

Leisure Preferences, Long-Run Risks, and Human Capital Returns*

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Abstract

We analyze the contribution of leisure preferences to a model of long-run risks in leisure and consumption growth. The marginal utility of consumption is affected by short- and long-run risks in leisure under nonseparable and recursive preferences, respectively. Our model matches equity risk premia and macroeconomic moments with plausible coefficients of relative risk aversion. Further, the incorporation of leisure in utility allows us to examine the optimal tradeoff between labor and leisure and derive model implications for the price of and return on human capital. Human capital exhibits returns that are significantly less volatile than and positively correlated with stock returns, implies expected returns that are between 45% and 60% of the equity premium, and has a Sharpe ratio that is 30% higher than that of the equity return.

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1 Introduction

A long-standing practice in the analysis of consumption, portfolio choice, and asset pricing in the endowment economy of Lucas (1978) is the measurement of the representative agent's utility over consumption of nondurable goods and services. This practice, popularized in Hansen and Singleton (1982) and Mehra and Prescott (1985) is justified on the basis of the assumption that intratemporal preferences are separable over consumption of the basket of nondurables and services and other sources of utility. This assumption can be justified in the standard framework of power utility, implying that asset prices are affected only by consumption of nondurable goods and services and not directly by other potential sources of utility. However, as noted in Uhlig (2010), this assumption is no longer valid under recursive preferences, such as those analyzed in Epstein and Zin (1989). With recursive preferences, the marginal utility of consumption depends not only on current consumption, but also on continuation utility. If agents derive utility from quantities other than consumption of nondurables and services, the marginal utility of consumption, and thus asset prices, will depend on these quantities through the continuation utility.¹

The issue of sources of marginal utility of consumption is particularly germane in the context of recent advances in asset pricing that rely on recursive preferences to generate implications for aggregate asset risk premia. In particular, Bansal and Yaron (2004) derive a model with persistent means of consumption growth and volatility that generates asset market phenomena consistent with the observed data under the assumption of recursive preferences. Persistence in these moments is also generated endogenously in general equilibrium economies with recursive preferences by Kaltenbrunner and Lochstoer (2010) and Croce (2012). These frameworks rely on measurement of marginal utility of consumption with respect only to consumption of nondurable goods and services. An open question is the degree to which preferences over quantities other than nondurable goods and services affect equilibrium asset prices. In this paper, we address this question through the analysis of the impact of preferences over the consumption of leisure on equilibrium in asset markets.

We concentrate on the impact of leisure in marginal utility for a number of different reasons. In endowment economy models, asset prices are traditionally determined by agents' allocation of wealth to consumption and investment. Allocating more wealth to investment results in a higher flow of future dividends available for consumption. Agents can also consume income derived through the provision of labor, but there is no explicit tradeoff between provision of work hours and utility. Consequently, agents will optimally provide all available work hours to maximize consumption, and

¹Implications of preferences over consumption outside of the standard bundle of nondurables and services have been explored previously in the literature. Eichenbaum, Hansen and Singleton (1988) examine implications of preferences over leisure in the context of a non-separable utility function. Yogo (2006) derives a model with non-separable preferences over durable goods and examines implications for the equity premium puzzle. Yang (2011) considers the contribution of preference over durable goods to the long run risk model.

the labor-leisure tradeoff will not affect marginal utility, nor, as a result, asset prices. Empirically, however, we observe considerable variation in the provision of labor hours, which is frequently modeled in general equilibrium by introducing leisure preferences, resulting in elastic labor supply. The implication in our context is that agents assess the tradeoff between provision of labor resulting in income flow for consumption, and the consumption of leisure. We analyze the importance of this tradeoff in determining equilibrium asset prices.

An additional benefit of considering preferences over consumption and leisure is in analyzing the return on human capital. The importance of human capital in asset pricing has generated significant attention in the recent literature, including Jagannathan and Wang (1996), Lettau and Ludvigson (2001), Lustig and Nieuwerburgh (2006), and Bansal et al. (2013). In these papers, labor income is viewed as a dividend to human wealth, but the portfolio choice decision in the allocation of the endowment of hours is not explicitly modeled. As a result, an equilibrium price of human capital is not endogenously determined, and the interaction between financial wealth, human wealth, and consumption of resources cannot be fully analyzed. By introducing utility over leisure into the model, we are able to provide an analysis of the risk and price of human capital and its resulting impact on equilibrium financial asset pricing. This analysis also contributes to a growing literature examining the impact of labor and asset pricing, including Favilukis and Lin (2013), Li and Palomino (2013), and Kuehn, Petrosky-Nadeau and Zhang (2013).

Last, introduction of preference for leisure generates implications for equilibrium dividends from firms in the economy. Endowment economy asset pricing models generally specify dividends and consumption as different exogenous processes, with dividend growth dynamics that generate more volatility than consumption growth. By introducing the resource constraints that state that consumption is funded by dividends and labor income with limits to the amount of labor that can be provided, we are able to derive an endogenous dividend growth process. We do not explicitly use this process as it links *total* dividends to consumption and labor income, rather than the dividends per share of equity ownership typically investigated in the literature. However, the endogenous process can be utilized to better understand the relation between consumption, dividends, and labor income, and the resulting relation between these quantities and asset prices.

We examine financial asset and human capital pricing through the lens of a long-run risk model with non-separable preferences between leisure and consumption. This framework allows us to analyze different degrees of substitutability of leisure and consumption, and resulting implications for macroeconomic and financial asset quantities. We calibrate the model to key moments of the data, guided by an empirical analysis of the joint dynamics of consumption, leisure and wages. In order to compare the impact of including leisure in preferences, we compare our calibrated model to a baseline calibration in Bansal, Kiku and Yaron (2007) in which agents derive utility only from consumption of nondurable goods and services. Additionally, we use the model calibrated to the

moments of macroeconomic and financial market data to generate new implications for the riskiness of investment in human capital and its resulting excess return.

Our empirical analysis indicates that consumption, leisure, and wages share a persistent common component with explanatory power for dividend dynamics. This source of long-run risk has opposite effects on consumption and leisure growth, driving the negative correlation in these two variables observed in the data. Additionally, the dynamics of these series display common time variation in volatility, supporting the modeling of asset prices with exposures to persistent risk in aggregate economic uncertainty.

In calibration, we find that the model incorporating preference over leisure performs about as well as the nondurable goods and services consumption-only model in matching the aggregate moments of asset returns and macroeconomic quantities, with some marginal improvements and additional insights. Like the calibrations in Bansal and Yaron (2004) and Bansal, Kiku and Yaron (2007), our model is able to match the equity risk premium with a reasonable degree of risk aversion. This coefficient of risk aversion is lower when computed relative to gambles over non-durable goods and services than when computed relative to aggregate wealth. The difference in these results is attributable to the fact that with leisure preferences, claims to the consumption bundle reflect only a fraction of total wealth. Incorporating the human capital risks implied in the labor-leisure tradeoff results in a computation of a higher degree of risk aversion.

We also find that the price-dividend ratio lacks predictive power for leisure, labor income and wage growth, in addition to consumption growth. However, the price-dividend ratio has predictive power for the volatility of these series. These results corroborate the calibration of Bansal, Kiku and Yaron (2007) in emphasizing the conditional volatility, rather than conditional mean as a source of long-run risk. Finally, we find that incorporating leisure preferences reduces the negative slope of the term structure of real interest rates relative to the consumption-only model. This alleviates, but does not eliminate, the criticism of Beeler and Campbell (2012) of negative long-term real yields implied by the long-run risk framework.

In addition to these comparisons with the existing long-run risk calibrations, we document novel implications for the price of human capital risk and the relation between the excess return on human capital and equity. We find that human capital claims to both labor income and wages are much less volatile than those of equities, resulting in a risk premium that is 45-60% of the risk premium on equity. However, while the risk premium is reduced, the reduction in volatility is even greater, such that the Sharpe ratio associated with human capital claims is approximately 30% larger than that associated with stock market investment. Further, we find that excess returns to human capital claims are positively correlated with excess returns on equities, consistent with the evidence in Bansal et al. (2013) and contrary to that in Lustig and Nieuwerburgh (2006).

The remainder of this paper is organized as follows. In Section 2, we discuss the construction and sample moments of the data to which we calibrate the model parameters. Additionally, we investigate the joint dynamics of consumption, leisure, and wage growth, and these variables' relation to aggregate dividend growth, in order to understand sources of risk and provide parameter estimates for model calibration. In Section 3, we present model solutions for prices of risk and financial asset prices. Calibration of the model and analysis relative to existing long run risk frameworks is presented in Section 4, with implications for the returns to human capital. Concluding remarks are provided in Section 5.

2 Empirical Analysis

We undertake an empirical analysis of the three principal variables in our economic framework: consumption, leisure, and wage growth. The purpose of this analysis is both to characterize the dynamics of these variables and to guide the implementation of our modeling and calibration strategy. Although the dynamic properties of consumption growth have been well documented, the properties of leisure and wage growth, in relation to asset pricing, have been less thoroughly explored.² We address the following questions: (i) what are the properties of the joint dynamics of consumption, leisure, and wage growth? (ii) is there evidence of persistence in the conditional mean and volatility of these series? (iii) if conditional moments are persistent, how many sources of conditional moment risk are present in the series? (iv) are the series sensitive to information about conditional moments in other series? and (v) what is the sensitivity of asset pricing quantities (dividends) to these moments?

2.1 Data Description and Construction

We use annual observations for consumption, leisure, labor income, and dividends from 1929-2011. Consumption is measured as per capita real consumption of nondurables and services, as in Bansal and Yaron (2004). Labor income is calculated as in Lettau and Ludvigson (2005) as per capita real after tax labor income. Specifically, pretax labor income is calculated as wages and salaries, plus personal current transfer receipts, plus employer contributions for employee pension and insurance funds, less the difference in domestic contributions for government social insurance and employer contributions for government social insurance. Taxes are calculated as wage and salary income times personal current taxes, divided by the sum of wage and salary income, proprietors income, rental income, and income receipts on assets. Data are sampled at the annual frequency from 1929 through 2011 and converted to real using the Personal Consumption Expenditure (PCE) deflator.

²For instance, Bansal, Khatchatrian and Yaron (2005) report evidence in time-varying uncertainty and conditional expected growth in consumption.

These data are obtained from the National Income and Product Account (NIPA) tables at the Bureau of Economic Analysis (BEA).

The leisure series is the series used in Ramey and Francis (2009*b*) from the Bureau of Labor Statistics (BLS), and obtained from Valerie Ramey's website.³ The series is constructed as the ratio of leisure hours to the total number of hours available for work and leisure activities. We assume that the total number of hours is $16 \times 7 = 112$ hours per week.⁴ Wages are inferred using the labor income series described above and hours worked. Specifically, wages are calculated by dividing the real per capita labor income series by number of hours worked to produce a measure of real per capita annual wages.

Asset market data are obtained from CRSP. Dividends per share are computed using the CRSP value-weighted index. We first compute the dividend yield as the difference in the monthly cum-dividend return on the index and the ex-dividend return on the index. The dividend per share is then calculated by multiplying the dividend yield by the lagged value of the cumulative capital gain on the index. Monthly data are summed to the annual frequency and converted to real using the PCE deflator. We use this per-share dividend series and the cumulative capital gain on the index to compute the price-dividend ratio. The real risk-free rate is computed using a simplified version of the procedure in Pflueger and Viceira (2011) and Beeler and Campbell (2012). This rate is obtained by subtracting an estimate of expected inflation from the nominal risk-free rate (one-month T-Bill rate). Expected inflation is measured by regressing future inflation on the current nominal rate and the current and lagged values of monthly inflation for one year.

Summary statistics for these four variables are presented in Table 1. Moments of consumption and dividend growth are familiar to readers of this literature; the mean of consumption growth is approximately 2% per annum, has low volatility of 2.25%, and is positively autocorrelated at the annual frequency, with an autocorrelation coefficient of 0.47. Dividend growth has a somewhat lower mean at 1.38% per annum, but is substantially more volatile at 10.82% per annum. Dividends are also less autocorrelated, with first- and second-order autocorrelations of 0.21 and -0.22, respectively. Moments of leisure and wage growth are perhaps less familiar. Leisure grows slowly, with an annual growth rate of 0.27%, and is less volatile than consumption growth, with a standard deviation of 1.08%. Wages have grown faster and are more volatile than consumption growth, with a mean of 2.70% and standard deviation of 3.46%. Neither series exhibits pronounced autocorrelation; leisure has somewhat higher first-order autocorrelation of 0.28, compared to 0.18 for growth in wages.

³We thank Valerie Ramey for making the data available at her website, <http://www.econ.ucsd.edu/~vramey/research.html>.

⁴In an earlier version of this paper, we utilized a leisure series from Ramey and Francis (2009*a*). These data differ from the standard measures of labor and leisure by accounting for hours spent in household production and education. The resulting leisure series exhibits less of an upward trend in the post-war data than alternative measures such as the measure used in this paper. We utilize the more standard series since our model does not incorporate household production and the data are available only through 2005.

2.2 Conditional Means of Consumption, Leisure, and Wage Growth

We specify a trivariate vector autoregression (VAR) for (log) consumption (Δc_t), leisure (Δl_t), and wage (Δw_t) growth,

$$\mathbf{y}_t = \mathbf{P}\mathbf{y}_{t-1} + \mathbf{u}_t, \quad (1)$$

where $\mathbf{y}_t = \{\Delta c_t - \overline{\Delta c_t}, \Delta l_t - \overline{\Delta l_t}, \Delta w_t - \overline{\Delta w_t}\}$. Innovations to this system, \mathbf{u}_t , are potentially affected by time-varying volatility, which we analyze later in this section. Under these dynamics, the conditional means of the growth rate in the three variables are given by

$$x_{c,t-1} = \mathbf{e}'_c \mathbf{P}\mathbf{y}_{t-1}, \quad x_{l,t-1} = \mathbf{e}'_l \mathbf{P}\mathbf{y}_{t-1}, \quad \text{and} \quad x_{w,t-1} = \mathbf{e}'_w \mathbf{P}\mathbf{y}_{t-1},$$

where $\mathbf{e}_c = \{1, 0, 0\}$, $\mathbf{e}_l = \{0, 1, 0\}$, and $\mathbf{e}_w = \{0, 0, 1\}$. Dividends are assumed to be levered claims to consumption, and consequently sensitive to the state variables in this system,

$$\Delta d_t - \overline{\Delta d_t} = \phi_d \mathbf{x}_{t-1} + u_{d,t}. \quad (2)$$

We estimate equations (1) and (2) using the generalized method of moments (GMM), allowing for autocorrelation using the Newey-West correction with a single lag. Point estimates and standard errors for the VAR, equation (1), and dividend sensitivity to the VAR variables, equation (2), are presented in Table 2. The evidence in the table suggests that there is some indication of persistent conditional mean in each of the three variables. VAR coefficients for the sensitivity of each variable's lag on its current realization are estimated at more than two standard errors from zero and the coefficient exceeds 0.25 for each variable. The table also suggests that each of the VAR variables influences the conditional mean of the other variables; leisure growth marginally statistically significantly forecasts consumption growth, wages negatively and statistically forecast future leisure growth, and leisure negatively and statistically forecasts wage growth.

The final row of Table 2 shows loadings of dividend growth on the conditional means of consumption, leisure, and wage growth. The table suggests that dividend growth loads positively on consumption growth and negatively on wage and leisure growth. However, none of the coefficients can be statistically distinguished from zero. We examine the correlation of the conditional means, and find that while the conditional mean of consumption growth has very low correlation with the conditional means of leisure and wage growth (0.07 and -0.24, respectively), the conditional means of leisure and wage growth are almost perfectly negatively correlated (correlation coefficient of -0.98). As a result, there is little independent information in these conditional means, and the results are affected by strong collinearity.

To formally analyze the degree to which there is commonality vs. independence in information about growth rates in the conditional means, we conduct a principal component analysis. Results

of this analysis are presented in Table 3. As shown in the table, there are two sources of variation in the conditional means of consumption, wage, and leisure growth. The first principal component explains 68% of the common variation in the variables, and loads positively on wage growth, and negatively on consumption and leisure growth. The wage and leisure growth loadings nearly offset one another, suggesting some degree of complementarity of these variables in determining the first principal component. Consumption growth loads on the second principal component with a loading of nearly one; leisure growth loads negatively and wage growth has virtually no loading on the component. The evidence suggests that there are two sources of conditional mean risk in the trivariate VAR.

We next examine a VAR of the first two principal components extracted from the conditional means of consumption, leisure, and wage growth. Results are shown in Panel B of Table 3. The first principal component exhibits mild first-order autocorrelation, with a coefficient of 0.23 (SE=0.11), and is not statistically forecast by the second principal component. The second principal component is more persistent, with an autocorrelation coefficient of 0.45 (SE=0.09), and is also forecast by the second principal component. The degree of persistence in this component is close to the persistence in long-run risk calibrated in Bansal, Kiku and Yaron (2012) of 0.97 at the monthly frequency. Since consumption loads with a coefficient of approximately one on this principal component, we think of this as the long-run risk associated with the conditional mean of consumption growth.

With this evidence, we re-estimate the VAR, equation (1), the dividend sensitivity to the conditional means of consumption and leisure growth, equation (2), and an additional moment to capture the sensitivity of leisure growth to the conditional mean in consumption growth,

$$\Delta l_t - \overline{\Delta l}_t = \phi_{lx_c} x_{c,t} + e_{l,t}. \quad (3)$$

Results for the dividend regression with $\phi_{dw} = 0$ and leisure regressions are presented in Table 4. The sensitivities of dividend growth to conditional means in consumption and leisure growth are now positive and statistically significant. The point estimate of $\phi_{dx_c} = 4.34$ (SE=1.82) is similar in magnitude to the parameter calibrated in Bansal and Yaron (2004). The estimates suggest an even larger sensitivity on leisure, with $\phi_{dx_l} = 6.22$ (SE=3.12). Additionally, as shown in the table, leisure loads negatively on the conditional mean of consumption growth, with $\phi_{lx_c} = -0.36$ (SE=0.16).

We conclude from the evidence in this section that, while there appear to be two sources of conditional mean variation in the consumption, leisure, and wage growth series, only one is sufficiently persistent to have the potential to contribute to long-run risk. As noted above, a principal component on which consumption growth loads with a coefficient of approximately 1.0 appears to exhibit a fairly high degree of autocorrelation, commensurate with the autocorrelation assumed in Bansal and Yaron (2004) and Bansal, Kiku and Yaron (2012). The second component, on which both leisure and consumption load, also exhibits a degree of persistence. However, the

estimated autocorrelation appears to be too low to generate meaningful long-run risk under either of the aforementioned calibrations. Dividend growth loads significantly on both sources of long-run risk, indicating sensitivity of dividend growth to components of total consumption, and leisure growth exhibits statistically significant exposure to the conditional mean of consumption growth.

2.3 Conditional Variance of Consumption, Leisure, and Wage Growth

We next focus on the conditional variance of innovations to consumption, leisure, and wage growth.⁵ Using the residuals from the VAR in the previous section, we analyze variance ratios for the absolute value of the residuals,

$$VR_k = \frac{Var\left(\sum_{j=0}^{J-1} |u_{k,t+j}|\right)}{J \cdot Var(|u_{k,t}|)} \quad (4)$$

for $k = \{\Delta c, \Delta l, \Delta w\}$. Under the null that variances of innovations are constant, the variance ratio should be close to one and flat with respect to the horizon. We compute variance ratios for horizons $J = 2, 5$, and 10 years. Results are tabulated in Panel A of Table 5. As shown in the table, there is evidence of time-varying volatility for all three innovations. At the 2-year horizon, the variance ratio is highest for the consumption growth innovation, with a ratio of 1.38, and weakest for the leisure growth innovation, with a ratio of 0.93. However, all three variance ratios increase with the horizon, rising to 1.71, 1.71, and 1.59 for consumption, leisure, and wage growth innovations, respectively.

As an alternative look at time-varying volatility in the innovations, we fit GARCH(1,1) models to the innovations. Results of this estimation are shown in Panel B of Table 5. The table suggests stronger evidence in favor of time-variation in the volatility of leisure and wage growth than in consumption growth. The GARCH coefficient for consumption growth of 0.29 is not statistically significant from zero, although the ARCH coefficient is reasonably large and statistically significant. In contrast, both leisure and wage growth exhibit highly persistent conditional volatility, with GARCH coefficients of 0.78 and 0.83, respectively. Taken together with the evidence from variance ratios, these results suggest that the null of homoskedastic volatility in the residuals of consumption, leisure, and wage growth is likely to be rejected.

The three volatility series are plotted in Figure 1. As shown in the plots, all three series exhibit high volatility associated with the pre-war period, and a gradual reduction throughout the post-war period. Volatility of all three series also tends to increase in correspondence with NBER recessions, depicted as grey bars in the figure. However, the volatility of consumption growth appears to

⁵Stock and Watson (2002) present evidence of changes in the volatility of a set of macroeconomic variables over time, and potential explanations. Justiniano and Primiceri (2008) provide an estimation of an equilibrium model that supports the importance of investment shocks for these changes in volatility.

somewhat lead volatility in the remaining two series. The figure suggests that while all three volatility series are positively correlated, volatility of wage and leisure innovations are particularly highly correlated. This is indeed the case; the correlations of consumption growth innovation volatility are 0.55 and 0.53 with leisure and wage growth innovation volatilities, respectively, while wage and leisure growth innovation volatilities exhibit a correlation coefficient of 0.91. Much like the conditional means, the conditional volatilities suggest two sources of common variation in conditional volatility. In untabulated results, we document two significant principal components of conditional volatility for the series. The first component explains 78% of common variation in the conditional volatilities, with each conditional volatility loading positively on the principal component. The second component explains an additional 21% of common variation; consumption volatility again loads on this component with a large positive loading (0.87), while leisure and wage volatility load with smaller negative signs.

3 Economic Model

The economic environment in which we model consumption, leisure, and portfolio decisions is very similar to that of Bansal and Yaron (2004), but incorporating felicity for leisure into preferences. The framework is an endowment economy with exogenous processes for consumption, leisure, and dividend growth. In this environment, we derive the equilibrium prices of risk, wages, and returns on various claims to the endowment.

3.1 Preferences on Consumption and Leisure

A representative agent maximizes lifetime utility given by Epstein and Zin (1989) preferences:

$$V_t = \left((1 - \beta)A_t^{1-\frac{1}{\psi}} + \beta Q_t^{1-\frac{1}{\psi}} \right)^{\frac{1}{1-\frac{1}{\psi}}}, \quad (5)$$

where β is a subjective time discount factor, and ψ is the elasticity of intertemporal substitution of consumption. Q_t is the certainty equivalent defined as

$$Q_t = \mathbb{E}_t \left[V_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}},$$

where γ captures risk aversion. A_t represents the total consumption bundle, defined over consumption of nondurable goods and services, C_t , and leisure, L_t , as

$$A_t = \left((1 - \alpha)C_t^{1-\frac{1}{\rho}} + \alpha(\zeta_t L_t)^{1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}}, \quad (6)$$

where ζ_t represents a “preference shock” to be defined later in this section. The role of the preference shock is to ensure that utility derived from leisure does not vanish as consumption of non-durable goods and services grows over time. We refer to the total consumption bundle as “total consumption.”

Leisure is measured as the fraction of time $L_t \equiv 1 - N_t$, where N_t is labor supplied by households to the production sector. The parameter ρ captures the elasticity of substitution between consumption of nondurables and services and leisure. To make comparisons with the nondurables and services consumption-only case, we define the fraction of total consumption relative to nondurables and services consumption $Z_t \equiv A_t/C_t$, such that

$$Z_t = \left(1 - \alpha + \alpha \left(\frac{\zeta_t L_t}{C_t} \right)^{1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}}. \quad (7)$$

Notice that the consumption aggregator implies, in general, non-separability in nondurables and services consumption and leisure. Three particular cases are worth noting. The case $\alpha = 0$ corresponds to utility from non-durable and services consumption only, the case $\rho = 1$ corresponds to the Cobb-Douglas aggregator where Z_t reduces to $(\zeta_t L_t/C_t)^\alpha$, and the case $\rho = \psi$ implies separable intertemporal preferences in nondurables and services consumption and leisure.

The representative agent faces the intertemporal budget constraint

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} M_{t,t+s} C_{t+s} \right] \leq \mathbb{E}_t \left[\sum_{s=0}^{\infty} M_{t,t+s} (W_{t+s} N_{t+s} + D_{t+s} + G_{t+s}) \right], \quad (8)$$

where $M_{t,t+s}$ is the pricing kernel that discounts cashflows in units of nondurable and services consumption from $t+s$ to time t , W_t is the wage earned from supplying a unit of labor to productive activities, D_t are the dividends from owning the production sector, and G_t captures other sources of income such as government transfers.

Maximization of utility with respect to the budget constraint yields the intertemporal marginal rate of substitution of consumption

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{Z_{t+1}}{Z_t} \right)^{\frac{1}{\rho} - \frac{1}{\psi}} \left(\frac{V_{t+1}}{Q_t} \right)^{\frac{1}{\psi} - \gamma}, \quad (9)$$

which represents the pricing kernel for the economy. It can also be expressed as

$$M_{t,t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{Z_{t+1}}{Z_t} \right)^{\frac{1}{\rho} - \frac{1}{\psi}} \right]^\theta \left[\frac{1}{R_{a,t+1}} \right]^{1-\theta}, \quad (10)$$

where $\theta = (1-\gamma)/(1-1/\psi)$, and $R_{a,t+1}$ is the return of the wealth portfolio. The wealth portfolio is a claim on all future total consumption, $C_t + W_t L_t$, which includes the opportunity cost of leisure. The price of the wealth portfolio is defined recursively as

$$S_{a,t} = \mathbb{E}_t [M_{t,t+1} (C_t + W_t L_t + S_{a,t+1})]. \quad (11)$$

Notice that the wealth portfolio becomes a claim only on non-durable and services consumption when $\alpha = 0$, as in Bansal and Yaron (2004), since $L_t = 0$.

Preference for leisure has two effects on the pricing kernel. The first effect is on the CRRA component of the pricing kernel, when $\gamma = 1/\psi$. This component is affected by the ratio Z_t as long as $\psi \neq \rho$. This is a result of the non-separability of nondurables and services consumption and leisure in preferences. An increase in the ratio Z_t increases (decreases) the marginal utility of nondurables and services consumption if $\psi > \rho$ ($\psi < \rho$). This additional term can be written in log form as⁶

$$\left(\frac{1}{\rho} - \frac{1}{\psi} \right) \Delta z_t = \left(\frac{1}{\rho} - \frac{1}{\psi} \right) (\Delta a_t - \Delta c_t).$$

If $\psi > \rho$, this component is positive as long as $\Delta a_t > \Delta c_t$. A total consumption growth higher than nondurables and services consumption growth is a state of high marginal utility if the elasticity of substitution between nondurables and services consumption and leisure is low enough (nondurables and services consumption and leisure tend to be complements), but it is a state of low marginal utility if this elasticity is high enough (nondurables and services consumption and leisure tend to be substitutes).

The second effect of leisure preferences on the pricing kernel is the result of the preference for resolution of uncertainty, when $\gamma \neq \frac{1}{\psi}$. In this case, the marginal rate of substitution of consumption also depends on the difference between the value function V_{t+1} and the certainty equivalent Q_t . This difference is captured by the return on the wealth portfolio, $R_{a,t+1}$. In the absence of leisure preferences, $R_{a,t+1} = R_{c,t+1}$. However, more generally, the riskiness of $R_{a,t+1}$ depends not only on nondurables and services consumption but also on the value of leisure $W_t L_t$. To see this, consider an approximation of the pricing kernel similar to that in Piazzesi and Schneider (2007) under the assumption of log-normality and constant volatility. The recursive utility term can be approximated

⁶Throughout the paper, we use lower case to denote the log of a variable and Δ to denote the difference operator.

as

$$\log\left(\frac{V_{t+1}}{Q_t}\right) \approx \text{constant} + \sum_{i=1}^{\infty} \beta^{i-1} (\mathbb{E}_{t+1} - \mathbb{E}_t)[\Delta a_{t+1+i}].$$

That is, the marginal utility of consumption depends on revisions on expectations of all future total consumption growth. Leisure preferences make the pricing kernel depend not only on the nondurables and services consumption growth process but also on the evolution of expectations of the value of leisure over time.

A useful alternative representation of the pricing kernel is

$$M_{t,t+1} = M_{t,t+1}^a \left(\frac{F_{t+1}}{F_t}\right)^{-1},$$

where

$$M_{t,t+1}^a = \left[\beta \left(\frac{A_{t+1}}{A_t}\right)^{-\frac{1}{\psi}} \right]^{\theta} \left[\frac{1}{R_{a,t+1}} \right]^{1-\theta}, \quad \text{and} \quad F_t = \frac{1}{1-\alpha} Z_t^{-\frac{1}{\rho}},$$

are the pricing kernel in units of total consumption, and the price of total consumption in units of non-durable and services consumption, respectively. Dividends and labor income in the economy are paid in terms of units of consumption of nondurable goods and services. Since households care about total consumption, rather than simply consumption of nondurable goods and services, the riskiness of dividend and labor income cash flows is affected by the evolution of the relative price of total consumption, F_t , over time.

It is worth noting that the presence of multiple goods in the consumption aggregator alters the measurement of several quantities of interest relative to the case in which preferences are defined over a single good. These quantities, such as the elasticity of intertemporal substitution and relative risk aversion coefficient, are defined relative to total consumption, rather than simply consumption of nondurables and services. As a result, empirical measurements of these quantities are altered relative to the case in which agents derive utility only through consumption of nondurables and services. Uhlig (2007) and Swanson (2012) examine differences in the elasticity of intertemporal substitution and measures of risk aversion, respectively, in models with leisure. In this model, the elasticity of intertemporal substitution of total consumption is given by ψ , and the coefficient of relative risk aversion relative to wealth is $R^a = \gamma$.⁷ An alternative measure of risk aversion, relative to gambles on non-durables and services consumption only, can be computed as

$$\mathcal{R}^c = \gamma \frac{C_t}{C_t + W_t L_t} < \gamma. \tag{12}$$

⁷In this particular model, the elasticity of intertemporal substitution of total consumption, $-\frac{\partial \log(A_{t+1}/A_t)}{\partial \log M_{t,t+1}^a}$, and the elasticity of substitution of consumption of non-durables and services, $-\frac{\partial \log(C_{t+1}/C_t)}{\partial \log M_{t,t+1}}$, are both equal to ψ .

For comparison purposes, we compute both measures of risk aversion in our calibrations.

3.2 Consumption, Leisure, and Dividend Growth

The stochastic processes for consumption and dividend growth are similar to those used in Bansal and Yaron (2004). We motivate the relation between these variables and leisure from the analysis in Section 2. Specifically, we assume that all three processes are affected by a single source of long-run (conditional mean) risk and one source of time-varying uncertainty. Growth in consumption of nondurables and services, leisure, and dividends are described by the processes

$$\begin{aligned}
 \Delta c_{t+1} &= \mu_c + x_t + \sigma_{c,t} \varepsilon_{c,t+1}, \\
 x_{t+1} &= \phi_x x_t + \sigma_{x,t} \varepsilon_{x,t+1}, \\
 \Delta l_{t+1} &= \phi_l x_t + \sigma_{l,t} \varepsilon_{l,t+1} + \sigma_{lc} \sigma_c \varepsilon_{c,t+1}, \\
 \Delta d_{t+1} &= \mu_c + b_{dc} (\Delta c_{t+1} - \mu_c) + b_{dl} \Delta l_{t+1} + \sigma_{d,t} \varepsilon_{d,t+1}.
 \end{aligned} \tag{13}$$

where x_t is the time-varying component of the conditional mean of nondurables and services consumption growth. We assume that leisure is stationary with unconditional mean $\mathbb{E}[l_t] = \bar{l}$. All innovations $\varepsilon_{k,t}$ are i.i.d. $\sim \mathcal{N}(0, 1)$.

Conditional volatilities in our framework are specified as

$$\sigma_{k,t} = \sigma_k (1 - I_k + I_k \nu_t)^{1/2}, \tag{14}$$

for $k = \{c, l, x, d\}$, where the process ν_t captures time variation in economic uncertainty. We assume that this process follows a conditional autoregressive gamma process with parameters $(\delta_\nu, \phi_\nu, \varsigma_\nu)$.⁸ The indicator I_k is 1 if the process k is affected by time-varying uncertainty, and 0 otherwise. Specifying the process in this manner allows us to quantify the contribution of time-varying volatility in each process to the results. We note also that this volatility process is different than the approximate square root process in Bansal and Yaron (2004) and Bansal, Kiku and Yaron (2007) specifically in that the volatility of our volatility process is also time varying.

Our specification of the dividend growth process differs slightly from the specification in Bansal and Yaron (2004). For comparison, the dividend growth process in the set of equations (13) can

⁸This process is the exact discrete-time counterpart of the Cox, Ingersoll and Ross process and avoids the possibility of negative values for volatility. It allows us to obtain tractable approximate closed-form expressions for the model solution. Its properties are described in Jasiak and Gourieroux (2006). Hsu and Palomino (2011) present a general solution for rational equilibrium models where uncertainty is described by Gaussian and autoregressive gamma processes. Le, Singleton and Dai (2010) apply autoregressive gamma process to the analysis of the term structure of interest rates.

also be written as

$$\Delta d_{t+1} = \mu_c + (b_{dc} + b_{dl}\phi_{lx})x_t + (b_{dc} + b_{dl}\sigma_{lc})\sigma_{c,t}\varepsilon_{c,t+1} + b_{dl}\sigma_{l,t}\varepsilon_{l,t+1} + \sigma_{d,t}\varepsilon_{d,t+1},$$

which is similar to the specification in Bansal and Yaron (2004). The advantage of the dividend growth process in (13) is that it links dividends to nondurables and services consumption and leisure. This link can be obtained endogenously from a resource constraint for the economy as shown in Appendix C, where dividends are linked to choices of consumption and leisure. However, it is not directly useful for our analysis as it relates to total dividends rather than dividends per share, which is the object of interest in asset pricing.

3.3 Wage and Labor Income Growth

The processes for wage and labor income growth are implied by the household’s optimality conditions. These processes allow us to compute and characterize the returns on human capital implied by the model. In an economy with frictionless labor markets, optimality implies that wages are determined by the marginal rate of substitution between leisure and consumption of nondurables and services,

$$MRS_{CL} = \left(\frac{\alpha}{1-\alpha} \right) \left(\frac{C_t}{L_t} \right)^{\frac{1}{\rho}} \zeta_t^{1-\frac{1}{\rho}},$$

Frictions in the labor market such as market power, wage rigidities, or unemployment can generate deviations from this rate. We exogenously capture these deviations by introducing a “wedge” process, ξ_t , such that the wage is

$$W_t = MRS_{CL}e^{\xi_t} = \left(\frac{\alpha}{1-\alpha} \right) \left(\frac{C_t}{L_t} \right)^{\frac{1}{\rho}} \zeta_t^{1-\frac{1}{\rho}} e^{\xi_t}, \quad (15)$$

We assume that the wedge is stationary, has zero unconditional mean, and follows the exogenous process⁹

$$\Delta \xi_{t+1} = \sigma_{\xi c}\sigma_{c,t}\varepsilon_{c,t+1} + \sigma_{\xi l}\sigma_{l,t}\varepsilon_{l,t+1}, \quad (16)$$

where the innovations are again *i.i.d.* $\sim \mathcal{N}(0, 1)$.

The wage equation (15) is affected by the preference shock, ζ_t . For parsimony, we define this

⁹Specifications where ξ_t is modeled in levels were also tested, but did not improve the model’s performance.

shock as¹⁰

$$\zeta_t \equiv C_t. \quad (17)$$

The specification for ζ_t ensures balanced growth in the economy.¹¹ To see this, we can rewrite equation (15) as

$$\frac{W_t}{C_t} = \left(\frac{\alpha}{1-\alpha} \right) L_t^{-\frac{1}{\rho}} e^{\xi_t}. \quad (18)$$

Notice that consumption of nondurables and services and wages share the same trend under the assumption that leisure and the wedge are stationary.

From equation (18) and the fact that log-labor income is $y_t \equiv \log(W_t(1 - L_t))$, wage and labor income growth can be approximated as

$$\Delta w_t = \Delta c_t + b_{wl}\Delta l_t + \Delta \xi_t, \quad \text{and} \quad \Delta y_t = \Delta c_t + b_{yl}\Delta l_t + \Delta \xi_t, \quad (19)$$

respectively, where

$$b_{wl} = -\frac{1}{\rho}, \quad \text{and} \quad b_{yl} = b_{wl} - \frac{e^{\bar{l}}}{1 - e^{\bar{l}}}.$$

3.4 Prices of Risk

Prices of risk in the economy are represented by the coefficients on innovations in the stochastic discount factor. To obtain analytical expressions for these coefficients, we first approximate equation (7) as

$$z_t = \mu_z + b_{zl}(l_t - \bar{l}), \quad (20)$$

¹⁰We also tried the specification $\zeta_t = C_t e^{\xi_t}$. In this case, the process ξ_t has the interpretation of a preference shock that affects the marginal rate of substitution of consumption and leisure, and then the pricing kernel. This specification provides similar results but makes less clear and more difficult to describe the effects of leisure on prices of risk.

¹¹Although ζ_t depends on consumption, we assume that this shock is “external” to the household, such that it is taken as given. This assumption ensures that the elasticity of substitution between consumption and leisure is

$$-\frac{d \log(L_t/C_t)}{d \log W_t} = \rho.$$

A specification where the shock is “internal,” generates a time-varying elasticity. An alternative specification that delivers a constant elasticity is $\zeta_t = C_0 \exp(\mu_c t)$. This specification involves a less parsimonious model with no clear improvement in performance.

where

$$\mu_z = \left(1 - \frac{1}{\rho}\right)^{-1} \log a_z, \quad b_{zl} = \frac{\alpha e^{\left(1 - \frac{1}{\rho}\right)\bar{l}}}{a_z},$$

and $a_z = 1 - \alpha + \alpha e^{\left(1 - \frac{1}{\rho}\right)\bar{l}}$. Given this approximation, we show in Appendix A that the innovation in the log pricing kernel can be expressed as

$$m_{t,t+1} - E_t[m_{t,t+1}] = -\lambda_c \sigma_{c,t} \varepsilon_{c,t+1} - \lambda_l \sigma_{l,t} \varepsilon_{l,t+1} - \lambda_x \sigma_{x,t} \varepsilon_{x,t+1} - \lambda_\nu (\nu_{t+1} - \mathbb{E}_t[\nu_{t+1}]),$$

where

$$\begin{aligned} \lambda_c &= \gamma + \left(\gamma - \frac{1}{\rho}\right) b_{zl} \sigma_{lc}, \\ \lambda_l &= \left(\gamma - \frac{1}{\rho}\right) b_{zl}, \\ \lambda_x &= \left(\frac{\gamma - \frac{1}{\psi}}{1 - \eta_a \phi_x}\right) \eta_a (1 + b_{zl} \phi_{lx}), \\ \lambda_\nu &= (1 - \theta) \eta_a p_{a,\nu}, \end{aligned} \tag{21}$$

where the approximation constant η_a , and the sensitivity of the wealth-consumption ratio to volatility, $p_{a,\nu}$, are defined in Appendix A.

There are several differences in the prices of risk relative to the single consumption good model of Bansal and Yaron (2004). First, non-separability in the utility of consumption and leisure implies that contemporaneous innovations to leisure growth (short-run leisure risk) are priced. It is reflected in an additional term in the price of contemporaneous innovations in consumption of nondurables and services (short-run consumption risk), λ_c , and an extra price of risk, λ_l . The additional term in λ_c represents the sensitivity of the ratio z_t to innovations in non-durables and services consumption growth. The quantitative impact of the term is determined by the sensitivity of leisure to these shocks and the weight of leisure in the felicity function, α . For $\gamma > \frac{1}{\rho}$, this sensitivity is lower than in the absence of leisure preferences if $b_{zl} \sigma_{lc} < 0$. Since $b_{zl} \approx \alpha > 0$, it implies that a negative σ_{lc} , reduces the price of consumption growth risk. That is, a negative correlation between consumption growth and leisure growth induced by these shocks reduces the sensitivity of the marginal utility of consumption to this risk. In addition, a greater weight of leisure in the utility function (higher α) amplifies this effect. On the other hand, the price of risk λ_l is positive as long as $\gamma > \frac{1}{\rho}$. In this case, an independent shock that increases leisure also increases the marginal utility of consumption.

Second, the price of long-run risk, λ_x , is equal to that in Bansal and Yaron (2004) but amplified by the sensitivity $b_{zl} \phi_{lx}$ that results from leisure preferences. A negative loading ϕ_{lx} reduces the price of this risk. Also, volatility risk is only priced if $\gamma \neq \frac{1}{\psi}$. If $\gamma > \frac{1}{\psi}$, the price of volatility risk

is negative if the wealth-consumption ratio decreases after a positive volatility shock ($p_{a,\nu} < 0$). The effect of leisure preferences on the magnitude of the price of volatility risk can be positive or negative depending on the magnitudes of α and the parameters describing the leisure growth process. In summary, leisure preferences affect prices of short- and long-run nondurables and service consumption and leisure risk as long as $\gamma \neq \rho$ and $\gamma \neq 1/\psi$, respectively. Shocks that induce a negative correlation between consumption and leisure have a hedging effect on the marginal utility of consumption as long as $\gamma > \rho$ and/or $\gamma > 1/\psi$, and then leisure preferences reduce their prices of risk.

3.5 Risk-Free Rate in Units of Consumption

The risk-free asset in the economy is an asset that pays a unit of total consumption with certainty. If agents have preference for leisure and $\rho \neq 1$, the risk-free asset will not be equivalent to an asset that pays a unit of nondurables and services consumption, since movements in the relative price of total consumption will make a risk-free bond issued in units of nondurables and services consumption risky. Since zero-coupon real Treasury debt pays a unit of nondurables and services consumption, it will not generally be a risk-free security. However, in accordance with past literature, we refer to this security as the risk-free asset.

The risk-free rate in units of consumption of nondurables and services, r_t , is the conditional expectation of the pricing kernel,

$$\exp(-r_t) = \mathbb{E}_t[M_{t,t+1}],$$

given by

$$r_t = \text{const}_r + \left[\frac{1}{\psi} + \left(\frac{1}{\psi} - \frac{1}{\rho} \right) b_{zlx} \right] x_t - \left[(1 - \theta)(1 - \eta_a \phi_\nu) p_{a,\nu} + \frac{\lambda_\nu^2 \phi_\nu \varsigma_\nu}{1 + \lambda_\nu \varsigma_\nu} + q_r \right] \nu_t.$$

where expressions for const_r , and q_r are provided in Appendix B. The sensitivity of the risk-free rate to long-run risk depends on the effect of x_t on expected consumption and total consumption. This sensitivity is not only affected by the elasticity of substitution ψ and expectations of future nondurables and services consumption growth, but also by $\left(\frac{1}{\psi} - \frac{1}{\rho} \right)$ and expectations of total consumption growth. The later effect depends on the elasticities ψ and ρ and the loading of leisure on long-run risk. Similarly, the sensitivity of the risk-free rate to volatility is affected by leisure preferences through its effects on the wealth-consumption ratio, $p_{a,\nu}$, and the precautionary savings term q_r .

3.6 Asset Returns

We price and compute expected returns of claims on all future consumption of nondurables and services, dividends, labor income, and wages. The claims on all future labor income and wages allow us to quantify the return on human capital. In models with no leisure preferences, $L = 0$, and the returns on labor income and wage claims are the same. In the presence of leisure preferences, claims on labor income do not depend only on wages but also on the household willingness to work in different states of the world. Therefore, the riskiness and expected returns of claims on wages and labor income can be different.

Our claims have cashflows $K_t = \{C_t, D_t, W_t, W_t N_t\}$. From equations (13) and (19), growth in these cashflows follow the process

$$\Delta k_t = \mu_k + b_{kx}x_{t-1} + \sigma_{k,t-1}\varepsilon_{k,t} + \sigma_{kc}\sigma_{c,t-1}\varepsilon_{c,t} + \sigma_{kl}\sigma_{l,t-1}\varepsilon_{l,t},$$

for appropriate coefficients defined in Appendix B. The appendix shows that log-returns for these claims can be approximated as

$$r_{k,t+1} = \bar{\eta}_k + \eta_k p_{k,t+1} + \Delta k_{t+1} - p_{k,t},$$

where the price-cashflow ratio has the form

$$p_{k,t} = \bar{p}_k + p_{k,x}x_t + p_{k,\nu}\nu_t.$$

Hsu and Palomino (2011) show that expected excess returns on these claims are

$$\begin{aligned} \log \mathbb{E}_t[\exp(xr_{k,t+1})] &= \lambda_c \sigma_{kc} \sigma_{k,t}^2 + \lambda_l \sigma_{kl} \sigma_{l,t}^2 \\ &+ \lambda_x \eta_k p_{k,x} \sigma_{x,t}^2 + \delta_\nu \log \left[\frac{1 + (\lambda_\nu - \eta_k p_{k,\nu}) \varsigma_\nu}{(1 + \lambda_\nu \varsigma_\nu)(1 - \eta_k p_{k,\nu} \varsigma_\nu)} \right] \\ &- \phi_\nu \varsigma_\nu \left[\frac{(\lambda_\nu - \eta_k p_{k,\nu})^2}{1 + (\lambda_\nu - \eta_k p_{k,\nu}) \varsigma_\nu} - \frac{\lambda_\nu^2}{1 + \lambda_\nu \varsigma_\nu} + \frac{\eta_k^2 p_{k,\nu}^2}{1 - \eta_k p_{k,\nu} \varsigma_\nu} \right] \nu_t, \end{aligned}$$

where $xr_{k,t+1} \equiv r_{k,t+1} - r_t$. In the absence of volatility shocks, the expected excess returns equation reduces to the familiar $-\text{cov}_t(m_{t,t+1}, r_{k,t+1})$. Notice that expected excess returns are time varying as a result of time-varying volatility. The last two terms capture the volatility premium. This premium is time-varying since there is time-varying volatility in the volatility process.

4 Analysis

Given the solutions to quantities of interest in Section 3, we calibrate the model to the data to highlight the contribution of leisure preferences to the price of risky claims in the economy. Our calibration provides insight into the marginal contribution of leisure preferences to the pricing of financial claims. Further, the presence of leisure preferences allows us to analyze the impact of aggregate quantities on the expected returns to human capital.

4.1 Calibration

We solve the model using the analytical approximations presented above as in Bansal and Yaron (2004) and Beeler and Campbell (2012).¹² We assume a monthly frequency, and simulate and aggregate the monthly dynamics to annual frequency to match select macroeconomic and asset pricing statistics of the United States annual data from 1930 to 2011 described in Section 2. The aggregation procedure from monthly to annual frequency for consumption, dividends, labor income, price-dividend and wealth-consumption ratios is identical to that described in Bansal and Yaron (2004). We describe here the aggregation procedure for annual leisure and wages. Since leisure is defined as a fraction of time, we compute annual leisure L_t^a as a weighted average of monthly leisure during the year. To compute the weights, notice that the annual wage W_t^a and the annual labor income $Y_t^a = W_t^a N_t^a$ are

$$W_t^a = \sum_{i=0}^{11} W_{t-i}, \quad \text{and} \quad Y_t^a = \sum_{i=0}^{11} W_{t-i} N_{t-i}.$$

It follows that $L_t^a \equiv 1 - N_t^a$ is

$$L_t^a = \sum_{i=0}^{11} \frac{W_{t-i}}{W_t^a} L_{t-i}.$$

For comparison purposes, we present five different calibrations. The first is a baseline calibration that corresponds to a model with preferences in consumption only ($\alpha = 0$), as in Bansal and Yaron (2004). The baseline calibration is similar to the one presented in Bansal, Kiku and Yaron (2010), updated to include data for recent years. This calibration highlights the contribution of the stochastic volatility channel for understanding asset returns. Beeler and Campbell (2012) show that this calibration improves the predictability properties of the long-run risk model. We then present four representative calibrations for the model with leisure preferences. The main difference

¹²The approximations for price-cashflow ratios are around their unconditional means. We compute these means using a fixed-point algorithm. These approximations are highly accurate even in the presence of autoregressive gamma shocks.

between these calibrations is that they use four different values for the elasticity of substitution between consumption and leisure, $\rho = \{0.5, 1, 1.5, 5\}$. Comparisons across these calibrations allow us to quantify the importance for the results of leisure preferences, the degree of substitution between consumption and labor, and the presence of a wedge between wages and the marginal rate of substitution of consumption and leisure. The case $\rho = 0.5$ captures complementarity between leisure and consumption. The case $\rho = 1$ implies a constant share of consumption relative to total consumption. In this setting, the return $R_{a,t}$ in the pricing kernel, equation (11), is equal to the return on the consumption claim, $R_{c,t}$. The case $\rho = 1.5$ implies separability in the intertemporal utility of consumption and leisure since we set $\psi = 1.5$. The case $\rho = 5$ captures substitutability between leisure and consumption.

Table 6 presents the parameter values that are common across calibrations. We set the parameter values of ψ and γ to 1.5 and 10, respectively, to be consistent with Bansal and Yaron (2004). We set $\phi_x = 0.975$, the value used in Bansal, Kiku and Yaron (2010). We choose parameter values for μ_c , σ_c , and σ_x to match the average, volatility, and first-order autocorrelation of consumption growth, given a set of volatility parameters δ_ν , ϕ_ν , and ς_ν described below. We assume that all variables have stochastic volatility except for the independent volatility component of dividend growth. That is, $I_k = 1$ for $k = \{c, x, l\}$, and $I_d = 0$. We normalize $\varsigma_\nu = \delta_\nu^{-1}(1 - \phi_\nu)$ such that $\mathbb{E}[\nu_t] = 1$. The persistence of the volatility process is set at $\phi_\nu = 0.995$. This value and the low value for δ_ν imply significant volatility in the volatility process that increases the volatility premium. We choose these values to match in our baseline calibration the equity premium in the data given the risk aversion parameter of $\gamma = 10$. We choose μ_l to match the average leisure in the data. The parameter values for ϕ_{lx} , σ_l , and σ_{lc} are chosen to match the volatility and first-order autocorrelation of leisure growth, and the correlation of consumption and leisure growth. The negative correlation between leisure and consumption in the data is captured by negative values for ϕ_{lx} and σ_{lc} , consistent with the empirical analysis.

To calibrate the dividend process in the baseline calibration, we set the loading of dividend growth on long-run risk to be 2.5, and select the remaining parameters in the dividend growth process to match the volatility of dividend growth and the correlation of this variable with consumption growth. For the remaining calibrations, we set the parameter values for $b_{dc} = 2.86$, $b_{dl} = -0.5$, and $\sigma_d = 0.03$ to capture the volatility of dividend growth and the correlations of dividend growth with consumption and leisure growth. The loadings of dividend growth on consumption and leisure growth imply a loading of dividend growth on long-run risk of $b_{dc} + b_{dl}\phi_{lx} = 2.965$.

Table 7 presents the parameter values that are specific to the model calibrations with leisure preferences. In order to match the average labor income - consumption ratio, α is constrained to

be the function of ρ ,

$$\alpha = \frac{1}{1 + \frac{1}{\overline{W/C} \bar{L}^{1/\rho}}},$$

where $\overline{W/C}$ and \bar{L} are the average W_t/C_t and L_t in the data, respectively. The parameter values describing the wedge process σ_{ξ_c} and σ_{ξ_l} are selected to simultaneously match the volatility of labor income and wage growth given a value for ρ . Calibrations with no wedge imply a volatility for wage growth that is too low relative to the volatility of labor income growth. The parameter value for β is chosen to match the average level of the risk-free rate as well as possible. The risk aversion parameter γ is chosen to match the equity premium in the data. Notice that models with leisure preferences require a level of risk aversion that is higher than 10 to match the premium, since the negative correlation between leisure and consumption reduces the prices of risk. However, the table shows that when risk aversion is measured relative to consumption gambles as in equation (12), the coefficient of risk aversion \mathcal{R}^c is below 4.

4.2 Pricing Financial Claims

We first examine the implications of the model for the pricing of financial assets. In Table 8, we present calibrated means, volatilities, first-, and second-order autocorrelations of macroeconomic variables common to the leisure and nondurables and services-only models. The first column presents the data moments, the second column presents moments implied the baseline calibration with preferences only over consumption of nondurables and services, and the third column presents moments implied by the model calibrations with leisure preferences. The baseline model and the four calibrations for the model with leisure preferences share the same dynamics for consumption growth. These dynamics reproduce the consumption growth moments in the data. All models also generate moments of dividend growth that are consistent with the data, despite the fact that dividends are a function of leisure growth in the models with leisure preferences. The table also shows that all models capture the correlation of consumption and dividend growth, and the models with leisure preferences also capture the correlation of leisure and dividend growth in the data. In addition, the models with leisure are also calibrated to match moments of leisure, wage, and labor income growth, discussed below.

Table 9 shows that leisure preferences do not have a large impact on the moments of asset pricing variables. Except in the case where $\rho = 0.5$, the model generates a mean risk-free rate that is identical to that in the baseline calibration. Like the baseline calibration, the model has difficulty matching the volatility of the risk-free rate and generates somewhat higher autocorrelation. The leisure model has similar implications for the mean and autocorrelation of the price-dividend ratio

as the baseline case, but generates modestly higher volatility, which is increasing in ρ . However, the volatility of the price-dividend ratio is still approximately half of that observed in the data. Finally, incorporating leisure preferences generates an equity premium consistent with both the data and the baseline calibration, and generates slightly more volatility in the equity return as ρ increases than the baseline case.

Some insight into how the incorporation of leisure preferences affects the pricing of financial claims can be obtained by decomposing the prices of risk into the fraction attributable to each component. We report the results of this decomposition in Table 10. The prices of risk are very similar across calibrations with and without leisure preferences, with the exception of the price of shocks to leisure. In the presence of leisure preferences, innovations in leisure growth have a positive price of risk, λ_l , since $\gamma > \frac{1}{\rho}$, and a small negative contribution to the equity premium. A negative innovation to leisure decreases the marginal utility of consumption, but has a positive effect on dividend growth, as captured by $b_{dl} < 0$. As the elasticity ρ increases, the prices of innovations to consumption λ_c and of long-run risks λ_x and λ_v decrease (in absolute value), while the price of innovations to leisure λ_l increase. The contribution to the equity premium of shocks to conditional means is larger in models with leisure, while the contribution of volatility shocks and innovations to consumption are smaller. These effects are amplified as the elasticity ρ increases but the differences are not quantitatively significant.

As in Bansal and Yaron (2004), the introduction of time-varying uncertainty generates time-varying expected excess asset returns. In turn, this time variation affects the predictability of macroeconomic variables and asset returns. Our modeling choice for volatility in equation (14) allows us to determine which source of time variation contributes most to this predictability by setting $I_k = 0$ for $k = \{c, x, l, d\}$. In untabulated results, available from the authors upon request, we show that the most significant contribution to the volatility premium arises from time-varying volatility in the conditional mean ($I_x = 1$), which combined with time variation in innovations to nondurable and services consumption growth ($I_c = 1$) improves results for predictability. In contrast, time-varying volatility in leisure growth ($I_l = 1$) results in a small deterioration of the predictability of macroeconomic variables and excess stock returns. However, we set $I_l = 1$ because it improves the predictability results for the volatility of leisure and wage growth. We also find that setting $I_d = 0$ slightly improves the lack of predictability of dividend growth.

Finally, we examine the implications of the model for the predictability of levels and volatility of growth in nondurables and services consumption, dividends, leisure, and wages and excess stock returns by the price-dividend ratio. Again, the model with leisure preferences has similar implications for the predictability as the baseline calibration without leisure for consumption and dividend growth. The same pattern is observed in the predictability of leisure and wage growth in models with leisure preferences. That is, the models imply more predictability of the levels of macroe-

conomic variables and less predictability of excess stock returns and volatility of macroeconomic variables than observed in the data. These predictability results are improved for all variables as the stochastic volatility channel becomes more important in the calibration. Because of the similarity of these results to those presented earlier in the literature, we do not tabulate them, but the tables are available upon request.

In summary, incorporating leisure preferences into the long-run risks model has little impact on the model’s ability to match the moments of financial asset data. Like the original calibration, the model generates a low-risk free rate, a high equity premium, and time-varying expected returns that are, to some degree, predictable by price-dividend ratios. However, the model continues to have difficulty matching the degree of predictability of excess returns, especially at long horizons, and the predictability of the volatility of macroeconomic variables. One possible remedy is the incorporation of an additional source of volatility, as in Zhou and Zhu (2013). Our principal conclusion is that while a model calibrated to leisure growth moments does not improve upon the pricing of financial assets, it does not hurt the model’s ability to price financial assets either, despite the negative effect of leisure on the prices of risk.

4.3 The Term Structure of Interest Rates

The yield on a bond that pays a unit of nondurable and services consumption at time $t + n$, $r_t^{(n)}$, is obtained from the conditional expectation of the pricing kernel,

$$\exp\left(-r_t^{(n)}\right) = \mathbb{E}_t[M_{t,t+n}].$$

This yield depends linearly on the sources of long-run risk and economic uncertainty, such that

$$r_t^{(n)} = \frac{1}{n} [\mathcal{A}_n + \mathcal{B}_{n,x}x_t + \mathcal{B}_{n,\nu}\nu_t],$$

with coefficients \mathcal{A}_n , $\mathcal{B}_{n,x}$, and $\mathcal{B}_{n,\nu}$ described in appendix B. As noted above, in the presence of preferences for leisure, these bonds are not actually risk free if held to maturity since they pay a unit of nondurable and services consumption rather than total consumption.

Beeler and Campbell (2012) report a significantly downward sloping term structure of real rates implied by the long-run risks model. The authors note that United States Treasury inflation protected securities (TIPS) have never exhibited a negative term slope.¹³ In our framework, the term structure is also downward sloping, but not as severely as in the baseline long-run risks model.

¹³While the TIPS term structure is upward sloping, spreads between 10- and 5-year UK inflation protected bonds have been negative for sustained periods of time, and the average spread is close to zero according to data from Global Financial Data. While our model is calibrated to U.S. rather than U.K. data, these yields suggest that a downward sloping real term structure is at least feasible empirically.

Figure 2 shows the term structure for annual maturities from 1 to 10 years for our calibrations. Our baseline calibration reproduces the Beeler and Campbell (2012) finding, with a spread between the 10-year bond yield and the 1-month risk-free rate of -1.09%. The model with leisure preferences and $\rho = 0.5$ exacerbates this problem, and implies a spread of -1.54%. Increasing the elasticity parameter results in a less steeply downward sloping term structure, and falls to -0.88% when $\rho = 5$. Preferences with a high substitution between consumption and leisure reduce the sensitivity of long-term bonds to sources of long-run risk and thus the hedging properties of these bonds.

4.4 Substitution of Consumption and Leisure and the Wealth Portfolio

A key parameter in our calibrations is the parameter of intratemporal substitution between consumption of nondurables and services consumption and leisure. The asset pricing effects of ρ can be understood from the set of equations (21) and their impact on the return on the portfolio of aggregate wealth. As in the baseline model of Bansal and Yaron (2004), recursive preferences ($\gamma \neq \frac{1}{\psi}$) result in the dependence of the pricing kernel on the return of the wealth portfolio, $R_{a,t+1}$, as in equation (10). However, the wealth portfolio is no longer a claim only on nondurable and services consumption, but rather a claim on total consumption $C_t + W_t L_t$. In this framework, the prices of risk depend on $\gamma - \frac{1}{\rho}$. This component captures the dependence of the pricing kernel on leisure. For values of ρ such that $\gamma > \frac{1}{\rho}$, a higher substitution between consumption and leisure increases the effect of leisure in prices of risk.

From equation (18), it follows that

$$C_t + W_t L_t = \frac{1}{1 - \alpha} C_t Z_t^{1 - \frac{1}{\rho}}.$$

When $\rho = 1$, nondurables and services consumption is a constant fraction of total consumption and the return on the wealth portfolio is the same as the return on nondurables and services consumption claims ($R_{a,t+1} = R_{c,t+1}$), as in the baseline case. However, for values of $\rho \neq 1$, consumption as a fraction of total consumption varies over time. The volatility of the wealth portfolio return can be higher or lower than the volatility of consumption claim returns, affecting the volatility of the pricing kernel. Table 11 shows that excess returns on the wealth portfolio are 1.35% and 1.16% in models without and with leisure preferences, respectively, suggesting that leisure in the utility function reduces the riskiness of the wealth portfolio. The expected excess returns on the wealth portfolio are similar across models with different ρ but the volatility of these returns is lower for higher values of ρ , which implies higher Sharpe ratios. For all of these calibrations, the Sharpe ratios of the wealth portfolio are higher than that of the stock market.

4.5 Human Capital Returns

The principal contribution of our model is that it allows us to quantify the risk in human capital and its associated expected returns. We analyze two claims on human capital: claims on all future labor income, and claims on all future wages. In a model with no leisure preferences, households provide labor inelastically and the two claims are the same. In the presence of leisure preferences, households have the ability to adjust leisure over time, affecting their labor income stream. Measuring human capital returns using claims on wages (per unit of time) provides an idea of the value and riskiness of human capital for a fixed amount of labor in the economy. Measuring human capital returns using claims on labor income is more appropriate for understanding portfolio choice decisions. Consequently, it is of interest to analyze the riskiness of both types of claims.

We first examine the calibration to correlations and moments of macroeconomic variables related to leisure preferences, specifically wages, labor income, and leisure. Results are presented in Table 12. As shown in the table, most moments are captured fairly well; the model captures means, volatilities, and autocorrelations of all variables with the exception of a slightly lower mean wage and leisure growth. The difficulty in capturing leisure growth is due to the fact that in the model, leisure is stationary, and so should exhibit no deterministic trend in its time series. However, the data series exhibits a pronounced trend, increasing from the early 1960s through the present. Correlations are also captured reasonably well. The model implies a somewhat higher correlation of consumption and wage growth, leisure and wage growth, dividend and wage growth, consumption and labor income growth, and dividend and labor income growth than observed in the data. However, all correlations are of the right sign and relatively close in magnitude to those in the data.

Table 13 shows properties of excess returns on labor income and wage claims. These claims have excess returns that are significantly less volatile than excess stock returns, their expected returns are lower, and their Sharpe ratios are around 30% higher. Across our four calibrations, expected excess returns on labor income claims range between 2.97% to 3.26%, and those of claims on wages range around 2.23% to 2.47%, in comparison to an expected excess stock return of 5.13%. The ability of households to adjust their labor supply increases the riskiness of labor claims relative to wage claims. Periods of high marginal utility coincide with periods not only of low wages but also of low labor provision, given the negative correlation between consumption and leisure. The table also shows that calibrations with a lower degree of substitution between consumption and leisure imply higher expected excess returns for these claims, despite the fact that all calibrations match the volatility of labor income and wage growth in the data.

Lustig and Nieuwerburgh (2006) obtain an estimate of the return on human capital and find that it is negatively correlated with the return on equity claims. However, Bansal, Tallarini and Yaron (2008) provide a model in which the correlation in human capital and equity returns is zero,

and Bansal et al. (2013) document a positive correlation between human capital and equity returns. We investigate the correlation in the returns on dividend and human capital claims and tabulate the results in Table 14. The table also documents the correlations of returns to total consumption claims and nondurables and services consumption claims with excess returns on equity. As shown in the table, the correlation of excess returns on equity with both measures of the return on human capital are positive and stable across parameterizations of ρ . Correlations of excess equity returns with total and nondurables and services consumption are also similar across calibrations and consistent with the baseline calibration of Bansal, Kiku and Yaron (2007), except in the case where leisure and nondurables and services consumption are substitutes.

We compare the contribution of different sources of risk in the risk premium on human capital claims in Table 15. As in the case of stocks, the expected return compensation for innovations in leisure is negative in both labor income and wage claims. However, while this compensation represents less than 1% of the premium in the case of equities, it contributes approximately 10% to the premium on human capital when measured as the claim to labor income. This contribution is particularly high in magnitude for low levels of ρ and decreases as the elasticity of substitution between leisure and consumption increases. The results suggest that this increase in relative importance is offset largely by an increase in the importance of the time-varying volatility premium and compensation for the transitory portion of nondurables and consumption risk. The table also suggests that the contribution of long-run risk to conditional means decreases as ρ increases. These results emphasize the difference in the nature of risk of labor and financial claims.

In summary, human capital claims have expected returns that are between 45% and 60% of expected stock returns with Sharpe ratios that are 30% higher than that of the market, consistent with Bansal, Tallarini and Yaron (2008). Excess returns on these claims are positively correlated with the excess returns on equity and are significantly affected by the presence of leisure in the utility function and the degree of substitutability of leisure and consumption of nondurables and services. These correlations are supportive of the evidence in Bansal et al. (2013), and run counter to the evidence of negative correlation in Lustig and Nieuwerburgh (2006).

5 Conclusion

Under time recursive preferences, quantities that provide utility such as leisure will matter for agents' marginal utility of consumption, even if intratemporal preferences are separable. We model asset prices in a framework with persistent moments in consumption, leisure, and wage growth, and recursive preferences, as in the long-run risk model of Bansal and Yaron (2004). We find that the model delivers similar results for financial asset prices as the calibrations of Bansal and Yaron (2004) and Bansal, Kiku and Yaron (2007). In particular, equity risk premia, risk-free rates and volatility

of financial assets consistent with that observed in the data can be generated using plausible risk aversion parameters, while matching macroeconomic moments. The model reproduces results on predictability of asset returns, macroeconomic variables, and volatility of macroeconomic variables in Bansal, Kiku and Yaron (2007). Finally, the model is able to generate a real term structure with a less pronounced negative slope than long-run risk calibrations with only consumption data.

A novel contribution of our analysis is the endogenous generation of the risk premium and price of human capital. The tradeoff between labor and leisure allows us to determine the price of a human capital claim in equilibrium. Similar to Bansal et al. (2013), but in contrast to Lustig and Nieuwerburgh (2006), returns on human capital are positively correlated with returns on financial assets. These human capital returns have a lower risk premium than returns on dividend and consumption claims, with lower loadings on shocks to the conditional mean and larger loadings on contemporaneous consumption shocks. Although the risk premia are lower, Sharpe ratios to human capital are approximately 30% higher than those to dividend claims. These results provide important guidance to the growing literature investigating the relation between labor and asset prices.

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A Model Solution

The pricing kernel in equation (10) depends on the portfolio return $R_{a,t}$, which in turn satisfies the pricing equation (11). The return on this portfolio can be written in terms of the wealth-consumption ratio $p_{a,t} \equiv \log S_{a,t} - \log A_t$. Since $A_t = \frac{1}{1-\alpha} C_t Z_t^{1-\frac{1}{\rho}}$, this return can be written as

$$R_{a,t+1} = (1 + e^{p_{a,t+1}}) \left(\frac{C_{t+1}}{C_t} \right) \left(\frac{Z_{t+1}}{Z_t} \right)^{1-\frac{1}{\rho}} e^{-p_{a,t}}.$$

The equation above can be approximated around $\bar{\ell}_a = \mathbb{E}[p_{a,t}]$ to obtain

$$r_{a,t+1} = \bar{\eta}_a + \eta_a p_{a,t+1} + \Delta c_{t+1} + \left(1 - \frac{1}{\rho}\right) \Delta z_{t+1} - p_{a,t},$$

where $\eta_a = \frac{\exp(\bar{\ell}_a)}{1 + \exp(\bar{\ell}_a)}$, and $\bar{\eta}_a = \log[1 + \exp(\bar{\ell}_a)] - \bar{\ell}_a \eta_a$. Notice that the solution for $\bar{\ell}_a$ involves a fixed point problem.

The pricing equation (11), the approximation above, and the approximation for z_t given in equation (20) imply a solution for the ratio above given by

$$p_{a,t} = \bar{p}_a + p_{a,x} x_t + p_{a,\nu} \nu_t,$$

where the coefficients satisfy

$$\begin{aligned} \bar{p}_a &= (1 - \eta_{p_a})^{-1} \left[\log \beta + \left(1 - \frac{1}{\psi}\right) \mu_c + \bar{\eta}_a + \bar{q}_a - \frac{\delta_\nu}{\theta} \log(1 - \theta \eta_a p_{a,\nu} \varsigma_\nu) \right], \\ p_{a,x} &= \left(\frac{1 - \frac{1}{\psi}}{1 - \eta_a \phi_x} \right) (1 + b_{z_l} \phi_{l_x}), \\ p_{a,\nu} &= q_a + \frac{\eta_a \phi_\nu p_{a,\nu}}{1 - \theta \eta_a \varsigma_\nu p_{a,\nu}}, \end{aligned}$$

for $\bar{q}_a = \frac{1}{2} \theta \left[\left(1 - \frac{1}{\psi}\right)^2 (1 + b_{z_l} \sigma_{l_c})^2 \sigma_c^2 (1 - I_c) + \left(1 - \frac{1}{\psi}\right)^2 b_{z_l}^2 \sigma_l^2 (1 - I_l) + \eta_a^2 p_{a,x}^2 \sigma_x^2 (1 - I_x) \right]$, and

$q_a = \frac{1}{2} \theta \left[\left(1 - \frac{1}{\psi}\right)^2 (1 + b_{z_l} \sigma_{l_c})^2 \sigma_c^2 I_c + \left(1 - \frac{1}{\psi}\right)^2 b_{z_l}^2 \sigma_l^2 I_l + \eta_a^2 p_{a,x}^2 \sigma_x^2 I_x \right]$. Notice that the coefficient on the volatility factor solves a quadratic equation. The solution is the one that makes $p_{a,\nu} = 0$ if $I_k = 0$ for all k .

B Asset Prices and Expected Returns

B.1 Expected Returns

The log-pricing kernel $m_{t,t+1} \equiv \log M_{t,t+1}$ can be expressed as

$$-m_{t,t+1} = \Gamma_0 + \Gamma_x x_t + \Gamma_\nu \nu_t + \lambda_c \sigma_{c,t} \varepsilon_{c,t+1} + \lambda_l \sigma_{l,t} \varepsilon_{l,t+1} + \lambda_x \sigma_{x,t} \varepsilon_{x,t+1} + \lambda_\nu (\nu_{t+1} - \mathbb{E}_t[\nu_{t+1}]),$$

where the prices of risk λ_k for $k = \{c, l, x, \nu\}$ are described in equation (21) and

$$\begin{aligned}\Gamma_0 &= -\log \beta + \frac{1}{\psi} \mu_c + (1 - \theta) \left[-\bar{q}_a + \frac{\delta_\nu}{\theta} \log(1 - \theta \eta_a p_{a,\nu} \varsigma_\nu) + \eta_a p_{a,\nu} \delta_\nu \varsigma_\nu \right], \\ \Gamma_x &= \frac{1}{\psi} + \left(\frac{1}{\psi} - \frac{1}{\rho} \right) b_{zl} \phi_{lx}, \\ \Gamma_\nu &= -(1 - \theta)(1 - \eta_a \phi_\nu) p_{a,\nu}.\end{aligned}$$

Consider a claim on all future cashflows K_t . We are interested in pricing claims on cashflows $K_t = \{C_t, D_t, W_t, W_t N_t\}$. Growth in these cashflows can be expressed as

$$\Delta k_t = \mu_k + b_{kx} x_{t-1} + \sigma_{k,t-1} \varepsilon_{k,t} + \sigma_{kc} \sigma_{c,t-1} \varepsilon_{c,t} + \sigma_{kl} \sigma_{l,t-1} \varepsilon_{l,t}.$$

A claim on all future cashflows has the no-arbitrage price

$$S_{k,t} = \mathbb{E}_t[M_{t,t+1}(K_{t+1} + S_{k,t+1})],$$

and return

$$e^{r_{k,t+1}} = \left(1 + \frac{S_{k,t+1}}{K_{t+1}} \right) \left(\frac{K_{t+1}}{K_t} \right) \left(\frac{K_t}{S_{k,t}} \right).$$

Denote the price-cashflow ratio by $p_{k,t} \equiv \log S_{k,t} - \log K_t$, and approximate the equation above around $\bar{\ell}_k = \mathbb{E}[p_{k,t+1}]$ to obtain

$$r_{k,t+1} = \bar{\eta}_k + \eta_k p_{k,t+1} + \Delta k_{t+1} - p_{k,t},$$

where $\eta_k = \frac{\exp(\bar{\ell}_k)}{1 + \exp(\bar{\ell}_k)}$, and $\bar{\eta}_k = \log(1 + \exp(\bar{\ell}_k)) - \bar{\ell}_k \eta_k$. Notice that the solution for $\bar{\ell}_k$ involves a fixed point problem.

The solution for the price-cashflow ratio has the form

$$p_{k,t} = \bar{p}_k + p_{k,x} x_t + p_{k,\nu} \nu_t,$$

with coefficients

$$\begin{aligned}\bar{p}_k &= \left(\frac{1}{1 - \eta_k} \right) \left\{ -\Gamma_0 + \lambda_\nu \varsigma_\nu \delta_\nu + \bar{\eta}_k + \mu_k + \bar{q}_k - \delta_\nu \log[1 + (\lambda_\nu - \eta_k p_{k,\nu}) \varsigma_\nu] \right\}, \\ p_{k,x} &= \left(\frac{b_{kx} - \Gamma_x}{1 - \eta_k \phi_x} \right) \\ p_{k,\nu} &= q_k - \frac{(\lambda_\nu - \eta_k p_{k,\nu}) \phi_\nu}{1 + (\lambda_\nu - \eta_k p_{k,\nu}) \varsigma_\nu},\end{aligned}$$

where $\bar{q}_k = \frac{1}{2} [(\lambda_c - \sigma_{kc})^2 \sigma_c^2 (1 - I_c) + (\lambda_l - \sigma_{kl})^2 \sigma_l^2 (1 - I_l) + (\lambda_x - \eta_k p_{k,x})^2 \sigma_x^2 (1 - I_x) + \sigma_k^2 (1 - I_k)]$, and $q_k = -\Gamma_\nu + \lambda_\nu \phi_\nu + \frac{1}{2} [(\lambda_c - \sigma_{kc})^2 \sigma_c^2 I_c + (\lambda_l - \sigma_{kl})^2 \sigma_l^2 I_l + (\lambda_x - \eta_k p_{k,x})^2 \sigma_x^2 I_x + \sigma_k^2 I_k]$. Notice that the coefficient on the volatility factor solves a quadratic equation. The solution is the one that makes $p_{k,\nu} = 0$ if $I_k = 0$ for all k .

The expected excess return of this claim is

$$\begin{aligned}\log \mathbb{E}_t[\exp(r_{k,t+1} - r_t)] &= \lambda_c \sigma_{kc} \sigma_{k,t}^2 + \lambda_l \sigma_{kl} \sigma_{l,t}^2 \\ &+ \lambda_x \eta_k p_{k,x} \sigma_{x,t}^2 + \delta_\nu \log \left[\frac{1 + (\lambda_\nu - \eta_k p_{k,\nu}) \varsigma_\nu}{(1 + \lambda_\nu \varsigma_\nu)(1 - \eta_k p_{k,\nu} \varsigma_\nu)} \right] \\ &- \phi_\nu \varsigma_\nu \left[\frac{(\lambda_\nu - \eta_k p_{k,\nu})^2}{1 + (\lambda_\nu - \eta_k p_{k,\nu}) \varsigma_\nu} - \frac{\lambda_\nu^2}{1 + \lambda_\nu \varsigma_\nu} + \frac{\eta_k^2 p_{k,\nu}^2}{1 - \eta_k p_{k,\nu} \varsigma_\nu} \right] \nu_t.\end{aligned}$$

For claims on dividends, $b_{dx} = b_{dc} + b_{dl}\phi_{lx}$, $\sigma_{dc} = b_{dc} + b_{dl}\sigma_{lc} + b_{d\xi}\sigma_{\xi x}$, and $\sigma_{dl} = b_{dl}$.

For claims on consumption $b_{cx} = 1$, $\sigma_{cc} = \sigma_{cl} = 0$.

For claims on wages $b_{wx} = 1 + b_{wl}\phi_{wx}$, $\sigma_{wc} = 1 + b_{wl}\sigma_{lc} + \sigma_{\xi c}$, $\sigma_{wl} = b_{wl} + \sigma_{\xi l}$, and $\sigma_w = 0$.

For claims on labor income $b_{yx} = 1 + b_{yl}\phi_{lx}$, $\sigma_{yc} = 1 + b_{yl}\sigma_{lc} + \sigma_{\xi c}$, $\sigma_{yl} = b_{yl} + \sigma_{\xi l}$, and $\sigma_y = 0$.

B.2 The Term Structure of Interest Rates

The risk-free rate is

$$r_t = \Gamma_0 + \delta_\nu \log(1 + \lambda_\nu \varsigma_\nu) - \lambda_\nu \varsigma_\nu \delta_\nu - \bar{q}_r + \Gamma_x x_t + \left[\Gamma_\nu - \frac{\lambda_\nu^2 \phi_\nu \varsigma_\nu}{1 + \lambda_\nu \varsigma_\nu} - q_r \right] \nu_t.$$

where $\bar{q}_r = \frac{1}{2}[\lambda_c^2 \sigma_c^2(1 - I_c) + \lambda_l^2 \sigma_l^2(1 - I_l) + \lambda_x^2 \sigma_x^2(1 - I_x)]$, and $q_r = \frac{1}{2}(\lambda_c^2 \sigma_c^2 I_c + \lambda_l^2 \sigma_l^2 I_l + \lambda_x^2 \sigma_x^2 I_x)$.

The yield of a bond with maturity at $t + n$, $r_t^{(n)}$ is obtained from

$$\exp(-r_t^{(n)}) = \mathbb{E}_t[M_{t,t+n}] = \mathbb{E}_t \left[M_{t,t+1} \exp(-r_{t+1}^{(n-1)}) \right].$$

It can be shown that

$$r_t^{(n)} = \frac{1}{n} [\mathcal{A}_n + \mathcal{B}_{n,x} x_t + \mathcal{B}_{n,\nu} \nu_t],$$

where the coefficients are obtained recursively as

$$\begin{aligned} \mathcal{A}_n &= \mathcal{A}_{n-1} + \Gamma_0 - \lambda_\nu \varsigma_\nu \delta_\nu + \delta_\nu \log[1 + (\lambda_\nu + \mathcal{B}_{n-1,\nu})\varsigma_\nu] - \bar{q}_{r,n}, \\ \mathcal{B}_{n,x} &= \Gamma_x + \mathcal{B}_{n-1,x} \phi_x, \\ \mathcal{B}_{n,\nu} &= \Gamma_\nu + \frac{(\lambda_\nu + \mathcal{B}_{n-1,\nu})\phi_\nu}{1 - (\lambda_\nu + \mathcal{B}_{n-1,\nu})\varsigma_\nu u} - q_{r,n}, \end{aligned}$$

where $\bar{q}_{r,n} = \frac{1}{2}[\lambda_c^2 \sigma_c^2(1 - I_c) + \lambda_l^2 \sigma_l^2(1 - I_l) + (\lambda_x + \mathcal{B}_{n-1,x})^2 \sigma_x^2(1 - I_x)]$, and $q_{r,n} = \frac{1}{2}[\lambda_c^2 \sigma_c^2 I_c + \lambda_l^2 \sigma_l^2 I_l + (\lambda_x + \mathcal{B}_{n-1,x})^2 \sigma_x^2 I_x]$, with initial conditions $\mathcal{A}_0 = \mathcal{B}_{0,x} = \mathcal{B}_{0,\nu} = 0$.

C Endogenous Dividend Growth

The dividend growth process in the set of equations (13) can be rationalized from a resource constraint

$$C_t = W_t N_t + D_t + G_t,$$

where G_t captures sources of income that are not distributed as labor income or dividends, such as debt payments or government transfers. This constraint can be written as

$$\frac{D_t}{C_t} = 1 - \frac{W_t N_t}{C_t} - \frac{G_t}{C_t}. \quad (22)$$

From the wage equation (18), an approximation to the labor income-consumption ratio $y_t - c_t \equiv \log(W_t N_t / C_t)$ can be written as

$$y_t - c_t = \mu_{yc} + b_{yl}(l_t - \bar{l}) + \xi_t,$$

where

$$\mu_{yc} = \log\left(\frac{\alpha}{1-\alpha}\right) - \frac{1}{\rho}\bar{l} + \log(1 - e^{\bar{l}}), \quad \text{and} \quad b_{yl} = -\left(\frac{1}{\rho} + \frac{e^{\bar{l}}}{1 - e^{\bar{l}}}\right).$$

We can assume that the unconditional mean of $\frac{G_t}{C_t}$ is \bar{G} and define the process for $\Delta g_t \equiv \log G_t - \log G_{t-1}$ as

$$\Delta g_t - \Delta c_t = b_{gc}(\Delta c_t - \mu_c) + b_{gl}\Delta l_t + b_{g\xi}\Delta\xi_t + \sigma_{gd}\sigma_{d,t-1}\varepsilon_{d,t}.$$

It can be shown that an approximation of the constraint (22) can be expressed as

$$d_t - c_t = b_{dy}(y_t - c_t) + b_{dg}(g_t - c_t),$$

where

$$b_{dy} = -\frac{e^{\mu_{yc}}}{1 - e^{\mu_{yc}} - \bar{G}}, \quad \text{and} \quad b_{dg} = -\frac{\bar{G}}{1 - e^{\mu_{yc}} - \bar{G}}.$$

The dividend growth equation in (13) follows by making

$$b_{dc} = 1 + b_{dg}b_{gc}, \quad b_{dl} = b_{dy}b_{yl} + b_{dg}b_{gl}, \quad b_{d\xi} \equiv 0 = b_{dy} + b_{dg}b_{g\xi}, \quad \text{and} \quad \sigma_d = b_{dg}\sigma_{gd}.$$

Table 1: Summary Statistics for Growth Rates

In Table 1, we present summary statistics for consumption, leisure, wage, and dividend growth. Consumption is defined as log real consumption of nondurable goods and services per capita, where consumption expenditures are obtained from the Bureau of Economic Analysis (BEA). Leisure is the fraction of non-sleeping hours ($16 \times 7 = 112$) devoted to leisure, or one minus the fraction of non-sleeping hours dedicated to work. Work hours are determined using data from the Bureau of Labor Statistics available on Valerie Ramey’s website and employed in Ramey and Francis (2009*b*). Wages are the natural log of real per capita aftertax labor income, as defined in Lettau and Ludvigson (2005), divided by the number of work hours per year. Labor income data is also obtained from the BEA. Dividends per share are constructed using the CRSP value-weighted portfolio cum- and ex-dividend return series. Dividends are the difference in the cum- and ex-dividend return multiplied by a cumulative capital gain index. Return data are sampled at the monthly frequency, summed to annual quantities, and deflated to real. Consumption and labor income are converted to per capita quantities using midperiod estimates of total population from the BEA. All variables are converted to real quantities using the Personal Consumption Expenditure (PCE) deflator. Data cover the period 1930-2011.

	Δc_t	Δl_t	Δw_t	Δd_t
Mean	1.99	0.27	2.70	1.38
Std. Dev.	2.25	1.08	3.46	10.82
AC(1)	0.47	0.28	0.18	0.21
AC(2)	0.15	0.13	0.11	-0.22

Table 2: **VAR of Consumption, Leisure, and Wage Growth**

Table 2 presents results of a vector autoregression (VAR) model for consumption, leisure, and wage growth,

$$\mathbf{y}_t = \mathbf{P}\mathbf{y}_{t-1} + \mathbf{u}_t,$$

and the loading of dividend growth on condition means of consumption and leisure growth,

$$\Delta d_t - \overline{\Delta d_t} = \phi_d \mathbf{x}_{t-1} + u_{d,t}.$$

where $\mathbf{y}_t = \{\Delta c_t - \overline{\Delta c_t}, \Delta l_t - \overline{\Delta l_t}, \Delta w_t - \overline{\Delta w_t}\}$, demeaned log consumption, leisure, and wage growth, respectively and $\Delta d_t - \overline{\Delta d_t}$ is demeaned dividend growth. The conditional mean variables, $\mathbf{x}_{t-1} = \{\mathbf{e}'_c \mathbf{P}\mathbf{y}_{t-1}, \mathbf{e}'_l \mathbf{P}\mathbf{y}_{t-1}, \mathbf{e}'_w \mathbf{y}_{t-1}\}$, where \mathbf{e}_c , \mathbf{e}_l , and \mathbf{e}_w are 3×1 vectors with ones in the first, second, and third elements, respectively, and zeros elsewhere. Consumption is defined as log real consumption of nondurable goods and services per capita, where consumption expenditures are obtained from the Bureau of Economic Analysis (BEA). Leisure is the fraction of non-sleeping hours ($16 \times 7 = 112$) devoted to leisure, or one minus the fraction of non-sleeping hours dedicated to work. Work hours are determined using data from the Bureau of Labor Statistics available on Valerie Ramey's website and employed in Ramey and Francis (2009b). Wages are the natural log of real per capita aftertax labor income, as defined in Lettau and Ludvigson (2005), divided by the number of work hours per year. Labor income data is also obtained from the BEA. Dividends per share are constructed using the CRSP value-weighted portfolio cum- and ex-dividend return series. Dividends are the difference in the cum- and ex-dividend return multiplied by a cumulative capital gain index. Return data are sampled at the monthly frequency, summed to annual quantities, and deflated to real. Parameters are estimated using GMM and standard errors are corrected for heteroskedasticity and autocorrelation using one Newey-West lag. Data are sampled at the annual frequency and cover the period 1930-2011.

	Δc_{t-1}	Δl_{t-1}	Δw_{t-1}
Δc_t	0.35	-0.40	0.03
SE	(0.15)	(0.12)	(0.07)
Δl_t	0.01	0.28	-0.11
SE	(0.07)	(0.10)	(0.04)
Δw_t	-0.21	-0.70	0.26
SE	(0.25)	(0.23)	(0.08)
Δd_t	2.61	-4.11	-4.00
SE	(3.99)	(16.21)	(7.17)

Table 3: **Principal Components Analysis**

We present an analysis of the principal components of the conditional mean of consumption, leisure, and wage growth in Table 3. Conditional means of growth rates are obtained from a vector autoregression,

$$\mathbf{y}_t = \mathbf{P}\mathbf{y}_{t-1} + \mathbf{u}_t,$$

where $\mathbf{y}_t = \{\overline{\Delta c}_t - \overline{\Delta c}_t, \overline{\Delta l}_t - \overline{\Delta l}_t, \overline{\Delta w}_t - \overline{\Delta w}_t\}$, demeaned log consumption, leisure, and wage growth, respectively and $\overline{\Delta d}_t - \overline{\Delta d}_t$ is demeaned dividend growth. The conditional mean variables are given by

$$\mathbf{x}_{t-1} = \{\mathbf{e}'_c \mathbf{P}\mathbf{y}_{t-1}, \mathbf{e}'_l \mathbf{P}\mathbf{y}_{t-1}, \mathbf{e}'_w \mathbf{y}_{t-1}\},$$

where \mathbf{e}_c , \mathbf{e}_l , and \mathbf{e}_w are 3×1 vectors with ones in the first, second, and third elements, respectively, and zeros elsewhere. Panel A presents the percentage of variation explained by each principal component and the component loadings. Panel B presents a vector autoregression (VAR) of the principal components. Consumption is defined as log real consumption of nondurable goods and services per capita, where consumption expenditures are obtained from the Bureau of Economic Analysis (BEA). Leisure is the fraction of non-sleeping hours ($16 \times 7 = 112$) devoted to leisure, or one minus the fraction of non-sleeping hours dedicated to work. Work hours are determined using data from the Bureau of Labor Statistics available on Valerie Ramey's website and employed in Ramey and Francis (2009b). Wages are the natural log of real per capita aftertax labor income, as defined in Lettau and Ludvigson (2005), divided by the number of work hours per year. Labor income data is also obtained from the BEA. Data are sampled at the annual frequency and cover the time period from 1930-2011.

Panel A: Principal Components Analysis

	<i>PC1</i>	<i>PC2</i>	<i>PC3</i>
% Explained	67.85	32.14	0.01
$x_{c,t}$	-0.22	0.97	0.12
x_l	-0.68	-0.24	0.69
x_w	0.70	0.07	0.72

Panel B: VAR of Principal Components

	$PC1_{t-1}$	$PC2_{t-1}$	$PC3_{t-1}$
$PC1_t$	0.23	0.00	14.44
SE	(0.11)	(0.16)	(7.42)
$PC2_t$	0.14	0.45	1.64
SE	(0.07)	(0.10)	(4.54)
$PC3_t$	0.00	0.00	0.19
SE	(0.00)	(0.00)	(0.11)

Table 4: Sensitivity of Dividend and Leisure Growth to Conditional Means

Table 4 presents the loadings of dividend growth on the conditional means of consumption and leisure growth, and loadings of leisure growth on the conditional mean of consumption growth. Conditional means are estimated using a vector autoregression,

$$\mathbf{y}_t = \mathbf{P}\mathbf{y}_{t-1} + \mathbf{u}_t,$$

where $\mathbf{y}_t = \{\Delta c_t - \overline{\Delta c}_t, \Delta l_t - \overline{\Delta l}_t, \Delta w_t - \overline{\Delta w}_t\}$, demeaned log consumption, leisure, and wage growth, respectively and $\Delta d_t - \overline{\Delta d}_t$ is demeaned dividend growth. The conditional mean variables are given by

$$\mathbf{x}_{t-1} = \{\mathbf{e}'_c \mathbf{P}\mathbf{y}_{t-1}, \mathbf{e}'_l \mathbf{P}\mathbf{y}_{t-1}, \mathbf{e}'_w \mathbf{y}_{t-1}\},$$

where \mathbf{e}_c , \mathbf{e}_l , and \mathbf{e}_w are 3×1 vectors with ones in the first, second, and third elements, respectively, and zeros elsewhere. Sensitivities are estimated from the following regressions:

$$\begin{aligned} \Delta d_t - \overline{\Delta d}_t &= \phi_{dx_c} x_{c,t-1} + \phi_{dx_l} x_{l,t-1} + e_{d,t} \\ \Delta l_t - \overline{\Delta l}_t &= \phi_{lx_c} x_{c,t-1} + e_{l,t}. \end{aligned}$$

VAR and sensitivities are estimated simultaneously using GMM, correcting for autocorrelation and heteroskedasticity using a single Newey-West lag. Consumption is defined as log real consumption of nondurable goods and services per capita, where consumption expenditures are obtained from the Bureau of Economic Analysis (BEA). Leisure is the fraction of non-sleeping hours ($16 \times 7 = 112$) devoted to leisure, or one minus the fraction of non-sleeping hours dedicated to work. Work hours are determined using data from the Bureau of Labor Statistics available on Valerie Ramey's website and employed in Ramey and Francis (2009b). Wages are the natural log of real per capita aftertax labor income, as defined in Lettau and Ludvigson (2005), divided by the number of work hours per year. Labor income data is also obtained from the BEA. Dividends per share are constructed using the CRSP value-weighted portfolio cum- and ex-dividend return series. Dividends are the difference in the cum- and ex-dividend return multiplied by a cumulative capital gain index. Return data are sampled at the monthly frequency, summed to annual quantities, and deflated to real. Data are sampled at the annual frequency and cover the period 1930-2011.

	ϕ_{dx_c}	ϕ_{dx_l}	ϕ_{lx_c}
Coeff.	4.34	6.22	-0.36
SE	(1.82)	(3.12)	(0.16)

Table 5: Conditional Variance of Consumption, Leisure, and Wage Growth

Table 5 presents results of the analysis of time variation in the volatility of consumption, leisure, and wage growth. Panel A presents variance ratios of residuals from a first stage VAR of consumption, leisure, and wage growth,

$$\mathbf{y}_t = \mathbf{P}\mathbf{y}_{t-1} + \mathbf{u}_t,$$

where $\mathbf{y}_t = \{\Delta c_t - \overline{\Delta c_t}, \Delta l_t - \overline{\Delta l_t}, \Delta w_t - \overline{\Delta w_t}\}$, demeaned log consumption, leisure, and wage growth, respectively. Variance ratios are computed as

$$VR_k = \frac{Var\left(\sum_{j=0}^{J-1} |u_{k,t+j}|\right)}{J \cdot Var(|u_{k,t}|)}$$

for $k = \Delta c, \Delta l, \Delta w$ and $J = 2, 5,$ and 10 years. Panel B presents estimates of an GARCH (1,1) model for the conditional variance of the innovations,

$$\sigma_{k,t}^2 = \kappa + \nu_k \sigma_{k,t-1}^2 + \vartheta u_{k,t-1}^2.$$

Consumption is defined as log real consumption of nondurable goods and services per capita, where consumption expenditures are obtained from the Bureau of Economic Analysis (BEA). Leisure is the fraction of non-sleeping hours ($16 \times 7 = 112$) devoted to leisure, or one minus the fraction of non-sleeping hours dedicated to work. Work hours are determined using data from the Bureau of Labor Statistics available on Valerie Ramey's website and employed in Ramey and Francis (2009b). Wages are the natural log of real per capita aftertax labor income, as defined in Lettau and Ludvigson (2005), divided by the number of work hours per year. Labor income data is also obtained from the BEA. Data are sampled at the annual frequency and cover the period 1930-2011.

Panel A: Variance Ratios

J	$u_{c,t}$	$u_{l,t}$	$u_{w,t}$
2	1.38	0.93	1.11
5	1.56	1.40	1.31
10	1.71	1.71	1.59

Panel B: GARCH Estimates

	$u_{c,t}$	$u_{l,t}$	$u_{w,t}$
GARCH	0.29	0.78	0.83
SE	(0.22)	(0.09)	(0.05)
ARCH	0.68	0.18	0.12
SE	(0.27)	(0.08)	(0.05)

Table 6: Common Parameter Values Across Model Calibrations

Common parameter values for five different model calibrations. The baseline model corresponds to the case of $\alpha = 0$. A model labeled “ $\rho = j$ ” is a model where the elasticity of substitution between consumption and leisure is set at $\rho = j$. For all specifications, $I_k=1$ for $k = \{c, l, x\}$, and $I_d = 0$.

Description	Parameter	Value
Elasticity parameter of intertemporal consumption	ψ	1.5
Average consumption growth	$\mu_c \times 10^3$	1.65
Volatility parameter of consumption growth	$\sigma_c \times 10^3$	6.31
Autocorrelation parameter of x_t	ϕ_x	0.975
Volatility parameter of x_t	$\sigma_x \times 10^4$	$0.0488\sigma_c$
Average log-leisure	\bar{l}	-0.4
Loading of leisure growth on x_t	ϕ_{lx}	-0.21
Volatility parameter of leisure growth	$\sigma_l \times 10^3$	3.62
Correlation parameter of leisure and consumption growth	σ_{lc}	-0.13
Autocorrelation parameter of time-varying volatility	ϕ_ν	0.995
Parameter of time-varying volatility	δ_ν	6.05
Parameter of time-varying volatility	$\varsigma_\nu \times 10^4$	8.26

Table 7: Model Specific Parameter Values for the Calibrations

Specific parameter values for five different model calibrations. The common parameter values across models are presented in table 6. The baseline model corresponds to the case of $\alpha = 0$. Dividend growth in the baseline model is specified as

$$\Delta d_{t+1} = \mu_c + \phi_{dx}x_t + \sigma_{d,t}\varepsilon_{d,t+1} + \sigma_{dc}\sigma_c\varepsilon_{c,t+1}.$$

The parameter values for the baseline model calibration are $\beta = 0.99964$, $\phi_{dx} = 2.5$, $\sigma_d = 4.75\sigma_c$, $\sigma_{dc} = 3.31$, and $I_d = 0$.

Models labeled “ $\rho = j$ ” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. The dividend growth for these models is specified as

$$\Delta d_{t+1} = \mu_c + b_{dc}(\Delta c_{t+1} - \mu_c) + b_{dl}\Delta l_{t+1} + \sigma_{d,t}\varepsilon_{d,t+1}.$$

The parameter values for these models are $b_{dc} = 2.86$, $b_{dl} = -0.5$, $\sigma_d = 4.7544\sigma_c$, and $I_d = 0$.

Description	Parameter	$\rho = 0.5$	$\rho = 1$	$\rho = 1.5$	$\rho = 5$
Subjective discount factor	β	0.99936	0.99973	0.99973	0.999725
Risk aversion parameter	γ	11.30	10.63	10.59	10.55
Loading of leisure in the utility function	α	0.5635	0.6583	0.6876	0.7262
Elasticity parameter of leisure and consumption	ρ	0.5	1	1.5	5
Volatility parameter of ξ_t	$\sigma_{\xi c}$	0.04	0.32	0.40	0.51
Volatility parameter of ξ_t	$\sigma_{\xi l}$	3.66	2.65	2.33	1.84
Coeff. of risk aversion (consumption gambles)	\mathcal{R}^c	3.86	3.63	3.62	3.61

Table 8: Moments of Growth Rates in Consumption and Dividends

This table contains data and model means, standard deviations, and autocorrelations for growth in log consumption of nondurables and services and dividends per share. $AC(\cdot, j)$ denotes the autocorrelation of order j . The model statistics are the median of 1,000 simulations of 984 months each, aggregated to the annual frequency. The “Baseline” column corresponds to the case of $\alpha = 0$. The “Leisure” column corresponds to the model calibrations with leisure preferences ($\alpha > 0$). Parameter values for the model calibrations are presented in Tables 6 and 7.

	Data	Model	
	1930-2011	Baseline	Leisure
$\mathbb{E}[\Delta c]$	1.99	1.97	1.97
$\sigma(\Delta c)$	2.25	2.25	2.26
$AC(\Delta c, 1)$	0.47	0.47	0.47
$AC(\Delta c, 2)$	0.15	0.22	0.22
$\mathbb{E}[\Delta d]$	1.38	1.98	2.03
$\sigma(\Delta d)$	10.82	10.81	10.81
$AC(\Delta d, 1)$	0.21	0.29	0.32
$AC(\Delta d, 2)$	-0.22	0.04	0.07
$\text{corr}(\Delta c, \Delta d)$	0.62	0.62	0.62
$\text{corr}(\Delta c, \Delta l)$	-0.35	-	-0.35
$\text{corr}(\Delta l, \Delta d)$	-0.25	-	-0.27

Table 9: Annualized Time Average Statistics for Financial Asset Variables

This table contains data and model means, standard deviations, and autocorrelations for financial asset pricing variables, specifically the risk-free rate, the price-dividend ratio, and the equity claim on dividends. $AC(\cdot, j)$ denotes the autocorrelation of order j . SR_b denotes the Sharpe Ratio for claims on cashflows b . The model statistics are the median of 1,000 simulations of 984 months each, aggregated at annual frequency. The baseline model corresponds to the case of $\alpha = 0$. Models labeled “ $\rho = j$ ” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. Parameter values for the model calibrations are presented in Tables 6 and 7.

	Data	Model				
	1930-2011	Baseline	$\rho = 0.5$	$\rho = 1$	$\rho = 1.5$	$\rho = 5$
Risk-free rate						
$\mathbb{E}[r]$	0.92	0.92	1.36	0.92	0.92	0.92
$\sigma(r)$	3.40	1.02	1.26	1.06	1.00	0.91
$AC(r, 1)$	0.66	0.80	0.79	0.80	0.80	0.80
Price-dividend ratio						
$\mathbb{E}[p_d]$	3.33	3.20	3.11	3.22	3.23	3.22
$\sigma(p_d)$	40.69	18.46	18.83	19.66	19.77	19.96
$AC(p_d, 1)$	0.86	0.72	0.73	0.73	0.73	0.73
Claim on dividends						
$\mathbb{E}[xr_d]$	5.13	5.15	5.13	5.13	5.13	5.14
$\sigma(xr_d)$	19.37	15.82	15.71	16.10	16.18	16.31
SR_d	0.37	0.39	0.39	0.38	0.38	0.38

Table 10: **Prices of Risk and Equity Risk Premia**

This table contains the market prices of risk for the four shocks affecting the model economy and the percentage contribution of each shock to the equity premium for different calibrations. The baseline model corresponds to the case of $\alpha = 0$. Models labeled “ $\rho = j$ ” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. Parameter values for the model calibrations are presented in Tables 6 and 7.

	Baseline	$\rho = 0.5$	$\rho = 1$	$\rho = 1.5$	$\rho = 5$
Prices of Risk					
λ_c	10.00	10.50	9.81	9.74	9.66
λ_l	-	6.12	6.34	6.53	6.81
λ_x	369.67	359.90	342.28	340.96	339.58
λ_ν	-2.33	-2.16	-2.14	-2.11	-2.08
Contribution to the Equity Premium (%)					
ε_c	26.06	24.42	22.78	22.58	22.31
ε_l	-	-0.80	-0.84	-0.86	-0.89
ε_x	44.59	50.04	51.34	52.10	53.08
ε_ν	29.35	26.34	26.73	26.18	25.50

Table 11: **The Portfolio of Aggregate Wealth**

This table contains data and model means, standard deviations, and autocorrelations for the portfolio of aggregate wealth and the ratio of aggregate wealth to consumption of nondurables and services. $AC(\cdot, j)$ denotes the autocorrelation of order j . SR_b denotes the Sharpe Ratio for claims on cashflows b . The model statistics are the median of 1,000 simulations of 984 months each, aggregated at annual frequency. The baseline model corresponds to the case of $\alpha = 0$. Models labeled “ $\rho = j$ ” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. Parameter values for the model calibrations are presented in Tables 6 and 7.

	Model				
	Baseline	$\rho = 0.5$	$\rho = 1$	$\rho = 1.5$	$\rho = 5$
Wealth-consumption ratio					
$\mathbb{E}[p_a]$	5.83	5.21	6.90	6.95	6.94
$\sigma(p_a)$	3.66	3.22	3.16	3.12	3.08
$AC(p_a, 1)$	0.73	0.68	0.71	0.71	0.71
Wealth portfolio					
$\mathbb{E}[xr_a]$	1.35	1.17	1.16	1.16	1.16
$\sigma(xr_a)$	2.82	2.87	2.63	2.60	2.60
SR_a	0.50	0.42	0.45	0.46	0.46

Table 12: Moments of Growth Rates in Leisure, Wages, and Labor Income

This table contains data and model means, standard deviations, and autocorrelations for macroeconomic variables. $AC(\cdot, j)$ denotes the autocorrelation of order j . The model statistics are the median of 1,000 simulations of 984 months each, aggregated to the annual frequency. The baseline model corresponds to the case of $\alpha = 0$. Models labeled “ $\rho = j$ ” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. Parameter values for the model calibrations are presented in Tables 6 and 7.

	Data	Model			
	1930-2011	$\rho = 0.5$	$\rho = 1$	$\rho = 1.5$	$\rho = 5$
$\mathbb{E}[\Delta l]$	0.27	0.00	0.00	0.00	0.00
$\sigma(\Delta l)$	1.08	1.07	1.07	1.07	1.07
$AC(\Delta l, 1)$	0.28	0.28	0.28	0.28	0.28
$AC(\Delta l, 2)$	0.13	0.03	0.03	0.03	0.03
$\mathbb{E}[\Delta w]$	2.70	1.98	1.97	1.97	1.96
$\sigma(\Delta w)$	3.46	3.45	3.45	3.45	3.45
$AC(\Delta w, 1)$	0.18	0.43	0.37	0.36	0.33
$AC(\Delta w, 2)$	0.11	0.18	0.12	0.10	0.08
$\mathbb{E}[\Delta y]$	2.22	1.97	1.97	1.97	1.98
$\sigma(\Delta y)$	3.82	3.82	3.82	3.82	3.82
$AC(\Delta y, 1)$	0.41	0.52	0.46	0.44	0.41
$AC(\Delta y, 2)$	0.16	0.27	0.20	0.19	0.16
$\mathbb{E}[y - c]$	-0.07	-0.07	-0.07	-0.07	-0.07
$\sigma(y - c)$	9.01	10.91	9.09	8.55	7.98
$AC(y - c, 1)$	0.95	0.99	0.98	0.98	0.98
$AC(y - c, 2)$	0.86	0.96	0.95	0.95	0.94
$\text{corr}(\Delta c, \Delta l)$	-0.35	-0.35	-0.35	-0.35	-0.35
$\text{corr}(\Delta c, \Delta w)$	0.58	0.88	0.88	0.87	0.87
$\text{corr}(\Delta l, \Delta d)$	-0.25	-0.27	-0.27	-0.27	-0.27
$\text{corr}(\Delta l, \Delta w)$	0.01	0.14	0.15	0.16	0.16
$\text{corr}(\Delta d, \Delta w)$	0.40	0.52	0.52	0.52	0.51
$\text{corr}(\Delta c, \Delta y)$	0.68	0.99	0.99	0.99	0.99
$\text{corr}(\Delta l, \Delta y)$	-0.47	-0.45	-0.44	-0.44	-0.44
$\text{corr}(\Delta d, \Delta y)$	0.48	0.63	0.62	0.62	0.62
$\text{corr}(\Delta w, \Delta y)$	0.88	0.83	0.83	0.83	0.83

Table 13: Annualized Time Average Statistics for Human Capital Returns

This table contains data and model means, standard deviations, and Sharpe Ratios for returns on claims to human capital, where SR_b denotes the Sharpe Ratio for claims on cashflows b . The model statistics are the median of 1,000 simulations of 984 months each, aggregated at annual frequency. Models labeled “ $\rho = j$ ” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. Parameter values for the model calibrations are presented in Tables 6 and 7.

	Model			
	$\rho = 0.5$	$\rho = 1$	$\rho = 1.5$	$\rho = 5$
Claim on labor income				
$\mathbb{E}[xr_y]$	3.26	3.12	3.05	2.97
$\sigma(xr_y)$	6.41	6.50	6.45	6.38
SR_y	0.53	0.50	0.49	0.49
Claim on wages				
$\mathbb{E}[xr_w]$	2.47	2.36	2.30	2.23
$\sigma(xr_w)$	5.10	5.19	5.17	5.09
SR_w	0.51	0.48	0.47	0.46

Table 14: Correlations of Human Capital Claims

This table contains data and model correlations for macroeconomic and asset pricing variables. The model statistics are the median of 1,000 simulations of 984 months each, aggregated at annual frequency. The baseline model corresponds to the case of $\alpha = 0$. Models labeled “ $\rho = j$ ” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. Parameter values for the model calibrations are presented in Tables 6 and 7.

	Baseline	Model			
		$\rho = 0.5$	$\rho = 1$	$\rho = 1.5$	$\rho = 5$
Excess Returns					
$\text{corr}(xr_a, xr_d)$	0.73	0.67	0.70	0.70	0.68
$\text{corr}(xr_c, xr_d)$	0.73	0.48	0.70	0.73	0.75
$\text{corr}(xr_y, xr_d)$		0.75	0.76	0.76	0.76
$\text{corr}(xr_w, xr_d)$		0.66	0.66	0.66	0.65

Table 15: **Human Capital Risk Premia**

This table contains the market prices of risk for the four shocks affecting the model economy and the percentage contribution of each shock to human capital premia for different calibrations. Models labeled “ $\rho = j$ ” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. Parameter values for the model calibrations are presented in Tables 6 and 7.

	$\rho = 0.5$	$\rho = 1$	$\rho = 1.5$	$\rho = 5$
Prices of Risk				
λ_c	10.50	9.81	9.74	9.66
λ_l	6.12	6.34	6.53	6.81
λ_x	359.90	342.28	340.96	339.58
λ_ν	-2.16	-2.14	-2.11	-2.08
Contribution to the Premium in Labor Income Claims (%)				
ε_c	26.30	27.34	28.08	29.07
ε_l	-13.03	-10.30	-9.55	-8.35
ε_x	50.22	45.85	45.12	44.04
ε_ν	36.51	37.11	36.35	35.23
Contribution to the Premium in Wage Claims (%)				
ε_c	29.66	31.13	32.14	33.43
ε_l	-8.72	-4.57	-3.18	-1.02
ε_x	39.86	33.99	32.70	30.83
ε_ν	39.20	39.44	38.33	36.76

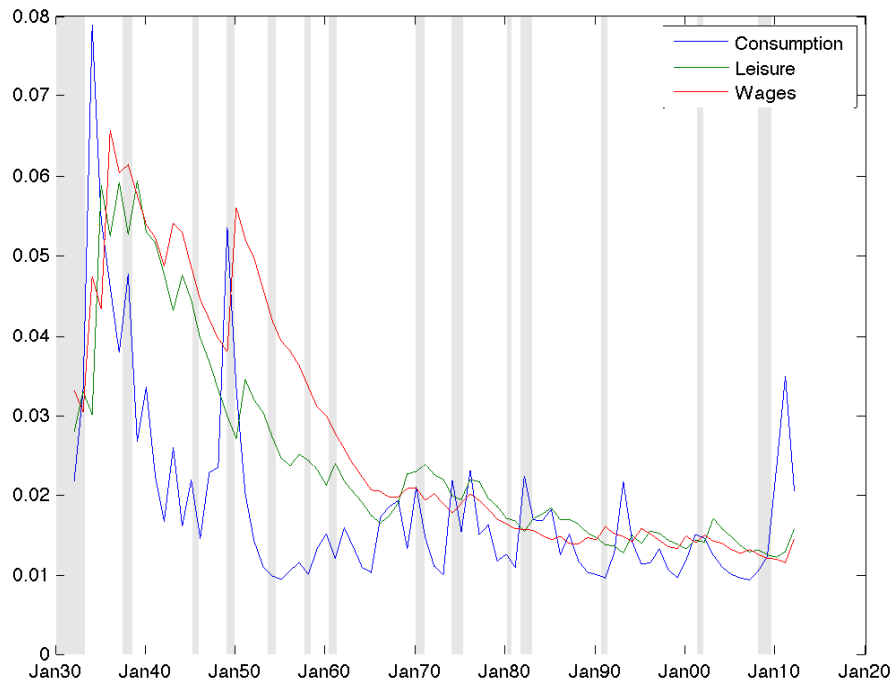


Figure 1: **Time Series of Conditional Volatility**

Figure 1 presents time series of the conditional volatility of consumption, leisure, and wage growth. Volatility is estimated using a GARCH (1,1) model on VAR residuals,

$$\begin{aligned} \mathbf{y}_t &= \mathbf{P}\mathbf{y}_{t-1} + \mathbf{u}_t, \\ \sigma_{k,t}^2 &= \kappa + \nu_k \sigma_{t-1}^2 + \vartheta u_{t-1}^2, \end{aligned}$$

where $\mathbf{y}_t = \{\Delta c_t - \overline{\Delta c_t}, \Delta l_t - \overline{\Delta l_t}, \Delta w_t - \overline{\Delta w_t}\}$, demeaned log consumption, leisure, and wage growth, respectively and $k = \Delta c, \Delta l, \Delta w$. Consumption is defined as log real consumption of nondurable goods and services per capita, where consumption expenditures are obtained from the Bureau of Economic Analysis (BEA). Leisure is the fraction of non-sleeping hours ($16 \times 7 = 112$) devoted to leisure, or one minus the fraction of non-sleeping hours dedicated to work. Work hours are determined using data from the Bureau of Labor Statistics available on Valerie Ramey's website and employed in Ramey and Francis (2009b). Wages are the natural log of real per capita aftertax labor income, as defined in Lettau and Ludvigson (2005), divided by the number of work hours per year. Labor income data is also obtained from the BEA. Data are sampled at the annual frequency and cover the period 1930-2011.

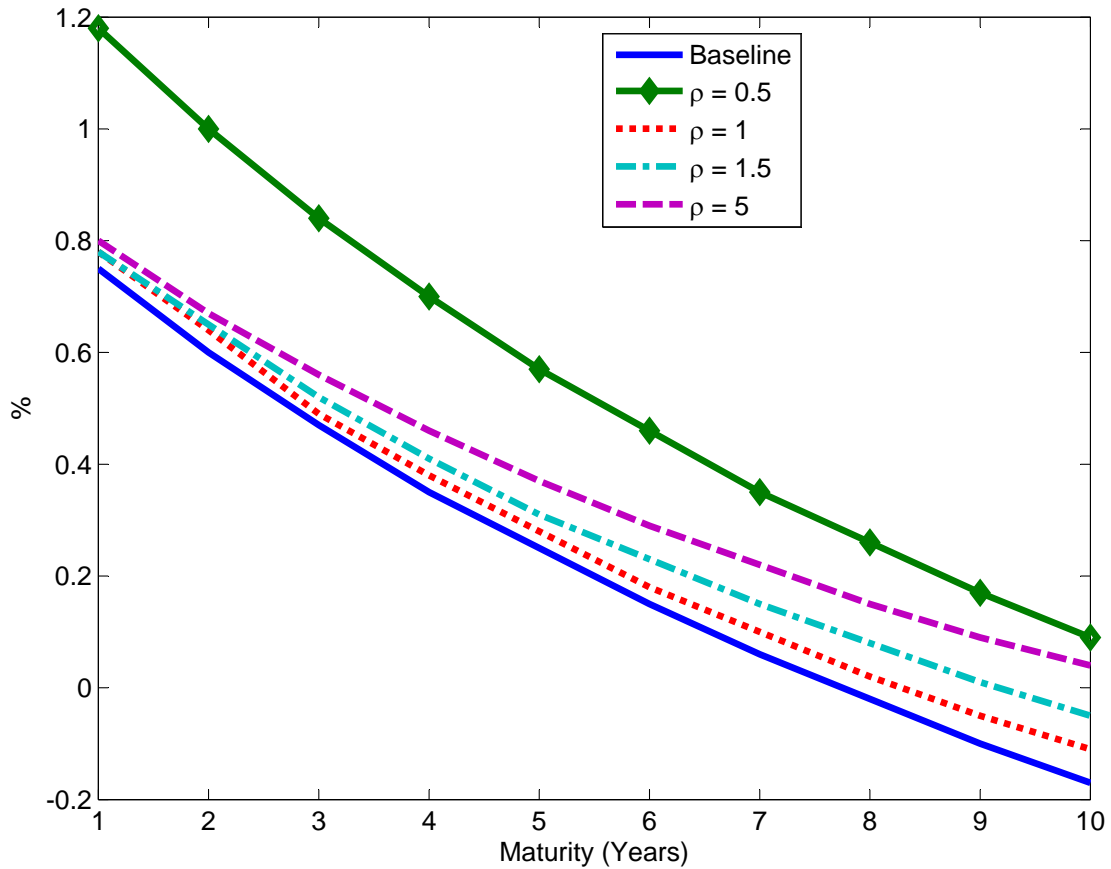


Figure 2: **Term Structure of Real Yields**

This figure plots real yields for the model calibrations for annual maturities from 1 to 10 years. The baseline model corresponds to the case of $\alpha = 0$. Models labeled “ $\rho = j$ ” are models where the elasticity of substitution between consumption and leisure is set at $\rho = j$. Parameter values for the model calibrations are presented in Tables 6 and 7.