Learning about the Neighborhood: The Role of Supply Elasticity for Housing Cycles*

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December 2014

Abstract

Motivated by an intriguing observation during the recent U.S. housing cycle that counties with housing supply elasticities in an intermediate range experienced the most dramatic price booms and busts, this paper develops a model to analyze information aggregation and learning in housing markets. In the presence of pervasive informational frictions, housing prices serve as important signals in the households’ learning of the economic strength of a neighborhood. Supply elasticity affects not only housing supply but also the informational noise in the price signal. Our model predicts that the housing price and the share of investment home purchases exhibit the greatest variability in areas with intermediate supply elasticities, which is supported by our empirical analysis.

*PRELIMINARY DRAFT. We wish to thank Nick Barberis, Pietro Veronesi, and seminar participants at NBER Financing Housing Capital Conference for helpful comments and discussion.
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Conventional wisdom posits that supply elasticity attenuates housing cycles. As a result, one expects housing prices to be more volatile in areas with more inelastic housing supplies. As noted by Glaeser (2013) and other commentators, however, during the recent U.S. housing cycle in the 2000s, some areas such as Las Vegas and Phoenix experienced more dramatic housing price booms and busts, despite their relatively elastic housing supply, compared to areas with more inelastic supply, such as New York and Los Angeles. Interestingly, by systematically examining the cross-section of the booms and busts experienced by different counties during this housing cycle, we find that the monotonically decreasing relationship between the magnitude of housing cycles and supply elasticity is more fragile than commonly perceived. If one simply sorts counties into three groups based on Saiz’s (2010) widely-used measure of supply elasticity, each with an equal number of counties, the average housing price boom in 2003-2006 and bust in 2006-2009 is monotonically decreasing across the inelastic, middle and elastic groups. As the inelastic group holds more than half of the population, however, this coarse grouping may disguise non-monotonicity under present finer parsings. Indeed, when we sort the counties into ten elasticity groups, each with an equal number of counties, or into either three or ten elasticity groups each with an equal population, we uncover a non-monotonic relationship between the magnitudes of the housing price booms and busts experienced by different counties and their supply elasticity. The most dramatic boom and bust cycle occurring in an intermediate range of supply elasticity.

This humped-shape relationship between housing cycle and supply elasticity, which we summarize in Section 1, is intriguing and cannot be explained by the usual supply-side mechanisms. In this paper, we develop a theoretical model to highlight a novel mechanism for supply elasticity to affect housing demand through a learning channel. We emphasize that home buyers observe neither the economic strength of a neighborhood, which ultimately determines the demand for housing in the neighborhood, nor the supply of housing. In the presence of these pervasive informational frictions, local housing markets provide a useful platform for aggregating information. This fundamental aspect of housing markets, however, has received little attention in the academic literature. It is intuitive that traded housing prices reflect the net effect of demand and supply factors. Supply elasticity determines the weight of supply-side factors in determining the housing prices, and therefore by extension determines the informational noise faced by home buyers in using housing prices as signals for the strength of demand.
Our model integrates the standard framework of Grossman and Stiglitz (1980) and Hellwig (1980) for information aggregation in asset markets with a housing market in a local neighborhood. This setting allows us to extend the insights of market microstructure analysis to explore the real consequences of informational frictions in housing markets. In particular, our model allows us to analyze how agents form expectations in housing markets, how these expectations interact with characteristics endemic to a neighborhood, and how these expectations feed into housing prices.

We first present a baseline setting in Section 2 to highlight the basic information aggregation mechanism with each household purchasing homes for their own consumption, and then extend the model in Section 3 to further incorporate purchases of investment homes. The baseline model features a continuum of households in a closed neighborhood, which can be viewed as a county. Each household’s productivity depends on the neighborhood’s common productivity, which is unobservable and ultimately determines the demand for housing in the neighborhood. Each household has a Cobb-Douglas utility function over consumption of housing and a numeraire consumption good. Despite each household’s housing demand being non-linear, the Law of Large Numbers allows us to aggregate their housing demand in closed-form and to derive a unique log-linear equilibrium for the housing market. Each household possesses a private signal regarding the neighborhood common productivity. By aggregating the households’ housing demand, the housing price aggregates their private signals. The presence of unobservable supply shocks, however, prevents the housing price from perfectly revealing the neighborhood strength and acts as a source of informational noise in the housing price.

Our model also builds in another important feature that households underestimate supply elasticity. By examining a series of historical episodes of real estate speculation in the U.S., Glaeser (2013) summarizes the tendency of speculators to underestimate the response of housing supply to rising prices as a key for understanding these historical experiences. In our model, underestimation of supply elasticity implies that households underestimate the amount of informational noise in the observed price signal, which in turn causes the households’ expectations of the neighborhood strength and housing demand to overreact to the housing price. The amplification of housing price volatility induced by such overreaction depends on the uncertainty faced by households and the informational content of the price, both of which are endogenously linked to the neighborhood’s supply elasticity.
It is useful to consider two polar cases. At one end with the supply being infinitely inelastic, the housing price is fully determined by the strength of the neighborhood and thus perfectly reveals it. At the other end, with housing supply being infinitely elastic, the housing price is fully determined by the supply shock and households’ uncertainty about the strength of the neighborhood does not interact with the housing price. In between these two polar cases, the households face uncertainty regarding the neighborhood strength and the uncertainty matters for the housing price. Consequently, households’ overreaction to the price signal has the most pronounced effect on their housing demand and the housing price in an intermediate range of supply elasticity, causing the price volatility to have a humped-shape relationship with supply elasticity. That is, housing price volatility is largest at an intermediate supply elasticity rather than when supply is infinitely inelastic. This key insight helps explain the aforementioned empirical observation that during the recent U.S. housing cycle, counties with supply elasticities in an intermediate range experienced the most dramatic price booms and busts.

We further extend the baseline model in Section 3 to incorporate immigrants who are attracted to the neighborhood by its economic strength in a later period, and the speculation of the current households in acquiring secondary homes in anticipation of selling/renting them to these immigrants. This model extension generates an additional prediction that the households’ learning effects can induce another non-monotonic relationship between the variability of secondary home purchases relative to primary home purchases with respect to supply elasticity. The intuition is similar to before. As secondary home purchases are more sensitive than primary home purchases to the households’ expectations of the neighborhood strength, informational frictions and the households’ overreaction to the price signal make households’ secondary home purchases most variable at an intermediate range of supply elasticity. Interestingly, we are able to confirm this new model prediction in the data by showing that counties in an intermediate range of supply elasticity indeed had the largest share of non-owner-occupied home (investment home) purchases in 2005, as opposed to counties with either the most elastic or inelastic supplies. This non-monotonic pattern observed in the share of investment home purchases provides evidence from a new dimension to support the important roles played by informational frictions and household learning in driving housing cycles.

The existing literature has emphasized the importance of accounting for home buyers’
expectations (and in particular extrapolative expectations) in understanding dramatic hous-
ing boom and bust cycles, e.g., Case and Shiller (2003); Glaeser, Gyourko, and Saiz (2008); and Piazzesi and Schneider (2009). Much of the analysis and discussions, however, are made in the absence of a systematic framework that anchors home buyers’ expectations on their information aggregation and learning process. In this paper, we fill this gap by developing a model for analyzing information aggregation and learning in housing markets. By doing so, we are able to uncover a novel effect of supply elasticity, beyond its role in driving housing supply, in determining the informational content of the housing price and households’ learning from the price signal. This learning effect implies non-monotonic patterns of housing price volatility and share of investment home purchases across neighborhoods with different supply elasticities, which are observed in the data. This learning mechanism also differentiates our model from Gao (2013), which shares a similar motivation as ours to explain the dramatic housing price booms and busts in the 2000s experienced by areas with intermediate supply elasticities, and which emphasizes a joint effect of time-to-build and housing speculators’ extrapolative expectations as an explanation.

In our model, households overreact to the housing price signal. Such overreaction is driven by their underestimation of supply elasticity. This overreaction mechanism, which depends on the informational frictions faced by households and the endogenous informational content of the housing price, is different from the commonly discussed mechanisms in the behavioral finance literature, such as overconfidence highlighted by Daniel, Hirshleifer, and Subrahmanyam (1998), slow information diffusion by Hong and Stein (1999), and extrapolation by Barberis, Shleifer and Vishny (1998) and Barberis et al. (2014).

Our model also differs from Burnside, Eichenbaum, and Rebelo (2013), which offers a model of housing market booms and busts based on the epidemic spreading of optimistic or pessimistic beliefs among home buyers through their social interactions. Our learning-based mechanism is also different from Nathanson and Zwick (2014), which studies the hoarding of land by home builders in certain elastic areas as a mechanism to amplify price volatility in the recent U.S. housing cycle. Informational frictions in our model anchor on the elasticity of housing supply, which is different from the amplification to price volatility induced by dispersed information and short-sale constraints featured in Favara and Song (2014). Furthermore, while our model does not differentiate learning of in-town and out-of-town home buyers, our framework can serve as a basis for future studies of out-of-town
speculators, which are shown to be important by a recent study of Chinco and Mayer (2013).

By focusing on information aggregation and learning of symmetrically informed households with dispersed private information, our study differs in emphasis from those that analyze the presence of information asymmetry between buyers and sellers of homes, such as Garmaise and Moskowitz (2004) and Kurlat and Stroebel (2014).

There are extensive studies in the housing literature highlighting the roles played by both demand-side and supply-side factors in driving housing cycles. On the demand side, Himmelberg, Mayer and Sinai (2006) focuses on interest rates, Poterba (1991) on tax changes, and Mian and Sufi (2009) on credit expansion. On the supply side, Glaser, Gyourko, Saiz (2008) emphasizes supply as a key force in mitigating housing bubbles, Haughwout, Peach, Sporn and Tracy (2012) provides a detailed account of the housing supply side during the U.S. housing cycle in the 2000s, and Gyourko (2009) systematically reviews the literature on housing supply. By introducing informational frictions, our analysis shows that supply-side and demand-side factors are not mutually independent. In particular, supply shocks may affect housing demand by acting as informational noise in household learning and thus influencing households’ expectations of the strength of the neighborhood.

1 Some Basic Facts

Before we present a model to analyze how supply elasticity affects learning in housing markets, we present some basic facts regarding the relationship between supply elasticity and the magnitudes of housing price booms and busts experienced by different counties during the recent U.S. housing cycle. Even though common wisdom holds that supply elasticity attenuates boom and bust cycles, the data does not support a robust, monotonic relationship between the magnitude of the housing cycle in a county and its supply elasticity. In fact, our analysis uncovers that counties with supply elasticities in an intermediate range had experienced more dramatic housing booms and busts than counties with the most inelastic supply.

Our county-level house price data comes from the Case-Shiller home price indices, which are constructed from repeated home sales. There are 420 counties in 46 states with a large enough number of repeat home sales to compute the Case-Shiller home price indices. We use the Consumer Price Index (CPI) from the Bureau of Labor Statistics to deflate the Case-Shiller home price indices. In addition, we also use population data from the 2000 U.S.
For housing supply elasticity, we employ the commonly-used elasticity measure constructed by Saiz (2010). This elasticity measure focuses on geographic constraints by defining undevelopable land for construction as terrain with a slope of 15 degrees or more and areas lost to bodies of water including seas, lakes, and wetlands. This measure provides an exogenous measure of supply elasticity, with a higher value if an area is more geographically restricted. Saiz’s measure is available for 269 Metropolitan Statistical Areas (MSAs). By matching counties with MSAs, our sample includes 326 counties for which we have data on both house prices and supply elasticity available from 2000 to 2010. Though our sample covers only 11 percent of the counties in the U.S., they represent 53 percent of the U.S. population and 57 percent of the housing trading volume in 2000.

Figure 1 displays the real home price indices for the U.S. and three cities, New York, Las Vegas, and Charlotte, from 2000 to 2010. We normalize all indices to 100 in 2000. The national housing market experienced a significant boom and bust cycle in the 2000s with the national home price index increasing over 60 percent from 2000 to 2006 and then falling back to the 2000 level through 2010. Different cities in the U.S experienced largely synchronized price booms and busts during this period, even though the magnitudes of the cycle varied across these cities. According to Saiz’s measure, the elasticity measure for New York, Las Vegas, and Charlotte are 0.76, 1.39, and 3.09, respectively. New York, which has severe geographic constraints and building regulations, had a real housing price appreciation of more than 80 percent during the boom, and then declined by over 25 percent during the bust. Charlotte, with its vast developable land and few building restrictions, had an almost flat real housing price level throughout this decade. Sitting in between New York and Charlotte, Las Vegas, with its intermediate supply elasticity, experienced the most pronounced price expansion of over 120 percent during the boom, and the most dramatic price drop of over 50 percent during the bust. Many commentators, including Glaeser (2013), have pointed out that the dramatic boom and bust cycles experienced by Las Vegas and other cities such as Phoenix are peculiar given the relatively elastic supply in these areas.

Are Las Vegas and Phoenix unique in experiencing these dramatic housing cycles despite their relatively elastic housing supply? We now systematically examine this issue by sorting different counties in our sample into three groups, an inelastic, a middle, and an elastic group, based on Saiz’s elasticity measure, each with the same number of counties.
2 plots the average price expansion and contraction experienced by each group during the housing cycle (the top panel), together with the total population in each group (the bottom panel). We measure the price expansion from 2003 to 2006, the period that is often defined as the housing bubble period, and the price contraction from 2006 to 2009. We have also used an alternative boom period from 2000 to 2006 and obtained qualitatively similar results as defining the boom from 2003 to 2006.

The top panel of Figure 2 shows that the inelastic group had the largest house price expansion from 2003 to 2006 and the largest price contraction from 2006 to 2009, the middle group experienced a milder cycle, and the elastic group had the most modest cycle. This pattern appears to be consistent with the aforementioned common wisdom that supply elasticity attenuates housing cycles.

It seems natural to sort the counties into several groups each with an equal number of counties. In fact, this is a common practice used in the literature to demonstrate a monotonic relationship between housing cycles and supply elasticity. Interestingly, the bottom panel of Figure 2 shows that the population is unevenly distributed across the three groups, with the inelastic group having more than half of the total population. This is consistent with the fact that inelastic areas tend to be densely populated. As the inelastic group pools together a large fraction of the population, there might be substantial heterogeneity between counties within the inelastic group. Indeed, both New York and Las Vegas fall into this inelastic group. This consideration motivates us to examine alternative ways of grouping the counties.

In Figure 3, we sort the counties into ten groups from the most inelastic group to the most elastic group, still with each group holding an equal number of counties. The top panel shows that the housing price expansion and contraction experienced by these ten groups are no longer monotonic with elasticity. In particular, group 3, which has the third most inelastic supply, experienced the largest price expansion during the boom, and the largest price contraction during the bust. Interestingly, Las Vegas falls into group 3, while New York into group 1. The bottom panel again shows that the population tends to be concentrated in the more inelastic groups. Taken together, Figure 3 shows that the commonly perceived, monotonic relationship between housing cycles and supply elasticity is not robust.

In Figure 4, we sort the counties into three groups based on supply elasticity in an alternative way. Instead of letting different groups have an equal number of counties, we let them have the same population. If the magnitude of the housing cycle is monotonically
decreasing with supply elasticity, whether we group the counties by number or population should not affect the monotonically decreasing pattern across the groups. In contrast, the top panel of Figure 4 shows that the middle group has the most pronounced housing cycle, with its price expansion during the boom being substantially more pronounced than that of the inelastic group, and its price contraction during the bust slightly greater than that of the inelastic group. The bottom panel shows that the inelastic group has only 40 counties, the middle group slightly below 120 counties, and the elastic group over 160 counties. Under this grouping, while New York remains in the inelastic group, Las Vegas is now in the middle group.

In Figure 5, we further sort the counties into ten groups from the most inelastic group to the most elastic group, with each group having the same population. This figure shows a finer non-monotonicity with groups 3 and 5 experiencing the most pronounced price expansions and contractions.

To further examine whether the more pronounced housing cycles experienced by the intermediate elasticity groups are robust to controlling for other fundamental factors, such as changes of income and population and fraction of subprime households, we adopt a regression approach. Specifically, we separately regress the housing price expansion in 2003-2006 and contraction in 2006-2009 on two dummy variables that indicate whether a county is in the middle elasticity group or the most elastic group, which are constructed in Figure 4, together with a list of control variables. This regression implicitly uses the inelastic group as the benchmark for the middle and elastic groups. The control variables include the fraction of subprime households in the county in 2005, which is computed based on individual mortgage loan applications reported by the “Home Mortgage Disclosure Act” (HMDA) dataset, as well as the contemporaneous population change and annualized per capita income change.

Table 1 reports the regression results. Columns 1 and 2 report the regressions of the housing price expansion in 2003-2006 without and with the controls. Columns 3 and 4 report the regressions of the housing price contraction in 2006-2009 without and with the controls. Among the control variables, the fraction of subprime households is significantly correlated with both the price expansion during the boom and the price contraction during the bust. This result is consistent with Mian and Sufi (2009), which shows that credit expansion to subprime households before 2006 was a key factor in explaining the recent housing cycle. The changes in population and income are insignificant in explaining either the
price expansion or the contraction across the cycle. More importantly, even after controlling
for these fundamental factors, the middle group experienced a significantly more pronounced
housing price expansion in 2003-2006 and a more pronounced price contraction in 2006-2009
relative to the inelastic group.

Taken together, Figures 2-5 and Table 1 show that the commonly perceived, monotonic
relationship between housing cycle and supply elasticity is not robust to finer groupings of
counties. Finer groupings and an alternative method of grouping counties by population
rather than county number, however, reveal a robust non-monotonic relationship in which
the counties in a median elasticity range experienced more pronounced price booms and busts
in the 2000s than counties with the most inelastic supply. This non-monotonic relationship
is intriguing and cannot easily be explained by the usual role of elasticity in affecting the
supply side of housing. In the next section, we present a simple model to illustrate a learning
mechanism through which supply elasticity affects the informational role of housing prices
and households’ learning from housing prices.

2 A Baseline Model

In this section we develop a simple model with two dates \( t = 1, 2 \) to analyze the effects of
informational frictions on the housing market equilibrium in a closed neighborhood. One
can think of this neighborhood as a county or a township. A key feature of the model
is that the housing market is not only a place for households to trade housing but also a
platform to aggregate private information about the unobservable economic strength of the
neighborhood. In addition to its direct role in affecting housing supply in the neighborhood,
supply elasticity also indirectly affects the informational noise in the housing prices.

2.1 Model setting

There are two types of agents in the economy: households looking to buy homes in the
neighborhood and home builders. Suppose that the neighborhood is new and all households
need to purchase homes at the same time.\(^1\) Each household cares about the quality of the
neighborhood in which it lives, as its utility depends on not only its own housing consumption
and its consumption of the final good it produces, but also on the housing consumption of

\(^1\)For simplicity, we do not consider the endogenous decision of households choosing their neighborhood,
and instead take the pool of households in the neighborhood as given. See Van Nieuwerburgh and Weill
(2010) for a systematic treatment of moving decisions by households across neighborhoods.
other households in their neighborhood. This assumption is motivated by the empirical findings of Ioannides and Zabel (2003). This leads to strategic complementarity in each household’s housing demand.\textsuperscript{2} The economic strength of this closed neighborhood is reflected by the aggregate productivity of its households. A strong aggregate productivity implies greater output by all households, and thus greater housing demand by them as well. In the presence of realistic informational frictions in gauging the strength of the neighborhood, the housing market provides an important platform for aggregating information about this aggregate productivity. As a consequence, the resulting housing price serves as a useful signal about the neighborhood’s strength.

Households purchase houses from home builders in a centralized market at $t = 1$ and consume both housing and consumption goods at $t = 2$. Each household will choose to purchase a bigger house in the first period if it expects to produce more goods in the second.

\subsection*{2.1.1 Households}

There is a continuum of households, indexed by $i \in [0, 1]$. Household $i$ has a Cobb-Douglas utility function over its own housing $H_i$, consumption good $C_i$, and the housing consumption of all other households in the neighborhood $\{H_j\}_{j \in [0, 1]}$, given by\textsuperscript{3}

$$U \left( \{H_j\}_{j \in [0, 1]} ; C_i \right) = \left\{ \frac{1}{1 - \eta_H} \left( \frac{H_i}{1 - \eta_c} \right)^{1-\eta_c} \left( \frac{\int_{[0,1]/i} H_j \, dj}{\eta_c} \right)^{\eta_c} \right\}^{1-\eta_H} \left( \frac{1}{\eta_H} C_i \right)^{\eta_H}. \quad (1)$$

The parameters $\eta_H \in (0, 1)$ and $\eta_c \in (0, 1)$ measure the weights of different consumption components in the utility function. A higher $\eta_H$ means a stronger complementarity between housing consumption and goods consumption, while a higher $\eta_c$ means a stronger complementarity between the housing of household $i$ and housing of the composite house $\int_{[0,1]/i} H_j \, dj$ purchased by the other households in the neighborhood.

The production function of household $i$ is $e^{A_i} l_i$, where $l_i$ is the household’s labor choice and $A_i$ is its productivity. $A_i$ is composed of a component $A$ common to all households in the neighborhood and an idiosyncratic component $\varepsilon_i$: $A_i = A + \varepsilon_i$.

\textsuperscript{2}There are other types of social interactions between households living in a neighborhood, which are explored, for instance, in Durlauf (2004) and Glaeser, Sacerdote, and Scheinkman (2010).

\textsuperscript{3}Our modeling choice of non-separable preferences for housing and consumption is similar to the CES specification of Piazzesi, Schneider, and Tuzel (2007).
where $A \sim \mathcal{N}(A, \tau_A^{-1})$ and $\varepsilon_i \sim \mathcal{N}(0, \tau_v^{-1})$ are both normally distributed. The common productivity $A$ represents the strength of the neighborhood, as a higher $A$ implies a more productive neighborhood. As $A$ determines the households’ aggregate demand for housing, it represents the demand-side fundamental.

As a result of realistic informational frictions, neither $A$ nor $A_i$ is observable to the households. Instead, each household observes a noisy private signal about $A$ at $t = 1$. Specifically, household $i$ observes

$$s_i = A + \nu_i,$$

where $\nu_i \sim \mathcal{N}(0, \tau_s^{-1})$ is signal noise independent across households. The parameter $\tau_s$ measures the precision of the private signal. As $\tau_s \to \infty$, the households’ signals become infinitely precise and the informational frictions about $A$ vanish.

We assume that each household experiences a disutility for labor $l_i^{1+\psi}$, and that it maximizes its utility subject to its budget constraint:

$$\max_{\{H_i, (C_i)_{i \in [0,1]}\}} E \left[ U \left( \{H_j\}_{j \in [0,1]}, C_i \right) - \frac{l_i^{1+\psi}}{1+\psi} \right] \mathbb{I}_i,$$

such that $P_H H_i + C_i = e^{A_i} l_i + \Pi_i$, where $\Pi_i$ is the income from building the house. We assume for simplicity that the home builder for household $i$ is part of the household, and that the builder brings home its wage $\Pi_i = P_H H_i$ to the household after construction has taken place. As a result, at $t = 2$, household $i$’s budget constraint satisfies $C_i(i) = e^{A_i} l_i$. The choices of labor and housing are made at $t = 1$ subject to each household’s information set $\mathcal{I}_i = \{s_i, P_H\}$, which includes its private signal $s_i$ and the housing price $P_H$.\footnote{We do not include the volume of housing transactions in the information set as a result of a realistic consideration that, in practice, people observe only delayed reports of total housing transactions at highly aggregated levels, such as national or metropolitan levels.}

### 2.1.2 Builders

Home builders face a convex labor cost

$$\frac{k}{1+k} e^{-\zeta} H_{S}^{\frac{1+k}{k}}$$

in supplying housing, where $H_S$ is the quantity of housing supplied, $k \in (0, \infty)$ is a constant parameter, and $\zeta$ represents a shock to the building cost. We assume that $\zeta$ is observed by
builders but not households,\(^5\) and that from the perspective of households \(\zeta \sim \mathcal{N}(\bar{\zeta}, 1)\), i.e.,
a normal distribution with \(\bar{\zeta}\) as the mean and unit variance.

Builders at \(t = 1\) maximize their profit subject to their supply curve

\[
\Pi(H_S) = \max_{H_S} \ P_H H_S - \frac{k}{1+k} e^{-\zeta H_{S}^{1+k}}. \tag{3}
\]

It is easy to determine the builders’ optimal supply curve:

\[
H_S = P_H^k e^{\xi}, \tag{4}
\]

where \(\xi = k\zeta\) has the interpretation of being a supply shock with normal distribution \(\xi \sim \mathcal{N}(\bar{\xi}, k^2)\), where \(\bar{\xi} = k\bar{\zeta}\). The parameter \(k\) measures the supply elasticity of the neighborhood. A more elastic neighborhood has a larger supply shock, i.e., the supply shock has greater mean and variance. In the housing market equilibrium, the supply shock \(\xi\) not only affects the supply side, but also the demand side, as it acts as informational noise in the price signal when the households use the price to learn about the common productivity \(A\).

We also incorporate a behavioral feature that households may underestimate the supply elasticity in the neighborhood, and incorrectly believe it to be \(\phi k\) rather than \(k\), where \(\phi \leq 1\). This feature is motivated by the observation made by Glaeser (2013) that agents tend to underestimate supply shocks during various episodes of real-estate speculation observed in U.S. history. When households underestimate the supply elasticity, they overestimate the information contained in the housing price, and consequently overreact to the price signal.

2.1.3 Equilibrium

Our model features a noisy rational expectations equilibrium, which requires clearing of the housing market that is consistent with the optimal behavior of both households and home builders:

- **Household optimization:** \(\{H_i\}_{i \in [0,1]}, C_i, l_i\) solves each household’s maximization problem in (2).

- **Builder optimization:** \(H_S\) solves the builders’ maximization problem in (3).

\(^5\)Even though we assume that builders perfectly observe the supply shock, a more realistic setting would have builders each observing part of the supply and thus needing to aggregate their respective information in order to fully observe the supply-side shock. We have explored this more general setting, which entails an additional layer of information aggregation on the builder side of the housing market. Nevertheless, it gives qualitatively similar insights as our current setting.
• At $t=1$, the housing market clears:

$$
\int_{-\infty}^{\infty} H_i(s_i, P_H) d\Phi(v_i) = P_H e^\xi,
$$

where each household’s housing demand $H_i(s_i, P_H)$ depends on its private signal $s_i$ and the housing price $P_H$. The demand from households is integrated over the idiosyncratic component of their private signals $\{\nu_i\}_{i \in [0,1]}$.

2.2 The equilibrium

We first solve for the optimal labor and housing choices for a household given its utility function and budget constraint in (2), which are characterized in the following proposition.

**Proposition 1** Households $i$’s optimal labor choice depends on its expected productivity:

$$
l_i = E [e^{A_i} | I_i]^{1/\psi},
$$

and its demand for housing is

$$
\log H_i = \frac{1}{(1-\eta_c)\eta_H + \eta_c} \log \left( \frac{1}{P_H} \cdot E \left[ \left( \int_{[0,1]/i} H_j dj \right)^{\eta_c(1-\eta_H)} \left( e^{A_i} E \left[ e^{A_i} | I_i \right]^{1/\psi} \right)^{\eta_H} \left| I_i \right] \right) \right) 
+ \frac{1}{(1-\eta_c)\eta_H + \eta_c} \log \left( \left( \frac{1-\eta_H}{\eta_H} \right)^{\eta_H} \left( \frac{1-\eta_c}{\eta_c} \right)^{\eta_c(1-\eta_H)} \left( 1-\eta_c \right)^{\eta_H} \right). \tag{5}
$$

Proposition 1 demonstrates that the labor chosen by a household is determined by its expected productivity, and that its housing demand is determined by not only its own productivity $e^{A_i}$ but also the aggregate housing consumption of other households. This latter component arises from the complementarity in the utility function of the household.

By clearing the aggregate housing demand of the households with the supply from the builders, we derive the housing market equilibrium. Despite the nonlinearity in each household’s demand and in the supply from builders, we obtain a tractable unique log-linear equilibrium. The following proposition summarizes the housing price and each household’s housing demand in this equilibrium.

**Proposition 2** At $t=1$, the housing market has a unique log-linear equilibrium: 1) The housing price is a log-linear function of $A$ and $\xi$:

$$
\log P_H = p_A A + p_\xi \xi + p_0, \tag{6}
$$
with the coefficients $p_A$ and $p_\xi$ given by

$$p_A = \frac{\eta_H}{1 + \eta_H k} \frac{1 + \psi}{\psi} - \frac{(1 - \eta_c) \eta_H + \eta_c}{1 + \eta_H k} \tau_A b > 0, \quad (7)$$

$$p_\xi = -\frac{\eta_H}{1 + k \eta_H} - \frac{(1 - \eta_c) \eta_H + \eta_c}{1 + k \eta_H} \tau_s^{-1} \left( \frac{b}{\phi k} \right)^2 < 0, \quad (8)$$

where $b \in \left[ 0, \frac{1 + \psi}{\psi} \left( \frac{1}{1 - \eta_c} \eta_H + \eta_c \tau_A + \eta_H \tau_s \right) \right]$ is the unique positive, real root of equation (28), and $p_0$ is given in equation (33).

2) The housing demand of household $i$ is a log-linear function of its private signal $s_i$ and log $P_H$:

$$\log H_i = h_s s_i + h_P \log P_H + h_0, \quad (9)$$

with the coefficients $h_s$ and $h_P$ given by

$$h_s = b > 0, \quad (10)$$

$$h_P = -\frac{1}{\eta_H} + \left( 1 + \frac{1}{\psi} + \frac{1 - \eta_H}{\eta_H} b \right) \frac{b^2 - \frac{1}{(\phi k)^2}}{\tau_A + \tau_s + b^2 - \frac{1}{(\phi k)^2} p_A}, \quad (11)$$

and $h_0$ given by equation (22).

Proposition 2 establishes that the housing price $P_H$ is a log-linear function of the neighborhood strength $A$ and the housing supply shock $\xi$, and that each household’s housing demand is a log-linear function of its private signal $s_i$ and the log housing price $\log P_H$. Similar to Hellwig (1980), the housing price aggregates the households’ dispersed private information to partially reveal $A$. The price does not depend on the idiosyncratic noise in any individual household’s signal because of the Law of Large Numbers. This last observation is key to the tractability of our model, and ensures that the housing demand from the households retains a log-normal distribution after aggregation.

In the presence of informational frictions, the housing supply shock $\xi$ serves the same role as noise trading in standard models of asset market trading with dispersed information. This feature is new to the housing literature and highlights an important channel for supply shocks to affect the expectations of potential home buyers. Since households cannot perfectly disentangle changes in housing prices caused by supply shocks from those brought about by shocks to demand, they partially confuse a housing price change caused by a supply shock to be a signal about the strength of the neighborhood.
To facilitate our discussion of the impact of learning, it will be useful to introduce a perfect-information benchmark in which all households perfectly observe the strength of the neighborhood $A$. The following proposition characterizes this benchmark equilibrium.

**Proposition 3** Consider a benchmark setting, in which households perfectly observe $A$ (i.e., $s_i = A, \forall i$.) There is also a log-linear equilibrium, in which the housing price is

$$
\log P_H = \frac{\eta_H}{1 + k \eta_H} \frac{1 + \psi}{\psi} A - \eta_H \frac{1}{1 + k \eta_H} \xi + \frac{\eta_H}{1 + k \eta_H} \log \left( \frac{1 - \eta_H}{\eta_H} \right) + \frac{1 - \eta_H}{1 + k \eta_H} \log \left( \frac{1 - \eta_c}{\eta_c} \right)
$$

and all households have the same housing demand

$$
\log H = \frac{1 + \psi}{\psi} A - \frac{1}{\eta_H} \log P_H + \log \left( \frac{1 - \eta_H}{\eta_H} \right) + \frac{1 - \eta_H}{\eta_H} \log \left( \frac{1 - \eta_c}{\eta_c} \right)
$$

Furthermore, the housing market equilibrium with information frictions characterized in Proposition 2 converges to this benchmark equilibrium as $s \rightarrow 1$; and the variance of the housing price $\text{Var} \left[ \log P_H \right]$ has a U-shaped relationship with the supply elasticity $k$.

It is reassuring that as the households’ private information becomes infinitely precise, the housing market equilibrium converges to the perfect-information benchmark. In this perfect-information benchmark, the housing price is also a log-linear function of the demand-side fundamental $A$ and the supply shock $\xi$, and each household’s identical demand is a log-linear function of the perfectly observed $A$ and the housing price $\log P_H$. Consistent with the standard intuition, a higher $A$ increases both the housing price and aggregate housing demand, while a larger supply shock $\xi$ reduces the housing price but increases aggregate housing demand. It is also easy to see that in this benchmark setting, as the supply elasticity $k$ rises from zero to infinity, the weight of $A$ (the demand-side fundamental) in the housing price decreases, while the weight of $\xi$ (the supply-side shock) increases.

Furthermore, in the perfect-information benchmark, the housing price variance has a U-shaped relationship with the housing supply elasticity $k$. This is because, as $k$ varies, it causes the housing price to assign different weights to the demand-side fundamental and the supply-side shock. The standard intuition from diversification implies that the price has the lowest variance when the weights of the two factors are balanced, i.e., the supply elasticity takes an intermediate value. This U-shaped price variance serves a benchmark to evaluate the housing price variance in the presence of informational frictions.
2.3 Impact of learning

In the presence of informational frictions about the strength of the neighborhood $A$, each household needs to use its private signal $s_i$ and the publicly-observed housing price $\log P_H$ to learn about $A$. As the housing price $\log P_H$ is a linear combination of the demand-side fundamental $A$ and the housing supply shock $\xi$, the supply shock interferes with this learning process. A larger supply shock $\xi$, by depressing the housing price, will have an additional effect of reducing the households’ expectations of $A$. This, in turn, reduces their housing demand and consequently further depresses the housing price. This learning effect thus causes the supply shock to have a larger negative effect on the equilibrium housing price than it would in the perfect-information benchmark. Similarly, this learning effect also causes the demand-side fundamental $A$ to have a smaller positive effect on the price than in the perfect-information benchmark because informational frictions cause households to partially discount the value of $A$. The following proposition formally establishes this learning effect on the housing price.

**Proposition 4** In the presence of informational frictions, coefficients $p_A > 0$ and $p_\xi < 0$ derived in Proposition 2 are both lower than their corresponding values in the perfect-information benchmark.

The precision of the households’ private information $\tau_s$ determines the informational frictions they face. The next proposition establishes that an increase in $\tau_s$ mitigates the informational frictions and brings the coefficient $p_A$ closer to its value in the perfect-information benchmark. In fact, as $\tau_s$ goes to infinity, the housing market equilibrium converges to the perfect-information benchmark (Proposition 3).

**Proposition 5** $p_A$ is monotonically increasing with the precision $\tau_s$ of each household’s private signal and decreasing with the degree of complementarity in households’ housing consumption $\eta_c$.

Each household’s housing demand also reveals how the households learn from the housing price. In the presence of informational frictions about $A$, the housing price is not only the cost of acquiring shelter but also a signal about $A$. The housing demand of each household derived in (9) reflects both of these effects. Specifically, we can decompose the price elasticity of each household’s housing demand $h_P$ in equation (11) into two components: The first
component $-\frac{1}{\eta_H}$ is negative and represents the standard cost effect (i.e., downward sloping demand curve), as in the perfect-information benchmark in Proposition 3, and the second component $\left(1 + \frac{1}{\psi} + \eta_c \frac{1-\eta_H}{\eta_H} b\right) \frac{b^2}{(\log b)^2} \frac{1}{\tau_A + \tau_s + \log^2 b} \frac{1}{\eta_H}$ is positive and represents the learning effect.

A higher housing price raises the household’s expectation of $A$ and induces it to consume more housing through two related but distinct learning channels. First, a higher $A$ implies a higher productivity for the household itself. Second, a higher $A$ also implies that other households demand more housing, which in turn induces each household to demand more housing. As a reflection of this complementarity effect, the second component in the price elasticity of housing demand increases with $\eta_c$, the degree of complementarity in the household’s utility of its own housing consumption and other households’ housing consumption.

As a result of the presence of the complementarity channel, $\eta_c$ also affects the impact of learning on the housing price. As $\eta_c$ increases, each household puts a greater weight on the housing price in its learning of $A$ and a smaller weight on its own private signal. This in turn makes the housing price less informative of $A$. In this way, a larger $\eta_c$ exacerbates the informational frictions faced by households. Indeed, Proposition 5 shows that the loading of $\log P_H$ on $A$ is decreasing with $\eta_c$.

Housing supply elasticity $k$ plays an important role in determining the informational frictions faced by the households, in addition to its standard supply effect. To illustrate this learning effect of supply elasticity, we consider two limiting economies as $k$ goes to 0 and $\infty$, which are characterized in the following proposition.

**Proposition 6** As $k \to \infty$, the housing price and each household’s housing demand converge to

$$\log P_H = -\zeta,$$

and

$$\log H_i = \frac{1}{\psi} \frac{\eta_H \tau_s}{(1 - \eta_c) \eta_H + \eta_c} \frac{\tau_A + \eta_H \tau_s}{s_i} \frac{1}{\eta_H} \log P_H + h_0.$$  

As $k \to 0$, the housing price and each household’s housing demand converge to

$$\log P_H = \frac{1}{\psi} \frac{1}{\eta_H} A + \frac{1}{2} \eta_H \left(\eta_H + \frac{1}{\psi}\right) \tau_s^{-1} + \eta_H \log \left(1 - \frac{1 - \eta_c}{\eta_H} \left(\frac{1 - \eta_c}{\eta_c} \right)^{\frac{1-\eta_H}{\eta_H}} \right),$$  

and $\log H_i = 0$.

At one end, as supply elasticity goes to zero, the housing price is completely driven by $A$ and thus fully reveals it. In this case, each household precisely learns $A$ from the
price, and as a result, both the housing price and each household’s housing demand coincide with their corresponding values in the prefect-information benchmark. At the other end, as supply elasticity goes to infinity, the housing price is completely driven by the supply shock $\xi$ and contains no information about $A$. In this case, each household has to rely on its own private signal to infer $A$. As the housing price, however, is fully determined by the supply shock and independent of the demand-side fundamental, informational frictions about $A$ do not matter for the housing price. Consequently, the housing price also coincides with that in the perfect-information benchmark, even though informational frictions still affect each household’s housing demand. Taken together, when housing supply is either perfectly elastic or inelastic, the housing price is not affected by informational frictions and coincides with that in the perfect-information benchmark.

The following proposition characterizes the housing price at an intermediate supply elasticity and, in particular, analyzes the role of the households’ underestimation $\phi$ of supply elasticity.

**Proposition 7** Consider an intermediate level of supply elasticity $k \in (0, \infty)$. 1) In the presence of informational frictions, both $p_A$ and $|p_\xi|$ are monotonically decreasing with $\phi$. 2) When $\phi = 1$, the housing price variance with informational frictions is lower than that of the perfect-information benchmark. 3) The variance of the housing price $\log P_H$ is monotonically decreasing with $\phi$, and a sufficient condition $1 - \frac{\eta}{(1-\eta)c_H + \eta_H} \leq \phi^2 \leq \frac{1}{2}$ ensures the price variance to be at least as large as its corresponding value in the perfect-information benchmark.

Proposition 7 shows that, at an intermediate supply elasticity, the households’ underestimation of supply elasticity causes them to over-interpret the information contained in the price signal and thus overreact to the price signal. Consequently, the positive loading of the equilibrium housing price $p_A$ on the demand-side fundamental $A$ becomes larger and the negative loading $p_\xi$ on the supply shock becomes more negative. That is, the housing price becomes more responsive to both demand and supply shocks.

Proposition 7 also shows that, in the absence of the households’ underestimation of supply elasticity, the presence of informational frictions reduces the housing price variance. This is because informational frictions make households less responsive to demand shocks, causing the housing price to load less on demand shocks. When households underestimate the supply elasticity ($\phi < 1$), their overreaction to the price signal amplifies the price effects of both
supply and demand shocks, and implies that the housing price variance is monotonically decreasing with $\phi$. In fact, Proposition 7 shows that when $\phi$ is sufficiently small, the housing price variance is at least as large as its value in the perfect-information benchmark.

Interestingly, the volatility amplification induced by the households’ overreaction to the housing price is most pronounced when the supply elasticity is in an intermediate range. This follows from our earlier discussion of the two limiting cases when the elasticity goes to either zero or infinity. At one end, when the supply is infinitely elastic, the households’ learning about the demand side is irrelevant for the price. At the other end, when the supply is infinitely inelastic, the price fully reveals the demand-side fundamental and there is no room for the households to overreact. In between these two limiting cases, the demand-side fundamental plays a significant role in determining the housing price and at the same time households face substantial uncertainty about the demand-side fundamental, which leaves room for their overreaction to amplify the price volatility.

In Figure 6, we provide a numerical example to illustrate how informational frictions and households’ overreaction jointly affect the housing price variance. The figure depicts the log-price variance $\text{Var} \{\log P_H\}$ against the supply elasticity under the following parameter values:

$$\tau_s = 1, \tau_A = \phi = 0.25, \eta_c = 0.5, \psi = \eta_H = 0.75.$$  

For comparison, it also depicts the log-price variance in the perfect-information benchmark, which is obtained as $\tau_s \to \infty$. As the supply elasticity $k$ rises from 0 to 1 (i.e., from infinitely inelastic to more elastic), the log-price variance decreases with the supply elasticity. In contrast, when the households face informational frictions with $\tau_s = 1$, Figure 6 shows that the log-price variance first increases with $k$, when $k$ is lower than an intermediate level around 0.1, and then decreases with $k$. The difference between this humped shape and the monotonically decreasing curve in the perfect-information benchmark illustrates the joint effect of informational frictions and the households’ overreaction to the price signal.

The humped log-price variance illustrated in Figure 6 provides an explanation for the aforementioned, non-monotonic relationship between the housing boom and bust cycles experienced by different U.S. counties in the 2000s and supply elasticity.

$^6$Outside the range of $k$ depicted in the figure, both of these two lines are decreasing and eventually converge to each other as $k \to \infty$, as derived in Proposition 6.
3 Elasticity and Housing Speculation

In this section, we further explore the effects of household learning on housing speculation. We first extend the baseline model presented in the last section to incorporate secondary homes and, in particular, to show that the same learning effect discussed earlier leads to a new prediction regarding a non-monotonic relationship between housing speculation and supply elasticity. Then, we examine this prediction in the data and provide some supportive evidence.

3.1 A model extension

We extend the model presented in the previous section to incorporate three types of agents in the economy: households, home builders, and immigrants looking to move into the neighborhood. These immigrants are the new addition to this extension. Suppose that these immigrants make their decision on whether to move into the neighborhood at $t = 1$, which determines the migration inflow into the neighborhood at $t = 2$. They provide additional labor to households, the wages from which they use to buy secondary homes owned initially by the households. For simplicity, we assume that these immigrants use all of their resources to consume housing at $t = 2$. Importantly, immigrants are outside the household community and therefore do not receive any private signal about the strength of the neighborhood. As a result, they rely on the housing price at $t = 1$ to infer the neighborhood’s strength.

At $t = 1$, households purchase primary and secondary houses from home builders from two separate markets and decide how much of their goods to produce at $t = 2$.

Households have the same preferences as in the baseline model. They supply their own labor $l_i$ and employ immigrants who migrate into the neighborhood at $t = 2$ and supply labor $L$ for a wage $w_iL$. The production function of household $i$ is $e^{A_i} (l_i + L)$, with the labor supplied by itself and immigrants being perfect substitutes. Households operate in perfectly competitive industries, and therefore, in equilibrium, $w_i = e^{A_i}$. Households again receive a private signal $s_i$ about the strength of the neighborhood.

Builders build two types of housing at $t = 1$ for households to purchase, one type as their primary residence and the other as secondary homes, which the households sell at $t = 2$ to the immigrants. Our separate treatment of primary and secondary homes is consistent with the fact that, in practice, primary homes tend to be single houses, while secondary homes tend to be apartments and condominiums. Another advantage of giving separate supply
curves to primary and secondary homes is that we are able to maintain a tractable log-linear equilibrium.

Let $H_i$ be the demand for primary housing by household $i$, and $h_i$ its demand for secondary housing. Each household now maximizes

$$\max_{H_i, C_i, l_i} E \left[ U \left( \{H_j(i)\}_{j \in [0,1]}, C_i \right) - \frac{l_i^{1+\psi}}{1+\psi} \right]_{I_i}$$

such that

$$P_H H_i + P_h h_i + C_i = e^{A_i} (l_i + L) - w_i L + R_{hi} + \Pi_{Hi} + \Pi_{hi},$$

where $P_H$ is the price of primary homes, $P_h$ is the price of secondary homes, $R_{hi} = E[w_i|A] L$ is the income for household $i$ to rent its secondary home to immigrants at $t = 2$, and $\Pi_{Hi}$ and $\Pi_{hi}$ are the income from building the primary and secondary houses, respectively, delivered at $t = 2$. We assume for simplicity that the primary home builder for household $i$ is part of the household, so that $\Pi_{Hi} = P_H H_i$ in equilibrium, and that the net builder for secondary homes consumes the net profit from the construction and sale of the secondary homes $\Pi_{hi} = (P_h - R_h) h_i$. The choice of goods consumption is made at $t = 2$, while the choices of labor and housing are made at $t = 1$ subject to each household’s information set $I_i = \{s_i, P_H, P_h\}$, which includes its private signal $s_i$ and the housing prices $P_H$ and $P_h$.

Since it is often costly to move and takes time to find a place to live, the immigrants have to make their moving decision based on the expected wage at $t = 1$ rather than the realized wage $L$ at $t = 2$. Specifically, immigrants move into the neighborhood at $t = 2$, but make a decision on the move at $t = 1$ based on their expectations of the neighborhood strength, by choosing how much labor to supply to the households in the neighborhood. Similar to the households, they face a disutility of labor $\frac{1+\psi}{\psi} L^{\frac{1+\psi}{\psi}}$ and solve the optimization problem

$$\max_L E \left[ w_i L | I_c \right] - \frac{\psi}{1+\psi} L^{\frac{1+\psi}{\psi}},$$

subject to each immigrant’s information set $I_c = \{P_H, P_h\}$, from which follows that

$$L = E \left[ w_i | I_c \right]^\psi = E \left[ e^{A_i} | I_c \right]^\psi.$$

The labor choice $L$ can be viewed as the size of the immigrant population moving into the neighborhood at $t = 2$. For simplicity, we assume that immigrants spend all of their wages on housing at $t = 2$ when they enter the neighborhood.
Home builders face separate production processes for building primary and secondary homes. Specifically, they face the following convex labor cost for building each type of home:

\[
\frac{k}{1 + k} e^{-\zeta_j} H_{S,j}^{1+k}
\]

where \( j \in \{H, h\} \) indicates the type of homes with \( H \) representing primary homes and \( h \) representing secondary homes, \( H_{S,j} \) is the quantity of type-\( j \) homes supplied, and \( \zeta_j \) represents a supply shock. We assume that \( \zeta_j \) is observed by builders but not households. From the perspective of households, there are two components in the supply shock of type-\( j \) homes:

\[
\zeta_j = \zeta + e_j, \ j \in \{H, h\}.
\]

The first component \( \zeta \) is common to the two types of homes. It has a normal distribution with \( \bar{\zeta} \) as the mean and unit variance. The second component \( e_j \) is idiosyncratic to type-\( j \) homes. It has a normal distribution with zero mean and \( \alpha \) as its standard deviation.

 Builders of each type of homes maximize their profit at \( t = 1 \):

\[
\Pi(H_{S,j}) = \max_{H_{S,j}} P_H H_{S,j} - \frac{k}{1 + k} e^{-\zeta_j} H_{S,j}^{1+k}.
\]

It is easy to determine the builders’ optimal supply curve:

\[
H_{S,j} = P_H^k e^{\xi_j},
\]

where \( \xi_j \sim N(\bar{\zeta}, (1 + \alpha^2) k^2) \) and \( \bar{\xi} = k \bar{\zeta} \).

We derive the noisy rational expectations equilibrium as in the baseline model. The equilibrium features the clearing of both primary and secondary homes, and the households’ learning from the prices of both primary and secondary homes. As the nature of the equilibrium and the key steps of deriving the equilibrium are similar to the baseline model, we leave the detailed description and derivation of the equilibrium in an Internet Appendix. Instead, we briefly summarize the key features of the extended model here.

There is a unique log-linear equilibrium where the primary home price is a log-linear function of \( A, \xi_H, \) and \( \log P_h \):

\[
\log P_H = p_A A + p_{\xi} \xi_H + p_{\log} \log P_h + p_0,
\]

and the secondary housing price is a log-linear function of \( A, \xi_h, \) and \( \log P_H \):

\[
\log P_h = \bar{p}_A A + \bar{p}_{\xi} \xi_h + \bar{p}_{\log} \log P_H + \bar{p}_0.
\]
All coefficients are given in the Internet Appendix. As a result of the separate supply shocks in the primary and secondary home markets, the prices of primary and secondary homes are not perfectly correlated. Both of them serve to aggregate the private information of households regarding the strength of the neighborhood $A$. In turn, each household, say household $i$, treats both prices $P_H$ and $P_h$ as useful signals, in addition to its private signal $s_i$, in forming its expectation of $A$.

Following our discussion of the baseline model, through this informational channel, informational frictions and households’ overreaction to the price signals can jointly lead to a humped-shape relationship between the log-price variance of both primary and secondary homes and the supply elasticity. To illustrate this relationship, we again use a numerical example based on the following parameter choices:

$$\tau_s = 1, \tau_A = \phi = .2, \eta_c = .5, \psi = \eta_H = .75, \alpha = 1.$$  \hspace{1cm} (15)

Figure 7 depicts the log-price variance of both primary and secondary homes against supply elasticity in the top panel. It shows humped-shapes for both curves, consistent with that in Figure 6 for the baseline model.

As the households’ demand for secondary homes is entirely driven by their expectation of the neighborhood strength and the future housing demand of immigrants, the households’ learning effects are particularly important on the demand for secondary homes. Thus, in this extended model, the households’ demand for secondary homes provides an additional dimension to examine learning effects. Specifically, the demand of household $i$ for primary and secondary homes are both log-linear function of its private signal $s_i$, log $P_H$, and log $P_h$:

$$\log H_i = h_s s_i + h_P \log P_H + h_p \log P_h + h_0,$$

$$\log h_i = \tilde{h}_s s_i + \tilde{h}_P \log P_H + \tilde{h}_p \log P_h + \tilde{h}_0,$$

with all coefficients given in the Internet Appendix. We are particularly interested in the ratio of demand for secondary homes relative to demand for primary homes log $\left( \frac{h_i}{H_i} \right)$, as this ratio is directly measurable in the data.

Figure 7 further depicts the variance of log $\left( \frac{h_i}{H_i} \right)$, which measures the variability of investment-driven housing demand relative to consumption-driven housing demand with respect to supply elasticity in the presence and absence of informational frictions. In the absence of informational frictions, $Var \left[ \log \left( \frac{h_i}{H_i} \right) \right]$ is monotonically increasing with supply
elasticity. This pattern is intuitive and reflects the households’ use of secondary homes to buffer housing supply shocks. The households will demand more secondary homes if the supply shock turns out to be positive, and less secondary homes if the supply shock turns out to be negative. In areas with more elastic supply, housing supply is more variable and consequently the ratio of demand of secondary homes relative to primary homes is also more variable.

Interestingly, in the presence of informational frictions, Figure 7 shows a humped-shape pattern of $Var \left[ \log \left( \frac{h_i}{H_i} \right) \right]$ with respect to supply elasticity. This humped-shape highlights the learning effects on the households’ demand for secondary homes. Building on the same insight from our earlier discussion, the demand for secondary homes is most variable in an intermediate range of supply elasticity because the joint effects of informational frictions and the households’ overreaction to the price signals are most influential in affecting the households’ expectation of the neighborhood strength, and thus their demand for secondary homes.

The non-monotonic relationship between the variability of secondary-home demand relative to primary-home demand and housing supply elasticity is in sharp contrast to the monotonic relationship in the perfect-information benchmark. This non-monotonic relationship provides a new prediction for us to explore in the data.

### 3.2 Empirical evidence

In this subsection, we examine how investment home purchases vary across the counties in our sample. Motivated by the extended model presented in the previous subsection, we test whether during the recent U.S. housing boom, the share of investment home purchases in the total home purchases of a county was most dramatic in counties with intermediate supply elasticities.

We construct the share of non-owner-occupied home purchases at the county level from the “Home Mortgage Disclosure Act” (HMDA) data set. The HMDA has comprehensive coverage for mortgage applications and originations in the U.S. We use mortgages originated for home purchases. Specifically, HMDA data identifies owner occupancy of the home for individual mortgages. We then aggregate the HMDA data to the county level and calculate the fraction of mortgage originations for non-owner-occupied homes in the total mortgage originations as our measure of the share of investment home purchases.
Figure 8 depicts the share of non-owner-occupied home purchases for the U.S. and for three cities, New York, Las Vegas, and Charlotte. At a national level, the share of non-owner-occupied home purchases rose steadily from a modest level of 7% in 2000 to peak at a level above 15% in 2005. It then fell gradually to less than 10% in 2010. The peak of the share of non-owner-occupied home purchases in 2005 was slightly in advance of the peak of the national home price index in 2006, as shown in Figure 1. Nevertheless, the rise and fall of the share of non-owner-occupied home purchases were roughly in sync with the boom and bust of home prices.

Across the three cities, it is interesting to note that Las Vegas had the most dramatic rise and fall in the share of non-owner-occupied home purchases, followed by Charlotte, and with New York having the most modest rise and fall. The most variable share of non-owner-occupied home purchases experienced by Las Vegas is particularly interesting as Vegas also had the most dramatic price cycle among these cities.

We now systematically examine the share of non-owner-occupied home purchases across counties with different housing supply elasticities. We focus on 2005, which is the peak year of the share at the national level. We have also examined the average share in an alternative period from 2003-2006, which leads to similar results.

In Figure 9, we sort the counties in our sample into three groups in the top panel and ten groups in the bottom panel using the Saiz’s elasticity measure, with each group having the same number of counties. The top panel shows that the fraction of non-owner-occupied home purchases decreases monotonically across the three groups, from inelastic to the middle and elastic groups. As we discussed before, this coarse grouping might hide finer non-monotonicity. Indeed, the bottom panel shows that the fraction of non-owner-occupied home purchases displays a non-monotonic pattern across ten elasticity groups with the largest share of non-owner-occupied home purchases in groups 3 and 4. This non-monotonic pattern is consistent with the prediction of the extended model.

In Figure 10, we sort the counties into elasticity groups each with an equal population rather than number of counties. Across either the three groups shown in the top panel or the ten groups shown in the bottom panel, there is a non-monotonic pattern in the share of non-owner-occupied home purchases across the elasticity groups, with the share peaking in the middle of the groups.

To further examine whether the largest share of non-owner-occupied home purchases in
the middle elasticity groups are robust to controlling for other fundamental factors, we also adopt a regression approach in Table 2. Similar to the regressions reported in Table 1, we regress the share of non-owner-occupied home purchases in 2005 on two dummy variables that indicate whether a county is in the middle elastic group or the elastic group, which are constructed in top panel of Figure 10, together with a list of control variables. This regression implicitly uses the inelastic group as the benchmark for the middle and elastic groups. The control variables include the fraction of subprime households in the county in 2005, the population change, and annualized per capita income change. Columns 1 and 2 of Table 2 report the regressions without and with the controls. In either regression specification, we observe the middle group has a significantly larger share of non-owner-occupied home purchases than the other groups. Furthermore, none of the control variables is significant.

Taken together, Figures 9 and 10 and Table 2 confirm the new prediction of the extended model that there is a non-monotonic relationship between the variability of investment home purchases relative to primary home purchases and housing supply elasticity.

4 Conclusion

This paper highlights a non-monotonic relationship between the magnitude of housing cycles and housing supply elasticity in the cross-section of U.S. county data during the U.S. housing cycle of the 2000’s. We develop a tractable model to analyze information aggregation and learning in housing markets to explain this phenomenon. In the presence of pervasive informational frictions regarding economic strength and housing supply of a neighborhood, households face a realistic problem in learning about these fundamental variables with housing prices serving as important signals. Our model highlights how the households’ learning interacts with characteristics endemic to local housing supply and demand to impact housing price dynamics. In particular, supply elasticity affects not only housing supply but also the informational noise in the price signal for the households’ learning of the neighborhood strength. Our model predicts that housing price and share of investment home purchases are both most variable in areas with intermediate supply elasticities, which is supported by our empirical analysis.
Appendix  Proofs of Propositions

A.1 Proof of Proposition 1

The first order conditions for household $i$’s choices of $H_i$ and $l_i$ at an interior point are

$$H_i : \frac{(1 - \eta_c)(1 - \eta_H)}{H_i} E \left[ U \left( \{H_j\}_{j \in [0,1]} , C_i \right) \bigg| I_i \right] = \lambda_i P_H, \tag{16}$$

$$l_i : l_i^\psi = \lambda_i E \left[ e^{A_i} \bigg| I_i \right]. \tag{17}$$

Taking expectation of equation (1) and imposing $\lambda_i = 1$ to equation (16), one arrives at

$$P_H H_i^{(1 - \eta_c)\eta_H + \eta_c} = \left( \frac{1 - \eta_H}{\eta_H (1 - \eta_c)} \right)^{\eta_H} \left( 1 - \frac{\eta_c}{\eta_c} \right)^{\eta_c (1 - \eta_H)} E \left[ \left( \int_{[0,1]/i} H_j dj \right) \eta_c (1 - \eta_H) \left( e^{A_i} l_i^\eta_H \right) \bigg| I_i \right].$$

From equation (17), it follows that

$$l_i = E \left[ e^{A_i} \bigg| I_i \right]^{1/\psi},$$

from which we see that

$$\log H_i = \frac{1}{\eta_H + \eta_c (1 - \eta_H)} \log \left( \frac{1}{P_H} E \left[ e^{\eta_H A_i} \left( \int_{[0,1]/i} H_j dj \right)^{\eta_c (1 - \eta_H)} \bigg| I_i \right] E \left[ e^{A_i} \bigg| I_i \right]^{\eta_H/\psi} \right)$$

$$+ \frac{1}{\eta_H + \eta_c (1 - \eta_H)} \log \left( \left( \frac{1 - \eta_H}{\eta_H} \right)^{\eta_H} \left( 1 - \frac{\eta_c}{\eta_c} \right)^{\eta_c (1 - \eta_H)} \right).$$

Note that integrating over the continuum of other households’ housing choices is equivalent to taking an expectation with respect to a representative household’s housing decision. We then obtain equation (5).

A.2 Proof of Proposition 2

We first conjecture that each household’s housing purchasing and the housing price take the following log-linear forms:

$$\log H_i = h_P \log P_H + h_s s_i + h_0, \tag{18}$$

$$\log P_H = p_A A + p_\xi \xi + p_0, \tag{19}$$

where the coefficients $h_0, h_P, h_s, p_0, p_A,$ and $p_\xi$ will be determined by equilibrium conditions.

Given the conjectured functional form for $H_i$, we can expand equation (5). It follows that

$$E \left[ \left( \int_{[0,1]/i} H_j dj \right)^{\eta_c (1 - \eta_H)} \left( e^{A_i} E \left[ e^{A_i} \bigg| I_i \right]^{1/\psi} \right)^{\eta_H} \bigg| I_i \right]$$

$$= e^{\eta_c (1 - \eta_H) \left(h_0 + h_P \log P_H + \frac{1}{2} \eta_H \rho_s^2 \right) + \frac{1}{2} \eta_H \left( \eta_H + \frac{1}{2} \right) \rho_s^2} E \left[ e^{(\eta_H + \eta_c (1 - \eta_H) h_s) A} \bigg| I_i \right] E \left[ e^{A} \bigg| I_i \right]^{\eta_H/\psi}.$$
where we use the fact that $A$ is independent of $\xi_j$ and exploit the Law of Large Number for the continuum when integrating over households, which still holds if we subtract sets of measure 0 from the integral.

Define

$$q \equiv \frac{\log P_H - p_0 - p_0 \xi}{p_A} = A + \frac{p_0}{p_A} (\xi - \bar{\xi}),$$

which is a sufficient statistic of information contained in $P_H$. Then, conditional on observing its own signal $s_i$ and the housing price $P_H$, household $i$’s expectation of $A$ is

$$E[A \mid s_i, \log P_H] = E[A \mid s_i, q] = \frac{1}{\tau_A + \tau_s + \frac{p_A^2}{p_\xi^2} \frac{1}{(\phi k)^2}} \left( \tau_A \tilde{A} + \tau_s s_i + \frac{p_A^2}{p_\xi^2} \frac{1}{(\phi k)^2} q \right),$$

and its conditional variance of $A$ is

$$\text{Var}[A \mid s_i, \log P_H] = \left( \tau_A + \tau_s + \frac{p_A^2}{p_\xi^2} \frac{1}{(\phi k)^2} \right)^{-1}.$$

Therefore,

$$\log \left( E \left[ e^{(\eta_H + \eta_c (1 - \eta_H) h_s)} A \mid \mathcal{I}_i \right] E \left[ e^{A} \mid \mathcal{I}_i \right]^{\eta_H/\psi} \right)$$

$$= \left( 1 + \frac{\eta_H + \eta_c (1 - \eta_H) h_s}{\psi} \right) \left( \tau_A + \tau_s + \frac{p_A^2}{p_\xi^2} \frac{1}{(\phi k)^2} \right)^{-1} \left( \tau_A \tilde{A} + \tau_s s_i + \frac{p_A^2}{p_\xi^2} \frac{1}{(\phi k)^2} q \right)$$

$$+ \frac{1}{2} \left( \frac{\eta_H + \eta_c (1 - \eta_H) h_s}{\psi} \right)^2 \left( \tau_A + \tau_s + \frac{p_A^2}{p_\xi^2} \frac{1}{(\phi k)^2} \right)^{-1}.$$
Substituting this expression into equation (5) and matching coefficients with the conjectured log-linear form in (18), it follows that

\[ h_s = \frac{1}{(1 - \eta_c) \eta_H + \eta_c} \left( \frac{1 + \frac{1}{\psi}}{\eta_H + \frac{1 - \eta_H}{\eta_H} h_s} \right) \left( \tau_A + \tau_s + \frac{p_A^2}{p_\xi^2} \left( \frac{1}{H} \right)^2 \right)^{-1} \tau_s, \quad (20) \]

\[ h_P = -\frac{1}{\eta_H} + \left( \frac{1 + \frac{1}{\psi} + \eta_c \frac{1 - \eta_H}{\eta_H} h_s}{\eta_H} \right) \left( \tau_A + \tau_s + \frac{p_A^2}{p_\xi^2} \left( \frac{1}{H} \right)^2 \right)^{-1} \frac{p_A}{p_\xi^2} \left( \frac{1}{H} \right)^2, \quad (21) \]

\[ h_0 = \left( \frac{1 + \frac{1}{\psi} + \eta_c \frac{1 - \eta_H}{\eta_H} h_s}{\eta_H} \right) \left( \tau_A + \tau_s + \frac{p_A^2}{p_\xi^2} \left( \frac{1}{H} \right)^2 \right)^{-1} \left( \tau_A - \frac{p_A}{p_\xi^2} \left( \frac{1}{H} \right)^2 (p_0 + p_\xi \xi) \right), \quad (22) \]

\[ + \frac{1}{2\eta_H} \left( \eta_H + \frac{1 - \eta_H}{\eta_H} h_s \right)^2 + \frac{\eta_H}{\psi} \left( \tau_A + \tau_s + \frac{p_A^2}{p_\xi^2} \left( \frac{1}{H} \right)^2 \right)^{-1} \left( \tau_A - \frac{p_A}{p_\xi^2} \left( \frac{1}{H} \right)^2 (p_0 + p_\xi \xi) \right), \]

\[ + \frac{1}{2\eta_H} \left( \eta_H (1 - \eta_H) h_s^2 \tau_s^{-1} + \eta_H \left( \frac{H + 1}{\psi} \right) \tau_s^{-1} \right) \]

\[ + \log \left( \frac{(1 - \eta_c) (1 - \eta_H)}{\eta_H} \right) \frac{H}{\eta_H} \left( \frac{1 - \eta_c \eta_H}{\eta_c} \right) \left( \frac{1 - \eta_c \eta_H}{\eta_c} \right) \right). \]

By aggregating households’ housing demand and the builders’ supply and imposing market clearing in the housing market, we have

\[ h_0 + h_P (p_0 + p_A A + p_\xi \xi) + h_s A + \frac{1}{2} h_s^2 \tau_s^{-1} = \xi + k (p_0 + p_A A + p_\xi \xi). \]

Matching coefficients of the two sides of the equation leads to the following three conditions:

\[ h_0 + h_P p_0 + \frac{1}{2} h_s^2 \tau_s^{-1} = k p_0, \quad (23) \]

\[ h_P p_A + h_s = k p_A, \quad (24) \]

\[ h_P p_\xi = 1 + k p_\xi. \quad (25) \]

It follows from equation (25) that

\[ p_\xi = -\frac{1}{k - h_P}, \quad (26) \]

and further from equation (24) that

\[ p_A = \frac{h_s}{k - h_P}. \quad (27) \]

Thus, by taking the ratio of equations (27) and (26), we arrive at

\[ \frac{p_A}{p_\xi} = -h_s. \]

29
Substituting $\frac{p_A}{p_c} = -h_s$ into equation (20), and defining $b = -\frac{p_A}{p_c}$, we arrive at

$$\frac{1}{(\phi k)^2} b^3 + \left( \tau_A + \frac{\eta_H}{(1 - \eta_c) \eta_H + \eta_c} \right) b - \frac{1 + \psi}{\psi} \frac{\eta_H}{(1 - \eta_c) \eta_H + \eta_c} = 0. \quad (28)$$

We see from equation (28) that $b$ has at most one positive root since the above 3rd order polynomial has only one sign change, by Descartes’ Rule of Signs. By setting $b \rightarrow -b$, we see that there is no sign change, and therefore $b$ has no negative root. Furthermore, by the Fundamental Theorem of Algebra, the roots of the polynomial (28) exist. Thus, it follows that equation (28) has only one real, nonnegative root $b \geq 0$ and 2 complex roots.\(^7\)

Furthermore, by dropping the cubic term from equation (28), one arrives at an upper bound for $b$:

$$b \leq \frac{1 + \psi}{\psi} \frac{\eta_H}{\eta_H} \left( \frac{b}{\phi k} \right)^2. \quad (29)$$

Since $h_s = -\frac{p_A}{p_c} = b$, we can recover $h_s = b > 0$ and $p_\xi = -\frac{1}{b}p_A < 0$. From equation (21) and $b = -\frac{p_A}{p_c}$, it follows that

$$h_p = -\frac{1}{\eta_H} + \left( 1 + \frac{1}{\psi} + \eta_c \frac{1 - \eta_H b}{\eta_H} \right) \frac{\left( \frac{b}{\phi k} \right)^2}{\tau_A + \tau_s + \left( \frac{b}{\phi k} \right)^2} \frac{1}{\tau_A}. \quad (29)$$

From equation (25), one also has that $h_p = k + p_\xi^1$. Since $p_\xi \leq 0$, it follows that $h_p < k$ whenever $k > 0$.

From $h_s = b$ and equations (27) and (29), we arrive at

$$p_A = \frac{\eta_H}{1 + k \eta_H} \left( b + \left( 1 + \frac{1}{\psi} + \eta_c \frac{1 - \eta_H b}{\eta_H} \right) \frac{\left( \frac{b}{\phi k} \right)^2}{\tau_A + \tau_s + \left( \frac{b}{\phi k} \right)^2} + \frac{1}{\tau_A} \right) > 0. \quad (30)$$

One arrives at $p_\xi$ from recognizing that $p_\xi = -\frac{1}{b}p_A$. Manipulating equation (28), we first we recognize that

$$1 + \frac{1}{\psi} + \eta_c \frac{1 - \eta_H b}{\eta_H} = \frac{(1 - \eta_c) \eta_H + \eta_c}{\eta_H} \left( \frac{b}{\phi k} \right)^2 + \tau_A + \tau_s \right) b \tau_s^{-1}. \quad (31)$$

Substituting equation (31) into equation (30), and invoking equation (28) to replace $k \tau_\xi b^3$, one arrives at

$$p_A = \frac{\eta_H}{1 + \eta_H k} \frac{1 + \psi}{\psi} - \frac{(1 - \eta_c) \eta_H + \eta_c \tau_s^{-1} \tau_A b}. \quad (32)$$

\(^7\)The uniqueness of the positive, real root also follows from the fact that the LHS of the polynomial equation is monotonically increasing in $b$. 

30
and from equation (30) and \( p_\xi = -\frac{1}{\xi} p_A \), one also has that
\[
p_\xi = -\frac{\eta_H}{1 + \eta_H k} - \frac{(1 - \eta_c) \eta_H + \eta_c \tau_s - 1}{1 + k \eta_H} \left( \frac{b}{\phi k} \right)^2 < 0.
\]
From \( h_s = b, b = -\frac{p_A}{p_\xi} \), and equations (29), (23) and (22), one also finds that
\[
p_0 = \frac{\eta_H}{1 + \eta_H k} \left( 1 + \frac{1}{\psi} + \eta_c \frac{1 - \eta_H}{\eta_H} b \right) \frac{\tau_A A + b \frac{1}{(\phi k)^2} \xi}{\tau_A + \tau_s + b^2 \frac{1}{(\phi k)^2}} + \frac{1}{2} \frac{\eta_H}{1 + \eta_H k} b^2 \tau_s - 1
+ \frac{1}{2} \frac{\eta_H}{1 + \eta_H k} \left( \eta_c \frac{1 - \eta_H}{\eta_H} b^2 \tau_s - 1 + \left( \eta_H + \frac{1}{\psi} \right) \tau_s - 1 \right)
+ \frac{\eta_H}{1 + \eta_H k} \log \left( \frac{1 - \eta_c}{\eta_H} \left( \frac{1 - \eta_c}{\eta_c} \right)^{1 - \eta_H} \right).
\]

Given \( p_0, p_A, \) and \( b = -\frac{p_A}{p_\xi} \), we can recover \( h_0 \) from equation (22).

Since we have explicit expressions for all other equilibrium objects as functions of \( b \), and \( b \) exists and is unique, it follows that an equilibrium in the economy exists and is unique.

### A.3 Proof of Proposition 3

When all households observe \( A \) directly, there are no longer information frictions in the economy. Since the households’ idiosyncratic productivity components are unobservable, they are now symmetric. Then, it follows that \( H_j = H_i = H \). Imposing this symmetry in equation (5), we see that each household’s housing demand is then given by
\[
\log H = \frac{1 + \psi}{\psi} A - \frac{1}{\eta_H} \log P_H + \log \left( \frac{1 - \eta_H}{\eta_H} \right) + \eta_c \frac{1 - \eta_H}{\eta_H} \log \left( \frac{1 - \eta_c}{\eta_c} \right)
+ \log (1 - \eta_c) + \frac{1}{2} \left( \eta_H + \frac{1}{\psi} \right) \tau_s - 1.
\]

By market clearing, \( \log H = \xi + k \log P_H \), it follows that
\[
\log P_H = \frac{\eta_H}{1 + k \eta_H} \frac{1 + \psi}{\psi} A - \frac{\eta_H}{1 + k \eta_H} \xi + \eta_H \frac{1}{1 + k \eta_H} \log \left( \frac{1 - \eta_H}{\eta_H} \right) + \eta_c \frac{1 - \eta_H}{1 + k \eta_H} \log \left( \frac{1 - \eta_c}{\eta_c} \right)
+ \frac{\eta_H}{1 + k \eta_H} \log (1 - \eta_c) + \frac{1}{2} \frac{\eta_H}{1 + k \eta_H} \left( \eta_H + \frac{1}{\psi} \right) \tau_s - 1.
\]

This characterizes the economy in the limit as information frictions dissipate.
To see that the economy with information frictions (finite $\tau_s$) converges to this perfect-information limit, we consider a sequence of $\tau_s$ that converges to $\infty$. From equation (28), it follows that, as $\tau_s \to \infty$, $b \to \frac{1 + \psi}{\psi}$. Since $h_s = b$, it follows that

$$h_s \to \frac{1 + \psi}{\psi}.$$  

Taking the limit $\tau_s \to \infty$ in equation (30), recognizing that $h_s = b$ remains finite in the limit, we see that

$$p_A \to \frac{\eta_H}{1 + \eta_H k} \frac{1 + \psi}{\psi}.$$  

Since $p_x = -\frac{1}{b} p_A$, it follows that

$$p_x \to -\frac{\eta_H}{1 + k \eta_H}.$$  

In addition, from equation (29), we find that as $\tau_s \to \infty$,

$$h_P \to -\frac{1}{\eta_H}.$$  

Finally, from equations (33) and (22), it follows that

$$p_0 \to \frac{\eta_H}{1 + \eta_H k} \log \left( (1 - \eta_c) \frac{1 - \eta_H}{\eta_H} \left( \frac{1 - \eta_c}{\eta_c} \right)^{\eta_c^{-1} - \eta_H^{-1}} + \frac{1}{2} \right),$$

$$h_0 \to \log \left( (1 - \eta_c) \frac{1 - \eta_H}{\eta_H} \left( \frac{1 - \eta_c}{\eta_c} \right)^{\eta_c^{-1} - \eta_H^{-1}} + \frac{1}{2} \right).$$

Thus, we see that the economy with information frictions converges to the perfect-information benchmark as $\tau_s \to \infty$.

Furthermore, the variance of the log housing price is given by

$$Var [\log P_H] = \left( \frac{\eta_H}{1 + \eta_H k} \frac{1 + \psi}{\psi} \right)^2 \tau_A^{-1} + \left( \frac{k \eta_H}{1 + k \eta_H} \right)^2,$$

from which follows that

$$\frac{\partial Var [\log P_H]}{\partial k} = 2 \left( \frac{\eta_H}{1 + k \eta_H} \right)^2 \frac{1}{1 + k \eta_H} \left( k - \frac{1 + \psi}{\psi} \right)^2 \frac{1}{\tau_A^{-1} \eta_H}.$$  

As $k \to 0$, $\frac{\partial Var [\log P_H]}{\partial k} < 0$. For $k > \left( \frac{1 + \psi}{\psi} \right)^2 \tau_A^{-1} \eta_H$, $\frac{\partial Var [\log P_H]}{\partial k} > 0$. Thus it follows that the log housing price is U-shaped in $k$.  

32
A.4 Proof of Proposition 4

From equation (32), it is clear that

\[ p_A = \frac{\eta_H}{1 + \eta_H k} \left( 1 + \psi \left( \frac{1 - \eta_c}{\eta_H} \right) \cdots \right) - \frac{(1 - \eta_c) \eta_H + \eta_c \tau - 1 \tau_A b}{1 + \eta_H k} < \frac{\eta_H}{1 + \eta_H k} \left( 1 + \psi \left( \frac{1 - \eta_c}{\eta_H} \right) \cdots \right). \]

Thus, it follows that \( p_A \) is always lower than its corresponding value in the perfect-information benchmark.

Similarly, since \( p_\xi = -\frac{1}{\tau} p_A \), it follows from equation (30) that we can express \( p_\xi \) as

\[ p_\xi = -\frac{\eta_H}{1 + k \eta_H} - \frac{\eta_H}{1 + k \eta_H} \left( 1 + \frac{1}{\psi + \eta_c \frac{1 - \eta_H b}{\eta_H}} \right) \frac{b \frac{1}{(\phi k)^2}}{\tau_A + \tau_s + b^2} < -\frac{\eta_H}{1 + k \eta_H}, \]

which is the corresponding value of \( p_\xi \) in the perfect-information benchmark.

A.5 Proof of Proposition 5

Note that \( b \) is determined by the polynomial equation (28). We define the LHS of the equation as \( G(b) \). By using the Implicit Function Theorem and invoking equation (28), we have

\[ \frac{\partial b}{\partial \eta_c} = -\frac{\partial G/\partial \eta_c}{\partial G/\partial b} = -\frac{\frac{1 + \psi - b}{\psi} (1 - \eta_H) \eta_H \tau_s}{3 \frac{1}{(\phi k)^2} b^2 + \tau_A + \frac{\eta_H}{(1 - \eta_c) \eta_H + \eta_c} \tau_s \left( (1 - \eta_c) \eta_H + \eta_c \right)^2} \]

\[ = \frac{\frac{1 + \psi - b}{\psi} (1 - \eta_H) \eta_H \tau_s}{2 \frac{1}{(\phi k)^2} b^2 + \frac{1 + \psi}{(1 - \eta_c) \eta_H + \eta_c} \tau_s \left( (1 - \eta_c) \eta_H + \eta_c \right)^2}. \]

Since, from Proposition 2, \( 0 \leq b \leq \frac{(1 - \eta_c) \eta_H + \eta_c}{(1 - \eta_c) \eta_H + \eta_c} \tau_s + \eta_H \tau_s \leq \frac{1 + \psi}{\psi} \), it follows that

\[ \frac{1 + \psi}{\psi} - b \geq 0 \]

Thus \( \frac{\partial b}{\partial \eta_c} < 0 \). Similarly,

\[ \frac{\partial b}{\partial \tau_s} = -\frac{\partial G/\partial \tau_s}{\partial G/\partial b} = -\frac{b - \frac{1 + \psi}{\psi} \eta_H b}{3 \frac{1}{(\phi k)^2} b^2 + \tau_A + \frac{\eta_H}{(1 - \eta_c) \eta_H + \eta_c} \tau_s \left( (1 - \eta_c) \eta_H + \eta_c \right)^2} \]

\[ = \frac{\frac{1 + \psi - b}{\psi} \eta_H b}{2 \frac{1}{(\phi k)^2} b^2 + \frac{1 + \psi}{(1 - \eta_c) \eta_H + \eta_c} \tau_s \left( (1 - \eta_c) \eta_H + \eta_c \right)^2} > 0. \]

From the expression for \( p_A \) in Proposition 2,

\[ \frac{\partial p_A}{\partial \eta_c} = -\frac{1 - \eta_H \tau_s^{-1} \tau_A b}{1 + \eta_H k} \frac{1}{1 + \eta_H k} \cdots - \frac{(1 - \eta_c) \eta_H + \eta_c \tau_A \tau_s^{-1} \tau_A b}{1 + \eta_H k} \frac{\partial b}{\partial \eta_c}. \]
Then, it follows that
\[
\frac{\partial p_A}{\partial \eta_c} = -\frac{2 (\phi k)^2 b^2 + \eta_H \tau_s}{2 (\phi k)^2 b^3 + 1 + \frac{(1 + \psi) \eta_H}{(1 - \eta_c) \eta_H + \eta_c} \tau_s} \left( 1 - \eta_H k \tau_s^{-1} \tau_A b^2 \right) < 0.
\]

Similarly, with respect to \( \tau_s \), we have
\[
\frac{\partial p_A}{\partial \tau_s} = \left( 1 - \eta_c \right) \eta_H + \eta_c \frac{b}{\tau_s^2 \tau_A b} \left( 1 - \frac{1}{b} \tau_s \frac{\partial b}{\partial \tau_s} \right) = \left( 1 - \eta_c \right) \eta_H + \eta_c \frac{b}{\tau_s^2 \tau_A b} \left( 2 (\phi k)^2 b^2 + \frac{1}{\psi} \frac{\eta_H}{(1 - \eta_c) \eta_H + \eta_c} \tau_s \right) > 0.
\]

### A.6 Proof of Proposition 6

We first consider the limiting case for the economy as \( k \to \infty \). Rewrite equation (28) as
\[
\left( \frac{b}{\phi k} \right)^3 + \left( \tau_A + \frac{\eta_H}{1 - \eta_c} \frac{\eta_H + \eta_c}{\eta_H + \eta_c} \tau_s \right) b - \frac{1}{\phi k} \left( 1 + \psi \right) \eta_H \frac{1}{\phi k} \tau_s = 0. \tag{34}
\]

Then it is apparent from equation (34) that, as \( k \to \infty \), that either \( \frac{b}{\phi k} = 0 \) or \( \frac{b}{\phi k} = \pm \sqrt{\tau_A + \frac{\eta_H}{1 - \eta_c} \frac{\eta_H + \eta_c}{\eta_H + \eta_c} \tau_s} \). Thus, as \( k \to \infty \), one has that \( \frac{b}{\phi k} \to 0 \), and therefore \( \frac{b}{\phi k} \to 0 \). Consequently, from equation (32), \( p_A \to 0 \) and the housing price is completely driven by the supply shock \( \xi \). From equation (29), then, since \( \frac{b}{\phi k} \to 0 \) and \( b \) is bounded from above by \( \frac{1 + \psi}{\psi} \), one has that

\[
h_P \to -\frac{1}{\eta_H}.
\]

In addition, from Proposition 2, one has that
\[
p_\xi k = -\eta_H k \left( 1 + \frac{1 + \psi}{\psi} \frac{(1 - \eta_c) \eta_H + \eta_c}{\eta_H} \tau_s^{-1} \tau_A b - \frac{(1 - \eta_c) \eta_H + \eta_c}{\eta_H} \tau_s^{-1} \frac{1}{(\phi k)^2} b^2 \right) \to -1,
\]

since \( b \) is bounded from above by \( \frac{1 + \psi}{\psi} \). Thus, \( \log P_H = -\zeta \).

From equation (20), it is straightforward to see that, as \( k \to \infty \),
\[
h_s = b \to \frac{1 + \psi}{\psi} \frac{\eta_H \tau_s}{((1 - \eta_c) \eta_H + \eta_c) \tau_A + \eta_H \tau_s}.
\]

Since \( h_s \) remains bounded in the limit, it is easy to see from equation (33) that \( p_0 \to 0 \) as
\( k \to \infty \). It further follows from equation (22) that in the limit

\[
h_0 \to \log \left( \left(1 - \eta_c\right) \frac{1 - \eta_H}{\eta_H} \left(1 - \eta_c\right) \left(1 - \eta_H\right) \right)  \\
+ \frac{1}{2} \left( \frac{1 + \psi}{\psi} \eta_H \tau_s \right) \eta_c \left(1 - \eta_H\right) \eta_H \tau_s  \\
+ \frac{1 + \psi}{\psi} \left( (1 - \eta_c) \eta_H + \eta_c \right) \tau_A + \eta_H \tau_s \bar{A}  \\
+ \frac{1}{2} \left( \frac{\eta_H + \frac{1 + \psi}{\psi} \eta_c \left(1 - \eta_H\right) \eta_H \tau_s}{\tau_A + \tau_s} \right)  \\
+ \frac{1}{2} \left( \eta_H + \frac{1 + \psi}{\psi} \eta_c \left(1 - \eta_H\right) \eta_H \tau_s \right) \tau_A + \eta_H \tau_s \bar{A} \\
\left(35\right)
\]

In the case \( k \to 0 \), it follows from equation (34) that \( b \to 0 \) and \( \frac{b}{k} \to \infty \). From equation (20), it follows that as \( k \to 0 \) one has that \( h_s = b \to 0 \). Furthermore, from equation (32), one has that

\[
p_A \to \frac{1 + \psi}{\psi} \eta_H.
\]

Since \( h_s \to 0 \), and \( p_A \) remain bounded as \( k \to 0 \), we also see from equation (24) that \( h_P \to 0 \). Thus as \( k \to 0 \), \( \frac{b}{k} \to \infty \), and therefore \( p_k k \to 0 \) and the demand shock \( A \) completely drives the housing price.

Since \( h_s \) remains bounded in the limit, it is easy to see from equation (33) that as \( k \to 0 \),

\[
p_0 \to \frac{1}{2} \eta_H \left( \eta_H + \frac{1}{\psi} \right) \tau_s^{-1} + \eta_H \log \left( \left(1 - \eta_c\right) \frac{1 - \eta_H}{\eta_H} \left(1 - \eta_H\right) \eta_H \right).
\]

It further follows from equation (23) that in the limit \( h_0 \to 0 \).

\section{A.7 Proof of Proposition 7}

We first prove that \( p_A \) is decreasing with \( \phi \) and \( p_k < 0 \) is increasing with \( \phi \). Note that \( b \) is determined by the polynomial equation (28). We define the LHS of the equation as \( G (b) \). Comparative statics of \( b \) with respect to \( \phi \) reveal, by the Implicit Function Theorem and invoking equation (28), that

\[
\frac{\partial b}{\partial \phi} = -\frac{\partial G}{\partial \phi} = \frac{2}{(\phi k)^{3}} b^3 + \frac{1}{(\phi k)^{4}} b^4  \\
= \frac{2}{(\phi k)^{3}} b^3 + \frac{1}{(\phi k)^{4}} b^4 > 0.
\]

35
From the expression for $p_A$ in Proposition 2,

$$\frac{\partial p_A}{\partial \phi} = -\frac{(1 - \eta_c) \eta_H + \eta_c}{1 + \eta_H k} \tau_s^{-1} \tau_A b \frac{\partial b}{\partial \phi} < 0.$$  

Furthermore, by the Implicit Function Theorem, it follows that

$$\frac{\partial p_A}{\partial \phi} = -\frac{2(1 - \eta_c) \eta_H + \eta_c}{1 + k \eta_H} \tau_s^{-1} \left( b \frac{\phi b}{\phi k} \left( \frac{\phi b}{\phi k} - 1 \right) \right) \left( \frac{1 + \psi}{\psi} (1 - \eta_c) \eta_H + \eta_c \right) \tau_s^2 \frac{1 + \psi}{1 + \psi} \eta_H + \eta_c \tau_s.$$  

Since $\phi \in [0, 1]$, it follows that $\frac{\partial p_A}{\partial \phi} > 0$.

The variance of the housing price $Var[\log P_H]$ is given by

$$Var[\log P_H] = p_A^2 \tau_s^{-1} + p_X^2 k^2,$$

from which follows that

$$\frac{\partial Var[\log P_H]}{\partial \phi} = 2p_A \tau_s^{-1} \frac{\partial p_A}{\partial \phi} + 2p_X k^2 \frac{\partial p_X}{\partial \phi} < 0,$$

since $p_A \frac{\partial p_A}{\partial \phi} < 0$ and $p_X \frac{\partial p_X}{\partial \phi} < 0$.

From Proposition 3, the variance of the housing price in the perfect-information benchmark is

$$Var[\log P_{H_{perf}}] = \left( \frac{\eta_H}{1 + k \eta_H} \right)^2 \left( \frac{1 + \psi}{\psi} \right)^2 \tau_s^{-1} + k^2.$$

It then follows, substituting for $p_A$ and $p_X$ with Proposition 2, that

$$Var[\log P_H] - Var[\log P_{H_{perf}}] = \left( p_A^2 - \left( \frac{\eta_H}{1 + k \eta_H} \right)^2 \left( \frac{1 + \psi}{\psi} \right)^2 \right) \tau_s^{-1} + \left( p_X^2 - \left( \frac{\eta_H}{1 + k \eta_H} \right)^2 \right) k^2 \tau_s^{-1} \tau_A b \left( \frac{\phi b}{\phi k} \right)^2 \left( \frac{2 \eta_H}{(1 - \eta_c) \eta_H + \eta_c} \right) + \left( \frac{1 - \eta_c}{1 + k \eta_H} \right)^2 \tau_s^{-1} \left( b \frac{\phi b}{\phi k} \right)^2 \left( \frac{2 \eta_H}{(1 - \eta_c) \eta_H + \eta_c} \right) \tau_s^{-1} \left( b \frac{\phi b}{\phi k} \right)^2,$$

from which follows, substituting with equation (28), that $Var[\log P_H] - Var[\log P_{H_{perf}}] \geq 0$ whenever

$$b \geq \left( \frac{\phi^2 - 1}{\tau_A} + \frac{\eta_H}{(1 - \eta_c) \eta_H + \eta_c} \tau_s \right)^{-1} \left( 2 \phi^2 - 1 \right) \frac{1 + \psi}{\psi} \frac{\eta_H}{(1 - \eta_c) \eta_H + \eta_c} \tau_s. \quad (37)$$
Since $b \geq 0$, it is thus sufficient for $1 - \frac{\eta_H}{(1-\eta_c)\eta_H + \eta_c} \frac{\tau_H}{\tau_A} \leq \phi^2 \leq \frac{1}{2}$ for the condition in (37) to be satisfied.

When $\phi = 1$, then the condition in (37) becomes $b \geq \frac{1+\psi}{\psi}$. Since

$$0 \leq b \leq \frac{1+\psi}{\psi} \left(\frac{\eta_H \tau_s}{(1-\eta_c) \eta_H + \eta_c} \frac{\tau_A}{\eta_H \tau_s} + \frac{1+\psi}{\psi}\right)$$

from Proposition 2, this condition can be satisfied only when $b = \frac{1+\psi}{\psi}$, which is the value of $b$ in the perfect-information benchmark, in which case $\text{Var} [\log P_H] = \text{Var} \left[\log P_H^{\text{perf}}\right]$. Thus, when $\phi = 1$, variance with informational frictions is always less than that of the perfect-information benchmark.

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Ioannides, Yannis and Jeffrey E. Zabel (2003), Neighbourhood Effects and Housing Demand, *Journal of Applied Econometrics* 18, 563-584.

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Figure 1: Case-Shiller Home Price Index

This figure plots the Case-Shiller home price index for the U.S. and three cities, New York, Las Vegas and Charlotte. The price index is deflated by the CPI and normalized to 100 in 2000.
Figure 2: Housing Cycle across Three Elasticity Groups with an Equal Number of Counties

This figure is constructed from sorting the counties in the U.S. into three groups based on the Saiz’s housing supply elasticity measure, with each group holding an equal number of counties. The top panel depicts the average housing price expansion during the boom years of 2003 to 2006 and the average housing price contraction during the bust years of 2006 to 2009 in each of the groups. The bottom panel depicts the population in each group in the 2000 U.S. census.
Figure 3: Housing Cycle across Ten Elasticity Groups with an Equal Number of Counties

This figure is constructed from sorting the counties in the U.S. into ten groups based on the Saiz’s housing supply elasticity measure, with each group holding an equal number of counties. The top panel depicts the average housing price expansion during the boom years of 2003 to 2006 and the average housing price contraction during the bust years of 2006 to 2009 in each of the groups. The bottom panel depicts the population in each group in the 2000 U.S. census.
This figure is constructed from sorting the counties in the U.S. into three groups based on the Saiz’s housing supply elasticity measure, with each group holding an equal population. The top panel depicts the average housing price expansion during the boom years of 2003 to 2006 and the average housing price contraction during the bust years of 2006 to 2009 in each of the groups. The bottom panel depicts the population in each group in the 2000 U.S. census.
Figure 5: Housing Cycle across Ten Elasticity Groups with an Equal Population

This figure is constructed from sorting the counties in the U.S. into ten groups based on the Saiz’s housing supply elasticity measure, with each group holding an equal population. The top panel depicts the average housing price expansion during the boom years of 2003 to 2006 and the average housing price contraction during the bust years of 2006 to 2009 in each of the groups. The bottom panel depicts the population in each group in the 2000 U.S. census.
Figure 6: Housing Price Variance in the Baseline Model

This figure depicts the log-price variance in the baseline model against the supply elasticity, based on the following parameters: \( \tau_s = 1, \tau_A = \phi = 0.25, \eta_c = 0.5, \psi = \eta_H = 0.75 \). The solid line depicts the log-price variance in the presence of informational frictions, while the dashed line depicts the log-price variance in the perfect-information benchmark.
Table 7: Variance of Housing Prices and Ratio of Secondary to Primary Homes in the Extended Model

This figure depicts the log-price variance of both primary and secondary homes in the extended model in the top panel and the variance of the log ratio of secondary to primary home demands in the bottom panel, based on the following parameters: $\tau_s = 1, \tau_A = \phi = 1, \eta_c = 0.5, \psi = \eta_H = 0.75, \alpha = 1$. 
This figure plots the share of non-owner-occupied home purchases for the U.S. and three cities, New York, Las Vegas and Charlotte.
We use the Saiz’s (2010) measure of supply elasticity measure to sort the counties in our sample into three groups in the top panel and ten groups in the bottom panel, with each group holding the same number of counties. Each bar measures the average share of non-owner-occupied home purchases in 2005 in each group. The share of non-owner-occupied home purchases in each county is computed from the “Home Mortgage Disclosure Act” data set.
Figure 10: The Share of Non-Owner-Occupied Home Purchases in 2005 across Elasticity Groups with an Equal Population

We use the Saiz’s (2010) measure of supply elasticity measure to sort the counties in our sample into three groups in the top panel and ten groups in the bottom panel, with each group holding the same population. Each bar measures the average share of non-owner-occupied home purchases in 2005 in each group. The share of non-owner-occupied home purchases in each county is computed from the “Home Mortgage Disclosure Act” data set.
Table 1: Housing Boom and Bust during the Recent Cycle

This table presents coefficient estimates from regressing the change in real house price from 2003 to 2006 (housing boom period) and from 2006 to 2009 (housing bust period) on the dummies indicating whether a county is in the middle-elasticity group or the elastic group, with the inelastic group as the benchmark and a list of control variables. Robust standard errors are in parentheses. ***, **, * indicate coefficient estimates statistically distinct from 0 at the 1%, 5%, and 10% levels, respectively.

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Table 2: Fraction of Non-Owner-Occupied Home Purchases in 2005

This table presents coefficient estimates from regressing the fraction of non-owner occupied home purchases in 2005 on the dummies indicating whether a county is in the middle-elasticity group or the elastic group, with the inelastic group as the benchmark and a list of control variables. Robust standard errors are in parentheses. ***, **, * indicate coefficient estimates statistically distinct from 0 at the 1%, 5%, and 10% levels, respectively.

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