A systemic approach to home loans: Continuous Workouts v.s. Fixed Rate contracts

Robert J. Shiller[1] Rafal M. Wojakowski[2] M. Shahid Ebrahim[3]

Mark B. Shackleton[4,5]

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[1]Cowles Foundation for Research in Economics, International Center for Finance, Yale University, New Haven CT 06520-8281; Phone: +1 203 432-3726; Email: robert.shiller@yale.edu

[2]Corresponding author: Faculty of Business, Economics and Law, University of Surrey, Guildford GU2 7XH, UK. Phone: +44(1483)683477; Email: r.wojakowski@surrey.ac.uk

[3]Durham University Business School, Durham, UK. Phone: +44(191)3345405, Email: m.s.ebrahim@durham.ac.uk

[4]Lancaster University Management School, Lancaster, UK. Phone: +44(1524)594131, Email: m.shackleton@lancaster.ac.uk

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Abstract

We take a systemic, market based approach, stipulating that mortgage values and payments should be linked to housing prices and adjusted downward to prevent negative equity. We advocate using Continuous Workout Mortgage (CWM), which is a two in one product: a fixed rate home loan coupled with negative equity insurance. We show that Continuous Workout Mortgages could be the optimal home financing instrument for many households.

Keywords: Systemic risk, Negative equity, House price index, Continuous Workout Mortgage (CWM), Repayment, Insurance, Cap and floor on continuous flow.

JEL: C63, D11, D14, D92, G13, G21, R31.

"Because [systemic] risk cannot be diversified away, investors demand a risk premium if they are exposed to it. There is no magic potion that can mitigate this kind of risk: somebody must hold this risk. Because crisis risk is endemic, it cannot be diversified away. To off-load this kind of risk, you have to purchase insurance, that is, induce others to take the risk off your hands by paying them a risk premium."

Jonathan Berk (in Fabozzi, Focardi, and Jonas (2010))

1 Introduction

The ad hoc measures taken to resolve the recent crisis involved giving away handfuls of money, yet not stopping the wave of foreclosures. Such short-term patches contradict mortgage contracts thus rejecting the rational principles of finance (Roubini, Roach, Smick, Shiller, and Baker (2009)). Moreover, volatility of house prices rose substantially in recent years (see Figure 1). In contrast, we come up with a mortgage contract which is robust to downturns. We demonstrate how Continuous Workout Mortgages can be offered to homeowners as an *ex ante* solution to non-anticipated real estate price declines.

PUT FIGURE 1 ABOUT HERE

Continuous Workout Mortgage (CWM, Shiller (2008b)) is a two-in-one product: a fixed rate home loan coupled with negative equity insurance. More importantly its payments are linked to housing prices and adjusted downward on regular basis to prevent negative equity. CWMs eliminate the expensive workout of a defaulting rigid plain vanilla mortgage contract. This subsequently reduces the risk exposure of financial institutions and thus the government to their bailouts. CWMs share the price risk of a home with the lender and thus provide automatic adjustments for changes in home prices. This feature eliminates the rational incentive to exercise the costly option to default which is embedded in the loan contract. Despite sharing the underlying risk, the lender continues to receive an uninterrupted stream of monthly payments. Moreover, this can occur without multiple and costly negotiations.

Unfortunately, prior to the current crisis CWMs have rarely been considered. The academic literature, with the exception of Ambrose and Buttimer, Jr. (2012), has not discussed its mechanics and especially its design. Shiller is the first researcher, who forcefully articulates the exigency of its employment. CWMs were conceived in his studies[1] (Shiller (2008b) and Shiller (2009)) as an extension of the well-known Price-Level Adjusted Mortgages (PLAMs), where the mortgage contract adjusts to a narrow index of local home prices instead of a broad index of consumer prices. In their recent study, Ambrose and Buttimer, Jr. (2012) numerically investigate properties of Adjustable Balance Mortgages which bear many similarities to CWMs. Alternatively, Duarte and McManus (2011) stipulate creation of derivative instruments written on credit losses of a reference mortgage pool. The model in Shiller, Wojakowski, Ebrahim, and Shackleton (2013) complements the more intricate one of Ambrose and Buttimer, Jr. (2012). Unlike a numerical grid, it relies on a methodology which allows valuation of optional continuous flows in closed form (see e.g. Carr, Lipton, and Madan (2000) and Shackleton and Wojakowski (2007)).

Continuous workout mortgages need indicators and markets for home prices and incomes. These markets and instruments already exist for lenders to hedge risks. The Chicago Mercantile Exchange (CME) for example, offers options and futures on single-family home prices. Furthermore, reduction of moral hazard incentives requires inclusion of the home-price index of the neighborhood into monthly payment formula. This is to prevent moral hazard stemming from an individual failing to maintain or, worse,

[1] See also Shiller Shiller (2003) for home equity insurance.

damaging the property just to reduce mortgage payments.

Finally, we observe that there is evidence (see Shiller (2014)) that the retirement trends are changing. More people are planning to sell their house and consume the proceeds in retirement. CWMs could help insure house values and preserve the welfare gain. Notably, the continuing care retirement community (CCRC) is a concept that is growing rapidly around the world. After the drop in home prices there is a CCRC crisis in the US today. CCRCs have a lot of vacancies, and this is no doubt because of the absence of CWMs.

In our approach the homeowner can choose from a classic 30-year Fixed Rate Mortgage (FRM) or an CWM. We show that Continuous Workout Mortgages could be the optimal home financing instrument for many households.

2 The contracts

2.1 The standard Fixed Rate Mortgage (FRM)

A major invention introduced during years of Great Depression were *fully amortizing* repayment mortgages. Typically, these mortgages are analysed in a discrete time setup with 12 fixed rate monthly payments per year and a $T = 30$ year time horizon. For our purposes we need to have a continuous time representation of Fixed Rate Mortgages (FRM). In place of the monthly payment we introduce a repayment flow rate R which is constant in time. For simplicity we will also assume that the term structure is flat i.e. rates for all maturities are constant and equal to r. For this repayment mortgage the mortgage balance Q decreases so as to become zero at maturity. By definition, the mortgage balance is equal to the amount owed to the lender at time t and can be computed as the present value of remaining payments

$$Q(R,t) = \int_t^T e^{-r(s-t)} R\, ds = \frac{R}{r}\left(1 - e^{-r(T-t)}\right) \ . \tag{1}$$

Differentiating (1) with respect to t we obtain the dynamics of Q

$$dQ = -Re^{-r(T-t)}dt = -R\left(1 - \frac{Qr}{R}\right) dt \ . \tag{2}$$

It is *deterministic* and is described by the ordinary differential equation

$$\frac{dQ}{dt} = rQ - R \tag{3}$$

with terminal condition $Q(R,T) = 0$. It says that the balance Q grows at rate r but is progressively repaid by a *constant* mortgage payment flow R. The mortgage can be repaid in full only if the interest flow on principal rQ is *lower* than the coupon flow R. In this case the net flow is negative: $rQ - R < 0$ i.e. funds are "poured back" to the lender and the mortgage is being repaid. The advantage and simplicity of FRM is that the required repayment flow parameter R can be computed and set once and for all at the beginning, when the loan contract is signed. To compute R we only need to know three numbers: the initial balance $Q_0 = Q(R,0)$ (the loan amount), the maturity T of the loan and the discount rate r

$$R = \frac{Q_0}{A(r,T)} \ , \tag{4}$$

where $A(r,T)$ is the annuity

$$A(r,T) = \int_0^T e^{-rt} dt = \frac{1 - e^{-rT}}{r} \tag{5}$$

i.e. the present value at rate r of a unit flow terminating after T years.

The simplicity of the FRM becomes problematic when a house becomes underwater. This was the case of many households in the US in years following the burst of the housing bubble in 2007. This is illustrated on Figure 2. A \$ 500 000 property is financed for 30 years and interest rates are 5%. However, a sharp price decline puts the house in negative equity. House values plunge well below the convex decreasing line representing the balance. In this example, although the balance in a FRM is set to be fully repaid at maturity, the house is underwater until year 11. Problems then re-surface around years 20 and then 28. The value of the property then plunges below the initial purchase price H_0.

PUT FIGURE 2 ABOUT HERE

In the next section we argue that a standard repayment FRM can be improved by making the payment lower in bad times. This is illustrated on Figure 3. In our example a substantial reduction, proportional to the drop in house prices, occurs up until year 11 and payments are also reduced in years 20 and 28.

PUT FIGURE 3 ABOUT HERE

2.2 The repayment Continuous Workout Mortgage (CWM)

In a repayment Continuous Workout Mortgage (CWM) the repayment flow R is no longer a constant. Instead it is linearly scaled down when the house price index ξ_t decreases

$$R(\xi_t) = \min\{\rho, \rho\xi_t\} = \rho \min\{1, \xi_t\} = \rho\left[1 - (1-\xi_t)^+\right] , \tag{6}$$

where ρ is an endogenous parameter (see Figure 4).

PUT FIGURE 4 ABOUT HERE

We assume that a house price index ξ_t is available[2] for all $t \in [0, T]$. Without loss of generality we also assume that this index is normalized to one initially, i.e. $\xi_0 = 1$. If $\overline{H}_t$ is the *absolute* value of the house price index at time t, we can then define ξ_t as

$$\xi_t = \frac{\overline{H}_t}{\overline{H}_0} \,. \tag{7}$$

When $\xi_t < 1$ the return on initial housing purchase is negative and the mortgage risks negative equity if the decline in value is higher than funds repaid so far. In the limiting case when the house price index drops to zero $\xi_t \to 0$, the CWM contract automatically produces a full workout i.e. $R(\xi_t) \to 0$. This is obvious from the shape of the payoff function illustrated on Figure 4. If the collateral becomes worthless, the homeowner should be fully compensated and shouldn't pay any interest or repay any principal. In other words there is full insurance against house price declines. In more realistic, intermediate situations when prices decline but not by too much, this contract provides automatic compensation to homeowners. Repayments of principal and the interest are reduced proportionally to the index. As a consequence it is no longer possible to express the current balance as a function of future payments. In fact, for CWMs, (1) no longer holds. Two quantities emerge which might become numerically different:

1. The actual **balance** to date, which is equal to initial balance minus payments done

[2] Such as the Case-Shiller index in the US or the Countrywide or Halifax index in the UK.

so far

$$Q_t^{CWM-} = Q_0 e^{rt} - \int_0^t R\left(\xi_s\right) e^{r(t-s)} ds \ ; \tag{8}$$

2. The **expected payments** to occur in future

$$Q_t^{CWM+} = E_t \left[\int_t^T R\left(\xi_s\right) e^{-r(s-t)} ds \right] \ . \tag{9}$$

The balance Q_t^{CWM-} is a path-dependent quantity, which can go up as well as down, reflecting the history $\{\xi_s\}_{s=0}^t$ of the house price index up to time t. If we know this history we can compute the balance as

$$Q_t^{CWM-} = \left[Q_0 - \rho A\left(r,t\right)\right] e^{rt} + \rho \int_0^t \left(1 - \xi_s\right)^+ e^{r(t-s)} ds \ . \tag{10}$$

In particular we note that

$$\left[Q_0 - \rho A\left(r,t\right)\right] e^{rt} \leq Q_t^{CWM-} \leq Q_0 e^{rt} \ , \tag{11}$$

so for $\rho > R$ the lower bound can become a negative number closer to maturity (see Appendix A.1). This occurs when home prices remained high, the workout did not kick in, the actual loan was repaid earlier and the mortgagee also collected the insurance premium over the lifetime of the contract. However, following a sharp decline in house prices the balance can also exponentially increase, as shown by the upper bound. Because the expected payments must decline to zero at maturity, the lender bears risk which he was paid for to hold. The borrower have been paying a premium to compensate for this shortfall.

We will require that the initial balance and the expected payments are equal

$$Q_0^{CWM-} = Q_0^{CWM+} = Q_0 \,. \tag{12}$$

This equilibrium condition is necessary to compute the endogenous repayment flow ρ of a CWM. Subsequently, the fair value of the insurance premium $\rho - R$ embedded in a CWM can also be obtained.

Proposition 1 *The expected present value of future payments of the Continuous Workout Mortgage (CWM) at time $t \in [0, T]$ is equal to*

$$Q_t^{CWM+} = \rho \left[A\left(r, T-t\right) - P\left(\xi_t, 1, T-t, r, \delta, \sigma\right) \right] \,. \tag{13}$$

*where $P \geq 0$ is the **floor** (i.e. collection of* put *options for all maturities from $t = 0$ to $t = T$) on continuous flow ξ_t capped at 1, expressed in closed form (see Appendix), and δ, σ are the service flow and the volatility of the house price.*

Proof. See Appendix. ■

The payment flow ρ which appears in (13) is a constant parameter which is computed at origination ($t = 0$) for the duration of the contract. It should not be confused with mortgage payment $R\left(\xi_t\right)$ given by equation (6) which is a function of randomly changing adjusted house price level ξ_t. Mortgage payments $R\left(\xi_t\right)$ *decrease* when home prices decline, whilst ρ is fixed ex ante.

Proposition 2 *The repayment flow of a CWM is capped by ρ, which is an endogenous parameter and can be computed explicitly as*

$$\rho = \frac{Q_0}{A\left(r, T\right) - P\left(1, 1, T, r, \delta, \sigma\right)} \,. \tag{14}$$

Proof. Set $t = 0$ in (13) and solve for ρ. $\blacksquare$

The endogenous parameter ρ provides the cap on repayment flow under CWM. Clearly, if the mortgage is fairly priced, parameter ρ must be greater than R because the issuer of the insurance must be rewarded. We can think of the difference $\rho - R > 0$ as equal to the price of the insurance to be paid in good states of nature for the continuous workouts to be automatically provided when bad states prevail. If there were no risk to be compensated for ($\sigma \to 0$), the present value of insurance puts represented by the floor function P would become zero and ρ would equal R. This is illustrated on Figure 5, where we also consider the case of a partial guarantee which only covers for $\alpha = \frac{1}{2}$ of the loss. Equation (14) is critical for potential originators of continuous workouts with repayment features. This pricing condition helps us evaluate the maximal annual payment for this mortgage. A broker can instantly compute this quantity on a computer screen and make an offer to a customer.

PUT FIGURE 5 ABOUT HERE

Furthermore, it is not immediately obvious whether expected payments of a CWM are lower or higher compared to the balance Q_t of an otherwise identical, standard 30-year FRM. Using (1) and (5) the latter can be written as

$$Q_t = Q\left(R, t\right) = Q_0 \frac{A(r, T-t)}{A(r, T)} \,. \tag{15}$$

Similarly, using (13) and (14) we can represent the expected payments of a CWM as

$$Q_t^{CWM+} = Q_0 \frac{A(r, T-t) - P\left(\xi_t, 1, T-t, r, \delta, \sigma\right)}{A(r, T) - P\left(1, 1, T, r, \delta, \sigma\right)} \,. \tag{16}$$

Clearly, when house prices are low $\xi_t < 1$ and t is relatively low we must have $Q_t^{CWM+} \leq Q_t$. This is because the insurance pays off. However, when house prices are high $\xi_t > 1$ and closer to maturity $t \to T$, we should expect a reversal $Q_t^{CWM+} \geq Q_t$. In this case the insurance puts expire out of the money but a CWM homeowner still has to pay the insurance premia, which results in a slightly higher remaining balance. However, even for very high values of home price index, expected payments Q_t^{CWM+} are always capped from above by

$$\bar{Q}_t^{CWM} = Q_0 \frac{A(r, T-t)}{A(r,T) - P(1,1,T,r,\delta,\sigma)} \geq Q_t^{CWM+} , \tag{17}$$

which has been computed as $\lim_{\xi \to \infty} Q_t^{CWM+}$. Note that $\bar{Q}_t^{CWM}$ and Q_t are similarly shaped (concave, decreasing to zero). However $\bar{Q}_t^{CWM}$ always dominates Q_t. The difference $\bar{Q}_t^{CWM} - Q_t > 0$ is relatively small and represents a "cushion" area above the standard repayment schedule Q_t. It is in this area where the insurance premium is collected in good times.

Figures 6 and 7 summarize the key differences between FRM and CWM. For FRM there is a fixed repayment schedule. In contrast, the repayment schedule for a CWM has a *fixed upper bound*. This upper bound is above (but very close to) the FRM's schedule, forming a "cushion" (see Figure 6). This upper bound or cushion gets quickly saturated in good times. We could say that this extra cushion needs to be there to collect the insurance premia in good times. However, in bad times, CWM's payments outstanding can dive well below FRM's outstanding balance. This self adjustment mechanism is key to remain away from negative equity, avoid the house becoming underwater and prevent foreclosures automatically (see Figure 7).

PUT FIGURES 6 AND 7 ABOUT HERE

We also investigate the behaviour of a CWM under alternative path scenarios. In the first scenario (Figure 8) prices drop and stay at 40% of initial value until maturity. In the second scenario (Figure 9) prices also drop to 40% of initial value but then jump and stay at $600000 which represents 110% of the initial value. In both cases expected payments evolve similarly, progressively decreasing to zero at maturity. Obviously, the starting point Q_t^{CWM+} for the second scenario is much higher than for the first. However, this starting point must always lie below the upper bound $\bar{Q}_t^{CWM}$, no matter how much prices appreciate in future.

PUT FIGURES 8 AND 9 ABOUT HERE

3 Data and calibration of house price index paths

We use S&P/Case-Shiller home price index. The "10-city" composite 318 not seasonally adjusted monthly observations starts in January 1987. We also check our estimates against the "20-city" composite, which is a shorter series with 162 monthly observations, starting in January 2000. Both series are normalized to 100 in January 2000 and span the period up until July 2013. Before proceeding we check for unit roots. Before using calibrated parameters in simulations we validate them against estimates obtained using a much longer time series data. This series, which we short name "Shiller", has been originally used to produce illustrations in *Irrational Exuberance* (Shiller (2005)) and *Subprime Solution* (Shiller (2008b)) books. It is regularly updated and available for download at `www.econ.yale.edu/~shiller/data/`. It has annual observations of nominal and real home price indices from 1892 till 1952. Observations are then quarterly. Either interpolating data pre-1953 or annualizing post-1952 sample we obtain similar results. We therefore only report results from interpolated data.

Using detailed data and tools researchers revealed inefficiencies in house prices (see e.g. Case and Shiller (1989), Tirtiroglu (1992)), structural breaks (see Canarella, Miller, and Pollard (2012)), market segmentation (Montañés and Olmos (2013)) and ripple effects (UK: Meen (1999) and US: Canarella, Miller, and Pollard (2012)). There is, however, evidence in favour of unit roots in house price index capital gains. Canarella, Miller, and Pollard (2012), for example, confirm unit roots in logarithmic differentials of seasonally adjusted regional S&P/Case–Shiller indices for 10 US cities, including the 10-city composite. If unit roots are present in these time series we must have $\phi = 1$ in the following regression

$$\Delta y_t = \alpha + \beta t + (\phi - 1)\, y_{t-1} + \epsilon_t \tag{18}$$

This means that the change in the capital gain $\Delta y_t = y_t - y_{t-1}$, where $y_t = \ln \xi_t$, does *not* depend on the capital gain at $t-1$. Taking exponential gives

$$\xi_t \approx \xi_{t-1} e^{\alpha + \beta t + \epsilon_t} \tag{19}$$

Therefore, as a first approximation, we could assume that the house price index is log-normally distributed. Consequently, in a first attempt to model the random behaviour of the house price index we calibrate a geometric Brownian motion for drift and volatility. Also, because of a possible bias toward nonrejection of the unit root hypothesis in seasonally adjusted data (see Ghysels and Perron (1993)), before proceeding we first check for unit roots using non seasonally adjusted and longer time series data.

The results of our calibration are reported in Table 1.

	10-city	**20-city**	**Shiller**	Simulations
	1987-	2000-	1890-	
$\hat{\mu}$	0.0395166	0.0367472	0.0310475	0.03
$\hat{\sigma}$	0.0325394	0.0397048	0.0404518	0.01–0.10

Table 1: S&P/Case-Shiller home price index calibration.

We set μ close to the long term trend 0.03 and we do not retain a particular value for the volatility in our simulations. As observed from a longer perspective, volatility is on average somewhere between 3% and 4%. However, we adopt a broader range in our simulations. Historically, there were periods of higher and lower volatility. Using interpolated annual data (pre 1953) and quarterly data (post 1953), a moving volatility estimate is obtained, quarter by quarter. Each estimate is using 40 most recent quarterly observations spanning the last 10 years. This is illustrated on figure 1.

4 Comparing Continuous Workouts with Fixed Rates

It is difficult to uniquely say what would constitute a good mortgage product. Every household may apply different criteria so it is safer to consider a range of alternative cases and specific situations.

To establish when a risk averse mortgagor will prefer CWM to FRM we come up with a utility-based argument. We assume that borrowers are unable to dynamically adjust their portfolios so as to finance their home purchase directly on financial markets. We also assume that prospective homeowners must choose either a CWM or a FRM i.e. they cannot use a mix of two smaller mortgages. Denote this choice by $m \in \{CWM, FRM\}$.

4.1 Intertemporal utilities

Risk-averse mortgagor maximizes the intertemporal utility[3]

$$\max_m V_m = \max_m \int_0^T e^{-\delta(t)t} u\left(x\left(t\right) - y_m\left(t\right)\right) dt \;, \tag{20}$$

where $\delta\left(t\right)$ is the subjective discount rate, $x\left(t\right)$ are funds available and $y_m\left(t\right)$ is the mortgage payment at time t. Implicitly, we assume that there is no risk of default and all mortgages are riskless. For simplicity we assume $\delta\left(t\right) = r$ and a constant wage $x\left(t\right) = x$. We require $x > y_m\left(t\right)$ for all t. This is readily achieved for FRM for which $y_{\mathrm{FRM}}\left(t\right) = R$ by requiring $x > R$ initially. Using R given by (4) we obtain

$$V_{\mathrm{FRM}} = \int_0^T e^{-rt} u\left(x - R\right) dt = A\left(r, T\right) u\left(x - \frac{Q_0}{A\left(r, T\right)}\right) \;. \tag{21}$$

Note that because monthly payments are all constant, known in advance from the moment when the contract is signed, FRM removes all randomness. So if nothing happens and payments continue, the instantaneous utilities are also constant. As a result there is no need for computing any expectations in (21).

This is, however, not the case for a CWM. Payments are capped by $\rho > R$, but the mortgagor doesn't know their amounts in advance. Therefore, for the CWM we must compute the expected utility

$$V_{\mathrm{CWM}} = E \int_0^T e^{-rt} u\left(x - R\left(\xi_t\right)\right) dt \;. \tag{22}$$

Note that the repayment CWM *does not* insure the homeowner against the loss of collat-

[3] Assuming the house is sold at maturity for the random terminal price $\tilde{H}_T$, the discounted expected utility of terminal wealth $e^{-\delta(T)} EU\left(\tilde{H}_T\right)$ is the same across the initial choice m of available mortgages. Therefore it will not affect the result of our comparisons. Under default the termination time would be random and the utility of final consumption would be undefined too.

eral value. What a CWM does is dynamically aligning the outstanding loan balance to changes in collateral value by "controlling" the repayment flow $R(\xi_t)$. In other words, if the house is sold at or after maturity for less than the original price H_0, the CWM doesn't pay the difference.

Note that unlike expectations computed in the risk neutral world to price the CWM, expectations in (22) require the "actual" probability $\mathcal{P}$. This means we should consider information about the actual drift μ of house prices or, equivalently, the house price risk premium

$$\lambda = \frac{\mu - r}{\sigma} \,. \tag{23}$$

For simplicity we assume the following form of the utility of consumption

$$u(c) = U(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma} \,, \tag{24}$$

where γ is the risk aversion parameter. For $\gamma = 0$ the individual is risk-neutral, while $\gamma = 1$ corresponds to logarithmic utility.

4.2 Simulation results

Expectations in (22) can easily be evaluated by numerical simulations. We simulate house price paths under $\mathcal{P}$. We do monthly simulations over 30 years, so one path requires 360 draws from a normal distribution. Each path determines intermediate and terminal cash-flows of the CWM mortgage. After simulating many paths we compute sample average utilities, and stop simulations when results stop changing significantly. We repeat expected utility computations for different parameter values. We set the market risk-premium λ and the available monthly wage x to constant values. The most relevant parameters to vary are then the mortgagor relative risk-aversion γ and the subjective discount rate ρ.

Empirical studies have confirmed constant relative risk-aversion. Coefficient estimates consistently range from 1 to 4 with average γ close to 2 (See e.g. Szpiro (1986b), Szpiro (1986a) and Chiappori and Paiella (2011).) In our simulations we change the value of γ in the range $0 < \gamma < 5$.

Results are illustrated on Figures 10 to 15 for two sets of simulations. We report findings for volatility levels 10%, 7.5% and 5%. Results for the historical volatility level (about 4% annually) and lower are quantitatively and qualitatively similar to those for $\sigma = 5\%$.

In the first simulation (Figures 10, 12 and 14), we compute the maximal dollar welfare gain of a CWM expressed as a percentage of FRM's annual repayment R

$$I_1 = \frac{\hat{R}_{CWM} - R_{CWM}}{R}, \tag{25}$$

where $\hat{R}_{CWM}$ is the maximal annual CWM's repayment amount the borrower is ready to afford in good times. If $I_1 > 0$ the borrower prefers CWM to FRM because the welfare improves. The expected utility of a CWM with R_{CWM} is higher than the utility[4] of a FRM with R.

PUT FIGURES 10, 12 AND 14 ABOUT HERE

In the second set of simulations (Figures 11, 13 and 15) we compute the net dollar welfare gain, defined as

$$NG = \underbrace{\left(R - \hat{R}\right)}_{gain} - \underbrace{\left(R_{CWM} - R\right)}_{cost} = 2R - \left(\hat{R} + R_{CWM}\right) . \tag{26}$$

[4] Equal to expected utility of a CWM with $\hat{R}_{CWM}$.

The *gain* refers to the annual dollar amount by which the payment of a FRM should be lowered for the borrower to enjoy the same welfare level as a holder of a CWM. The *cost* equals the annual payment of a CWM minus the annual payment of an otherwise identical FRM. The actual cost is likely to be lower in bad times because automatic workouts will be active.

PUT FIGURES 11, 13 AND 15 ABOUT HERE

5 Conclusions

This paper attempts to mitigate the fragility of the economy to fragile plain vanilla mortgages by advocating Continuous Workout Mortgages (CWMs). As pointed out by Shiller (2008b), Shiller (2008a) and Shiller (2009), these are related to Price Level Adjusting Mortgages (PLAMs), advocated by Modigliani (1974) for high inflation regimes. CWMs share some positive attributes with PLAMs (such as purchasing power risk) and mitigate other negative attributes (such as default risk, interest rate risk and prepayment risk). The liquidity risk can be alleviated by employing CWMs in sufficient volume to warrant their securitization.

We evaluated the expected utility of the mortgagor by using the expected utility formula (22). This captures the welfare from the demand side of loans. Our results reveal a substantial welfare gain to creating CWMs. In our simulations CWMs appear to be more attractive to FRMs at current volatility levels and for typical values of risk aversion. At these levels almost all agents prefer CWMs to FRMs.

The results of our simulations are encouraging, because both measures of the welfare gain are positive in most cases. However, we do not advocate CWMs for everybody. As any insurance product, CWMs may turn out to be more costly if only good times occur in future. The embedded insurance premia will have to be paid making CWMs more costly ex post, compared to FRMs.

Furthermore, our framework captures only the key features and simulations are very simple. In real world an increase in volatility of house prices, for example, would normally impact the level or volatility of incomes (or vice versa). Similarly, the FRMs rates are not dependent on volatility levels in our framework. As a result, for higher levels of volatility in our model, the more risk-averse homeowners will, perversely, prefer not paying the insurance premia embedded in the CWM. Instead, they may prefer locking in the comfort of the fixed monthly payments that the FRMs provide. While we are conscious of these shortcomings, we believe that the current state of the Housing Finance should be open to debate.

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Appendix

A Proofs

A.1 Proof of relationships (10) and (11)

Between $s = 0$ and $s = t$ the dynamics of Q_s^{CWM-} is given by

$$\frac{dQ_s^{CWM-}}{ds} = rQ_s^{CWM-} - R\left(\xi_s\right) \tag{27}$$

For initial condition $Q_0^{CWM} = Q_0$ we have observed ξ_s up to time $t > s$, therefore this ODE has solution

$$\begin{aligned}
Q_t^{CWM-} &= Q_0 e^{rt} - \int_0^t R\left(\xi_s\right) e^{r(t-s)}\, ds && (28)\\
&= Q_0 e^{rt} - \int_0^t \rho \min\left\{1, \xi_s\right\} e^{r(t-s)}\, ds && (29)\\
&= Q_0 e^{rt} - \int_0^t \rho \left[1 - \left(1 - \xi_s\right)^+\right] e^{r(t-s)}\, ds && (30)\\
&= Q_0 e^{rt} - \rho \int_0^t e^{r(t-s)}\, ds + \rho \int_0^t \left(1 - \xi_s\right)^+ e^{r(t-s)} ds && (31)\\
&= Q_0 e^{rt} - \rho \frac{e^{rt} - 1}{r} + \rho \int_0^t \left(1 - \xi_s\right)^+ e^{r(t-s)} ds && (32)\\
&= \left[Q_0 - \rho A\left(r, t\right)\right] e^{rt} + \rho \int_0^t \left(1 - \xi_s\right)^+ e^{r(t-s)} ds && (33)
\end{aligned}$$

In particular note that considering two limiting cases $\xi_s \to 0$ and $\xi_s \geq 1$ (or $\xi_s \to +\infty$) for all $s \in [0, t]$ we conclude that there must be a lower and an upper bound to Q_t^{CWM-}

$$\left[Q_0 - \rho A\left(r, t\right)\right] e^{rt} \leq Q_t^{CWM-} \leq Q_0 e^{rt}\,.$$

When $\rho > R = \frac{Q_0}{A(r,T)}$ and for t close to maturity T, the lower bound can become a negative number, because of the strict inequality in

$$Q_0 - \rho A\left(r,t\right) < Q_0 - \frac{Q_0}{A\left(r,T\right)} A\left(r,t\right) = Q_0 \left[1 - \frac{A\left(r,t\right)}{A\left(r,T\right)}\right] \xrightarrow{t \to T} 0\,. \tag{34}$$

B Floor and Put formulae

Floors P on flow s with strike flow level k for finite horizon T can be computed using the following closed-form formula (see Shackleton and Wojakowski (2007)):

$$\begin{aligned} P\left(s_0, k, T, r, \delta, \sigma\right) = {} & A s_0^a \left(\mathbf{1}_{s_0<k} - N\left(-d_a\right)\right) - \frac{s_0}{\delta}\left(\mathbf{1}_{s_0<k} - e^{-\delta T} N\left(-d_1\right)\right) \\ & + \frac{k}{r}\left(\mathbf{1}_{s_0<k} - e^{-rT} N\left(-d_0\right)\right) - B s_0^b \left(\mathbf{1}_{s_0<k} - N\left(-d_b\right)\right), \end{aligned} \tag{35}$$

where

$$\begin{aligned} A &= \frac{k^{1-a}}{a-b}\left(\frac{b}{r} - \frac{b-1}{\delta}\right), \\ B &= \frac{k^{1-b}}{a-b}\left(\frac{a}{r} - \frac{a-1}{\delta}\right), \end{aligned} \tag{36}$$

and

$$a, b = \frac{1}{2} - \frac{r-\delta}{\sigma^2} \pm \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}, \tag{37}$$

whereas the cumulative normal integrals $N\left(\cdot\right)$ are labelled with parameters d_β

$$d_\beta = \frac{\ln s_0 - \ln k + \left(r - \delta + \left(\beta - \frac{1}{2}\right)\sigma^2\right) T}{\sigma\sqrt{T}} \tag{38}$$

(different to the standard textbook notation) for elasticity β which takes one of four values $\beta \in \{a, b, 0, 1\}$.

Standard Black and Scholes (1973) put on S with strike value of K can be computed using

$$p\left(S_0, K, T, r, \delta, \sigma\right) = Ke^{-rT}N\left(-d_0\right) - S_0 e^{-\delta T}N\left(-d_1\right) \tag{39}$$

where d_0 and d_1 can be computed using formula (38) in which *values* S_0 and K can (formally) be used in place of *flows* s_0 and k.

Both floor (35) and put (39) formulae assume that the underlying flow s or asset S follows the stochastic differential equation

$$\frac{ds_t}{s_t} = \frac{dS_t}{S_t} = \left(r - \delta\right) dt + \sigma dZ_t \tag{40}$$

with initial values s_0 and S_0, respectively. Clearly, (40) describes a geometric Brownian motion under risk-neutral measure where Z_t is the standard Brownian motion, σ is the volatility, r is the riskless rate and δ is the service flow. We assume that (40) describes the dynamics of the repayment flow s. Similarly, (40) also defines the dynamics of the value S of the real estate property.

C Figures

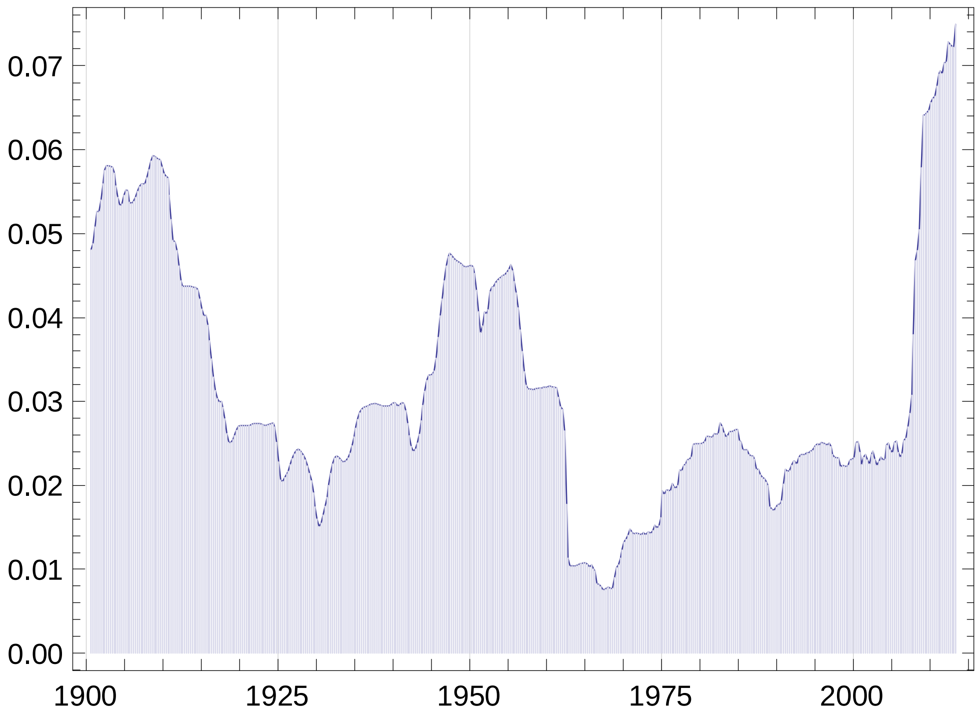

Figure 1: House price index volatility estimates. Each quarterly observation involves 40 datapoints covering the most recent 10 years.

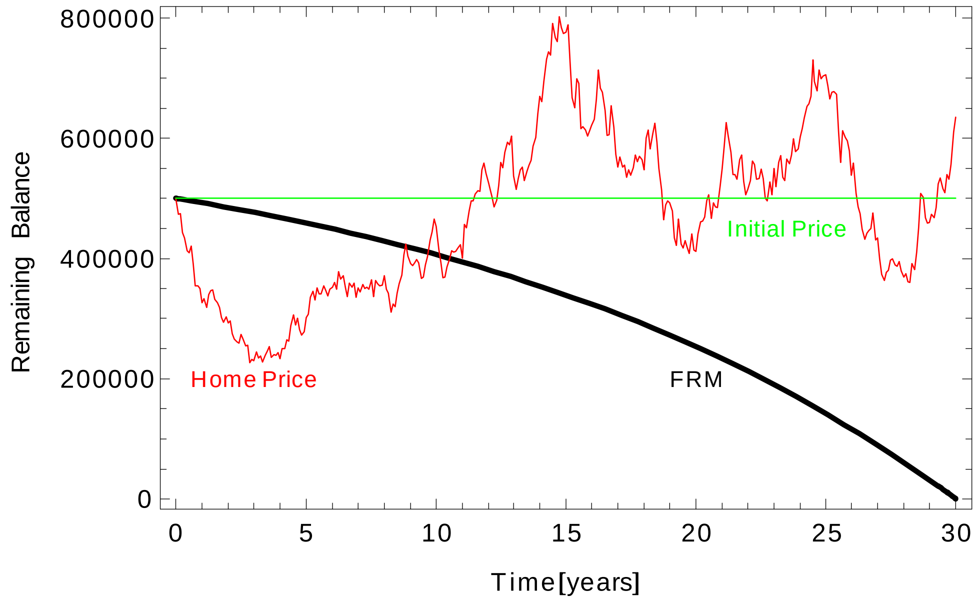

Figure 2: Financing a $ 500 000 property for 30 years with a Fixed Rate Mortgage (FRM) when interest rates are 5%. Drop in prices puts the house in **negative equity.**

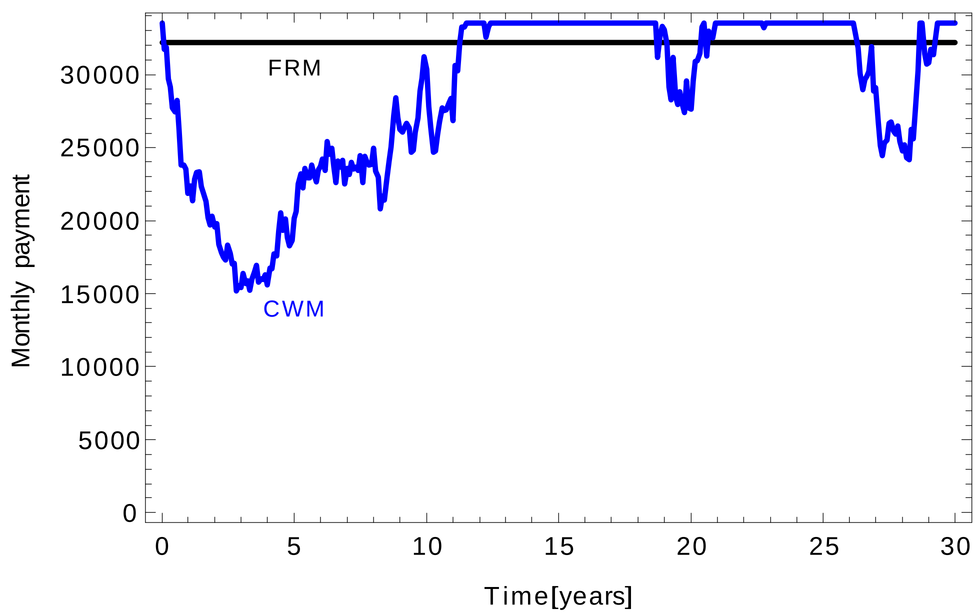

Figure 3: Monthly payments of a CWM are reduced in bad times.

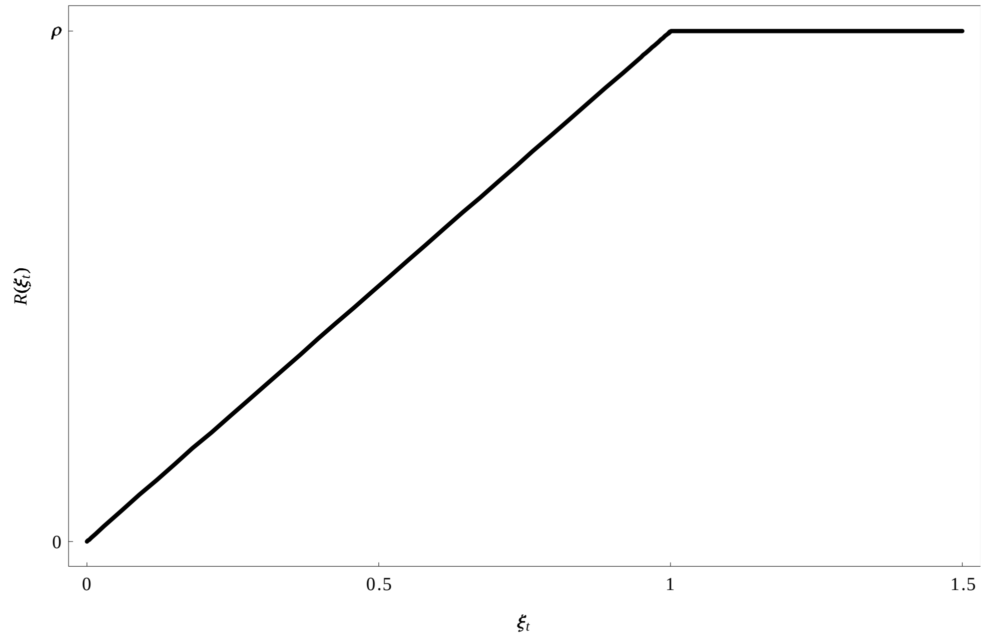

Figure 4: Continuous Workout Mortgage (CWM): The repayment flow R as a function of the house price index ξ_t.

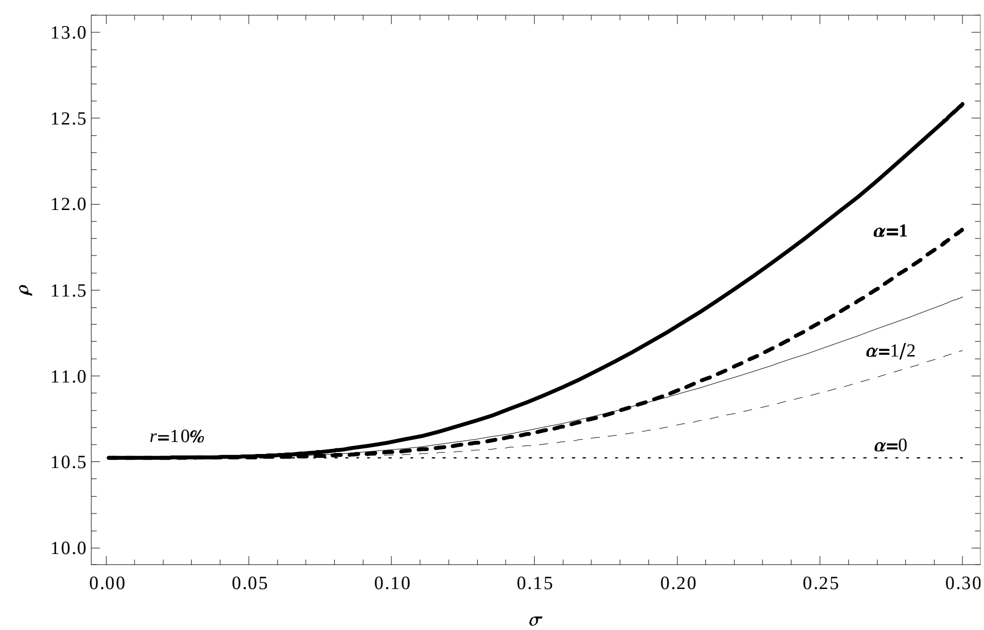

Figure 5: Payment ρ as a function of risk σ. Full workout (CWM, $\alpha = 1$, thick), no workout (FRM, $\alpha = 0$, dotted). $T = 30$, service flow rate $\delta = 1\%$ (solid), 4% (dashed).

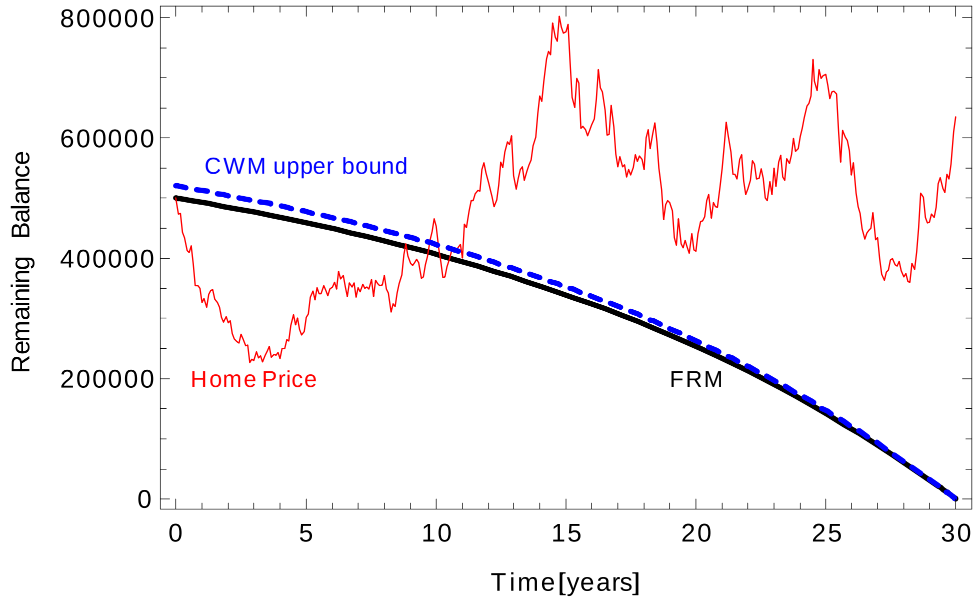

Figure 6: The CWM's upper bound on expected future payments v.s. FRM's scheduled remaining balance.

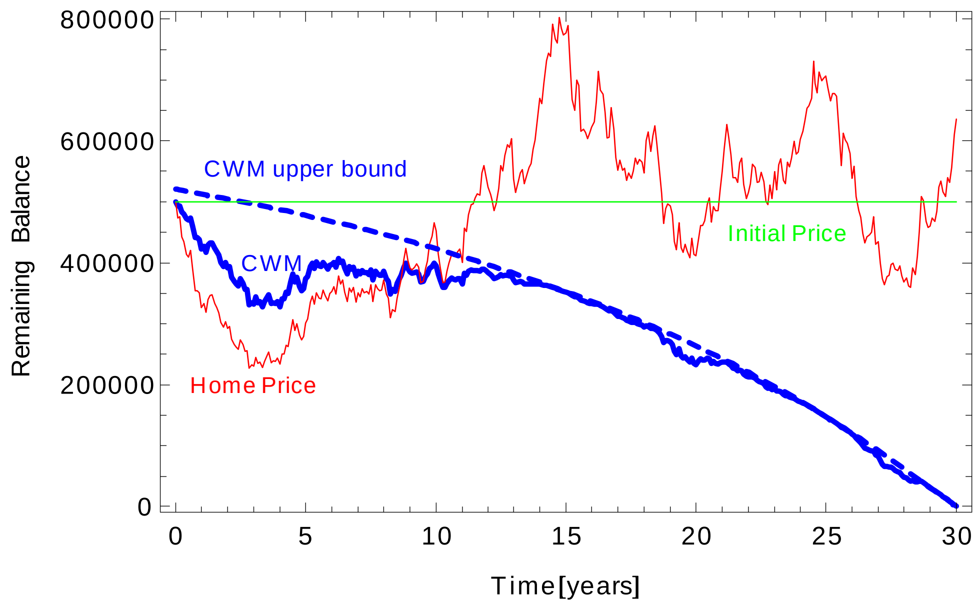

Figure 7: Expected future payments of a CWM, Q_t^{CWM+}, as a function of the current house price levels measured by a Home Price index ξ_t and Initial Price H_0. The dashed line is the CWM upper bound $\bar{Q}_t^{CWM}$.

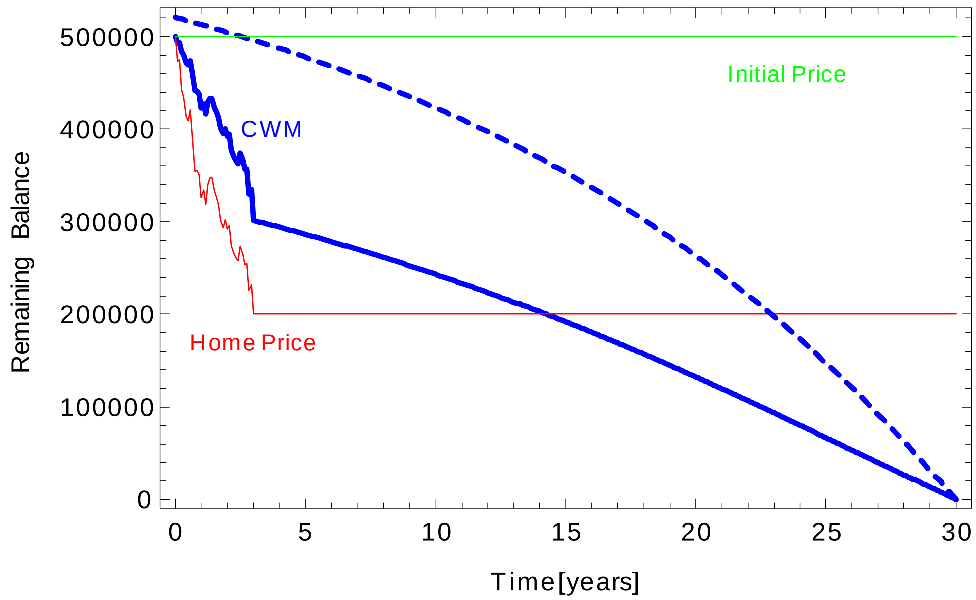

Figure 8: The expected future payments of a CWM when house prices decrease to 40% of initial value and then remain at this level until term.

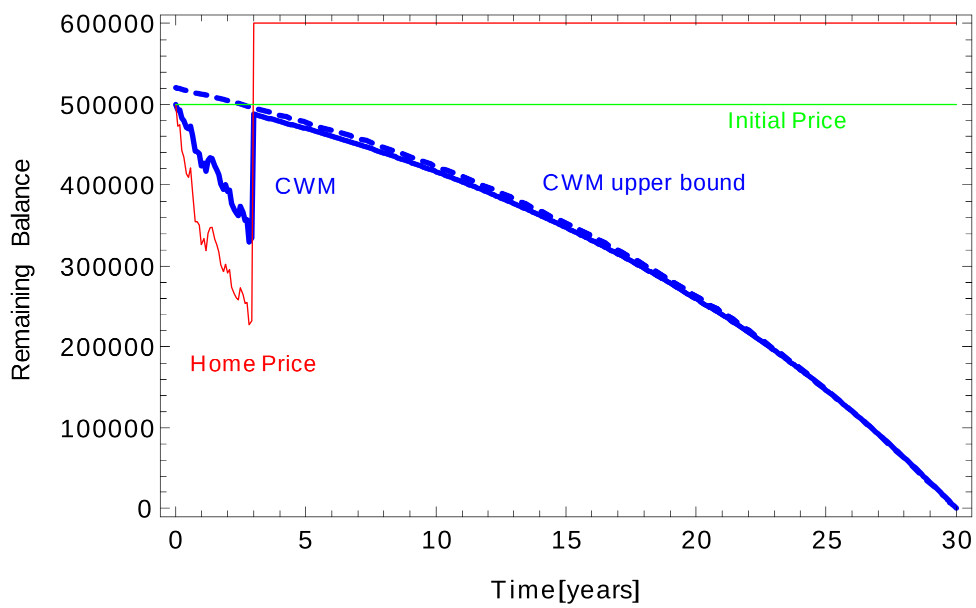

Figure 9: The expected future payments of a CWM when house prices first decrease to 40% of initial value, then instantly appreciate to 110% of initial value and remain at this level until term.

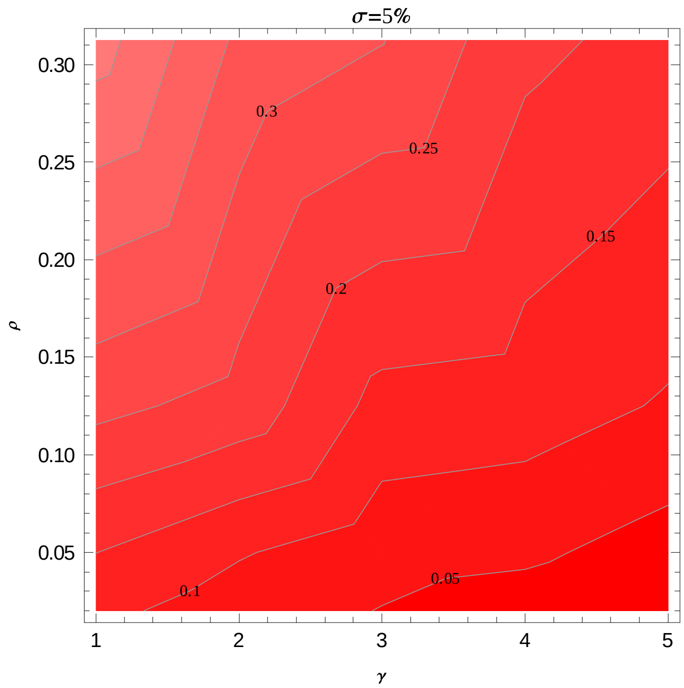

Figure 10: CWM v.s. FRM: The maximal percentage welfare gain for $\sigma = 5\%$. In red areas CWM is preferred. In blue areas FRM is preferred.

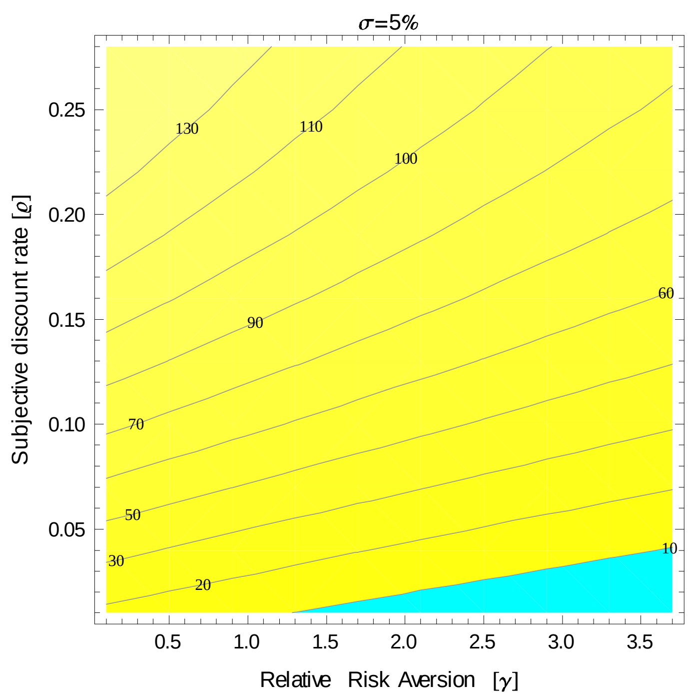

Figure 11: CWM v.s. FRM: The net dollar welfare gain for $\sigma = 5\%$. In yellow areas CWM is preferred. In cyan areas FRM is preferred.

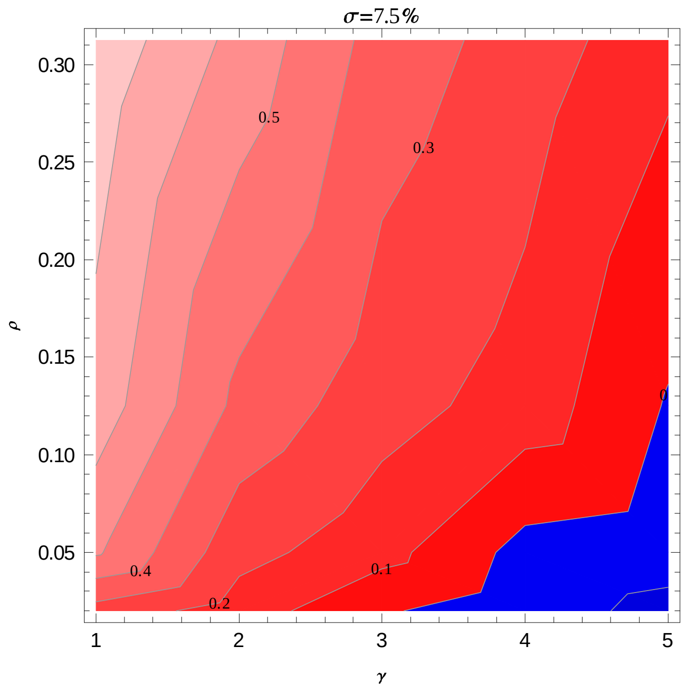

Figure 12: CWM v.s. FRM: The maximal percentage welfare gain for $\sigma = 7.5\%$. In red areas CWM is preferred. In blue areas FRM is preferred.

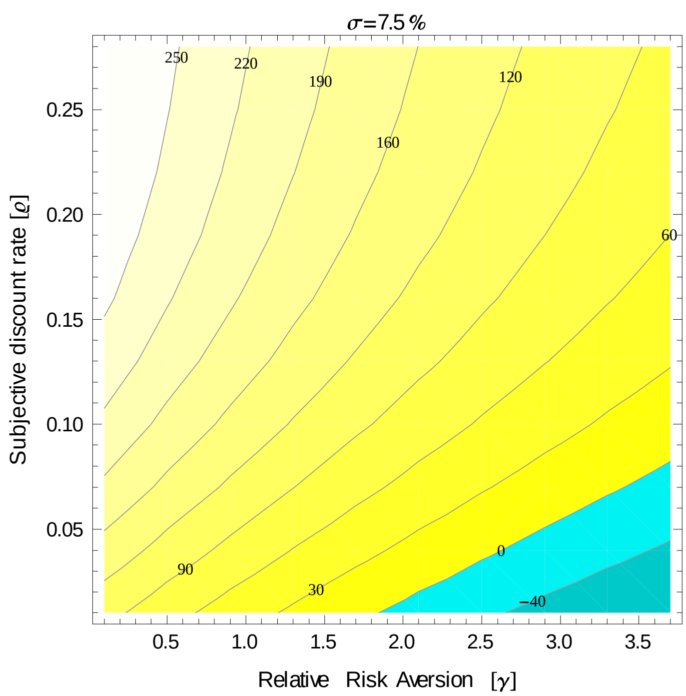

Figure 13: CWM v.s. FRM: The net dollar welfare gain for $\sigma = 7.5\%$. In yellow areas CWM is preferred. In cyan areas FRM is preferred.

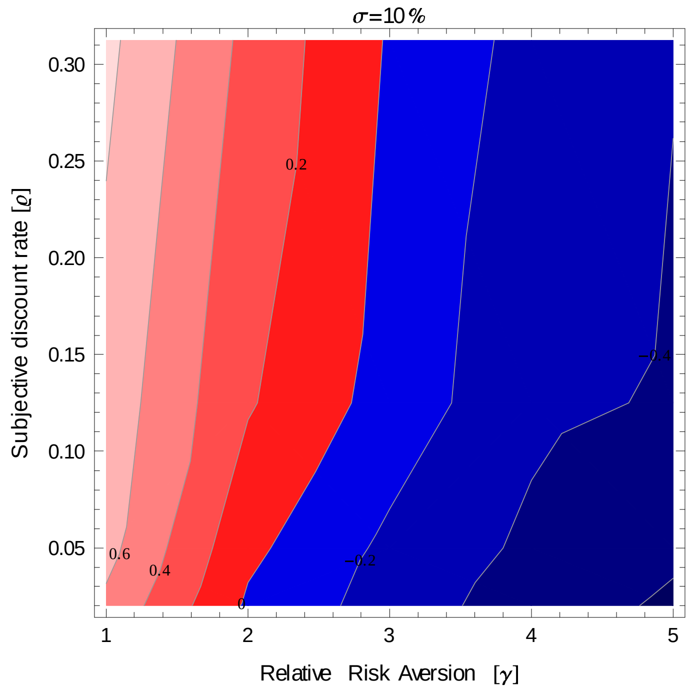

Figure 14: CWM v.s. FRM: The maximal percentage welfare gain for $\sigma = 10\%$. In red areas CWM is preferred. In blue areas FRM is preferred.

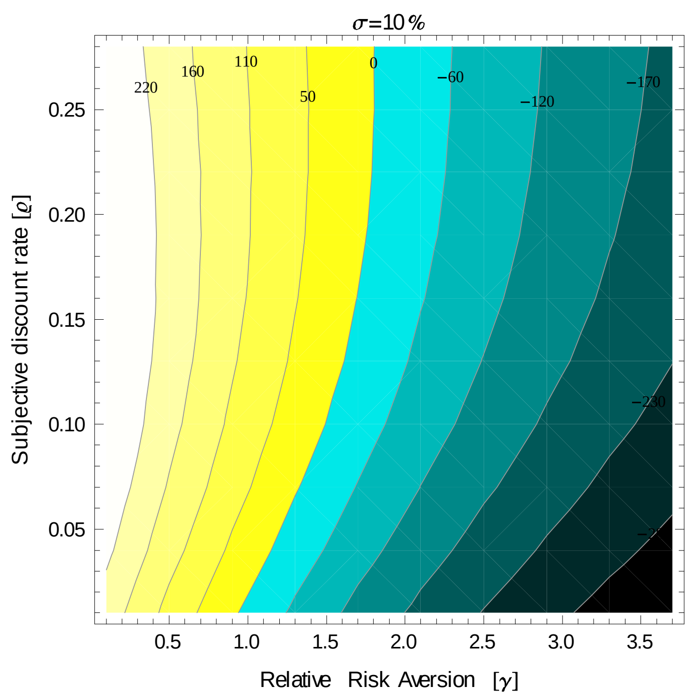

Figure 15: CWM v.s. FRM: The net dollar welfare gain for $\sigma = 10\%$. In yellow areas CWM is preferred. In cyan areas FRM is preferred.

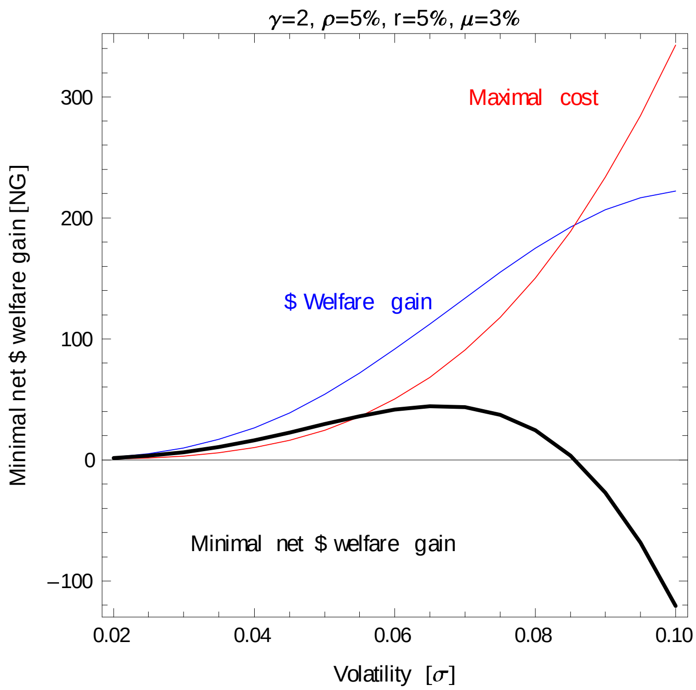

Figure 16: Minimal dollar welfare gain v.s. dollar-equivalent utility gain and maximal cost.