Delegating relational contracts to corruptible intermediaries*

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First Draft

December 31, 2014

Abstract

This article explores the links between productive relational contracts and corruption. The model considers a context where responsibility for a relational contract is delegated to a supervisor who cares in part about the profit of the relationship, and in part about the kickbacks paid by the agent. The agent can both exert effort that increases production and make corrupt kickbacks to the supervisor; the incentives for both come through self-enforcing contracts. We show that delegation to such a supervisor may increase social welfare by easing the time-inconsistency problem of paying ex-post incentive payments, even though corruption occurs in equilibrium. We also establish the connection between the agent’s compensation scheme, his effort and the associated kickbacks.

1 Introduction

Self-enforcing relational contracts form an essential part of a wide range of important economic activities (MacLeod, 2007). Frequently, responsibility for relational contracts is delegated to an intermediary - firms delegate to managers, governments delegate to bureaucrats. Yet such delegation carries risks, since these intermediaries

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*We are grateful to Simon Board, Mikhail Drugov, Florian Englmaier, Matthias Fahn, Elisabetta Iossa, Rocco Macchiavello, Jim Malcolmson, David Martimort, Gerard Padró i Miquel, and the participants of ASSET, ISNIE, GREThA, CMPO and EEA conferences and the seminars at HSE, LMU Munich, NES, Oxford and Paris I for many insightful comments. Any remaining errors are our own.

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may extract kickbacks in the form of bribes or non-monetary private benefits. Moreover, since these forms of corruption are not legally enforceable, they themselves also depend on self-enforcing relational contracts (Rose-Ackerman, 1999a; Lambsdorff and Teksoz, 2005).

These observations naturally lead us to ask the following question - can the same relationship facilitate both productive and corrupt contracts? If so, does this create a potential trade-off between corruption and productivity? And, in such contexts, when it is optimal to delegate? This paper aims to answer these questions by understanding how the possibility of corruption may influence the use of incentive relational contracts.

We build a model where an incentive relational contract is needed to induce an agent to exert a hidden effort. The compensation scheme offered by the principal consists of an enforceable fixed payment and a discretionary ‘bonus’. The principal can choose either to control discretionary bonuses herself, as in a standard model of relational contracting, or delegate control of the bonuses to an intermediary supervisor.¹ We assume that the only difference between the principal and the agent is that the supervisor may care less than the principal about potential profit, and the principal may restrict the supervisor’s discretion by capping the maximum bonus the supervisor can pay. If the supervisor doesn’t accrue the entire profit, then she will be motivated to ask the agent for kickbacks.² The optimal principal-agent relational contract a la Levin (2003) is calculated as a benchmark, and we then investigate the levels of effort, bonuses and bribes in the optimal contract between the supervisor and the agent. This allows us to then analyze how the supervisor-agent contract varies as a function of key parameters, and hence derive when it is optimal for the principal to delegate, as well as how much discretion she should give the supervisor and how much surplus she should transfer to the agent.

This article fits into an increasing body of work on relational incentive contracts

¹The principal and the supervisor are female, while the agent is male.
²The terms side-payments, bribes and kickbacks are used interchangeably.
that follows the seminal article of Levin (2003) - see Malcomson (2013) for a recent survey. Typically, such models focus on situations with only two players - a principal and agent - and hence there is no possibility for delegating to an intermediary. Where delegation has been considered, it is generally direct delegation of decisions to the agent - see, for example, Goldlücke and Kranz (2012) and Li et al. (2014). In such contexts, it is not possible to consider corruption, which is most often modeled as collusion between an agent and an intermediary against the interests of a principal - see Banerjee et al. (2013) for a recent survey. Meanwhile, the literature on corruption has typically abstracted from dynamic commitment problems (Tirole, 1986; Strausz, 1997; Celik, 2009). An important exception is the paper of Martimort (1999), who considers a supervisor and an agent who can only collude through self-enforcing contracts. He shows that relaxing the assumption of contract enforcement generates important dynamic results, since the principal can fight corruption in the present through reducing the potential for self-enforcing contracts in the future. However, in this model the productive activity can be explicitly contracted upon by the principal, and hence there is no positive value of relational contracts.

The major contribution of this article is therefore to build a model where corrupt relational contracts exist alongside productive ones. To our knowledge, this is the first paper that builds such a framework.\footnote{In a similar vein, Hermalin (2014) builds a model whereby the relationship between two firms' managers sustains productive activity while also allowing collusion against shareholders. However, unlike in our paper, collusion is not sustained through relational contracting, since it only occurs when such an activity is costless for the managers.} Our first result therefore is that it is indeed possible for the same relationship to simultaneously be responsible for incentivizing valuable activity and kickbacks. Moreover, our analysis allows us to characterize the optimal contract between supervisor and agent, which we show is stationary and maximizes joint supervisor-agent surplus. We then describe the optimal contract as a function of the fixed payment provided to the agent and the maximum bonus that the supervisor is allowed to pay. We show that, when wages are high relative to the bonus cap, the contract that maximizes supervisor-agent surplus uses variation in
bribes to generate effort incentives, with the maximum bonus being paid regardless of the performance. As the fixed wage decreases, so does the average bonus that the supervisor can credibly pay. The supervisor then prefers to gradually give more incentives through bonuses and less through bribes. Eventually, when the wage is very low, only bonuses are used as an incentive device and a fixed kickback is asked for regardless of the performance.

A second contribution of the article is then to provide insight as to when delegating will generate higher effort, and when it will be optimal for the principal. We find that, when the principal is constrained by relational contracting, delegating to a corruptible supervisor can indeed increase effort in equilibrium. This is because the supervisor has a comparative advantage over the principal when enforcing the relational contract. In particular, the supervisor has more credibility when paying promised bonuses - she cares less about making such payments compared to the principal and yet values the relationship because of the expected stream of future kickbacks. More precisely, the agent punishes the supervisor by not paying the promised kickback if the supervisor does not honor her promises. Our model thus can be seen as an example of how delegation can be used as a solution to commitment problems, as explored by Rogoff (1985), Vickers (1985) and Melumad and Mookherjee (1989) amongst others. However, to our knowledge no paper in this literature has explicitly considered corruption as a potential tool to influence supervisors’ payoffs, and hence our contribution here is to draw out insights on potential trade-offs between this commitment effect and corruption.

Our analysis also allows us to characterize the optimal form of delegation for the principal. A noteworthy result is that, unlike in the case of principal-agent relational contracting, the agent’s effort is non-monotonic in the value of supervisor-agent relationship. Hence, in terms of maximizing effort, the principal faces a trade-off when deciding on the agent’s wage level. If the wage is too low, then side-payments will be small, and the relationship will not be sufficiently valuable for the supervisor.
to credibly promise large incentive payments. On the other hand, if the wage is too high, then the supervisor will be unconstrained by relational contracting problems, and hence it will be credible for the supervisor pay large bonus payments regardless of output, and hence the incentives to induce effort will be lower. We also show that there exists a similar trade-off if the principal has some control over the supervisor’s objective function. If the supervisor cares too little about the principal’s payoff, then they will demand little effort from the agent, instead using the relational contract to extract larger kickbacks. On the other hand, if the supervisor cares too much about the principal’s payoffs, then she will face a similar credibility problem to the principal when it comes to paying bonuses in the relational contract. Overall, therefore, the principal faces a trade-off between reducing corruption and increasing the credibility of the relational contract.

We begin in the following section by discussing various instances where such ’dual’ relational contracts are at work. We argue that such relationships occur frequently both within and between firms, as well as in public procurement systems. Section 3 then sets out the basic benchmark case where the principal contracts with the agent directly, which uses a version of the model provided by Levin (2003). We then extend this model in section 4 by introducing a supervisor and characterizing the optimal supervisor-agent relational contract. We solve the model analytically and provide a numerical example to allow us to display comparative statics graphically. We then proceed in section 5 to analyze the benefit to the principal of delegating, and how they will choose the paramaters ex-ante to optimize their expected payoff. Finally, section 6 concludes, drawing empirically testable hypotheses and discussing avenues for future work. Mathematical proofs of all lemmas and propositions are given in the appendix.
2 Examples of ‘dual’ relational contracts

Before detailing our model, we find it useful to consider a range of examples of relational contracts that potentially sustain both productive and corrupt activities. In particular, we believe there are three domains where our analysis is particularly salient: relationships between firms delegated to sales or purchasing managers, relationships between firms and employees delegated to managers, and relationships between government and firms delegated to procurement officers or regulators.

2.1 Inter-firm relationships

It is now well established that relational contracts play a key role in transactions between firms, particularly when courts are weak or trade is international (McMillan and Woodruff, 1999; Johnson et al., 2002; Greif, 2005; Fafchamps, 2006; Gil and Marion, 2012; Macchiavello and Morjaria, 2013; Antras and Foley, 2014). A typical example is a firm purchasing goods where it is difficult or impossible to fully observe the quality of a good before purchase. In this case, the purchasing firm instead may rely on a relational contract, inducing the selling firm to produce high quality goods through the threat of partial non-payment or the termination of the relationship. Many of these relationships are delegated to intermediaries who do not fully own the firm, particularly when firms become large and have many relationships. A purchasing manager is a typical example of such an intermediary. In many situations, a purchasing manager has some discretion as to the client she purchases from, or the price that is paid, and hence the purchasing manager can be seen as having control of the relational contract in the way modelled in this paper.

It is well known that such delegation carries risks of kickbacks or other corrupt behavior. Indeed, Bloom et al. (2013) provides evidence that firm owners “did not trust non-family members. For example, they were concerned if they let their plant managers procure yarn they may do so at inflated rates from friends and receive


Another example concerns the Chinese practice of Guanxi, which has been noted to have negative effects in addition to the positive value for businesses of enhancing relationships. Warren et al. (2004) provides evidence that such relationships are associated with giving bribes in order to sustain relationships, often at the cost of the firm. Hermalin (2014) shows how the use of entertainment budgets - so called ’wining and dining’ - may be used to encourage productive cooperation between firms, but may also simply be used to benefit firms’ managers even when cooperation does not occur.

In this context, Cole and Tran (2011) provide some evidence that these two aspects of relational contracts can be interlinked. In particular, they explore the bribe-paying behavior of two firms who provide goods to other organizations and can thus be considered as the agent in our model. In their examples, they describe kickbacks that are made to intermediaries within the purchasing organization. In on case, they note that when relational contracts are needed because quality is not contractible, “the supplier allows the client to hold back roughly 20 percent of the contract value until one month after delivery, until the client is satisfied that the product meets the specified quality”. Then, apparently in order to encourage the final transfer, the ”kickback is paid only after all contract payments have been made.” (p.411). In another case, where it is “difficult to verify the quantity and quality” (p. 419), the agent ”usually specifies the kickback amount in advance but typically does not start paying until the first deposit is made” (p.420). As we will see, making kickbacks conditional on the supervisor releasing payments is a key aspect of the model below.

2.2 Labor relations and organizational structure

A large portion of the relational contracting literature has focused on labor relations within an organization (MacLeod and Malcomson, 1989; Gillan et al., 2009; Gibbons and Henderson, 2012). In this case, the relationship in question is between employers and employees, with the problem being to incentivize employees to induce effort for
performance that is not easily verifiable. Employees are frequently rewarded with promotions, wage increases or bonuses based on unverifiable subjective performance evaluations, rather than contracted measures of output. In many organizations, such relational contracts are delegated to intermediary managers, who have a substantial amount of control over such incentives given to employees under their supervision.

The risks of delegation in this setting are also well known. For instance, Milgrom (1988), Milgrom and Roberts (1988), Fairburn and Malcomson (2001) and Thiele (2013) each consider the possibility of employees wastefully engaging in ‘influence activities’ or collusion, and argue that the nature of delegation within organizations is often designed to limit such behavior. These papers however implicitly assume such influence activities or collusion to be automatically enforced, and hence do not explore how such behavior relates to relational contracts.\(^4\)

The dual nature of relational contracts in this context lead to conflicting implications within the literature as to their value. For instance, Francois and Roberts (2003) argue that factors enabling relational contracts increase employee productivity and innovation, while Martimort and Verdier (2004) argue that the same factors increase the ability of supervisors and employees to collude and hence dampen economic growth. A similar dispute arises when discussing the use of ‘travel allowances’ in African government bureaucracies which, due to their size compared to wages and weak accounting standards in many countries, serve effectively as cash which managers can hand out to lower-ranked civil servants (Søreide et al., 2012; Nkamleu and Kamgnia, 2014). On the one hand, these are argued to provide an important mechanism through which managers can use relational contracts to incentivize unverifiable effort amongst employees. On the other hand, it is argued that repeated interactions allow civil servants simply to collude to extract the payments. Clearly both interpretations are possible, but it is difficult to evaluate which is more potent

\(^4\)Thiele (2013) considers a principal who may operate a relational contract with the agent, but assumes that delegation to a corruptible supervisor results in all contracts become court-enforceable, and does not consider collusion as a relational contract.
without understanding how both types of contracts may operate within the same relationship.

2.3 Regulation and public procurement

It is increasingly recognized that implicit relational contracts have an important role to play in systems of government regulation and public procurement (Zheng et al., 2008; Calzolari and Spagnolo, 2009; Iossa and Spagnolo, 2011). In such a situation, discretionary performance incentives can be payment installments, favoritism for future contracts, regulatory rulings or the non-application of explicit contract clauses. Spagnolo (2012) gives evidence that allowing discretion improves the quality of government procurement, because governments are limited in their ability to enforce contract performance otherwise. In regulation, judiciaries are often unable to constrain government actions, and hence the ability to effectively expropriate is a discretionary tool available to the government to enforce conducive behavior from the regulated firm (Gilbert and Newbery, 1994; Wren-Lewis, 2013). In many contexts, control of the discretionary tools are delegated to bureaucracies including procurement departments and independent regulators.

Of course, the potential for collusion between bureaucrats and firms through regulation and procurement is well known, and is perhaps one of the most studied forms of corruption (Rose-Ackerman (1999b); Estache and Wren-Lewis (2011); Piga (2011)). Anecdotal evidence indeed suggests that corrupt deals in these sectors are maintained by relational contracts (Lambsdorff and Teksoz, 2005). This thus creates a tension, since policies that hamper corrupt relational contracts typically also hamper productive ones. For instance, the US government started awarding contracts through “full and open competition” in the mid 80s. In order to reduce corruption, the evaluation panel was instructed to ignore subjective information, such as prior performance (Board (2011)).

Within the computer procurement, Kelman (1990) finds that the government’s
lack of loyalty resulted in overpromises made by private contractors which lead to more dissatisfaction and formal disputes. Kelman states that “Lacking the ability to recoup transaction specific investments through the assurance of repeat business, vendors fail to make investments that require conscious effort and expenditure of resources” (? , p. 72).” Similarly, in an effort to reduce corruption, Russia changed its procurement rules in the mid-2000s to reduce the discretion of bureaucrats. Whilst there appears to have been some limited success in reducing the importance of government ‘connections’ in winning bids, evidence suggests that the new system has serious difficulties preventing contract breaches when reputation cannot be taken into account (Podkoloizina and Voytova, 2011; Yakovlev and Demidova, 2012).

Overall, the dual nature of relational contracts in these sectors appears to be important to account for, with Lambsdorff and Teksoz (2005) explicitly noting that “pre-existing legal relationships can lower transaction costs and serve as a basis for the enforcement of corrupt arrangements”.

3 Benchmark: Principal-agent contracting

Let us start by providing a benchmark where the principal implements an incentive relational contract with the agent. We begin by setting up the basic model, which corresponds to the moral hazard model of Levin (2003) simplified by considering the case of binary output (i.e output is high or low). We then solve for the optimal principal-agent relational contract as in Levin (2003).

3.1 The basic model

Consider the situation where a principal (P) oversees the performance of an agent (A) in an infinitely repeated relationship. As in Levin (2003), the agent’s compensation consist of a fixed wage $w$ and a non-contractible payment $b_t(Y_t)$, which depends on the output produced by the agent $Y_t$. 
At the beginning of each period, the principal offers the agent the compensation scheme \( b_t(Y_t) \). The agent either accepts or rejects. Let \( d_t \in \{0, 1\} \) denote the agent’s decision. If the agent rejects the offer, then principal and agent get their outside options (\( \pi \) and \( u \), respectively). If instead the agent accepts, he chooses an effort \( e_t \in [0, \infty) \) incurring a cost \( c(e_t) \). The agent’s cost is increasing and weakly convex. The agent’s effort generates a binary stochastic output \( Y_t \in \{0, y\} \) where \( 0 < y \). The output is high \( Y_t = y \) with probability \( p(e_t) \) where \( 1 > p(0) \geq 0, p'(\cdot) > 0 \) and \( p''(\cdot) \leq 0 \). Furthermore, we assume that \( \frac{c'(e)}{p'(e)} - p(e)y \) is an increasing function of \( e \), in order to rule out uninteresting multiple equilibria.

The information structure corresponds to the one of moral hazard. The effort \( e_t \) is the agent’s private information. Everyone observes the output \( Y_t \). We summarize the timing at any date \( t \) in Figure 3.1.

All the players share the same discount factor \( \delta \in (0, 1) \). At the beginning of any
period $t \geq 1$, the principal and agent’s payoff functions respectively are:

$$\pi_t = (1 - \delta) \sum_{\tau = t}^{\infty} \delta^{\tau-t} \{ d_\tau [Y_\tau - b_\tau(Y_\tau) - w] + (1 - d_\tau) \pi \}$$

$$u_t = (1 - \delta) \sum_{\tau = t}^{\infty} \delta^{\tau-t} \{ d_\tau [w + b_\tau(Y_\tau) - c(e_\tau)] + (1 - d_\tau) u \}$$

### 3.2 The optimal principal-agent contract

Note first that, if the output $Y_t$ were contractible, the first best value of $e_t$ would be that which maximises every period the joint surplus, $yp(e_t) - c(e_t)$, which gives

$$\frac{c'(e^{FB})}{p'(e^{FB})} = y$$

A possible way to implement this effort is to pay a bonus $b = y$ only when the output is high, with the wage $w$ being used to split the total surplus according to the relative bargaining powers of the principal and agent. When the output is not verifiable (and hence not contractible), a relational contract between the principal and agent is needed. We follow Levin (2003) in defining a self-enforcing contract as optimal if no other self-enforcing contract generates higher expected surplus. Levin (2003) shows that, if we are concerned with optimal contracts, there is no loss of generality in focusing on stationary optimal contracts (Theorem 2), and that the optimal contract implements the following effort:

**Proposition 1.** When the future relationship is very valuable, then the principal-agent relational contracting achieves the first best. Otherwise, the equilibrium effort induced in the optimal principal-agent relational contract will be lower, and will be constrained by the maximum bonus the principal can credibly promise.

**Proof.** See Theorem 6 of Levin (2003).
when output is high, i.e. we require that the following inequality holds

$$y \leq \frac{\delta}{1 - \delta} (p(e^{FB})y - c(e^{FB}) - \pi - u)$$

(1)

The right hand side of the this equation is essentially the discounted joint-surplus generated by the principal-agent relationship. When the future relationship is not valuable enough - for instance, because discounting is high - then the principal cannot credibly pay the amount of bonus needed to implement the first best effort. Instead, the bonus $b$ will be the largest which can be credibly promised given this discounted future value, and since $\frac{c'(e)}{p'(e)} = b$, we will have downward distortion in the effort $e^{PA} < e^{FB}$, given by the following differential equation:

$$\frac{c'(e^{PA})}{p'(e^{PA})} = \frac{\delta}{1 - \delta} (p(e^{PA})y - c(e^{PA}) - \pi - u)$$

(2)

One possible way to implement this equilibrium effort is to have bonuses take one of two values: if $Y_t = y$, then $b_t = b^*$ and if $Y_t = 0$, then $b_t = 0$, where $b^* = \frac{c'(e^{PA})}{p'(e^{PA})}$. Furthermore, effort $e^{PA}$ is induced every period, and the relationship never breaks down. The wage $w$ is then chosen to split surplus appropriately.

### 4 Delegation: Supervisor-agent contracting

We now extend the model beyond that of Levin (2003) by introducing a supervisor with a different payoff function from the principal. After describing the new version of the game, we proceed to solve for the optimal supervisor-agent contract.

#### 4.1 Introducing the supervisor

Let us now consider that relational contracting takes place between the agent and a supervisor whose payoff function differs from the principal’s. In particular, we change the model in four ways. First, we assume that the supervisor discounts the elements in
the principal’s payoff by a function of $\alpha \in (0, 1]$. That is, an output $Y$ only produces a benefit of $\alpha Y$ for the supervisor, and if the compensation given to the agent is $b_t + w$, then this only produces a negative payoff of $\alpha(b_t + w)$ for the supervisor.

Second, we assume that at each time $t$ a side transfer can be made between the agent and the supervisor, which costs the agent $s_t$ and gives the supervisor a benefit of $s_t$. The side-transfer is a kick-back, that is, it is paid by the agent to the supervisor after the agent is paid the entire compensation scheme that period. Third, we assume that wage $w$ is fixed and cannot be set by the supervisor. Fourth, we allow for the possibility that the bonus the supervisor can give to the agent may be capped, such that $b_t \leq \bar{b}$.

In this new game, at the beginning of each period, the supervisor offers the agent the compensation scheme $b_t(Y_t)$ and asks for a discretionary side-transfer $s_t$ from the agent. The new timeline is given in figure 4.1, and the payoff functions are given as follows:

$$v_t = (1 - \delta) \sum_{t=\tau}^{\infty} \delta^{\tau - t} \{d_{\tau}[\alpha(Y_{\tau} - b_{\tau} - w) + s_{\tau}] + (1 - d_{\tau})\underline{v}\}$$

$$u_t = (1 - \delta) \sum_{\tau=t}^{\infty} \delta^{\tau - t} \{d_{\tau}[w + b_{\tau}(Y_{\tau}) - s_{\tau} - c(e_{\tau})] + (1 - d_{\tau})\underline{u}\}$$

where $v_t$ is the payoff of the supervisor in period $t$, and $\underline{v}$ is her outside option. Let the surplus generated by the relationship between the supervisor and the agent be $g_t = v_t + u_t$.

Note that, although we have made several changes to the model given in section 3, the two models are entirely equivalent if $\alpha = 1$. To see this, note that when $\alpha = 1$, side-transfers, bonuses and wages all become equivalent tools for making transfers between the two players. The fact that bonuses are capped and wages are fixed therefore becomes an irrelevance, because side-transfers can be used as a perfect substitute. Hence we can think of the benchmark principal-agent model as simply the model given in this section when $\alpha$ takes the value 1.
We assume that only the supervisor and the agent observe the side-transfer $s_t$, and that it is not verifiable. Finally, we assume that there is a one-off side transfer between the supervisor and the agent $s_0$ before the game starts. We can think of $s_0$ as an upfront fee needed for trade to start. We make this assumption to remove uninteresting dynamic patterns.  

### 4.2 The optimal supervisor-agent contract

We now solve for the optimal supervisor-agent contract treating $w$, $b$ and $\alpha$ as exogenous, since these are not chosen by either the supervisor or the agent. We will investigate which values of these variables the principal would like to choose in Section 5.

There are two important differences between the supervisor agent relational con-

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5See the proof of Lemma 1. Alternatively, we could simply assume that the agent has all the bargaining power in the supervisor-agent relationship.
tract and that between the principal and the agent. First, the surplus generated by the relationship between the supervisor and the agent depends on the compensation scheme as well as the effort level. This is because the supervisor only pays for part of the cost of the bonus and fixed wage, but the agent receives both in their entirety. Bonuses therefore serve a dual purpose of both incentivizing effort and generating surplus directly. Second, since the level of the wage $w$ is exogenous to the relationship, it cannot be used as a tool to split the generated surplus between the parties.

Because of these two differences, we cannot instantly assume that contracts will be similar to the principal-agent case. In the following lemmas however, we show that the possibility of side-transfers between the two parties restores us to a situation where we can focus upon optimal stationary contracts in a similar way to Levin.

**Lemma 1.** If there is a self-enforcing contract between the supervisor and agent that generates expected surplus $g_t \geq u + v$, then there are self-enforcing contracts that give as expected payoffs at $t = 0$ any pair $(u_0, v_0)$ satisfying $u_0 \geq u$ and $v_0 \geq v$ and $u_0 + v_0 \leq g$.

This lemma tells us that the initial side payment $s_0$ can be used to split the surplus between the two parties, and hence we can focus on contracts that generate the largest possible total surplus. We therefore follow Levin in defining a self-enforcing contract as optimal if no other self-enforcing contract generates higher expected surplus.

**Lemma 2.** If an optimal contract exists, there are stationary contracts that are optimal.

The intuition behind this stationarity result is that any variation in promised continuation values can be transferred into side-payments, in the same way that Levin shows any variation can be transferred to bonus payments. In our case, note that bonus payments cannot be used in an equivalent way to continuation values,
because changes in the bonus levels directly influence the surplus which exists between
the supervisor and the agent.

Note that effort is determined by the equation

$$\frac{c'(e)}{p'(e)} = b(y) - b(0) - s(y) + s(0) \quad (IC_e)$$

Expected payoffs for $t > 0$ are given according to the following equations

$$u \equiv (1 - \delta)\mathbb{E}_Y [w + b(Y) - s(Y) - c(e)|e] + \delta u$$

$$v \equiv (1 - \delta)\mathbb{E}_Y [\alpha(Y - w - b(Y)) + s(Y)|e] + \delta v$$

In the first period, the initial side-transfer $s_0$ is used to divide the total surplus, and
hence expected payoffs are:

$$u_0 \equiv (1 - \delta)\mathbb{E}_Y [-s_0 + w + b(Y) - s(Y) - c(e)|e] + \delta u$$

$$v_0 \equiv (1 - \delta)\mathbb{E}_Y [s_0 + \alpha(Y - w - b(Y)) + s(Y)|e] + \delta v$$

Define $g(e, b(y), b(0), w)$ as the expected supervisor-agent surplus:

$$g(e, b(y), b(0), w) = \alpha p(e)y + (1 - \alpha)(w + p(e)b(y) + (1 - p(e))b(0)) - c(e) - u - v \quad (3)$$

Note that, holding effort constant, $g$ is increasing in the compensation scheme.

Before the next proposition, it is useful to prove a sequence of lemmas which help
to characterize the supervisor-agent optimal contracts:

**Lemma 3.** In any optimal contract, bonuses are always non-negative, i.e. $b(Y) \geq 0$
$\forall Y$

The proof of this lemma comes from the fact that, if the supervisor wants to take
surplus from the agent, she prefers to do so using bribes, since bribes and bonuses are
equivalent for the agent, but the supervisor captures the whole value of any bribes given.

From this lemma, we can see that there will only be two incentive compatibility constraints that may be binding. In other words, it is only the supervisor who has a reason to deviate when it comes to the bonus payment (because receiving a bonus payment always increases the agent’s payoff), and only the agent that may deviate when it comes to paying the side-transfer (because if the supervisor does not wish to pay the side payment, she already would have deviated by not paying the bonus). These IC constraints are then as follows:

\[
(1 - \delta) (-\alpha b(Y) + s(Y)) + \delta v(Y) \geq \delta u \quad (IC_S)
\]

\[
-(1 - \delta) s(Y) + \delta u(Y) \geq \delta u \quad (IC_A)
\]

This then gives us the following lemma, which simply states that if bonuses or bribes depend on output, they will do so in a way which encourages effort:

**Lemma 4.** In any optimal contract with positive effort, bonuses are weakly higher when output is high \((b(y) \geq b(0))\) and side transfers are weakly lower \((s(y) \leq s(0))\).

This lemma essentially tells us that the incentive compatibility constraints that we need to be concerned about are \(IC_S\) when output is high, and \(IC_A\) when output is low.

Before we provide the optimal relational contract between the supervisor and the agent, it is useful to consider what would be the optimal contract if explicit contracting between the two were possible. From equation (3), it is clear to see that joint surplus is increasing in \(b(0)\) and \(b(y)\), and hence these would both be at the maximum, i.e. \(\bar{b}\). Effort would then be set at \(e_{FB}^{SA}\), where \(\frac{c'(e_{FB}^{SA})}{p'(e_{FB}^{SA})} = \alpha y\). The following lemma then tells us that whether or not such a relational contract is possible is essentially a function of how high the wage \(w\) is:
Lemma 5. If the wage \( w \) is sufficiently high, then the first-best supervisor-agent contract will be self-enforcing. However, if the wage is sufficiently small, then, in any optimal stationary contract, the supervisor’s incentive compatibility constraint will be binding when output is high, and the agent’s incentive compatibility constraint will be binding when output is low.

In particular, the condition on \( w \) such that the first-best contract is self-enforcing is

\[
\alpha y + \alpha \bar{b} \leq \frac{\delta g(e_{FA}, \bar{b}, \bar{b}, w)}{1 - \delta}
\]

Note the similarity between this inequality and the corresponding one in the principal-agent game, inequality (1). In both cases, on the right-hand-side is the total discounted future surplus in the relationship, whilst we can consider the left-hand-side as the effective cost of the largest discretionary payment that needs to be made. In the supervisor-agent case, this is the cost of paying a bonus \( \bar{b} \) (which costs the supervisor \( \alpha \bar{b} \)) and a bribe of \( \alpha y \) to induce the first-best level of effort.

If this inequality is not met, then the fact that contracts are relational will be a constraint for the supervisor and agent. Lemma 5 tells us precisely which constraints are binding, and hence we can sum together the two constraints to form the joint incentive compatibility constraint:

\[
\alpha b(y) - s(y) + s(0) = \frac{\delta g(e, b(y), b(0), w)}{1 - \delta}
\]

We can substitute out for the difference in bribes using the effort equation \( (IC_e) \), which then gives us

\[
\frac{e'(e)}{p'(e)} + \alpha b(y) - [b(y) - b(0)] = \frac{\delta g(e, b(y), b(0), w)}{1 - \delta}
\]

Comparing this equation to the equivalent in the principal-agent case, equation (2), we can see that the requirement for contracts to be self-enforcing has a slightly more
complex impact on the supervisor-agent game. In particular, as the total surplus in the relationship decreases, a reduction in effort is now only one possible effect. Alternatively, the supervisor and agent may choose to reduce the size of the high output bonus, or to increase the difference in the bonuses (keeping effort constant). In particular, the latter makes relational contracting easier since it is more credible for the supervisor to induce effort using bonuses than bribes, since paying bonuses is less costly. The following proposition thus characterizes the different optimal contracts as a function of the wage level:

**Proposition 2.** Within the optimal supervisor-agent contract, the tools used to incentivize the agent to make effort depend on the wage level in the following way:

1. **High wage:** Only bribes are used to incentivize effort, with bonuses always kept at the maximum - i.e. $b(y) = b(0) = \bar{b}$, $s(y) < s(0)$

2. **Intermediate wage:** Bonuses and bribes are used to incentivize effort - i.e. $b(y) > b(0)$ and $s(y) < s(0)$

3. **Low wage:** Only bonuses are used to incentivize effort - i.e. $b(y) > b(0)$ and $s(y) = s(0)$

Moreover, effort is a continuous non-monotonic function of the wage level. When the wage is high or low, effort is weakly increasing in the wage, but, when the wage is intermediate, effort is weakly decreasing in the wage.

The full proof of the proposition is given in the appendix. In order to give the intuition and understand further the nature of the optimal supervisor-agent contract, we now characterize the three different types of contract given in the proposition.

### 4.2.1 Optimal supervisor-agent contract with a high wage

As already stated in Lemma 5, if the wage is very large then the bonuses will always be at the maximum and the effort level will be at $e_{FB}^{SA}$. However, if wages are slightly
below this level, then this contract is not self-enforcing, and effort will be lower as a result. In particular, for a high wage, effort will be given according to the equation:

\[
\frac{c'(e)}{p'(e)} = \min \left\{ \alpha y, \delta g(e, \bar{b}, \bar{b}, w) \right\}
\]

We can see that this definition of the wage comes straight from equation (4) when the bonus levels are the same.

The reason that a fall in \( w \) first leads to a reduction in the effort level, rather than the bonuses, is because for very large \( w \) the effort level is at its first best \( e_{FA} \). The cost to the joint supervisor-agent surplus is therefore second-order, whilst the cost of reducing the bonuses is not (since the supervisor-agent surplus is always strictly increasing in the bonus levels for a given level of effort).

4.2.2 Optimal supervisor-agent contract with an intermediate wage

As the wage falls further, at some point paying continually high bonuses no longer becomes optimal for two reasons. First, paying high bonuses when effort is low reduces the agent’s incentive to make effort and the supervisor cannot compensate for this by asking for higher bribes, since it is not credible that the agent pays such bribes. Second, since bonuses are discretionary, the supervisor is simply not able to promise to pay high bonuses. However, they will not reduce both bonus levels simultaneously. Instead, they will first cut the bonus given when output is low, since cutting this bonus further incentivizes the agent to make effort. This thus allows the difference in bribes asked for to be reduced, and hence further relaxes the incentive compatibility constraint. In this zone, therefore, \( b(y) = \bar{b} \).

In deciding upon the bonus given when output is low, \( b(0) \), the players thus face a tradeoff. A higher \( b(0) \) generates greater surplus, whilst a lower \( b(0) \) increases the effort induced. Note however that the bonuses will never be negative (from Lemma
\[ b(0) = \max \left\{ 0, \frac{1 - \alpha}{\alpha} \left( 1 - p(e) \right) \frac{d}{de} c'(e) - y + \frac{1}{\alpha} \delta g(e, b(y), b(0), w) \right\} \] (6)

The first term in the non-zero part of this expression stems from the loss in supervisor-agent surplus that a reduction in \( b(0) \) produces - the more likely negative output is to occur, the higher this loss. The second term, \( y \), then stems from the change in expected output a change in \( b(0) \) produces, through the change in effort induced. The final term represents the relational contracting constraint, and the fact that a higher wage allows \( b(0) \) to be higher.

Effort is then given according to the following equation:

\[ \frac{c'(e)}{p'(e)} = \bar{b} - b(0) + \frac{\delta g(e, \bar{b}, b(0), w)}{1 - \delta} - \alpha \bar{b} \]

Note from this equation, it is clear that bribes are doing some work to incentivize effort, and in particular we must have \( s(0) - s(y) = \frac{\delta g(e, \bar{b}, b(0), w)}{1 - \delta} - \alpha \bar{b} \). Furthermore, note that if we substitute in the equation for \( b(0) \), we can see that effort is weakly decreasing in the wage level. This is because, as the wage decreases, then \( b(0) \) decreases and hence the agent and supervisor have a further incentive to increase effort.

4.2.3 Optimal supervisor-agent contract with a low wage

Finally, as the wage becomes lower, the supervisor can no longer credibly promise to pay the maximum bonus even when output is high. The bonus given when output is high is therefore the maximum it can credibly promise, i.e.

\[ b(y) = \frac{1}{\alpha} \frac{\delta g(e, b(y), b(0), w)}{1 - \delta} \] (7)
The low bonus is again given according to equation (6). When the wage is so low, bribes will not be used to induce effort. If they were, the supervisor would simultaneously offer a higher bonus when output was high along with demanding a higher bribe. Hence effort in this case is entirely determined by the difference in the bonus levels, i.e.

\[
\frac{c'(e)}{p'(e)} = b(y) - b(0)
\]

(8)

5 When and how should the principal delegate?

In this section, we first add a principal into the supervisor-agent game, who may have some control over the parameters including \(w, \tilde{b}\) and \(\alpha\). We then compare the optimal contract in the supervisor-agent game to that in the principal-agent game to examine when delegation can be beneficial for the principal.

We assume that, if delegation occurs, the surplus is shared between the principal and the supervisor, such that the principal’s payoff function is as follows:

\[
\pi_t = (1 - \delta) \sum_{\tau=t}^{\infty} \delta^{\tau-t} \{d_{\tau} [(1 - \alpha)(Y_{\tau} - b_{\tau}(Y_{\tau}) - w_{\tau})] + (1 - d_{\tau}) \pi\}
\]

The supervisor and agent’s payoff functions are as given in the previous section.

5.1 How should the principal delegate

5.1.1 Optimal \(w\) and \(\tilde{b}\) for principal

Let us now consider how the principal would set were some parameters under her control. To begin with, let us take \(\alpha\) as given and consider the optimal values of \(w\) and \(\tilde{b}\). Let \(\tilde{\alpha}\) be the following critical value of \(\alpha\):

\[
(1 - \tilde{\alpha})\delta(1 - p(\tilde{e})) = \tilde{\alpha}(1 - \delta)
\]

where \(\tilde{e}\) is defined in (9) in the appendix.

**Proposition 3.** If \(\alpha\) is below a critical value \(\tilde{\alpha}\), then the principal will set the maximum bonus to zero. Otherwise, it is optimal for the principal to set a bonus cap such
that i is never a constraint for the supervisor. In both cases, the wage will be set to reflect a trade-off between the cost the principal bears for paying the wage and the benefit she obtains from allowing the supervisor-agent relationship to sustain a higher level of effort.

If \( \alpha \) is below \( \tilde{\alpha} \), then there will be no supervisor-agent optimal contracts with \( b(0) < b(y) \). This is because increasing \( b(0) \) increases the total supervisor-agent surplus sufficiently that the supervisor can increase \( s(y) \) by the same amount, thus keeping effort constant. In other words, the wage will always be ‘high’ in the sense of Proposition 2. In this case, the optimal contract has the following effort

\[
\frac{c'(e_A)}{p'(e_A)} = \min \left\{ \alpha y - \frac{1 - \delta}{\delta p'(e_A)} \frac{d}{de} \left[ \frac{c'(e_A)}{p'(e_A)} \right], \frac{1 - \alpha}{\alpha p'(e)} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right] \right\}
\]

If \( \alpha \) is above this critical value, then the principal can set the wage such that it will be ‘low’ in the sense of Proposition 2, and in particular such that \( b(0) = 0 \). The principal has no incentive to set a higher wage, since effort reaches its peak at the boundary between the ‘low’ and ‘intermediate’ wage contracts. Then the principal will set \( b \) and \( w \) such that the optimal contract has effort \( e \) where

\[
\frac{c'(e)}{p'(e)} = y - \max \left\{ \frac{\alpha (1 - \delta)}{\delta p'(e)} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right], \frac{(1 - \alpha)(1 - p'(e))}{\alpha p'(e)} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right] \right\}
\]

### 5.1.2 Optimal \( \alpha \) for principal

**Proposition 4.** The optimal value of \( \alpha \) for the principal lies strictly between 0 and 1

In particular, the optimal level of \( \alpha \) is set such that

\[
\frac{c'(e)}{p'(e)} = y - \alpha \frac{1 - \delta}{\delta p'(e)} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right] - \alpha \left( \frac{1 - \delta}{\delta} \frac{c'(e)}{p'(e)} + \tau + \nu \right) \left( \frac{(1 - \alpha)}{p'(e)} \frac{d^2}{de^2} \left[ \frac{c'(e)}{p'(e)} \right] - \frac{p'(e)^2}{p'(e)^2} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right] \right)
\]
The direct negative effect of choosing a larger $\alpha$ is that the share of the principal’s profit shrinks. There is also an strategic trade-off behind this choice. On one hand, a larger $\alpha$ is positive because it makes the supervisor care more about effort, but on the other, it also makes the supervisor more concerned about paying the bonus.

### 5.2 When should the principal delegate?

It is useful to consider when the supervisor-agent relational contract might generate greater effort than the principal-agent relational contract. For comparison, Proposition 1 says that effort in the principal agent relationship will be given by $e^{PA}$ where

$$
\frac{c'(e^{PA})}{p'(e^{PA})} = \min \left\{ y, \frac{\delta g(e^{PA}, y, 0, -\pi)}{1 - \delta} \right\}
$$

where we have assumed that $v = \alpha \pi$. Since $g(e, b(y), b(0), w)$ must be less than the surplus in the principal-agent relationship, the following corollary follows instantly

**Corollary 1.** Delegation can only lead to greater induced effort if the optimal supervisor-agent contract is of the low or intermediate wage type i.e. where $b(0) < b(y)$. Therefore, if $(1 - \alpha)\delta(1 - p(\tilde{e})) \geq \alpha(1 - \delta)$, delegation cannot lead to higher effort for any values of $\tilde{b}$ and $w$.

The intuition behind this result is that delegation can increase the amount of effort if the supervisor exploits the fact that paying bonuses is relatively easier for her to do. If, however, $b(0) = b(y)$, then effort is incentivized only through bribes, and hence the relational contracting constraints on the supervisor are no less harsh than those on the principal.

### 6 Conclusions and future work

Overall, the paper has considered a situation where an incentive relational contract takes place between a supervisor and an agent. Unlike the standard principal-agent
setting, we have assumed that the supervisor cares only partially about the production that they are incentivizing, and partly about side-payments that they may receive from the agent. We have shown that such an assumption substantially changes the form of the optimal relational contract between the two players, particularly when there is a cap on bonuses. In particular, bonuses become a tool for extracting rent from the principal as well as incentivizing effort, and this leads effort to be non-monotonic in the relationship surplus. As we have argued in the paper, situations where relational contracts are delegated to corruptible agents are extremely common, and hence we believe we have made an important further step into understanding how such relational contracts may operate.

In some cases, the principal may be forced to delegate the relational contracts - if, for example, they are at the head of a large organization and there are many relationships to manage. In this case, we have considered how they can setup the game between the supervisor and the agent to best manage the tension between reducing corruption and encouraging effort. In particular, we have shown that both the incentives of the supervisor and the surplus given to the agent are important tools in managing this relationship.

We have also shown that the effect of delegating on the principal’s welfare is dependent on the particular context. Hence there may be situations where delegation is indeed not used due to fear of corruption. Consider, for example, the small business owner who is unable to grow due to lack of ability to trust intermediary managers. However, there may also be situations where delegation in fact improves the principal’s payoff, precisely because the supervisor is willing to accept side-transfers. This may help us to understand why governments continue to employ corrupt bureaucrats, despite the apparent cost.

A next step will be to use the results of this paper to draw finer testable hypotheses that could be used in empirical work. We could generate results, for instance, both on when we would expect delegation to occur, and how we would expect corruption
and effort to vary depending on the nature of the supervisor-agent relationship.

Finally, there are a number of important extensions that would be valuable to undertake. We have considered the case whereby the principal may choose the amount by which the supervisor discounts profits, but there may also be cases where the principal can gear more precise the supervisor’s relative valuation of output compared to agent compensation. This is important since such fine-tuning may work as a substitute to the value of corruption. Furthermore, by assuming that side-transfers are costless, we have stayed in the world of stationary contracts as in Levin. However, if we were to assume that there was some non-linearity in the cost of side-transfers (as is commonly argued), then the optimal relational contract may become strictly dynamic. Exploring ‘dual’ relational contracts in a dynamic setting would be extremely valuable in providing insights as to how corruption and effort might co-evolve over time.

7 References

References


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8 Appendix: Proofs of propositions and lemmas

This appendix proves the Lemmas and Propositions in the text above. We first begin by proving Proposition 1. Since in this case there is no supervisor, our model is equivalent to a case considered in Levin (2003), and we can simply apply the theorems derived in that paper.

Proof of Proposition 1. From Levin (2003), we can restrict ourselves to stationary contracts (Theorem 2) where the bonus takes one of two values - 0 and $b$, where incentive compatibility implies

$$b \leq \frac{\delta}{1 - \delta}(p(e)y - c(e) - \pi - u)$$

The agent optimizes his payoff given this bonus, which gives us

$$p'(e)b = c'(e)$$

and hence $b = \frac{c'(e)}{p'(e)}$. In order for the principal to be able to implement first best, we therefore require that

$$y \leq \frac{\delta}{1 - \delta}(p(e^{FB})y - c(e^{FB}) - \pi - u)$$

If $y > \frac{\delta}{1 - \delta}(p(e^{FB})y - c(e^{FB}) - \pi - u)$, then the first best is not implementable, and instead we will have the level of effort determined by equation (2).

The remaining propositions consider the case where there exists a supervisor, and hence we can no longer directly apply the results of Levin (2003). Instead, consider a contract that in its initial period calls for payments $w$, $b(Y)n s(Y)$ and effort $e$. If the offer is made and accepted and the discretionary payments made, the continuation contract gives payoffs $u(Y)$, $v(Y)$ as a function of the observed outcome $Y$. A deviation - an unexpected offer or rejection or a refusal to make the
discretionary payment - implies reversion to the static equilibrium.

Let \( u, v \) be the expected payoffs under this contract:

\[
\begin{align*}
  u &\equiv (1 - \delta) \mathbb{E}_Y [w + b(Y) - s(Y) - c(e)|e] + \delta \mathbb{E}_Y [u(Y)|e] \\
  v &\equiv (1 - \delta) \mathbb{E}_Y [\alpha(Y - w - b(Y)) + s(Y)|e] + \delta \mathbb{E}_Y [v(Y)|e]
\end{align*}
\]

except for the first period:

\[
\begin{align*}
  u_0 &\equiv (1 - \delta) \mathbb{E}_Y [-s_0 + w + b(Y) - s(Y) - c(e)|e] + \delta \mathbb{E}_Y [u(Y)|e] \\
  v_0 &\equiv (1 - \delta) \mathbb{E}_Y [s_0 + \alpha(Y - w - b(Y)) + s(Y)|e] + \delta \mathbb{E}_Y [v(Y)|e]
\end{align*}
\]

Let \( g \) be the expected supervisor-agent surplus:

\[
g = (1 - \delta) \mathbb{E}_Y [\alpha Y + (1 - \alpha)(w + b(Y)) - c(e)|e] + \delta \mathbb{E}_Y [g(Y)|e] - \overline{u} - \overline{v}
\]

where \( g(Y) = u(Y) + v(Y) - \overline{u} - \overline{v} \) is the continuation surplus following outcome \( Y \).

Note that \( g \) increases in the compensation scheme.

This contract is self-enforcing if and only if the following conditions hold:

i The parties are willing to initiate the contract: \( u \geq \underline{u} \) and \( v \geq \underline{v} \)

ii The agent is willing to choose \( e \): \( e \in \arg \max_e \mathbb{E}_Y [b(Y) - s(Y) + \frac{\delta}{1 - \delta} u(Y)|e] - c(e) \)

iii For all \( Y \), both parties are willing to make the discretionary payments \( b \):

\[
(1 - \delta) - \alpha b(Y) + s(Y) + \delta v(Y) \geq \delta \underline{u} \quad (ICb_S)
\]
\[
(1 - \delta)b(Y) - s(Y) + \delta u(Y) \geq \delta \underline{u} \quad (ICb_A)
\]
iv For all \( Y \), both parties are willing to make the side-payment:

\[
(1 - \delta)s(Y) + \delta v(Y) \geq \delta v \\
-(1 - \delta)s(Y) + \delta u(Y) \geq \delta u
\]

\((ICsS)\) \hspace{2cm} \((ICsA)\)

v Each continuation contract is self-enforcing - i.e. the pair \( u(Y), v(Y) \) correspond to a self-enforcing contract that will be initiated in the next period.

**Proof of Lemma 1.** Consider changing the initial side payment \( s_0 \) in the contract above. This changes the expected payoffs \( u_0, v_0 \), but not the joint surplus. The new contract is also self-enforcing provided that \( u \geq u_0 \) and \( v \geq v_0 \).

Let \( g^* \) be the maximum supervisor-agent surplus generated by any self-enforcing contract.

**Lemma 6.** Any optimal contract is sequentially optimal.

**Proof.** Consider increasing \( v(Y) \) for some \( Y \) in the above contract. This does not make any of the self-enforcement constraints more binding so long as \( u(Y) + v(Y) \leq g^* \). But this change increases the expected surplus, and hence by contradiction we must have \( u(Y) + v(Y) = g^* \) for all \( Y \).

**Proof of Lemma 2.** Take an optimal contract as defined above - i.e. where, in the initial period, bonuses, side-payments and the agent’s continuation value are given by the functions \( b(Y), s(Y) \) and \( u(Y) \), where \( Y \) is the first period output. Then, we define new side payments, \( s^*(Y) \), and a continuation value \( u^* \) such that:

\[
s^*(Y) = s(Y) - \frac{\delta}{1 - \delta} u(Y) + \frac{\delta}{1 - \delta} u^* \\
u^* = \mathbb{E}_Y [w + b(Y) - s^*(Y) - c(e)|e]
\]

This generates a contract that is stationary and gives an expected continuation value of \( u^* \). It therefore remains to check that this contract is self-enforcing and optimal.
We now check the self-enforcing conditions.

i It requires that \( u^* \geq u \) and \( v^* \geq v \). From Lemma ??, we have that \( u^* \geq \min_Y u(Y) \), and we know \( u(Y) \geq u \) \( \forall Y \), since the participation constraint must have held in the original self-enforcing contract. Similarly, \( v^* = g^* - u^* \geq g^* - \max_Y u(Y) \geq v \).

ii Effort is chosen to maximize the expression

\[
\mathbb{E}_Y \left[ b(Y) - s^*(Y) + \frac{\delta}{1 - \delta} u^*|e \right] - c(e)
\]

which is the same expression that is being maximized in the original optimal contract.

iii For all \( Y \), both parties are willing to make the discretionary payments \( b \):

\[
(1 - \delta)(-\alpha b(Y) + s^*(Y)) + \delta v^*(Y) = (1 - \delta)(-\alpha b(Y) + s(Y)) + \delta v(Y) \geq \delta u
\]

\[
(1 - \delta)(b(Y) - s^*(Y)) + \delta u^* = (1 - \delta)(-\alpha b(Y) + s(Y)) + \delta v(Y) \geq \delta u
\]

iv For all \( Y \), both parties are willing to make the side-payment:

\[
(1 - \delta)s^*(Y) + \delta v^* = (1 - \delta)s(Y) + \delta v(Y) \geq \delta u
\]

\[
-(1 - \delta)s^*(Y) + \delta u^* = -(1 - \delta)s(Y) + \delta u(Y) \geq \delta u
\]

v Each continuation contract is self-enforcing - i.e. the pair \( u^* \), \( v^* \) corresponds to a self-enforcing contract.

The contract is optimal since it generates surplus \( g^* \) each period (since bonuses and effort are unchanged). Finally, we can then define the one-off payment at time 0, \( s_0^* \), such that the expected payoffs are \((u_0, v_0)\). \( \Box \)
Proof of Lemma 3. Consider an optimal contract with \(b(Y) < 0\) for some \(Y\). Then consider an alternative contract with \(b'(Y) = 0\) and \(s'(Y) = s(Y) - b(Y)\). It is simple to check that all the self-enforcing constraints are still satisfied. However, this contract has a higher surplus, and therefore the original contract cannot be optimal. \(\square\)

Proof of Lemma 4. Consider the case of bribes first. Suppose the opposite, i.e. \(s(y) > s(0)\). If positive effort is being made, we must therefore have \(b(y) > b(0)\). Moreover, since \(s(y) \leq \frac{\delta}{1-\delta} (u-u)\), we instantly have \(s(0) < \frac{\delta}{1-\delta} (u-u)\) - i.e. \((ICs_A)\) is not binding when \(Y = 0\). Now, consider an alternative contract with \(b'(0) = b(0) + \epsilon\) and \(s'(0) = s(0) + \epsilon\), and other values as before. In this alternative contract, surplus is greater since effort is unchanged and bonuses are higher. Yet, since \((ICs_A)\) is not binding when \(Y = 0\), then for some \(\epsilon > 0\) this contract is self-enforcing and hence the original contract is not optimal.

Now consider the case of bonuses. Suppose the opposite, i.e. \(b(y) < b(0)\). Given that effort is positive, we must therefore have \(s(y) < s(0)\). This implies that \((ICs_A)\) is not binding when \(Y = y\). Now, consider an alternative contract with \(b'(y) = b(y) + \epsilon\) and \(s'(y) = s(y) + \epsilon\), and other values as before. In this alternative contract, surplus is greater since effort is unchanged and bonuses are higher. Yet, since \((ICb_S)\) is not binding when \(Y = y\), then for some \(\epsilon > 0\) this contract is self-enforcing and hence the original contract is not optimal. \(\square\)

Lemma 7. In any optimal contract with positive effort and \(b(0) < b(y)\), then \(s(0) = \frac{\delta}{1-\delta} (u-u)\).

Proof of lemma 7. We show proof by contradiction. Suppose \(ICs_A\) is not binding when \(Y = 0\) (i.e. \(s(0) < \frac{\delta}{1-\delta} (u-u)\)). Now, consider an alternative contract with \(b'(0) = b(0) + \epsilon\) and \(s'(0) = s(0) + \epsilon\), and other values as before. In this alternative contract, surplus is greater since effort is unchanged and bonuses are higher. Yet, since \((ICs_A)\) is not binding when \(Y = 0\), then for some \(\epsilon > 0\) this contract is self-
enforcing and hence the original contract is not optimal.

Lemma 8. In any optimal contract with positive effort and \( b(y) < \bar{b} \), we have \( s(y) = s(0) \).

Proof of Lemma 8. Suppose \( s(y) < s(0) \). Now, consider an alternative contract with \( b'(y) = b(y) + \epsilon \) and \( s'(y) = s(y) + \epsilon \), and other values as before. In this alternative contract, surplus is greater since effort is unchanged and bonuses are higher. Yet, since \((ICs_A)\) is not binding when \( Y = 0 \), then for some \( \epsilon > 0 \) this contract is self-enforcing and hence the original contract is not optimal.

Proof of Lemma 5. Suppose that \( w \) is sufficiently small that \( \alpha b \geq \delta g(e_{SAFB}, b, b, w) 1 - \delta - \alpha y \). We then consider an optimal contract of the form described above and proceed through proof by contradiction.

First, suppose that \((ICb_S)\) is not binding when \( Y = y \). Note that it must be that \( e \geq e_{SAFB}^S \), since otherwise we can consider an alternative contract with \( s'(y) = s(y) - \epsilon \), which is also self-enforcing but induces higher effort. This implies \( b(y) - s(y) - b(0) + s(0) = \frac{e'(\epsilon)}{p'(\epsilon)} \geq \alpha y > 0 \). Suppose that \( b(y) < \bar{b} \). Then, from Lemma 8, \( s(y) = s(0) \) and hence \( b(0) < b(y) \), which means that \((ICb_S)\) is not binding when \( Y = 0 \). Then we can consider a contract with \( b'(y) = b(y) + \epsilon \) and \( b'(0) = b(0) + \epsilon \), and other values as before. Since this contract is self-enforcing for some \( \epsilon > 0 \) and has higher surplus, it must be that \( b(y) = \bar{b} \). Finally, suppose that \( b(0) < \bar{b} \). From Lemma 4, \( s(0) \geq s(y) \) and hence \((ICb_S)\) is not binding when \( Y = 0 \). Then consider a contract with \( b'(0) = b(0) + \epsilon \) and \( s'(y) = s(y) - \epsilon \), and other values as before. Since this contract is self-enforcing for some \( \epsilon > 0 \) and has higher surplus, it must be that \( b(0) = \bar{b} \).

Second, suppose that \((ICs_A)\) is not binding when \( Y = 0 \). Note that it must be that \( e \geq e_{SAFB}^S \), since otherwise we can consider an alternative contract with \( s'(0) = s(0) + \epsilon \), which is also self-enforcing but induces higher effort. Furthermore, Lemma 7 tell us that \( b(0) = \bar{b} \). If \( b(0) < \bar{b} \), then consider a contract with \( b'(0) = b(0) + \epsilon \) and
\( s'(0) = s(0) + \epsilon, \) and other values as before. Since this contract is self-enforcing for some \( \epsilon > 0 \) and has higher surplus, it must be that \( b(0) = b(y) = \bar{b}. \)

Hence if either constraint is not binding, we must have \( b(y) = b(0) = \bar{b} \) and \( e \geq e_{FB}^{SA} \). Moreover, summing the two constraints together then gives us that

\[
(1 - \delta)(-\alpha \bar{b} + s(y) - s(0)) + \delta(v + u) > \delta v + u
\]

Hence we have

\[
\alpha \bar{b} + \frac{c'(e)}{p'(e)} < \frac{\delta}{1 - \delta} (v + u - v - u) \leq \frac{\delta g(e_{FB}^{SA}, \bar{b}, \bar{b}, w)}{1 - \delta}
\]

But since \( e \geq e_{FB}^{SA} \), we must have \( \frac{c'(e)}{p'(e)} \geq \alpha y \), which leads us to a contradiction of the initial assumption. This thus completes our proof. \( \square \)

We now have all the tools required to prove the proposition describing the optimal contract between the supervisor and agent. Define \( \hat{e} \) and \( \tilde{e} \) such that

\[
\frac{c'(\hat{e})}{p'(\hat{e})} = \alpha y - (1 - \alpha) \left( \frac{1 - p(\hat{e})}{p'(\hat{e})} \frac{d}{dc} \left[ \frac{c'(e)}{p'(e)} \right]_{e=\hat{e}} \right)
\]

\[
\frac{c'(\tilde{e})}{p'(\tilde{e})} = \alpha y + (1 - \alpha) \tilde{b} - (1 - \alpha) \left( \frac{1 - p(\tilde{e})}{p'(\tilde{e})} \frac{d}{dc} \left[ \frac{c'(e)}{p'(e)} \right]_{e=\tilde{e}} \right)
\]

(9)

where \( \tilde{b} = \min \left\{ \bar{b}, \frac{1}{\alpha} \frac{\delta g(\hat{e}, \bar{b}, \bar{b}, w)}{1 - \delta} \right\} \) and \( \hat{b} = \min \left\{ \bar{b}, \frac{1}{\alpha} \frac{\delta g(\tilde{e}, \hat{b}, \bar{b}, w)}{1 - \delta} \right\} \).

**Proof of Proposition 2.** Lemma 5 tells us that bonuses are maximal when the wage is sufficiently high such that \( \alpha \tilde{b} + \alpha y \geq \frac{\delta g(e_{FB}^{SA}, \bar{b}, \bar{b}, w)}{1 - \delta} \), which proves the proposition for \( w \) above this threshold. We therefore proceed in the rest of the proof to consider the case where \( \alpha \bar{b} + \alpha y < \frac{\delta g(e_{FB}^{SA}, \bar{b}, \bar{b}, w)}{1 - \delta} \).

When \( \alpha \bar{b} + \alpha y < \frac{\delta g(e_{FB}^{SA}, \bar{b}, \bar{b}, w)}{1 - \delta} \), Lemma 5 tells us that both IC constraints must now be binding which gives us equation (4). Furthermore, Lemmas 3 and 4 give us the
following constraints:

\[ s(y) \leq s(0) \]
\[ 0 \leq b(y) \leq \bar{b} \]
\[ 0 \leq b(0) \leq \bar{b} \]

We know from Lemma 8 that at least one of these constraints must be binding. Let us first therefore consider the set of optimal contracts where only one of the constraints is binding. In this case, we essentially have three unknowns \((e, b(y)\) and \(b(0)\)) and two equations (the joint IC constraint in (4) and whichever of the three above is binding). In terms of \(b(y)\), we note that surplus and effort are increasing in \(b(y)\) and hence \(b(y)\) must be at an upper bound, with either \(b(y) = \bar{b}\) or \(b(y)\) determined according to the joint IC.

We therefore maximize with respect to \(e\) and \(b(0)\) under the joint IC constraint (4). This can be expressed in the following Lagrangian:

\[
\mathbb{L}(e, b(0)) = \alpha p(e)y + (1 - \alpha)(w + p(e)b(y) + (1 - p(e))b(0)) - c(e) \\
+ \mu [\alpha p(e)y + (1 - \alpha)(w + p(e)b(y) + (1 - p(e))b(0)) - c(e)] \\
- \nu - u - \frac{1 - \delta}{\delta} \left( \alpha b(y) + \frac{c'(e)}{p'(e)} - b(y) + b(0) \right)
\]

Differentiating the Lagrangian with respect to \(e\) and setting to 0 gives

\[
\mu \frac{1 - \delta}{\delta} \frac{d}{de} \left( \frac{c'(e)}{p'(e)} \right) = (1 + \mu) (\alpha p'(e)y + (1 - \alpha)(p'(e)b(y) - p'(e)b(0)) - c'(e))
\]

Hence

\[
\frac{1 - \delta}{\delta} \frac{d}{de} \left( \frac{c'(e)}{p'(e)} \right) = \frac{\mu}{1 + \mu} \frac{1 - \delta}{\delta} \frac{d}{de} \left( \frac{c'(e)}{p'(e)} \right) \tag{10}
\]

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Differentiating the Lagrangian with respect to $b(0)$ gives us

$$0 = (1 + \mu)(1 - \alpha)(1 - p(e)) - \mu \frac{1 - \delta}{\delta}$$  \hspace{1cm} (11)

Substituting (11) into (10) and rearranging then gives the following possible interior solution:

$$\frac{c'(e)}{p'(e)} = \alpha y + (1 - \alpha)(b(y) - b(0)) - (1 - \alpha) \frac{1 - p(e)}{p'(e)} \frac{d}{de} \left( \frac{c'(e)}{p'(e)} \right)$$  \hspace{1cm} (12)

with $b(0)$ given according to the joint IC, i.e.

$$b(0) = (1 - \alpha)b(y) - \frac{c'(e)}{p'(e)} + \frac{\delta g(e, b(y), b(0), w)}{1 - \delta}$$  \hspace{1cm} (13)

Equation (12) makes clear the trade-offs faced by the players. Increasing effort brings two benefits - high output is more probably, and high bonuses are more probable - with the weighting between these benefits depending on $\alpha$. There is now an additional cost in inducing effort though, corresponding to the last term, which is that it involves a lower bonus when output is low. Hence we can see that this term is higher when low output is more probably - $p(e)$ is lower - and when the supervisor bears less of the cost of bonuses - i.e. $\alpha$ is lower.

In order to complete the proof, we now note that the conditions for this contract type to be optimal are therefore the conditions for this solution to be interior. We therefore consider the various boundary constraints that might be met, and what happens in each case.

From Lemma 4, we require that $b(y) \geq b(0)$. From equation (13), this means we require $\alpha b(y) \leq \frac{\delta g(e, b(y), b(0), w)}{1 - \delta} - \frac{c'(e)}{p'(e)}$. Note that this is not possible if $b(y)$ is bounded by the joint surplus, and hence can only occur when $b(y) = \bar{b}$. This therefore forms a upper bound on $w$ for the interior solution to be feasible. When $w$ is above this level, we must have $b(y) = b(0) = \bar{b}$, and hence we are in the ‘high wage’ case described in
the proposition. From the definition of $e^{SA}_{FB}$, we know that the supervisor and agent would like to induce this effort if possible, so if the relational constraint binds in this zone then effort will be at the largest value possible when $b(y) = b(0) = \tilde{b}$. This is given by the joint IC, and hence effort when wages are high will be as in equation (5). This effort is achieved by setting $s(y) = s(0) - \frac{c'(e)}{p'(e)}$ and $s(0)$ such that $IC_A$ binds, i.e. $s(0) = \frac{\delta}{1-\delta}(u - y)$.

From Lemma 3, we also require that $b(0) \geq 0$. From equation (13), this means we require $\alpha b(y) \geq b(y) + \frac{\delta g(e, b(y), 0, w)}{1-\delta} - \frac{c'(e)}{p'(e)}$. If the wage is below this value, then we will have $b(0) = 0$, rather than the value given by (13). Note that this critical value of $w$ may be above or below the boundary between ‘intermediate’ and ‘low’ wage contracts as classified in the proposition. Substituting equation (12) into this condition gives us the definition of $b(0)$ used in the ‘intermediate’ and ‘low’ wage cases, as described in equation (6).

Finally, it remains to consider the boundary between what is classified in the proposition as an ‘intermediate’ wage and a ‘low’ wage. This is given by the requirement from Lemma 4 that $s(y) \leq s(0)$ and hence $b(y) \leq \frac{\delta g(e, b(y), 0, w)}{1-\delta}$. The value of $w$ such that $\tilde{b} = \frac{\delta g(e, \tilde{b}(0), 0, w)}{1-\delta}$ then gives the boundary between the two types of contract. □

**Proof of Proposition 3.** If $(1-\alpha)\delta(1-p(\tilde{e})) \geq \alpha(1-\delta)$, by Corollary 1 incentives are not provided through bonuses, then we can assume $\tilde{b} = 0$, and hence we have

$$\frac{c'(e)}{p'(e)} = \min \left\{ \alpha y, \frac{\delta g(e, 0, 0, w)}{1-\delta} \right\}$$

Optimization leads to

$$\frac{c'(e)}{p'(e)} = \min \left\{ \alpha y, y - \frac{1-\delta}{\delta p'(e)} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right] \right\}$$

Let us now $(1-\alpha)\delta(1-p(\tilde{e})) < \alpha(1-\delta)$. In this case, we begin by showing that $w$
and \( \bar{b} \) will be set in such a way that the optimal contract is of type 2 or intermediate wage.

Firstly, suppose that the parameters were such that the optimal contract was of type 1. In this case, the principal can adjust \( b \) and \( w \) such that \( b = \alpha y \) and \( \alpha b = \frac{\delta g(e, \bar{b}, 0, w)}{1 - \delta} \). Now \( g(e, \bar{b}, 0, w) = g(e, \bar{b}, \bar{b}, w) - (1 - \alpha)(1 - p(e))\bar{b} \), and hence \( \frac{\delta g(e, \bar{b}, 0, w)}{1 - \delta} = \alpha \bar{b} + \frac{\delta(1 - \alpha)}{1 - \delta}(1 - p(e))\alpha y \). Hence, since \( (1 - \alpha)\delta(1 - p(\bar{e})) < \alpha(1 - \delta) \) and \( e \geq \bar{e} \), the principal will not set parameters such that the optimal contract is of type 1.

Secondly, note that amongst contracts of type 3, effort is decreasing in the supervisor-agent surplus. The principal will therefore reduce surplus (through reducing \( w \)) until we arrive at an equilibrium of type 2.

Let us now consider the best optimal contract of type 2 for the principal. Without loss of generality, we can assume that \( \bar{b} \) is set such that \( \alpha \bar{b} = \frac{\delta g(e, \bar{b}, 0, w)}{1 - \delta} \). The principal then sets \( w \) to maximise their payoff, which is:

\[
\pi = (1 - \alpha)(p(e)(y - \bar{b}) - w)
= p(e)y - c(e) - g(e, \bar{b}, 0, w) + \overline{\nu} + \overline{\nu}
= p(e)y - c(e) - \frac{\alpha(1 - \delta)}{\delta} \frac{c'(e)}{p'(e)} + \overline{\nu} + \overline{\nu}
\]

Differentiating with respect to \( e \) and setting to zero gives

\[
\frac{c'(e)}{p'(e)} = y - \frac{\alpha(1 - \delta)}{\delta} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right]
\]

Note however, for a contract of type 2 to be optimal we require that \( b(0) \leq 0 \), which translates into

\[
\frac{\alpha(1 - \delta)}{\delta} \geq \frac{1 - \alpha}{\alpha} (1 - p(e))
\]
Proof of Proposition 4. The principal wishes to maximise the payoff function

\[ \pi = (1 - \alpha)(p(e)(y - \bar{b}) - w) \]
\[ = p(e)y - c(e) - g(e, \bar{b}, 0, w) + \bar{v} + \bar{u} \]
\[ = p(e)y - c(e) - \alpha \frac{1 - \delta}{\delta} \frac{c'(e)}{p'(e)} + \bar{v} + \bar{u} \]

From the previous proposition, we have that

\[ \frac{c'(e)}{p'(e)} = y - \max \left\{ \frac{\alpha(1 - \delta)}{\delta p'(e)} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right], \frac{(1 - \alpha)(1 - p(e))}{\alpha p'(e)} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right] \right\} \tag{14} \]

and \( \bar{b} = \frac{c'(e)}{p'(e)} = \frac{1}{\alpha} \frac{\delta g(e, \bar{b}, 0, w)}{1 - \delta} \).

Now, the value of \( \alpha \) that maximizes \( e \) is that such that the two terms in the maximum expression are equal. There is now an extra incentive to reduce \( \alpha \) (and effort), and hence \( \alpha \) will be smaller than the value which attains this maximum, i.e.

\[ \frac{\alpha(1 - \delta)}{\delta p'(e)} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right] < \frac{(1 - \alpha)(1 - p(e))}{\alpha p'(e)} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right]. \]

Hence

\[ \frac{c'(e)}{p'(e)} = y - \frac{(1 - \alpha)(1 - p(e))}{\alpha p'(e)} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right] \]

We can therefore consider the Lagrangian:

\[ L = p(e)y - c(e) - \alpha \frac{1 - \delta}{\delta} \frac{c'(e)}{p'(e)} + \bar{v} + \bar{u} \]
\[ + \mu \left( \frac{c'(e)}{p'(e)} - y + \frac{(1 - \alpha)(1 - p(e))}{\alpha p'(e)} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right] \right) \]

Differentiating with respect to \( \alpha \) gives

\[ 0 = -\frac{1 - \delta}{\delta} \frac{c'(e)}{p'(e)} - \mu \left( \frac{1 - p(e)}{\alpha^2 p'(e)} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right] \right) \]
Differentiating with respect to \(e\) gives

\[
0 = p'(e)y - c'(e) - \frac{1 - \delta}{\delta} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right]
\]

\[
+ \mu \left( \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right] + \frac{(1 - \alpha)(1 - p(e))}{\alpha p'(e)} \frac{d^2}{de^2} \left[ \frac{c'(e)}{p'(e)} \right] - \frac{(1 - \alpha)(p'(e)^2 + (1 - p(e))p''(e))}{\alpha p'(e)^2} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right] \right)
\]

\[
= p'(e)y - c'(e) - \frac{1 - \delta}{\delta} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right]
\]

\[
+ \mu \left( \frac{(1 - \alpha)(1 - p(e))}{\alpha p'(e)} \frac{d^2}{de^2} \left[ \frac{c'(e)}{p'(e)} \right] - \frac{p'(e)^2/(1 - p(e)) + (1 - \alpha)p''(e)}{p'(e)^2} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right] \right)
\]

Hence

\[
\frac{c'(e)}{p'(e)} = y - \alpha \frac{1 - \delta}{\delta p'(e)} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right]
\]

\[
+ \mu \left( \frac{1 - p(e)}{p'(e)} \frac{d^2}{de^2} \left[ \frac{c'(e)}{p'(e)} \right] - \frac{p'(e)^2/(1 - p(e)) + (1 - \alpha)p''(e)}{p'(e)^2} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right] \right)
\]

\[
y = \alpha \frac{1 - \delta}{\delta p'(e)} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right]
\]

\[
- \alpha \left( \frac{1 - \delta}{\delta} \frac{c'(e)}{p'(e)} + \nu + \bar{\nu} \right) \left( \frac{1 - \alpha}{p'(e)} \frac{d^2}{de^2} \left[ \frac{c'(e)}{p'(e)} \right] - \frac{p'(e)^2/(1 - p(e)) + (1 - \alpha)p''(e)}{p'(e)^2} \frac{d}{de} \left[ \frac{c'(e)}{p'(e)} \right] \right)
\]

\[
\square
\]

**Proof of Corollary 1.** If \((1 - \alpha)\delta(1 - p(\hat{e})) \geq \alpha(1 - \delta)\), then \(\frac{\delta g(e,b,\delta,w)}{1 - \delta} \leq \frac{\delta g(e,b,\epsilon,w)}{1 - \delta} - \alpha \epsilon\) for all \(e \leq \hat{e}\). Therefore, for any contract, with \(b(0) < b(y)\), we can consider a new contract with \(b'(0) = b(0) + \epsilon\) and \(s'(0) = s(0) + \gamma \epsilon\) and \(s'(y) = s(y) - (1 - \gamma) \epsilon\). For some \(\gamma\), this contract will also be self-enforcing as the surplus has increased by at least as large amount as the joint IC constraints. Since surplus is higher, the original contract cannot be optimal, and hence all optimal contracts must have \(b(0) = b(y)\). \(\square\)