Abstract

Why do financial crises lead to very slow recoveries? First, I document that firms which are intensive in innovation (intangible capital formation) are less able to engage in volatile external financing flows. The effect is primarily due to debt financing; equity financing acts as a partial substitute. Then, I develop a business cycle model with endogenous innovation that incorporates these facts in order to explain the short and medium-run effects of financial shocks. The increases in the cost of debt and venture capital financing during the Great Recession can explain an important part of the ensuing Great Deviation of output from trend, as the reduction in innovation amplifies persistence.
1 Introduction

Why do financial crises lead to very slow recoveries (see Reinhart and Rogoff (2009))? Conditional on the magnitude of the fall in output, recessions that are accompanied by default and/or restructuring episodes in the financial sector tend to last longer (Boissay et al. (2013)). A prominent example is the crisis that started in the US in late 2007. As of the second quarter of 2013, output is still well below its long-run trend, as shown in Figure 1. What started as the Great Recession has become the Great Deviation: growth is positive but not strong enough to bring the economy back to its previous trend. The goal of this paper is to develop a model to explain the impact of financial crises —defined as increases in the cost of firms’ external financing— on short- and medium-run economic fluctuations through their effect on innovation. Such a model will allow me to quantify the extent to which the Great Deviation is due to financial shocks.

Figure 1: US real GDP per capita vs trend

In order to guide the modeling assumptions, I describe the key empirical patterns on the financing of ideas. First, I document two novel facts regarding the relationship
between external financing and innovation with firm-level data. One, that firms which are more intensive in innovation are less able to raise external financing flows in general. Two, that innovation intensity predicts lower debt financing, but equity financing acts as a partial substitute. The rationale for these empirical results is that innovation produces intangible assets (ideas) that are not easy to pledge as debt collateral. Equity financing, which does not include a claim to a company’s assets in case of default, does not seem to be as sensitive to the collateralizability of assets. Thus, even if equity financing has other costs, it will be preferentially used as a source of external financing by the innovation-intensive firms which do not have internal funds. In addition, I summarize the stylized facts about the financing of start-up firms and the recent evolution of the spreads or premiums related to each type of external financing.

Next, I develop a business cycle model with endogenous innovation in which the financial structure of different firms depends on the nature of their investment, as seen in the data. In the model, final goods producers, who invest in tangible assets, are able to raise debt. Innovators, who invest in intangible assets, have to rely instead on internal financing or on equity financing when internal funds are not available. Incumbent innovators can use accumulated past and current profits to finance innovation, but new innovator entrants, which have not accumulated profits yet, must raise external equity.

External financing (debt and equity) is provided by a financial intermediary with real costs per unit of financing issued. The cost of each type of external financing varies exogenously and is the source of economic fluctuations. A shock to these costs affects the number of final good producers and the number of innovators who will be active in the following period. This in turn affects present and future output and thus the incentives to invest in physical capital and ideas. Endogenous innovation amplifies the effect of shocks over the medium run. Therefore, the model will permit me to quantify the short- and medium-run effects on economic aggregates of the movements in spreads that we observe in the data.

In reality, there may be other kinds of shocks simultaneously affecting spreads and output. What my model computes is the fall in output during the Great Deviation which is directly caused by the change in external financing costs. In other words, it determines the additional effect compared to a world where firms were not subject to financial disruptions.
The rest of the paper is organized as follows. Section 2 highlights my main contributions to the literature. Section 3 presents the empirical facts about the financing of ideas. Section 4 develops the model, defines an equilibrium, describes the calibration and presents the main results. Section 5 concludes and establishes paths for future research.

2 Related Literature

My research question lies at the intersection of two large fields. On the one hand, there is an ample body of research studying the impact of financial frictions on the economic cycle, starting with Bernanke and Gertler (1989). However, this literature does not consider the medium-run effects (frequencies between 8 and 50 years) that arise when innovation is endogenized. On the other hand, another branch of the literature (e.g. Rajan and Zingales (1998)) uses endogenous growth models to study the relationship between long-run growth and finance.

My contribution tries to fill the gap in between by recognizing that changes in innovation can lead to the medium-term fluctuations that are not usually studied in the business cycle literature. The reference work on this point is Comin and Gertler (2006), hereafter CG, who develop a real business cycle model with endogenous growth and adoption of technologies to explain the short and medium-term cycles. However, CG do not introduce financial frictions. I will take their framework as a baseline and model the channels through which different types of firms obtain financing more realistically, justified by a previous empirical study. Limiting the extent of external financing to the values observed in the data is a step beyond many recent papers on financial intermediation, which require the intermediaries, and in particular banks, to channel all firms’ financing in order to generate sizable output effects of financial crises (e.g. Gertler and Karadi (2011)). Even in models of financial frictions without an explicit financial intermediary, the fraction of capital owned by the agents with access to the productive technology (the equivalent of internal financing) usually needs to take an unrealistically low value (a popular example is Bernanke et al. (1999)). Moreover, the sources of fluctuations in my model, financial spreads, are empirically measurable variables, whereas CG use a more abstract wage markup shock which has to be indirectly cali-
brated. On top of producing business cycle statistics, I can determine the quantitative effects of specific historical episodes of financial crises.

The spirit of my approach is similar to Hall (2013) in that financial wedges are taken as an exogenous source of shocks and the emphasis is on studying the response of real aggregate variables to these shocks. Hall focuses on unemployment; I will focus on the interplay between production of final goods and innovation. Gilchrist and Zakrajšek (2012) use a reduced form approach to show that their measure of the spreads on debt financing has predictive power over real aggregate outcomes, but they are silent on the structural mechanism.

The methods used in the empirical part of this paper are similar to those in the corporate finance literature that measures the correlation between the capital structure of firms on one side and tangibility or book-to-market ratios on the other (e.g. Frank and Goyal (2004)). However, I am interested in describing the properties of the flows of external financing rather than the structure of the stock of capital because the latter does not contain information on the dynamics of financing. The dispersion of external financing flows is a better indicator of the degree to which a firm can raise external financing at the moment an investment opportunity materializes.

3 Empirical Analysis: The Financing of Ideas

The role of the empirical facts that I document in this section is to guide the assumptions of the model and its calibration. My ultimate goal is to understand how shocks in the financial sector of the economy can have effects over the short and medium-term business cycle through their impact on the adoption of new ideas, which is a persistent source of fluctuations. In order to do so, it is necessary to first describe the financing process of firms as a function of their innovation intensity. Table I summarizes the equivalence between firms in the data and in the model, as well as their main sources of financing. The last column in the table is both a stylized summary of the empirical patterns and a description of my modeling assumptions. In this section, I will firstly focus in the distinction between final good and incumbent innovator firms. In the data, as I explain below, these two categories are defined by the tangibility of a firm’s assets. Next, I will review the existing evidence on the financing of entrant innovators, more
commonly known as technology start-ups. Finally, I will comment on the time series of the spreads or premiums to the types of external financing that are relevant to my model.

Table 1: Equivalence between firms in the data and in the model

<table>
<thead>
<tr>
<th>Data</th>
<th>Example</th>
<th>Model</th>
<th>Financing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Publicly listed, high tangibility</td>
<td>General Motors Co.</td>
<td>Final good producers</td>
<td>Debt &amp; Internal Eq.</td>
</tr>
<tr>
<td>Publicly listed, low tangibility</td>
<td>Google Inc.</td>
<td>Incumbent innovators</td>
<td>Internal Eq.</td>
</tr>
<tr>
<td>Venture capital start-ups</td>
<td>WhatsApp Inc.</td>
<td>Entrant innovators</td>
<td>External Eq. (VC)</td>
</tr>
</tbody>
</table>

3.1 Final Good Firms versus Incumbent Innovator Firms

3.1.1 Theoretical Motivation:

In line with the recent growth-accounting frameworks in the literature (Corrado et al. (2012), etc.), I define innovation as a broader concept than formally reported R&D, including any expenditures in intangibles aimed at increasing the long-run (horizons longer than one year) productivity of a firm. Therefore, throughout this section, I will use intangibility as a synonym of innovative intensity. Still, R&D is positively correlated with my measure of intangibility at the industry level.

In terms of financing, my first hypothesis is that firms are able to raise more external funds by collateralizing their tangible assets than by collateralizing their intangibles. This is relevant for the distinction between final good and innovator firms because investment by final good firms is in physical capital, which is a tangible asset, while investment by innovator firms is in ideas, which are intangible. Innovation, as opposed to formation of tangible capital, generates less collateralizable assets to issue external financing against.

The effects of tangibility on the financing structure of a firm have been extensively studied in the literature. On the one hand, corporate finance economists have carefully documented the relationship between the capital structure of a firm and the market-to-book ratio or tangibility of assets, concluding that such variables have low explanatory power but are significant (e.g. Rauh and Sufi (2010)). On the other hand, growth
economists have tried to relate the growth of an industry to its flow external financing needs (e.g. Rajan and Zingales (1998)).

I will adapt the empirical analysis to my particular goal, which is to understand the consequences of the constraints faced by different types of firms when trying to raise new external financing. Then, it is useful to follow the flow approach to external financing, instead of looking at the stock capital structure, which does not contain information about the timing of flows. Besides, I want to avoid some of the pitfalls of the growth literature, such as looking at average net financial flows during a period of time, which may only capture temporary growth opportunities in a sector and should not differ substantially in the long run. Instead, I look at the ability of firms to engage in more volatile or wider financial flows, particularly at the upper end (positive external financing).

In accounting terms, the net (of outstanding debt) amount of tangible assets corresponds to the book value of the equity of a firm minus the book value of its intangible assets. I denote this difference $B$. Then, the difference between market value $M$ and net tangible book value $B$ reflects the net value of intangible assets. This is, I take into account both intangible assets which are recorded in the book value of a firm and those which are not, since innovation is only recorded in the books as long as it involves a purchase from another firm. Items such as own innovation taking place within the firm, organizational capital or brand value do not appear in the books but are captured by the difference between book and market value$^{1}$ and they constitute the lion’s share of intangible assets. My measure of tangible assets also includes financial or current assets, which are easily collateralizable. The qualitative results remain if I define tangibility as the ratio between Property Plant and Equipment (PPE) and total productive assets (PPE plus total intangible assets). Yet, I do not use this measure in the analysis because it does not reflect assets net of outstanding debt, as I have no information on the fraction of PPE which has already been pledged as collateral.

Let $i$ index a firm and $t$ a time period. The hypothesis that external financing of firms $XF$ is limited by their net tangible assets more than by their net intangible assets

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$^{1}$One could argue that the intangible assets that have enough legal structure to be recorded in the book value can also be pledged as collateral. None of the qualitative results change remarkably if intangible assets in the books are instead classified as tangibles.
can be formalized as:

\[ XF_{it} \leq \beta_1 B_{it} + \beta_2 (M_{it} - B_{it}), \quad \beta_1 > \beta_2 \geq 0 \]

Moreover, on average firms must display non-positive external financing flows in the long run (otherwise they would be valueless for external investors). Normalizing both sides of the inequality by the market value, which is equal to total net assets, the reduced-form implication is then that firms with a higher tangible-book-to-market ratio \( \frac{B}{M_{it}} \) will be able to engage in more volatile relative external financing flows \( \frac{XF}{M_{it}} \). Figure 2 depicts this hypothesis in a simple diagram. The red line at the top marks the maximum positive external financing that an individual firm can issue in a period given its current tangible-book-to-market value, where \( \alpha = \beta_2 \) and \( \beta = \beta_1 - \beta_2 \). The shaded area below is the region where firm-period observations live: firms with a higher \( \frac{B}{M_{it}} \) should display a wider dispersion of values for \( \frac{XF}{M_{it}} \).

Figure 2: Hypothesized external financing flows by tangible-book-to-market ratio

External financing is the sum of debt financing (DF) and equity financing (EF). Debt contracts, even if they have a preferential position in the pecking order of corporate financing, require a higher degree of collateralizability of assets than equity issuances. Therefore, my second hypothesis is that firms with a higher innovation in-

\footnote{The external financing flow is defined as investment minus the cash flow after repayment of labor, taxes and other operating expenses (see exact accounting definition in Rajan and Zingales (1998)). Investment is defined as capital expenditures plus R&D expenditures.}
tensity will be less able to rely on debt financing, and will have to (partially) substitute with equity financing.

3.1.2 Data

I use Compustat data on 3,179 non-financial publicly listed companies. I have an unbalanced panel for the period 1988-2012. As is common in the literature, I do not include outliers or observations with values inconsistent with the theoretical model of the firm. In particular, I drop observations with $M_{it}, B_{it-1}$ or lagged total assets ($S_{it-1}$) smaller than $10$ million; $\frac{B_{it-1}}{M_{it-1}} \notin (0,2); \frac{XF_{it}}{M_{it}} \notin (-2,2)$; negative investment or R&D expenditures; non-positive total investment or less than 2 years of age. This leaves 19,780 valid observations in total. Reasonable alterations in these thresholds do not lead to substantial changes in the number of surviving observations or the estimated statistics. Table 2 displays descriptive statistics for the main variables.

Table 2: Descriptive statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$XF_{it}$</td>
<td>19780</td>
<td>.029</td>
<td>.21</td>
<td>-1.9</td>
<td>2</td>
</tr>
<tr>
<td>$DF_{it}$</td>
<td>17873</td>
<td>.016</td>
<td>.21</td>
<td>-1.9</td>
<td>2.7</td>
</tr>
<tr>
<td>$EF_{it}$</td>
<td>17873</td>
<td>.0094</td>
<td>.093</td>
<td>-2.5</td>
<td>1.7</td>
</tr>
<tr>
<td>$\frac{B_{it-1}}{M_{it-1}}$</td>
<td>19780</td>
<td>.51</td>
<td>.39</td>
<td>.001</td>
<td>2</td>
</tr>
<tr>
<td>ln$S_{it-1}$</td>
<td>19778</td>
<td>5.6</td>
<td>1.9</td>
<td>-3.9</td>
<td>12</td>
</tr>
<tr>
<td>$age_{it}$</td>
<td>19780</td>
<td>16</td>
<td>12</td>
<td>2</td>
<td>53</td>
</tr>
</tbody>
</table>

3.1.3 Empirical Results

**Dispersion of external financing flows by tangibility** Since the pattern I want to emphasize is about the dispersion of financing flows, the most informative statistic is the empirical equivalent of Figure 2, which is shown in Figure 3. The lines correspond to 5th and 95th percentiles of relative external financing $\frac{XF_{it}}{M_{it}}$ by tangible-book-to-market bins $\frac{B_{it-1}}{M_{it-1}}$ while the mean for each side of the sample, $XF_{it} \geq 0$ and $XF_{it} < 0$, is depicted with a dot. I include a bin for firms with tangible-book-to-market between one and two for transparency, but these observations are not consistent with my model, since they imply a negative value of intangible assets. The shaded areas
around the lines represent asymptotic 95 percent confidence intervals, counting each firm as a single observation. An obvious issue when analyzing the correlation between $XF_{M_{it}}$ and $B_{M_{it}}$ is that the absolute value of both variables depends positively on the absolute value of investment, since external financing equals investment minus cash flow whereas tangible capital measured at the end of a period includes investment during the period. Hence, what I calculate is the average flow of external financing $XF_{M_{it}}$ by one-year-lagged tangible-book-to-market bin $B_{M_{it-1}}$. Since within-firm variation in $B_{M_{it-1}}$ is low and the average external financing flow is relatively stable, the correlation is mainly capturing differences in the cross-sectional and time-series dispersion of flows between firms, for different types of firms. I also decompose the effects between debt financing $DF_{M_{it}}$ and equity financing $EF_{M_{it}}$, given that: $XF_{M_{it}} = DF_{M_{it}} + EF_{M_{it}}$.  

The main takeaway is that the density of firms in the data lives in a region which is qualitatively similar to the shaded area in Figure 2. Firms with higher tangibility (higher B/M) have more disperse external financing flows. This is true for debt financing as well, but not for equity financing. Firms with very low tangibility of assets have wider equity financing flows. The dependence on tangibility seems stronger for firms with positive financial inflows, which is consistent with firms being uncertain about their future external financing needs. In other words, the relationship is stronger for firms which are constrained in the current period. It is also important to mention that equity financing only constitutes on average a fifth of total external financing for all firms getting positive external financing. Thus, higher equity financing for low tangibility firms does not compensate lower debt financing.

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3 Compustat has direct data on sales and purchases of equity. The difference of the two equals external equity financing. Debt financing is defined as the residual external financing.
Controlling for other determinants of external financing  There are many reasons other than the collateralizability of assets why external financing may be related to the tangible book value of a firm. Hence, in Figure 4 I provide a version of Figure 3 with the OLS residuals of external financing regressed on controls for other factors that have been deemed important in the corporate finance literature, instead of the raw dependent variable. Rajan and Zingales (1998) show that external financing is very high for the first years after the IPO and then decays hyperbolically until it stabilizes. To capture this pattern, I control for age and the inverse of age, where the age of a firm is the number of years since its IPO. Another factor discussed in the literature (e.g. Frank and Goyal (2004)) is the size of the firm. Large firms are supposed to have more flexibility and opportunities to raise external financing. Thus, I also control for the
natural log of total assets, which has more explanatory power than the level. To avoid capturing any cyclical effects at the aggregate level, I introduce time fixed effects. And to avoid capturing technological differences across industries I include industry (GIC group) fixed effects. Finally, I include tangibility-bin fixed effects so that the average residual in each bin is zero by construction.

The result is that the picture remains very similar after introducing the controls: the dispersion of residual external financing flows is increasing in tangibility and this relationship is driven by debt financing, with external equity financing becoming less disperse as tangibility increases. The conclusion is qualitatively the same if I plot 10th to 90th percentiles, introduce 2, 3 or 4-year lags, use different functional specifications for the controls or drop the years around the so-called Dot-Com Bubble (1999-2002).

Figure 4: Empirical 5th and 95th percentiles of regression residuals $\frac{XF}{M_t}$ by $\frac{B}{M_{t-1}}$ bins

Residuals after regressing $\frac{XF}{M_t}$ on the natural log of total assets, age, the inverse of age, lagged B/M dummies, GIC group dummies and time fixed effects.
Controlling for division bias  One could suspect the existence of a potential spurious correlation (a time-series form of division bias) between $\frac{XF}{M_{it}}$ and $\frac{B}{M_{it-1}}$, as $M_{it}$ is serially correlated and it appears in the denominator of the two variables of interest. To rule out the possibility that this spurious correlation is driving the results, I run a regression of $XF_{it}$ on $B_{it-1}$ and $(M_{it-1} - B_{it-1})$ in levels, for $XF_{it} > 0$ and $XF_{it} \leq 0$ separately, including year and GIC group fixed effects. For both sides of the sample, I obtain that the coefficient on tangibles is significantly greater in absolute value than the one on intangibles, and that only the former is significantly different from zero, with standard errors clustered at the firm level. An additional unit of tangible assets allows a firm to have about 0.1 additional units of financing flows, in both directions, whereas an additional unit of intangible assets appears to be useless as collateral. A log-log specification leads to a similar conclusion. Hence, the results shown above are not a mere consequence of division bias.

External financing supply vs demand  The reason why firms with more net tangible assets have wider external financing flows can be driven by differences in the supply or in the demand for external financing. My interpretation is that tangible assets are more easily collateralizable and so there is a greater supply of external financing to firms with a high $\frac{B}{M_{it-1}}$. Yet, the analysis so far does not rule out the possibility that these firms have an inherently more volatile productivity or consumer demand and are consequently forced to demand wider flows of external financing.

If that was the case, these firms should feature more volatile profits. However, the difference between the 1st and 3rd quartile of cash flows normalized by market value divided by the median of the same variable is actually decreasing in tangibility. Therefore, firms with more tangibility do not access a wider range of external financing flows because they have more volatile profits, but because they have greater/cheaper access to external financing. I use this measure of dispersion instead of standard deviations because Compustat cash flows are equal to profits minus investment in intangibles (which is classified as an operating expense). Thus, innovation-intensive firms will have by construction smaller cash flows and, assuming that profits and investment in intangibles are positively correlated, a smaller variance in cash flows.

To be clear, these results do not imply that my theoretical motivation is the only
mechanism behind the facts in the data. I simply provide a plausible rationalization of the facts and a general equilibrium model (in Section 4) which is qualitatively and quantitatively consistent with both the underlying motivation and the facts.

3.2 Entrant Innovator Firms

Kaplan and Strömberg (2003), Hall and Woodward (2007) and Hall and Woodward (2010) provide a comprehensive description of the data on the financing of start-ups, focusing on venture capital. These firms correspond to entrants in the innovation sector. Here I briefly summarize the key facts with regard to my model:

1. The most prevalent form of start-up financing is venture capital, which holds more than 50% of ownership rights. These financing contracts take the form of preferred stock.

2. The distribution of the payoffs of innovation projects is extremely skewed to the right. About three quarters of the projects financed never pay out anything to the original owners and 81% pay less than $3 million, compared to a mean payoff of $50 million.

3. The returns to venture capital imply a substantial premium over the stock market (27% on average in Hall and Woodward (2010)’s sample).

3.3 Costs of External Financing

Firms use different sources of external financing and each source entails a particular time-varying cost process. In Figure 5 I show the evolution of a measure of the spread or expected premium associated with each type of external financing around the years of the Great Recession (2002-2012). For debt financing, I plot the Gilchrist Zakrajšek (GZ) annual spread, which is shown to be negatively correlated with future output in Gilchrist and Zakrajšek (2012). For equity financing of venture capital (VC), I plot the one-year-ahead premium in venture capital predicted by the current dividend-price ratio in the stock market in the spirit of Cochrane (1999)’s predictability regressions.

The stock market index used is the annual S&P 500 weighted index from WRSD data.
The dependent variable in the OLS regression is the realized excess return on venture capital as measured by Sand Hill Econometrics between 1992 and 2011. The reason why I look at predicted values for VC returns is that the spread or premium must be known at the time when the financial contract is stipulated, before the actual payoff is realized. I use an annual frequency because in the model that will be the length of time between the first injection of capital in an entrant innovator and its IPO or exit. For comparability, I also plot the predicted premium for external equity in general (EQ).

Figure 5: Empirical spreads for debt (GZ), venture capital (VC) and equity (EQ)

GZ spreads include a default component on top of the pure premium. This is, the expected excess return is less than the GZ spread because some debt holders will default. Yet, the default component is much smaller and less volatile, both in the model and in the data, than the overall spread. VC and EQ spreads, instead, are equivalent to excess returns or premiums.

It is clear from the time series that the Great Recession was accompanied by a considerable increase in the costs of the two types of external financing as measured by the

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prematures. The central goal of this paper is to quantify the effects of such an increase over the short and medium-run level of the main aggregate variables, abstracting from other causes that may be explaining both the premiums and the aggregates. For example, a negative technology shock may have directly caused a fall in output and an increase in financing costs. I would then measure the additional fall in output which has been caused by the increase in financing costs.

4 Model: Endogenous Innovation and External Financing Shocks

I next develop a business cycle model with endogenous innovation to assess the impact on the aggregate economic variables of realistic financial shocks. The baseline is a simplified version of Comin and Gertler (2006) with only one final goods production sector, no shocks to the labor supply, only one representative household, and an exogenous growth process for the stock of basic ideas. The novelty with respect to CG is in the financial structure of the economy. It is still the case that the household owns the capital and the firms. However, firms can only raise external financing through a representative financial intermediary, also owned by the household, which has an exogenous time-varying resource cost which differs across the two types of external financing: debt and venture capital.

Firms need to raise external financing for two purposes. First, final good firms are required to meet a fixed tangible capital investment one period ahead of production. This can be thought of as a reduced form for a periodic liquidity requirement. They will meet this financing need with debt from the financial intermediary. Second, entrants in the innovation sector need to raise funds to invest in the adoption of ideas. They will do so by issuing preferred stock to the financial intermediary, which is able to partially monitor the otherwise unobservable investment of innovators. Incumbent innovators also invest in adopting ideas, but they finance internally with accumulated profits. Hence, the model is consistent with the empirical corporate finance facts previously described. Moreover, these two types of contracts will be optimal given the nature of investment in each sector, as will become clear below. I will now describe
the problem of each agent in the model and then define a competitive equilibrium.

4.1 Agents

4.1.1 The Household

There is a representative household that solves the typical consumption-savings problem of the Neoclassical Growth Model. The household owns the capital and all the firms in the economy, including the financial intermediary. Hence, it maximizes the net present value of utility flows subject to a budget constraint and a capital accumulation equation:

\[
\max_{\{C_{t+i}, L_{t+i}, K_{t+i}\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} \beta^i \left[ \ln C_{t+i} - \frac{(L_{t+i})^{1+\varsigma}}{1 + \varsigma} \right] \tag{1}
\]

s.t.

\[
C_t + I_t = W_t L_t + (1 + r^K_t)K_t + \Pi_t \tag{2}
\]

\[
K_{t+1} = I_t + (1 - \delta)K_t \tag{3}
\]

\(C_t\) denotes consumption, \(I_t\) investment in physical capital, \(L_t\) labor, \(W_t\) wages, \(K_t\) physical capital, \(r^K_t\) the net return on physical capital and \(\Pi_t\) profits from all firms, including the financial intermediary. The stochastic discount factor of the household \(\Lambda_{t+1}^{HH}\) is then given by:

\[
\Lambda_{t+1}^{HH} = \beta^{\frac{C_{t+1} - \theta}{\Pi_t}} \tag{4}
\]

4.1.2 The Financial Intermediary

There is a representative financial intermediary that charges an exogenous spread or premium for every unit of financing that it extends. The existence of a spread could be motivated by real operating costs or by an agency problem between financial institutions and households, as in Gertler and Karadi (2011). For simplicity, I will keep this variable as an exogenous source of shocks to the economy and model it as a real
The spread or premium depends on the nature of the investment which is being financed. The financial intermediary extends loans to pay for the initial fixed tangible capital investment needs of final good producers and owns venture capital in entrant innovators, as will become clear in the next subsections. The first type of financing is intended to model firms receiving liquidity shocks for which external debt acts as a more prompt source of funds than external equity. In the second type of financing the intermediary could also be thought of as a hedge fund owning a venture capital fund. What matters is that both are financial services that households cannot provide on their own and that cost physical resources.

I denote the cost per unit of financing associated with debt of final good producers as $r_{s,D}^t$ and the one associated with venture capital financing of entrant innovators as $r_{s,VC}^t$. The most realistic timing assumption is that these costs are known at the moment any financial contract is stipulated. The costs $r_t^s = \begin{pmatrix} r_{s,D}^t \\ r_{s,VC}^t \end{pmatrix}$ evolve as a VAR(1) process:

$$\hat{r}_t^s = R\hat{r}_{t-1}^s + \varepsilon_t, \quad \varepsilon_t \sim N(0, \Sigma), \quad R = \text{diag}(\rho^x)$$

(5)

where variables with hats denote deviations with respect to the mean. The financial intermediary has the same underlying intertemporal discounting as the household, but it must earn a higher equilibrium return on investments to pay for its operation cost. Thus, the intermediary effectively discounts the payoffs of each of the investments $x = \{D, VC\}$ at a rate:

$$\Lambda_t^{HH} = \frac{\Lambda_{t+1}^{HH}}{(1 + r_t^{s,x})}$$

(6)

There is perfect competition in the financial sector. Hence, the net present value of future repayment must equal the amount of financing issued for each of the two investments.

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6Modeling spreads as real resource costs is not a numerically crucial assumption. The impulse response functions of a model where spreads are rebated to households as a lump-sum transfer are very similar. Except for the first period, when the shock kicks in, the response of the aggregate variables to a shock stays between 85% and 100% of the response in the model with real costs and ends up converging as more periods elapse.
4.1.3 Final Goods Production

Aggregate output is defined as a CES composite of the existing $N_t^{fin}$ final good varieties:

$$Y_t = \left( \int_0^{N_t^{fin}} \left( Y_{t,j} \right)^{1/\mu} \text{d}j \right)^{\mu}$$

(7)

where $\mu > 1$. The composite $Y_t$ is the numeraire good in the economy. Each final good producer $j$ is active just for one period and produces a final good variety with a Cobb-Douglas production function:

$$Y_{t,j} = \left[ (K_{t,j})^\alpha (L_{t,j})^{1-\alpha} \right]^{1-\gamma} \left[ M_{t,j} \right]^{\gamma}$$

(8)

where the production factors are capital $K_{t,j}$, labor $L_{t,j}$ and a CES composite $M_{t,j}$ of all intermediate good varieties $A_{t,i}$:

$$M_{t,j} = \left( \int_0^{A_{t,i}} \left( M_{t,j}^{i,k} \right)^{1/\vartheta} \text{d}k \right)^{\vartheta}$$

(9)

with $\vartheta > 1$. Each final goods producer takes factor prices as given and sells its variety in monopolistic competition. Capital and labor are rented from households. Intermediate inputs are bought from innovator firms.

So far, the set-up is the same as in CG. In order to introduce a need for debt financing, I force firms to incur a fixed capital cost $\Psi_{t+1}$ one period before the period in which production occurs. There is a constant pool of potential producers in every period $\overline{N}$. They all pay the fixed cost, as ex-ante expected profits are zero, but only a fraction of them are going to produce in equilibrium. This fraction will be determined in the period when production takes place in order to satisfy free entry in production.

The debt contract is structured as follows. Each potential entrant borrows $\Psi_{t+1}$ from the financial intermediary in the period before production. If in the following period it decides to produce, it must repay a gross interest of $(1 + r_t^{fin})$ to the financial intermediary, i.e., the interest is specified when the loan is issued. If it does not produce, the financial intermediary can recover a fraction of the fixed capital investment $\eta \Psi_{t+1}$ and the firm owner is left with zero payoffs. Hence, by the free-entry condition,
potential entrants will enter final good production until profits net of repayment of the loan are zero. Due to observability of investment in tangible capital, debt financing is optimal in the final goods sector, and it will be strictly preferable to equity financing as long as the latter entails higher exogenous costs \( r_t^D > r_t^{VC} \), which is always the case in the data.

In order to obtain a balanced growth equilibrium I assume that the fixed cost is proportional to the current aggregate capital stock:

\[
\Psi_t = \psi K_t
\]  

This is, individual firms do not internalize the change in fixed costs when investing in capital, even if fixed costs paid today \( \Psi_{t+1} \) depend on the aggregate current investment \( I_t \), which fully determines the capital level in the next period \( K_{t+1} \).

### 4.1.4 Innovation

**Basic Research**  This model aims to describe the response of the economy to financial crises over the medium term, but not the very long-term trend, so it features endogenous innovation, not endogenous growth. The generation of basic ideas is kept exogenous as the (unique) source of long-run-trend growth for simplicity. The stock of basic ideas \( Z_t \) grows deterministically as follows:

\[
Z_t = \exp(g_z) Z_{t-1}
\]  

where \( g_z \) is the instantaneous rate of generation of new basic ideas. Innovators can pick these ideas for free and transform them into useful intermediate inputs, the process which is known as adoption. I view the generation of basic ideas as an activity conducted by universities and research centers which are heavily subsidized by the government and whose financial structure is very different from that of innovation firms. What firms really do is take basic ideas and convert them into productive items.

**Entrant Innovators**  All innovators (entrants and incumbents) have the capacity to freely pick an unadopted idea from the basic research pool and invest physical re-
sources in the probability of adopting it, i.e., turning it into an intermediate input. Entrant innovators choose the amount $H_t^{ent}$ to invest in the probability $\lambda^{ent}(\Gamma_t H_t^{ent})$ of adoption, where $\lambda^{ent}(\cdot)$ is a non-negative, strictly increasing and strictly concave function between zero and one. The scaling factor $\Gamma_t$ is given by

$$\Gamma_t = \frac{A_t}{K_t}$$

where $A_t$ is the stock of adopted ideas and $K_t$ the stock of physical capital. This scaling implies the presence of a positive spill-over from the stock of ideas adopted in the past relative to the current complexity of the economy as measured by the capital stock, and guarantees the existence of a balanced growth path.

Entrants finance their investment by issuing preferred stock. The financial contract is similar to the contracts that are usually observed in venture capital financing: it limits the liability of owners at the lower tail without claiming all the dividends at the upper tail, is linear on the outcome and provides monitoring services. It is structured as follows. Each adoption entrant raises $H_t^{ent}$ from the financial intermediary, which can partially monitor the investment of the firm. For each unit of funds raised that the firm diverts from investment, it can obtain $\chi$ consumption goods. Monitoring permits to have $\chi < 1$. The firm can be successful or unsuccessful in adopting an idea. If it is successful, in the following period it is bought by the representative household at a price $P_{t+1}^{ent}$ and must repay the venture capitalist a fraction $\nu_t P_{t+1}^{ent}$ of the sale. If it is not successful, it exits with zero liquidation value, since its investment is in intangible assets, which are fully firm-specific. The reason why entrants are bought by the representative household is that it has cash available, which allows entrants to innovate even in the period when they have to repay initial investors. The household is not able to monitor investment, so it cannot provide financing ex-ante, but it can observe the output of a firm and buy it ex-post. If I did not allow for this transaction, innovation of successful entrants would be limited by their reduced availability of cash and by the higher discounting that applies to external financing. Since competition between households bids away any rents from buying entrants, in equilibrium $P_t^{ent} = V_t^{inc}$. A successful entrant bought by a household automatically becomes a new incumbent.

Entrant innovators choose the amount to invest $H_t$ in order to maximize expected
profits given that they have raised $H_t^{ent}$ funds:

$$\max_{H_t \leq H_t^{ent}} \lambda^{ent}(\Gamma_t H_t) (1 - \nu_t) E_t \left[ \Lambda^{HH} \rho^{ent}_{t+1} \right] + \chi (H_t^{ent} - H_t)$$ (13)

Investment increases the probability of a successful sale of the firm (an IPO) but leaves less funds to divert into consumption. In order to make sure that no diversion occurs and the optimal investment $H_t = H_t^{ent}$ is implemented, the following incentive compatibility condition must be satisfied at the margin for entrant innovators:

$$\frac{\partial \lambda^{ent}(\Gamma_t H_t^{ent})}{\partial H_t^{ent}} (1 - \nu_t) E_t \left[ \Lambda^{HH} \rho^{ent}_{t+1} \right] \geq \chi$$ (14)

In general, financing entrant innovators with external equity is optimal because a debt contract would not give venture capitalists the right incentives to monitor the start-up whenever there are more than two possible outcomes (see Admati and Pfleiderer (1994)). The monitoring decision is not endogenous here, but the contract is consistent with this theoretical result.

**Incumbent Innovators** Incumbent innovators choose the amount $H_t^{inc}$ to invest in the probability $\lambda^{inc}(\Gamma_t H_t^{inc})$ of adopting an additional idea, where the function $\lambda^{inc}(\cdot)$ has the same properties as $\lambda^{ent}(\cdot)$. Yet, it is helpful in terms of the simulation to allow for different parametrizations across incumbents and entrants. If incumbent innovators succeed, they give birth to a new incumbent firm (a spin-off) which is also bought by the household at a price equal to $V_t^{inc}$, so that all incumbent firms at a given point in time are identical. Adopted ideas become obsolete with exogenous probability $(1 - \phi)$. Since every successful innovator produces only one idea, $(1 - \phi)$ is also the exit rate of innovator firms. Actually, the distribution across firms of the portfolio of intermediate varieties produced by incumbents is irrelevant. I just assume that an incumbent is equivalent to a variety for clarity of exposition.

Since incumbents are obtaining profits in the current period, they can finance $H_t^{inc}$ with internal funds$. Hence, the value of an incumbent is:

---

$^7$In the calibration of the model, the parameters chosen are such that $\Pi_t^{m} \geq H_t^{inc}$ for all $t$, so incumbents never need to raise external financing. Otherwise, incumbent firms would not be homogeneous.
\[
V_{t}^{inc} = \max_{H_{t}^{inc} \geq 0} \{ \Pi_{t}^{m} - H_{t}^{inc} + (\phi + \lambda^{inc} (\Gamma_{t} H_{t}^{inc})) E_t [\Lambda_{t+1} H_{t+1}^{inc}] \} \quad (15)
\]

Summing up the contribution to innovation of all firms, the evolution of the stock of adopted ideas is given by:

\[
A_{t+1} = (\lambda^{inc} (\Gamma_{t} H_{t}^{inc}) + \lambda^{ent} (\Gamma_{t} H_{t}^{ent})) (Z_{t} - A_{t}) + \phi A_{t} \quad (16)
\]

This is, innovation from entrants and incumbents transforms unadopted basic ideas into adopted ideas, which survive with probability \( \phi \). Note that entrants who are successful carry out “incumbent innovation” in the following period.

### 4.2 Equilibrium

I focus on a symmetric market equilibrium where all firms of a given type are equal. The endogenous state variables are the aggregate capital stock \( K_{t} \), the stock of adopted ideas \( A_{t} \), the fixed cost paid in the last period \( \Psi_{t} \) (which is proportional to \( K_{t} \)) and the financing costs vector in the previous period \( \hat{r}_{s}^{t} \). The equilibrium is characterized by the following system of difference equations:

**Resource constraints and technology** Aggregate value added \( Y_{t}^{net} \) is given by total output minus expenditures in intermediate inputs, production costs paid in the current period (to produce in the following period) and financial costs, which are the exogenous per unit cost times the quantity of financing provided by the financial intermediary in each investment:

\[
Y_{t}^{net} = Y_{t} - A_{t}^{1-\theta} M_{t} - H_{t}^{ent} r_{s}^{VC} (Z_{t} - A_{t}) - (1 + r_{s}^{D}) \bar{N} \Psi_{t+1} \quad (17)
\]

\( Y_{t}^{net} \) is used in consumption, investment in physical capital and investment in adoption of unadopted basic ideas:

\[
Y_{t}^{net} = C_{t} + I_{t} + (H_{t}^{ent} + H_{t}^{inc}) (Z_{t} - A_{t}) \quad (18)
\]
The law of motion of capital is given in equation (5) and the production function for \( Y_t \) in equations (7) and (8).

**Factor market clearing**  From the FOCs of the household and the firm with respect to labor, an equilibrium in the labor market must satisfy:

\[
\frac{(1 - \alpha)(1 - \gamma)}{\mu} \frac{Y_t}{L_t} = W_t = L_t^\varphi C_t^{\theta} \tag{19}
\]

Similarly, the capital market equalizes the marginal product of capital to the price of capital plus depreciation times a mark-up:

\[
\alpha(1 - \gamma) \frac{Y_t}{K_t} = \mu (1 + r^K_t + \delta) \tag{20}
\]

and the market for intermediate inputs equalizes the marginal product of the intermediate CES composite \( M_t \) to its price \( P^M_t \) times a mark-up:

\[
\gamma \frac{Y_t}{M_t} = \mu P^M_t \tag{21}
\]

From the definition of the intermediate good composite in equation (9) and imposing symmetry:

\[
P^M_t = \vartheta A_t^{1 - \theta} \tag{22}
\]

**Household’s intertemporal optimality**  The household’s Euler equation for capital is given by:

\[
E_t \left[ \Lambda^HH_{t+1} (1 + r^K_{t+1}) \right] = 1 \tag{23}
\]

Moreover, households also own the firms. Hence, \( \Lambda^HH_{t+1} \) will price the intertemporal trade-offs of all firms except for the financial intermediary, which takes into account its financing costs.

**Final goods intertemporal optimality**  Final good producers who have made the required initial fixed capital investment can freely enter the market for final good varieties. If they enter, they will have to repay the loan, so the number of final good firms
producing in equilibrium \( N_{t}^{\text{fin}} \) will be pinned down by the free-entry condition:

\[
\Pi_{t}^{\text{fin}} \equiv (\mu - 1)MC_{t}Y_{t}\left(N_{t}^{\text{fin}}\right)^{-\mu} = \Psi_{t-1}\left(1 + r_{t-1}^{\text{fin}}\right) \tag{24}
\]

where the marginal cost per unit of output \( MC_{t} \) is obtained from the solution to the cost minimization problem of the final good producer:

\[
C_{t}(q) = \min_{L,K,M} \left\{ W_{t}L + (1 + r^{K}_{t})K + P_{t}^{M}M \right\} \quad \text{s.t.} \quad \left[K^{\alpha L^{1-\alpha}}\right]^{1-\gamma}M^{\gamma} = q \tag{25}
\]

The solution given factor prices is:

\[
MC_{t} = \frac{\partial C_{t}(q)}{\partial q} = \left(P_{t}^{M} / \gamma\right) \left(1 + r^{K}_{t}\right)^{\alpha(1-\gamma)} \left(W_{t} / (1-\alpha)(1-\gamma)\right) \tag{26}
\]

Perfect competition between financial intermediaries implies that in final-good-firm debt financing, the discounted net present value of repayment must equal the financing cost:

\[
\bar{N} = E_{t}\left[ \frac{\Lambda_{t+1}^{HH}}{1 + r_{t}^{\text{s,VC}}} \left(\bar{N} - N_{t+1}^{\text{fin}}\right)\eta + N_{t+1}^{\text{fin}}\left(1 + r_{t}^{\text{fin}}\right)\right] \tag{27}
\]

Equations (24) and (27) together determine the equilibrium number of effective final good producers \( N_{t}^{\text{fin}} \) and the interest rate of debt \( r_{t}^{\text{fin}} \).

**Innovator entrants intertemporal optimality**  Similarly, perfect competition between financial intermediaries in entrant-innovator venture-capital financing implies that:

\[
H_{t}^{\text{ent}} = \lambda^{\text{ent}}(\Gamma_{t}H_{t}^{\text{ent}})V_{t}E_{t}\left[ \frac{\Lambda_{t+1}^{HH}}{1 + r_{t}^{\text{s,VC}}}V_{t+1}^{\text{inc}} \right] \tag{28}
\]

This condition together with the incentive compatibility constraint in equation [14] imply that the unique equilibrium venture capital contract is given by:
$$V_t^{opt} = \frac{\varepsilon \left(1 + r_t^{VC}\right)}{\varepsilon \left(1 + r_t^{VC}\right) + \chi}$$

(29)

This is the highest $V_t$ that satisfies incentive compatibility, i.e., the contract that permits a higher innovation level and thus is closer to the first best. Note that if $\chi = 0$ (stealing is useless) the moral hazard problem would be absent and it would be efficient to completely transfer ownership to the external investor. Also, the fraction $V_t$ that the financial intermediary requires to be indifferent is increasing in the cost of innovator entrants financing and in the elasticity of the probability of innovation with respect to investment in innovation.

For both types of external financing, the equilibrium return required by the financial intermediary includes four components. First, the risk-free intertemporal discounting. Second, compensation for the probability of default. This compensation is decreasing in the tangibility of assets because, in line with my empirical hypothesis, more-tangible assets have a higher liquidation value. Third, the exogenous real resource cost that the intermediary bears. And last, compensation for covariance with the stock market. Since I will solve the model by first-order log-linearization, I will only consider the first three components and excess compensation for covariance with the stock market will be captured by the exogenous real resource cost imputed from the data for each type of external financing. The solution method also implies that the return to the capital owned by the households $r_t^K$ will be equivalent to the risk-free rate.

**Innovator incumbents intertemporal optimality** Incumbent innovator’s optimality is given by the FOC of the Bellman equation (15):

$$1 = \frac{\partial \lambda^{inc}(\Gamma_t H_t^{inc})}{\partial H_t^{inc}} E_t \left[ \Lambda_{t+1} \Psi_t^{inc} \right]$$

(30)

The symmetric monopolistic competition solution for $\Pi_t^m$ is:

$$\Pi_t^m = \frac{\vartheta - 1}{\vartheta} \frac{\gamma}{\mu} Y_t$$

(31)

The law of motion for the stock of ideas is given in equation (16). The definitions of
the fixed cost and the scaling factor in equations (10) and (12) respectively, in addition to the stochastic process for the costs of external financing in equation (5) complete the characterization of the equilibrium.

The market equilibrium differs from the social planner’s solution due to multiple market imperfections. As in the classical expanding varieties endogenous growth model (Romer (1990)), market power of monopolistic competitors and innovation spillovers make the level of investment in innovation suboptimal. However, in my model there are additional sources of inefficiency. First, the external financing costs will not apply to the social planner if they are interpreted as an agency problem. Second, unobservability of investment in innovation requires the presence of rents to innovator entrants, which further reduces the decentralized rate of adoption of ideas.

I solve the model by first-order log-linearization around the steady state of the system after it is transformed to be stationary. The stochastic simulation of the model is done with Dynare, using the Anderson Moore Algorithm. I do not apply any filter because the time period is one year (the highest frequency is relevant) and I am interested in the medium-term response (the lowest frequencies are also relevant).

4.3 Calibration

Table 3 summarizes the choice of values for every parameter and the target for the choice. I individually calibrate the parameters which have several precedents in the literature or for which there exist straightforward empirical estimates. The rest of parameters are jointly calibrated by targeting moments concerning the financing of innovation and the role of tangible and intangible capital in the economy at the BGP. I focus on BGP moments instead of second moments because the shocks that I consider are not necessarily a complete characterization of all the sources of volatility in the data. The system is exactly identified.

I will start describing the choice of values for individual calibration. With respect to the parameters that are common in the business cycle literature, such as the intertemporal discount factor $\beta$, the depreciation rate $\delta$, the elasticity of labor supply $\zeta$, the capital/labor share $\alpha$ and the share of intermediates in production $\gamma$, I use the typical values. The elasticity of substitution between final good varieties $\mu$ is taken from the
empirical estimates of Basu and Fernald (1997). The percentage of the fixed investment recovered in case of default \( \eta \) directly maps to the data on recovery rates of secured corporate bonds, which is 41.7% from 1978 to 1995 according to Altman and Kishore (1996). The exogenous growth rate of basic ideas \( g_z \) is set to match the trend annual growth rate of real output per capita between 1947 and 2013: 2.22%. For the elasticity of adoption of ideas with respect to investment in innovation \( \varepsilon \), Griliches (1990) estimates an interval between 0.6 and 1. I choose the middle value, 0.8, and also calibrate the model with 0.7 and 0.9 as a robustness check. A higher \( \varepsilon \) magnifies the amplitude, and more significantly, the persistence of the effects of shocks to the financing of innovation. In the case with \( \varepsilon = 0.9 \), the shocks have almost permanent consequences, similar to the unit-root process that would emerge if the generation of new ideas was endogenous. Time will tell whether the US economy eventually reverts to the previous long-run growth trend or whether the Great Deviation is permanent. The survival rate of adopted ideas \( \phi \) is set to match the average exit rate of establishments operated by firms 0 to 5 years old in the Longitudinal Business Database 2011 data: 17.1%. I do not target the whole firm population because all incumbent growth in my model leads to creation of new firms or establishments (the two concepts are equivalent in the model), not expansion of existing ones, while in reality the exit probability of an establishment decreases as it grows in size. In other words, I intend to target the exit rate of establishments which have not given birth to additional varieties yet.

The parameters that drive the exogenous shocks are chosen as follows. The mean value of the resource cost of final good producers’ debt \( r^{s,D} \) is set such that the average spread generated by the model, which includes a default component, equals the average Gilchrist Zakrajšek (GZ) annual spread between 1973 and 2012. The mean value of the resource cost of venture capital \( r^{s,VC} \) is equal to the average predicted annual excess return on venture capital between 1991 and 2010. The prediction comes from an OLS regression of realized returns on the previous period dividend-price ratio in the stock market, as I explained in Section 3.3.

Parameters governing the shock dynamics are chosen in order to compute relevant impulse response functions (IRFs). I use the data on the deviations of the premium to bonds in excess of exit risk and VC predicted returns from 2008 to 2010 in order to estimate the peak and the autocorrelation of a geometrically decaying process by
minimizing the sum of squared errors. This is an approximation to the realized values after 2008, when external financing costs peaked, as seen in Figure 5.

First, I am interested in analyzing the effects of a one-period shock to each spread separately that mimics the deviation from the mean at the peak of the Great Recession. The reason I first introduce purely temporary shocks instead of matching the empirical autocorrelation is that I want to emphasize how endogenous innovation generates persistence, so in this case $R$ is just a matrix of zeros. Next, I am going to consider the combined effects of the two shocks, maintaining the same period zero magnitude of the shocks, and adding the autocorrelation that is estimated from the data. In this case I will use the values for $R$ in Table 3. I do not try to estimate diagonal entries because I am simply interested in replicating the decaying shape of the two types of spreads to generate realistic IRFs.

I now turn to the six parameters that are jointly calibrated. I aim at minimizing the sum of squared errors of six model moments at the BGP compared to their empirical counterparts. Even though all these parameters are interconnected, two of them are more directly related to final good firm dynamics: the size of the pool of potential entrants $N$ and the ratio of the fixed cost to the current capital stock $\psi$. These two parameters are mainly identified with the empirical default rate for Moody’s Ba-rated corporate bonds 8 (see Elton et al. (2001)), 1.5%, and the ratio of debt financing flows to market value of firms with external financing inflows ($XF_{it} > 0$) in the highest tangible-book-to-market bin in Figure 3 (which correspond to final good producers in my model), 17.4%.

I assume that the innovation production functions take the following form: $\lambda^{ent}(\Gamma_t, H_t) = \min\{\lambda^{ent} \cdot (\Gamma_t, H_t)^{\epsilon}, 1\}$ and $\lambda^{inc}(\Gamma_t, H_t) = \min\{\lambda^{inc} \cdot (\Gamma_t, H_t)^{\epsilon}, 1\}$ for entrants and incumbents respectively. Then, the remaining four parameters are the elasticity of substitution across intermediate varieties $\vartheta$; the fraction of venture capital raised which can be stolen and consumed by innovator entrants $\chi$; and the constant in the innovation production function for entrants $\lambda^{ent}$ and for incumbents $\lambda^{inc}$. These four parameters are mainly identified by the following four moments: the ratio of equity financing

---

8Matching lower default rates from higher investment grade bonds does not alter the results of the model. However, a higher exit rate gives enough room for more firms to enter in response to a realistic fall in financing costs without reaching the problematic point where all potential entrants become active producers.
flows to market value of firms with $XF_{it} > 0$ in the lowest book-to-market bin in Figure 4 (which correspond to innovators in my model), 9.8%\footnote{The actual share may be higher, because Figure 3 is based on a sample of publicly owned firms while innovators in the model include start-ups. So I take this moment as a conservative starting point, given that the aggregate effect of shocks will be larger the higher is the exposure of these firms to external financing. Again, this is in contrast to many quantitative models in the literature, which assume unrealistically high levels of intermediation to generate sizable aggregate effects of financial disruptions.} tangible and intangible gross capital formation in 2007 (prior to the simulated shocks in 2008) as measured by Corrado et al. (2012)\footnote{Corrado et al. (2012)'s measure of intangible investment is much broader than reported R&D expenditures. It includes the proportion which qualifies as investment (generating payoffs more than one year ahead) of private expenditures in computerized information (software and databases), innovative property (R&D, design, product development in financial services, mineral exploration and spending on the production of artistic originals) and economic competencies (market research, advertising, training and organizational capital).} 10 and 15.4 percent of GDP respectively; and the fraction of venture capital projects exiting with a value less than $3$ million for entrepreneurs according to Hall and Woodward (2010), 81 percent\footnote{The fraction exiting with zero value for entrepreneurs is 75 percent. I do not have data for the distribution of total repayment to investors, but the probability that the overall value is zero could be lower given that external investors have preference over original owners in venture capital contracts.}.

Importantly, the set-up of the model and its calibration ensure that the simulated economy will reproduce the empirical means plotted in Figure 3 for the firms with $XF_{it} > 0$ in the 0-0.1 and 0.9-1 bins of tangible-book-to-market value. The overlapping nature of firms in the model generates the empirical cross-sectional dispersion and composition of external financing flows for firms with different innovation intensity. Moreover, the steady-state ownership share of innovators $ν^{opt}_{SS} = 0.40$ is very close to the empirical equivalent in Kaplan and Strömberg (2003), between 0.29 and 0.37, despite not targeting this particular moment.

### 4.4 Results

I first show the response of the model to a one-period shock in the cost of each type of investment implied by the yearly shock to the spreads observed at the peak of the Great Recession. Figure 6 shows the effects on the main endogenous variables of a shock to the cost of debt financing and Figure 7 the effects of a shock to the cost of venture capital financing. Then, in Figure 8 I present the combined effects of the two shocks.
<table>
<thead>
<tr>
<th>parameter</th>
<th>concept</th>
<th>value</th>
<th>source/target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>disc. factor</td>
<td>0.95</td>
<td>literature</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depr. rate</td>
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<td>literature</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>el. lab. supply</td>
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<td>literature</td>
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<tr>
<td>$\alpha$</td>
<td>K/L share</td>
<td>1/3</td>
<td>literature</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>M share</td>
<td>1/2</td>
<td>literature</td>
</tr>
<tr>
<td>$\mu$</td>
<td>markup fin. goods</td>
<td>1.1</td>
<td>Basu and Fernald (1997)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>fixed cost recovered</td>
<td>0.417</td>
<td>Altman and Kishore (1996)</td>
</tr>
<tr>
<td>$g_c$</td>
<td>growth basic ideas</td>
<td>0.051</td>
<td>output p.c. trend growth rate: 2.22 percent</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>el. innov. fct.</td>
<td>0.8</td>
<td>Griliches 1990</td>
</tr>
<tr>
<td>$\phi$</td>
<td>survival rate ideas</td>
<td>0.829</td>
<td>exit rate firms 0-5 years old: 17.1 percent</td>
</tr>
<tr>
<td>$(\rho^D, \rho^VC)$</td>
<td>mean cost</td>
<td>(0.009, 0.123)</td>
<td>all data GZ spreads and VC returns</td>
</tr>
<tr>
<td>$R$</td>
<td>persistence shocks</td>
<td>$\begin{pmatrix} 0.330 &amp; 0 \ 0 &amp; 0.799 \end{pmatrix}$</td>
<td>data GZ spreads and VC returns 2008-2010</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>shocks to cost for IRFs</td>
<td>$\begin{pmatrix} 0.0115^2 &amp; 0 \ 0 &amp; 0.1013^2 \end{pmatrix}$</td>
<td>data GZ spreads and VC returns 2008-2010</td>
</tr>
<tr>
<td>$\Xi$</td>
<td>pool fin. good prod.</td>
<td>4.037</td>
<td>default rate Ba bonds: 1.5 percent</td>
</tr>
<tr>
<td>$\psi$</td>
<td>fixed K cost</td>
<td>0.160</td>
<td>share debt financing highest BTM bin: 17.4 percent</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>markup intern. goods</td>
<td>1.268</td>
<td>tangible GCF share in 2007: 10.0 percent</td>
</tr>
<tr>
<td>$\chi$</td>
<td>stealing utility</td>
<td>0.594</td>
<td>share equity financing lowest BTM bin: 9.8 percent</td>
</tr>
<tr>
<td>$\lambda^e$</td>
<td>ct. innov. fct. ent.</td>
<td>0.536</td>
<td>81 percent entrants die</td>
</tr>
<tr>
<td>$\lambda^i$</td>
<td>ct. innov. fct. inc.</td>
<td>0.249</td>
<td>intangible GCF share in 2007: 15.4 percent</td>
</tr>
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</table>
adding the empirical persistence to each of them. While the first method is useful in disentangling the consequences of each shock and illustrating the transmission mechanism, the second method is helpful to quantify the total effect of financial frictions during the Great Deviation. The effects of each shock on other endogenous variables are shown in the Appendix. The units of the y-axis are percentage points and those of the x-axis years.

For each shock, I also plot in the same graphs the IRFs of a model without endogenous innovation in order to assess the quantitative relevance of this channel with respect to medium-term fluctuations. In the exogenous innovation model, the level of adopted ideas is set exogenously to equal the steady state value of the main model in every period, but the parameters are unchanged. The only financial friction that applies in this case is the one affecting final goods fixed cost financing, which is satisfied with debt issuances.

Let us start by analyzing the effects of a shock to the cost of final goods debt financing. There are two results to remark upon. First, the impact on the aggregate variables of the shock to final good financing observed at the peak of the crisis is not so large. This financial friction alone generates a fall of about 0.7 percent in value added output. Second, a one-year lasting shock generates effects that are relatively temporary, lasting only four years. The mechanism in a crisis is the following. The increase in the spread reduces the number of final good producers that will engage in production in the next period. Households anticipate this negative wealth shock and increase current labor supply (see Figure 10 in the Appendix), which slightly mitigates the fall in current output. The anticipation of lower production in the future, and therefore lower demand for factors, reduces all kinds of investment: in physical capital \( I_t \) and in ideas \( H_t^{inc} \) and \( H_t^{int} \). In the following period, required repayment will be higher and less firms will enter final good production. After one period, the direct effect of higher financing

\[ I_t \]

I have also computed a version of the exogenous innovation model where I extend the role of physical capital to target the empirical total investment rate (tangibles plus intangibles). To do so, it is necessary to increase the depreciation rate. Otherwise, the steady-state investment rate is too low even if the whole intermediate input share is attributed to capital, since investment in the fixed capital cost must decrease to maintain the debt share of final good producers. In any case, the IRFs of the extended-tangible-capital case are not wider than in the original exogenous-innovation case. This reinforces the conclusion that intangibility per se is necessary to generate persistence and amplification, as its contribution is not isomorphic to increasing the tangible capital share.
costs is gone, but the reduction in the stocks of physical capital and adopted ideas due to the initial fall in investment will keep the economy depressed for more periods.

Figure 6: IRFs to a one-period shock to $r^{x, fin}_t$

- **Legend:** Ynet=value added, C=consumption, I=investment in physical capital, He=investment in innovation by entrants, Hi=investment in innovation by incumbents, A=stock of adopted ideas, K=capital stock, rsD=cost of debt financing.

However, for this one-period shock alone, endogenous adoption of ideas adds little amplification compared to the model with exogenous innovation. This is because the fall in the capital stock reduces the complexity in the economy, which means that the scaling factor $\Gamma_t$ increases and adoption of new ideas does not fall substantially even if investment in innovation does.

These are not modeled here, but any shock impacting directly the final goods sector, such as a shock to the fixed cost, disembodied productivity, labor wedges or tangible
capital wedges, would have similar consequences to a shock to the costs of final good producers external financing. This is, it would not significantly increase persistence via endogenous innovation. As we will see below, financial crises are only more persistent than other crises insofar as they tend to affect the innovation decision directly by bringing larger venture-capital-financing costs.

Figure 7: IRFs to a one-period shock to \( r^x_{t,ent} \)

I now turn to the analysis of the effects of a shock to the cost of venture capital. This type of shock has a more persistent impact on aggregate variables because it has a direct negative impact on the stock of ideas that accumulates over time. Changes in the adoption rate of ideas are the source of medium-run fluctuations. The mechanism here
works as follows. An increase in the cost of the financial intermediary decreases the number of innovator entrants in the current period until their return is high enough to compensate the higher cost per unit of financing. Since competition in the intermediate goods sector is decreased, innovation by incumbents increases, but the aggregate innovation effort still falls appreciably, as incumbents are responsible for a small fraction of total innovation according to the calibration. Reduced entry lowers the adoption of ideas, which is an input in final good production. Thus, in the following period production will be lower. Again, the anticipation of a lower productivity level (TFP) reduces investment in physical capital, which reinforces the fall in innovation by entrants. Investment in innovation remains low even after the shock is over because demand from final good producers is still depressed due to the reduced stock of capital and ideas.

The effects of this shock in the model with exogenous innovation are obviously nonexistent, as there is no need to invest in order to adopt ideas.

Finally, I consider the combination of the two previous temporary shocks, plus the empirical persistence. As I explained in Section 4.3, the size and autocorrelation of the shocks is set to minimize the sum of squared errors with respect to the empirical values between 2008 and 2010. The mechanisms discussed above still apply. There is feedback between decreased productivity and lower demand for innovation. The fall in output and consumption at the trough is almost 2 percent of GDP and persistence in the shocks leads to a much more persistent fall in those variables and to a very sluggish recovery. By comparison, the IRFs of the exogenous innovation model are almost economically insignificant, in terms of size and especially in terms of persistence.

Hence, the main message with regard to the financial frictions literature is that allowing for endogenous innovation is key to understand how financial shocks of a realistic size can have such durable effects on aggregate variables. Although financial shocks can only explain a small fraction of the fall in output from trend observed during the Great Deviation (14.3 in 2009), the increase in the cost of external financing had a prominent contribution in slowing the recovery of aggregate variables.
The model also has quantitative implications for the evolution of variables related to innovation. In Table 4 I compare the percentage fall at the heyday of financial disruptions (2009) and at the most recent data point available with respect to the 1995-2007 linear trend in the simulation and in the data. I do not use any filter to prevent the trend from including the endogenous medium-term component. The model explains about half of the 2009 fall in the variables connected to productivity: utilization-adjusted TFP and intangible capital investment; and a smaller fraction of the change in tangible capital investment and GDP. This is probably because the latter variables are more directly affected by other types of shocks that were part of the Great Recession but
are not modeled here (e.g. labor market distortions). Regarding the most recent data, if anything, the model seems to understated the astonishing degree of persistence observed, but it is way ahead of the results from typical financial frictions models. I also compare empirical R&D expenditures of firms below and above 5 years of age to $H^{ent}_i$ and $H^{inc}_i$ respectively. I already commented on the restrictiveness of R&D as a measure of investment in innovation, but it is reassuring if this subcategory in the data behaves according to the model prediction for total innovation investment. Indeed, both in the model and in R&D data, the consequences of a financial crisis are worse for young firms’ innovation, but the data does not support the model’s result that incumbents actually increase innovation. Still, the model predicts shocks affecting the final goods sector to decrease incumbent innovation, so it would come closer to the data under the presence of other shocks. The simulation also replicates the empirical regularity that R&D is more volatile and more procyclical for young firms.

Table 4: Percentage fall with respect to trend (1995-2007)

<table>
<thead>
<tr>
<th>variable</th>
<th>source</th>
<th>data</th>
<th>model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2009</td>
<td>most recent</td>
</tr>
<tr>
<td>GDP</td>
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<td>-14.3</td>
<td>-16.1 (2012)</td>
</tr>
<tr>
<td>TFP (utiliz. adj.)</td>
<td>Fernald (2012)</td>
<td>-2.8</td>
<td>-7.7 (2012)</td>
</tr>
<tr>
<td>Tang. Inv.</td>
<td>Corrado et al. (2012)</td>
<td>-22.4</td>
<td>-25.8 (2010)</td>
</tr>
<tr>
<td>R&amp;D young</td>
<td>Compustat</td>
<td>-55.8</td>
<td>15.0 (2011)</td>
</tr>
<tr>
<td>R&amp;D old</td>
<td>Compustat</td>
<td>-21.5</td>
<td>-23.8 (2011)</td>
</tr>
</tbody>
</table>

5 Conclusions

Firms with different innovation intensity also differ in their access to external financing. Innovation-intense firms are less able to finance their investments externally, and the restriction is mainly on debt financing. In fact, these firms use more external equity financing. Moreover, the cost of external financing moves countercyclically and is different for debt and equity financing. These elements together imply that financial crises affect the investment capacity of firms, and that the extent of the effect depends on the innovation intensity of the firm.
Using a business cycle model with endogenous innovation and a financial structure consistent with the patterns in the data, I quantify the effects of realistic shocks to external financing on the short and medium-run level of aggregate economic variables. According to the model, the increase in the spreads observed during the Great Recession led to a fall of almost 2 percent in GDP and a very sluggish recovery, with GDP being more than 1.5 percent below trend for more than a decade. The Great Deviation following the Great Recession may be partially being caused by the long-lasting effects of temporary financial shocks.

I believe that the following paths are worth pursuing in the future. Regarding the empirical analysis, I am going to analyze the dependence of debt financing costs on the innovation intensity of a firm. This should strengthen the case that firms investing in tangible capital do not engage in more disperse external financing flows because of greater volatility in technology or consumer demand, but because it is cheaper for them to do so.

With respect to the model calibration, I will adapt it to the European periphery economies whose non-financial firms are still suffering from high external financing costs. This is an interesting case due to the particularities of these economies, which are both more dependent on banks and less intense in intangible capital. According to my model, we should expect a more pronounced dip but a relatively faster recovery than in the US.

Taking into account endogenous innovation and its medium-run effects also provides an improvement upon existing frameworks when addressing a relevant and current policy question: what is the optimal macroprudential policy? And more specifically: how should bank capital requirements be designed? In a model with endogenous innovation and financial intermediaries, requiring financial institutions to maintain minimum levels of capital over assets creates a trade-off with respect to economic growth. On the one hand, such a policy may limit real investment opportunities and hinder growth. On the other hand, it may avoid financial market freezes and therefore insure the corporate sector against the need to save in liquid assets, allowing it to focus on productive investment and innovation. I plan to endogenize the role of financial agents in my model to provide an answer to these questions.
References


Appendix

(For Online Publication)

Impulse Response Functions to other variables under each shock:

Figure 9: IRFs to a one-period shock to $r_i^{s,fin}$

Legend: Profm=profits of intermediate good producers, Psi=fixed cost final goods, Nfin=mass final good producers, Nent=ratio mass entrants to incumbents, SDF=stochastic discount factor of the household, rK=net return to physical capital, rfin=interest rate on debt, MC=marginal cost final goods.
Figure 10: IRFs to a one-period shock to $r_t^{s, fin}$

Legend: W=wage, L=labor, M=CES composite intermediate good, TFP=total factor productivity, Vi=value of an incumbent, Scaling=scaling factor in innovation function.
Figure 11: IRFs to a one-period shock to $r_t^{s, ent}$

Legend: Profm=profits of intermediate good producers, Psi=fixed cost final goods, Nfin=mass final good producers, Nent=ratio mass entrants to incumbents, SDF=stochastic discount factor of the household, rK=net return to physical capital, rfin=interest rate on debt, MC=marginal cost final goods.
Figure 12: IRFs to a one-period shock to $r_t^{s, ent}$

Legend: W=wage, L=labor, M=CES composite intermediate good, TFP=total factor productivity, Vi=value of an incumbent, Scaling=scaling factor in innovation function, nu=optimal contract.
Figure 13: IRFs to a persistent shock to the two spreads

Legend: C=consumption, Profm=profits of intermediate good producers, Psi=fixed cost final goods, Nfin=mass final good producers, Nent=ratio mass entrants to incumbents, SDF=stochastic discount factor of the household, rK=net return to physical capital, rfin=interest rate on debt.
Figure 14: IRFs to a persistent shock to the two spreads

Legend: MC=marginal cost final goods, W=wage, L=labor, M=CES composite intermediate good, TFP=total factor productivity, Vi=value of an incumbent, Scaling=scaling factor in innovation function, nu=optimal contract.