Identifying REITs Asset-pricing Bubbles: A Modified CCAPM Approach

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Abstract

This paper develops a model for the pricing of US Equity Real Estate Investment Trusts (REIT’s), based upon economic fundamentals. The analysis compares the theoretical price generated by our model with actual prices in order to examine the existence of REIT’s pricing bubbles. Our price modeling employs a novel approach by applying a special case of the CCAPM, inclusive of stochastic taxation, to derive the price dividend ratio as a function of the parameters of the log normal distribution of the rate of growth of after-tax dividends, stochastic dividend taxation, and the coefficient of relative risk aversion. We find that Equity REITs, during our sample period, 1972-2013, were overvalued for a substantial portion of the time period. During this forty-two year span, Equity REITs were underpriced according to our model for only nine years. More surprisingly, contrary to a popular belief, during and after the so-called, “Great Recession,” 2008-2013, equity REITs were underpriced for only one year (i.e., 2008).

**Keywords:** Bubbles, Equity Premium, REITs, Risk Aversion, Stochastic Tax

I. Introduction

This paper, employing a modified version of the Consumption Capital Asset Pricing Model (CCAPM), develops a model for the pricing of US Equity Real Estate Investment Trusts, based upon economic fundamentals. The pricing model utilizes a novel approach by applying a special case of the CCAPM, inclusive of stochastic taxation, to derive the price-dividend ratio as a function of the parameters of the log-normal distribution of the rate of growth of after-tax dividends, stochastic taxation, and the coefficient of relative risk aversion. The statistical analysis suggests that equity REITs, during the sample period, 1972-2013, are overvalued for the substantial bulk of the time period. During this forty-two year span, Equity REITs were underpriced, according to the model, for only nine years. Between 2000 and 2013, REITs were overpriced (i.e., a positive bubble) in all but one year (2008), during the nadir of the Great Recession. This analysis and approach intersect with three interrelated strands of the economic-finance literature: the equity premium puzzle of corporate finance; publicly traded real estate pricing; and economic asset market bubbles. In order to provide a context for the current research, a brief, selective literature review for these three subjects follows.

The Equity Premium Puzzle

The equity premium puzzle was originally identified by Mehra and Prescott (1985), using historical data for the stock market portfolio $\beta = 1$. The traditional Capital Consumption Asset Pricing Model (CCAPM), with an isoelastic Constant Relative Risk Aversion (CRRA) utility function and an expected equity risk premium of 6% for the S&P 500, using average historical stock returns, produces a coefficient of relative risk aversion of roughly $47.6$. This unbelievably high coefficient of relative risk aversion constitutes the so-called "equity premium puzzle".

Among many approaches for resolving the equity premium puzzle, Fama and French (2002) have suggested one of the most promising paths for deriving a solution. They observe that historical stock market trends will overestimate the expected equity risk premium for stocks because there were large unexpected capital gains during 1951–2000. They conclude that the application of the dividend growth model produces an estimate that should be superior (in terms of the standard error and the stability of the Sharpe ratio) to the traditional methods of using historical stock data averages.

However, as Magin (2014) demonstrates, the use of the Fama and French estimate of the expected rate of returns is not sufficient to resolve the equity premium risk puzzle for stock market portfolios, $\beta = 1$. Magin (2014) calculates that the coefficient of relative risk aversion implied by the expected equity premium of 2.55% (obtained by Fama and French (2002) using the dividend growth model) is still too high: 20.40.

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2See DeLong and Magin (2009) for a review, for example.
Magin (2014), recognizing that taxation uncertainty plays a major role for investors, introduced a modified CCAPM with a stochastic tax rate $\tau_t$ imposed on the income and capital wealth of stock holders. Using this modified model, he finds that for an average investor, who realizes after-tax dividend income as well as short-term and long-term gains in accordance with historical patterns, the coefficient of relative risk aversion is 3.76. Since earlier studies imply that a coefficient of relative risk aversion, $a$, between 2 and 4 would seem reasonable, the Magin estimate for $a = 3.76$ is believable.

**Publicly Traded Real Estate Pricing**

The risk premium puzzle for asset classes other than $\beta = 1$ stock market portfolios has been largely unexplored. The known exceptions for real estate assets are Shilling (2003) and Edelstein and Magin (2013) and (2014). In his study, Shilling (2003) deploys the CCAPM and two different real estate value data sets; but he does not take into account the possible impacts of taxation. He confirms the existence of the equity risk premium puzzle for real estate assets, and concludes that the "puzzle" is even more pronounced for real estate than for general stock market.

On the other hand, using a novel modeling twist by applying the CCAPM with stochastic taxation to NAREIT data, Edelstein and Magin (2013) demonstrate that, for a range of reasonable stochastic tax burdens, the coefficient of relative risk aversion for US Equity REITs shareholders is likely to fall within the interval of 4.32 to 6.29, values significantly lower than those reported in most prior studies for real estate and other asset markets.

Using a database and methodology similar to those employed by Edelstein and Magin (2013) for estimating the expected risk premium for US Equity REITs, Edelstein and Magin (2014) estimate the expected after-tax risk premium for US CMBS REITs. They find that, for plausible levels of tax burdens, the coefficient of relative risk aversion for CMBS REITs investors will be in the interval between 7.43 and 10.59, a reasonable range for risk aversion for CMBS REITs.

**Economic Asset Bubbles, Booms, and Busts**

An extensive, evolving and ever-growing literature has emerged for identifying and explaining asset market pricing bubbles. These discussions examine boom and bust cycles, where asset pricing seems to differ significantly from the underlying fundamentals. Perhaps the classic tome in this area was written by the economic historian, Charles Kindleberger, Manias (2011)\(^4\), *Panics and Crashes: The History of Financial Crises*. His book traces various bubbles over the last five hundred years. A more recent scholarly study about bubbles, and boom-bust cycles has been prepared by Reinhart and Rogoff (2009), *This Time is Different: Eight Centuries of Financial Folly*. They assemble interesting data for examining boom and bust cycles for over 700 years. The recent

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\(^3\)Mehra (2003), Mehra and Prescott (2003)

\(^4\)6th Edition
book by Mian and Sufi (2014), *House of Debt: How They (and You) Caused the Great Recession And How We Can Prevent It From Happening Again*, addresses the root causes of the housing bubble that occurred during the 2008-2009 Great Recession.

While asset pricing bubbles have been documented for a wide range of asset classes (e.g., houses, tulip bulbs, CDO’s, stock equities” across many countries and over the centuries, the omnipresent driver appears to be the supply of credit. Asset pricing bubbles are typically associated with credit growth at a rate substantially above economic output.

Friedman and Schwartz (1965), in their now classic book, *The Great Contraction, 1929-1933*, suggest credit supply effects are symmetric. The Great Contraction of Credit during the 1930’s Depression engendered asset price implosions in the stock market, housing market, and other asset markets worldwide; financial institutions and government policy were linchpins for the contraction of credit, as was the failure of financial institutions.

Akerlof and Shiller (2009), in their influential book, echo the belief that sentiment (animal spirits) can play an important role in determining economic activity. This book crystalizes and presents the view that sentiment plays a vital role in the dynamics, the amplitude, and volatility of booms and busts. Akerlof and Shiller take the position that animal spirits, at times, can be the driving forces behind booms, busts, and bubbles.

The remainder of the paper is organized as follows. Section II defines and reviews the analytics for the CCAPM, the coefficient of relative risk aversion and the Equity Risk Premium Puzzle. Section III derives the price/dividend ratio as a function of the parameters of the lognormal distribution of the rate of growth of after-tax dividends, stochastic dividend tax and the coefficient of relative risk aversion. This section also computes theoretical-fundamental prices for REITs, which are then utilized to identify possible asset-pricing bubbles. Section IV concludes.

II. The CCAPM and the Equity Premium Puzzle

The capital-consumption asset pricing model (CCAPM) is one of the central concepts in financial economics and is a significant generalization of the capital asset pricing model (CAPM). Unlike the CAPM, where economic agents optimize by simply distributing resources between different financial assets, the CCAPM focuses on multiperiod consumption-saving decisions under uncertainty. Following Rubinstein (1976) and Lucas (1978), we define the CCAPM. Consider an infinite horizon model with $n - 1$ risky assets and the $n^{th}$ risk-free asset. Let $p_{kt}$ be the price per share of asset $k$ at period $t$, $d_{kt}$ be the dividend paid per share of asset $k$ at period $t$, $z_{kt}$ be the number of shares of asset $k$ held by an agent at period $t$, $c_t$ be the agent’s consumption at period $t$. Let the investor’s one-period utility function be $u(c_t)$. Consider the investor’s optimization problem:
\[
\max_{\{c_{t+T}\}_{T=0}^\infty} E \left[ \sum_{T=0}^\infty b^T u(c_{t+T}) \right],
\]

where \(0 < b < 1\) and \(u(\cdot)\) is such that \(u'(\cdot) > 0\) and \(u''(\cdot) < 0\), subject to

\[
c_{t+T} = \sum_{k=1}^n (p_{k_{t+T}} + d_{k_{t+T}}) z_{k_{t+T}} - \sum_{k=1}^n p_{k_{t+T}} z_{k_{t+T}+1}. \tag{2}
\]

Taking the first-order condition we obtain

\[-u'(c_t)p_{kt} + bE [u'(c_{t+1}) (p_{kt+1} + d_{kt+1})] = 0 \text{ for } k = 1, ..., n. \tag{3}\]

Hence,

\[p_{kt} = E \left[ \frac{bu'(c_{t+1})}{u'(c_t)} (p_{kt+1} + d_{kt+1}) \right] \text{ for } k = 1, ..., n - 1. \tag{4}\]

Thus,

\[E \left[ \frac{bu'(c_{t+1})}{u'(c_t)} R_{kt+1} \right] = 1 \text{ for } k = 1, ..., n - 1, \tag{5}\]

and

\[E \left[ \frac{bu'(c_{t+1})}{u'(c_t)} \right] R_f = 1, \tag{6}\]

where \(R_{kt+1} = \frac{p_{kt+1} + d_{kt+1}}{p_{kt}}\) is the total rate of return for asset \(k\) and \(R_f\) is the total risk-free rate.

Equations (5) and (6) are known as Euler equations.

Rubinstein (1976) demonstrated that if \(c_{t+1}\) and \(R_{kt+1}\) are bivariate lognormally distributed and \(u(c) = \frac{c^{1-a}}{1-a}\), then

\[
\ln (E[R_{kt+1}]) - \ln (R_f) = a \cdot COV [\ln(R_{kt+1}), \ln \left( \frac{C_{t+1}}{C_t} \right)], \tag{7}
\]

where \(a\) is the coefficient of relative risk aversion. The coefficient of relative risk aversion measures agents’ propensity to take risk. The higher is the coefficient of agent’s relative risk aversion, the lower is agent’s propensity to take risk. Generally, for an agent with utility function \(u(\cdot)\) we define the coefficient of agent’s relative risk aversion as

\[rr(c) = \left[ -\frac{u''(c)c}{u'(c)} \right].\]
If \( u(c) = \frac{c^{1-a}}{1-a} \), then \( u'(c) = c^{-a} \) and \( u''(c) = -a \cdot c^{-a-1} \). So clearly

\[
rr(c) = \left[ -\frac{a \cdot c^{-a-1} - c}{c^{-a}} \right] = a.
\]

Therefore, the major conclusion of the CCAPM is that the expected risk premium for a risky asset is equal to the covariance of the logarithms of the asset’s return and consumption in the period of the return multiplied by the agents’ coefficient of relative risk aversion.

The traditional CCAPM without insecure property rights, and with the current expected equity premium of 6% for the S&P 500 (\( \beta = 1 \)) portfolio, calculated by Mehra (2003), using the average stock return, yields a coefficient of risk aversion of roughly 50:

\[
a = \frac{\ln \left( E[R_{kt+1}] \right) - \ln(R_f)}{COV \left[ \ln(R_{kt+1}), \ln \left( \frac{C_{t+1}}{C_t} \right) \right]} = \frac{0.07 - 0.01}{0.00125} = 47.6.
\]

This unbelievably high value for the coefficient of relative risk aversion constitutes the so-called "equity premium puzzle". It was first identified by Mehra and Prescott (1985) using historical data for the stock market portfolio.

Subsequent research by Magin (2014), Edelstein and Magin (2013) utilizing this traditional CCAPM with stochastic taxation imposed on stock holders’ wealth resolves a substantial part of the Equity Premium Puzzle for various asset classes.

Contrary to the model’s name, most of the discussion in the literature regarding the CCAPM is focused on its properties in terms of the rates of return. However, for the purposes of this paper we need to explore the CCAPM’s asset-pricing properties. Using iteration we can write (4) as

\[
p_{kt} = E \left[ \sum_{T=1}^{\infty} b^T u'(c_t) \frac{d_{kt+T}}{u'(c_{t+T})} \right] + \lim_{T \to \infty} E \left[ \frac{b^T u'(c_{t+T})}{u'(c_t)} p_{kt+T} \right] \forall k = 1, \ldots, n.
\]

By imposing the Transversality Condition

\[
\lim_{T \to \infty} E \left[ \frac{b^T u'(c_{t+T})}{u'(c_t)} p_{kt+T} \right] = 0 \forall k = 1, \ldots, n,
\]

we obtain

\[
p_{kt} = E \left[ \sum_{T=1}^{\infty} b^T u'(c_t) \frac{d_{kt+T}}{u'(c_{t+T})} \right] \forall k = 1, \ldots, n.
\]

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The right-hand side of equation (10) represents the "theoretical" prices of an asset or an asset’s fundamentals. Under the assumptions of Mehra and Prescott (1985) we can write (10) as

\[ p_{kt} = E \left[ \sum_{t=1}^{\infty} b^T \left( \frac{d_{kt+T}}{d_{kt}} \right)^{1-a} \right] \cdot d_{kt} \; \forall k = 1, \ldots, n. \] (11)

Finally, Mehra (2003) and Mehra and Prescott (2003) derived that under the assumptions of Rubinstein (1976), the CCAPM can be stated as

\[ p_{kt} = \left[ e^{\mu_e + \frac{1}{2} \sigma_e^2} \right] \cdot d_{kt}, \] (12)

where

\[ \ln(b^{(1 - \tau_{ct+1})}) = \ln(b^{(1 - \tau_{ct+1})}) \sim N(\mu_e, \sigma_e) \; \forall k = 1, \ldots, n \] and

III. Stochastic Taxation and Asset Pricing

Following Magin (2014), we introduce stochastic taxes into the CCAPM.\(^5\) Let \( \tau_{ct} \) be the effective tax rate imposed on the dividends paid to Equity REITs holders. Then the price per share of an equity REIT \( k \) is given by

\[ p_{kt} = \left[ e^{\mu_e + \frac{1}{2} \sigma_e^2} \right] \cdot (1 - \tau_{ct}) \cdot d_{kt}, \] (13)

where

\[ \ln(b^{(1 - \tau_{ct+1})d_{kt+1+1}}) = \ln(b^{(1 - \tau_{ct+1})d_{kt+1}}) \sim N(\mu_e, \sigma_e) \; \forall l = 0, \ldots, \infty \]

and

\[ \mu_e = E \left[ \ln(b^{(1 - \tau_{ct+1})d_{kt+1+1}}) \right] = \ln(b) + (1 - a) \cdot E \left[ \ln(b^{(1 - \tau_{ct+1})d_{kt+1+1}}) \right], \]

\[ \sigma_e^2 = \text{VAR} \left[ \ln(b^{(1 - \tau_{ct+1})d_{kt+1+1}}) \right] = (1 - a)^2 \cdot \text{VAR} \left[ \ln(b^{(1 - \tau_{ct+1})d_{kt+1+1}}) \right]. \]

Historically\(^6\),

\[ E \left[ \ln(b^{(1 - \tau_{ct+1})d_{kt+1+1}}) \right] = 0.02, \]

\[ \text{VAR} \left[ \ln(b^{(1 - \tau_{ct+1})d_{kt+1+1}}) \right] = 0.00125. \]

\(^5\)See also Sialm (2006) and (2009).

\(^6\)Mehra (2003) and Mehra and Prescott (2003)
Set
\[ b = \frac{1}{R_f} = \frac{1}{1.01} = 0.99. \]

Therefore, we estimate
\[ \mu_c = \ln(0.99) + (1 - a) \cdot 0.02, \]
\[ \sigma_c^2 = (1 - a)^2 \cdot 0.00125. \]

Thus,
\[ \mu_c + \frac{1}{2} \sigma_c^2 = \ln(0.99) + (1 - a) \cdot 0.02 + \frac{1}{2} \cdot (1 - a)^2 \cdot 0.00125. \]

Hence,
\[ e^{\mu_c + \frac{1}{2} \sigma_c^2} = e^{\ln(0.99) + (1 - a) \cdot 0.02 + \frac{1}{2} \cdot (1 - a)^2 \cdot 0.00125}. \]

Edelstein and Magin (2013) estimated that for Equity REITs holders
\[ 4.32 < a < 6.29. \]

Therefore, it is reasonable for the purposes of our calculations to set
\[ a = 5. \]

We obtain the following expression for the price of Equity REITs
\[ p_{kt} = \left[ \frac{e^{\ln(0.99) + (1 - 5) \cdot 0.02 + \frac{1}{2} \cdot (1 - 5)^2 \cdot 0.00125}}{1 - e^{\ln(0.99) + (1 - 5) \cdot 0.02 + \frac{1}{2} \cdot (1 - 5)^2 \cdot 0.00125}} \right] \cdot (1 - \tau_{re,t}^d) d_{kt} \quad (14) \]

As in Edelstein and Magin (2013) and (2014), we assume that the typical investor in REITs, who has below average ordinary income tax rates, pays an overall effective dividend tax rate \( \tau_{re,t}^d \) of half of that of an investor in general stocks.\(^7\)

Figure 1 below charts theoretical (fundamentals) and actual market prices for Equity REITs for the period of 1972–2013.\(^8\)

\(^7\)Effective dividend tax rates for investors in general stocks can be found at http://users.nber.org/~taxsim/marginal-tax-rates/af.html

\(^8\)Calculations are based on monthly NAREIT ALL EQUITY REITs INDEX data for Equity REITs prices and dividends.
The difference between the actual market price of an asset and its theoretical price constitutes the asset-pricing bubble. Formally, we define the asset-pricing bubble for an arbitrary asset $k$ at period $t$ as follows

$$B_{kt} = \overline{p}_{kt} - \left[ \frac{e^{\mu_c + \frac{1}{2} \sigma_c^2}}{1 - e^{\mu_c + \frac{1}{2} \sigma_c^2}} \right] \cdot (1 - \pi^d_t) d_{kt}, \tag{15}$$

where $\overline{p}_{kt}$ is the actual market price of an arbitrary asset $k$ at period $t$. Depending on the sign of $B_{kt}$, we say that

$$B_{kt} = \begin{cases} 
B < 0, & \text{asset } k \text{ is underpriced,} \\
B > 0, & \text{asset } k \text{ is overpriced,} \\
B = 0, & \text{asset } k \text{ is priced correctly.}
\end{cases}$$

Figure 2 plots the asset pricing bubble for Equity REITs for the period of 1972-2013. During this time period, the asset pricing bubble has been negative (REITs were underpriced) for only 9 of the 42 years: 1973–1975, 1987,
FIGURE 2: ASSET-PRICING BUBBLE FOR EQUITY REITs FOR 1972-2013
During 1973-1975, actual market prices of Equity REITs shares fell significantly below theoretical (model-predicted) prices. The collapse of the bubble was a part of the broader 1973-1975 recession, and the 1973-1974 stock market crash. GDP declined by 3.2%. Unemployment reached 9%. Between January 11, 1973 and December 6, 1974, the Dow Jones Industrial Average declined more than 45%. The recession and the crash were the resultant of many forces, including a quadrupling of oil prices combined with higher government spending, taxation, as well as price and wage controls and stimulative monetary policy. The economy ultimately lapsed into a stagflation.

During 1987 and 1988–1989, actual market prices of Equity REITs declined relative to prices implied by the CCAPM. The collapse of the bubble appears to have been an aftershock of the broader October 1987 Stock Market Crash.

During 1998–1999, actual market prices of Equity REITs dropped well below fundamentals. The dot-com bubble of 1997–2000 seems to be the chief suspect. The dot-com bubble manifested itself by a significant rise in the demand for growth stocks at the expense of high dividend yield stocks. Thus, the demand for Equity REITs dropped causing a decline in REITs prices.

In 2008, actual market prices of Equity REITs shares again diverged significantly below model-predicted prices. The collapse of the bubble was part of the broader 2008-2009 Great Recession. GDP declined by 4.3%. Unemployment mushroomed to over 10%. The U.S. housing market lost roughly 30% of its value, and the S&P 500 declined approximately 50% by early 2009. The recession was, in part, blamed on the subprime mortgage crisis, leading to the collapse of the housing bubble. Root causes of the crisis are still the subject of a fierce discussion. But unwise lending and underwriting practices, often directly and indirectly enhanced by lax government regulation, seem to be principal suspects. Some economists argue that without the herculean intervention by the world’s central banks, a great depression similar to that of the 1930’s might have occurred.

A major economic transition appears to have occurred during the world financial crisis and the Great Recession, with real and financial markets disrupted by substantial financial and economic dislocations. The world’s central banks were designed to avert implosions of the asset bubbles in many real and financial markets. The injection of cash into troubled (and not so troubled) financial institutions, often lead to a partial or even complete nationalization of financial institutions. The creation of liquidity required the US and other governments to engage in massive borrowing, raising many sovereign debts to unprecedented levels. In such periods of great economic and policy uncertainty actual market prices would be expected to decline significantly vis-à-vis theoretical prices. Surprisingly, for the period during and since the Great Recession, Equity REITs, using our pricing model, have been underpriced for only 1 year: 2008. In fact, for the 2000-2013 time horizon, an era of prolonged accommodative monetary policy, a negative bubble occurs only in 2008.

Since the investors in REITs are likely to have lower than average dividend
tax rates, we allow the effective dividend tax rates for REITs shareholders to vary from 25% to 100% that of general stock holders. We find that if the effective dividend tax rates for REITs shareholders were 25% that of general stock holders, then equity REITs would be underpriced for 12 out 42 years. If the effective dividend tax rates for REITs shareholders were 75% that of general stock holders, then equity REITs would be underpriced for 7 out 42 years. Finally, if the effective dividend tax rates for REITs shareholders were 100% that of general stock holders, then equity REITs would be underpriced for only 3 out 42 years.

These computations suggest that Equity REITs asset-pricing bubbles are very sensitive to changes in stochastic tax burdens imposed on fundamentals. As the effective dividend tax rate increases, the theoretical prices of Equity REITs decline relative to the actual market prices, inflating the bubbles.

IV. Conclusion

This paper develops a model for the pricing of US Equity Real Estate Investment Trusts, based upon economic fundamentals. The analysis compares the theoretical price generated by our model with actual prices in order to examine the existence of Real Estate Investment Trust pricing bubbles. Our price modeling employs a novel approach by applying a special case of the CCAPM, inclusive of stochastic taxation, to derive the price dividend ratio as a function of the parameters of the log normal distribution of the rate of growth of after tax dividends, stochastic dividend taxation, and the coefficient of relative risk aversion. We find that Equity REITs, during our sample period, 1972-2013, were overvalued for a substantial portion of the time period. During this forty-two year span, Equity REITs were underpriced according to our model for only nine years. More surprisingly, contrary to a popular belief, during and after the so-called, “Great Recession,” 2008-2013, equity REITs were underpriced for only one year (i.e., 2008). The negative REITs pricing bubbles typically occur either during recessions and/or points in time with significant financial and economic dislocations. In contrast, the 2000-2013 era is characterized as a prolonged period for accommodative monetary policy, and a concomitant prolonged REIT pricing bubble (except for 2008). Subject to future verifying research, it appears that the 2000-2013 REITs bubble is, in part, linked to prolonged loose monetary policy.
REFERENCES


