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Monocentric City Redux

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Abstract

This paper argues that centralized employment remains an empirically relevant stylization of midsize U.S. metros. It extends the monocentric model to explicitly include leisure as a source of utility but constrains workers to supply fixed labor hours. Doing so sharpens the marginal disutility from longer commutes. The numerical implementation calibrates traffic congestion to tightly match observed commute times in Portland, Oregon. The implied geographic distribution of CBD workers' residences tightly matches that of Portland. The implied population density, land price, and house price gradients approximately match empirical estimates. Variations to the baseline calibration build intuition on underlying mechanics.

Keywords: Urban Land Use, Commuting, Leisure, Value of Time

JEL classifications: R12, R14, R41

1 Introduction

The Alonso-Muth-Mills (AMM) framework of circular metro area with centralized employment has been a workhorse model for almost 50 years (Alonso, 1964; Mills, 1967; Muth, 1969). At its core, AMM gives insight into the tradeoff by which decreasing house prices compensate workers for the monetary and time costs of commuting. With just a few general assumptions, stripped-down models achieve closed-form solutions describing the falloff in population density, land prices, and house prices as residents live increasingly far from a central business district (CBD). The framework can also be extended to allow for a range of salient features such as income heterogeneity (Alonso, 1964; Muth, 1969), taste heterogeneity (Anas, 1990), multiple labor inputs (Brueckner, 1978), multiple modes of commuting (Anas and Moses, 1979; LeRoy and Sonstelie, 1983), and spatial

*The views expressed herein are those of the author and do not necessarily reflect the views of the Federal Reserve Bank of Kansas City or the Federal Reserve System. Thank you to David Albuoy and Santiago Pinto for feedback and suggestions. Daniel Molling provided excellent research assistance.

variations in consumption amenities (Brueckner, Thisse, and Zenou, 1999; De Bartolome and Ross, 2003).[1]

Numerical implementations of AMM that tried to match observed metro structure soon followed the theory. Notwithstanding generic criticism that these were “black box” exercises, many build intuition into the dependence of metro outcomes on deep structure such as the ability to substitute between land and capital in the production of housing (Muth, 1975; Arnott and MacKinnon, 1977a) and between housing and numeraire consumption in maximizing utility (King, 1977; Richter, 1978).

But from the perspective of quantitatively matching observed metros, the numerical implementations disappointed. Most effected outer commutes of 7 miles or less (e.g., Muth, 1975; King, 1977; Sullivan, 1983, 1985, 1986). Those that achieved more plausible outer commutes effected other implausible outcomes such as density and price gradients that are far steeper than empirical estimates (Mills, 1972; Arnott and MacKinnon, 1977a) or a maximum population density that is an order-of-magnitude too low (Steen, 1987). These first-order misses likely contributed to the withering of this numerical literature beginning in the late 1980s.[2]

A more fundamental criticism of AMM is that except for Arnott and MacKinnon (1977b) and Fujita (1989), the many variations of it do not include leisure as an explicit source of utility. Doing so would seem—and proves—a critical element for a framework that focuses on the tradeoffs associated with longer commutes.

A second fundamental criticism of AMM is the seemingly self-evident non-monocentricity of observed metro employment. In this case, however, empirical research suggests that centralized employment continues to be a relevant stylization of midsize U.S. metros (McMillen and Smith, 2003; Brinkman, 2013; Baum-Snow, 2014). Moreover, such criticism misses the necessity in all economic modeling to starkly simplify a complicated world. The stylization of monocentric land use is likely no worse than the stylization of a world populated by homogeneous individuals and firms. It is also straightforward to extend the AMM framework to allow for employment outside the CBD, either diffused throughout the entire residential area (Solow, 1973; Brueckner, 1978, 1979; Wheaton, 2004; Larson and Yezer, 2014) or clustered in one or a few exogenously-specified locations (White, 1976; Sullivan, 1986).

An alternative framework to AMM models employment location as arising *endogenously* from the tradeoff of localized agglomeration among firms with costly commuting by workers (Fujita and Ogawa, 1982; Anas and Kim, 1996; Lucas and Rossi-Hansberg, 2002). With sufficiently strong agglomeration relative to commuting costs, centralized employment can be a unique equilibrium. But a wide range of equilibrium land use patterns are also possible. A disadvantage of these models

[1] Bruekner (1987) presents an excellent integrated treatment of these and other variations of AMM. Duranton and Puga (2015) do the same for urban land use more generally.

[2] Arnott and MacKinnon (1977b) stands out as an exception to this critique. Its failure to spur much follow-up research at the time may reflect its rich setup, which includes worker heterogeneity, multiple commute modes, leisure as an explicit source of utility, and a semi-circular shape.

is that they trade off much of the richness of AMM to remain tractable. For example, utility is assumed to be Cobb Douglas with land proxying for housing.[3]

The present paper remains firmly within the AMM framework. It extends the framework to explicitly include leisure as a source of utility while requiring individuals to supply fixed weekly work hours. Doing so sharpens the marginal disutility from longer commutes. The model is calibrated to tightly match the distribution of commute times of CBD workers in Portland, Oregon. The implied geographic distribution of residences tightly matches that of these CBD workers. The implied population-density distribution and gradient approximately match those of Portland. The implied land- and house-price gradients are consistent with empirical estimates. A key element to achieving this match is targeting the observed land area and outer commute distance of Portland and so allowing an otherwise circular metro to span less than 360°.

Section 2 below presents the empirical evidence on the relevance of monocentric land use. Section 3 develops the theoretical model. Section 4 calibrates it. Section 5 describes quantitative results. Section 6 varies the baseline calibration to build intuition on the model's underlying mechanics.

2 Empirical Relevance

The monocentric stylization has never accurately described urban land use. Even before suburbanization, a wide range of service occupations closely complemented residential location. Then, during the 1950s through 1990s, less-complementary jobs eventually followed people away from principle cities but to a much smaller extent. Baum-Snow (2014) calculates that the *share* of urban jobs that shifted to suburbs was only one third the share of residents that shifted there. For jobs likely to benefit from agglomerative spillovers—such as in finance, insurance, and real estate—the shift to the suburbs was minimal.

For workers in agglomerative occupations, the share of metro employment in the CBD remained moderately high in 2000 (Table 1, top horizontal block). Among midsize metros (those with population of 1 to 2 million), the mean CBD share was 29 percent. In selected metros it was even higher: 36 percent in Portland Oregon and 43 percent in Pittsburgh. Among agglomerative occupations, lawyers were especially tied to the CBD. On average more than half worked there; in Pittsburgh and Austin, nearly two-thirds did.

Correspondingly, proximity among agglomerative workers remained far higher in CBDs than in the remainder of midsize metros (Table 1, second horizontal block). On average, agglomerative workers in a CBD experienced a density that was 21 times higher; in Pittsburgh, they experienced a

[3]More recently, Chatterjee and Eyigungor (2013) develop a hybrid of the endogenous location and AMM frameworks and Brinkman (2013) extends it to allow for mixed land usage and commuting congestion. Ahlfeldt and Nicolai (2013) establish empirically that localized agglomeration helps sustain moncentric land use as transporation costs fall.

	metros with pop 1 to 2 mn (mean)	Portland	Denver	Columbus	Pitts-burgh	Sacra-mento	Austin
CBD Share							
all workers	0.186	0.235	0.186	0.177	0.234	0.180	0.265
agglom occupations	0.294	0.365	0.290	0.309	0.425	0.307	0.340
legal occupations	0.524	0.590	0.569	0.606	0.646	0.498	0.659
land	0.012	0.019	0.012	0.011	0.008	0.013	0.016
residents	0.021	0.037	0.027	0.016	0.031	0.027	0.033
Relative Density							
(CBD to remainder)							
all workers	14	18	19	11	30	17	14
agglom occupations	21	32	27	16	83	25	15
legal occupations	31	55	69	55	65	31	65
1990 Urban Subcenters							
(McMillen and Smith, 2003)	0.8	2	1	0	1	0	0
Commute Mode (to CBD)							
all workers:							
car	0.864	0.750	0.796	0.878	0.726	0.874	0.895
public	0.099	0.190	0.157	0.074	0.223	0.089	0.062
walked/bicycle/home	0.037	0.060	0.047	0.049	0.050	0.037	0.042
agglom occupations:							
car	0.886	0.792	0.839	0.913	0.786	0.870	0.899
public	0.042	0.102	0.067	0.028	0.133	0.046	0.026
walked/bicycle/home	0.069	0.102	0.090	0.056	0.076	0.079	0.073

Table 1: Empirical Monocentricity in 2000. Metro and CBD delineations are described in the text. Agglomerative occupations combine business and financial operations; computers and mathematical; life, physical, and social sciences; legal; and arts, design, entertainment, sports, and media. Densities are constructed as the weighted average of raw densities within each census tract; weights are the number of workers whose density is being compared. Mean midsize-metro employment subcenters excludes San Antonio, which has 4, and San Jose and Virgina Beach, which are not in the sample. Sources: Hollian and Kahn (2012); McMillen and Smith (2003); Census Transportation Planning Package 2000; U.S. Decennial Census 2000 summary files.

density that was more than 80 times higher.[4] Consistent with this concentration, Brinkman (2013) shows that employment density in 2000 in Columbus, Philadelphia, and Houston fell off sharply approximately 3 miles from the city center.

More generally, firms located in CBDs tend be larger and more productive than firms located elsewhere in metros (Brinkman, Coen-Priani, and Sieg, 2014). This joint centralization of agglomerative workers and more productive firms is likely to anchor the geographic distribution of residents throughout midsize metros (Black, Kolesnikova, and Taylor, 2014).

Nor are midsize metros especially polycentric. McMillen and Smith (2003) identify urban employment subcenters for a sample of 62 large urban areas in 1990 (Table 1, third horizontal block). They estimate that a metro with average traffic congestion and other characteristics is expected to have only one employment sub-center until its population exceeds 4 million.[5]

Lastly, most workers in midsize-metro CBDs commute there by automobile (Table 1, bottom horizontal block). Among all CBD workers, 86 percent did so in 2000. Among CBD workers in agglomerative occupations, 89 percent did so. This suggests that the stylization of a single commute mode remains empirically relevant.

Constructing these measures of monocentricity requires judging which parcels of land within officially-delineated metropolitan areas are truly "metro" in character and which among these form the CBD. These judgments are also critical for the numerical calibration below.

From a theory perspective, I interpret a metro to be a geographically-integrated labor market that encompasses the residences of most of its workers while excluding primarily agricultural or unoccupied land. In practice, I construct metros by combining all census tracts in a Core-Based Statistical Area (CBSA) with a population density of at least 500 persons per square mile *or* an employment density of at least 1000 workers per square mile. For the resulting midsize metros, this union on average captures 84 percent of CBSA population, 89 percent of CBSA workers, but just 17 percent of CBSA land area (Table 2).[6]

As an example, population density in the resulting Portland metro is shown in Figure 1. Its land area, which is made up of the shaded census tracts, equals 8 percent of CBSA land area. Its CBD, the delineation of which will be described presently, is centered around the black dot within the downtown highway loop. Population density is highest just to the east and north of this downtown loop.

CBDs are geographically anchored by centroids constructed by Holian and Kahn (2012). These

[4] The underlying densities for this comparison are constructed as worker-weighted means across census tracts. Doing so measures density as it is "experienced" by workers. This can be interpreted as the frequency of bumping into another worker. The more standard calculation of density, total workers divided by total land area, measures average worker density as experienced by land parcels (Glaeser and Kahn, 2004; Rappaport 2008a).

[5] McMillen and Smith identify subcenters as spikes in employment density above a smooth fitted surface.

[6] This residential threshold corresponds to the Census Bureau criteria for delineating urbanized areas. The employment threshold typically adds only a handful of additional census tracts and so results are robust to setting it higher. Employment density is calculated using the Census Transportation Planning Package 2000, which re-tabulates the 2000 decennial census based on individuals' place of work. The definition of "midsize" as corresponding to a population of 1 to 2 million applies to this constructed delineation of metros rather than to the CBSA delineation.

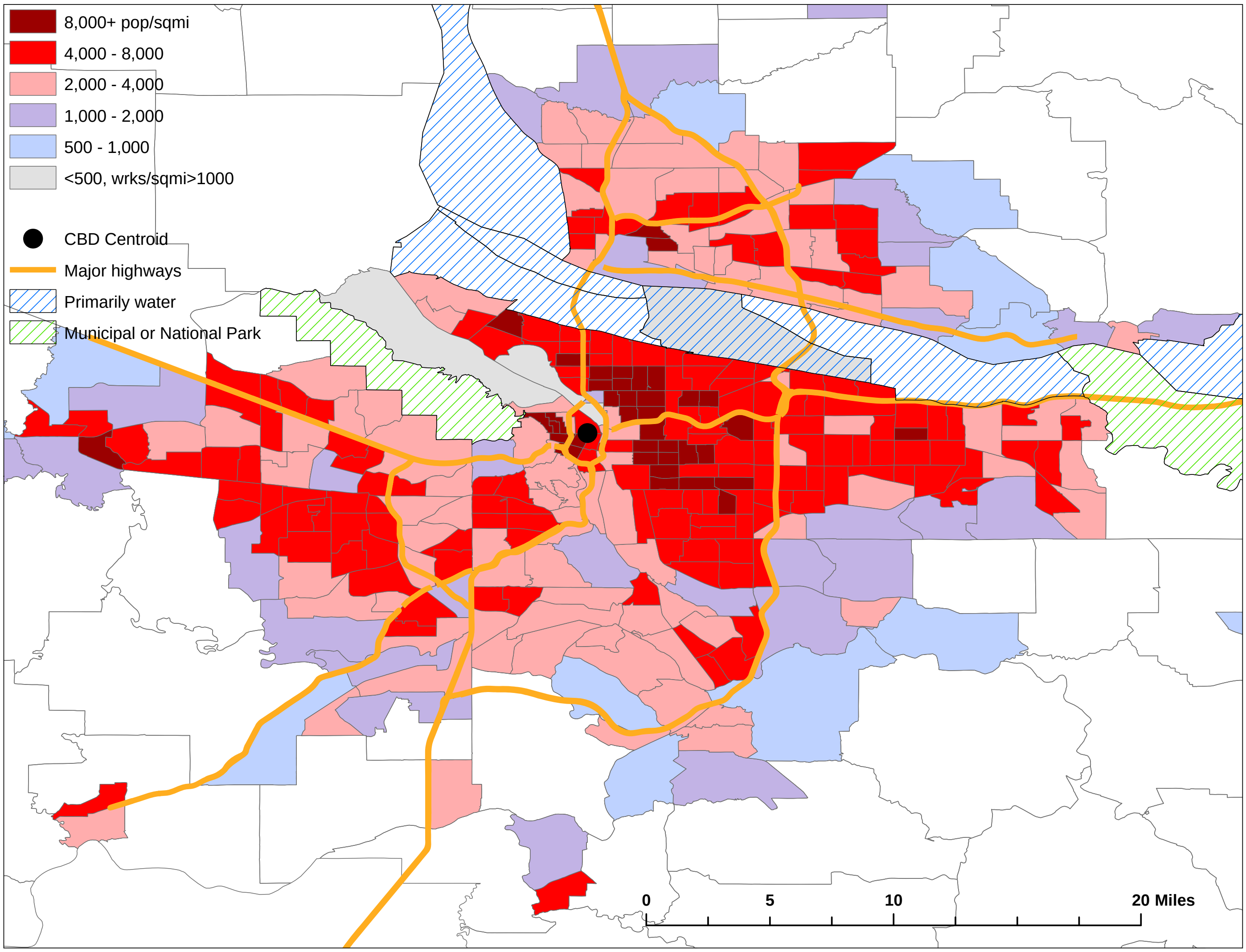

Figure 1: Portland Oregon Metro Population Density in 2000. Metro is constructed as the union of all Portland CBSA census tracts with population per square mile of at least 500 or employment per square mile of at least 1000. A few isolated census tracts that meet one of these criteria lie outside the displayed area. Grey-shaded tracts meet the employment criterion but not the population one. The black dot is the centroid returned by Google Earth for the city of Portland. Sources: Holian and Kahn (2012); Census Transportation Planning Package 2000; U.S. Decennial Census 2000 summary files.

	metros with pop 1 to 2 mn	Portland	Denver	Columbus	Pitts-burgh	Sacra-mento	Austin
METRO AREA							
Population	1,480,000	1,610,000	1,970,000	1,260,000	1,780,000	1,560,000	910,000
U.S. rank	21 to 37	25	21	35	23	27	41
share of CBSA pop	0.84	0.83	0.91	0.78	0.73	0.87	0.73
Workers	760,000	860,000	1,030,000	720,000	900,000	700,000	570,000
share of CBSA workers	0.89	0.89	0.92	0.85	0.83	0.87	0.87
Land Area (sq.mi)	580	540	600	560	860	530	400
share of CBSA land	0.17	0.08	0.07	0.14	0.16	0.10	0.09
CBD							
Land Area (sq.mi)	6.2	10.3	7.2	5.9	6.8	6.7	6.2
share of metro land	0.012	0.019	0.012	0.011	0.008	0.013	0.016
Commute Distances (mi)							
90th pctile	13.0	12.3	12.7	12.0	14.2	15.1	13.8
95th pctile	15.8	13.5	13.9	13.6	18.0	17.8	16.1
98th pctile	19.3	14.7	15.3	15.0	21.1	21.8	18.8
99th pctile	22.4	17.1	17.2	24.3	24.4	24.9	27.2

Table 2: Metro Geography in 2000. Metros are constructed as the union of CBSA census tracts with a density of at least 500 persons per square mile *or* an employment density of at least 1000 workers per square mile. Central business districts are constructed as the union of census tracts with a minimum employment density of 8000 workers per square mile *and* with centroids within 5 miles the Google Earth centroid of the largest principle city. Sources: Holian and Kahn (2012); Census Transportation Planning Package 2000; U.S. Decennial Census 2000 summary files.

are located at the latitude and longitude returned by Google Earth upon entering each CBSA's largest principle city (the first city in each CBSA's official name). The algorithm by which Google Earth selects these points is unclear. But extensive inspection shows that the centroids closely correspond to subjective judgements of CBD location.[7] For each metro, all census tracts within 5 miles of the Google centroid that have an employment density of at least 8 thousand workers per square mile together make up the CBD. For midsize metros, the resulting CBDs average 6 square miles.

The resulting CBD for Portland is shown in Figure 2. It is made up of the contiguous dark and bright red census tracts immediately surrounding the Google centroid. Only four census tracts with density above the 8 thousand threshold are not included in the CBD.

[7]For example, for the centroid of New York City, Google Earth returns Broadway and Chambers; for Los Angeles, it returns First and Main; for Chicago, Jackson and Federal; for San Francisco, Market and Van Ness.

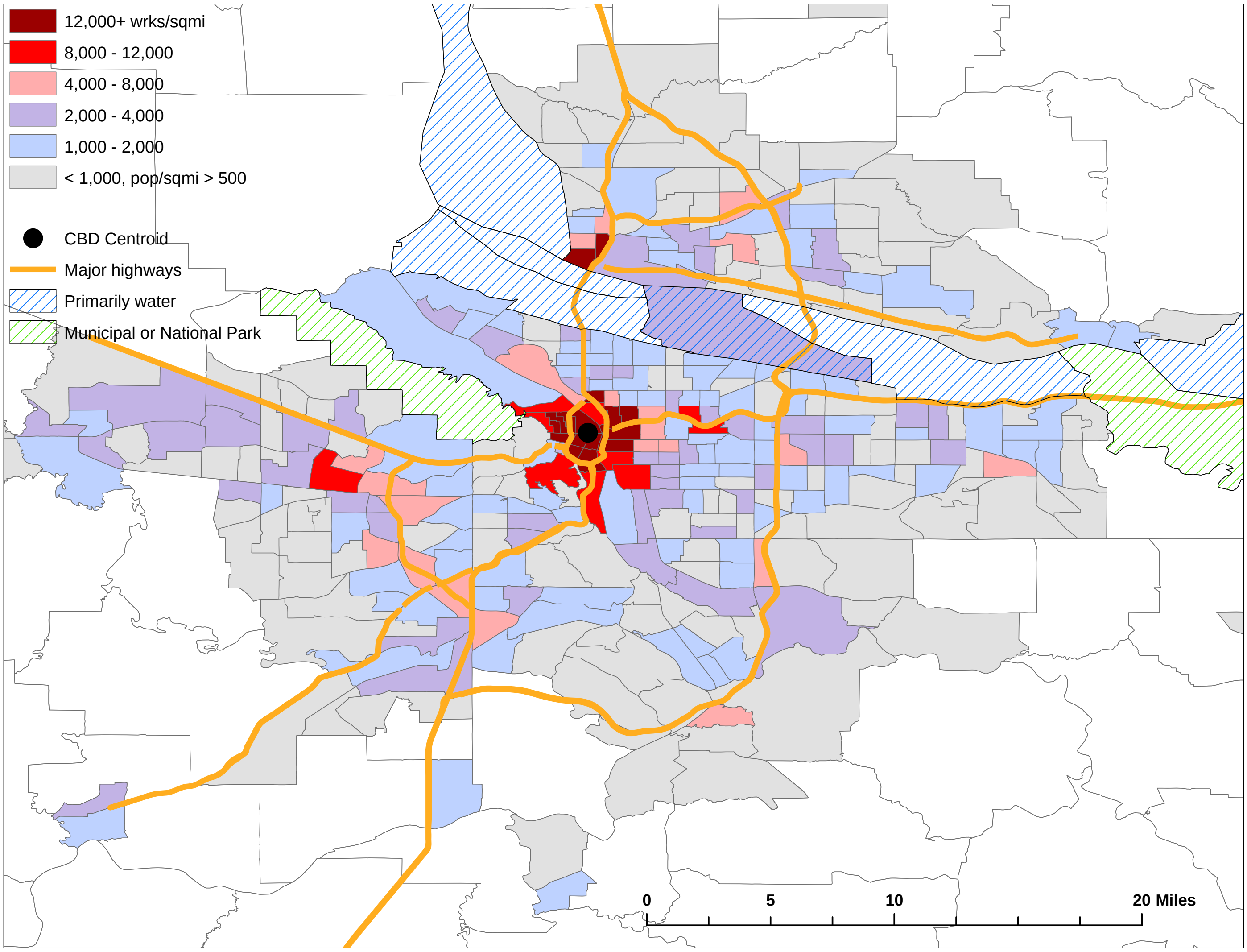

Figure 2: Portland Oregon Metro Employment Density. Metro is constructed as the union of all Portland CBSA census tracts with population per square mile of at least 500 or employment per square mile of at least 1000. A few isolated census tracts that meet one of these criteria lie outside the displayed area. Grey-shaded tracts meet the population criterion but not the employment one. The black dot is the centroid returned by Google Earth for the city of Portland. The surrounding tracts in dark and bright red make up the CBD. Sources: Holian and Kahn (2012); Census Transportation Planning Package 2000; U.S. Decennial Census 2000 summary files.

3 The Model

The setup is static and so should be interpreted as a long-run outcome. The metro consists of a central business district where production of a numeraire good takes place and a finite number of concentric residential rings surrounding it.[8] Total population and land area are exogenously specified. So too are the radius of the CBD, the number of residential rings, and the width of each. In order to simultaneously match observed metro land areas and commute distances, the metro is allowed to occupy an exogenous span, $\widehat{\theta} \leq 360°$. Figure 3 illustrates the setup.[9]

3.1 Production

Numeraire production, which takes place exclusively in the CBD (ring 0), is Cobb Douglas in land, capital, and aggregate labor hours. Each factor is paid its marginal product,

$$X = L_0^{\alpha_L} K_0^{\alpha_K} N^{1 - \alpha_L - \alpha_K} \tag{1}$$

$$r_0^L = \frac{\partial X}{\partial L_0} \qquad r_0^K = \frac{\partial X}{\partial K_0} \qquad w = \frac{\partial X}{\partial N}$$

Capital in the CBD is determined residually by achieving an exogenously-specified required rent, $r_0^K = \widehat{r}^K$. Aggregate labor hours are the sum of labor hours supplied by residents in each residential ring, j,

$$N = \sum_{j=1}^{J} POP_j \cdot n_j \tag{2}$$

Housing in each residential ring is produced with constant elasticity of substitution between land and capital, with each factor being paid its marginal revenue product

$$H_j = \left(\eta_L Lj^{\frac{\sigma_L - 1}{\sigma_L}} + (1 - \eta_L) K_j^{\frac{\sigma_L - 1}{\sigma_L}} \right)^{\frac{\sigma_L}{\sigma_L - 1}} \tag{3}$$

$$r_j^L = p_j \cdot \frac{\partial H_j}{\partial L_j} \qquad r_j^K = p_j \cdot \frac{\partial H_j}{\partial K_j}$$

The capital input can equivalently be interpreted as structure. As in the CBD, the quantity of capital in each ring is residually determined such that $r_j^K = \widehat{r}^K$.

For both types of production, factor payments to land and capital are paid to absentee owners. Rebating land and capital payments to individuals on a lump-sum basis, regardless of the ring in which they live, achieves similar quantitative results.

[8] Modeling space as discrete is standard in the numerical implementations of AMM cited herein. The reason is that a number of structural variables, such as the weights in utility and vectors of endogenous outcomes across residential rings are co-determined. In contrast, simplified AMM variations can often be solved as continuous gradients by a system of differential equations.

[9] A quick guide to notation: Decorative hats denote an exogenous variable. For example, $\widehat{d}_0$ is the radius of the CBD. Decorative tildes denote variables that are experienced when commuting through a ring. For example, $\widetilde{s}_j$ is the speed of commute traffic through ring j. The combination of a hat on top of a tilde denotes an exogenous variable experienced when commuting through a ring. For example, $\widehat{\widetilde{d}}_j$ is the width of the jth residential ring. A decorative overbar denotes a population-weighted mean value. For example, $\overline{n}$ is the population-weighted mean number of work hours $((\sum_{j=1}^{J} \widehat{POP}_j \cdot n_j)/\widehat{POP})$.

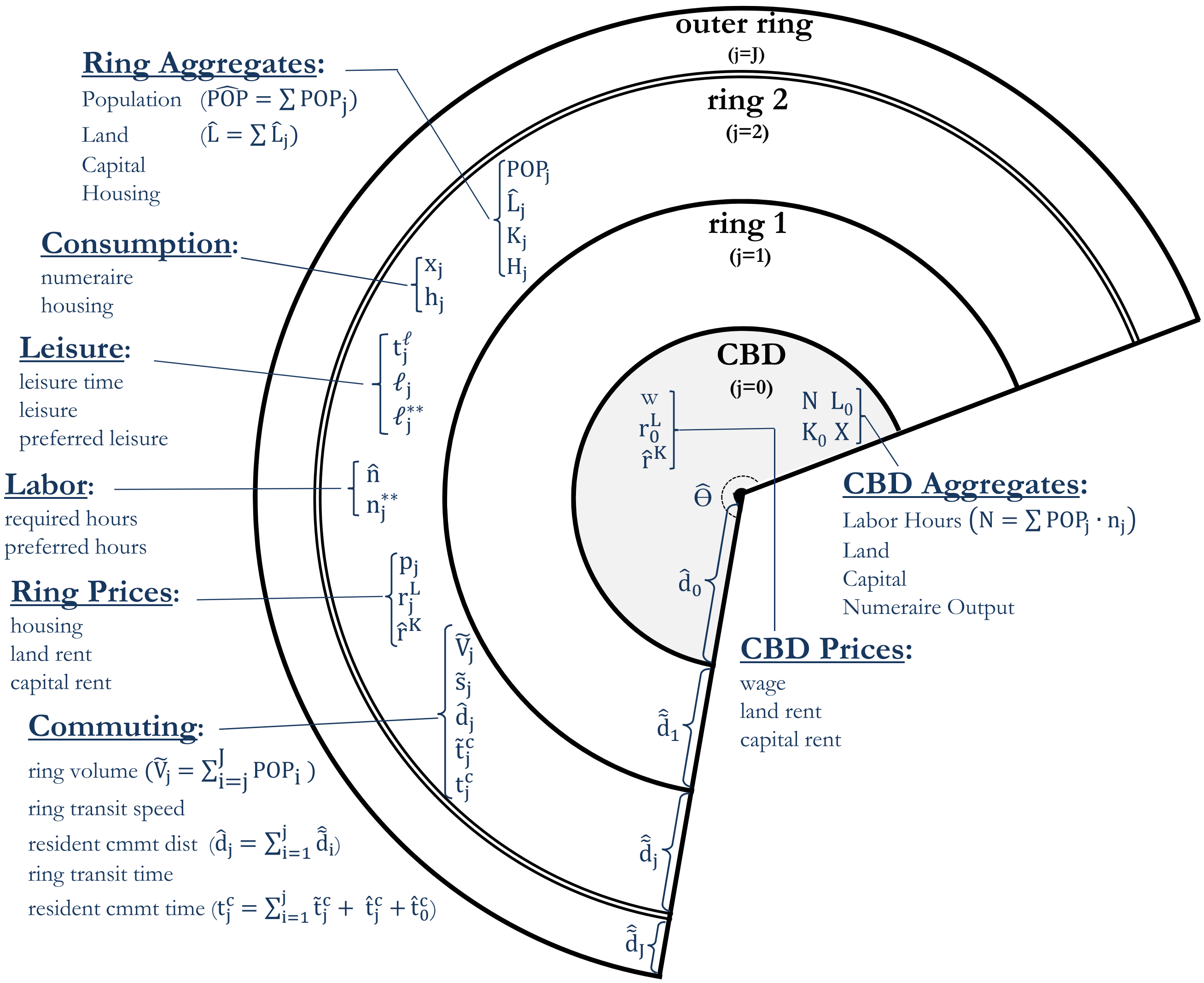

Figure 3: A Monocentric Metro. Residents live in ring $j \in \{1, 2, ..., J\}$ and commute to work in the CBD (ring 0). Decorative hats denote an exogenous variable. Decorative tildes denote a variable that applies to all commuters who pass through the subscripted ring.

3.2 Individuals

Utility is nested constant elasticity of substitution. Housing services, h, and numeraire consumption, x, are combined in an inner nesting. Leisure, ℓ, and the housing-numeraire hybrid are then combined in an outer nesting,

$$Z_j = \left(\eta_h \, h_j^{\frac{\sigma_h - 1}{\sigma_h}} + (1 - \eta_h) \, x_j^{\frac{\sigma_h - 1}{\sigma_h}} \right)^{\frac{\sigma_h}{\sigma_h - 1}} \tag{4a}$$

$$U_j = \left(\eta_\ell \, \left(\ell_j - \ell^{min}\right)^{\frac{\sigma_\ell - 1}{\sigma_\ell}} + (1 - \eta_\ell) \, Z_j^{\frac{\sigma_\ell - 1}{\sigma_\ell}} \right)^{\frac{\sigma_\ell}{\sigma_\ell - 1}} \tag{4b}$$

The CES specification of leisure and the numeraire-housing composite generalizes Arnott and MacKinnon (1977b) and Fujita (1989), who model utility as Cobb Douglas. All other variations of AMM of which I am aware implicitly model leisure as perfectly substitutable with numeraire consumption. The required minimum leisure (Stone-Geary specification) allows for necessities such as sleeping and eating. Equivalently, individuals' time budget could be reduced by ℓ^{min}.

Leisure is derived both from explicit leisure time, t_j^ℓ, and from commute time, t_j^c,

$$\ell_j = t_j^\ell + \lambda \cdot t_j^c \qquad \lambda \leq 1 \tag{5}$$

The leisure component to commuting time makes it possible to match empirical estimates that individuals' marginal willingness to pay (MWTP) to shorten their commute time is typically below their wage rate (Small, Winston, and Yan, 2005; Small and Verhoef, 2007). The present specification implies that commuters who can consume their preferred bundle have an MWTP equal to $(1-\lambda)\cdot w$. A possible interpretation is that drivers enjoy listening to their radio and talking on their cell phone (hands free).[10]

Define disposable income to be total wage income less numeraire commute costs. Let d_j denote the distance of each one-way commute and δ denote the per mile numeraire cost. Then,

$$y_j^d = w \cdot n_j - \delta \cdot d_j \cdot \text{trips} \tag{6}$$

Individuals face the numeraire budget constraint that their consumption expenditure not exceed their disposable income. Similarly, they face the time constraint that the sum of their weekly work hours, commute hours, and leisure-time hours not exceed physical hours,

$$x_j + p_j \cdot h_j \leq y_j^d \tag{7}$$

$$n_j + t_j^c \cdot \text{trips} + t_j^\ell \leq 24 \cdot 7 \tag{8}$$

[10] Modeling commuting as including a positive leisure component contrasts with surveys that find that commuting is among the least-liked uses of time (Krueger et al., 2009). One partial reconciliation is that leisure content may be a decreasing function of traffic congestion (Rappaport, 2014).

Individuals' *preferred* consumption bundle, $\{x_j^{**}, h_j^{**}, \ell_j^{**}\}$, equates the marginal utility relative to price for each of numeraire, housing, and leisure consumption,

$$\partial U_j/\partial x_j^{**} = \frac{\left(\partial U_j/\partial h_j^{**}\right)}{p_j} = \frac{\left(\partial U_j/\partial \ell_j^{**}\right)}{w} \tag{9}$$

Under the baseline calibration below, individuals are constrained to work fixed hours $\widehat{n}$. This residually implies that their actual consumption bundle, $\{x_j, h_j, \ell_j\}$, will typically differ from their preferred one. As will be illustrated below, fixing work hours considerably sharpens the disutility of long commutes. Notwithstanding the assumed fixity, the numerical implementation must solve for individuals' preferred bundle in order to calibrate the weight on leisure, η_ℓ. In particular, the baseline calibrates η_ℓ such that the desired leisure of residents in the first ring equals their actual leisure, $\ell_1^{**} = \ell_1$. Equivalently, individuals in the first residential ring prefer to work their required hours.

With fixed hours, only the first equality in (9) will hold,

$$\partial U_j/\partial x_j = \frac{\left(\partial U_j/\partial h_j\right)}{p_j} \tag{10}$$

Both the preferred and actual consumption bundles are determined at the house price that matches aggregate housing supply in each ring with *actual* aggregate housing demand in that ring,

$$H_j = POP_j \cdot h_j \tag{11}$$

Lastly, assumed perfect mobility implies that realized utility must be equal across residential rings. And an adding up constraint requires that the sum of endogenously-determined population in each ring equals the exogenously-specified metro population.

$$U_{j\neq 1} = U_1 \tag{12}$$

$$\sum_{j=1}^{J} POP_i = \widehat{POP} \tag{13}$$

3.3 Commuting

Individuals drive directly from the outer perimeter of their residential ring to the outer perimeter of the CBD. The model thus abstracts from the number and placement of highway rays. Commutes also include a fixed-time component, which can be interpreted as capturing arterial portions. As described above, the numeraire cost of a one-way commute is the per-mile cost, δ, times the radial distance to the CBD, d_j.[11] Distance is simply the sum of the ring widths through which a commuter must pass, $d_j = \sum_{i=1}^{j} \widetilde{d}_i$.

[11] More realistically, the per mile cost would depend on the speed at which a commuter travels (Larson and Yezer, 2014).

Under the baseline calibration, commute speed decreases with the volume of traffic. Let $\widetilde{V}_j$ denote the volume of commuters passing through residential ring j during each daily commute,

$$\widetilde{V}_j = \sum_{i=j}^{J} POP_i \tag{14}$$

The speed through any residential ring also depends on highway capacity through that ring, $\widetilde{V}_j^K$, and free-flow traffic speed, $\widehat{s}^f$, according to a standard formula (Small and Verhoef, 2007),

$$\frac{1}{\widetilde{\widetilde{s}}_j} = \frac{1}{\widehat{s}^f} \cdot \left(1 + a \cdot \left(\frac{\widetilde{V}_j}{\widetilde{V}_j^K}\right)^b\right) \qquad a, b > 0 \tag{15a}$$

A speed limit below the free-flow speed gives an extra degree of freedom with which to match observed commute times,

$$\widetilde{s}_j = \min\left(\widetilde{\widetilde{s}}_j \, , \, s^{max}\right) \tag{15b}$$

Highway capacity is assumed to endogenously depend on commute volume according to,

$$\widetilde{V}_j^K = \widehat{V} \cdot \left(\frac{\widetilde{V}_j}{\widehat{V}}\right)^{\sigma_V} \qquad 0 \leq \sigma_V \leq 1 \tag{16}$$

The term $\widehat{V}$ is an exogenously-specified value at which road capacity equals commute volume. Higher values of $\widehat{V}$ imply a larger volume of commuters can be accommodated before congestion sets in. The term σ_V is the elasticity of highway capacity with respect to volume. Parameterizing σ_V to equal 1 implies that speed is constant. Parameterizing σ_V to equal 0 implies that highway capacity is constant. Speed falls off more rapidly with commute volume as σ_V is increasingly below 1. Volume and capacity are interdependent in the sense that an exogenous increase in capacity directly increases commute speed and so pulls residents further away from the CBD. But this pull increases commute volumes at further distances and so partly offsets the increase in speed there, which is consistent with Duranton and Turner (2011).[12]

The time to commute through a ring, $\widetilde{t}_j^c$, is just the reciprocal of the speed through that ring. Total commute time sums the cumulative time to pass through each required residential ring together with two fixed components, one that is specific to the ring in which one lives and one that is experienced by all commuters,

$$t_j^c = \sum_{i=1}^{j} \widetilde{t}_i^c + \widehat{t}_j^c + \widehat{t}_0^c \tag{17}$$

The fixed times can be interpreted as commute time from a residence to a highway and from a highway to a workplace.

[12]Analagously, $\widehat{V}$ can itself be modeled as having an elasticity with respect to *metro* population to calibrate differences in speed across metros (Couture, Duranton, and Turner, 2014; Rappaport, 2014).

4 Baseline Calibration and Solution

The model requires specifying values for a large number of parameters. Metro population, land area, and maximum commute distance are set to match those of Portland, which also serves as benchmark against which to evaluate the model's fit. Alternatively calibrating and benchmarking the model to Denver achieves a very similar fit and quantitative results.[13]

Most remaining parameters are set based on empirical estimates of the parameter itself or of a single data moment that pins down its value. A few parameters are set to stylized values such as a required work week of 40 hours and ten one-way weekly commutes. Lastly, the parameters determining highway speed, highway capacity, and the fixed time components of commuting are jointly calibrated to best match fitted commute times for Portland.

4.1 Population and Geography

As is standard, the monocentric metro is modeled as "closed".[14] Its three key aggregate moments—population, land area, and maximum commute distance—are exogenously set to match those of Portland. Specifically, population is set to 1.6 million; land area, to 540 square miles; maximum commute distance, to 15 miles (Table 3). The latter corresponds to the 98th percentile distance of workers that commute alone by car to the CBD, departing home between 5am and 9am (Table 2).[15]

The radius of the CBD is exogenously set to 2 miles implying that the CBD accounts for 1.4 percent of metro land, slightly low compared to 1.9 percent in Portland. The exogenously-specified land area, outer commute distance, and CBD radius together imply that the metro must occupy 214°. The excluded 146° accounts for 41 percent of circular potential land area, approximately matching the 38 percent of metro Portland land that Saiz (2010) classifies as undevelopable. The residential portion of the metro is partitioned into five inner rings with 1-mile width and five outer rings with 2-mile width. Results are insensitive to a finer partition and to varying the CBD radius by several miles while keeping the outer commute distance unchanged.[16]

[13] A possible concern with the choice of Portland is its urban growth boundary, a set of zoning restrictions that demarcates the boundary between land used for residential use and land reserved for agriculture use. But for present purposes, an urban growth boundary serves as a rationale for the assumed exogenous metro radius. The similarity in the fit to Denver reflects that its ratio of population to land area and its 98th percentile commute distance are nearly the same as those of Portland. In consequence, the main quantitative difference from calibrating to Denver is a geographic span of 238° rather than 214°.

[14] The distinction between numerically modeling a stand-alone metro area as closed or open is largely one of solution strategy. Using an open strategy, a metro's population and land area are mechanically determined to match an exogenously-specified reservation utility level and perimeter land price. But utility is ordinal and choosing an appropriate numeraire price target for land, especially in a static context, is an extreme challenge. In contrast, a closed solution strategy cleanly matches three well-measured moments (population, land area, and outermost commute).

[15] The empirical distribution of commute distances and times for CBD workers is calculated using the Census Transportation Planning Package 2000. It reports tract-to-tract commute flows and the median travel time by commute mode for each of these. The distance of each flow is calculated using a great-circle formula between tract geographic centroids.

[16] The insensitivity of results to the partition reflects that all endogenous outcomes are smooth. The insensitivity

Description	Notation	Value/Target	Rationale
Population & Geography			
population	POP	1.6 million	Portland OR
land area	L	540 sq.mi	Portland OR
outermost commute	d_J	15 miles	Portland OR, 98th percentile
CBD radius	d_0	2 miles	Portland OR (approx matches CBD share of metro land)
span of settlement	θ	214°	residually implied
rings	J	10	without loss of generality
ring widths	$\{\tilde{d}_1, ..., \tilde{d}_5\}$	1 mile	without loss of generality
	$\{\tilde{d}_6, ..., \tilde{d}_{10}\}$	2 miles	without loss of generality
Numeraire Production			
land factor share	α_L	0.016	Jorgenson et al.
capital factor share	α_K	0.328	Jorgenson et al.
required wkly work hours	$\widehat{n}$	40	
Housing Production			
CES, L and K	σ_L	0.90	Jackson, Johnson, and Kaserman (1984); Thorsnes (1997)
weight on land	η_L	$\overline{\left(\frac{r^L \cdot L}{r^K \cdot K + r^L \cdot L}\right)} = 0.35$	Davis and Heathcote (2007)
Utility			
CES, h and x	σ_h	0.75	Davis and Ortalo-Magne (2011); Albuoy, Ehrlich, and Liu (2014)
CES, h-x and ℓ	σ_ℓ	0.33	s.t. mean Frisch elasticity = 0.20 (Reichling and Whalen, 2012)
weight on housing	η_h	$\overline{\left(\frac{p \cdot h}{x + p \cdot h}\right)} = 0.17$	housing nominal expenditure share of market PCE, average 1990-2000, U.S. NIPA
weight on leisure	η_ℓ	$n_1^{**} = 40$	residents in inner ring *prefer* to work required hours
min wkly leisure	ℓ^{min}	70 hrs	

Table 3: Non-Commuting Calibration. Bar decoration denotes a population-weighted mean; tilde decoration denotes a parameter that applies to all commuters who pass through the subscripted ring.

4.2 Production

Assumed Cobb Douglas production of numeraire requires parameterizing the factor income shares accruing to land, capital, and labor, $\{\alpha_L, \alpha_K, 1 - \alpha_L - \alpha_K\}$. The land share is set to 1.6 percent. This value corresponds to a weighted average across a large number of industries using intermediate input shares estimated by Jorgenson, Ho, and Stiroh (2005).[17] It is nearly identical to the 1.5% land share that Ciccone (2002) suggests is reasonable for the manufacturing sector. One third of remaining factor income is assumed to accrue to capital; two thirds are assumed to accrue to labor (Gollin, 2002). Under the baseline calibration, individuals are required to work 40 hours per week.

Production of housing services requires calibrating the elasticity of substitution between land and structure, σ_L, and the relative weight on land, η_L. The former is set to 0.90, which is meant to balance the wide range of empirical estimates. A survey by McDonald (1981) reports preferred estimates from twelve different studies ranging from 0.36 to 1.13. Updating this research, Jackson, Johnson, and Kaserman (1984) estimate the elasticity to lie somewhere between 0.5 and 1. More recently, Thorsnes (1997) argues that a unitary elasticity of substitution cannot be rejected. The weight on land is calibrated such that the household-weighted mean share of housing factor income, $\nu \equiv (r^L \cdot L)/(r^L \cdot L + r^K \cdot K)$, equals 0.35 as suggested in Davis and Heathcote (2007).[18]

4.3 Utility

The utility specification, (4a) and (4b), requires calibrating four key parameters: two elasticities of substitution, which together describe the curvature of the tradeoffs among numeraire, housing, and leisure; and two weights, which determine housing expenditure shares and desired work hours.

The elasticity of substitution between housing and the numeraire good, σ_h, is set to 0.75. Median metro rents and housing expenditure shares reported in Davis and Ortalo-Magné (2011) yield maximum-likelihood estimates of σ_h between 0.67 and 0.80. Albuoy, Ehrlich, and Liu (2014) report estimates that range from 0.42 to 0.76. Estimates using micro data are typically much lower. For example, Li et al. (2012), using simulated method of moments applied to a structural model of life-cycle housing consumption, estimate σ_h to be 0.32.

The elasticity of substitution between leisure and the housing-numeraire hybrid, σ_ℓ, is set to 0.33. Doing so implies that the population-weighted mean compensated elasticity of *preferred* work hours with respect to wages (the "Frisch elasticity") equals 0.20, which is the central value from a comprehensive survey of empirical studies reported in Reichling and Whalen (2012).

The weight on housing, η_h, is calibrated such that the population-weighted mean housing

to the radius of the CBD captures that the fixed supply of land in numeraire production affects only the nominal wage but not relative prices.

[17]The industry-specific intermediate input estimates, which are not included in the publication, were kindly provided by the authors.

[18]Davis and Heathcote find that between 1975 and 2004, land accounted for an average of 47 percent of the sales value of the aggregate U.S. housing stock. Adjusting for the fact that structures depreciate but land does not brings the land share down to approximately 35 percent.

share of consumption expenditures, $\mu \equiv (p \cdot h)/(x + p \cdot h)$, equals 17 percent. This matches the aggregate U.S. ratio of nominal rent plus owners' equivalent rent relative to nominal market personal consumption expenditures throughout the 1990s and early 2000s. The weight on leisure, η_ℓ, is calibrated such that residents in the innermost ring *prefer* to work the required number of hours, $n_1^{**} = \widehat{n} = 40$.

Lastly, it is assumed that individuals require ten hours each day for basics such as eating and sleeping, $\ell^{min} = 70$. Quantitative results are insensitive to moderately large variations around this.

4.4 Commuting

Baseline values of commuting parameters are reported in Table 4. Individuals make ten weekly one-way commutes. The numeraire cost per mile, δ, is set such that the population-weighted mean ratio of commute costs to wage income equals 0.05 (Albouy and Lue, 2014).

Stated and revealed preferences suggest that individuals' MWTP to shorten their commute time is about half their wage rate (Small, Winston, and Yan, 2005; Small and Verhoef, 2007). Thus the leisure content of commuting time, λ, is set to 0.50.

Remaining commuting parameters are calibrated to match fitted commute times of workers who drive alone to the Portland CBD departing from home between 5am and 9am. Table 5 reports results from regressing median tract-to-tract commute time on tract-to-tract great circle distance using a linear spline of four 4-mile segments. The regression is weighted by the tract-to-tract flows. Coefficients should be interpreted as the marginal time in minutes to drive an additional mile through each segment.

Maximum highway speed, $\widehat{s}^{max}$, is set to 60 mph. This matches the estimated marginal commute time per mile in Portland through distances at least 12 miles from the CBD.

Four criteria are used to jointly calibrate $\widehat{s}^f$, $\widehat{V}$, σ_V, and the the ring-specific fixed time components, $\widehat{t}_j^c$. First, commute speed through the innermost ring (1 mile) is required to equal 26 mph, which matches Portland commuters' fitted marginal commute time through the four miles closest to the CBD. Second, speed through the outermost two rings (4 miles) is required to be at the calibrated maximum. Third, the calibration relies as much as possible on variations in highway speed rather than the fixed time components to match variations in fitted times. Fourth, *increments* to the fixed time components, $\widehat{t}_j^c - \widehat{t}_{j-1}^c$, are kept "algorithmic." Specifically, fixed time increases by 1 minute for each additional mile commute distance through ring 6 (whose residents have a 7 mile commute). Above this, the increment to fixed time smoothly tapers to zero through ring 10.

These criteria, together with endogenously-determined commute volumes, achieve a tight match to the fitted commute times of Portland CBD workers (Figure 4, left panel). The baseline time gradient, which includes the ring-specific fixed components, catches up from about 3 minutes less than the fitted Portland gradient at near distances to within 1 minute of them at further distances. By construction, the model gradient's slope matches that of Portland across the

Description	Notation	Value/Target	Rationale
General			
weekly 1-way commutes	$trips$	10	
per mile cost	δ	$\overline{\left(\frac{\delta \cdot d_j}{w \cdot n_j}\right)} = 0.05$	Albouy and Lue (2014)
leisure content	λ	0.50	Small and Verhoef (2007)
Highway Speed			
maximum	$\widehat{s}^{max}$	60 mph	s.t. outer distance marginal commute time = 1 min/mi (Table 5 estimate)
free-flow	$\widehat{s}^{f}$	75 mph	to match Portland OR fitted commute times with $\tilde{s}_1 = 26$ mph
benchmark capacity	$\widehat{V}$	525 ths	to match Portland OR fitted commute times with $\tilde{s}_1 = 26$ mph
elasticity, capacity to volume	σ_V	0.80	to match Portland OR fitted commute times with $\tilde{s}_1 = 26$ mph
technical parameters	a, b	0.2, 10	Small and Verhoef (2007)
Fixed Time (Arterial Portion of Commute)			to match Portland OR fitted commute times with $\tilde{s}_1 = 26$ mph
CBD	$\widehat{t}^c_0$	4.0 min	
ring 1	$\widehat{t}^c_1$	1.0 min	
ring 2	$\widehat{t}^c_2$	2.0 min	
ring 3	$\widehat{t}^c_3$	3.0 min	
ring 4	$\widehat{t}^c_4$	4.0 min	
ring 5	$\widehat{t}^c_5$	5.0 min	
ring 6	$\widehat{t}^c_6$	7.0 min	
ring 7	$\widehat{t}^c_7$	8.5 min	
ring 8	$\widehat{t}^c_8$	9.5 min	
ring 9	$\widehat{t}^c_9$	10.0 min	
ring 10	$\widehat{t}^c_{10}$	10.0 min	

Table 4: Commuting Calibration Bar decoration denotes a population-weighted mean; tilde decoration denotes a parameter that applies to all commuters who pass through the subscripted ring. As described in Table 3, residential rings 1 through 5 have a width of 1 mile and rings 6 through 10 have a width of 2 miles. The calibrated fixed commute time increases by 1 minute for each additional 1 mile of distance for rings 1 through 6 and then tapers to no additional time in ring 10.

Median Tract-to-Tract Commute Time	(1) Portland	(2) Denver	(3) Colum-bus	(4) Pitts-burgh	(5) Sacra-mento	(6) Austin
Constant	8.38	8.00	8.31	9.20	8.87	7.92
	(0.81)	(0.73)	(0.72)	(0.94)	(1.17)	(0.78)
marginal minutes per mile						
0-to-4 miles:	2.40	2.89	2.42	3.08	1.98	2.70
	(0.26)	(0.25)	(0.23)	(0.30)	(0.39)	(0.26)
4-to-8 miles:	2.44	2.29	1.32	2.64	1.91	1.72
	(0.16)	(0.17)	(0.13)	(0.20)	(0.27)	(0.17)
8-to-12 miles:	1.16	1.96	1.21	1.11	1.32	0.96
	(0.19)	(0.18)	(0.14)	(0.22)	(0.27)	(0.18)
above 12 miles:	1.03	0.97	1.35	0.96	1.09	0.97
	(0.18)	(0.20)	(0.09)	(0.10)	(0.13)	(0.10)
Total Pairwise Obs	3,212	2,480	957	2,292	1,944	1,189
R^2	0.341	0.492	0.632	0.439	0.308	0.570

Table 5: Marginal Commute Times. Table reports estimated coefficients from regressing median tract-to-tract commute time on a four-segment spline of tract-to-tract distance for individuals who work in the CBD and commute there alone by car departing from home between 5am and 9am. Observations are weighted by the number of commuters making up each tract-to-tract flow. Corresponding average speed through each commute segment equals the reciprocal of the marginal time multiplied by 60 minutes per hour. Source: Census Transportation Planning Package 2000.

outermost four miles. Commuting inbound from the farthest suburb (right panel, right to left), highway speed remains at 60 mph from miles 15 to 11. It then gradually slows to 40 mph at mile 5 and to its calibrated minimum, 26 mph, at mile 1.

4.5 Solving

The model admits a large number of endogenous variables. These can be reduced to an exactly-identified system of 54 equations and unknowns. Four equations calibrate the structural parameters, $\{\eta_L, \eta_h, \eta_\ell, \delta\}$, to hit targets enumerated in Tables 3 and 4.

The remaining equations correspond to five 10-by-1 vectors, $P\vec{O}P$, $\vec{\ell}^{**}$, $\vec{h}^{**}$, $\vec{h}$, and $\vec{p}$. The associated equations are twenty first order conditions determining preferred leisure and housing consumption, (9); ten first order conditions determining actual housing consumption (10); ten

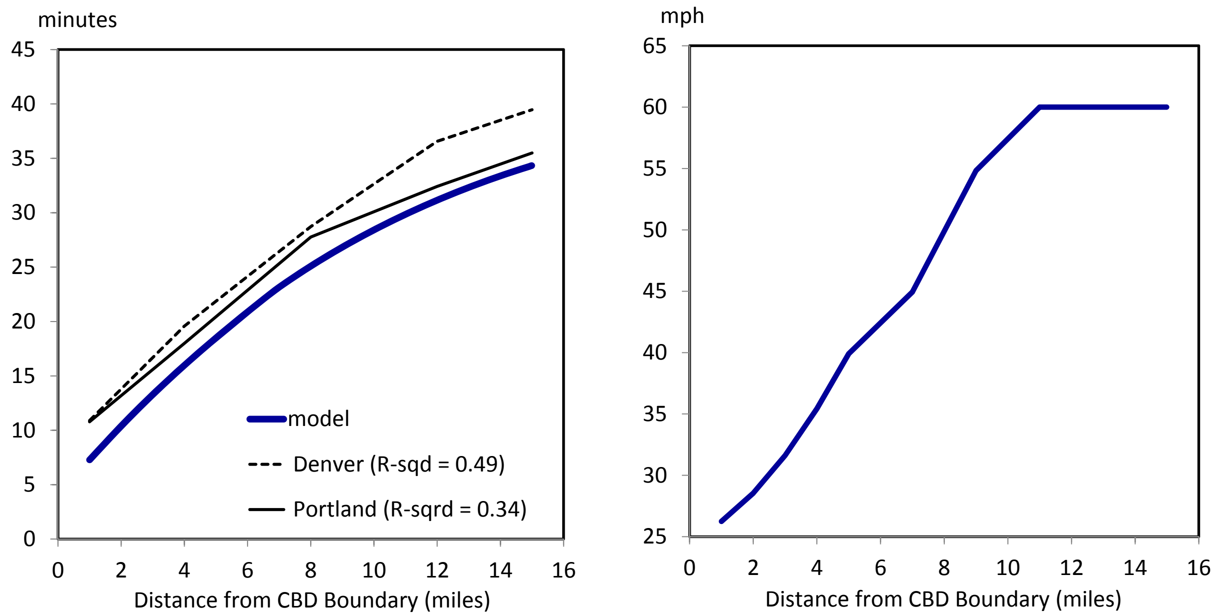

Figure 4: Commute Time and Speed. Left panel shows one-way commute times. The fitted gradients for Portland and Denver are based on tract-to-tract flows of individuals who drive alone to work in the CBD, departing from home between 5am and 9am. The tract-to-tract median commute time is regressed on a four-way spline of tract-to-tract straight-line distance weighted by the tract-to-tract flow. Right panel shows the modeled speed through each residential ring.

housing market clearing conditions, (11); nine utility matching equations, (12); and the single population adding-up equation, (13). The system is solved incrementally. First, the four parameters and the endogenous variables are solved for a metro with a single residential ring. The solved values are then used as guesses to solve for a metro with two residential rings. This is repeated until the metro comprises ten residential rings.

5 Baseline Quantitative Results

Baseline quantitative results establish that the model is consistent with observed metro land use in Portland, Denver, and other midsize U.S. metro areas. The calibrated metro tightly matches the geographic and commute-time distributions of workers who commute by car to Portland's CBD. It approximately matches the population density gradient and distribution of Portland. Its land and house price gradients are consistent with empirical estimates. So too are its house demand and supply price elasticities.

This consistency with observed land use is surprising given the strong simplifications of the model: no decentralized employment, no land use restrictions, no durability of the housing stock, no alternative modes of commuting, no consumption amenities, no heterogeneity of individuals and firms, and more. Because of these simplifications, reported quantitative values should be interpreted

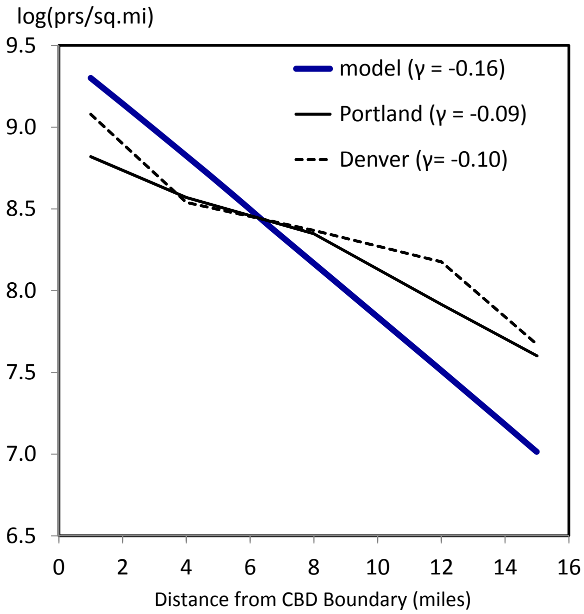

Figure 5: Population Density Gradients and Cumulative Distribution. Left panel shows modeled and fitted density gradients. Fitted ones are based on population-weighted regressions of log tract density on a four-segment spline. Average semi-elasticity of density with respect to distance is reported in parentheses. R-squared values for Portland and Denver are, respectively, 0.25 and 0.22. Right panel shows modeled and actual cumulative distributions of metro population and commute distance to the CBD.

as being quite imprecise.

More positively, the consistency with observed outcomes validates the empirical relevance of the monocentric framework. And the quantitative results illuminate the interplay of forces driving metro land use.

5.1 Population

The baseline population density gradient falls off more steeply than those of Portland and Denver (Figure 5, left panel). Correspondingly, the baseline cumulative distribution of metro population with respect to distance exceeds those of Portland and Denver (Figure 5, right panel, blue versus black lines). In contrast, the baseline CDF almost perfectly matches the CDFs of Portland and Denver residents who drive alone to work in their metros' CBD (blue versus red lines). Note that these fits are implied by the baseline calibration but *not* targeted by it.[19]

The model also approximately matches observed density when it is not conditioned on distance. The population-weighted mean and median density are within 5 percent of that of Portland (Table

[19] The slope of the density gradients measures the semi-elasticity with respect to distance. The semi-elasticity of -0.16 for baseline population density compares to semi-elasticities of -0.09 for Portland and -0.10 for Denver. Paulson (2012) estimates similar semi-elasticities using a a large cross section of metros in 2000. In contrast, Macauley (1985) and Jordan, Ross, and Usowski (1998) respectively estimate elasticities for U.S urbanized areas in 1980 and U.S. metros in 1990 centered on the modeled value.

	Model	Portland	Denver	Columbus	Pitts-burgh	Sacra-mento	Austin
mean	**4,500**	4,700	5,400	4,200	4,300	5,200	4,200
distribution:							
5th pctile	**1,100**	1,100	1,100	700	700	800	800
10th pctile	**1,500**	1,500	1,600	900	800	1,300	1,000
25th pctile	**2,200**	2,700	3,100	1,900	1,500	2,900	1,800
50th pctile	**4,100**	4,300	5,000	3,500	3,500	5,100	3,500
75th pctile	**6,800**	6,100	7,000	5,800	5,900	6,900	5,300
90th pctile	**9,400**	7,800	9,100	7,700	8,900	8,600	8,300
95th pctile	**10,900**	9,000	10,700	9,700	11,400	9,900	10,800
98th pctile	**10,900**	11,500	15,900	13,900	14,600	11,800	17,300

Table 6: Model Fit: Population Density Not Conditioned on Distance. Modeled density percentiles are weighted by ring population. Observed density percentiles are weighted by tract population.

6). The population-weighted density distribution is within 20 percent of that of Portland from the 5th through the 98th percentiles. The modeled distribution also remains especially close to that of Denver from the 75th through the 95th percentiles.

5.2 Commuting

The tight match of the baseline commute times to fitted commute times of Portland CBD workers, described in the previous section, is an explicit target of the calibration rather than an implied result. Even so, the four criteria for setting the speed and fixed-time parameter values are fairly restrictive and so achieving a tight match contributes to the validation of the model.[20]

The distribution of commute times not conditioned on distance also closely matches that of Portland from the 5th through the 90th percentiles (Table 7). But at the upper tail, modeled commute times fall considerably short of observed ones.

Table 8 enumerates some additional summary statistics on commuting. The median individual commutes 7 miles on a radial highway to work at an average speed of 35 mph. For individuals living in the outermost ring, average highway speed is 44 mph. Taking account of the fixed time component of commuting, average speeds are considerably lower. The numeraire commute cost, which is calibrated to equal 5 percent of income on average, rises to 10 percent of income in the outermost ring.

[20] Alternatively, the ring-specific fixed commute times could have been chosen to *exactly* match the fitted commute times. In this case, the match to fitted commute times would be strictly by construction.

	Model	**Portland**	**Denver**	**Colum-bus**	**Pitts-burgh**	**Sacra-mento**	**Austin**
mean (minutes)	**22.5**	23.3	25.2	21.8	27.6	22.9	22.5
distribution:							
5th pctile	**7.3**	8.9	10.1	10.6	10.5	7.9	10.3
10th pctile	**10.4**	10.7	10.7	15.0	13.0	10.5	10.9
25th pctile	**16.0**	15.5	15.7	15.9	20.1	15.5	15.5
50th pctile	**23.2**	20.6	25.2	20.6	25.9	20.5	20.6
75th pctile	**29.8**	30.4	30.7	25.6	35.1	30.2	30.3
90th pctile	**32.3**	35.9	40.7	30.3	45.2	35.6	30.9
95th pctile	**34.3**	45.2	45.2	30.5	50.1	45.1	40.3
98th pctile	**34.3**	45.8	45.8	36.2	60.4	50.2	45.2

Table 7: Model Fit: One-Way Commute Times. Empirical times are for individuals who commute alone by car to the CBD, departing home between 5am and 9am. Underlying data are median tract-to-tract commute times. The reported mean and percentile times are the flow-weighted average of these median times.

		ring		
	mean	inner (j=1)	median (j=6)	outer (j=10)
distance	7.5 mi	*1.0 mi*	7.0 mi	*15.0 mi*
time	22.5 min	7.3 min	23.2 min	34.3 min
average highway speed	35 mph	*26 mph*	35 mph	44 mph
average speed (including fixed time)	18 mph	8 mph	18 mph	26 mph
weekly cost (relative to income)	*0.050*	0.007	0.046	0.100

Table 8: Commute Summary Statistics. Italics denote data moments explicitly targeted by the baseline calibration.

5.3 Prices and Consumption

The baseline land-price gradient is moderately steeper than the population-density gradient (Figure 6, left panel, dark blue line). This is easier to understand in terms of moving closer to the CBD (from right to left). As distance decreases, housing production becomes more capital intensive and the price of housing increases. Both of these increase the marginal revenue product of the land input

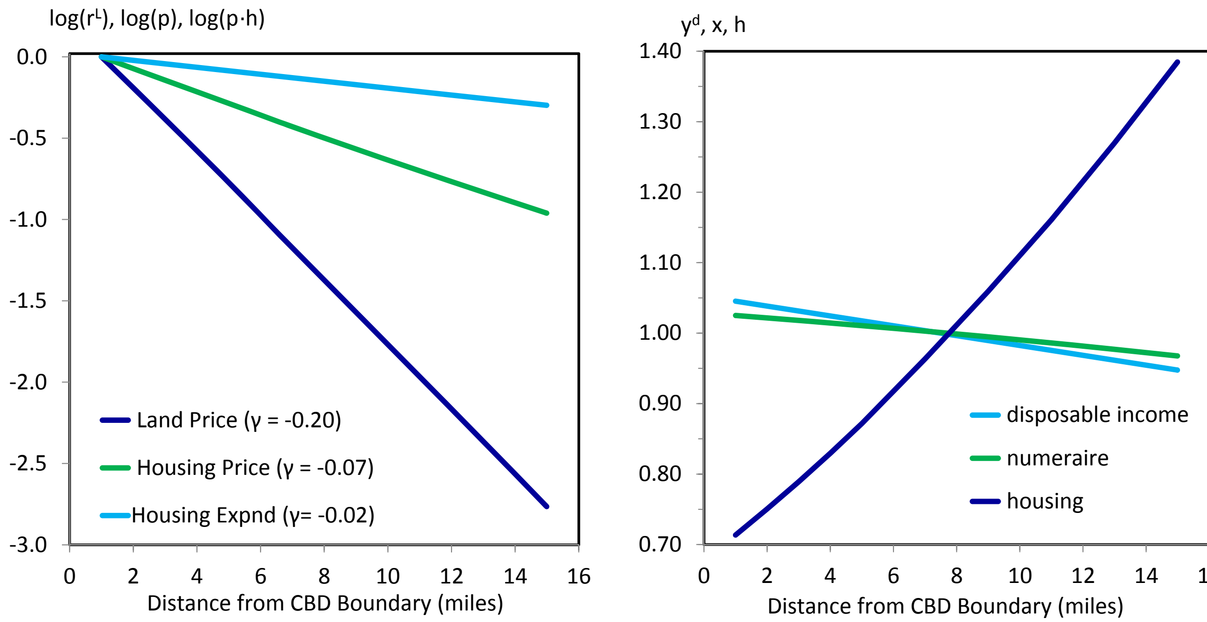

Figure 6: Prices, Consumption, and Disposable Income. Semi-elasticity with respect to distance is reported in parentheses.

and so its price. The land price gradient is steeper than the population density gradient because of the calibrated complementarity between land and capital and between housing and numeraire consumption, $\sigma_L < 1$ and $\sigma_h < 1$. The substitution from land to capital and from housing to numeraire cause the price of housing to increase less steeply (green line). The latter substitution causes the expenditure on housing to increase still less steeply (light blue line).[21]

In response to lower house prices, individuals sharply increase their consumption of housing services as they live further from the CBD (right panel, dark blue line). Per capita consumption of housing in the outermost ring is nearly twice that of the innermost ring. Because of the calibrated complementarity between housing and numeraire consumption, individuals decrease their numeraire consumption by less than the decrease in their disposable income (green line versus light blue line).[22]

Figure 7 gives some measure of the physical characteristics of housing. Aggregate housing services per unit of land is 5 times higher in the innermost ring compared to the outermost one. Housing capital per unit land is 12 times higher. The obvious interpretation is that residential structures are between 5 and 12 times taller.

Table 9 enumerates some additional summary statistics on housing consumption and produc-

[21]The land price gradient has a semi-elasticity of -0.20. For comparison, McMillen (1997) and Colwell and Munneke (1997) estimate the semi-elasticities for Chicago circa 1990 to be in the range -0.11 to -0.14. The modeled house price gradient has a semi-elasticity of -0.07. For comparison, McMillen (2003) estimates the corresponding semi-elasticity in municipal Chicago during 1980s and 1990s was in the range of -0.04 to -0.08.

[22]If housing and numeraire consumption were instead calibrated to be substitutes, the numeraire consumption gradient would be steeper than the disposable income gradient. If, in addition, there were no numeraire cost to commuting, the house expenditure gradient would slope upward.

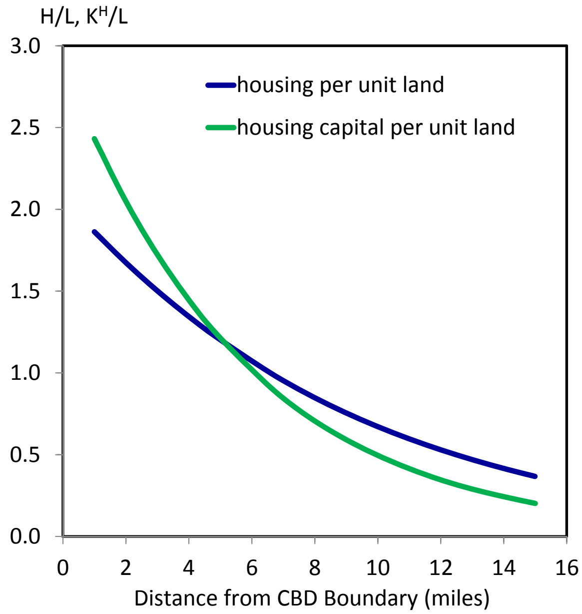

Figure 7: Housing Services and Housing Structure per unit of Land. Values are normalized to equal 1 at the population-weighted mean.

tion. Both the housing expenditure share and the land factor income share, μ and ν, moderately decline with commute distance. The uncompensated price elasticity of housing demand remains approximately constant at -0.79 throughout the metro. This is high in absolute value compared to empirical estimates, which typically find the uncompensated price elasticity to be somewhere in the range of -0.40 to -0.70 (Pollinsky and Ellwood, 1979; Hanushek and Quigley, 1980; Goodman, 1988, 2002; Albouy, Ehrlich, and Liu, 2014). The price elasticity of housing supply, ϵ_h^s, rises from just below 1.5 in the innermost ring to almost 2.0 in the outermost one. This range is moderately high compared to Saiz (2010), who estimates long-run supply elasticities of 1.07 for Portland and 1.53 for Denver.

5.4 Leisure

Including leisure as a source of utility is the key theoretical innovation of the present model. Because the baseline calibration fixes work hours, the leisure *time* gradient (Figure 8, left panel, green line) simply mirrors the commuting time gradient above. Because of the leisure component to commute time, leisure falls off at half the rate of leisure time (dark blue line).

The right panel of Figure 8 shows individuals' marginal valuation of leisure normalized by the metro wage (dark blue line). This is simply the numeraire compensation needed to keep utility constant while marginally decreasing leisure,

$$\mathrm{MV}(\ell) = \frac{\partial U/\partial \ell}{\partial U/\partial x}$$

		ring		
	mean	inner (j=1)	median (j=6)	outer (j=10)
house consumption				
house expnd share (μ)	*0.170*	0.186	0.171	0.153
house demand elasticity:				
uncompensated	-0.79	-0.80	-0.79	-0.79
compensated	-0.62	-0.61	-0.62	-0.63
house production				
land factor share (ν)	*0.35*	0.38	0.35	0.32
supply elasticity	1.68	1.47	1.66	1.94

Table 9: House Consumption and Production. Italics denote data moments explicitly targeted by the baseline calibration. House demand elasticities hold commute time constant.

The marginal valuation of leisure *time* equals the marginal valuation of leisure because the former contributes one-for-one to the latter.

Also shown are indviduals' normalized MWTP to shorten their commute (green line). This equals $(1 - \lambda)$ times their marginal valuation because they "lose" λ units of leisure per marginal unit of shorter commute time. The model is calibrated so that individuals in the innermost ring prefer to work the required hours and so their normalized marginal valuation is exactly 1 and their normalized MWTP is exactly 0.5.

Residents of more distant rings would like to partly offset their increased commute time with decreased work hours. In other words, their preferred leisure falls off more gradually than does their actual leisure (left panel, dashed versus solid blue line). For example, residents of the outermost ring spend 4.5 more hours per week commuting than do residents of the innermost ring and so have 2.25 less hours of leisure. These outermost residents prefer to work just under 39 hours per week. Because they cannot, their normalized marginal valuation of leisure exceeds 1.

The downward sloping preferred leisure gradient captures the income effect of the per mile numeraire commute cost. In its absence, individuals would prefer to consume the same amount of leisure regardless of where the live and so cut their weekly work hours by $(1 - \lambda)$ for each additional weekly hour of commute time.

The right panel of Figure 8 also shows the average willingness to pay (AWTP) to have a commute time of zero (light blue line). This is calculated using the total willingness to pay (TWTP) for a zero commute time, which is the compensating variation needed to hold utility constant while continuing to face the same disposable income and house price of one's residential ring. AWTP

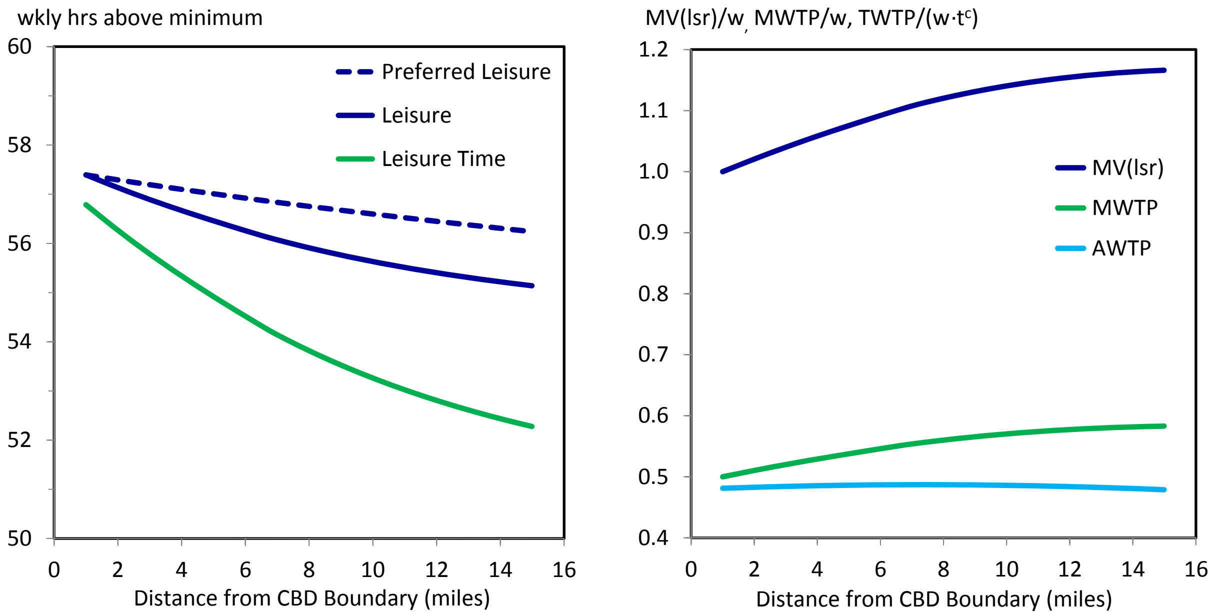

Figure 8: Leisure and Its Valuation. Leisure and leisure time are the amount above the weekly minimum. The marginal valuation of leisure and the willingness to pay to marginally shorten one's commute (MWTP) are normalized by the wage rate. The average willingness to pay (AWTP) is the total amount a resident would pay to have a commute of zero time normalized by the total wages foregone during their actual commute. MWTP and AWTP assume that the numeraire costs of commuting are unchanged. House prices and numeraire commute costs vary by residential ring and so the marginal valuation and willingness-to-pay curves do not have their typical "cumulative" interpretation.

divides TWTP by the time of the actual commute. As displayed in the figure, average willingness is further normalized by the metro wage rate. Equivalently, normalized AWTP is the ratio of TWTP to the additional income that could be earned if one were to increase work hours by the saved commute time.

Under the baseline calibration, normalized AWTP is approximately 0.5 in all residential rings.[23] In consequence, dropping leisure from the utility function and instead assuming a numeraire time cost at half the wage rate generates density and price gradients very similar to those under the baseline. But for a range of plausible alternative calibrations, such as a lower σ_ℓ or a lower λ or $n_1^{**} < \widehat{n}$, the AWTP gradient becomes upward sloping and the density and price gradients appreciably steepen.

[23] The approximate constancy of AWTP may seem incongruent with an MWTP that lies above it. The explanation is that the curves are measured with numeraire and housing consumption that vary across residential rings. More typically, depictions of marginal and average curves hold other outcomes constant.

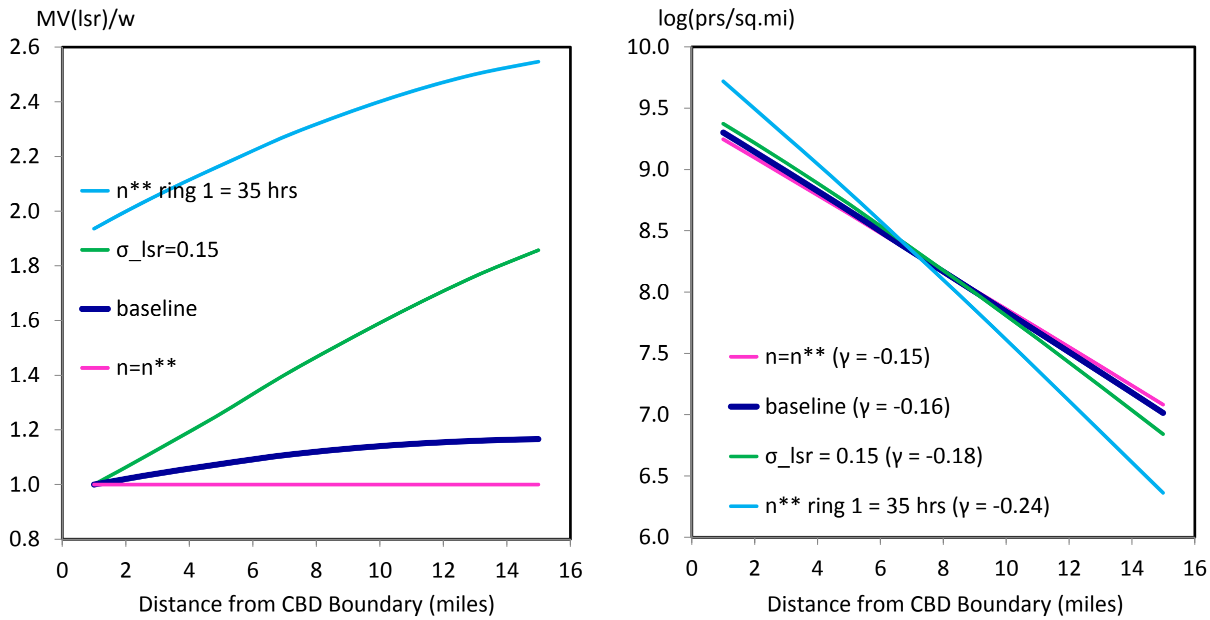

Figure 9: Alternative Calibrations of Leisure. Left panel shows marginal valuation of leisure normalized by metro wage. Right panel shows population density gradient. Fitted semi-elasticity of population density with respect to distance is reported in parentheses.

6 Alternative Calibrations

The calibration of leisure in individual utility, the assumed leisure content of commute time, and the dependence of driving speed on traffic volume each play a key role in driving quantitative outcomes. Three alternative sets of calibrations illustrate how they do so.

6.1 Leisure

The two parameters that most directly mediate the role of leisure are its elasticity of substitution with the hybrid of numeraire and housing consumption, σ_ℓ, and its weighting, η_ℓ. Decreasing the elasticity of substitution, from 0.33 under the baseline to 0.15, considerably steepens the marginal valuation of leisure as commute time increases (Figure 9, green line versus baseline in dark blue, left panel). As is intuitive, the house price gradient must steepen to compensate individuals for the otherwise sharper decrease in utility, which in turn requires the population density gradient to steepen (right panel). But these steepenings are quite modest because the calibration adjusts the weight on leisure, η_ℓ, downward so that inner-ring residents continue to desire to work 40 hours per week.[24]

In contrast, leaving σ_ℓ at its baseline value and instead calibrating η_ℓ so that inner-ring residents

[24]Similarly, the calibration adjusts η_h to changes in σ_h so that the mean expenditure share hits its 17 percent target.

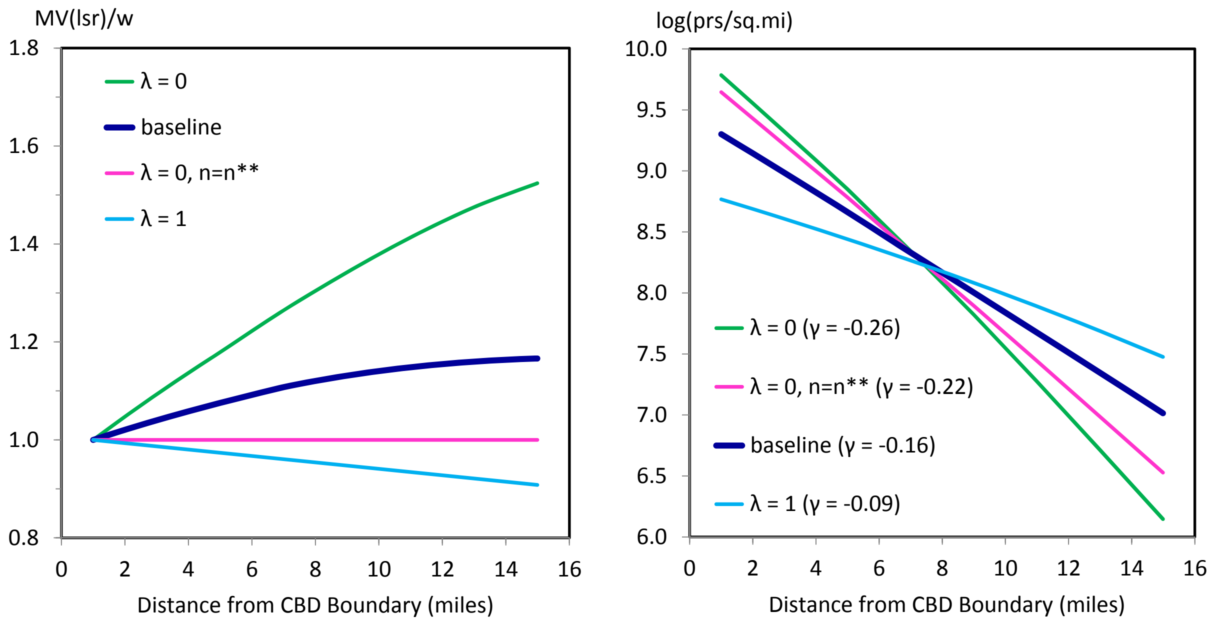

Figure 10: Alternative Leisure Content of Commute Time. Left panel shows marginal valuation of leisure normalized by metro wage. Right panel shows population density gradient. Fitted semi-elasticity of population density with respect to distance is reported in parentheses.

prefer to work 35 hours a week (while still requiring them to work 40) considerably steepens the density gradient (right panel, light blue line). Correspondingly, the marginal valuation of leisure shifts considerably upward at all distances (left panel).

As described in the previous section, individuals who can choose their own work hours do so such that the normalized marginal valuation of leisure equals 1 at all commute distances (left panel, magenta line). But this flexibility only modestly flattens the density gradient (right panel) because baseline preferred hours remain close to required ones throughout the metro.

6.2 Leisure from Commuting

Under the baseline, commute time has a 50 percent leisure content. This dampens the disincentive to living further from the CBD. As is intuitive, alternatively calibrating commute time to have no leisure content significantly steepens the marginal valuation of leisure as commute time increases (Figure 10, green line, left panel). Correspondingly, the population density steepens significantly (right panel).

With no leisure content to commute time, allowing individuals to choose their own work hours significantly flattens the population density gradient, especially at long commute distances (right panel, magenta versus green line).

Conversely, calibrating commute time to have the same leisure content as explicit leisure time

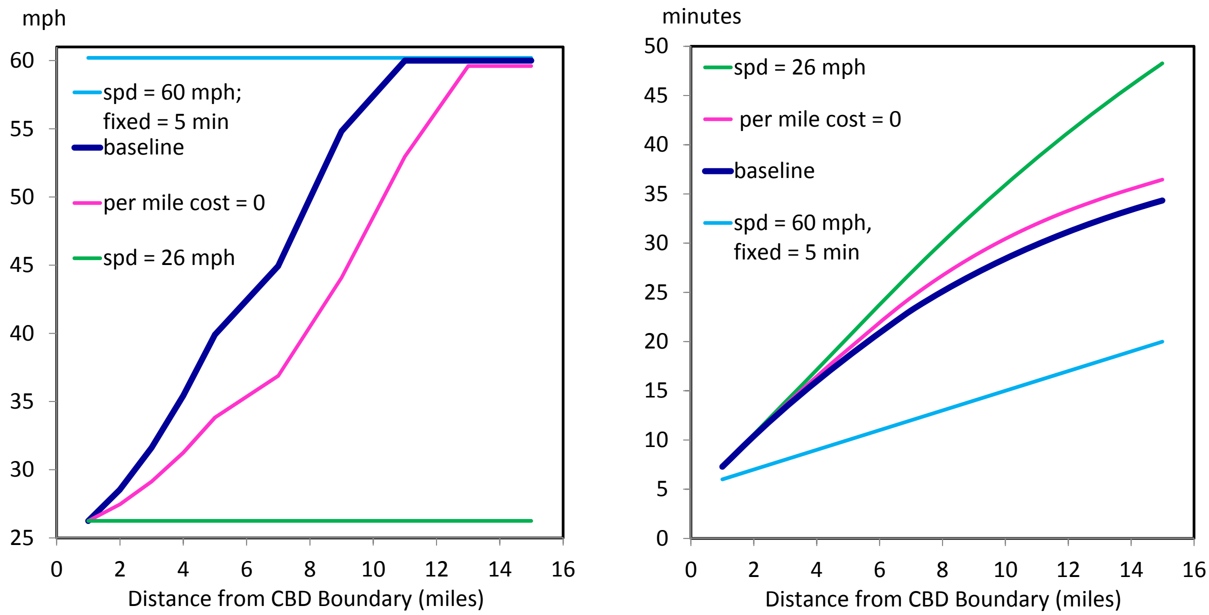

Figure 11: Alternative Speed and Time. Left panel shows highway speed. When numeraire commute cost is zero, speed is slower than under the baseline because a larger share of population lives at further distances. Right panel shows corresponding total commute times.

significantly flattens the density gradient (light blue line versus dark blue line). The downward slope of the density gradient now arises solely from the per mile numeraire cost to commuting, which increasingly lowers disposable income and increases the marginal utility of numeraire consumption as distance increases. In this case, individuals prefer to decrease their leisure consumption as their commutes become longer. Because they are unable to do so, their normalized marginal valuation of leisure falls increasingly below 1.

6.3 Commute Speed and Cost

A final set of calibrations illustrate the role of speed and the numeraire commute cost. A "slow" alternative calibrates highway speed to remain constant at its baseline minimum while retaining the baseline schedule of fixed-time components. By construction, highway speed is constant at 26 mph (Figure 11, green line, left panel). Total commute time is thus considerably higher than under the baseline at further commute distances (right panel).

In contrast, a "fast" alternative calibrates highway speed to remain constant at its baseline maximum and shortens fixed commute time—the sum of the home-to-highway and the highway-to-workplace components—to equal 5 minutes for all residential rings (light blue line, left panel). Commute time is now strictly less than under the baseline at all distances, considerably so at further distances (right panel).

A zero-cost alternative assumes there is no numeraire cost to commuting. This alternative

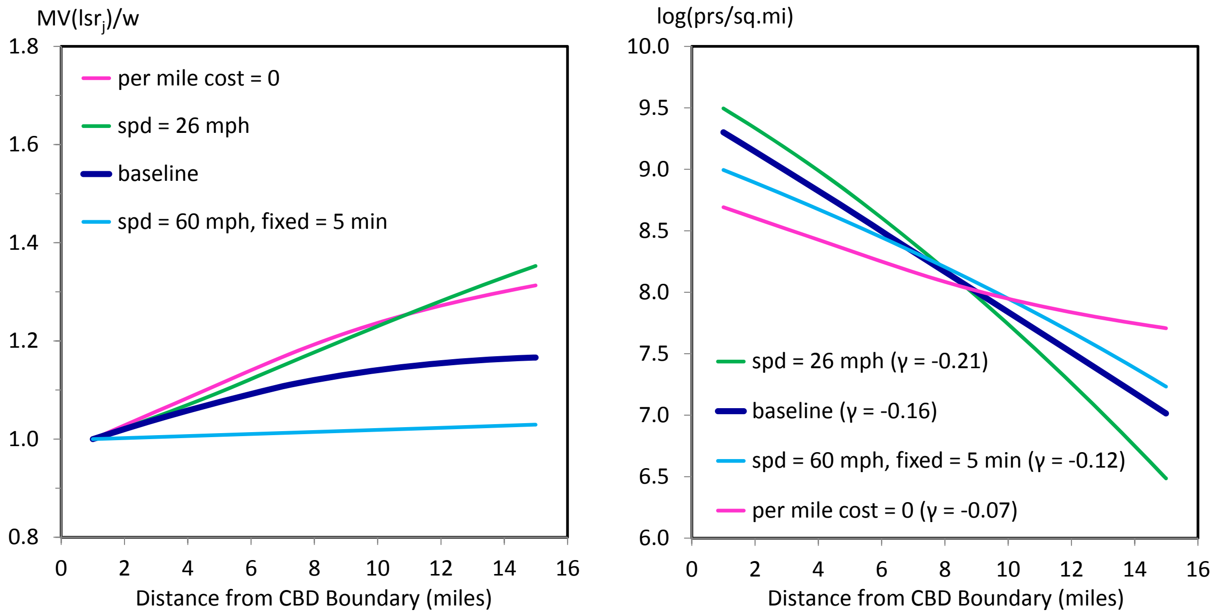

Figure 12: Leisure Valuation and Population Density with Alternative Speed and Time. Left panel shows marginal valuation of leisure normalized by metro wage. Right panel shows population density gradient. Fitted semi-elasticity of population density with respect to distance is reported in parentheses.

pulls population out to further distances and so endogenously slows commute speeds (left panel, magenta line) and increases commute times (right panel).

As is intuitive, the slow-speed calibration considerably increases the marginal valuation of leisure and steepens the population density gradient (Figure 12, green lines). Conversely, the fast-speed calibration results in marginal valuations of leisure only slightly above 1 and considerably flattens the population density (light blue lines).

As is also intuitive, a zero numeraire cost considerably flattens the density gradient (magenta line, right panel). Less intuitively, a zero numeraire cost considerably steepens the marginal valuation of leisure (left panel). This steepening captures the increase in disposable income at further distances, relative to the baseline, which in turn decreases the marginal utility of numeraire consumption (the denominator of marginal valuation). But as reflected in the flatter density gradient, the actual disutility from commuting decreases.

The zero-numeraire-cost density gradient is similar to the density gradient above for which there is no leisure cost to commuting ($\lambda = 1$; Figure 10, right panel, light blue line). This suggests that the numeraire cost and the 50-percent time cost make about equal quantitative contributions to the disutility of commuting.

7 Conclusions

This paper has argued that the AMM monocentric city framework remains an empirically relevant stylization of midsize U.S. metros. To be sure, the majority of jobs are geographically dispersed throughout these metros. But the one third or more of agglomerative jobs that are located in the CBD are likely to disproportionately affect the geographic distribution of residents. The post-war suburbanization of these agglomerative jobs has been minimal. And the polycentricity of midsize U.S. metros is typically limited to just a single employment cluster outside the CBD.

Motivated by this continuing empirical relevance, the paper extends the AMM theoretical framework to include leisure as an explicit source of utility while requiring workers to supply a fixed number of labor hours. Doing so sharpens the marginal disutility and increases the marginal valuation of leisure from long commutes.

The quantitative implementation exogenously specifies land area, CBD radius, and outer commute distance. An otherwise circular metro area is residually allowed to span less than a full 360°. Highway capacity and fixed-time components are jointly calibrated to tightly match observed commute times in Portland, Oregon. The implied geographic distribution of workers' residence tightly matches that of auto commuters to Portland's CBD. The implied population-density distribution and gradient approximately match those of Portland. The implied land- and house-price gradients are consistent with empirical estimates.

The approximate match to observed outcomes suggests that the present model can bring quantitative analysis to range of urban topics. To do so, Rappaport (2014) embeds the standalone monocentric metro herein as "representative" within a system of open monocentric metros. It determines the reservation utility and perimeter land price that must be matched in all other metros. This setup allows for a rich description of how metro population, land area, and internal structure quantitatively depend on variations in productivity, transportation infrastructure, and calibrated parameters.

For example, the higher productivity *required* to support increases in population above the representative level significantly exceeds empirical estimates of the agglomerative productivity *caused* by increases in population. Some non-agglomerative source of TFP or else some variation in consumption amenities must make up the difference. Conversely, at population levels moderately below that of the representative metro, the elasticity of agglomerative TFP with respect to population exceeds the elasticity of required TFP. For smaller metros, then, path dependence may be an especially important determinant of size and form.

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