives for cost reductions. In KMZ, as in AJ, for any given $\lambda \in [0,1]$ we have that $\partial q^*/\partial\beta > 0$. More interestingly, x^* decreases (respectively, increases) with β for low (high) values of λ because here $\chi > 0$. In the constant elasticity model, we have that for any positive λ, $\partial q^*/\partial\beta > 0$ (if $\lambda = 0$, then $\partial q^*/\partial\beta = 0$). Furthermore, as in the KMZ model, x^* decreases (respectively, increases) with β for low (high) values of λ (since here $\chi > 0$). Therefore, while in both KMZ and CE the presence of spillover effects tends to reduce the incentives for cost reductions, cooperation among firms alleviates the problem by restoring such incentives.

Degree of cooperation. As we discussed earlier, the impact of λ on q^* and x^* depends on the intensity of spillover effects. In particular, using the results of Section 2 we can determine $sign\{\partial q^*/\partial\lambda\}$ and $sign\{\partial x^*/\partial\lambda\}$. (See Lemma 6 in Appendix B.) From Lemma 6 we obtain the threshold values that determine RI, RII and $RIII$ (see Table 4)[23], and therefore apply Proposition 1 to each model specification.

In the analysis that follows, we will examine the optimal degree of cooperation (or optimal degree of toughness of the antitrust policy) and how it depends on the spillover level, the number of firms and the relevant parameters of demand and cost.

[22] The same result is obtained in Dasgupta and Stiglitz (1980) for $\lambda = \beta = 0$ and free entry.

[23] It is simple to verify that under the three model specifications the condition $\Delta > 0$ guarantees that $\beta' > 1/[1 + n + \Lambda\delta]$. Proposition 4(iii) extends this result to the general framework.

	Spillover Thresholds	
	$1/(1+n+\Lambda\delta)$	β'
AJ	$1/(1+n)$	$[-1+\sqrt{1+4b\gamma(n-1)}]/[2(n-1)]$
KMZ	$1/(1+n)$	γb
CE	$1/[(n-\varepsilon)-\lambda(n-1)(1+\varepsilon)]$	$\varepsilon(\alpha+1)/[\alpha(n-\varepsilon)-(2\alpha+1)(n-1)\lambda\varepsilon]$

Table 4: Spillover Thresholds.

4 Welfare analysis

Welfare in equilibrium is given by the sum of consumer surplus (CS) and industry profits:

$$W(\lambda) = \int_0^{Q^*} f(Q)dQ - c(Bx^*)Q^* - n\Gamma(x^*).$$

We are interested in studying the effect of λ on welfare. Using the equilibrium conditions (2)-(3), we have

$$W\,'(\lambda) = \left(-\Lambda f'(Q^*)\frac{\partial q^*}{\partial \lambda} - (1-\lambda)\beta(n-1)c'(Bx^*)\frac{\partial x^*}{\partial \lambda}\right) Q^*. \tag{11}$$

Allowing for some additional cooperation alters equilibrium values of quantities and R&D investments, and each additional unit of output and R&D has respectively social value equal to $\Lambda(-f'(Q^*))Q^*$ and $(1-\lambda)\beta(n-1)(-c'(Bx^*))Q^*$. Therefore, the socially optimal value of λ depends on the magnitude of the change in decision variables at equilibrium as a response to a change in λ. In this sense, Proposition 1 turns out to be fruitful here. In RI we have that $W'(\lambda) < 0$ since $\partial x^*/\partial\lambda \leq 0$ and $\partial q^*/\partial\lambda < 0$; in RII, however, the impact of λ on welfare can be positive or negative depending on whether the positive effect of cooperation on R&D dominates or not the negative effect of cooperation on output; clearly, in $RIII$: $W'(\lambda) > 0$. Furthermore, since

$$sign\left\{CS'(\lambda)\right\} = sign\left\{\frac{\partial q^*}{\partial \lambda}\right\},$$

the impact of λ on consumer surplus is positive (i.e., $CS'(\lambda) > 0$) only in $RIII$. Thus, while in RI and RII consumers suffer from a higher degree of cooperation (because a lower output implies a higher price in equilibrium), in $RIII$ consumers benefit from cooperation. Therefore, antitrust policy will tend to be tougher under CS standard. The next proposition provides conditions under which the antitrust authority prefers allowing or not some cooperation among the firms.

PROPOSITION 4 *Suppose that $\delta > -n/\Lambda$, under assumptions A.1-A.4.,* *(i) Total surplus is maximized with $\lambda = 0$ when $\beta < \underline{\beta}$, where $\underline{\beta} = 1/(1 + n(1 + \delta))$ if $\delta \geq 0$ and $\underline{\beta} = 1/(1 + n + \delta)$ if $\delta \leq 0$. However, total surplus increases with λ if the spillover is larger than the threshold value $\hat{\beta} > \underline{\beta}$, where $\hat{\beta}$ is the unique positive solution to the equation*

$$H(\beta) - B = \frac{n - \Lambda}{\Lambda} \left(nP'(c)^{-1}\beta - \tau\right)$$

with $nP'(c)^{-1}\beta - \tau = (n + 1 + \delta\Lambda)\beta - 1$. Furthermore, $\hat{\beta} < 1$ if

$$H(1) - n < \frac{n - \Lambda}{\Lambda}(n + \delta\Lambda).$$

(ii) Considering β' as a function of λ, $\beta'(\lambda)$, we have: (a) If $\beta'(\lambda)$ is decreasing in λ, then

$$\beta \leq \beta'(1) \Rightarrow \lambda^*_{CS} = 0 \text{ and } \beta > \beta'(0) \Rightarrow \lambda^*_{CS} = \lambda^*_{TS} = 1;$$

(b) If $\beta'(\lambda)$ is increasing in λ, then

$$\beta \leq \beta'(0) \Rightarrow \lambda^*_{CS} = 0 \text{ and } \beta > \beta'(1) \Rightarrow \lambda^*_{CS} = \lambda^*_{TS} = 1;$$

(c) If β' is independent of λ, then

$$\beta \leq \beta' \Rightarrow \lambda^*_{CS} = 0 \text{ and } \beta > \beta' \Rightarrow \lambda^*_{CS} = \lambda^*_{TS} = 1.$$

We have that $\beta' < 1$ when $n - H(1) > 0$;

(iii) $\underline{\beta} < \hat{\beta} < \beta'$. Furthermore, $\partial\underline{\beta}/\partial n < 0$, $\partial\beta'/\partial n < 0$ if $\beta' > \partial H(\beta')/\partial n$, and $\partial\hat{\beta}/\partial n < 0$ if $\hat{\beta} > \partial(H(\hat{\beta}) - \partial g(\hat{\beta}))/\partial n$, where $g(\beta) = ((n - \Lambda)/\Lambda)(nP'(c)^{-1}\beta - \tau)$ and $\partial g(\hat{\beta})/\partial n > 0$.

Proof. See Appendix A. ■

Therefore, when $\beta < \underline{\beta}$ competition authorities that seek to maximize total surplus should not allow firms to cooperate. We observe that the threshold value $\underline{\beta}$ decreases with n. The reason is that the incentives for firms to "free ride" are larger when the number of firms increases because each firm can then appropriate R&D efforts of a higher number of participants. If demand is linear, then $\underline{\beta} = 1/(1 + n)$, whereas if demand has constant elasticity, ε^{-1}, so $\delta = -(1 + \varepsilon)$, then $\underline{\beta} = 1/(1 - n\varepsilon)$ when $\delta > 0$ ($\varepsilon < -1$) and $\underline{\beta} = 1/(n - \varepsilon)$ when $\delta < 0$ ($\varepsilon > -1$). Nevertheless, Proposition 4 also says that there may exist a threshold value, $\hat{\beta}$, lower than one and above which increasing the degree of cooperation is welfare improving. A corollary of Proposition 4 is:

COROLLARY 1 *There exists a threshold value* $\bar{\beta} \in (\underline{\beta}, 1)$ *above which allowing for some cooperation is socially optimal if*

$$(n + (n-1)(\delta + n)) - H(1) > 0. \tag{12}$$

Under condition (12), $W'(0) > 0$ holds for $\beta = 1$, so $\bar{\beta} = \hat{\beta}\Big|_{\lambda=0} < 1$ and therefore $\lambda^*_{TS} > 0$ whenever spillover is larger than the threshold value $\bar{\beta}$. For the functional forms assumed in the paper, below we will obtain the threshold value $\bar{\beta}$, and the condition that guarantees that such a threshold is below 1. In particular, under such model specifications $W'(0)$ is a quadratic function, and therefore $\bar{\beta}$ is given by the largest root of $W'(0) = 0$.[24]

The parameter β' is the threshold value above which consumers benefit from cooperation. Part ii of Proposition 4 distinguishes among three cases, and for each of them identifies threshold value below which no cooperation is optimal in terms of consumer surplus, and threshold value above which full cooperation is optimal in terms of consumer and total surplus. Note that if $H(\beta)$ is increasing (respectively decreasing) in λ, then $\beta'(\lambda)$ is also increasing (resp. decreasing) in λ.[25] In the CE model, $H(\beta)$ is increasing in λ, so case 'b' of Proposition 4 holds. In AJ and KMZ, however, $H(\beta)$ is independent of λ, so case 'c' of Proposition 4 holds and as a result under both model specifications the consumer surplus solution is bang-bang.

Finally, while more firms in the market decreases the threshold value below which no cooperation is optimal ($\underline{\beta}$), the impact of the number of firms on the threshold values above which some cooperation increases total surplus ($\hat{\beta}$) and consumer surplus (β') is less clear. The reason is that the impact of the number of firms on H depends on the functional forms. (Below we discuss the impact of entry on $\bar{\beta}$ and β' under each model specification.)

Examples. We are interested in determining for each model specification the threshold $\bar{\beta}$, which is the spillover level above which some (output and R&D) cooperation among firms is desirable for welfare:

[24] See proof of Lemma 4 in Appendix C.

[25] If β' is the unique positive solution to the equation $H(\beta) - B = 0$ and $H(\beta)$ is increasing in λ, then when λ increases, β' must also increase so as to reduce the difference $H(\beta) - B$ as B is independent of λ and, by Assumption A.4., $\partial(H(\beta) - B)/\partial\beta < 0$.

LEMMA 4 ***(Threshold Values)*** *(i) In AJ we have*

$$\bar{\beta}^{AJ} \equiv \frac{(n-2)+\sqrt{(n-2)^2+4b\gamma(n+2)(n-1)}}{2(n+2)(n-1)},$$

with $\bar{\beta}^{AJ} \leq 1$ *if* $\gamma b \leq n^2$. *(ii) In KMZ we have*

$$\bar{\beta}^{KMZ} \equiv \frac{(n-2)+b\gamma(n-1)+\sqrt{(n-2)^2+b\gamma(n-1)(b\gamma(n-1)+6n+4)}}{2(n+2)(n-1)},$$

with $\bar{\beta}^{KMZ} \leq 1$ *if* $\gamma b \leq n$. *(iii) In CE,* $\bar{\beta}^{CE}$ *is the threshold value above which*

$$(n-\varepsilon)\alpha\beta(B+(n-1)(\beta(n-\varepsilon)-1))-\varepsilon(\alpha+1)B>0.$$

Proof. See Appendix C. ■

In the AJ model the second-order condition requires that $\gamma b > 1/2$ (so $\bar{\beta}^{AJ} < 1$ in the region: $1/2 < \gamma b < n^2$), whereas in the KMZ model the second-order condition requires that $\gamma b > 1/2$ when $\lambda = 0$ (so $\bar{\beta}^{KMZ} < 1$ in the region: $1/2 < \gamma b < n$). As in these two cases, in the constant elasticity model we can derive the threshold $\bar{\beta}^{CE}$ above which it is socially optimal to allow some cooperation. However, as in this case $\bar{\beta}^{CE}$ takes a long expression, we are giving the threshold value in implicit form.

As mentioned above, the impact of entry on β' depends on the function H. For example, from Table 4 we have that with the model specification used in d'Aspremont and Jacquemin (1988), $\partial H/\partial n = 0$, so increasing the number of firms decreases the threshold $(\partial \beta'_{AJ}/\partial n < 0)$, but with the model specification used in Kamien, Muller and Zang (1992) introduced above, the threshold is constant (i.e., $\partial H(\beta')/\partial n = \beta'$), so increasing n has no impact on β' $(\partial \beta'_{KMZ}/\partial n = 0)$. Finally, in the constant elasticity model, the direction of the impact of n on the threshold depends on the value of λ: β'_{CE} decreases (respectively increases) with n when λ is low (high) $(\partial \beta'_{CE}/\partial n > 0$ for $\lambda > \alpha/[\varepsilon(2\alpha+1)])$. Note that β'_{CE} is increasing in λ (case b in Proposition 4). Thus, if $\beta < \beta'_{CE}(0)$, then $\lambda^*_{CS} = 0$. However, since $\beta'_{CE}(0)$ decreases with n, entry may facilitate that $\lambda^*_{CS} > 0$. On the other hand, if $\beta > \beta'_{CE}(1)$, then $\lambda^*_{CS} = \lambda^*_{TS} = 1$, but since $\beta'_{CE}(1)$ increases with n, entry may reduce the socially optimal level of cooperation.

Fig. 2a (respectively Fig. 2b) shows the value for $\bar{\beta}$ under the AJ (KMZ) model specifications as a function of the number of firms and for different values of γb. As the figure makes clear, $\bar{\beta}^{AJ}$ and $\bar{\beta}^{KMZ}$ decrease with n: when there are more firms in the market, there is more need to increase cooperation in order to internalize the additional externalities. We also have that $\bar{\beta}^{AJ}$ and $\bar{\beta}^{KMZ}$ decrease with γb, and that $\bar{\beta}^{AJ}$ may take values above 1 when $n < 3$, although this is not the case if γb is low enough. In particular, from Lemma 4 it is easy to conclude that under the AJ (respectively, KMZ) model specifications, in a duopoly $\bar{\beta} \leq 1$ when $\gamma b \leq 4$ ($\gamma b \leq 2$). That is, having $\bar{\beta}$ lower than 1, requires lower values of γb in the KMZ model than in the AJ model. Fig. 2a and 2b depict the threshold β' evaluated at the lowest value $b\gamma$ considered in each figure.

Threshold values $\bar{\beta}$, above which some cooperation is socially optimal

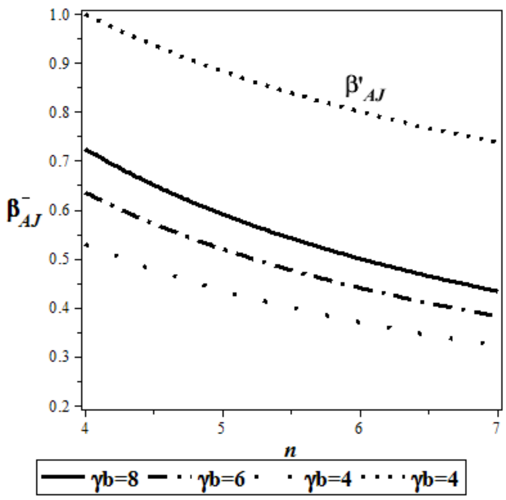

Fig. 2a. AJ model specification.

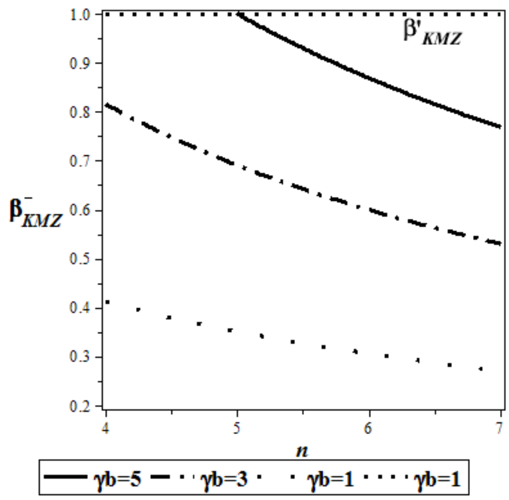

Fig. 2b. KMZ model specification.

Threshold values $\bar{\beta}$, above which some cooperation is socially optimal

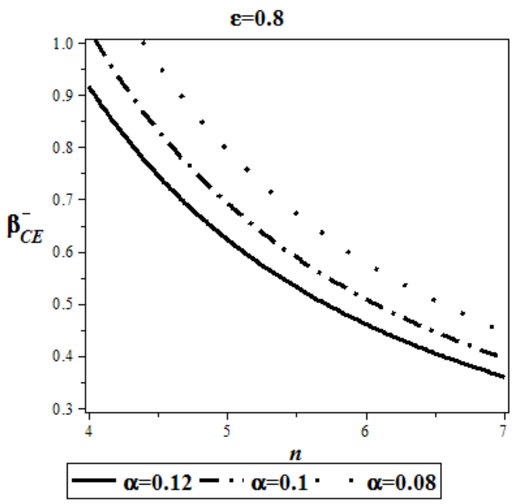

Figure 3a. Constant elasticity model.

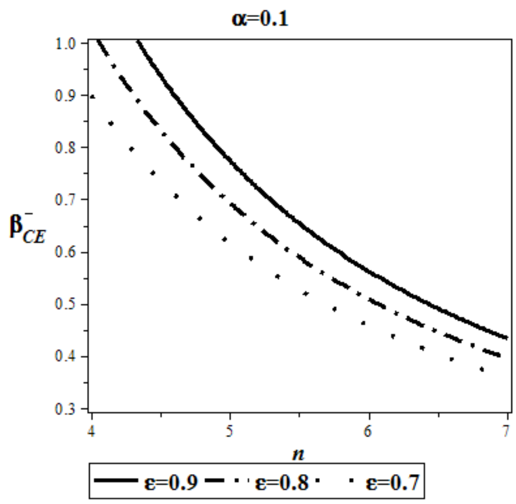

Figure 3b. Constant elasticity model.

Fig. 3a and Fig. 3b depict $\bar{\beta}^{CE}$ as a function of n and for different values for α and ε. In Appendix B, we shall examine the region for the values of α, ε and n such that feasible conditions for the existence of the equilibrium (provided in Proposition 8, Appendix B) are satisfied. A glance at these figures shows that $\bar{\beta}^{CE}$ decreases again with n (for given ε and α). In addition, Fig. 3a tells us that for given n and ε, $\bar{\beta}^{CE}$ decreases with the elasticity of the innovation function, α, whereas Fig. 3b shows that for given n and α, $\bar{\beta}^{CE}$ increases with ε, so it decreases with the elasticity of demand. We also have that for the (feasible) combination of parameters (α, ε) considered here, $\bar{\beta}^{CE} \geq 1$ when there are two or three firms in the market. β'_{CE} takes value above one in Fig. 3a and 3b. For example, with $\varepsilon = 0.8$, $\alpha = 0.1$ and $\lambda = 0$, $\beta'_{CE} < 1$ only if $n \geq 10$.

As mentioned above, in AJ and KMZ, case 'c' of Proposition 4 holds, thus if spillover is sufficiently large, so that $\beta > \beta'$ ($\beta^{AJ\prime}$ and $\beta^{KMZ\prime}$ can be found in Table 4), then $\lambda^*_{CS} = \lambda^*_{TS} = 1$. A key result is the following Proposition:

PROPOSITION 5 *In AJ, a social planner that seeks to maximize total surplus or consumer surplus will allow for full cooperation if $\beta > \beta'_{AJ}$ (or, equivalently, if $n > \frac{\gamma b - \beta(1-\beta)}{\beta^2}$) with $\beta'_{AJ} < 1$ if $\gamma b < n$, which together with second-order and regularity condition evaluated at $\lambda = \beta = 1$ require that $n/2 < \gamma b < n$. In KMZ, consumer surplus and total surplus are maximized with full cooperation if $\beta > \beta'_{KMZ}$ with $\beta'_{KMZ} < 1$ if $\gamma b < 1$, which together with second-order and regularity condition evaluated at $\lambda = 1$ require that $1/2 < \gamma b < 1$. In the CE model full cooperation is never socially optimal.*

Proof. See Appendix C. ■

Therefore, in terms of consumer surplus in AJ and CE it is optimal to no allow for some cooperation when entry is insufficient: in AJ when $n < \gamma b$ (since $\beta'_{AJ} > 1$) and in CE when $n < \varepsilon(2\alpha + 1)/\alpha$ (since $\beta'_{CE} > 1$ when $n < \varepsilon(2\alpha + 1)\Lambda/\alpha$). On the other hand, we cannot exclude the possibility that inducing full cooperation maximizes social welfare under the AJ and KMZ model specifications (Figure 6a and 7a show this case), however conditions are restrictive and only in AJ entry widens the region where this case arises. To get some insights into the socially optimal degree of cooperation, we will conduct some numerical simulations.[26] As will be clear below, simulations show that the socially optimal degree of cooperation in terms of total surplus is usually much lower in the constant elasticity demand model than under the

[26] Values for parameters are chosen so that the second-order conditions and the regularity condition are satisfied, and such that in equilibrium q, x, π, f, and c are non-negative.

AJ model specification. Moreover, as the above proposition asserts, it is never optimal to have full cooperation in the constant elasticity demand model. We first examine the constant elasticity model, and then turn to discuss numerical simulations under the AJ and KMZ model specifications. Table 5 summarizes the results of the static model.

Figure 4a and 4b depict the socially optimal value of λ, denoted as λ^*, in terms of total surplus (solid line) and consumer surplus (dashed line) for the constant elasticity model. When the number of firms is small (less than 4 in our example), it is never optimal to allow firms to cooperate (RI). As the spillover effects and the number of firms increase, λ^*_{TS} increases, but λ^*_{CS} is much flatter. This is RII, in which firms benefit and consumers suffer from a higher degree of cooperation because $\partial q^*/\partial\lambda < 0$ (and therefore the price in equilibrium increases with λ). Nevertheless, the overall impact of λ on welfare is positive because the positive impact on x^* dominates the negative impact on q^*. Finally, we find that slightly raising λ may be optimal from the consumer's point of view when the number of firms in the market is sufficiently large ($RIII$).

Optimal degree of cooperation in terms of total surplus and consumer surplus

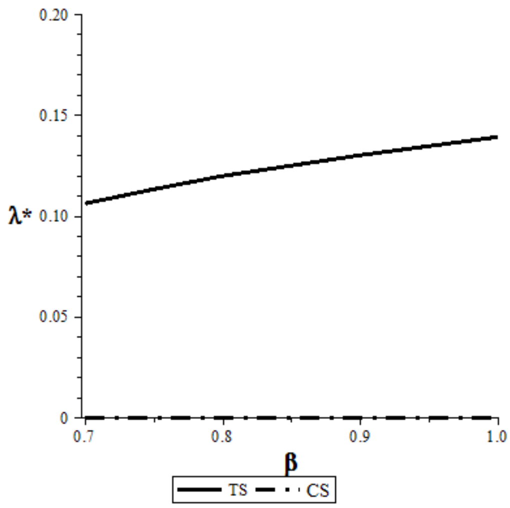

Figure 4a. Constant elasticity model. (Numerical values: $\alpha = 0.1$, $\varepsilon = 0.8$, $\sigma = \kappa = 1$, $n = 8$.)

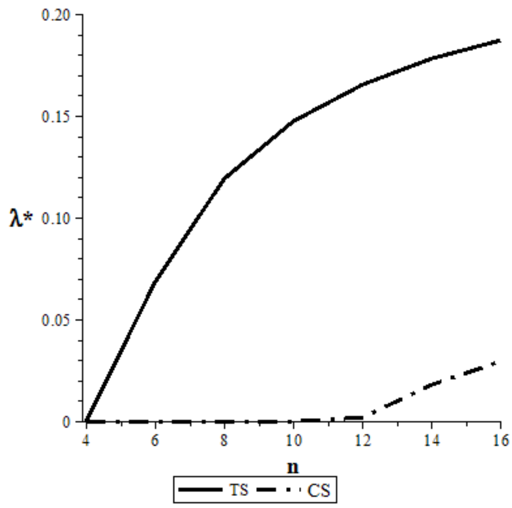

Figure 4b. Constant elasticity model. (Numerical values: $\alpha = 0.1$, $\varepsilon = 0.8$, $\sigma = \kappa = 1$, $\beta = 0.8$.)

Optimal degree of cooperation in terms of total surplus and consumer surplus

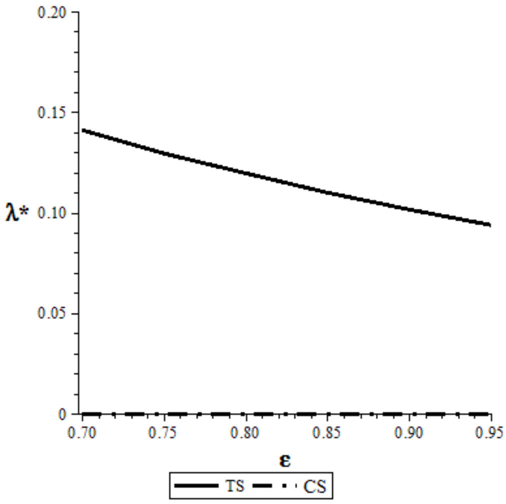

Figure 5a. Constant elasticity model.

(Numerical values: $\alpha = 0.1$, $\sigma = \kappa = 1$,

$n = 8$, $\beta = 0.8$.)

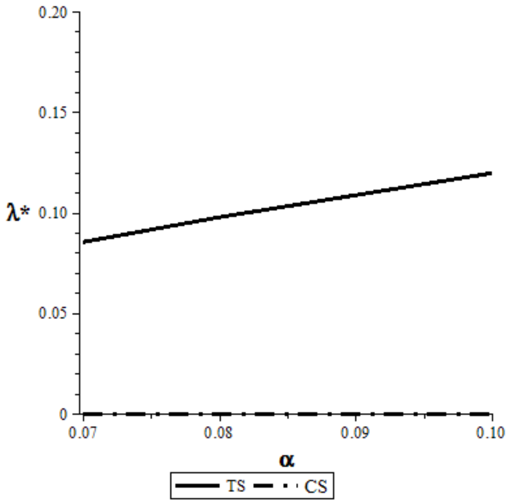

Figure 5b. Constant elasticity model.

(Numerical values: $\varepsilon = 0.8$, $\sigma = \kappa = 1$,

$n = 8$, $\beta = 0.8$.)

Fig. 5a and 5b show that the greater is the elasticity of demand, ε^{-1}, or the elasticity of the unit cost function, α, the greater should be the degree of cooperation if the social planner seeks to maximize total surplus; however, if the objective is to maximize consumer surplus, then for the same parameter range, $\lambda^*_{CS} = 0$.

We now explore the model under the AJ model specification. Fig. 6a shows that consumer surplus and total surplus are maximized with full cooperation when spillovers are sufficiently large. This result illustrates our previous points. We have shown that under the AJ model specification both $\partial x^*/\partial\lambda$ and $\partial q^*/\partial\lambda$ can be positive in the feasible region of parameters if β is large enough ($RIII$). In this case, increasing λ boosts R&D investment, which in turn stimulates further output (and therefore lowers price in equilibrium). As a result, both consumers and firms may benefit from a higher degree of cooperation. Fig. 6b shows that λ^*_{TS} also increases with the number of firms, although λ^*_{CS} is much flatter: only if the number of firms is large enough (in our example $-$ where $\beta = 0.8-$ we need more than 6 firms), it is socially optimal (in terms of total surplus and consumer surplus) to allow firms to fully cooperate. Therefore, increasing β and n raises λ^*_{TS}, but $\lambda^*_{CS} = 0$ unless $\beta > 0.8$ or $n \geq 7$. Finally, we can also explore the socially optimal degree of cooperation as a function of γ. Numerical results show that λ^*_{TS} decreases with, and λ^*_{CS} is neutral to, the value of γ.

Optimal degree of cooperation in terms of total surplus and consumer surplus

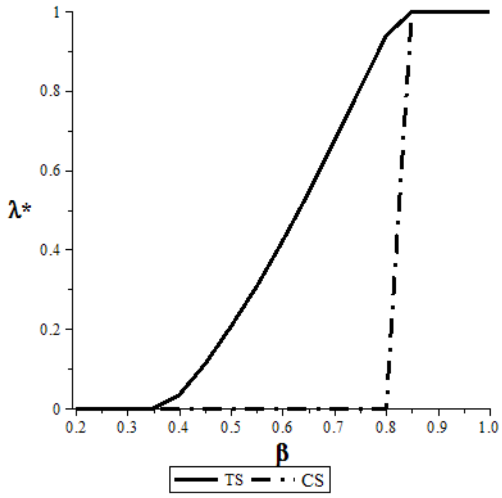

Fig 6a. AJ model specification.

(Numerical values: $a = 700$, $c = 500$,

$\gamma = 7$, $n = 6$ and $b = 0.6$.)

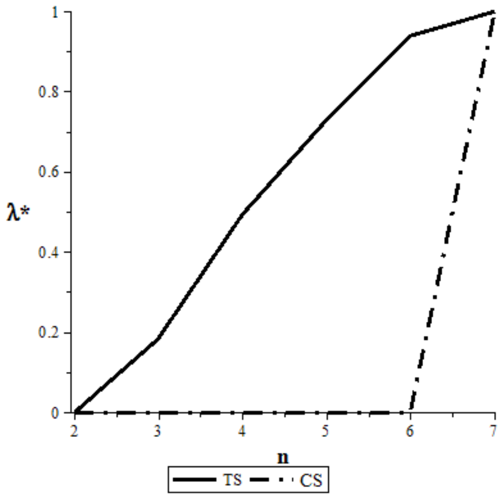

Fig. 6b. AJ model specification.

(Numerical values: $a = 700$, $c = 500$,

$\gamma = 7$, $\beta = 0.8$, $b = 0.6$.)

Optimal degree of cooperation in terms of total surplus and consumer surplus

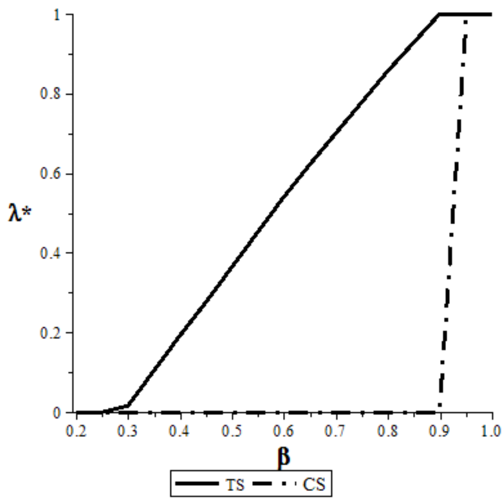

Fig 7a. KMZ model specification.

(Numerical values: $a = 700$, $c = 500$,

$\gamma = 3$, $n = 6$ and $b = 0.3$.)

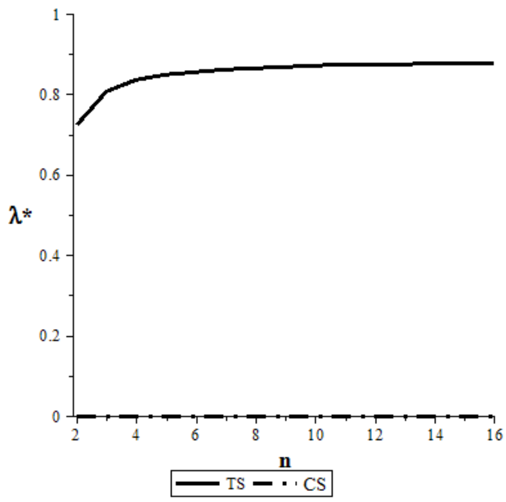

Fig. 7b. KMZ model specification.

(Numerical values: $a = 700$, $c = 500$,

$\gamma = 3$, $\beta = 0.8$, $b = 0.3$.)

Fig. 7a depicts the socially optimal degree of cooperation for different values of β under the KMZ model specification with $\gamma b = 0.9$, so full cooperation is socially optimal when $\beta > \beta'_{KMZ} = 0.9$ (see Proposition 5). Considering values of γb above 1, we find that λ^*_{TS} still

	$\bar{\beta}$			λ^*_{TS}			λ^*_{CS}		
	AJ	KMZ	CE	AJ	KMZ	CE	AJ	KMZ	CE
n	$-$	$-$	$-$	$+$	$+$	$+$	$(+)$	0	$(+)$
$b\gamma$	$+$	$+$							
α			$-$			$+$			$[+]$
ε^{-1}			$-$			$+$			$[+]$
β				$+$	$+$	$+$	$(+)$	$(+)^*$	$[+]$
γ	$+$	$+$		$-$	$-$		$-$	$-$	

Table 5: Impact of parameters in the one-stage model. *(+): the impact is positive only if β* and *n are sufficiently large, otherwise the impact is zero; $(+)^*$: the impact is positive only if the parameter is sufficiently large and γb is sufficiently small, otherwise the impact is zero; [+]: the impact is positive when n is sufficiently large, otherwise the impact is zero.*

increases with β, however, as expected, $\lambda^*_{CS} = 0$ even if $\beta = 1$. Fig. 7b considers different number of firms with $\beta = 0.8$. In terms of total surplus, λ^* is above 0.8 when there are four or more firms in the market. As we discussed above, in KMZ increasing the number of firms does not alter the $sign\{\partial q^*/\partial\lambda\}$, and therefore has no impact on the $sign\{CS'(\lambda)\}$. Therefore, in this case consumer surplus is constantly decreasing with the degree of cooperation.

5 The two-stage model

In this section we consider two stages. In the first stage, every firm i commits to invest an amount x_i in R&D. In the second stage, and for given R&D expenditures, firms compete in the product market. We will solve for the subgame-perfect equilibrium of the model with $\lambda-$degree of cooperation in the two stages.

Let $\mathbf{x} = [x_1, x_2, ..., x_n]$ and $\mathbf{q} = [q_1, q_2, ..., q_n]$ be the first-stage R&D and second-stage output profiles. The concavity of ϕ^i with respect to q_i ($\alpha_q < 0$) at a symmetric equilibrium is guaranteed by the condition: $\delta > -2n/\Lambda$. Note that for $\lambda = 1$ the condition reduces to $\delta > -2$ (or $-\delta < 2$ – i.e., the convexity of inverse demand must not be too high), which in turn implies that marginal revenue is strictly decreasing in output. Let $q^*_i(\mathbf{x})$ denote the firm-i output equilibrium value of the second-stage game associated to the R&D profile $\mathbf{x}$, then for all i:

$$\frac{\partial}{\partial q_i}\phi_i(\mathbf{q}^*(\mathbf{x}), \mathbf{x}, \lambda) = 0. \tag{13}$$

In the first-stage, the first-order necessary conditions are (for $i \neq j$ and $i, j = 1, 2, ..., n$)

$$\frac{\partial}{\partial x_i}\phi_i(\mathbf{q}^*(\mathbf{x}), \mathbf{x}, \lambda) + \sum_{j \neq i} \frac{\partial}{\partial q_j}\phi_i(\mathbf{q}^*(\mathbf{x}), \mathbf{x}, \lambda)\frac{\partial}{\partial x_i} q_j^*(\mathbf{x}) = 0. \tag{14}$$

The equilibrium R&D profile $\mathbf{x}^*$ is characterized by the system of equations (13)-(14) (provided second-order conditions hold). Let $\mathbf{q}^* = \mathbf{q}^*(\mathbf{x}^*)$, then $\{\mathbf{x}^*, \mathbf{q}^*\}$ is the subgame-perfect equilibrium of the two-stage game. The second term of equation (14) is the strategic effect of investment on profits. Building on Suzumura (1992) and Leahy and Neary (1997), it can be shown that the term $\partial q_j^*/\partial x_i$ evaluated at a symmetric equilibrium, where $q_i^* = q^*$ and $x_i^* = x^*$ for all i, is given by

$$\frac{\partial q_j^*}{\partial x_i} = -c'(Bx^*)\frac{\alpha_q}{(\alpha_q - \rho_q)(\alpha_q + (n-1)\rho_q)}\left(\tilde{\beta}(\lambda) - \beta\right), \tag{15}$$

with $\tilde{\beta}(\lambda) = \rho_q/\alpha_q < 1$ for $\lambda < 1$.[27] Therefore, only if production decisions are strategic substitutes ($\rho_q < 0$), we have that $\tilde{\beta}(\lambda) > 0$. Using Assumption A.1., it is easy to reduce the above expression to the following (for $\lambda < 1$):

$$\frac{\partial q_j^*}{\partial x_i} = \frac{-c'(Bx^*)}{f'(Q^*)}\left(\frac{1}{n(1-\lambda)}\right)\left(\frac{2n + \Lambda\delta}{n + \Lambda(\delta+1)}\right)\left(\tilde{\beta}(\lambda) - \beta\right),$$

where

$$\tilde{\beta}(\lambda) = \frac{n(1+\lambda) + \Lambda\delta}{2n + \Lambda\delta}.$$

Evaluating $\partial\phi_i/\partial q_j$ at the symmetric equilibrium we can rewrite the strategic effect of investment as follows:

$$\psi \equiv \frac{\partial\phi_i}{\partial q_j}\left(\frac{\partial q_j^*}{\partial x_i}\right) = (-c'(Bx^*))q^*\omega(\lambda)\left(\tilde{\beta}(\lambda) - \beta\right) \tag{16}$$

with[28]

$$\omega(\lambda) = \frac{\Lambda}{n}\left(\frac{2n + \Lambda\delta}{n + \Lambda(1+\delta)}\right) > 0.$$

By equation (16), we may write the first-order necessary condition (14) in the following manner (for $\lambda \in [0, 1)$)

$$-c'(Bx^*)\left[\tau + (n-1)\omega(\lambda)(\tilde{\beta}(\lambda) - \beta)\right](Q^*/n) - \Gamma'(x^*) = 0. \tag{17}$$

[27] Since $\alpha_q - \rho_q = f'(Q^*)(1-\lambda) < 0$ for $\lambda < 1$.

[28] The second-order condition, $\alpha_q < 0$, requires that $2n + \Lambda\delta > 0$, and the stability condition, $\alpha_q + (n-1)\rho_q < 0$, requires that $n + \Lambda(1+\delta) > 0$. Therefore, $\omega(\lambda) > 0$.

When second-stage production decisions are strategic substitutes, we have that $\partial q_i^*/\partial x_i > 0$. Thus, if a firm increases its investment in R&D in the first stage, then it will increase its output in the second stage. On the other hand, from equation (15) we have that $sign\{\partial q_j^*/\partial x_i\} = sign\{\beta - \tilde{\beta}(\lambda)\}$, then it is clear that $\partial q_j^*/\partial x_i > 0$ when quantities are strategic complements; in the case of strategic substitutes, however, $\partial q_j^*/\partial x_i > 0$ only if $\beta > \tilde{\beta}(\lambda)$. Intuitively, when a firm increases the amount invested in R&D, it exerts two opposite effects on the output decision of rival firms. There is a positive effect on the output of rival firms because they become more efficient thanks to the presence of spillovers. And at the same time there exists a negative effect because the natural reaction of firms to the higher quantity of firm i is to reduce their output. When spillover effects are strong such that $\beta > \tilde{\beta}(\lambda)$, the positive effect outweighs the negative effect implying that $\partial q_j^*/\partial x_i > 0$. These two effects are standard in the R&D literature, but we can also conduct comparative statics on the threshold value $\tilde{\beta}(\lambda)$, which is crucial in determining the sign of the strategic effect (ψ). Assumption A.1. facilitates this task because under this assumption $\tilde{\beta}$ takes a simple form. In particular, from the expression of $\tilde{\beta}$ it is straightforward to show that

PROPOSITION 6 *The threshold $\tilde{\beta}$ decreases (respectively increases) with the number of firms if demand is concave (convex), increases with the degree of cooperation if $\delta > -2$ (i.e., $\partial\tilde{\beta}/\partial\lambda > 0$ if $\delta > -2$)*[29]*, and decreases with the curvature of the inverse demand function $-\delta$ (i.e., $\partial\tilde{\beta}/\partial\delta > 0$).*

Proposition 6 reveals that the impact of the number of firms on the threshold, $\tilde{\beta}$, depends on the shape of the demand, and that $\tilde{\beta}$ increases with the degree of cooperation and decreases with the curvature of the inverse demand function.

Since $\partial\phi_i/\partial q_j < 0$, the sign of the strategic effect is the opposite to the sign of $\partial q_j^*/\partial x_i$, i.e.,

$$sign\{\psi\} = -sign\{\partial q_j^*/\partial x_i\} = sign\{\tilde{\beta}(\lambda) - \beta\}.$$

Therefore, the strategic effect is positive if production decisions are substitutes and β is below the threshold $\tilde{\beta}(< 1)$. Then, as shown in Leahy and Neary (1997, Proposition 1) in the case of $\lambda = 0$, equations (13)-(17) imply that output and R&D are higher in the two-stage model than in the static model.[30] Intuitively, if $\beta < \tilde{\beta}$, then each firm anticipates that a higher

[29] Note that the second-order condition with respect to q, requires that $\delta > -2$ when $\lambda = 1$.

[30] The result requires uniqueness and that the profit function in the two models satisfy the Seade stability

investment in R&D in the first stage will reduce the output of rival firms in the second stage. This implies that $\psi \equiv (\partial\phi_i/\partial q_j)\left(\partial q_j^*/\partial x_i\right) > 0$, each firm is then lead to invest more in R&D in the first stage, which in turn boosts output in the second stage ($\partial q_i^*/\partial x_i > 0$). The result that output and R&D are higher in the two-stage model than in the static model is no longer true if quantities are strategic complements or strategic substitutes with $\beta > \tilde{\beta}$.

Next we analyze how the degree of cooperation affects equilibrium decisions of output and R&D. Let $\psi_z \equiv \partial\psi/\partial z$ with $z = q, x, \lambda$. Then, if the regularity condition

$$\tilde{\Delta}(Q^*, x^*) = \Delta_q\left(\Delta_x + \psi_x(n-1)\right) - \varphi_{xq}\left(\varphi_{xq}\tau + \psi_q(n-1)\right)B > 0 \tag{18}$$

holds, we may state:

LEMMA 5 *In the two-stage model:*

(i) If $\partial x^*/\partial\lambda \leq 0$, then $\partial q^*/\partial\lambda < 0$.

(ii) The sign$\{\partial x^*/\partial\lambda\}$ is given by

$$\begin{aligned} sign\left\{\frac{\partial x^*}{\partial\lambda}\right\} &= sign\{\beta\left[(1-\omega'(\lambda))P'(c)^{-1}n + (n-1)(\omega(\lambda)-\lambda)\right] \\ &\quad +(\omega'(\lambda)\tilde{\beta}(\lambda) + \omega(\lambda)(d\tilde{\beta}/d\lambda))P'(c)^{-1}n - 1 - (n-1)\omega(\lambda)\tilde{\beta}(\lambda)\}. \end{aligned} \tag{19}$$

(iii) The sign$\{\partial q^*/\partial\lambda\}$ is given by

$$sign\left\{\frac{\partial q^*}{\partial\lambda}\right\} = sign\left\{[1-\omega'(\lambda) + \left(\omega'(\lambda)\tilde{\beta}(\lambda) + \omega(\lambda)(d\tilde{\beta}/d\lambda)\right)/\beta]B - H(\beta)\right\}. \tag{20}$$

Proof. See Appendix D. ■

Therefore, again we have that allowing for some cooperation will increase output only if it also boosts R&D. In particular, from (19), $\partial x^*/\partial\lambda > 0$ if and only if

$$\beta > \underline{\beta}^{2S} \equiv \frac{1 - (\omega'(\lambda)\tilde{\beta}(\lambda) + \omega(\lambda)\tilde{\beta}'(\lambda))P'(c)^{-1}n + \omega(\lambda)(n-1)\tilde{\beta}(\lambda)}{(1+n+\Lambda\delta) + (n-1)\omega(\lambda) - P'(c)^{-1}n\omega'(\lambda)}.$$

We may now derive the threshold values of spillover that determine the sign of the impact of λ on the R&D and output in equilibrium. *RI* (where $\partial x^*/\partial\lambda \leq 0$ and $\partial q^*/\partial\lambda < 0$) takes place when $\beta \leq \underline{\beta}^{2S}$. Assuming that $\beta = \beta^{2S\prime}$ is the unique positive solution to the equation

condition with respect to R&D: the marginal profit of each firm with respect to R&D must be decreasing in a uniform increase in R&D by all firms.

$[(1-\omega') + \left(\omega'\tilde{\beta} + \omega\tilde{\beta}'\right)/\beta]B - H(\beta) = 0$ with $\partial q^*/\partial\lambda < 0$ for $\beta \in [0, \beta^{2S\prime}]$ and $\partial q^*/\partial\lambda > 0$ for $\beta \in (\beta^{2S\prime}, 1]$,[31] we have that RII (where $\partial q^*/\partial\lambda \leq 0$ and $\partial x^*/\partial\lambda > 0$) occurs when $\underline{\beta}^{2S} < \beta \leq \beta^{2S\prime}$, and $RIII$ (where $\partial q^*/\partial\lambda > 0$ and $\partial x^*/\partial\lambda > 0$) occurs when $\beta > \beta^{2S\prime}$. (These results extend Proposition 1 to a model with two stages.) A direct application of (19) and (20) allows us to derive the threshold values for each of the model specifications considered in the present paper. Appendix D contains for each particular specification the expressions for $sign\{\partial x^*/\partial\lambda\}$, $sign\{\partial q^*/\partial\lambda\}$, and equilibrium values q^*, x^*, from which one can get the aforementioned threshold values and therefore obtain results analogous to Table 4. We may establish,[32]

PROPOSITION 7 *Under assumptions A.1.-A.4., in the two-stage model: (i) Total surplus is maximized with* $\lambda^* = 0$ *when* $\beta \leq \inf\{\underline{\beta}^{2S} : \lambda \in [0,1]\}$. *However, there exists a sufficiently large spillover value for which some cooperation is socially optimal (*$\lambda^* > 0$*) if for* $\beta = 1$

$$(1 + s'(0))n + (1 - s(0))(n-1)((1 + s'(0))(1 + \delta + n) - (1 + (n-1)s(0)) - H(1) > 0, \quad (21)$$

where

$$s(\lambda) = \omega(\lambda)\left(\tilde{\beta}(\lambda) - \beta\right).$$

Proof. See Appendix A. ■

For sufficiently low spillovers, namely when $\beta \leq \underline{\beta}^{2S}$ for all λ, we have that $\partial x^*/\partial\lambda \leq 0$ and therefore $\partial q^*/\partial\lambda < 0$, so from the welfare point of view no cooperation is optimal. Conversely, proof of Proposition 7 in Appendix D also shows that $W'(0)|_{\beta=1} > 0$ when condition (21) holds, in which case there exists a sufficiently large spillover value for which some degree of cooperation is socially optimal.[33] In particular, for the model specifications considered in this paper and from condition $W'(0) > 0$ we can obtain the threshold value $\bar{\beta}^{2S}$ above which $\lambda^*_{TS} > 0$.[34] Fig. 8a and 8b depict, respectively, the threshold $\bar{\beta}^{2S}$ under AJ and KMZ model specifications. Fig. 8b reveals that in KMZ, $\bar{\beta}^{2S}$ tends to be above 1 if we consider the same values as in AJ. In particular, only if $b\gamma$ is low enough, we have that $\bar{\beta}^{2S} < 1$ (this result is in line with the static

[31] Below simulations show that this is the case in AJ and KMZ.

[32] The explicit expressions of $s(0)$ and $s'(0)$ can be found in Appendix D.

[33] Condition (21) involves condition (12): both are identical when $s = 0$. The reason is that the first-order condition with respect to R&D of the two-stage model (given by equation (17)) involves the corresponding first-order condition of the simultaneous model (given by equation(3)). That is, from the former we can obtain the latter by setting $s = 0$. (The first-order necessary condition with respect to output is the same in the simultaneous and two-stage model.)

[34] The expression of $W'(\lambda)$ in the two-stage model is provided in the proof of Proposition 7 (Appendix D).

case). Also, we observe that under the AJ and KMZ model specifications, $\bar{\beta}^{2S}$ decreases with the number of firms and increases with γb. Figures 9a (respectively 9b) depict the threshold $\bar{\beta}^{2S}$ for the CE model specification and for a given ε (α) and different values of n and α (ε). As in the static case: the threshold value decreases with n, the elasticity of the innovation function, α, and the elasticity of demand ε^{-1}.

Threshold values $\bar{\beta}$, above which some cooperation is socially optimal

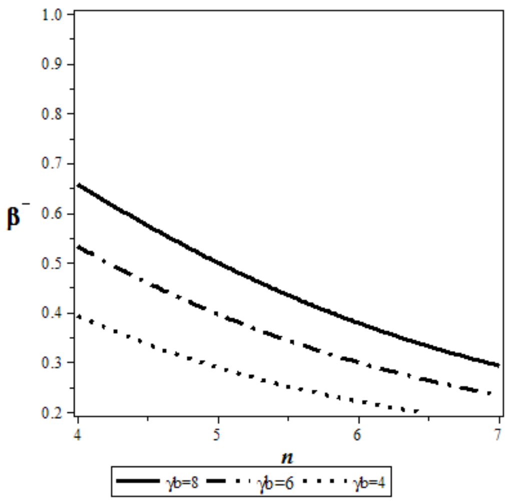

Fig. 8a. AJ model specification.

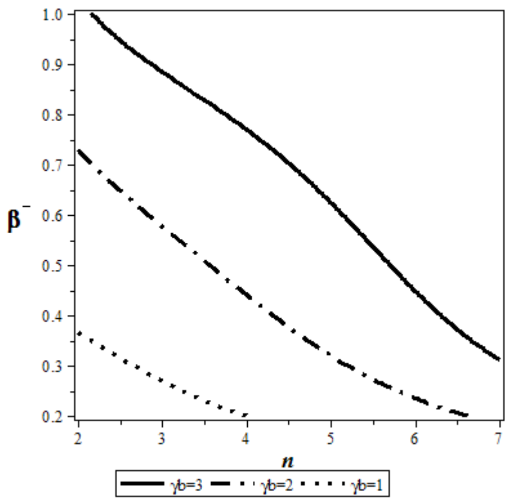

Fig. 8b. KMZ model specification.

Threshold values $\bar{\beta}$, above which some cooperation is socially optimal

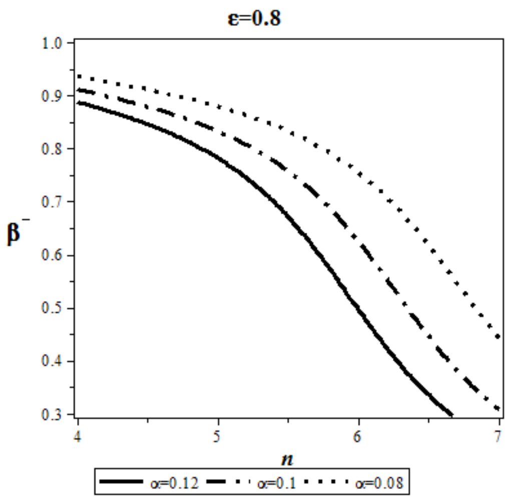

Fig. 9a. Constant elasticity model.

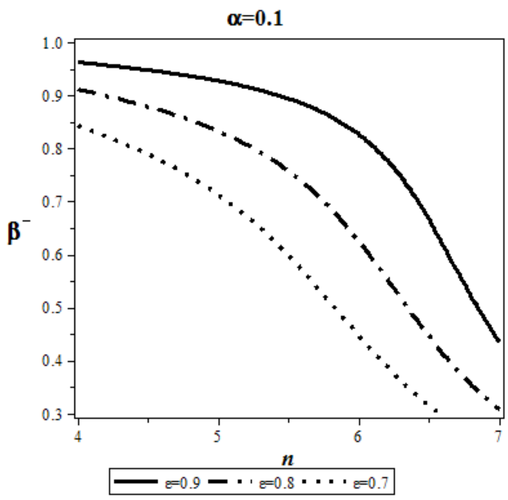

Fig. 9b. Constant elasticity model.

Figure 10a and 10b depict the socially optimal degree of cooperation (λ^*) in terms of total surplus and consumer surplus for the constant elasticity model. As in the static case we observe that when the number of firms is less than 4 we are in RI, implying that no cooperation is socially optimal. However, as the number of firms and the intensity of spillover effects increase, the socially optimal degree of cooperation (in terms of total surplus) increases (and faster than it does in the static model). Results with respect to α and ε^{-1} are similar to those of the static model, but in contrast to the static model, λ^*_{CS} may be positive if spillovers are sufficiently large, and it is neutral with respect to the number of firms.

Optimal degree of cooperation in terms of total surplus and consumer surplus

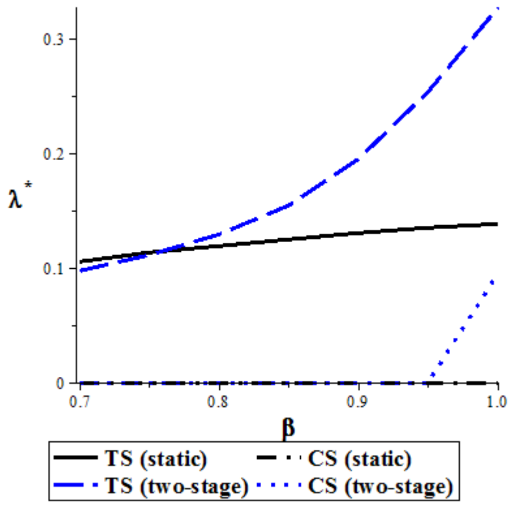

Fig. 10a. Constant elasticity model. (Numerical values: $\alpha = 0.1$, $\varepsilon = 0.8$, $\sigma = \kappa = 1$, $n = 8$.)

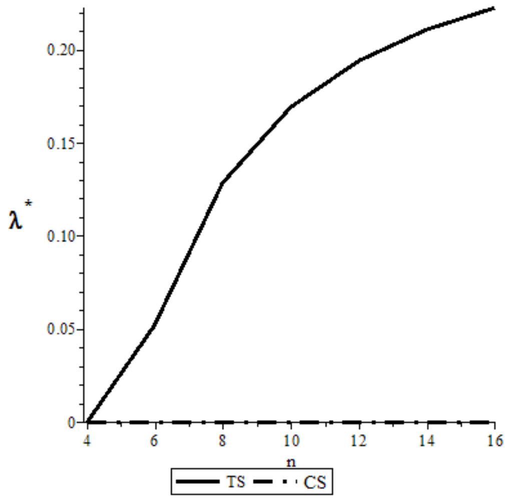

Fig. 10b. Constant elasticity model. (Numerical values: $\alpha = 0.1$, $\varepsilon = 0.8$, $\sigma = \kappa = 1$, $\beta = 0.8$.)

Figure 11a and 11b depict the socially optimal degrees of cooperation under the AJ model specification. As in the static model, the impact of λ on total surplus and consumer surplus can be positive if spillovers are sufficiently high as to enter in $RIII$. In particular, when $\beta > 0.8$ the planner will allow full cooperation. Additionally, we find that in the two-stage model the size of spillovers or the number of firms above which we are in $RIII$ is lower than in the static model. In this sense, Fig. 11a reveals that in contrast to the static model, the threshold value above which full cooperation is optimal for consumers is lower than the threshold value above which it is optimal in terms of total surplus perspective. In particular, $\lambda^*_{CS} > \lambda^*_{TS}$ for intermediate values of spillover: when β takes values larger than 0.6, consumer surplus, which is strictly convex with respect to λ, is maximized with full cooperation but total surplus is not. Intuitively, total surplus is not maximized with full cooperation because it implies too much production, which goes at the expense of firms' profit since they are not sufficiently efficient. Only when spillover is large enough ($\beta > 0.75$), firms are efficient enough to benefit from higher production quantities. Finally, under the KMZ model specification the optimal degree of cooperation in terms of total surplus is also increasing in β. As for consumer surplus, spillover effects tend to have no impact on the optimal degree of cooperation unless γb is sufficiently small (close to 1), as we assume in Fig. 12a, in which case full cooperation may be optimal with respect to CS and TS as it occurs in the static case. Since for $\beta = 0.8$ and $\gamma b = 0.9$, $\lambda^*_{TS} = \lambda^*_{CS} = 1$, in Fig. 12b we assume that $\gamma b = 1.4$ and observe that entry raises λ^*_{TS}. The pattern of results in the comparative statics analysis is the same as the one found in the one-stage game (see Table 5) with the only exception that γ seems to have no impact on λ^*_{CS} under the AJ model specification, and that for $\gamma b \in (1, 1.3)$, entry could slightly reduce λ^*_{TS} in KMZ. Figure 10a, 11a and 12a show that the socially optimal degree of cooperation in the two-stage model tends to be above the socially optimal degree of cooperation in the static model. This result indicates that toughness of antitrust policy (in terms of limiting cross-ownership or degree of collusion) should be moderated in the two-stage model when spillovers are high.

Optimal degree of cooperation in terms of total surplus and consumer surplus

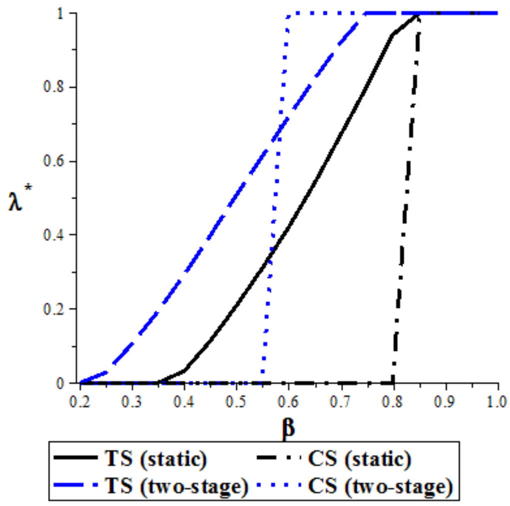

Fig 11a. AJ model specification.

(Numerical values: $a = 700$, $c = 500$, $\gamma = 7$, $n = 6$ and $b = 0.6$.)

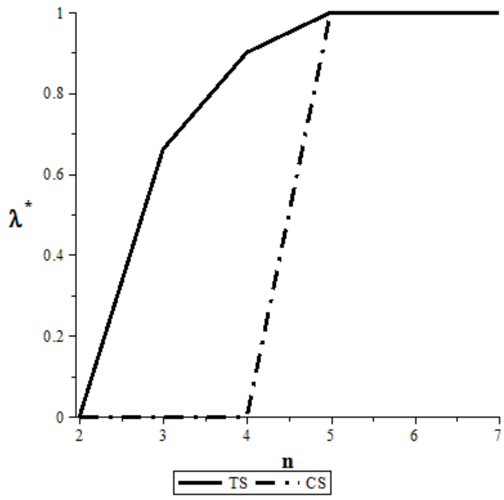

Fig. 11b. AJ model specification.

(Numerical values: $a = 700$, $c = 500$, $\gamma = 0.7$, $\beta = 0.8$ and $b = 0.6$.)

Optimal degree of cooperation in terms of total surplus and consumer surplus

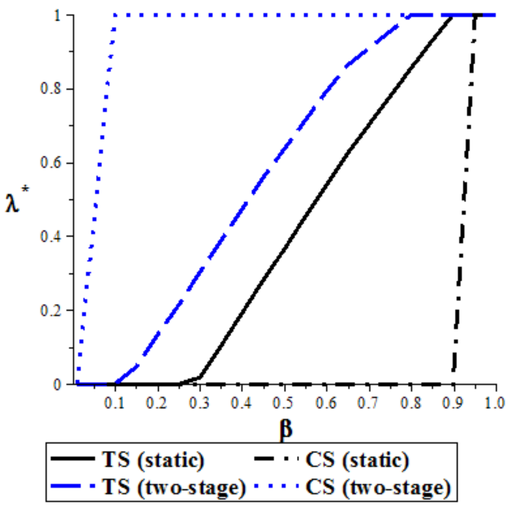

Fig. 12a. KMZ model specification.

(Numerical values: $a = 700$, $c = 500$,

$\gamma = 3$, $n = 6$ and $b = 0.3$.)

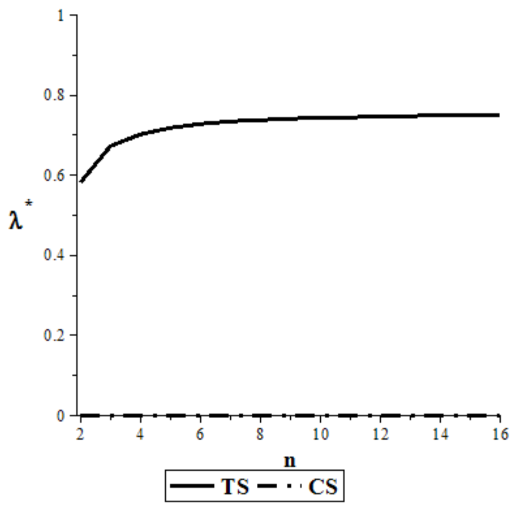

Fig. 12b. KMZ model specification.

(Numerical values: $a = 700$, $c = 500$,

$\gamma = 4$, $\beta = 0.8$ and $b = 0.4$.)

Remark. In the two-stage model under the AJ specification it is possible that the optimal degree of cooperation with respect to CS is higher than with respect to TS (as shown in Fig. 11a for intermediate values of spillover). The reason is that in the two-stage model and for intermediate values of spillover, $\partial q^*/\partial\lambda$ becomes strictly convex with respect to λ and has a critical point for some $\lambda \in (0,1)$.[35] Then, consumer surplus is also strictly convex with respect to λ, and allowing full cooperation may well be optimal for consumers. However, for the same intermediate values of spillover, total surplus is concave with critical value for some intermediate degree of cooperation (where $\partial q^*/\partial\lambda < 0$).[36] In the static model and under the AJ model specification, if β is lower (respectively, higher) than β', then $\partial q^*/\partial\lambda$ is always decreasing (increasing) with λ, i.e., $\partial^2 q^*/(\partial\lambda)^2$ is negative (positive) for any feasible λ. Then, when $\beta < \beta'$, no cooperation is optimal for consumers, while in terms of total surplus some degree of cooperation may be optimal. Instead, when $\beta > \beta'$, full cooperation is optimal for consumers and also for total surplus (case 'c' of Proposition 4). Under the KMZ model specification, case 'c' of Proposition 4 also holds, thus as in AJ, whenever full cooperation is optimal for consumers, it is also optimal for total surplus.

[35]In particular, simulation results show that $\partial q^*/\partial\lambda < 0$ for all λ when $\beta < 0.5$. But, when $\beta \geq 0.5$, q^* is convex with respect to λ: it is decreasing (respectively increasing) when λ is low (high).

[36]From equation (11), we have that $W'(\lambda) = 0$ for any $\lambda \in (0,1)$ with $\partial x^*/\partial\lambda > 0$ only if $\partial q^*/\partial\lambda < 0$.

6 Concluding remarks

We consider a general oligopoly Cournot model of process (cost-reducing) R&D investments with spillovers in which cooperation in R&D and output cannot be disentangled (and collusion may be imperfect) because of financial interests or because cooperation in R&D extends to cooperation in product market.

We show that, independently of the intensity of the R&D spillover effects, firms always have incentives to increase the degree of cooperation in the industry; the socially optimal degree of cooperation, however, depends on the number of firms, the elasticity of demand and innovation function, and the spillover levels. In particular, we derive and characterize the threshold value below which a social welfare maximizer planner will choose no cooperation at all, and the threshold value above which some degree of cooperation is socially optimal. We also obtain the threshold value above which consumers benefit from cooperation. Regarding total surplus, simultaneous cooperation in R&D and output can be optimal when spillovers are large enough (and the scope is larger the more firms there are in the market). However, if the objective is to maximize consumer surplus, then the scope for cooperation is greatly reduced, and entry moreover need not optimally induce more cooperation.

We interpret toughness of antitrust policy as limiting cooperation: cross-shareholdings or with increased degree of activism of competition policy when cooperation in R&D and output go together. The competition-reducing effect of silent financial interests gives support to policy intervention. However, passive investments may be welfare enhancing, and even increase consumer surplus, when the industry exhibits sufficiently large R&D spillovers. In the extreme, to form an RJV and cooperate fully may be socially optimal under some parametric assumptions. Competition authorities put increasing weight on consumer surplus and at the same time allow more cooperative R&D. Unless R&D spillovers are sufficiently large, this practice would be contradictory according to our results as long as R&D cooperation leads to the same degree of cooperation in output levels. Our results also indicate that toughness of antitrust policy should decrease with the number of firms, the elasticity of demand and innovation function, and the intensity of spillover effects.

We extend the static model to the two-stage model and find that our results are robust to this extension. It turns out that firms produce and invest in R&D less than in the static case when spillovers are above a given threshold so that the strategic effect becomes negative.

In this case, the social gains from a higher degree of cooperation that induces firms to invest and produce towards the socially optimal levels are even higher. We also characterize how the number of firms, the degree of cooperation and the curvature of the inverse demand function affect such a threshold value. Finally, numerical simulations show that toughness of antitrust policy should be moderated in the two-stage model as compared to that of the static model when spillovers are high.

7 Appendix

7.1 Appendix A

Proof of Lemma 3. From Lemma 1 we have that if $\delta \leq -(1+n)/\Lambda$, so $n+1+\delta\Lambda < 0$, then $dx^*/d\lambda \leq 0$, which, using equation (8), in turn implies that $dq^*/d\lambda < 0$: for all β only RI exists; (ii) if $\delta > -(n+1)/\Lambda$, then in addition to RI, RII exists only if $\delta > -n/\Lambda$ also holds. The reason is that when $1+n+\delta\Lambda > 0$ holds, from Lemma 1 we have that $\partial x^*/\partial\lambda > 0$ only if $\beta > 1/(1+n+\delta\Lambda)$. However, $1/(1+n+\delta\Lambda) < 1$ only if $\delta > -n/\Lambda$, in which case there exists some region of feasible spillover values for which $\partial x^*/\partial\lambda > 0$. Note that for a given n, the condition $\delta > -n/\Lambda$ is stricter than the condition $\delta > -(n+1)/\Lambda$. Furthermore, the stability condition $\Delta_q < 0$, which requires that $\delta > -(n+\Lambda)/\Lambda$, is stricter than (respectively equal to) the second-order condition $\alpha_q < 0$, which requires that $\delta > -2n/\Lambda$, for $\lambda \in [0,1)$ (respectively $\lambda = 1$). Thus, for $-(n+\Lambda)/\Lambda \leq \delta \leq -n/\Lambda$ only RI exists, while for $\delta > -n/\Lambda$ RI and RII exist. Finally, $RIII$ will only exist if $\beta' < 1$. The condition $n - H(1) > 0$ guarantees that $\beta' < 1$: $H(\beta) \geq 0$ with $H(0) = \infty$ (when $c'' > 0$ and/or $\Gamma'' > 0$), moreover A.4. implies that $\partial(H(\beta) - B)/\partial\beta < 0$, thus $\exists\beta' \in (0,1)$ if $(B - H(\beta))|_{\beta=1} > 0$, i.e., if $n - H(1) > 0$.

Proof of Proposition 2. Profit per firm as a function of λ at equilibrium is given by

$$\pi^*(\lambda) = [f(Q^*) - c(Bx^*)]q^* - \Gamma(x^*).$$

By differentiating π^* with respect to λ, we obtain

$$\pi^{*\prime}(\lambda) = f'(Q^*)n\frac{\partial q^*}{\partial\lambda}q^* - c'(Bx^*)B\frac{\partial x^*}{\partial\lambda}q^* + [f(Q^* - c(Bx^*)]\frac{\partial q^*}{\partial\lambda} - \Gamma'(x^*)\frac{\partial x^*}{\partial\lambda}.$$

Using that in equilibrium $f(Q^*) - c(Bx^*) = -f'(Q^*)\Lambda q^*$ and $\Gamma'(x^*) = -c'(Bx^*)q^*\tau$, we can rewrite the above expression as

$$
\begin{aligned}
\pi^{*\prime}(\lambda) &= f'(Q^*)n\frac{\partial q^*}{\partial \lambda}q^* - c'(Bx^*)B\frac{\partial x^*}{\partial \lambda}q^* - f'(Q^*)\Lambda q^*\frac{\partial q^*}{\partial \lambda} + c'(Bx^*)q^*\tau\frac{\partial x^*}{\partial \lambda} \\
&= f'(Q^*)(n-\Lambda)q^*\frac{\partial q^*}{\partial \lambda} + c'(Bx^*)(\tau - B)q^*\frac{\partial x^*}{\partial \lambda} \\
&= f'(Q^*)(n-1)(1-\lambda)q^*\frac{\partial q^*}{\partial \lambda} - c'(Bx^*)\beta(n-1)(1-\lambda)q^*\frac{\partial x^*}{\partial \lambda},
\end{aligned}
$$

or

$$
\pi^{*\prime}(\lambda) = (n-1)(1-\lambda)q^*[f'(Q^*)\frac{\partial q^*}{\partial \lambda} - c'(Bx^*)\beta\frac{\partial x^*}{\partial \lambda}].
$$

Therefore,

$$
sign\{\pi^{*\prime}(\lambda)\} = sign\{-c'(Bx^*)\beta\frac{\partial x^*}{\partial \lambda} + f'(Q^*)\frac{\partial q^*}{\partial \lambda}\}.
$$

In RII, we have that $\partial x^*/\partial\lambda > 0$ and $\partial q^*/\partial\lambda < 0$. Hence from (10) it is clear that $\pi^{*\prime}(\lambda) > 0$. To determine $sign\{\pi^{*\prime}(\lambda)\}$ in RI and $RIII$, the following ratio will be useful:

$$
\Omega \equiv \frac{\partial q^*/\partial \lambda}{\partial x^*/\partial \lambda}.
$$

Using equations (5) and (6) we have

$$
\begin{aligned}
\Omega &= \frac{\varphi_{\lambda x}\varphi_{xq}B - \varphi_{\lambda q}\Delta_x}{\varphi_{\lambda q}\varphi_{xq}\tau - \varphi_{\lambda x}\Delta_q} \\
&= \frac{\varphi_{\lambda x}\varphi_{xq}\left[B - \left(\varphi_{\lambda q}/\varphi_{\lambda x}\right)\left(\Delta_x/\varphi_{xq}\right)\right]}{\varphi_{\lambda q}\varphi_{xq}\left[\tau - \left(\varphi_{\lambda x}/\varphi_{\lambda q}\right)\left(\Delta_q/\varphi_{xq}\right)\right]}.
\end{aligned}
$$

Recall that $H(\beta) = \left(\varphi_{\lambda q}/\varphi_{\lambda x}\right)\left(\Delta_x/\varphi_{xq}\right)$. Let $H_q(\beta) = \left(\varphi_{\lambda x}/\varphi_{\lambda q}\right)\left(\Delta_q/\varphi_{xq}\right)$, then

$$
\Omega = \frac{\varphi_{\lambda x}}{\varphi_{\lambda q}}\left[\frac{B - H(\beta)}{\tau - H_q(\beta)}\right].
$$

Noting that $\Delta_q = f'(Q^*)(n + \Lambda(\delta+1))$, $\varphi_{xq} = -c'(Bx^*)$, $\varphi_{\lambda q} = f'(Q^*)(n-1)q^*$, and $\varphi_{\lambda x} = -\beta(n-1)c'(Bx^*)q^*$, we can write

$$
\begin{aligned}
H_q(\beta) &= -\frac{\beta(n-1)c'(Bx^*)q^*}{f'(Q^*)(n-1)q^*}\left[\frac{f'(Q^*)(n+\Lambda(\delta+1))}{-c'(Bx^*)}\right] \\
&= \beta(n + \Lambda(\delta+1)) \\
&= \beta n P'(c)^{-1}.
\end{aligned}
$$

Therefore, we can rewrite Ω as follows:

$$\begin{aligned}\Omega &= -\frac{\beta(n-1)c'(Bx^*)q^*}{f'(Q^*)(n-1)q^*}\left[\frac{B-H(\beta)}{\tau-\beta nP'(c)^{-1}}\right]\\ &= \frac{\beta c'(Bx^*)}{f'(Q^*)}\left[\frac{H(\beta)-B}{\tau-\beta nP'(c)^{-1}}\right].\end{aligned}$$

Since $\tau-\beta nP'(c)^{-1}=1-(n+1+\delta\Lambda)\beta$, we have that the term $\tau-\beta nP'(c)^{-1}$ is positive in RI (where $\beta\leq 1/(1+n+\Lambda\delta)$), and negative in RII and $RIII$.

Consider RI, which for $1+n+\Lambda\delta>0$ exists only if

$$\beta\leq\frac{1}{(1+n+\Lambda\delta)}. \tag{22}$$

Suppose first that the inequality (22) holds strictly, then $\partial x^*/\partial\lambda<0$ and, using equation (10), $\pi^{*\prime}(\lambda)>0$ if $-c'(Bx^*)\beta+f'(Q^*)\Omega<0$, that is, if

$$-c'(\cdot)\beta+f'(Q^*)\frac{(-c'(\cdot))\,\beta}{f'(Q^*)}\left[\frac{B-H(\beta)}{\tau-\beta nP'(c)^{-1}}\right]<0$$

$$1+\left[\frac{B-H(\beta)}{\tau-\beta nP'(c)^{-1}}\right]<0. \tag{23}$$

In RI: $\tau-\beta nP'(c)^{-1}>0$ and $B-H(\beta)<0$. In particular, condition (23) can be rewritten as

$$2(1-\beta)-\Lambda\delta\beta<H(\beta). \tag{24}$$

Condition (24) is trivially satisfied when $\beta=0$. Let us consider $\beta>0$, then we can rewrite the regularity condition in terms of H as follows: $\Delta=(\Lambda(1+\delta)+n)\,\beta H(\beta)-\tau B>0$, with $\Lambda(1+\delta)+n>0$ as $\Delta_q<0$. Thus, if the equilibrium is regular:

$$H(\beta)>\frac{\tau B}{(\Lambda(1+\delta)+n)\beta}.$$

We only have to show that in RI:

$$\frac{\tau B}{(\Lambda(1+\delta)+n)\beta}>2(1-\beta)-\Lambda\delta\beta,$$

or, equivalently,

$$\tilde{g}(\beta)\equiv\tau B>\tilde{h}(\beta)\equiv[2(1-\beta)-\Lambda\delta\beta][\Lambda(1+\delta)+n]\beta$$

holds. Note that $\tilde{g}(0) = 1$, $\tilde{g}'(\beta) > 0$, $\tilde{g}''(\beta) > 0$ for $\beta > 0$ and $\tilde{g}''(\beta) = 0$ for $\beta = 0$. On the other hand, $\tilde{h}(0) = 0$ and

$$\tilde{h}'(\beta) = 2[\Lambda(1+\delta) + n][1 - (2 + \Lambda\delta)\beta].$$

Furthermore, it can be shown that solving the equation $\tilde{g}(\beta) = \tilde{h}(\beta)$ for β yields the following two solutions:

$$\beta_1 = \frac{1}{\Lambda\delta + n + 1} \text{ and } \beta_2 = \frac{1}{\Lambda(\delta + 1) + 1}.$$

Note that when $\Lambda\delta + n + 1 > 0$ holds, β_1 is the threshold value that determines RI: for $\beta < \beta_1$, $\partial x^*/\partial\lambda < 0$. If however $\Lambda\delta + n + 1 < 0$, then $\partial x^*/\partial\lambda < 0$ for all λ.

Suppose that $2 + \Lambda\delta > 0$, i.e., $\delta > -2/\Lambda$, then $\Lambda\delta + n + 1 > 0$ and $\Lambda(\delta + 1) + 1 > 0$: $\beta_1 > 0$ and $\beta_2 > 0$. Furthermore, $\beta_1 < \beta_2$ (for $\lambda < 1$) and $\tilde{h}''(\beta) < 0$. Therefore, $\tilde{g}(\beta) > \tilde{h}(\beta)$ for $0 < \beta < \beta_1$.

Suppose that $2 + \Lambda\delta < 0$ and $\Lambda(\delta+1)+1 > 0$, i.e., $-(1+\Lambda)/\Lambda < \delta < -2/\Lambda$ (so $\Lambda\delta + n + 1 > 0$ also holds), then $\beta_1 > 0$, $\beta_2 > 0$, $\beta_1 < \beta_2$ (for $\lambda < 1$) and $\tilde{h}''(\beta) > 0$: $\tilde{g}(\beta) > \tilde{h}(\beta)$ for $0 < \beta < \beta_1$.

Suppose that $\Lambda\delta + n + 1 > 0$ and $\Lambda(\delta + 1) + 1 < 0$, i.e., $-(n+1)/\Lambda < \delta < -(1+\Lambda)/\Lambda$, then $\beta_1 > 0$, $\beta_2 < 0$ and $\tilde{h}''(\beta) > 0$: $\tilde{g}(\beta) > \tilde{h}(\beta)$ for $0 < \beta < \beta_1$.

Suppose that $\Lambda\delta + n + 1 < 0$, i.e., $\delta < -(n+1)/\Lambda$, then $\beta_1 < 0$, $\beta_2 < 0$, $\beta_1 > \beta_2$ (for $\lambda < 1$) and $\tilde{h}''(\beta) > 0$: $\tilde{g}(\beta) > \tilde{h}(\beta)$ for all β.

Suppose now that $\beta = \beta_1$, so condition (22) is binding, then $\partial x^*/\partial\lambda = 0$, so $sign\{\pi^{*\prime}(\lambda)\} = sign\{f'(Q^*)\partial q^*/\partial\lambda\}$, which is positive in RI since in this region: $\partial q^*/\partial\lambda < 0$.

Consider $RIII$, since here $x^{*\prime}(\lambda) > 0$, using equation (10) we have that $\pi^{*\prime}(\lambda) > 0$ if $-c'(Bx^*)\beta + f'(Q^*)\Omega > 0$, that is, if

$$-c'(\cdot)\beta + f'(Q^*)\frac{(-c'(\cdot))\beta}{f'(Q^*)}\left[\frac{B - H(\beta)}{\tau - \beta n P'(c)^{-1}}\right] > 0,$$

or, equivalently, if

$$1 + \left[\frac{B - H(\beta)}{\tau - \beta n P'(c)^{-1}}\right] > 0.$$

In $RIII : \tau - \beta n P'(c)^{-1} < 0$ and $B - H(\beta) > 0$. Thus, $\pi^{*\prime}(\lambda) > 0$ if

$$1 - (n + 1 + \delta\Lambda)\beta < H(\beta) - B, \tag{25}$$

which is equivalent to condition (24). Note that $RIII$ may exist only if $\delta > -n/\Lambda$, in which case $\delta > -(n+1)/\Lambda$, so $\beta_1 > 0$.

Suppose that $\delta > -2/\Lambda$, then $\beta_1 > 0$, $\beta_2 > 0$, $\beta_1 < \beta_2$ (for $\lambda < 1$) and $\tilde{h}''(\beta) < 0$. Hence, $\tilde{g}(\beta) > \tilde{h}(\beta)$ for $0 < \beta < \beta_1$, $\tilde{g}(\beta) < \tilde{h}(\beta)$ for $\beta_1 < \beta < \beta_2$, and $\tilde{g}(\beta) > \tilde{h}(\beta)$ for $\beta > \beta_2$. Thus, we only have to show that $\beta' > \beta_2$, so that $\tilde{g}(\beta) > \tilde{h}(\beta)$ for any $\beta \geq \beta'$. To see this, note that $\beta' > \beta_1$.[37] Furthermore, condition (25) holds at $\beta = \beta'$: $H(\beta') - (1 + \beta'(n-1)) = 0 > 1 - (n + 1 + \delta\Lambda)\beta$. Therefore, $\beta' > \beta_2$: for any $\beta > \beta'$, $\tilde{g}(\beta) > \tilde{h}(\beta)$.

Suppose now that $\delta < -2/\Lambda$, when $-n/\Lambda > -(\Lambda+1)/\Lambda$ (i.e., $\Lambda > n-1$), the feasible range is $-n/\Lambda < \delta < -2/\Lambda$, where $\beta_1 > 0$, $\beta_2 > 0$, $\beta_1 < \beta_2$ (for $\lambda < 1$) and $\tilde{h}''(\beta) > 0$. As in the previous case, we can conclude that $\beta' > \beta_2$: for any $\beta > \beta'$, $\tilde{g}(\beta) > \tilde{h}(\beta)$.

Suppose again that $\delta < -2/\Lambda$ but $-n/\Lambda < -(\Lambda+1)/\Lambda$, we can distinguish between two cases: (i) when $-(\Lambda+1)/\Lambda < \delta < -2/\Lambda$, then again we have $\beta_1 > 0$, $\beta_2 > 0$, $\beta_1 < \beta_2$ (for $\lambda < 1$) and $\tilde{h}''(\beta) > 0$, so $\beta' > \beta_2$: for any $\beta > \beta'$, $\tilde{g}(\beta) > \tilde{h}(\beta)$; (ii) when $-n/\Lambda < \delta < -(\Lambda+1)/\Lambda$, in which case $RIII$ does not exist. To see this note that in this case $\beta_1 > 0$, $\beta_2 < 0$, and $\tilde{h}''(\beta) > 0$: $\tilde{g}(\beta) > \tilde{h}(\beta)$ only for $\beta < \beta_1$. If $\beta' < 1$, then condition (25) holds at β', i.e., $\tilde{g}(\beta) > \tilde{h}(\beta)$ for $\beta \geq \beta'$. Therefore, $\beta' < \beta_1$, however as commented above $\beta' > \beta_1$, a contradiction.

Proof of Propositions 4. (i) In the simultaneous model, from (7) we have that $\partial x^*/\partial\lambda \leq 0$ if $\beta < 1/(1+n+\Lambda\delta)$. If $\delta > 0$, then $\inf\{1/(1+n+\Lambda\delta) : \lambda \in [0,1]\} = 1/(1+n(1+\delta))$, however if $\delta < 0$, then $\inf\{1/(1+n+\Lambda\delta) : \lambda \in [0,1]\} = 1/(1+n+\delta)$. Thus, if $\beta \leq \underline{\beta} = 1/(1+n(1+\delta))$ when $\delta > 0$, or $\beta < \underline{\beta} = 1/(1+n+\delta)$ when $\delta < 0$, it follows that $\dfrac{\partial x^*}{\partial\lambda} < 0$ for all λ, which implies that $\dfrac{\partial q^*}{\partial\lambda} < 0$ for all λ by equation (8) and therefore $W'(\lambda) < 0$ for all λ by equation (11): total surplus is maximized with $\lambda^* = 0$.

We now derive the condition that determines the threshold $\hat{\beta}$. The following ratio will be helpful:

$$\Omega \equiv \frac{\partial q^*/\partial\lambda}{\partial x^*/\partial\lambda} = \frac{\beta c'(Bx^*)}{f'(Q^*)}\left[\frac{H(\beta) - B}{\tau - \beta n P'(c)^{-1}}\right]. \tag{26}$$

which is derived in proof of Proposition 2. Since $\tau - \beta P'(c)^{-1}n = 1 - (n+1+\delta\Lambda)\beta$, we have that the term $\tau - \beta n P'(c)^{-1}$ is positive in RI (where $\beta < 1/(n+1+\Lambda\delta)$), and negative in RII

[37] Suppose that $\beta' < \beta_1$, then from Lemma 2 we have that $\partial q^*/\partial\lambda > 0$ for $\beta > \beta'$. However, from Lemma 1 we have that $\partial x^*/\partial\lambda < 0$ for $\beta < \beta_1$. Furthermore, if $\partial x^*/\partial\lambda < 0$, then $\partial q^*/\partial\lambda < 0$. Thus, $\partial q^*/\partial\lambda < 0$ for $\beta' < \beta < \beta_1$, a contradiction.

and $RIII$. From Lemma 3, RII and $RIII$ may exist only if $\delta > -n/\Lambda$, which guarantees that

$$\delta\Lambda + 1 + n > 0. \tag{27}$$

If condition (27) holds, in RII and $RIII$, where $\partial x^*/\partial\lambda > 0$, we have that $W'(\lambda) > 0$ if

$$-\Lambda f'(Q^*)\Omega - (1-\lambda)\beta(n-1)c'(Bx^*) > 0.$$

Inserting (26) into the above condition we obtain

$$-\Lambda f'(Q^*)\frac{\beta c'(\cdot)}{f'(Q^*)}\left[\frac{H(\beta)-B}{\tau-\beta nP'(c)^{-1}}\right] - (1-\lambda)\beta(n-1)c'(Bx^*) > 0,$$

or,

$$-\Lambda\left[\frac{H(\beta)-B}{\tau-\beta nP'(c)^{-1}}\right] - (1-\lambda)(n-1) < 0,$$

which is equivalent to

$$\left[\frac{H(\beta)-B}{\beta nP'(c)^{-1}-\tau}\right] < \frac{(1-\lambda)(n-1)}{\Lambda}.$$

Since in RII and $RIII$, $\beta nP'(c)^{-1} - \tau > 0$ and $(1-\lambda)(n-1) = n-\Lambda$, we have that $W'(\lambda) > 0$ if

$$H(\beta) - B < \frac{n-\Lambda}{\Lambda}\left[\beta nP'(c)^{-1} - \tau\right],$$

or, equivalently, if

$$H(\beta) - B < \frac{n-\Lambda}{\Lambda}\left[(n+1+\delta\Lambda)\beta - 1\right]. \tag{28}$$

Let $h(\beta) = H(\beta) - B$ and $g(\beta) = ((n-\Lambda)/\Lambda)[(n+1+\delta\Lambda)\beta - 1)$. Since $H(0) = \infty$ and (by Assumption A.4.) $\partial(H(\beta)-B)/\partial\beta < 0$, we have that $h(0) = \infty$ and $h'(\beta) < 0$. On the other hand, $g'(\beta) > 0$ since (27) holds when RII and $RIII$ exist, moreover $g(0) < 0$ and $g\left(\underline{\beta}\right) = 0$. Thus, there exists a unique positive threshold $\hat{\beta}$ that solves the equation

$$H(\beta) - B = ((n-\Lambda)/\Lambda)[(n+1+\delta\Lambda)\beta - 1], \tag{29}$$

and for any $\beta > \hat{\beta}$ condition (28) holds, that is, $W'(\lambda) > 0$. Then, $\hat{\beta} > \underline{\beta}$, otherwise for any $\beta \in (\hat{\beta}, \underline{\beta}]$, we have that $\partial x^*/\partial\lambda \leq 0$, which from equation (8) implies that $\partial q^*/\partial\lambda < 0$, as a result and using equation (11): $W'(\lambda) < 0$, a contradiction. Furthermore, if condition (28) holds at $\beta = 1$, then $\hat{\beta} < 1$. Thus, $\hat{\beta} < 1$ if $H(1) - n < ((n - \Lambda)/\Lambda)(n + \delta\Lambda)$.

(ii) Since $CS(\lambda) = \int_0^{Q^*} f(Q)dQ - f(Q^*)Q^*$, we have that $CS'(\lambda) = -f'(Q^*)(\partial q^*/\partial\lambda)n^2 q^*$, thus $sign\{CS(\lambda)\} = sign\{\partial q^*/\partial\lambda\}$. Let $\beta'(\lambda)$ be the solution to $B - H(\beta) = 0$ for a given λ. Suppose that $\beta'(\lambda)$ is decreasing in λ for all λ, then: if $\beta \leq \beta'(1)$, we have that $\partial q^*/\partial\lambda < 0$ for all λ, so $\lambda^*_{CS} = 0$; if $\beta > \beta'(0)$, then $\partial q^*/\partial\lambda > 0$ for all λ. Since $\partial q^*/\partial\lambda > 0$ implies that $\partial x^*/\partial\lambda > 0$ by equation (8), we have that $W'(\lambda) > 0$ for all λ by equation (11). Therefore, $\lambda^*_{CS} = \lambda^*_{TS} = 1$. Suppose now that $\beta'(\lambda)$ is increasing in λ for all λ, then: if $\beta \leq \beta'(0)$, we have that $\partial q^*/\partial\lambda < 0$ for all λ, so $\lambda^*_{CS} = 0$; if $\beta > \beta'(1)$, $\partial q^*/\partial\lambda > 0$ for all λ. Since $\partial q^*/\partial\lambda > 0$ implies that $\partial x^*/\partial\lambda > 0$ by equation (8), we have that $W'(\lambda) > 0$ for all λ by equation (11). Therefore, $\lambda^*_{CS} = \lambda^*_{TS} = 1$. Finally, suppose that β' is independent of λ, then: if $\beta \leq \beta'$, we have that $\partial q^*/\partial\lambda < 0$ for all λ, so $\lambda^*_{CS} = 0$; if $\beta > \beta'$, then $\partial q^*/\partial\lambda > 0$ for all λ. Since $\partial q^*/\partial\lambda > 0$ implies that $\partial x^*/\partial\lambda > 0$ by equation (8), we have that $W'(\lambda) > 0$ for all λ by equation (11). Therefore, $\lambda^*_{CS} = \lambda^*_{TS} = 1$.

(iii) $\underline{\beta} < \beta'$: Note that if $\beta > \beta'$, then $\partial q^*/\partial\lambda > 0$. Suppose that $\beta' < \underline{\beta}$, then for any $\beta > \beta'$, $\partial q^*/\partial\lambda > 0$, but for any $\beta \in (\beta', \underline{\beta})$ we have that $\partial x^*/\partial\lambda \leq 0$, which, from equation (8), implies that $\partial q^*/\partial\lambda < 0$, a contradiction. From part (i) we also know that $\hat{\beta} > \underline{\beta}$. Let us now show that $\beta' > \hat{\beta}$. Suppose that $\hat{\beta} > \beta'$, then from (9) we have that for $\beta \in (\beta', \hat{\beta})$ it holds that $\partial q^*/\partial\lambda > 0$. Thus, from equation (8) it also holds that $\partial x^*/\partial\lambda > 0$, which implies from equation (11) that $W'(\lambda) > 0$. However, from equation (29) we have that $W'(\lambda) < 0$ for $\beta < \hat{\beta}$, a contradiction. Suppose now that $\hat{\beta} = \beta'$, then $H(\beta) - B|_{\beta=\beta'} = 0$, thus from equation (29) we have that $\hat{\beta} = \beta' = 1/(n + 1 + \delta\Lambda)$, which implies that $\partial x^*/\partial\lambda = 0$ (see Table 6), and from equation (8) this in turn implies that $\partial q^*/\partial\lambda < 0$. However, at $\beta = \beta'$, $B - H(\beta) = 0$, so $\partial q^*/\partial\lambda = 0$ (see Table 6), a contradiction.

It is immediate that $\underline{\beta}$ decreases with n. Note that $\partial B/\partial n = \beta \geq 0$, thus if $\beta' - \partial H(\beta')/\partial n > 0$, then the increase in the left-hand side of equation $B = H(\beta)$ is higher than the increase in its right-hand side, which in turn implies that $1 + \beta'(n-1) > H(\beta')$. If β'' is the new solution to the equation $B = H(\beta)$, then it must be that $\beta'' < \beta'$ since $\partial B/\partial\beta = n - 1 > 0$ and by Assumption A.4., $\partial H(\beta)/\partial\beta < n - 1$, thus $\partial(B - H(\beta))/\partial\beta > 0$, i.e., decreasing β will reduce the difference $B - H(\beta)$. Finally, we examine the impact of entry on $\hat{\beta}$. Let $h(\beta) = H(\beta) - B$ and $g(\beta) = ((n - \Lambda)/\Lambda)[(n + 1 + \delta\Lambda)\beta - 1)$. We have that

$$\frac{\partial g(\beta)}{\partial n} = \frac{1 - \lambda}{\Lambda^2}\left[(n + 1 + \delta\Lambda)\beta - 1\right] + \frac{n - \Lambda}{\Lambda}\beta(1 + \lambda\delta).$$

The second term is positive if $\delta > -1/\lambda$, which is guaranteed by the condition $\delta > -n/\Lambda$, whereas the first term is positive in RII and $RIII$. We also have that $g'(\beta) > 0$, $g(0) = -(n-\Lambda)/\Lambda$ and $g(\beta) = 0$ at $\beta = 1/(n+1+\delta\Lambda)$, which is decreasing in n. Thus, when n increases, $g(\beta)$ rotates counterclockwise, i.e., $\partial g(\hat{\beta})/\partial n > 0$. Suppose that n increases, if $\hat{\beta}^*$ is the new solution to the equation $h(\beta) = g(\beta)$ and $\partial h(\hat{\beta})/\partial n = \partial H(\hat{\beta})/\partial n - \hat{\beta} < \partial g(\hat{\beta})/\partial n$, then it must be that $\hat{\beta}^* < \hat{\beta}$ so as to satisfy the equation $h(\beta) = g(\beta)$ since $h(\beta)$ decreases with β and $g(\beta)$ increases with β.

7.2 Appendix B

$\alpha_q = (\partial^2/\partial q_i^2)\phi_i\big\|_{q^*,x^*} =$	$f'(Q^*)(2+\delta\Lambda/n)$
$\rho_q = (\partial^2/\partial q_i \partial q_j)\phi_i\big\|_{q^*,x^*} =$	$f'(Q^*)(1+\lambda+\delta\Lambda/n)$
$\alpha_x = (\partial^2/\partial x_i^2)\phi_i\big\|_{q^*,x^*} =$	$-(c''(Bx^*)\tilde{\lambda}q^* + \Gamma''(x^*))$
$\rho_x = (\partial^2/\partial x_i \partial x_j)\phi_i\big\|_{q^*,x^*} =$	$-c''(Bx^*)\beta q^*(1+\lambda(1+(n-2)\beta))$
$\varphi_{xq} = (\partial^2/\partial x_i \partial q_i)\phi_i\big\|_{q^*,x^*} =$	$-c'(Bx^*)$
$\varphi_{\lambda q} = (\partial^2/\partial \lambda \partial q_i)\phi_i\big\|_{q^*,x^*} =$	$f'(Q^*)(n-1)q^*$
$\varphi_{\lambda x} = (\partial^2/\partial \lambda \partial x_i)\phi_i\big\|_{q^*,x^*} =$	$-\beta(n-1)c'(Bx^*)q^*$
$\alpha_q\alpha_x - (\varphi_{qx})^2 =$	$f'(Q^*)(2+\Lambda\delta/n)(c''(Bx^*)(Q^*/n)\tilde{\lambda} + \Gamma''(x^*)) - c'(Bx^*)^2$
$\Delta_q = \alpha_q + \rho_q(n-1) =$	$f'(Q^*)(n+\Lambda(\delta+1))$
$\Delta_x = \alpha_x + \rho_x(n-1) =$	$-(c''(Bx^*)B\tau q^* + \Gamma''(x^*))$
$\Delta(Q^*,x^*) =$	$-\left[c''(Bx^*)B\tau(Q^*/n) + \Gamma''(x^*)\right]\left[f'(Q^*)(\Lambda(1+\delta)+n)\right] - c'(Bx^*)^2\tau B$
$H(\beta) =$	$-[f'(Q^*)/(\beta c'(Bx^*)^2)][(-c''(Bx^*)B\Gamma'(x^*)/c'(Bx^*)) + \Gamma''(x^*)]$
$\partial x^*/\partial\lambda =$	$((n-1)(Q^*/n)f'(Q^*)c'(Bx^*)/\Delta)\left[\beta(\Lambda(1+\delta)+n) - \tau\right]$
$\partial q^*/\partial\lambda =$	$((n-1)(Q^*/n)/\Delta)\left[c'(Bx^*)^2\beta B + f'(Q^*)\left(c''(Bx^*)(Q^*/n)B\tau + \Gamma''(x^*)\right)\right]$
$=$	$((n-1)(Q^*/n)/\Delta)\, c'(Bx^*)^2\beta\,(B - H(\beta))$

, with $B = 1+\beta(n-1)$, $\Lambda = 1+\lambda(n-1)$, $\tau = 1+\lambda(n-1)\beta$, and $\tilde{\lambda} = 1+\lambda(n-1)\beta^2$.

Table 6: Summary of basic expressions.

In this appendix we first provide the second-order conditions and the regularity condition, $\Delta > 0$, for each of the model specifications considered in the text (AJ, KMZ, CE). We then discuss the feasible region for the constant elasticity model. Finally, Lemma 6 determines $sign\{\partial q^*/\partial\lambda\}$ and $sign\{\partial x^*/\partial\lambda\}$ under AJ, KMZ and CE model specifications. To start with, let us rewrite the regularity condition as follows

$$\Delta(Q^*,x^*) = -\left[c''(Bx^*)B\tau(Q^*/n) + \Gamma''(x^*)\right]\left[f'(Q^*)(\Lambda(1+\delta)+n)\right] - (c'(Bx^*))^2\tau B > 0. \quad (30)$$

In particular, for $\beta > 0$ the above condition can be rewritten as $\Delta(Q^*,x^*) = (\Lambda(1+\delta)+n)\beta H(\beta) - \tau B > 0$. Second-order conditions are: (i) $\alpha_q = 2f'(Q) + \Lambda(Q/n)f''(Q) = f'(Q)(2+$

$\Lambda\delta/n) < 0$, so $\alpha_q < 0$ if $\delta > -2n/\Lambda$; (ii) $\alpha_x < 0$, which is trivially satisfied by Assumptions A.2 and A.3; and (iii) $\alpha_q \alpha_x - \left(\varphi_{qx}\right)^2 > 0$, which is equivalent to

$$c'(Bx^*)^2 + f'(Q^*)(2 + \Lambda\delta/n)(c''(Bx^*)(Q^*/n)\tilde{\lambda} + \Gamma''(x^*)) < 0, \tag{31}$$

where $\tilde{\lambda} = 1 + \lambda(n-1)\beta^2$. Noting that $\rho_q = f'(Q^*)(1+\lambda) + f''(Q^*)\Lambda q^* = f'(Q^*)(1+\lambda+\delta\Lambda/n)$, we have that

$$\alpha_q + \rho_q(n-1) = f'(Q^*)(n + \Lambda(\delta+1)) < 0,$$

which is satisfied if $\delta > -(n+\Lambda)/\Lambda$. Similarly, noting that $\alpha_x = -c''(Bx^*)\tilde{\lambda}q^* - \Gamma''(x^*)$ and $\rho_x = -c''(Bx^*)\beta q^*(1 + \lambda(1 + (n-2)\beta))$, it is straightforward to show that

$$\alpha_x + \rho_x(n-1) = -(c''(Bx^*)B\tau q^* + \Gamma''(x^*)) < 0,$$

which is satisfied by Assumptions A.2 and A.3.

In AJ and KMZ it is immediate that $\alpha_q = -2b < 0$. Furthermore, in AJ: $\alpha_q\alpha_x - (\varphi_{qx})^2 = 2b\gamma - 1$, since $c''(\cdot) = 0$ and $\Gamma''(x) = \gamma$, so $\alpha_x = -\gamma$ and $\varphi_{qx} = -c'(\cdot) = 1$. In KMZ, condition (31) can be written as

$$\left[\frac{1}{\gamma^2}\left(\frac{2}{\gamma}(Bx^*)\right)^{-1}\right] - 2b\left[\frac{1}{\gamma^2}\left(\frac{2}{\gamma}(Bx^*)\right)^{-3/2}\right] q^*\tilde{\lambda} < 0. \tag{32}$$

From first-order condition (3) we have that in equilibrium

$$q^* = \frac{\Gamma'(x^*)}{-c'(Bx^*)\tau} = \frac{1}{\left[(1/\gamma)((2(Bx^*)/\gamma)^{-1/2}\tau\right]}. \tag{33}$$

Inserting the above equation into condition (32), after some manipulations, it reduces to $1 - 2b\gamma\tilde{\lambda}/\tau < 0$. (Note that if $\gamma b > \tau/2$ holds, then the condition $\gamma b > \tau/(2\tilde{\lambda})$ is satisfied.) In AJ and from (30), it is immediate that $\Delta = \gamma b(\Lambda + n) - \tau B$ since $c''(\cdot) = \delta = 0$, $f'(Q) = -b$ and $\Gamma'(x) = \gamma x$. In KMZ we have:

$$\Delta = -\left[\frac{1}{\gamma^2}\left(\frac{2}{\gamma}Bx^*\right)^{-3/2} B\tau\frac{\Gamma'(x^*)}{(1/\gamma)(2Bx^*/\gamma)^{-1/2}\tau}\right][-b(\Lambda+n)] - \frac{1}{\gamma^2}\left(\frac{2}{\gamma}Bx^*\right)^{-1}\tau B.$$

Inserting (33) into the above equation, after some manipulations, we obtain

$$\Delta = \frac{1}{\gamma}\left(\frac{2}{\gamma}Bx^*\right)^{-1}\left(Bb(\Lambda + n) - \frac{\tau B}{\gamma}\right).$$

Therefore, in KMZ $\Delta > 0$ if $\gamma b > \tau/(\Lambda + n)$. Regarding the constant elasticity model we have:

PROPOSITION 8 *(Constant elasticity model) For a given positive integer n and a non-negative λ, then at the equilibrium second-order conditions together with the condition of non-negative profits require that*

(i) $\max\{\varepsilon\Lambda, \Lambda(1+\varepsilon)/2\} < n \le \varepsilon\Lambda(B + \alpha\tau)/(\alpha\tau)$*,*

(ii) $\varepsilon(1+\alpha)/\alpha > n(n - \varepsilon\Lambda)/(\tilde{\lambda}(2n + \Lambda\delta))$*, with* $\tilde{\lambda} \equiv 1 + \lambda(n-1)\beta^2$*.*

Furthermore, the equilibrium is regular if and only if $(1+\alpha)/\alpha > 1/\varepsilon$*.*

Proof. From the first-order condition (2) we need that

$$n > \varepsilon\Lambda, \tag{34}$$

otherwise the system (2) and (3) will not have a solution. This condition also guarantees that Q^* and x^* are both positive. Notice that $\alpha_q < 0 \Leftrightarrow (f'(Q^*)/n)(2n + \Lambda\delta) < 0$. Since $\delta = -(1+\varepsilon)$, $\alpha_q < 0$ if and only if

$$n > \Lambda(1+\varepsilon)/2. \tag{35}$$

Since $\Lambda \in [1, n]$, we have that the latter condition is always satisfied for $\varepsilon < 1$. By construction $\alpha_x < 0$. Furthermore, second-order condition $\alpha_q\alpha_x - \left(\gamma_{qx}\right)^2 > 0$, which is given by (31), reduces to

$$-\frac{\varepsilon\sigma}{n}Q^{*-(\varepsilon+1)}(2n + \Lambda\delta)(\alpha(\alpha+1)\kappa(Bx^*)^{-(\alpha+2)}(Q^*/n)\tilde{\lambda}) + (\alpha\kappa)^2(Bx^*)^{-2(\alpha+1)} < 0. \tag{36}$$

From the first-order condition (2) we have that at the symmetric equilibrium

$$Q^* = (\sigma(n - \varepsilon\Lambda)/(n\kappa))^{1/\varepsilon}(Bx^*)^{\alpha/\varepsilon}. \tag{37}$$

By substituting (37) into (36), after some manipulations, we obtain

$$(Bx^*)^{-2(\alpha+1)}\alpha\kappa^2\left[-(\varepsilon/(n - \varepsilon\Lambda))(2n + \Lambda\delta)(\alpha+1)\tilde{\lambda}/n + \alpha\right] < 0.$$

The above condition is satisfied if and only if $\varepsilon(\alpha+1)/\alpha > (n(n-\varepsilon\Lambda))/((2n+\Lambda\delta)\tilde{\lambda})$, which proves statement (ii) of the Proposition.

From (30) we have that $\Delta > 0$ if and only if

$$\begin{aligned} 0 &< \left[-\alpha(\alpha+1)\kappa(Bx^*)^{-(\alpha+2)}(Q^*/n)\tau B\right]\left[\varepsilon(1+\varepsilon)\sigma Q^{*-(\varepsilon+2)}\Lambda Q^* - \varepsilon\sigma Q^{*-(\varepsilon+1)}(\Lambda+n)\right] \\ &\quad -(\alpha\kappa)^2(Bx^*)^{-2(\alpha+1)}\tau B, \end{aligned}$$

or,

$$\begin{aligned} 0 &< Q^{*-(\varepsilon+1)}\left[-\alpha(\alpha+1)\kappa(Bx^*)^{-(\alpha+2)}(Q^*/n)\tau B\right]\left[\varepsilon(1+\varepsilon)\sigma\Lambda - \varepsilon\sigma(\Lambda+n)\right] \\ &\quad -(\alpha\kappa)^2(Bx^*)^{-2(\alpha+1)}\tau B. \end{aligned}$$

Substituting (37) in the above expression, we obtain

$$\begin{aligned} 0 &< \left(\frac{\sigma(n-\varepsilon\Lambda)}{n\kappa}\right)^{-(\varepsilon+1)/\varepsilon}(Bx^*)^{-(\varepsilon+1)\alpha/\varepsilon}\left[-\alpha(\alpha+1)\kappa(Bx^*)^{-(\alpha+2)}\left(\frac{\sigma(n-\varepsilon\Lambda)}{n\kappa}\right)^{1/\varepsilon}(Bx^*)^{\alpha/\varepsilon}\frac{\tau B}{n}\right] \\ &\quad \left[\varepsilon(1+\varepsilon)\sigma\Lambda - \varepsilon\sigma(\Lambda+n)\right] - (\alpha\kappa)^2(Bx^*)^{-2(\alpha+1)}\tau B, \end{aligned}$$

rearranging terms yields

$$0 < (Bx^*)^{-2(\alpha+1)}\left[\frac{n\kappa}{\sigma(n-\varepsilon\Lambda)}\left(-\alpha(\alpha+1)\frac{\kappa\tau B}{n}\right)\left(-\varepsilon\sigma n + \varepsilon^2\sigma\Lambda\right) - (\alpha\kappa)^2\tau B\right],$$

or, equivalently,

$$0 < (Bx^*)^{-2(\alpha+1)}\alpha\kappa^2\tau B\left[\varepsilon(\alpha+1)-\alpha\right].$$

Therefore, $\Delta > 0$ holds if and only if $(1+\alpha)/\alpha > 1/\varepsilon$, or, equivalently, $\varepsilon - \alpha(1-\varepsilon) > 0$.

We turn now to deriving the condition under which profits in equilibrium are nonnegative. At the symmetric equilibrium, each firm's profit is given by $\pi(Q^*/n, x^*) = (f(Q^*) - c(Bx^*))(Q^*/n) - x^*$. Then, $\pi(Q^*/n, x^*) \geq 0$ if and only if $\bar{\pi} \equiv (f(Q^*) - c(Bx^*))(Q^*/(x^*n)) \geq 1$. Write

$$\psi \equiv \sigma\left(\frac{\tau\alpha}{n}\right)^{\varepsilon}\kappa^{\varepsilon-1}\left(\frac{n-\varepsilon\Lambda}{n}\right).$$

Then $Q^* = (n/(\alpha\kappa\tau))\psi^{(1+\alpha)/(\varepsilon-\alpha(1-\varepsilon))}$, $x^* = (1/B)\psi^{1/(\varepsilon-\alpha(1-\varepsilon))}$, and condition $\bar{\pi} \geq 1$ can be expressed as

$$\Big[\sigma(\frac{n}{\alpha\kappa\tau})^{-\varepsilon}\psi^{-\varepsilon(1+\alpha)/(\varepsilon-\alpha(1-\varepsilon))}-\kappa\psi^{-\alpha/(\varepsilon-\alpha(1-\varepsilon))}\Big]\frac{1}{\alpha\kappa\tau}\psi^{(1+\alpha)/(\varepsilon-\alpha(1-\varepsilon))}B\psi^{-1/(\varepsilon-\alpha(1-\varepsilon))}\geq 1.$$

Rearranging terms appropriately, and replacing ψ into the above expression, one obtains $(\varepsilon\Lambda/(n-\varepsilon\Lambda))(B/(\alpha\tau))\geq 1$. It follows that $\bar{\pi}\geq 1$ if and only if

$$\left(\frac{\varepsilon\Lambda}{\alpha\tau}\right)(B+\alpha\tau)\geq n. \tag{38}$$

Combining conditions (34), (35) and (38) yields statement (i).

Feasible region for the constant elasticity model with $\lambda=0$. From Proposition 8 we have that $\Delta>0\Leftrightarrow(1+\alpha)/\alpha>1/\varepsilon$. We are considering the case for which $\lambda=0$. The LHS of condition (i) is then trivially satisfied for any $n\geq 2$, moreover the RHS of condition (i) can be rewritten as follows $n\leq\xi(\beta)=\varepsilon(1+\alpha-\beta)/(\alpha-\varepsilon\beta)$. Since $\xi'>0$ (as we are also imposing that $\Delta>0$), the latter condition will hold for all β if $n\leq\varepsilon(1+\alpha)/\alpha$. Last, condition (ii) for $\lambda=0$ writes as $\varepsilon(1+\alpha)/\alpha>n(n-\varepsilon)/(2n-(1+\varepsilon))$. Therefore, for $\lambda=0$ we only have to consider the RHS of condition (i) and condition (ii). These two conditions are depicted in Fig. 13; the grey area are combinations (α,ε) with $n=7$ for which both conditions are satisfied (these combinations of parameters also satisfy both conditions for $n\leq 7$).

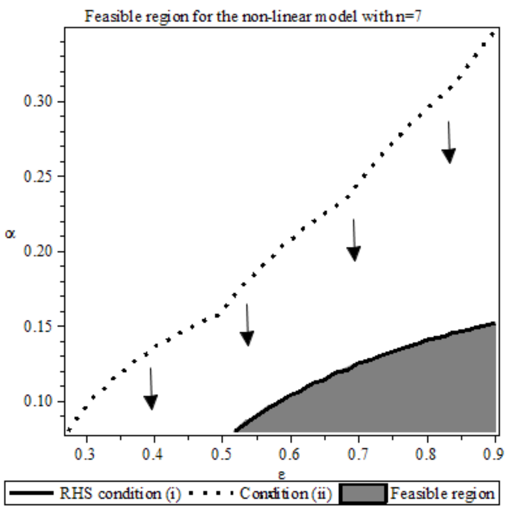

Fig. 13. Feasible region for the constant elasticity demand model.

Determination of $sign\{\partial q^*/\partial\lambda\}$ **and** $sign\{\partial x^*/\partial\lambda\}$ **in AJ, KMZ and CE.** Using the results of Section 2, and noting that equation (8) can also be written in the following manner

$$\frac{\partial q^*}{\partial\lambda}=\frac{(n-1)(Q^*/n)}{\Delta}\left[c'(Bx^*)^2\beta B+f'(Q^*)\left(c''(Bx^*)(Q^*/n)B\tau+\Gamma''(x^*)\right)\right],$$

after some calculations, it is simple to verify that in the static model:

LEMMA 6 *(i) With the AJ model specification:* $sign\{\frac{\partial q^*}{\partial\lambda}\}=sign\{\beta(1+\beta(n-1))-b\gamma\}$ *and* $sign\{\frac{\partial x^*}{\partial\lambda}\}=sign\{\beta(n+1)-1\}$*;*

(ii) With the KMZ model specification: $sign\{\frac{\partial q^*}{\partial\lambda}\}=sign\{\beta-\gamma b\}$ *and* $sign\{\frac{\partial x^*}{\partial\lambda}\}=sign\{\beta(n+1)-1\}$*;*

(iii) In the CE model: $sign\{\frac{\partial q^*}{\partial\lambda}\}=sign\{\beta\left[\alpha(n-\varepsilon\Lambda)-\lambda(n-1)\varepsilon(\alpha+1)\right]-\varepsilon(\alpha+1)\}$ *and* $sign\{\frac{\partial x^*}{\partial\lambda}\}=sign\{\beta\left[(n-\varepsilon)-\lambda(n-1)(1+\varepsilon)\right]-1\}$*.*

7.3 Appendix C

Proof of Lemma 4 Equations (5) and (6) can be written as

$$\frac{\partial q^*}{\partial\lambda}=\frac{(n-1)(Q^*/n)}{\Delta}[c'(Bx^*)^2\beta B+f'(Q^*)\left(c''(Bx^*)(Q^*/n)B\tau+\Gamma''(x^*)\right)] \tag{39}$$

$$\frac{\partial x^*}{\partial\lambda}=\frac{(n-1)(Q^*/n)f'(Q^*)c'(Bx^*)}{\Delta}[\beta(\Lambda(1+\delta)+n)-\tau] \tag{40}$$

If we insert equations (39) and (40) into equation (11), after some manipulations we obtain

$$\begin{aligned}W'(\lambda) \;=\; & \frac{(n-1)(Q^*)^2/n}{\Delta}(-f'(Q^*))\left[\Lambda\left(c'(Bx^*)^2\beta B+f'(Q^*)(c''(Bx^*)(Q^*/n)B\tau\right.\right. \\ & \left.\left.+\Gamma''(x^*))\right)+c'(Bx^*)^2(1-\lambda)\beta(n-1)(\beta(\Lambda(1+\delta)+n)-\tau)\right].\end{aligned} \tag{41}$$

Let F^{AJ} denote the expression in the square brackets of equation (41) for the AJ model specification. By noting that in AJ: $f'=-b$, $\delta=0$, $c'=-1$, $c''=0$ and $\Gamma''=\gamma$, it then follows that

$$\begin{aligned}F^{AJ}\big|_{\lambda=0} &= \beta B-b\gamma+\beta(n-1)(\beta(1+n)-1)\\ &= (n-1)(n+2)\beta^2-(n-2)\beta-b\gamma.\end{aligned}$$

By solving $\digamma^{AJ}|_{\lambda=0} = 0$ for β we obtain the expression for $\bar{\beta}^{AJ}$. Notice that $\bar{\beta}^{AJ} < 1$ if

$$(n-2) + \sqrt{(n-2)^2 + 4b\gamma(n+2)(n-1)} < 2(n+2)(n-1),$$

or

$$(n-2)^2 + 4b\gamma(n+2)(n-1) < (2(n+2)(n-1) - (n-2))^2,$$

which can be rewritten as $4b\gamma(n+2)(n-1) < 4n^2(n+2)(n-1)$. Thus, $\bar{\beta}^{AJ} < 1$ if $b\gamma < n^2$. In KMZ we have $c = \bar{c} - \sqrt{(2/\gamma)(x_i + \beta\sum_{j\neq i} x_j)}$, $f' = -b$, $\delta = 0$ and $\Gamma'' = 0$. Let $\digamma^{KMZ}$ denote the expression in the square brackets of equation (41) for the KMZ model specification, then

$$\begin{aligned} \digamma^{KMZ}|_{\lambda=0} &= \frac{\beta}{2\gamma x^*} + \frac{-bq^*B}{\gamma^2\,(2Bx^*/\gamma)^{3/2}} + \frac{\beta(n-1)(\beta(1+n)-1)}{2\gamma B x^*} \\ &= \frac{1}{B}\left[\frac{-bq^*B^{1/2}}{\gamma^2\,(2x^*/\gamma)^{3/2}} + \frac{\beta}{2\gamma x^*}\,(B + (n-1)(\beta(1+n)-1))\right]. \end{aligned}$$

By replacing q^* and x^* into the above expression, after some calculations we get

$$\digamma^{KMZ}|_{\lambda=0} = \frac{(b\gamma(1+n)-1)^2}{\gamma(a-\bar{c})^2}[-bB + \frac{\beta}{\gamma}(B + (n-1)(\beta(1+n)-1))].$$

It is then immediate that in the static case: $\digamma^{KMZ}|_{\lambda=0} > 0 \Leftrightarrow \beta > \bar{\beta}^{KMZ}$. Notice that $\bar{\beta}^{KMZ} < 1$ if

$$[(n-2)^2 + b\gamma(n-1)(b\gamma(n-1) + 2(3n+2))]^{1/2} < 2(n+2)(n-1) - n + 2 - b\gamma(n-1),$$

which can be rewritten as

$$4n(n+2)(n-1)(-n+b\gamma) < 0.$$

In the constant elasticity model $f = \sigma Q^{-\varepsilon}$, $c = \kappa(x_i + \beta\sum_{j\neq i} x_j)^{-\alpha}$ and $\Gamma(x) = x$. Let $\digamma^{CE}$ denote the expression in the square brackets of equation (41) in the constant elasticity model, then

$$\begin{aligned} \digamma^{CE}|_{\lambda=0} &= (\alpha\kappa)^2(Bx^*)^{-2(\alpha+1)}\beta B - \varepsilon\sigma(Q^*)^{-\varepsilon-1}(\alpha(\alpha+1)\kappa)(Bx^*)^{-(\alpha+2)}q^*B \\ &\quad +(\alpha\kappa)^2(Bx^*)^{-2(\alpha+1)}\beta(n-1)(\beta(-\varepsilon+n)-1). \end{aligned}$$

By replacing q^* and x^* into the above expression, we obtain

$$
\begin{aligned}
\digamma^{CE}\big|_{\lambda=0} &= \alpha^2\kappa^2 z^{-2(1+\alpha)}\beta B - \varepsilon\sigma\left(n/(\alpha\kappa)\right)^{-(1+\varepsilon)} z^{-(1+\alpha)(1+\varepsilon)}(\alpha+1)z^{-(\alpha+2)}z^{\alpha+1}B \\
&+\alpha^2\kappa^2 z^{-2(1+\alpha)}\beta(n-1)(\beta(-\varepsilon+n)-1),
\end{aligned} \tag{42}
$$

where

$$
z \equiv \left[\sigma\left(\frac{\tau\alpha}{n}\right)^{\varepsilon}\kappa^{\varepsilon-1}\left(1-\varepsilon/n\right)\right]^{1/(\varepsilon-\alpha(1-\varepsilon))}.
$$

By noting that $z^{-(\alpha+1)(1+\varepsilon)-(\alpha+2)+(\alpha+1)} = z^{-\varepsilon+\alpha(1-\varepsilon)}z^{-2(1+\alpha)}$ we can re-write equation (42) as follows

$$
\digamma^{CE}\big|_{\lambda=0} = z^{-2(1+\alpha)}\alpha\kappa^2\left[\alpha\beta B + \alpha\beta(n-1)(\beta(-\varepsilon+n)-1) - \varepsilon(\alpha+1)B/(n-\varepsilon)\right].
$$

Hence $\digamma^{CE}\big|_{\lambda=0} > 0$ if and only if

$$
(n-\varepsilon)\alpha\beta(B+(n-1)(\beta(n-\varepsilon)-1)) - \varepsilon(\alpha+1)B > 0.
$$

Proof Proposition 5 With the AJ model specification $sign\left\{CS'(\lambda)\right\} = sign\left\{\partial q^*/\partial\lambda\right\} = sign\left\{\beta(1+\beta(n-1)) - b\gamma\right\}$, so β'_{AJ} is the unique positive solution to the equation $\beta(1+\beta(n-1)) - b\gamma = 0$ (see Table 4). β'_{AJ} is independent of λ (since $H(\beta)$ is independent of λ), thus case 'c' of Proposition 4 holds: if $\beta > \beta'_{AJ}$, then $\lambda^*_{CS} = \lambda^*_{TS} = 1$. From Table 4 we have that $\beta'_{AJ} < 1$ if $\sqrt{1+4b\gamma(n-1)} < 2(n-1)+1$, or, equivalently, if $b\gamma < n$. Furthermore, when $\lambda = \beta = 1$, the second-order condition writes as $b\gamma > 1/2$, and $\Delta > 0$ holds if $b\gamma > n/2$; it can also be shown that $\pi^* > 0$ if $b\gamma > n/2$. Consider now the KMZ model specification: $sign\left\{CS'(\lambda)\right\} = sign\left\{\partial q^*/\partial\lambda\right\} = sign\left\{\beta - b\gamma\right\}$, thus $CS'(\lambda) > 0$ for $\beta > \beta'_{KMZ} = b\gamma$ (case 'c' of Proposition 4). We have that $\beta'_{KMZ} < 1$ if $b\gamma < 1$. Furthermore, when $\lambda = \beta = 1$, the second-order condition requires that $b\gamma > 1/2$, and $\Delta > 0$ holds if $b\gamma > 1/2$, moreover it can be shown that $\pi^* > 0$ if $b\gamma > 1/2$. As for the constant elasticity model, from Table 4 we have that β'_{CE} is increasing in λ (case 'b' of Proposition 4). For $\lambda = 1$, $\beta'_{CE} < 1$ if $\varepsilon < \alpha/(1+2\alpha)$, however $\Delta > 0$ if $\varepsilon > \alpha/(1+\alpha)$, a contradiction (since $\varepsilon, \alpha > 0$). Thus, $\partial q^*/\partial\lambda|_{\lambda=1} < 0$ for any $\beta \in [0,1]$, implying that $CS'(1) < 0$ and $W'(1) < 0$ (see equation (11)) for any $\beta \in [0,1]$.

7.4 Appendix D

Proof of Lemma 5. First, note that the first-order condition with respect to output is identical to the one associated to the static case. Therefore, by totally differentiating the first-order condition with respect to output and solving for $\partial q^*/\partial\lambda$, again one obtains equation (8), which implies that if $\partial x^*/\partial\lambda \leq 0$, then $\partial q^*/\partial\lambda < 0$.

Using (16), by totally differentiating the system formed by (13)-(14) in a symmetric equilibrium, and solving for $\partial q^*/\partial\lambda$ and $\partial x^*/\partial\lambda$, we obtain

$$\frac{\partial q^*}{\partial\lambda} = \frac{1}{\tilde{\Delta}}[(\varphi_{\lambda x} + (n-1)\psi_\lambda)\,\varphi_{xq}B - \varphi_{\lambda q}\,(\Delta_x + \psi_x(n-1))] \tag{43}$$

$$\frac{\partial x^*}{\partial\lambda} = \frac{1}{\tilde{\Delta}}[\varphi_{\lambda q}(\varphi_{xq}\tau + (n-1)\psi_q) - (\varphi_{\lambda x} + (n-1)\psi_\lambda)\Delta_q], \tag{44}$$

where $\psi_z \equiv \partial\psi/\partial z$ with $z = q, x, \lambda$, and

$$\tilde{\Delta}(Q^*, x^*) = \Delta_q\,(\Delta_x + \psi_x(n-1)) - \varphi_{xq}\left(\varphi_{xq}\tau + \psi_q(n-1)\right)B,$$

which in a regular equilibrium is assumed to be positive. By rewriting equation (44) as follows

$$\frac{\partial x^*}{\partial\lambda} = \xi f'(Q^*)c'(Bx^*)[(\beta + s'(\lambda))(\Lambda(1+\delta) + n) - (\tau + (n-1)s(\lambda))], \tag{45}$$

where $\xi \equiv (n-1)(Q^*/n)/\tilde{\Delta}$ and $s(\lambda) = \omega(\lambda)(\tilde{\beta}(\lambda) - \beta)$, we get that $sign\,\{\partial x^*/\partial\lambda\}$ is given by (19). Let us now turn to the impact of λ on output in equilibrium. Similarly, equation (43) can be rewritten as follows

$$\frac{\partial q^*}{\partial\lambda} = \xi[(\beta + s'(\lambda))c'(Bx^*)^2B + f'(Q^*)\left(c''(Bx^*)(Q^*/n)B(\tau + (n-1)s(\lambda)) + \Gamma''(x^*)\right)]. \tag{46}$$

Inserting the first-order necessary condition (14) evaluated at the symmetric equilibrium into the above expression, after some manipulations, we get that $sign\,\{\partial q^*/\partial\lambda\}$ is given by (20).

Proof of Proposition 7. From equation (19) we have that $\partial x^*/\partial\lambda \leq 0$ if $\beta \leq \underline{\beta}^{2S}$. Thus, if $\beta \leq \inf\{\underline{\beta}^{2S} : \lambda \in [0,1]\}$, then $\partial x^*/\partial\lambda \leq 0$ for all λ, which by Lemma 5 (i) implies that $\dfrac{\partial q^*}{\partial\lambda} < 0$ for all λ. Then, by equation (11) we have that $W'(\lambda) < 0$ for all λ: social welfare is maximized with $\lambda^* = 0$.

If we use equations (45) and (46) in equation (11), after some manipulations we obtain

$$\begin{aligned} W'(\lambda) \;=\; & \xi Q^*(-f'(Q^*))[\Lambda(c'(Bx^*)^2(\beta+s'(\lambda))B + f'(Q^*)(c''(Bx^*)(Q^*/n)B(\tau+(n-1)s(\lambda)) \\ & +\Gamma''(x^*))) + c'(Bx^*)^2((1-\lambda)\beta - s(\lambda))(n-1)((\beta+s'(\lambda))(\Lambda(1+\delta)+n) \\ & -(\tau+(n-1)s(\lambda)))]. \end{aligned} \tag{47}$$

Then $W'(0)|_{\beta=1} > 0$ if and only if

$$\begin{aligned} 0 \;<\; & (c'(nx^*))^2[(1+s'(0)|_{\beta=1})n + (1-s(0)|_{\beta=1})(n-1)((1+s'(0)|_{\beta=1})(1+\delta+n) \\ & -(1+(n-1)\,s(0)|_{\beta=1}))] + f'(Q^*)(c''(nx^*)Q^*(1+(n-1)\,s(0)|_{\beta=1}) + \Gamma''(x^*)). \end{aligned} \tag{48}$$

From equation (17) we have that in equilibrium and for $\lambda = 0$ and $\beta = 1$:

$$Q^*|_{\lambda=0,\beta=1} = -\frac{n\Gamma'(x^*)}{c'(nx^*)(1+(n-1)\,s(0)|_{\beta=1})}.$$

Substituting $Q^*|_{\lambda=0,\beta=1}$ into (48) and using the definitions for $\chi(Bx^*)$ and $\eta(Q^*, x^*)$, we obtain the condition for the two-period model:

$$[(1+s'(0)|_{\beta=1})n+(1-s(0)|_{\beta=1})(n-1)((1+s'(0)|_{\beta=1})(1+\delta+n)-(1+(n-1)\,s(0)|_{\beta=1}))]-H(1) > 0,$$

where

$$s(0) = \frac{(2n+\delta)[(n+\delta)/(2n+\delta)-\beta]}{n(n+1+\delta)}$$

and

$$s'(0) = -\frac{(2n^2+\delta(2n+1)+\delta^2)(n-1)\beta - \delta^2(n-1) - \delta(2n^2-1) - n(n^2+1)}{(n+1+\delta)^2 n}.$$

Thus, $s'(0)|_{\beta=1} = (1+\delta-n(n-2))/(n+1+\delta)^2$. Note that by setting $s = s' = 0$, we obtain the condition for the simultaneous case, that is, condition (12).

Next we derive equilibrium values of output and R&D of the two-stage model under each model specification.

The d'Aspremont and Jacquemin's model specification. From first-order necessary conditions (13)-(14) and using the expression for the strategic term (16) we obtain

$$q^* = \frac{\gamma(a-\bar{c})}{\tilde{\Delta}} \text{ and } x^* = \frac{((n-1)(\frac{\Lambda}{n+\Lambda})(1+\lambda-2\beta)+\tau)(a-\bar{c})}{\tilde{\Delta}}$$

where

$$\tilde{\Delta} = \frac{\gamma b(\Lambda+n)^2 - B((n-1)\Lambda(1+\lambda-2\beta)+(n+\Lambda)\tau)}{\Lambda+n}.$$

In this case, $H = b\gamma/\beta$, thus using (13) we have

$$sign\{\frac{\partial q^*}{\partial \lambda}\} = sign\left\{B\beta(n+\Lambda) + B\left(\frac{1+\lambda-2\beta}{n+\Lambda}(n-1)n+\Lambda\right) - b\gamma(n+\Lambda)\right\}$$

and using (14) we get

$$\begin{aligned} sign\{\frac{\partial x^*}{\partial \lambda}\} &= sign\{\beta\left(\Lambda+n+(n-1)(\omega(\lambda)-\lambda)\right) \\ &\quad +(\frac{1+\lambda-2\beta}{n+\Lambda}(n-1)n+\Lambda) - 1 - (n-1)\omega(\lambda)\tilde{\beta}(\lambda)\}, \end{aligned} \tag{49}$$

where we have used that

$$\left(\omega'(\lambda)(\tilde{\beta}(\lambda)-\beta)+\omega(\lambda)\tilde{\beta}'(\lambda)\right)(\Lambda+n) = \frac{1+\lambda-2\beta}{n+\Lambda}(n-1)n+\Lambda.$$

The Kamien, Muller and Zang's model specification. The output and R&D values in equilibrium are given by

$$q^* = \frac{\gamma(a-\bar{c})}{\gamma b(\Lambda+n)-v} \text{ and } x^* = \frac{1}{2}\frac{(a-\bar{c})^2 v^2 \gamma}{B(b\gamma(\Lambda+n)-v)^2}$$

with $v = (n-1)s(\lambda)+\tau$. Since $s(\lambda) = \omega(\lambda)(\tilde{\beta}(\lambda)-\beta)$, we can write

$$v = (n-1)\Lambda\left(\frac{1+\lambda-2\beta}{n+\Lambda}\right)+\tau.$$

In this case, $H = b\gamma B/\beta$, then using (13) we have

$$sign\{\frac{\partial q^*}{\partial \lambda}\} = sign\{\beta(n+\Lambda)+(\frac{1+\lambda-2\beta}{n+\Lambda}(n-1)n+\Lambda)-b\gamma(n+\Lambda)\}$$

and $sign\{\frac{\partial x^*}{\partial \lambda}\}$ is again given by (49).

Constant elasticity model. The output and R&D values in equilibrium are given by

$$Q^* = \frac{n}{\alpha\kappa((n-1)s(\lambda)+\tau)}\left[\sigma\left(\frac{((n-1)s(\lambda)+\tau)\alpha}{n}\right)^{\varepsilon}\kappa^{\varepsilon-1}\left(1-\frac{\varepsilon\Lambda}{n}\right)\right]^{(1+\alpha)/[\varepsilon-\alpha(1-\varepsilon)]}$$

and

$$x^* = \frac{1}{B}\left[\sigma\left(\frac{((n-1)s(\lambda)+\tau)\alpha}{n}\right)^{\varepsilon}\kappa^{\varepsilon-1}\left(1-\frac{\varepsilon\Lambda}{n}\right)\right]^{1/[\varepsilon-\alpha(1-\varepsilon)]},$$

where $s(\lambda) = \omega(\lambda)(\tilde{\beta}(\lambda) - \beta)$. It can be shown that in the constant elasticity model:

$$H = \frac{B}{\beta}\left(\frac{\alpha+1}{\alpha}\right)\frac{\varepsilon}{n-\varepsilon\Lambda}((n-1)s(\lambda)+\tau).$$

Hence, we have

$$sign\{\frac{\partial q^*}{\partial\lambda}\} = sign\{(\beta + s'(\lambda)) - \frac{\alpha+1}{\alpha}\frac{\varepsilon}{n-\varepsilon\Lambda}((n-1)s(\lambda)+\tau\}.$$

And, $sign\{\partial x^*/\partial\lambda\}$ is given by (19) with $\delta = -(1+\varepsilon)$.

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