Currency Risk Factors in a Recursive Multi-Country Economy

R. Colacito **M. M. Croce** **F. Gavazzoni** **R. Ready***

(Preliminary, Comments Welcome)

Abstract

We study a risk-sharing model featuring multiple countries with recursive preferences defined over bundles of consumption goods whose supply is subject to both global and local short- and long-run shocks. First, we quantify the extent of contagion and insurance possibilities as we vary the number of countries in the economy. Second, we introduce persistent heterogeneous exposure to global long-run news shocks and analyze the properties of several carry trade strategies in the context of our model (Lustig et al. 2011 HML-FX, and Della Corte et al. (2013) HML-NA). The average excess returns of these strategies can be rationalized in a recursive risk-sharing scheme with long-run global shocks to output growth.

JEL classification: C62; F31; G12.
This draft: December 23, 2014.

---

*Colacito and Croce are affiliated with the University of North Carolina at Chapel Hill, Kenan-Flagler Business School. Gavazzoni is affiliated with INSEAD. Ready is affiliated with the Simon School at the University of Rochester. We thank Bernard Dumas, Tarek Hassan, and seminar participants at INSEAD for useful comments and insights. All errors remain our own.

1 Introduction

A growing empirical literature in international finance examines the structure of risk in the cross-section of currency returns (see, among others, Lustig and Verdelhan 2007, Lustig et al. 2014, and Della Corte et al. 2013). These studies sort currencies on various criteria and highlight the empirical relevance of several economic and financial factors. This paper develops a structural dynamic equilibrium model that can account for these factor structures in the context of a consumption based asset-pricing model.

We analyze an economy populated by multiple countries engaged in a frictionless recursive risk-sharing scheme, in the spirit of Colacito and Croce (2013). This model features long-run growth news, which are directly priced by Epstein and Zin (1989) recursive preferences. This setting is of particular interest given its documented ability of accounting for several empirical regularities of the joint dynamics of international asset prices and quantities in a two-country setting (see Colacito 2008, Colacito and Croce 2011, and Bansal and Shaliastovich 2013). We expand and generalize this setting in at least two relevant directions.

In a first step, we use this setting to explore risk-sharing features as we depart from a two-country-two-good world by increasing the number of countries. We show that bi-lateral correlations and exchange rate volatility may be misleading measures of risk-sharing. Trade-weighted measures, in contrast, are more appropriate to capture the extent of risk-sharing opportunities. Furthermore, we show that the ability of the Colacito and Croce (2013) model to replicate the failure of the uncovered interest parity is not sufficient to produce a risk premium in the cross-section of interest-rate sorted currencies. That is, the mean of the Lustig et al. (2011) HML-FX factor is zero.

In a second step, we follow Lustig et al. (2011) and directly introduce heterogeneous exposure of shocks in the cross-section of countries. Specifically, we modify our baseline setup by modeling a very persistent stochastic heterogeneity in the exposure of country-level endowments to long-run global growth news. The long-run wealth distribution in this economy is well defined since we are still adopting a symmetric calibration (see Colacito and Croce 2012). In finite samples our countries feature substantial heterogeneity, consistent with the empirical investigation by Hassan and Mano (2014). These heterogeneous loadings are a reduced form way of capturing fundamental differences across countries such as size (Hassan 2013), commodity-intensity (Ready et al. 2012), and monetary policy rules (Backus et al. 2010).

When we calibrate the degree of heterogeneity to be stable over a 30 years horizon, we are able to produce an average HML annual spread of about 4%. We also show that this setting can replicate the empirical distribution of currency-portfolio betas on the HML factor. In addition, we document that in our model sorting countries on interest rates is equivalent to sorting on exposure to long-run global growth news, and net foreign assets positions. This suggests that the factors proposed by Della Corte et al. (2013) and Lustig et al. (2011) may be the risk-sharing outcome of just one fundamental source of heterogeneity in exposure to global long-run output growth.

Our analysis helps to shed light on the connection between currency risk and country-level characteristics related to international trade. On the one hand, we provide equilibrium foundations to the reduced form analysis of Lustig et al. (2011). On the other hand, we reconcile currency risk factors with macroeconomic fundamentals by analyzing directly the role of international asset positions (Gourinchas and Rey 2007 and Caballero et al. 2008).

The paper is organized as follows. In the next section we present our model and our equilibrium conditions. In section 3 we discuss key implications of our model for risk sharing and international variables of interest as we increase the number of countries. Section 4 presents our results when we introduce heterogeneous exposure to global long-run growth news, and section 5 concludes.

2 The Economy: Preferences and Endowments

The economy consists of N countries, and N goods, $\{X_i\}_{i=1}^{N}$. Agents' preferences are defined over consumption aggregates of the N goods as follows.

Consumption aggregate. Let $x_{i,t}^{j}$ denote the consumption of good j in country i at date t. Let $\alpha \in (0,1)$. The consumption aggregates in the N countries are

$$C_t^i = (x_{i,t}^i)^{\alpha} \prod_{j \neq i} (x_{j,t}^i)^{\frac{1-\alpha}{N-1}}. \tag{1}$$

The parameter α captures the degree of bias of the consumption of each representative agent. In what follows we assume that each country i receives a stochastic endowment of good $X_{i,t}$ at each point in time. Following some of the international macro-finance articles surveyed by Lewis (2011), we assume that α is larger than 0.5. This allows us to build consumption home bias into the model.

Preferences. As in Epstein and Zin (1989), agents' preferences are recursive but non-time-separable:

$$U_{i,t} = \left[(1-\delta) \cdot (C_{i,t})^{1-1/\psi} + \delta E_t \left[(U_{i,t+1})^{1-\gamma} \right]^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}, \quad \forall i \in \{1,\ldots,N\}. \tag{2}$$

The coefficients γ and ψ measure the relative risk aversion (RRA) and the IES, respectively.

The main departure from the constant RRA case often analyzed in the literature lies in the fact that these preferences allow agents to be risk averse in future utility as well as future consumption. The extent of such utility risk aversion depends on the preference for early resolution of uncertainty measured by $\gamma - 1/\psi > 0$. To better highlight this feature of the preferences, we focus on the ordinally equivalent transformation

$$V_t = \frac{U_t^{1-1/\psi}}{1-1/\psi}$$

and document in appendix B that

$$\begin{aligned} V_t &= (1-\delta)\frac{C_t^{1-1/\psi}}{1-1/\psi} + \delta E_t\left[V_{t+1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \\ &\approx (1-\delta)\frac{C_t^{1-1/\psi}}{1-1/\psi} + \delta E_t\left[V_{t+1}\right] - \frac{\delta}{2}\frac{\theta}{E_t\left[V_{t+1}\right]}Var_t\left[V_{t+1}\right], \end{aligned} \tag{3}$$

where $\theta \equiv \frac{\gamma-1/\psi}{1-1/\psi}$. Note that the sign of $\left(\frac{\theta}{E_t[V_{t+1}]}\right)$ depends on the sign of $(\gamma - 1/\psi)$. When $\gamma = 1/\psi$, the agent is utility-risk neutral and preferences collapse to the standard time-additive case. When the agent prefers early resolution of uncertainty, that is, when $\gamma > 1/\psi$, the coefficient θ is positive: uncertainty about continuation utility reduces welfare and generates an incentive to trade off future expected utility, $E_t\left[V_{t+1}\right]$, for future utility risk, $Var_t\left[V_{t+1}\right]$. This mean-variance trade-off is an appealing feature of these preferences, and one that is absent when agents have standard time-additive preferences. This trade-off drives international allocations and exchange rate adjustments in our economy, and it represents the most important element of our analysis. Our study is the first to fully characterize trade with Epstein and Zin (1989) preferences in an economy composed by an arbitrary number of countries.

Since there is a one-to-one mapping between utility, $U_{i,t}$, and lifetime wealth, the optimal risk-sharing scheme can also be interpreted in terms of mean-variance trade-off of wealth. For this reason, in what follows we will use the terms "wealth" and "continuation utility" interchangeably.

Endowments. We choose to endow each country with a stochastic supply of its most-preferred good. Endowments are co-integrated processes and embody predictive variables as follows:

$$\log X_t^i = \mu_x + \log X_{t-1}^i + z_{i,t-1} - \tau \left[\log X_{t-1}^i - \frac{1}{N} \log \left(\sum_{j=1}^{N} X_{i,t} \right) \right] + \varepsilon_{i,t}^X \tag{4}$$

where $\tau \in (0,1)$ determines the extent of co-integration, and the process z_i are modeled as highly persistent AR(1) processes,

$$z_{i,t} = \rho_i z_{i,t-1} + \varepsilon_{i,t}^z, \forall i \in \{1, 2, \ldots, N\}. \tag{5}$$

Throughout the paper, we refer to $\varepsilon_{i,t}^z$ as the long-run shocks, due to their long-lasting impact on the growth rates of the endowments. Similarly, we will call $\varepsilon_{i,t}^X$ short-run shocks. Shocks are jointly log-normal. We abstract from exogenous time-varying volatility in endowments to better quantify the amount of endogenous consumption and asset-price volatility generated by our recursive risk-sharing mechanism with complete markets.

Market Structure. At each date trade occurs in a complete set of one-period-ahead claims to state-contingent consumption. Financial and goods markets are assumed to be frictionless. The budget constraints of the two agents can be written as

$$\sum_{j=1}^{N} p_{j,t} x_{i,t}^j + \int_{\zeta^{t+1}} A_{i,t+1}\left(\zeta^{t+1}\right) Q_{t+1}(\zeta^{t+1}) \quad = \quad A_{i,t} + p_{i,t} X_{i,t} \tag{6}$$

where $p_{i,t}$ denotes the price of good i, $A_{i,t}\left(\zeta^t\right)$ denotes country i's claims to time t consumption of good X_i, and $Q_{t+1}(\zeta^{t+1})$ gives the price of one unit of time $t+1$ consumption of good X_1 contingent on the realization of ζ^{t+1} at time $t+1$. We shall normalize the price of good 1 to 1 and interpret the remaining $p_{i,t}$ as the terms of trade. In equilibrium, the market for international state-contingent claims clears, implying that $\sum_i A_{i,t} = 0, \forall t$.

Allocations. Since markets are complete, we can compute efficient allocations by solving the associated Pareto problem. The planner attaches date 0 nonnegative Pareto weights $\{\mu_i\}_{i=1}^{N}$ to the consumers and chooses the sequence of allocations $\{x_{i,t}^j\}_{t=0}^{+\infty}$, $\forall i$ and $j \in \{1, ..., N\}$ to maximize

$$\Lambda = \sum_{i=1}^{N} \mu_i \cdot U_{i,0},$$

subject to the following sequence of economy-wide feasibility constraints:

$$\sum_{j=1}^{N} x_{i,t}^j \quad = \quad X_{i,t}, \quad \forall t \geq 0 \quad and \quad \forall i \in \{1, ..., N\},$$

where the state-dependent notation is omitted for the sake of clarity. In characterizing the equilibrium, we follow Anderson (2005) and Colacito and Croce (2013) and formulate the problem using the ratio of time-varying pseudo-Pareto weights, $S_{j,t} = \mu_{j,t}/\mu_{1,t}$, as an additional state variable. This technique enables us to take into account the non-separability of the utility functions. We show in appendix A that the first-order necessary conditions imply the following allocations:

$$\begin{aligned} x_{i,t}^i &= \left(1 + \frac{1-\alpha}{\alpha(N-1)} \sum_{j \neq i} \frac{S_{j,t}}{S_{i,t}}\right)^{-1} X_{i,t}, \quad \forall i \in \{1, 2, ..., N\} \\ x_{i,t}^j &= \frac{1-\alpha}{\alpha} \frac{1}{N-1} \frac{S_{j,t}}{S_{i,t}} x_{i,t}^i, \quad \forall i \neq j \in \{1, 2, ..., N\} \end{aligned} \tag{7}$$

where

$$S_{j,t} = S_{j,t-1} \cdot \frac{M_{j,t}}{M_{1,t}} \cdot \left(\frac{C_{j,t}/C_{j,t-1}}{C_{1,t}/C_{1,t-1}} \right), \quad \forall t \geq 1 \tag{8}$$

and $S_{j,0} = 1$, as we start the economy from an identical allocation of wealth and endowments. This is consistent with the ergodic distribution of the model, which implies that on average all countries consume an identical share of world resources because of symmetry.

Prices. The stochastic discount factor that is used to discount future uncertain payoffs is

$$M_{i,t+1} = \delta \left(\frac{C_{i,t+1}}{C_{i,t}} \right)^{-\frac{1}{\psi}} \left(\frac{U_{i,t+1}^{1-\gamma}}{E_t \left[U_{i,t+1}^{1-\gamma} \right]} \right)^{\frac{1/\psi-\gamma}{1-\gamma}}. \tag{9}$$

Since markets are assumed to be complete, the log growth rate of the real exchange rate between the consumption bundles of country i and j is

$$\Delta e_{i,t}^{j} = \log M_{j,t} - \log M_{i,t}. \tag{10}$$

and the relative price of the good j and good 1 is $p_{j,t} = \frac{(1-\alpha)}{\alpha(N-1)} \frac{x_{1,t}^{1}}{x_{j,t}^{1}}$.

Bilateral Imports and Exports. At each point in time, the exports of country 1 toward country j are equal to $EXP_{1,t}^{j} = x_{j,t}^{1}$, and the imports of country 1 from country j are equal to $IMP_{1,t}^{j} = p_{j,t} x_{1,t}^{j}$, where $x_{j,t}^{1}$, $p_{j,t}$, and $x_{1,t}^{j}$ are defined above. It follows that the bilateral volume of trade and the bilateral net exports rescaled by total output are equal to

$$\begin{aligned} \frac{Vol_{1,t}^{j}}{X_{1,t}} &= \frac{(1-\alpha) \cdot (1 + S_t^j)}{\alpha(N-1) + (1-\alpha) \sum_{j \neq 1} S_t^j} \\ \frac{NX_{1,t}^{j}}{X_{1,t}} &= \frac{(1-\alpha) \cdot (S_t^j - 1)}{\alpha(N-1) + (1-\alpha) \sum_{j \neq 1} S_t^j}, \end{aligned} \tag{11}$$

TABLE 1: Calibration

Description	Parameter	Value
Relative Risk Aversion	γ	8
Intertemporal Elasticity of Substitution	ψ	1.5
Subjective Discount Factor	δ	0.9825
Degree of Home Bias	α	0.97
Mean of Endowment Growth	μ	0.02
Long-Run Risk Autocorrelation	ρ	0.985
Short-Run Risk Volatility	σ	1.87%
Long-Run Risk Volatility	σ_z	0.08σ
Cross-correlations of Short-Run Shocks	ρ_X	0.05
Cross-correlations of Long-Run Shocks	ρ_z	0.90

Notes - All parameters are calibrated at annual frequency.

respectively. Detailed derivations are reported in Appendix B.

3 A calibrated economy

In this section we report the results of a calibrated economy. Table 1 reports our baseline calibration. The parameters governing the dynamics of the growth rates of the endowments are chosen to reflect an average annual growth rate of 2%, a volatility of 2%, and a modest degree of autocorrelation (see Table 2 for the corresponding moments). We follow Colacito and Croce (2011) and proceed to calibrate the cross-country correlation of short-run shocks to a small number, and the cross-country correlation of long-run shocks to a number close to 1. This calibration is consistent with the small extent of correlation of output that is typically observed in the cross-section of major industrialized countries at quarterly or annual frequencies. All preference parameters are set in the spirit of the long-run risks literature (see Bansal and Yaron 2004, Bansal et al. 2010, and Colacito and Croce 2013).

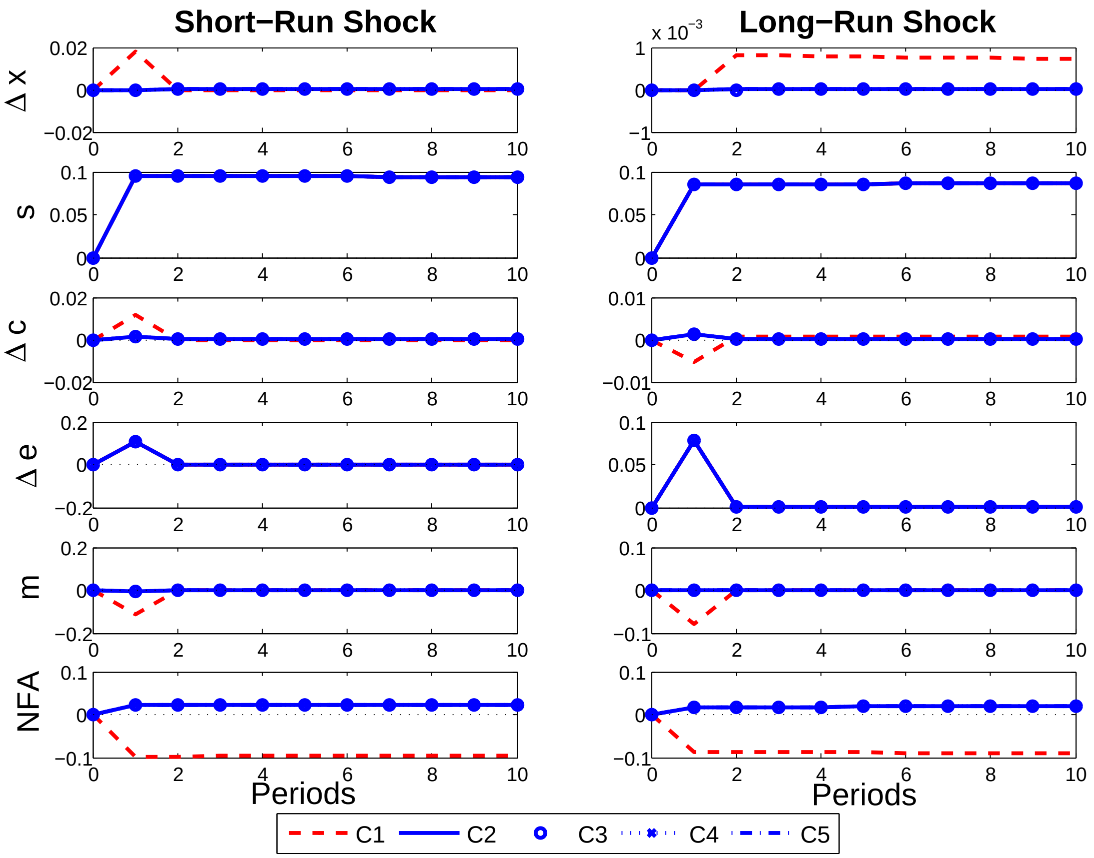

FIG. 1 - Impulse Response Functions. The left panels report the response of endowment growth, relative pareto weights, consumption growth, exchange rate growth, stochastic discount factors, and net foreign assets to a one standard deviation short-run shock to the endowment of country 1. The right panels depict the response of the same set of variables to a one standard deviation long-run shock to the endowment of country 1. All panels refer to the case in which the economy consists of 5 countries.

Response to shocks. Figure 1 reports the response of some variables of interest to short- and long-run shocks. For the sake of the exposition we consider the case in which only country 1 receives a positive shock (panels labeled Δx). The remaining countries experience no shocks and hence both their endowments and their pseudo-Pareto weights are identical, since they all share the same calibration. Because of our symmetric calibration, we find it convenient to refer to country 1 as the home country and to the remaining countries as the foreign countries.

The risk-sharing arrangement in this economy prescribes that in response to both a positive short- and long-run shock to the home country, the foreign countries experience an equal increase of their share of world consumption (panels labeled s). Specifically, upon the arrival of a positive endowment shock, the marginal utility of country 1 declines substantially because of home bias. In order to re-establish the equality of marginal utilities across countries, goods are reallocated toward the foreign countries. In both cases the real exchange rate of country 1 depreciates because of the larger current (expected) supply of the domestic good associated to a positive short-run (long-run) shock (panels labeled Δe).

The panels of Figure 1 labeled Δc document a different response of consumption growth rates with respect to short- and long-run shocks. When a positive short-run shock materializes, consumption increases in the entire cross-section of countries, as only part of country 1's additional resources are being redistributed abroad. In response to positive long-run news, in contrast, consumption drops in the home country as the risk-sharing redistribution effect dominates. The foreign countries experience a positive growth rate of consumption which is required to equalize their marginal utilities to that of the home country.

This differential response of consumption to short- and long-run endowment shocks results in the spread between country 1's and other countries' consumption being positive in one case and negative in the other case. Since country 1's exchange rate depreciates in response to both types of shocks, the model produce a less than perfect correlation between currency movements and consumption differentials, as in the data (Backus and Smith 1993).

The last two panels in Figure 1 report the response of the Net Foreign Assets (henceforth NFA) to the two types of shock. The country that receives either a positive short- or long-run shock acts as insurance provider to the other countries. As a consequence, it experiences a drop in its NFA.

Time-varying correlations. Figure 2 documents the ability of the model to produce endogenous time-varying correlations. In each panel, the conditional correlations of log-utilities are plotted against the log-share of consumption of country 1. The correlations in the first panel are plotted under the assumption that countries 2 and 3 have an identical log-share of consumption. The correlations in the second (third) panel refer to the case in which the log-share of consumption of country 2 is proportionally larger (smaller) than country 1. We can think of the second (third) panel as reflecting the situation in which country 2 (country 3) **is** affected by worse shocks compared to country 3 (country 2).

Several things are worth noticing. First of all, with recursive preferences the conditional correlation of the log-marginal utilities is almost entirely driven by the conditional correlation of the continuation utilities. These correlations reach their largest value when at least one of the countries is close to a zero share of world consumption. If we interpret high correlations of marginal utilities as a manifestation of a large extent of risk sharing, this pattern suggests that risk sharing is at its peak exactly when it is needed the most, i.e., when the distribution of wealth is very unequal.

Focusing on the first panel on the left, we observe that as country 1's share of consumption declines, i.e., as country 1 receives a sequence of positive relative shocks, country 1 becomes increasingly more correlated with both country 2 and 3. In contrast, the correlation between countries 2 and 3 remains relatively moderate. This is due to the fact that country 1 is providing insurance to the rest of the world by redistributing abroad part of the abundant supply of its output.

Turning our attention to either the second or the third panel, we see three general patterns. First, the bilateral correlations with the country with the lowest consumption share are the highest. Second, the bilateral correlations **with** the country with the lowest consumption share tend to be close to each other, meaning that they are

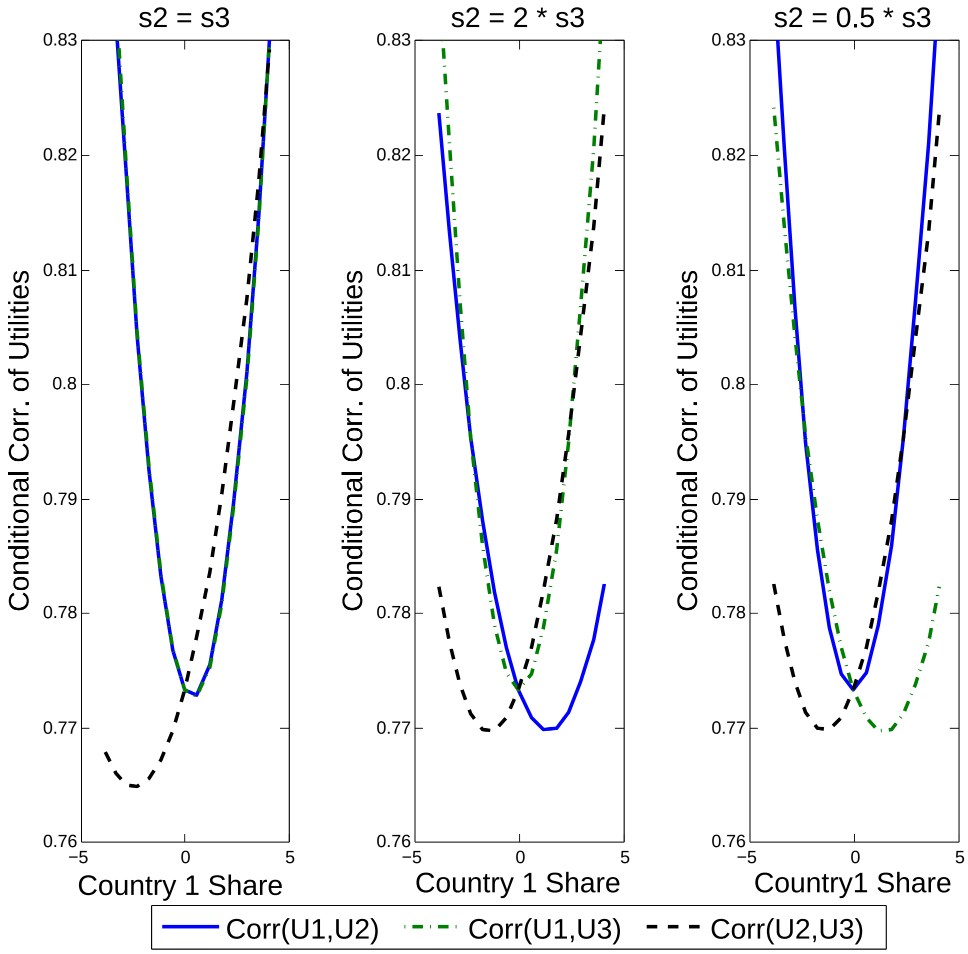

FIG. 2 - Conditional Correlations of Utilities. This figure shows the conditional correlation of utilities against a measure of the consumption share of country 1, $-(\log(S_2) + \log(S_3))$, for the case of a three country economy. The left panel refers to the case in which the log-Pareto weights of country 2 and 3 are equal, $\log(S_2) = \log(S_3)$. In the middle (right) panel, country 2 (3) log-Pareto weight is twice as large as that of country 3 (2). At zero, all countries have the same relative share of world consumption, $S_1 = S_2 = S_3$.

not very sensitive to distribution of wealth among the other countries. Third, the bilateral correlations between the two countries with the highest consumption share are always the smallest.[1]

Simulated moments. Table 2 reports the results of our analysis. Specifically, we analyze how several moments of interest change as the number of countries in the

[1] In the middle panel, when country 1's log-share is very negative, country 1 is the low-consumption share country, whereas country 2 and 3 feature the biggest shares. In the same panel, when country 1's log-share is very positive, country 1 and 2 are the ones featuring the biggest consumption shares, whereas country 3 is the one with the smallest consumption share.

TABLE 2: Simulated Moments

	$NC = 2$	$NC = 3$	$NC = 4$	$NC = 5$
Std(Δy)	1.9	1.91	1.91	1.91
ACF$_1$(Δy)	0.06	0.05	0.05	0.06
corr(Δy_t^h,Δy_t^f)	0.09	0.09	0.08	0.08
Single Country Moments				
Std(Δc)	1.72	1.62	1.57	1.54
ACF$_1$ (Δc)	0.06	0.05	0.05	0.06
Bilateral Moments				
corr(Δc_t^h,Δc_t^f)	0.41	0.38	0.33	0.3
Std Δe	16.51	17.31	17.53	17.66
corr(m, m^f)	0.8	0.78	0.78	0.77
corr($\Delta c^h - \Delta c^f$, Δe)	-0.16	0.14	0.25	0.3
Std[NX/X]	1.47	1.47	1.48	1.45
Std[$\mu/\sum \mu$]	8.86	7.31	6.04	5.02
Trade-weighted Moments				
Std Δe^w	16.51	16.29	16.02	15.76
corr(m, m^w)	0.8	0.83	0.84	0.85
corr(rx_t^h, rx_t^w)	0.68	0.71	0.73	0.73
Financial Variables				
E[r_f]	1.83	1.83	1.86	1.87
Std[r_f]	0.33	0.34	0.33	0.34
corr(r_f^h, r_f^f)	0.83	0.84	0.84	0.84
Std[M] /E[M]	27.05	27.06	26.8	26.95
Std[NFA/X]	46.29	46.66	47.29	46.47
corr(rx_t^h, rx_t^f)	0.68	0.65	0.63	0.62
β_{UIP}	-3.09	-3.22	-2.74	-3.12
E[HML]	n.a.	0	0	0

Notes - The table reports several moments of interest as the number of countries in the economy (NC) ranges from 2 to 5. The first panel reports the calibrated moments for the dynamics of endowments' growth rates. The panel labeled "Single Country Moments" reports the moments of consumption growth rate within each country. The panel label "Bilateral Moments" reports the cross-country moments for each country pair. The panel labeled "Trade-weighted Moments" reports the moments for the case in which each foreign country is weighted in proportion to its volume of trade with country 1. The panel labeled financial variables reports the moments for risk-free rates (r_f), stochastic discount factors (M), Net Foreign Assets (NFA), excess returns (rx), slope coefficient of the UIP regressions (β_{UIP}), and average currency risk premium (E[HML]).

economy (labeled as NC) increases from 2 to 5. We obtain several relevant results as the number of countries increases.

First, country-level volatilities decrease. This is a direct reflection of the better risk sharing opportunities that are available in a multi-country economy. A similar argument can be used to explain the decline in the volatility of Net Exports and pseudo Pareto weights. Second, the bilateral correlations of consumption growth rates, stochastic discount factors, and stock market excess returns decline. Under complete markets, less correlated discount factors immediately imply more volatile bilateral exchange rates. At a country-level hence better risk-sharing opportunities are not necessarily accompanied by higher correlations of consumption profiles and smoother exchange rates (Brandt et al. 2006).

Third, international correlations and exchange rate volatility are very sensitive to the trade dynamics arising from our recursive risk-sharing. We construct trade weighted variables by weighting each country's stochastic discount factor and exchange rate in proportion to its volume of trade with country 1 (see equation 11). As documented in the panel labeled "Trade-weighted Moments", when the number of countries rises the trade-weighted correlation of stochastic discount factors increases, in contrast to what obtained for bilateral correlations. The intuition for this result is that correlations tend to be larger between countries that are major trading partners. By no-arbitrage, a higher correlation between the home country and the trade-weighted rest of the world results in a smoother exchange rate.

For completeness, we report moments related to other key financial variables usually studied in international macro-finance. Our model can produce a low and smooth risk free rate and a large equity Sharpe ratio (as proxied by the volatility of the stochastic discount factor). Both the stock market returns and the risk-free rates are highly correlated across countries. Furthermore, the volatility of the net foreign asset position in each country is consistent with the data, as recursive preferences and long-run risks make the valuation channel as strong as in the data (see Gourinchas and Rey

2013 and Colacito and Croce 2013) . The model can also account for the almost complete lack of correlation between consumption growth differentials and exchange rate fluctuations (Backus and Smith 1993 puzzle). This is the result of the opposite response of consumption growth differentials to short- and long-run shocks.

We conclude our quantitative analysis noticing that the ability of the model to produce a negative slope of the uncovered interest rate parity regressions (β_{UIP}) does not automatically produce a positive risk premium in the cross-section of risk-free rate-sorted currencies ($E[HML]$). This finding is consistent with the analysis of Hassan and Mano (2014). In section 4, we propose a model with persistent heterogeneity in the exposure of countries long-run shocksthat simultaneously accounts for both the forward premium anomaly and the risk premium observed in the cross-section of currencies (carry trade).

CRRA case. In Table 3 we report some moments of interest for the special case of constant relative risk aversion preferences (henceforth CRRA). This case is obtained by imposing that the intertemporal elasticity of substitution (ψ) equals the inverse of the relative risk aversion coefficient (γ). Specifically, we set $\gamma = 8$, as in our benchmark calibration, and $\psi = 1/8$.

This experiment shows that increasing the number of countries does not immediately result in a larger trade-weighted correlation of stochastic discount factors. This is due to the fact that the highly cross-country correlated long-run news are not priced with CRRA preferences. As a consequence, the long-run risk-sharing opportunities that arise with an increasing number of countries are overshadowed by the reduced correlation of consumption bundles due to the addition of poorly correlated short-run shocks. Indeed, as we increase the number of countries, the consumption bundle of each country features a larger number of poorly correlated goods.

TABLE 3: Simulated Moments (CRRA case)

	$NC = 2$	$NC = 3$	$NC = 4$	$NC = 5$
Bilateral Moments				
corr(Δc_t^h,Δc_t^f)	0.67	0.55	0.45	0.41
Std Δe	10.47	11.68	12.63	12.93
corr(m, m^f)	0.67	0.55	0.45	0.41
corr($\Delta c^h - \Delta c^f$, Δe)	1	1	1	1
Trade-weighted Moments				
Std Δe^w	10.47	10.93	11.42	11.36
corr(m, m^w)	0.67	0.62	0.55	0.54

Notes - CRRA case. The table reports several moments of interest as the number of countries in the economy (NC) ranges from 2 to 5. The panel label "Bilateral Moments" reports the cross-country moments for each country pair. The panel labeled "Trade-weighted Moments" reports the moments for the case in which each foreign country is weighted in proportion to its volume of trade with country 1.

This outcome has important implications for the exchange rates dynamics. With recursive preferences an increasing number of countries results in smoother trade weighted exchange rate's fluctuations, whereas the opposite is true for the case of CRRA preferences.

4 The Cross-section of Currency Risk Premia

The endogenous dynamics of world wealth produced by our recursive risk sharing scheme rationalizes the failure of the uncovered interest rate parity, but it is unable to produce a risk-premium in the cross-section of interest rate-sorted currencies. In this section, we show that accounting for persistent heterogeneity in the exposure to world shocks can produce sizeable cross-sectional currency premia. Furthermore, our model produces equivalent results when sorting countries on i) nearly permanent heterogenous exposure to endowment shocks (Hassan and Mano 2014), ii) net foreign asset positions (Della Corte et al. 2013), or iii) the level of their risk-free rate (Lustig et al. 2011).

Endowments. We introduce cross-country variation in the exposure to global long-run endowment shocks, $\beta^z_{i,t}$. We focus on long-run shocks because they are the primary driver of our risk-sharing mechanism. Specifically, in each country we decompose our long-run shocks ($\epsilon^z_{i,t}$) in a common global component anda country-specific component as follows

$$\epsilon^z_{i,t} = (1 + \beta^z_{i,t-1})\epsilon^z_{global,t} + \epsilon^{\tilde{z}}_{i,t},$$

with the shocks to the two components being orthogonal to each other

$$\textbf{corr}(\epsilon^z_{global,t}, \epsilon^{\tilde{z}}_{i,t}) = \textbf{corr}(\epsilon^{\tilde{z}}_{i,t}, \epsilon^{\tilde{z}}_{f,t}) = 0.$$

The volatilities of $\epsilon^z_{global,t}$ and $\epsilon^{\tilde{z}}_{i,t}$ are set to replicate both the unconditional standard deviation and correlation of the long-run shocks, $\epsilon^z_{i,t}$, described in the previous section. Country-specific sensitivity coefficients are modeled as a slowly-moving AR(1) process,

$$\beta^z_{i,t} = \rho^\beta_z \beta^z_{i,t-1} + \epsilon^{\beta,z}_{i,t}$$

with $\epsilon^{\beta,z}_{i,t} \sim i.i.d.N(0, \sigma_{\beta,z})$. These shocks are both very long-lived ($\rho^\beta_z \approx 1$) and uncorrelated to other shocks, as they are meant to approximate nearly unconditional differences in the exposure of countries to global news. Countries with high $\beta^z_{i,t}$ have relatively more risky endowments, in the sense that their local growth processes are more exposed to shocks to world wide long-run growth.

Our way to model country-specific exposure to shocks produces a twofold benefit. First of all, it enables us to study an economy with ex-ante symmetrically calibrated countries for which a well defined equilibrium exists (Colacito and Croce 2012). Second, it allows us to study the characteristics of a cross-section of countries that are substantially heterogenous in finite samples. This can be achieved by first simulating a history of heterogenous exposure shocks ($\epsilon_{i,t}^{\beta,z}$). We can think of the $\beta_{i,t}^{z}$ coefficients as devices to capture the heterogeneity documented by Ready et al. (2012), Backus et al. (2010), and Hassan and Mano (2014) in a parsimonious reduced form manner.

Calibration and simulations. We work with small samples of monthly currency data to be consistent with the empirical evidence that our models aims to describe. For this reason, we re-calibrate our model at a monthly frequency as documented in table 4. Most of the parameters are chosen to be the quarterly counterpart of our annual parameters detailed in table 1. Other parameters are unchanged due to time-aggregation issues (see Bansal et al. 2010).

We set $\rho_z^\beta = .99$ to create nearly permanent heterogeneity in exposure to world output shocks. We set $\sigma_{\beta,z} = 0.01\%$ to have moderate conditional volatility of our exposure parameters while being able to match the cross-sectional dispersion of the net foreign asset position across countries documented in Lane and Milesi-Ferretti (2007) and Della Corte et al. (2013). Specifically, we simulate our model on a pre-sample of 600 months (50 years) and make sure to get enough cross-sectional dispersion in the net foreign asset position of our countries by the end of the pre-sample. From this point onwards, we simulate our economy and compute key cross-sectional statistics on the remaining sample. The sample consists of thirty years of data, consistent with the sample size of the data set used in Lustig et al. (2011).

Figure 3 reports our main findings. The left panels refer to the key characteristics of our five countries sorted according to their ex-post realized exposure to long-run world output shocks. In this section we use the term exposure and beta interchangeably. By construction, country 1 is the safest, i.e., the one with lowest long-run exposure,

TABLE 4: Calibration: Heterogenous Exposure to Global LRR

Description	Parameter	Value
Relative Risk Aversion	γ	8
Intertemporal Elasticity of Substitution	ψ	1.5
Subjective Discount Factor	δ	$0.9825^{\frac{1}{12}}$
Degree of Home Bias	α	0.97
Mean of Endowment Growth	μ	0.02/12
Long-Run Risk Autocorrelation	ρ	0.985
Short-Run Risk Volatility	σ	$1.87\%/\sqrt{12}$
Cross-correlations of Short-Run Shocks	ρ_X	0.05
Volatility of Global Long-Run Shocks	σ_z^{global}	$.076\sigma$
Volatility of Local Long-Run Shocks	$\sigma_{\tilde{z}}$	$.025\sigma$
Autocorrelation of $\beta_{i,t}^z$	ρ_z^β	0.99
Volatility of shocks to $\beta_{i,t}^z$	$\sigma_{\beta,z}$	0.01%

Notes - All parameters are calibrated at a monthly frequency.

whereas country 5 is the riskiest country (second panel, left column).

Each country has a loading of one on global short-run shocks. The first panel on the left of Figure 3 shows that the exposure of monthly country-specific output growth (ΔX_t^i) to monthly global growth ($\frac{1}{N}\sum_i^5 \Delta \log X_t^i$) is close to one as well. This coefficient is slightly increasing as we move from country 1 to country 5, because over time differential in exposure to long-run growth news (shown in the second panel of the left-column) turns into realized short-run growth adjustments captured by $\frac{1}{5}\sum_i^5 \Delta \log X_t^i$. Given these modest differences across of countries, we would be unable to identify any significant form of heterogeneity if we were to compare countries only according to their exposure to short-run global output growth.

The third and fourth panels on the left in Figure 3 focus on consumption growth. The exposure of country-specific consumption growth with respect to global output growth inherits a similar pattern to that of country-specific output growth. Hence

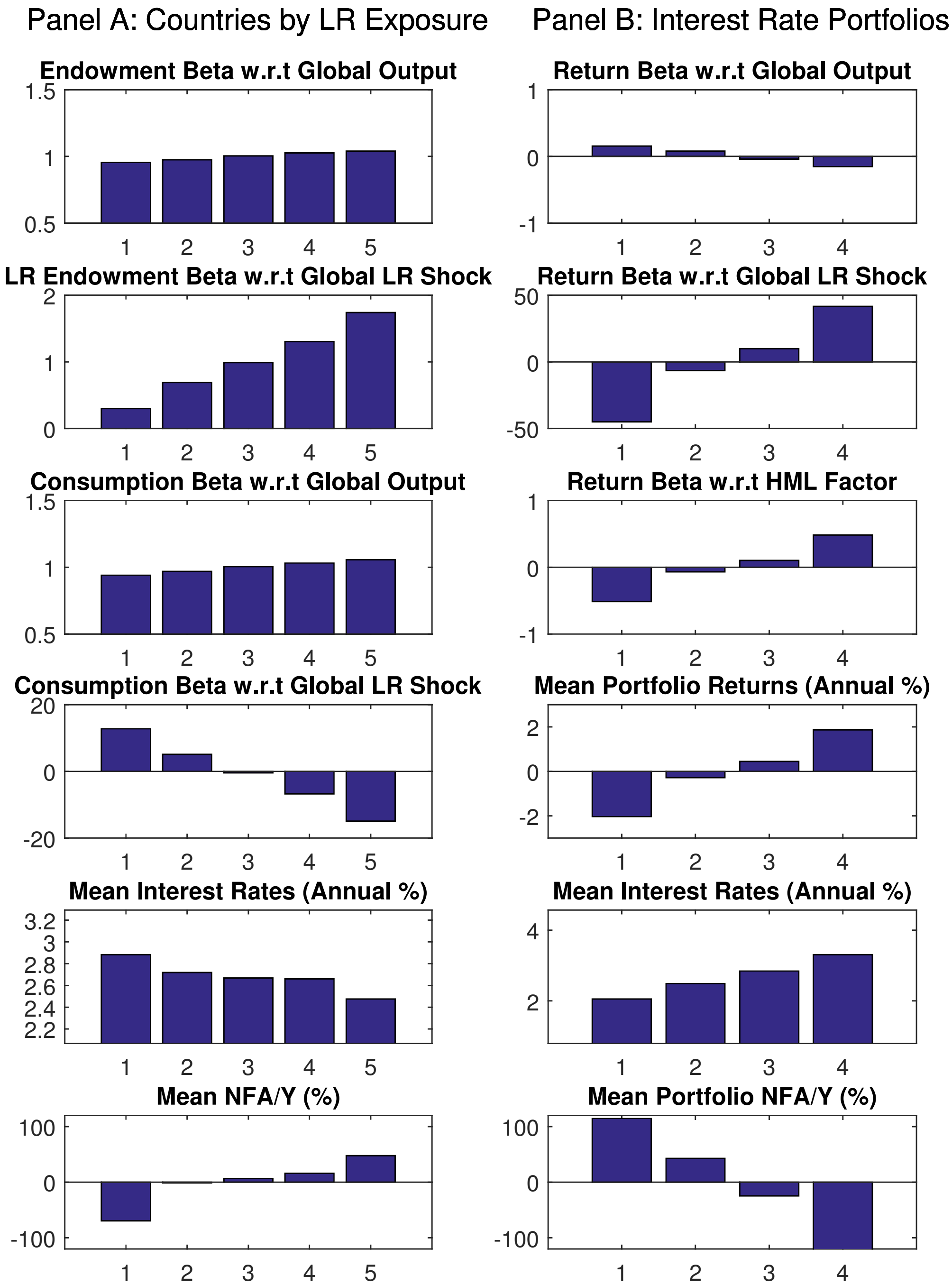

FIG. 3 - Cross Sectional Risk (EZ Case). Panel a) shows the cross-sectional characteristics of five countries sorted on their exposure to the global long-run shock (β_i^z). Panel b) shows the cross-sectional characteristics of four portfolios formed by sorting each period our countries on their lagged interest rate differential with respect to a fixed numeraire country. All the parameters are calibrated to the monthly values reported in table 4. Statistics are the averages across 100 simulations of 360 monthly observations.

it is a potentially poor measure of heterogeneity. Measuring exposure of country-specific growth to global long-run news instead produces a clear sorting, but it goes in the exact opposite direction of the fundamental output betas. This is a result of our documented risk-sharing scheme with respect to long-run news. The country with the highest exposure benefits the most from the arrival of positive long-run global news, implying that it is going to reduce its current consumption to redistribute resources abroad. We conclude that in our model, exposure of consumption growth to contemporaneous long-run shocks is not the appropriate metric to assess country-level riskiness.

The second panel from the bottom shows there is an inverse relationship between country exposure to global long-run news and the average level of its risk-free rate. This relationship can be explained by precautionary saving motives: country 5 is the one with the most volatile stochastic discount factor and the lowest risk-free rate.

The lowest panel on the right hand side of Figure 3 shows that there is a negative relationship between net foreign asset position and average risk-free rate: countries that are net lenders have a lower risk-free rate than countries that are net borrowers. This sorting arises immediately from risk-sharing. Specifically, in our setting all countries are risk averse and buy insurance against shocks that increase their exposure to long-run world growth news. Ex-post, country 5 is by construction the country that has experienced the most adverse path of exposure shocks. As a result of its financial portfolio allocation strategy, this country accumulates wealth against all the remaining countries and it pays a lower risk-free rate, due to its fundamental high riskiness with respect to output shocks. From a qualitative point of view, no additional financial frictions are required to obtain this sorting.

We now turn our attention to the right panels of figure 3. Without loss of generality, we pick country 3 (the one with average exposure) as our numeraire country and focus on the remaining four bi-later exchange rates to form four currency portfolios sorted on interest rate differentials, $i_{j,t} - i_{3,t}$ with $j \neq 3$. The top panel on the right hand side shows that the currency portfolio returns feature an almost complete lack of exposure to global growth shocks ($\frac{1}{5}\sum_i^5 \Delta \log X_t^i$) with virtually no cross-sectional heterogeneity.

The second panel on the right hand side, in contrast, shows that the results improve dramatically, once we focus on the exposure to long-run growth news. Indeed, high endowment-beta countries have low currency-beta, i.e., their currencies depreciate in global good times. This mechanism is sufficient to generate a cross-section of loadings on currency returns to the Lustig et al. (2011) HML factor which are consistent with the data (third panel). The implied average currency returns have an annual spread of about 4% (fourth panel).

The last two panels on the right hand side of Figure 3 confirm that sorting countries on interest rates is equivalent to sorting them on either their net foreign liabilities, or their currency exposure to global long-run news.

The CRRA case. We conclude this section by showing the results for the special case in which we set IES = 1/RRA = 1/8, i.e., the CRRA configuration. Figure 4 documents that a number of counterfactual results arise in this particular setup. First, since long-run news are not directly priced and produce no immediate movements in the marginal utilities of our countries, the consumption growth betas in this case are zero across all countries, even though their output growth exposures continue to be heterogeneous (panel four, left column).

Second, the amount of financial trade in the economy is much more limited than before, as documented by the limited spread in the average net foreign asset positions of our five countries (left column, bottom panel). Furthermore, with this particularly low value of the IES, the risk-free rates are too high (Weil 1989) and basically constant

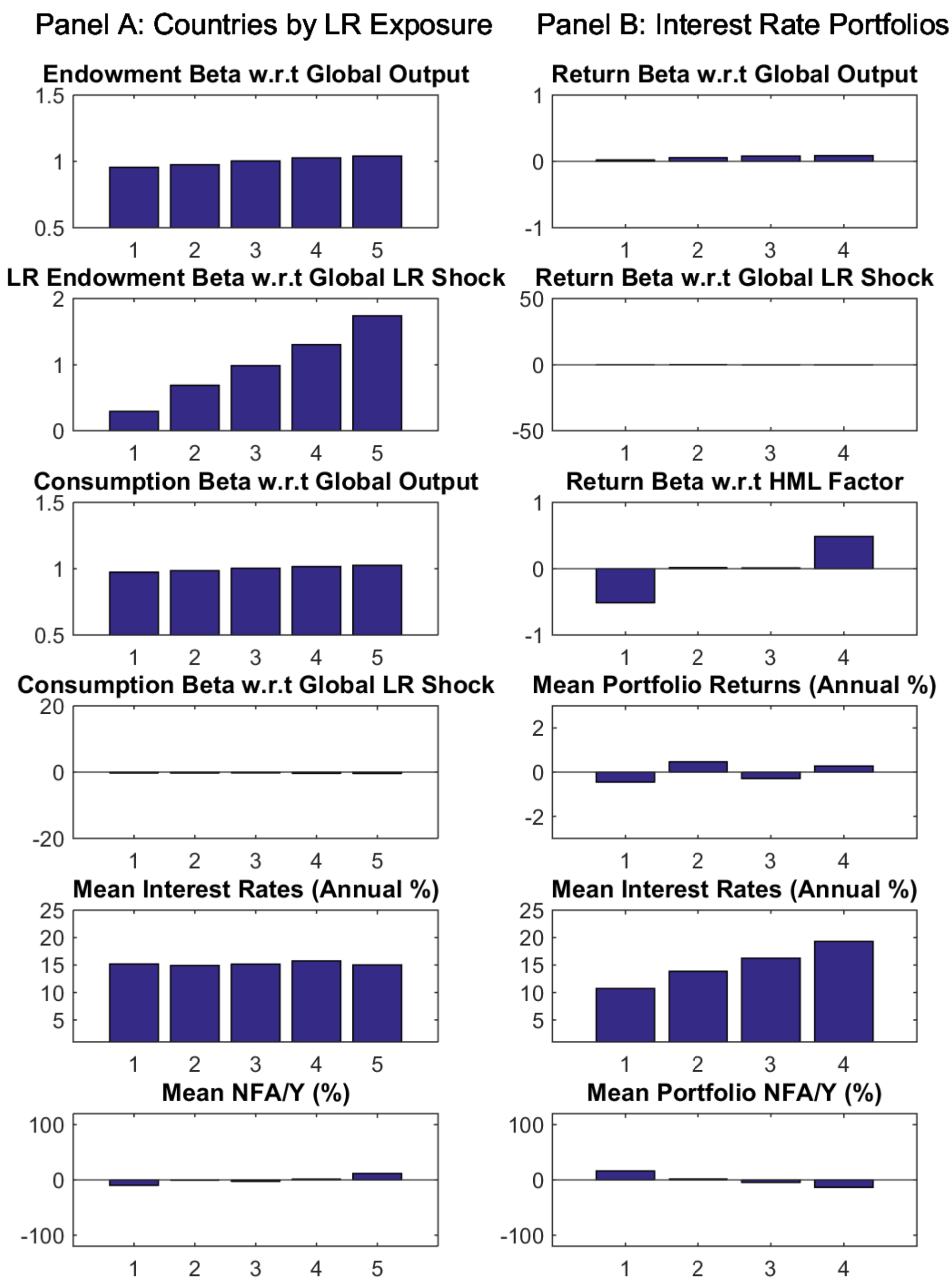

FIG. 4 - Cross Sectional Risk (CRRA Case). Panel a) shows the cross-sectional characteristics of five countries sorted on their exposure to the global long-run shock (β_i^z). Panel b) shows the cross-sectional characteristics of four portfolios formed by sorting each period our countries on their lagged interest rate differential with respect to a fixed numeraire country. All the parameters are calibrated to the monthly values reported in table 4, except the IES which is set to 1/8, the inverse of the risk aversion coefficient. Statistics are the averages across 100 simulations of short-samples (360 monthly observations).

across countries (left column, second panel from the bottom). As aconsequence, the average returns on currency portfolios sorted according to interest rate differentials have an irregular pattern. The same statements apply to the exposure of currency portfolios returns with respect to both global long-run shocks and the HML factor.

Summarizing, long-run global growth news can be an important driver of multiple phenomena in the cross-section of currency, provided that agents price them directly. Epstein and Zin (1989) preferences enable news shocks to be priced and generate a recursive risk sharing scheme that can explain key features of trade and international asset prices both at a country-pair level and in the cross-section of countries.

5 Concluding Remarks

In this paper we have analyzed a general-equilibrium model with long-run risk and recursive preferences populated by a cross-section of multiple countries. We have documented that expanding the analysis from a two-country economy to a larger dimensional setting comes with an array of important economic implications. Furthermore, we have shown that introducing heterogenous exposure to global long-run output growth risk allows us to simultaneously account for many currency risk-factor structures that have been proposed in the literature.

In addition to testing some of the empirical predictions of the model, future developments should focus on extending this setting to international real business cycle models in order to study the role of international investment flows and international frictions for the cross-section of currency risk premia.

References

Anderson, E. 2005. The dynamics of risk-sensitive allocations. *Journal of Economic Theory* 125(2):93–150.

Backus, D., and G. Smith. 1993. Consumption and real exchange rates in dynamic exchange economies with nontraded goods. *Journal of International Economics* 35: 297–316.

Backus, D. K., F. Gavazzoni, C. Telmer, and S. E. Zin. July 2010. Monetary policy and the uncovered interest parity puzzle. (16218). Working Paper, NBER.

Bansal, R., and I. Shaliastovich. 2013. A long-run risks explanation of predictability puzzles in bond and currency markets. *Review of Financial Studies* 26(1):1–33.

Bansal, R., and A. Yaron. 2004. Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance* 59:1481–1509.

Bansal, R., D. Kiku, and A. Yaron. 2010. Risks for the long run: Estimation and inference. *Working Paper.*

Brandt, W., J. Cochrane, and P. Santa-Clara. 2006. International risk sharing is better than you think, or exchange rates are too smooth. *Journal of Monetary Economics* 53(4):671–698.

Caballero, R. J., E. Farhi, and P.-O. Gourinchas. 2008. An equilibrium model of ”global imbalances” and low interest rates. *American Economic Review* 98(1):358–93.

Colacito, R. 2008. Six anomalies looking for a model. a consumption based explanation of international finance puzzles. *Working Paper, Department of Finance, University of North Carolina, Chapel Hill NC.*

Colacito, R., and M. Croce. 2013. International asset pricing with recursive preferences. *Journal of Finance* 68(6):2651–2686.

Colacito, R., and M. M. Croce. 2011. Risks for the long run and the real exchange rate. *Journal of Political Economy* 119(1):153–182.

Colacito, R., and M. M. Croce. 2012. Recursive allocations and wealth distribution with multiple goods. Working paper.

Della Corte, P., S. J. Riddiough, and L. Sarno. 2013. Currency premia and global imbalances. Working paper.

Epstein, L. G., and S. E. Zin. July 1989. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica* 57(4):937–69.

Gourinchas, P.-O., and H. Rey. 2007. International financial adjustment. *Journal of Political Economy* 115(4):665–703.

Gourinchas, P.-O., and H. Rey. 2013. External adjustment, global imbalances and valuation effects. *Handbook of International Economics* 4.

Hassan, T. 2013. Country size, currency unions, and international asset returns. *The Journal of Finance* 68(6):2269–2308.

Hassan, T., and R. Mano. 2014. Forward and spot exchange rates in a multi-currency world. *IMF and University of Chicago Working Paper*.

Lane, P. R., and G. M. Milesi-Ferretti. 2007. The external wealth of nations mark ii: Revised and extended estimates of foreign assets and liabilities, 19702004. *Journal of International Economics* 73:223–250.

Lewis, K. 2011. Global asset pricing. *Annual Reviews of Financial Economics*.

Lustig, H., and A. Verdelhan. 2007. The cross section of foreign currency risk premia and consumption growth risk. *American Economic Review* 97:89–117.

Lustig, H., N. Roussanov, and A. Verdelhan. 2011. Common risk factors in currency markets. *Review of Financial Studies* 24(11):3731–3777.

Lustig, H., N. Roussanov, and A. Verdelhan. 2014. Counter-cyclical currency risk premia. 111.

Ready, R., N. Roussanov, and C. Ward. 2012. Commodity Trade and the Carry Trade: A Tale of Two Countries. Working Paper, the Wharton School.

Weil, P. January 1989. The equity premium puzzle and the riskfree rate puzzle. *Journal of Monetary Economics* 24(3):401–422.

Appendix

A: Allocations as a Function of Pareto weights

Let $W_t^i = W(C_t^i, U_{t+1}^i)$ be the right-hand side of equation (2). If we denote the partial derivatives of the aggregator W^i as follows,

$$W_{1,t}^i := \frac{\partial W_t^i}{\partial C_t^i}, \quad W_{2,t}^i := \frac{\partial W_t^i}{\partial U_{t+1}^i},$$

the stochastic discount factor is equal to:

$$M_{t+1}^i = \frac{W_{2,t}^i W_{1,t+1}^i}{W_{1,t}^i} \quad \forall i = \{h, f\}. \tag{A.1}$$

The optimality condition for the allocation of good $X_{j,t}$ for $t = 1, 2, ...$ in each possible state is:

$$\mu_0^j \cdot \left(\prod_{k=0}^{t-1} W_{2,k}^j \right) \cdot W_{1,t}^j C_t^j \frac{\alpha}{x_{j,t}^j} = \frac{(1-\alpha)}{(N-1)} \cdot \frac{1}{x_{j,t}^i} C_t^i W_{1,t}^i \cdot \left(\prod_{k=0}^{t-1} W_{2,k}^i \right) \cdot \mu_0^j \tag{A.2}$$

for all countries $i \neq j$. Define the date t Pareto weights as:

$$\begin{aligned} \mu_t^i &= \mu_0^i \cdot \left(\prod_{j=0}^{t-1} W_{2,j}^i \right) \cdot W_{1,t}^i C_t^i \\ &= \mu_{t-1}^i \cdot W_{2,t-1}^i \cdot \frac{W_{1,t}^i}{W_{1,t-1}^i} \cdot \frac{C_t^i}{C_{t-1}^i} = \mu_{t-1}^i \cdot M_t^i \cdot \exp\left\{\Delta c_t^i\right\}, \quad \forall i \in \{h, f\} \end{aligned}$$

It follows that equation (A.2) can be rewritten as:

$$\mu_t^j \cdot \frac{\alpha}{x_{j,t}^j} = \frac{(1-\alpha)}{(N-1)} \cdot \frac{1}{x_{j,t}^i} \cdot \mu_t^i \tag{A.3}$$

Let $S_{j,t} := \mu_{j,t}/\mu_{1,t}$. Then the optimality condition in equation (A.3) combined with the feasibility constraint can be represented by the following system of recursive equations:

$$\begin{aligned} x^i_{i,t} &= \left(1 + \frac{1-\alpha}{\alpha(N-1)} \sum_{j \neq i} \frac{S_{j,t}}{S_{i,t}}\right)^{-1} X_{i,t}, \quad \forall i \in \{1, 2, ..., N\} \\ x^j_{i,t} &= \frac{1-\alpha}{\alpha} \frac{1}{N-1} \frac{S_{j,t}}{S_{i,t}} x^i_{i,t}, \quad \forall i \neq j \in \{1, 2, ..., N\} \\ S_{j,t} &= S_{j,t-1} \cdot \frac{M_{j,t}}{M_{1,t}} \cdot \left(\frac{C_{j,t}/C_{j,t-1}}{C_{1,t}/C_{1,t-1}}\right), \quad \forall t \geq 1. \end{aligned} \tag{A.4}$$

We use perturbation methods to solve our system of equations (1)–(10). We compute our policy functions using the `dynare++4.2.1` package. All variables are expressed in log-units.

B: Derivations of Terms of Trade, Imports, and Exports

We normalize the price of good 1 to 1. The terms of trade can be obtained from the intratemporal condition:

$$-p_{j,t} \frac{C^1_t}{x^1_{1,t}} \alpha + \frac{1-\alpha}{N-1} \frac{C^1_t}{x^1_{j,t}} = 0, \quad \forall j = \{2, 3, ..., N\},$$

which implies:

$$p_{j,t} = \frac{1-\alpha}{\alpha(N-1)} \frac{x^1_{1,t}}{x^1_{j,t}}, \quad \forall j = \{2, 3, ..., N\}.$$

Consider country 1 and country j. The exports of country 1 to country j are

$$Exp^j_{1,t} = x^j_{1,t},$$

where $x^j_{1,t}$ is defined in (7). The imports of country 1 from country j are

$$\begin{aligned} Imp^1_{j,t} &= p_{j,t} \cdot x^1_{j,t} = \frac{1-\alpha}{\alpha}\frac{1}{N-1}\frac{x^1_{1,t}}{x^1_{j,t}}x^1_{j,t} \\ &= \frac{1-\alpha}{\alpha}\frac{1}{N-1}\left(1+\frac{1-\alpha}{\alpha(N-1)}\sum_{j\neq 1} S_{j,t}\right)^{-1} X_{1,t}. \end{aligned}$$

It follows that the volume of trade between countries 1 and j normalized by the endowment of country 1 is

$$\begin{aligned} \frac{Vol^j_{1,t}}{X_{1,t}} &= \frac{\frac{1-\alpha}{\alpha}\frac{1}{N-1}(1+S^j_t)}{1+\frac{1-\alpha}{\alpha}\frac{1}{N-1}\sum_{j\neq 1} S^j_t} \\ &= \frac{(1-\alpha)\cdot(1+S^j_t)}{\alpha(N-1)+(1-\alpha)\sum_{j\neq 1} S^j_t}. \end{aligned} \tag{B.5}$$

Similarly, the net exports-output ratio between countries 1 and j is

$$\begin{aligned} \frac{NX^j_{1,t}}{X_{1,t}} &= \frac{\frac{1-\alpha}{\alpha}\frac{1}{N-1}(S^j_t-1)}{1+\frac{1-\alpha}{\alpha}\frac{1}{N-1}\sum_{j\neq 1} S^j_t} \\ &= \frac{(1-\alpha)\cdot(S^j_t-1)}{\alpha(N-1)+(1-\alpha)\sum_{j\neq 1} S^j_t}. \end{aligned} \tag{B.6}$$