Capital Account Intervention and Currency Wars*

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Abstract

Capital account interventions generate international spillover effects that have recently raised concerns about global currency wars. This paper analyzes the welfare effects and the desirability of global coordination of such policy measures. We find that if controls are designed to correct for domestic externalities, the resulting equilibrium is Pareto efficient and there is no role for global coordination, i.e. a global planner would impose the same measures. We illustrate this for a range of externalities that have recently been invoked as reasons for imposing capital controls: aggregate demand externalities in a liquidity trap, learning externalities, and pecuniary externalities arising from financial constraints. On the other hand, if controls are designed to manipulate a country’s terms-of-trade or if policymakers face an imperfect set of instruments, such as targeting problems or costly enforcement, then multilateral coordination is desirable in order to mitigate the inefficiencies arising from such imperfections.

JEL Codes: F34, F41, H23
Keywords: capital flows, capital controls, currency wars, externalities, global policy coordination

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1 Introduction

Capital account interventions generate international spillover effects that have led to considerable controversy in international policy circles in recent years (see e.g. Ostry et al, 2012; Stiglitz, 2012) and have raised concerns about global currency wars. This paper determines the welfare effects of such measures in a general equilibrium model of the world economy and analyzes under what conditions global coordination of capital account policies is desirable.

We describe the spillover effects from capital account intervention in an intertemporal benchmark model of a global economy in which individual countries engage in borrowing and lending. If one country imposes capital controls in the form of taxes on foreign borrowing, it reduces both borrowing and consumption and pushes down the world interest rate, leading to greater inflows to other countries. In an augmented model, it also depreciates its real exchange rate and appreciates the real exchange rate of other countries. Furthermore, we show an isomorphism between capital controls and reserve accumulation: any level of capital controls can be replicated by a corresponding level of reserve accumulation when the capital account is closed to private transactions.

Next we study several types of externalities that have recently been invoked as reasons why individual countries may want to impose capital controls: learning externalities, aggregate demand externalities in a liquidity trap, and pecuniary externalities arising from external financial constraints. For each of these domestic distortions, a national planner can improve domestic welfare by imposing capital controls that offset the externality, even though such controls create international spillover effects.

The resulting global equilibrium is Pareto efficient as long as national planners behave competitively and impose capital controls that offset domestic externalities. A global planner who internalizes all international spillover effects cannot improve on the described allocation. By the same token, if national planners refrain from imposing capital controls to correct for domestic externalities, global welfare is reduced. Conceptually, we can view the national planners that internalize domestic externalities in different countries as competitive agents to which the welfare theorems apply. Changes in the world interest rate that stem from capital controls constitute purely pecuniary externalities that cancel out and do not impede Pareto efficiency. We also find that a seeming “arms race” of escalating capital controls does not necessarily indicate inefficiency but may be the tatonnement process through which multiple countries optimally adjust their capital controls.

On the other hand, capital controls to manipulate a country’s terms-of-trade constitute a beggar-thy-neighbor policy and are Pareto inefficient. A national planner in a large country may face incentives to exert market power over the country’s intertemporal terms of trade, i.e. the world interest rate. For example, if a large lending country is worried about the low return it earns on its assets and restricts its lending, it benefits from an increase in the world interest rate. Such monopolistic behavior re-
duces the global gains from intertemporal trade and is Pareto-inefficient. If countries engage in monopolistic behavior, it is desirable to come to a global agreement that interventions aimed at manipulating the world interest rate will not be used.

The lesson for international policy coordination is that it is important to distinguish between ‘corrective’ capital controls that are imposed to offset domestic externalities and ‘distortive’ capital controls that are designed to manipulate a country’s terms of trade. The former are generally desirable, whereas the latter are always undesirable.

An additional motive for coordinating capital control policies arises when policymakers face restrictions on the set of available policy instruments. For example, if capital controls not only correct distorted incentives to borrow/lend but also impose an additional cost arising from costly implementation or corruption, then there is scope for global coordination of capital account policies: a global planner recognizes that adjusting all capital controls worldwide by the same factor may reduce the distortions created by capital controls but would leave the marginal incentives of all actors in the world economy unaffected.

**Literature** There is a growing recent literature that finds that capital controls may improve welfare from the perspective of a single country if they are designed to correct domestic externalities. An important example are prudential capital controls that reduce the risk of financial crises, as analyzed in the small open economy literature by Korinek (2007, 2010, 2011b), Ostry et al. (2010, 2011) and Bianchi (2011). This paper provides a normative analysis of the resulting general equilibrium effects and discusses whether global coordination of such policies is desirable.\(^1\) We find that in a benchmark case in which national regulators can optimally control domestic externalities, coordination is not indicated. By contrast, Bengui (2011) studies the role for coordination between national regulators in a multi-country framework of banking regulation. He shows that liquidity in the global interbank market is a global public good. In the presence of such global externalities, there exists a case for global coordination of liquidity requirements.

Earlier work by MacDougall (1960), Kemp (1962), Hamada (1966), Jones (1967) and Obstfeld and Rogoff (1996) investigated how a national planner of a large country in the world economy may impose capital controls to exert monopoly/monopsony power over intertemporal prices. As in optimal tariff theory, such policies are beggar-thy-neighbor, i.e. they improve national welfare at the expense of reducing overall global welfare. In a recent contribution to this literature, Costinot et al. (2011) analyze the optimal time path of monopolistic capital controls under commitment and show how they can be used to distort relative prices in goods markets. Our paper contrasts the global welfare effects of distortive (monopolistic) capital controls with corrective capital controls that are designed to offset domestic externalities, as

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\(^1\) Ostry et al. (2012) discusses the multilateral aspects of policies to manage the capital account from a policy perspective.
was invoked by a rising number of countries that have imposed such controls in recent years. Jeanne et al. (2012), Gallagher et al. (2012) and Ostry et al. (2012) discuss the multilateral implications of capital controls from a policy perspective without providing a formal welfare analysis.

Persson and Tabellini (1995) show that coordination of national fiscal and/or monetary policies is desirable if countries have incentives to employ such policies to exert monopoly power over international prices. Korinek (2011a) analyzes the positive implications of prudential capital controls in a multi-country setting.

The link between reserve accumulation and real exchange rate valuation is also investigated in Rodrik (2008) and Korinek and Serven (2010). Ghosh and Kim (2009) and Jeanne (2012) show how a combination of capital controls and tax measures can be used to undervalue a country’s real exchange rate. These papers look at the exchange rate effects of various capital account policies in a small open economy, whereas we focus explicitly on global general equilibrium effects.

Magud et al. (2011) provide a survey of the empirical literature on the effects of capital controls on the country imposing the controls. Forbes et al. (2011) and Lambert et al. (2011) investigate the spillover effects of capital controls empirically. They find evidence that when Brazil imposed capital controls, there was diversion of capital flows to other countries that were expected to maintain free capital flows. To the extent that the capital controls imposed by Brazil were imposed to correct a domestic distortion, our analysis suggests that this was a Pareto-efficient equilibrium response and did not introduce distortions in the global allocation of capital.

2 Baseline Intertemporal Model

We describe a world economy with $N \geq 2$ countries indexed by $i = 1, \ldots, N$ and a single homogenous tradable consumption good. Time is indexed by $t = 0, \ldots, T$. The mass of each country $i$ in the world economy is $\omega^i \in [0, 1]$, where $\sum_{i=1}^{N} \omega^i = 1$. (A country with $\omega^i = 0$ corresponds to a small open economy.)

2.1 Country Setup

Country $i$ is inhabited by a unit mass of identical consumers indexed by $z \in [0, 1]$ who value the consumption $c_{i,z}^t$ of a tradable good according to the utility function

$$\sum_{t=0}^{\infty} \beta^t u(c_{i,z}^t)$$

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Forbes et al. (2011) also document negative spillover effects on countries that were likely to follow the example of Brazil to impose controls.
where \( u(\cdot) \) is a standard neoclassical period utility function and \( \beta < 1 \) is a time discount factor, which we assume constant across countries.\(^3\) For simplicity we drop the index \( z \) of individual consumers from our notation.

A representative consumer in country \( i \) starts period \( t \) with an endowment of \( y^i_t \) of tradable goods and financial net worth \( b^i_t \), where the initial financial assets \( b^i_0 \) in period \( 0 \) are given. He chooses how much to consume and how much to save by purchasing \( b^i_{t+1} \) zero coupon bonds at a price \( 1/R^t_{t+1} \) that pay off one unit of tradable good in period \( t + 1 \), where \( R^t_{t+1} \) represents the gross world interest rate between periods \( t \) and \( t + 1 \). The budget constraint of the representative consumer in period \( t \) captures that consumption and net bond purchases need to be financed by output and transfers \( T^i_t \),

\[
c^i_t + (1 - \tau^i_{t+1}) \frac{b^i_{t+1}}{R^t_{t+1}} = y^i_t + b^i_t + T^i_t \tag{2}
\]

where the variable \( \tau^i_{t+1} \) is a proportional subsidy to bond purchases \( b^i_{t+1}/R^t_{t+1} \). We assume that the required revenue is raised as a lump-sum tax \( T^i_t = -\tau^i_{t+1} b^i_{t+1}/R^t_{t+1} \)

so as to make the measure wealth-neutral.

| \( \frac{b^i_{t+1}}{R^t_{t+1}} > b^i_t \) (net saver) | \( \tau^i_{t+1} > 0 \) | outflow subsidy | outflow tax |
| \( \frac{b^i_{t+1}}{R^t_{t+1}} < b^i_t \) (net borrower) | \( \tau^i_{t+1} < 0 \) | inflow tax | inflow subsidy |

Table 1: Interpretation of capital control \( \tau^i_{t+1} \)

Depending on the signs of \( \frac{b^i_{t+1}}{R^t_{t+1}} - b^i_t \) and \( \tau^i_{t+1} \), we can interpret the policy measure \( \tau^i_{t+1} \) in a number of different ways, as captured by Table 1: If the country is a net saver, \( \frac{b^i_{t+1}}{R^t_{t+1}} > b^i_t \), then \( \tau^i_{t+1} > 0 \) constitutes a subsidy to saving, i.e. a subsidy to capital outflows and \( \tau^i_{t+1} < 0 \) constitutes a tax on outflows. If the country dis-saves, \( \frac{b^i_{t+1}}{R^t_{t+1}} < b^i_t \), then a policy measure \( \tau^i_{t+1} > 0 \) can be interpreted as a tax on foreign borrowing, or a tax on capital inflows. Conversely, \( \tau^i_{t+1} < 0 \) constitutes a subsidy to foreign borrowing. To ensure that bond demand is bounded, we impose the assumption that \( \tau^i_{t+1} < 1 \) \( \forall i, t \). In the following, we will loosely refer to \( \tau^i_{t+1} \) as the “capital control” imposed in period \( t \). In the current section, we analyze the behavior of private agents for given capital controls. The ensuing sections will analyze why policymakers may want to impose capital controls.

Since there is a single representative consumer and a single homogenous good every period, the consumer’s borrowing/saving decisions map directly into the trade statistics of the economy. Substituting for the transfer \( T^i_t \), the budget constraint of

\(^3\)In our baseline model, we motivate international borrowing and saving by differences in endowments or output and intertemporal consumption smoothing considerations. Temporary differences in discount factors would offer an alternative route.
the economy implies a resource constraint,

\[ c^i_t - y^i_t = m^i_t = b^i_t - \frac{b^i_{t+1}}{R_{t+1}} \]

where we denote the difference between consumption and output as the agent's net imports \( m^i_t \), which are financed by decumulating savings \( b^i_t - \frac{b^i_{t+1}}{R_{t+1}} \). If the country decumulates savings \( b^i_t > b^i_{t+1}/R_{t+1} \) then it has positive net imports \( m^i_t > 0 \) and a positive capital control \( \tau^i_{t+1} > 0 \) can be interpreted as a net import tariff whereas \( \tau^i_{t+1} < 0 \) corresponds to a net import subsidy. By the same token, if the country accumulates savings \( b^i_{t+1}/R_{t+1} > b^i_t \) then it is a net exporter \( m^i_t < 0 \), and a positive capital control \( \tau^i_{t+1} > 0 \) can be interpreted as a subsidy to net exports, whereas a negative \( \tau^i_{t+1} < 0 \) constitutes an export tax. For future use, we note that the vector of net capital imports \( \{m^i_t\} \) is a sufficient statistic for the interactions of country \( i \) with the rest of the world.

To relate our trade statistics to the current account, observe that \( b^i_t \) represents the gross return on savings that the consumer receives at the beginning of period \( t \). The fraction \( b^i_t / R_t \) captures how much the consumer saved in period \( t \) in order to receive \( b^i_t \) units of goods in period \( t \). Therefore the interest earnings in period \( t \) are \( b^i_t (1 - 1/R_t) \). The trade balance is the negative of net imports \( tb^i_t = -m^i_t \). The current account balance \( ca^i_t \) is the sum of the trade balance and interest earnings, \( ca^i_t = tb^i_t + b^i_t (1 - 1/R_t) = b^i_{t+1}/R_{t+1} - b^i_t/R_t \), and corresponds to the increase in the net asset position of the country in period \( t \). Observe that a balanced current account requires that a country's net imports equal its interest earnings, \( m^i_t = b^i_t (1 - 1/R_t) \).

**Optimization Problem** A representative consumer takes the series of \( R_{t+1}, T^i_t \) and \( \tau^i_{t+1} \) as given and maximizes consumer utility (1) subject to the series of budget constraints (2). The resulting Euler equation is

\[ (1 - \tau^i_{t+1}) u'(c^i_t) = \beta R_{t+1} u'(c^i_{t+1}) \]

For given \( b_t \), the Euler equation implies a bond supply function \( b^i_{t+1} \left( R_{t+1}; \tau^i_{t+1} \right) \) that is strictly increasing in the capital control \( \tau^i_{t+1} \). Strictly speaking, bond supply \( b^i_{t+1} \) is an equilibrium object that depends on the entire path of future interest rates and capital controls, but it is useful to focus in particular on its dependence on \( \left( R_{t+1}, \tau^i_{t+1} \right) \). We impose the following assumption:

**Assumption 1 (Elasticity of Intertemporal Substitution)** The elasticity of intertemporal substitution is greater than the borrowing/consumption ratio of country \( i \),

\[ \sigma(c^i_t) > -\frac{b^i_{t+1}/R_{t+1}}{c^i_t} \]
This common assumption guarantees that for given $b_t$, bond supply $b_{i+1}^t(R_{i+1}^t; \tau_{i+1}^t)$ is strictly increasing in $R_{i+1}^t$ and can be inverted into an indirect bond supply function $R_{i+1}^t(b_{i+1}^t; \tau_{i+1}^t)$. The assumption is satisfied for all countries that are net savers and for net borrowers as long as their borrowing is not too large in comparison to consumption. See Appendix A.1 for a detailed derivation.

For given $b_t$, the effect of an increase in the world interest rate on saving $b_{i+1}^t/R_{i+1}^t$ (as opposed to bond holdings $b_{i+1}^t$) depends on two terms:

$$
\frac{\partial (b_{i+1}^t/R_{i+1}^t)}{\partial R_{i+1}^t} = \frac{\partial b_{i+1}^t/\partial R_{i+1}^t}{R_{i+1}^t} - \frac{b_{i+1}^t}{(R_{i+1}^t)^2} \left( \eta_b R_{i+1}^t - 1 \right)
$$

The first term in the expression in the middle captures the substitution effect – a higher interest rate makes it more desirable to save, as we assumed. The second term captures the income effect. For net borrowers, both terms are positive. For large savers, the income effect may offset the substitution effect and may lead to smaller net savings $b_{i+1}^t/R_{i+1}^t$ in response to an increase in the world interest rate.

For net borrowers and modest net savers, a rise in the world interest rate is associated with a decline in consumption, which is necessary so net savings can rise, $\partial c_i^t/\partial R_{i+1}^t < 0$. For large savers, the inequality may be reversed.

## 2.2 Equilibrium

In the following, we use the following naming conventions: we denote aggregate variables in a given economy by upper-case letters. For example, we denote the bond holdings of an individual (representative) agent by $b_i^t$ but aggregate bond holdings of country $i$ by $B_i^t$. We know that $b_i^t = B_i^t$ in equilibrium since all agents within a country are identical, but the distinction will matter below when we consider externalities from capital flows. Furthermore, we use upper-case variables without a country-specific superscript to denote world-wide aggregates, for example $B_t = \sum_{i=1}^N \omega_i B_i^t$ for world-wide bond supply. Finally, we denote rest-of-the-world aggregate variables by the superscript $-i$, for example $B_t^{-i} = \sum_{j \neq i} \omega_j b_j^t$ and similarly for all other variables.

### Definition 1 (Competitive Equilibrium)

For given initial bond holdings $\{B_i^0\}_i$ and a sequence of capital controls $\{\tau_{i+1}^t\}_{i,t}$, a competitive equilibrium of the world economy is described by consumption allocations $\{C_i^t\}_{i,t}$ and bond holdings $\{B_i^t\}_{i,t}$ as well as interest rates $\{R_t\}_t$ such that private consumers in each country $i$ solve their optimization problem (1) subject to their budget constraint (2) and the global bond market clears,

$$
B_{t+1} := \sum_{i=1}^N \omega_i B_{i+1}^t = 0 \quad \forall t
$$

(4)
where $\tau_{t+1} = \{\tau_{t+1}^i\}_i$ is the vector of capital controls across countries and $B_{t+1}$ is the global excess supply of bonds in period $t$, which is by Assumption 1 strictly increasing in $R_{t+1}$.4

2.3 Effects of Capital Controls

Let us now focus on the effects of changes in capital controls on the equilibrium of the world economy. We perform a comparative static exercise in which we assume that the national planner in country $i$ increases her capital control by $d\tau_{t+1}^i > 0$.

Lemma 1 (Effects of Capital Controls) For given $\{B_t^i\}_i$, an increase in the capital control $d\tau_{t+1}^i > 0$ in country $i$

1. increases bond holdings $B_{t+1}^i$ and saving $B_{t+1}^i/R_{t+1}$ and reduces consumption $C_t^i$ in country $i$ for a given world interest rate $R_{t+1}$,

2. if $\omega^i > 0$, it reduces the world interest rate $R_{t+1}$ and reduces bond holdings $B_{t+1}^{-i}$ and saving $B_{t+1}^{-i}/R_{t+1}$ while increasing consumption $C_{t+1}^{-i}$ in the rest of the world.

3. The decline in the world interest rate benefits all borrowing countries and hurts all saving countries.

Proof. Point 1 follows from implicitly differentiating the Euler equation of the consumer in appendix A.1 to express $\partial B_{t+1}^i/\partial \tau_{t+1}^i > 0$. We divide by $R_{t+1}$ and apply the period $t$ budget constraint to obtain the statements about saving and consumption.

For point 2, we apply the implicit function theorem to the global market clearing condition $\sum_{i=1}^N \omega^i B_{t+1}^i (R_{t+1}; \tau_{t+1}^i) = 0$ to obtain

$$\frac{dR_{t+1}}{d\tau_{t+1}^i} = -\frac{\omega^i B_t^i}{B_R} < 0 \quad (5)$$

where the partial derivatives satisfy $B_R = \sum_i \omega^i B_t^i > 0$ and $B_\tau = \omega^i B_t^i > 0$. The decline in rest-of-the-world bond holdings $B_{t+1}^{-i}$ and saving $B_{t+1}^{-i}/R_{t+1}$ and the increase in rest-of-the-world consumption $C_{t+1}^{-i}$ follow from market clearing.

Point 3 is obtained by taking the derivative of the welfare function of country $j$ as defined by the representative agent’s utility (1)

$$\frac{dU^j}{dR_{t+1}} = \beta' u' (C_t^j) \frac{B_{t+1}^j}{(R_{t+1})^2} \geq 0 \text{ depending on } B_{t+1}^j \geq 0$$

4This is our analogon of the Marshall-Lerner condition that an increase in the world interest rate increases the global excess demand for bonds.
Intuitively, capital controls introduce a wedge into the Euler equation of consumers that raises desired bond holdings while reducing consumption today. This shifts the global supply of bonds $B_{t+1} (R_{t+1}; \tau_{t+1})$ outwards. For the global bond market to clear, a decline in the world interest rate is required, which makes the rest of the world supply fewer bonds (i.e. save less) and consume more. The decline in the interest rate benefits borrowers because they obtain credit at lower rates and hurts lenders because they earn less in interest.

[⁎] Figure 1 illustrates our findings graphically for a world with two countries $i, j$ of equal mass. $R^{j} (b^{j})$ depicts the inverse bond supply of country $j$, $R^{i} (-b^{i})$ represents the inverse bond demand in country $i$ in the absence of capital controls. The intersection of the two, marked by $R^{LF}$ and $b^{LF}$, indicates the laissez faire equilibrium of the economy. However, suppose that there is a negative externality associated with borrowing by country $i$. Then a competitive national planner would demand less borrowing, as indicated by $R^{i*} (-b^{i})$, and impose a capital control $\tau^{i*}$ on borrowing to make private agents internalize the externality. The resulting equilibrium exhibits less borrowing/lending $b^{NP}$ and a lower world interest rate $R^{NP}$. Country $j$ looses the surplus that is marked by the shaded area in the figure.

2.4 Numerical Illustration

To illustrate the effects of changes in capital controls numerically, assume a world economy in which all agents have a CES period utility function $u (c) = c^{1-1/\sigma} / (1 - 1/\sigma)$, an identical discount factor $\beta$, no initial net wealth so $B^{i}_0 = 0 \forall i$ and period income
that is constant over time but may differ across countries. In the absence of intervention, i.e. if \( \tau_{i,t+1} = 0 \forall i,t \), all agents will consume their income every period so \( C^i_t = Y^i \forall i,t \) and \( \beta R = 1 \). We call this the no-intervention steady-state.

Now we assume that an economy \( i \) of mass \( \omega^i \) increases its capital control by \( d\tau^i_{t+1} > 0 \) and compare how the allocations change in comparison to the no-intervention steady-state. The two partial derivatives of the saving/output ratio \( B^i_{t+1}/Y^i \) with respect to the capital control and the interest rate are

\[
\frac{\partial B^i_{t+1}/Y^i}{\partial \tau^i_{t+1}} = B^i_\tau/Y^i = \sigma \\
\frac{\partial B^i_{t+1}/Y^i}{\partial R_{t+1}} = B^i_R/Y^i = \beta \sigma
\]

An increase in the capital control or an increase in the world interest rate both increase the net savings of the country by the intertemporal elasticity of substitution. (The second expression is pre-multiplied by \( \beta \) because interest is compounded in period \( t+1 \) whereas the capital control is imposed in period \( t \).) For the standard value of the elasticity of substitution \( \sigma = 1/2 \), both an increase in the capital control or the interest rate result in an increase in domestic savings by approximately half a percent of GDP.\(^5\)

Global bond supply as a fraction of world output \( B_{t+1}/Y \) satisfies

\[
\frac{\partial B_{t+1}/Y}{\partial R_{t+1}} = B^i_R/Y = \beta \sigma
\]

We combine this with the expression \( B^i_\tau/y^i = \sigma \) in equation (5) to find that the effect of capital controls in country \( i \) on the world interest rate is

\[
\frac{dR_{t+1}/R}{d\tau^i_{t+1}} = -\frac{B^i_\tau/R}{B^i_R} = -\omega^i
\]

In short, if a country that has a relative share \( \omega^i \) of world GDP imposes a 1\% capital control, the world interest rate will decline by \( \omega^i \% \). Observe that this expression is independent of the intertemporal elasticity of substitution (as long as it is constant across countries).

Accounting for the adjustment in the world interest rate, the general equilibrium effect of a capital control in country \( i \) is reduced to a fraction \((1 - \omega^i)\) of the partial equilibrium effect,

\[
\frac{dB^i_{t+1}/Y^i}{d\tau^i_{t+1}} = B^i_\tau/Y^i + B^i_R/y^i \cdot \frac{dR_{t+1}}{d\tau^i_{t+1}} = (1 - \omega^i) \sigma.
\]

\(^5\)We note that there is considerable disagreement among economists about the value of the intertemporal elasticity of substitution. See e.g. Bansal and Yaron (2004) for a discussion. The formulas we derived deliver transparent results for any value of the intertemporal elasticity of substitution preferred by the reader.
<table>
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<th>Country</th>
<th>$GDP^i$</th>
<th>$\Delta b^i , / , R$</th>
<th>$\Delta R , / , R$</th>
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<td>$\cdots$</td>
<td>$-1%$</td>
</tr>
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</tbody>
</table>

Table 2: Effects of 1% capital control on saving and the world interest rate
(Source: IMF 2011 IFS data and author’s calculations)

Taken together, the previous two equations illustrate that a fraction \((1 - \omega^i)\) of the adjustment to increased capital controls occurs via a change in the quantity of a country’s capital flows \(B^i_{t+1}/Y^i\) and the remaining fraction \(\omega^i\) of the adjustment occurs via a change in the world interest rate \(R_{t+1}\). (For example, for a small open economy with \(\omega^i = 0\), all the adjustment takes place via the quantity of flows \(B^i_{t+1}\); in a world in which there is a single country that makes up \(\omega^i = 100\%\) of the world economy, a change in the capital control would lead to an equiproportional change in the world interest rate \(R_{t+1}\) but would leave capital flows \(B^i_{t+1}\) unaffected – capital cannot flow anywhere else.)

In Table 2, we illustrate the effects of capital controls on bond holdings and the world interest rate for a number of countries that were important players in global capital markets and/or currency wars in recent years. For example, Brazil represents 3.6% of the world economy. If the country increases a 1% capital control, it will reduce capital inflows by $12.2bn, which in turn lowers the world interest rate by 0.036% according to our calibration.

### 2.5 Reserve Accumulation

We extend our framework to study reserve accumulation. Assume a planner in country \(i\) accumulates bond holdings \(a_t\) on behalf of domestic consumers, where any net accumulation/decumulation \(a_{t+1}/R_{t+1} - a_t\) is financed/rebated via lump-sum transfers. We may think of these bond holdings as reserves. This changes the period \(t\) budget constraint of consumers to

\[
c^i_{t,t} + a^i_{t+1} + \frac{(1 - \tau^i_{t+1}) b^i_{t+1}}{R_{t+1}} = y^i_{t,t} + a^i_t + b^i_t + T^i_t
\]

In the following, we distinguish two diametrically opposed cases. We describe the capital account in an economy \(i\) as open when domestic consumers can trade international bonds \(b^i_{t+1}\), as we have assumed so far. By contrast, we call the capital account
closed when domestic consumers are forbidden from borrowing or saving abroad. This imposes the constraint $b_{t+1}^i = 0 \forall t$.

**Proposition 1 (Reserve Accumulation)**  
(i) Under open capital accounts, domestic consumers undo any reserve holdings $a_{t+1}^i$ by adjusting their private bond holdings such that $b_{t+1}^i = \tilde{b}_{t+1}^i - a_{t+1}^i$, where $\tilde{b}_{t+1}^i$ corresponds to the optimal choice of consumers in the absence of reserves.

(ii) Under closed capital accounts, reserve accumulation cannot be undone and reduces domestic consumption one-for-one: $\partial c_{t, t}^i / \partial (a_{t+1}^i / R_{t+1}) = -1$. If the mass of the country is positive, it also reduces the world interest rate $\partial R_{t+1} / \partial a_{t+1}^i < 0$.

(iii) There is a one-to-one correspondence between a given level of capital controls $\tau_{t+1}^i$ under open capital accounts and a given amount of reserve accumulation $a_{t+1}^i$ under closed capital accounts.

**Proof.** For part (i), assume an equilibrium with zero reserves $a_{t+1}^i = 0 \forall t$ and denote the associated level of private bond holdings by $\tilde{b}_{t+1}^i$. If a planner accumulates a non-zero level of reserves $a_{t+1}^i \neq 0$ in some periods, then an allocation in which private bond holdings satisfy $b_{t+1}^i = \tilde{b}_{t+1}^i - a_{t+1}^i$ will leave all other variables unchanged and will therefore satisfy the optimality conditions of the consumer.

If consumers have unconstrained access to capital markets, then reserve accumulation is ineffective, even if the planner has imposed price controls $\tau_{t+1}^i$ on international capital flows. What matters for the real allocations of the consumer is solely the level of capital controls $\tau_{t+1}^i$, not the level of reserves $a_{t+1}^i$. This is a form of Ricardian equivalence – a representative consumer internalizes that government bond holdings are equivalent to private bond holdings.\(^6\)

In part (ii), under closed capital accounts, private agents are restricted to a zero international bond position $b_{t+1}^i = 0$ and international capital flows are solely determined by reserve accumulation. Reserve accumulation/decumulation constitutes forced saving/dissaving. The effects of reserve accumulation therefore mirror the effects of private capital flows in Proposition 1.

To show point (iii), we observe that a capital control $\tau_{t+1}^i$ under open capital accounts leads private consumers to accumulate $b_{t+1}^i (R_{t+1}; \tau_{t+1}^i)$ bonds and is therefore equivalent to reserve accumulation $a_{t+1}^i = b_{t+1}^i (R_{t+1}; \tau_{t+1}^i)$ under closed capital accounts. Since bond holdings $b_{t+1}^i (R_{t+1}; \tau_{t+1}^i)$ are strictly decreasing in $\tau_{t+1}^i$ and their range is $\mathbb{R}$, any level of reserve accumulation can be replicated by a commensurate capital control $\tau_{t+1}^i$. \(\blacksquare\)

**Numerical Illustration**  
We continue our numerical illustration to investigate the isomorphism between reserve accumulation and capital controls. Consider a small

\(^6\)The result is therefore subject to the same limitations as Ricardian equivalence. In particular, it critically relies on the assumption that consumers can access bond markets at the same conditions as governments.
economy that is in steady state. An increase in reserve accumulation as a fraction of GDP $a^i/y^i$ if the economy’ s capital account is closed is equivalent to an increase in capital controls if the economy’ s capital account is open of

$$\frac{d\tau^i}{da^i/y^i} = \frac{1}{\sigma}$$

For the standard value of the intertemporal elasticity of substitution $\sigma = 1/2$, this term is approximately $\partial\tau^i/\partial(a^i/y^i) \approx 2$. In short, accumulating an extra percent of GDP in reserves under closed capital accounts is equivalent to imposing a 2% capital control under open capital accounts or, vice versa, a 1% capital control is equivalent to accumulating half a percent of GDP in reserves.

For numerical results, we refer back to Table 2, in which we illustrated how much a 1% capital control improves the current account. Given the isomorphism, we can read the table in both directions. If, for example, China accumulates an extra $26bn$ in foreign reserves and its capital account is closed, this is equivalent to a 1% capital control under an open capital account. Similarly, if Brazil accumulates an extra $10bn$ in foreign reserves under closed capital accounts, it is equivalent to a 1% capital control under fully open capital accounts. In practice, many developing countries that have liberalized their capital accounts exhibit intermediate values of capital account openness and various other financial frictions so that only part of their reserve accumulation is undone by private agents.

3 General Model

We will next introduce a more general description of the problem in Arrow-Debreu formulation, which proves to be useful both to characterize the efficiency properties of equilibrium and to generalize our results to a broad class of open economy macro models. Instead of following the external bond position $b^i_{t+1}$ of a representative agent in country $i$ over time, we describe the intertemporal trade of the agent by the vector of net imports $m^i = \{m^i_0, m^i_1, \ldots\}$ which are traded in the world market at prices denoted by a vector $Q = \{Q_0, Q_1, \ldots\}$. This formulation nests our baseline model if $m^i_t$ is scalar and captures the net decumulation of bonds $m^i_t = b^i_t - b^i_{t+1}/R_{t+1}$ every period and if we denote the intertemporal goods prices by the numeraire $Q_0 = 1$ and $Q_{t+1} = Q_t/R_{t+1}$. However, our general formulation also encompasses the case that $m^i_t$ itself consists of a vector of net imports of different goods or of goods in different states of nature.

3.1 General Setup

We describe the utility of a representative agent in country $i$ by a general function

$$U^i(x^i)$$

(7)
where \( x^i \) denotes a collection of domestic variables that may include, for example, a vector of consumption of goods or leisure. We assume that the representative agent is subject to a collection of constraints

\[
f^i (m^i, M^i, x^i, X^i, \zeta^i, Z^i) \leq 0
\]

These constraints may include domestic budget constraints, financial constraints or incentive/selection constraints as well as the restrictions imposed by domestic policy measures. Observe that we include both individual variables \((m^i, x^i)\) and aggregate variables \((M^i, X^i)\) in the constraint in order to capture the potential for externalities from aggregate behavior that are not internalized by individual agents. (We will provide examples of such externalities in the following section.) In equilibrium, the behavior of all individual agents is symmetric so \( m^i = M^i \) and \( x^i = X^i \) will hold. However, individual agents do not internalize this consistency requirement in their optimization problem.

The difference between the variables \((m^i, M^i)\) and \((x^i, X^i)\) is that the former capture the transactions of the representative domestic agent in country \( i \) with the rest of the world. Given \((m^i, M^i)\), the variables \((x^i, X^i)\) only affect the domestic agent. Furthermore, \( Z^i \) represents a collection of exogenous state variables, for example endowments, productivity shocks or initial parameters and \( \zeta^i \) captures domestic policy instruments, which are choice variable of policymakers in country \( i \) but taken as given by the representative agent. They may include taxes/subsidies or constraints on domestic transactions.

Assuming an initial external net worth of \( b_0^i \) and a vector of tax instruments \( \xi^i \) on international transactions \( m^i \), we denote the external budget constraint of a representative agent in economy \( i \) by

\[
\frac{Q}{1 - \xi^i} \cdot m^i - T^i \leq b_0^i
\]

where the division of the price vector \( Q \) by the tax vector \((1 - \xi^i)\) is element-by-element, \( \frac{Q}{1 - \xi^i} \cdot m^i \) is the inner product of the two vectors \( \frac{Q}{1 - \xi^i} \) and \( m^i \), and the fiscal revenue is raised/rebated as a lump sum transfer \( T^i = \frac{\xi^i Q}{1 - \xi^i} \cdot M^i \). If the planner does not intervene in external transactions, then the budget constraint reduces to \( Q \cdot m^i \leq b_0^i \). Furthermore, the planner recognizes that the external budget constraint of the economy is \( Q \cdot M^i \leq B_0^i \).

**Example 1 (Baseline Model)** Our general setup nests the baseline model from section 2 as follows: We collect the consumption process in \( x^i = \{c^i_0, c^i_1, \ldots\} \) and the exogenous endowment process in \( Z^i = \{y^i_0, y^i_1, \ldots\} \). The utility function takes the standard form \( U^i = \sum_{t=0}^{\infty} \beta^t u(c^i_t) \) and the constraint function \( f^i (\cdot) \) simply captures the series of period budget constraints \( f^i_t (\cdot) = c^i_t - y^i_t - m^i_t \leq 0 \forall t \). In our baseline model there are no domestic policy instruments so \( \zeta^i = \emptyset \). The capital flow taxes in
our baseline model map into the general model if we recursively define $\xi_0^i = 0$ and
$1 - \xi_{t+1}^i = (1 - \xi_t^i) / (1 - \tau_{t+1}^i)$. Intuitively, $\tau_{t+1}^i$ is the tax wedge (capital control)
imposed between two consecutive periods $t$ and $t+1$ whereas $\xi_{t+1}^i$ captures the cumulative
tax wedges between periods 0 and $t + 1$. It can also be interpreted as a capital control on long-term investments between period 0 and $t + 1$.

**Example 2 (Uncertainty)** To incorporate uncertainty, assume that a state of nature $s_t \in \Omega_i(s_{t-1})$ is realized at the beginning of each period $t$ where $s_0$ is given and the probability of $s_t$ is denoted by $\pi(s_t|s_{t-1})$ where $\sum_{s_t \in \Omega_i(s_{t-1})} \pi(s_t|s_{t-1}) = 1$. Let us denote random variables as functions of the state $s_t$, for example, $m_t^i(s_t)$ for the stochastic net imports of economy $i$ in period $t$, which trade at state price $Q_t(s_t)$. Collecting the realizations of all variables across time and states of nature in vector notation and assuming that there exists a complete market to trade $m^i$ and a complete set of external tax instruments $\xi^i$ for the planner, this framework maps into our general setup above, and the external budget constraint of the representative agent can be denoted by (9). Observe that a security that pays off a state-contingent return of $a_t^i(s_t)$ in period $t$ will trade at a world market price $E[Q_t \cdot a_t^i|s_0]$ in the initial Arrow-Debreu market and can be purchased by the representative agent in country $i$ at a local price $E[Q_t \cdot a_t^i|s_0]$. Similarly, in state of nature $s_k$ at time $k < t$, the market price of the security would be $E[Q_t \cdot a_t^i|s_k]$.

**Example 3 (Multiple Traded Goods)** The general model also nests models with multiple traded goods if we interpret $m_t = (m_{t,1},...,m_{t,h})$ as a vector of $h$ different goods that are purchased or sold on the world market. Assuming a complete set of tax instruments would require that the planner can impose differential taxes/subsidies on each good $h$. Alternatively, assuming that the planner can only differentiate taxes by time period would amount to a restriction on the set of instruments of $\xi_{t,1}^i = \xi_{t,2}^i = ... = \xi_{t,h}^i \forall t$. We will discuss the role of such restrictions and the existence of optimal allocations for which tax instruments are nonetheless effectively complete below.

### 3.2 Domestic Optimization Problem

We analyze the optimization problem in our general model in two steps. The first step
is the domestic optimization problem of the economy for a given external allocation
$(m^i, M^i)$ and is described in the current subsection. In the ensuing subsections, we solve for the optimal external allocation in a second step.

**Representative Agent** We describe the domestic optimization problem of a representative agent in country $i$ for given external allocations $(m^i, M^i)$ and given aggregate
control variables, policy variables and exogenous state variables \((X^i, \zeta^i, Z^i)\). We dis-
tinguish between individual \(m^i\) and aggregate \(M^i\) because the representative agents
does not internalize that his choices will affect \(M^i\) (even though \(m^i = M^i\) will hold
in equilibrium). Specifically, we define the reduced-form utility of the representative
agent as

\[
v^i \left( m^i; M^i, X^i, \zeta^i, Z^i \right) = \max_{x^i} U \left( x^i \right) \quad \text{s.t.} \quad f^i \left( m^i, M^i, x^i, X^i, \zeta^i, Z^i \right) \leq 0
\]

Denoting the shadow price on the constraint by \(\lambda^i\), the optimality condition of a
domestic representative agent is

\[
U_x = \lambda^i f^i_x
\]

**Domestic Planner** For any aggregate external allocation \(M^i\) and exogenous state
variables \(Z^i\), a domestic planner in economy \(i\) chooses the optimal domestic policy
measures \(\zeta^i\) and aggregate choice variable \(X^i\) where she internalizes the domestic
consistency condition \(x^i = X^i\) for the representative agent. The planner’s problem is
subject to the set of constraints (8) and the implementability condition (11),

\[
\max_{X^i, \zeta^i} U \left( X^i \right) \quad \text{s.t.} \quad f^i \left( M^i, M^i, X^i, X^i, \zeta^i, Z^i \right) \leq 0,
\]

This defines optimal domestic policy measures and aggregate allocations \(\zeta^i(M^i)\) and
\(X^i(M^i)\). (For ease of notation we omit the argument \(Z^i\).)

**Optimal Domestic Allocation** For a given aggregate external allocation \(M^i\) and
exogenous state variables \(Z^i\), the optimal domestic allocation in economy \(i\) consists of
a consistent domestic allocation \(x^i = X^i\) and domestic policy measures \(\zeta^i\) that solve
the first-stage optimization problem (10) of a domestic planner.

**Definition 2 (Reduced-Form Utility)** We define the reduced-form utility func-
tion of a representative agent in economy \(i\) for any pair \((m^i, M^i)\) by

\[
V^i \left( m^i, M^i \right) = v^i \left( m^i, M^i, X^i(M^i), \zeta^i(M^i), Z^i \right)
\]

Observe that the optimal domestic allocation also constitutes a solution to the
first-stage optimization problem (10) of the representative agent since the planner
observes the implementability constraint (11). Whereas the planner only cares about
aggregate allocations in which the consistency condition \(m^i = M^i\) is automatically sat-
sified, the reduced-form utility function \(V^i(m^i, M^i)\) is also defined for off-equilibrium
allocations since individual agents are in principle free to choose any allocation of \(m^i\).
Again, for ease of notation we omit the exogenous parameters \(Z^i\) from the function
\(V^i(\cdot)\).
For the remainder of our analysis, we will focus on the case where the partial derivatives of this reduced-form utility function satisfy \( V_{m_i}^i > 0 \) and \( V_{M_i}^i + V_{M_M}^i > 0 \) \( \forall i \): ceteris paribus, a marginal increase in individual imports \( m_i^i \) increases the welfare of the representative consumer and a simultaneous marginal increase in both individual and aggregate imports \( m^i = M^i \) also increases the welfare of a representative consumer. These are fairly mild assumptions that hold for the vast majority of open economy macro models, including our baseline model. (For concreteness, the reduced-form utility functions in our baseline model are \( V_i(m_i^i; M_i^i) = P_t t u_i(y_i^i + m_i^i) \), satisfying the above marginal utility conditions since \( V_{m_i}^i = \beta^i u_i'(c_i^i) > 0 \) and \( V_{M_i}^i = 0 \).)

For our applications below, the reduced-form utility function \( V_i(m_i^i, M_i^i) \) contains all the information that is required to describe global equilibria.

### 3.3 External Allocations

**Representative Agent** Given the reduced-form utility \( V_i^i(m_i^i, M_i^i) \), initial external wealth \( b_i^0 \) and a vector of tax instruments \( \xi^i \) on international transactions, we describe the optimization problem of a representative consumer in country \( i \) as

\[
\max_{m_i^i} V_i^i(m_i^i, M_i^i) \quad \text{s.t.} \quad (9) \quad (12)
\]

The associated optimality condition

\[
(1 - \xi^i) V_{m_i}^i = \lambda^i Q
\]

describes the excess demand for each component of \( m_i^i \) of the representative agent as a function of the vector of world market price \( Q \), where the tax vector \( (1 - \xi^i) \) pre-multiplies the vector of marginal utilities of \( m_i^i \) in an element-by-element fashion.

**Free Capital Flows** We define the allocation that prevails when \( \xi^i = 0 \) \( \forall i \) as the free capital flows allocation.

**Competitive Planner** Next we consider how the policy instruments on international transactions \( \xi^i \) are determined. We assume a planner \( CP_i \) who acts competitively on world markets in the sense that she takes world market prices \( Q \) as given, which we term a competitive planner. There are two potential interpretations for such behavior. First, country \( i \) may represent a small economy with \( \omega_i \approx 0 \) so that it is not possible for the country to affect world market prices. Secondly, the planner may choose \( \xi^i \) to correct a domestic distortion while acting with benign neglect towards international markets. This benign neglect may be the consequence of an explicitly domestic objective of the policymaker, or because the policymaker observes a multilateral agreement to abstain from monopolistic behavior and disregard world-wide
terms-of-trade effects.\footnote{For example, the US Federal Reserve claims to follow a policy of acting with benign neglect towards external considerations such as exchange rates, as articulated for example by Bernanke (2013). Furthermore, the G-7 Ministers and Governors proclaimed in a Statement after their March 2013 summit that “we reaffirm that our fiscal and monetary policies have been and will remain oriented towards meeting our respective domestic objectives using domestic instruments, and that we will not target exchange rates.” See G-7 (2013).} (We will analyze the behavior of a monopolistic planner $MP_i$ who internalizes her market power in the following section. We will also discuss some guidelines about how to distinguish between monopolistic and competitive behavior of policymakers.)

Analytically, a competitive planner $CP_i$ who faces the reduced-form utility function $V^i(m^i, M^i)$ and initial external wealth $B^i_0$ solves

$$\max_{M^i} V^i(M^i, M^i) \quad \text{s.t.} \quad Q \cdot M^i \leq B^i_0 \tag{14}$$

Assigning a shadow price $\lambda^i$ to the international budget constraint, the optimality condition of the planner is

$$V^i_m + V^i_M = \lambda^i Q \tag{15}$$

**Lemma 2 (Implementation)** The planner can implement the allocation that solves her optimization problem (14) by setting the vector of policy instruments

$$\xi^i = -V^i_M / V^i_m \tag{16}$$

where the division $V^i_M / V^i_m$ is performed element-by-element.

**Proof.** Substituting the optimal $\xi^i$ from (16) into the optimality condition of private agents $(1 - \xi^i) V^i_m = \lambda^i Q$ yields the planner’s optimality condition (15). □

According to this implementation, the planner does not intervene in time periods/states of nature/goods for which $V^i_{M,t} = 0$, i.e. for which the marginal benefit is fully internalized by private agents. If $V^i_{M,t} > 0$ then $\xi^i_t < 0$ so the planner subsidizes $m^i_t$ and vice versa for $V^i_{M,t} < 0$.

**Indeterminacy of Implementation** Observe that there is an indeterminacy of implementation. The allocation implemented by the planner is unchanged if the policy instruments are scaled up or down by a positive constant $k > 0$: any vector $(1 - \tilde{\xi}) V^i_m = \lambda^i Q$ will implement the same real allocation and will simply rescale the shadow price in the optimality condition (15) by $1/k$. The intuition is that the incentive of a representative agent to shift consumption across time/states of nature/goods only depends on the relative price of consumption goods. Multiplying all prices by a constant amounts to changing the numeraire.

By the same token, if $\exists h \in (-1, \infty)$ s.t. $V^i_M = h V^i_m$, then it is not necessary for a planner to intervene since the vector of policy instruments $\tilde{\xi}^i = 0$ will implement
the same equilibrium as the vector $\xi^i = -V^i_M/V^i_m = -h$. This can easily be verified by setting $k = \frac{1}{1+h}$ and applying the argument in the preceding paragraph. In the following, we will assume that $\exists h$ that satisfies this requirement when we speak of a country that exhibits externalities $V^i_M \neq 0$.

**Sequential Trading** The formulation of problem (12) assumed – in Arrow-Debreu fashion – that all intertemporal trade is determined in period 0. If trading takes place sequentially, i.e. period after period, as it did in our baseline model, then the planner can implement her optimal allocation by imposing the relative tax wedge $(1 - \tau^i_{t+1}) = (1 - \xi^i) / (1 - \xi^i_{t+1})$ on trading between any two consecutive periods or $(1 - \tau^i_{t,t+s}) = (1 - \xi^i_t) / (1 - \xi^i_{t+s})$ on long-term financial instruments that mature after $s$ periods.

4 **Global Equilibrium and Welfare**

4.1 **Competitive Behavior**

We now turn to the determination of equilibrium at the global level. We describe the global equilibrium among competitive planners assuming that each individual country $i$ is governed by a competitive planner $CP^i$ who solves the optimization problem (14) and implements the resulting allocation:

**Definition 3 (Global Equilibrium among Competitive Planners $CP$)** For a given set of initial bond holdings $\{B^i_0\}$ and reduced-form welfare functions $\{V^i(m^i, M^i)\}$, the global equilibrium among competitive planners $CP$ is described by a collection of net imports $\{M^i\}$ and intertemporal prices $Q$ such that the planner in each country $i$ solves her optimization problem (14) and global markets clear $\sum_{i=1}^N \omega^i M^i = 0$.

Capital controls have significant spillover effects, as we illustrated for example in our baseline model. Let us next analyze the global efficiency of capital account interventions $\xi^i$ by competitive planners.

**Proposition 2 (Efficiency of Unilaterally Correcting Externalities)** The global equilibrium among competitive planners $CP$ as per Definition 3 is Pareto efficient.

**Proof.** An allocation is Pareto efficient if it maximizes the weighted sum of welfare of all countries for some vector of welfare weights $\{\phi^i \geq 0\}_{i=1}^N$ subject to the global resource constraint and the individual domestic constraints of each country,

$$\max_{\{M^i, X^i, \xi^i\}} \sum_i \phi^i \omega^i U^i (X^i) \quad \text{s.t.} \quad \sum_i \omega^i M^i = 0, \quad \sum_i \omega^i M^i = 0, \quad (11),$$

$$f^i(M^i, X^i, X^i, \xi^i, Z^i) \leq 0 \quad \forall i$$
By the definition of $V^i(m^i, M^i)$, we can restate this problem in terms of reduced-form utilities for any optimal collection of $\{M^i\}$,

$$\max_{\{M^i\}} \sum_i \phi^i \omega^i V^i(M^i, M^i) \quad \text{s.t.} \quad \sum_i \omega^i M^i = 0$$

Assigning the shadow price $\theta$ to the vector of resource constraints, the optimality condition of the global planner is

$$\phi^i (V^i_m + V^i_M) = \theta \quad \forall i$$

Any global equilibrium among competitive planners $CP$ as per Definition 3 satisfies these optimality conditions if we assign the welfare weights $\phi^i = 1/\lambda^i$ and the shadow price $\theta = Q$. Therefore any such equilibrium is Pareto efficient. 

**Remark** The proposition is a version of the first welfare theorem. Since the planner $CP^i$ has a complete set of tax instruments $\xi^i$, she can fully determine the efficient excess demand for $M^i$ given the world market price $Q$. If the planner acts competitively in determining $M^i$, then all the conditions of the first welfare theorem apply and the resulting competitive equilibrium is Pareto efficient. Given that the planner has internalized all domestic externalities, the excess demand $M^i$ of the country correctly reflects the country’s social marginal valuation of capital flows. The marginal rates of substitution of all domestic planners are equated across countries, and the resulting equilibrium is Pareto efficient.

Compared to the global laissez-faire equilibrium, capital account interventions do create significant spillover effects, but this is not a sign of Pareto inefficiency. The spillover effects constitute pecuniary externalities that are intermediated in a complete market for $M^i$. As exemplified by the numerical illustrations in section 2.4 on the interest rate effects of capital controls, they can be quantitatively quite large. However, Pareto optimality is independent of redistributive concerns. Even though the spillover effects entail redistributions between borrowing and lending countries, proposition 2 establishes that they do not lead to Pareto inefficiency.

**Efficiency of Laissez-Faire Equilibrium** A straightforward corollary to Proposition 2 is that the laissez faire equilibrium is generally not Pareto efficient if there are countries subject to externalities from international capital flows.

**Tatonnement and Arms Race of Capital Account Interventions** The equilibrium adjustment (tatonnement) process may sometimes involve dynamics that look like an arms race. For example, assume that several countries experience negative externalities $V^i_{M,t} < 0$ from capital inflows $M^i$ and that the absolute magnitude of these externalities increases in a convex fashion $V^i_{MM,t} < 0$ in period $t$. Then an
exogenous shock that makes one country increase its optimal degree of intervention, will lead to greater capital flows to all other countries, which in turn increases their externalities and induces them to respond with greater intervention. This in turn will deflect capital flows back into the original country, triggering further intervention there, and so on.

Such dynamics may give the appearance of an arms race but are nonetheless efficient. As long as the conditions of Proposition 2 are satisfied, this “arms race” is simply the natural mechanism through which the efficient equilibrium is restored. In the described example, each successive round of spillovers will be smaller and the degree of intervention will ultimately converge towards its efficient levels, which involves greater intervention in all affected countries.

**Pareto-Improving Capital Account Intervention** If the objective of a global planner is not only to achieve Pareto efficiency but the more stringent standard of achieving a Pareto improvement, then capital controls generally need to be accompanied by transfers that compensate the countries that lose from changes in world prices/interest rates. Lump-sum transfers enable a global planner to always implement a Pareto improvement when correcting for the domestic externalities:

**Proposition 3 (Pareto-Improving Capital Account Intervention, Transfers)**

Starting from the laissez faire equilibrium, a global planner who identifies domestic externalities \( V_i^m \neq 0 \) in some countries can achieve a Pareto improvement by setting the capital account interventions \( \xi^i = -V_i^i/V_i^m \) in all countries and providing compensatory international transfers \( \hat{T}_i \leq 0 \) that satisfy \( \sum_i \hat{T}_i = 0 \).

**Proof.** Denote the net imports and world prices in the laissez faire equilibrium by \( \{M_{i,LF}\}_i \) and \( Q_{LF} \) and in the global planner’s equilibrium that results from imposing optimal capital controls \( \xi^i \) and transfers by \( \{M_{i,GP}\}_i \) and \( Q_{GP} \). Assuming the planner provides transfers \( \hat{T}_i = Q_{GP} \cdot (M_{i,LF} - M_{i,GP}) \), then \( \sum \hat{T}_i = 0 \) since both sets of allocations (LF and GP) satisfy market clearing. Furthermore, given these transfers, consumers in each country \( i \) can still afford the allocation that prevailed in the laissez faire equilibrium. For non-zero interventions \( \{\xi^i\} \), the allocation differs from the laissez faire equilibrium since the optimality conditions (13) for \( \xi^i = 0 \) and \( \xi^i \neq 0 \) differ. Given that the old allocation is still feasible but is not chosen, revealed preference implies that every country is better off under the new allocation.

In an international context, compensatory transfers may be difficult to implement. As an alternative, we show that a planner who can coordinate the capital control policies of both inflow and outflow countries can correct the domestic externalities of individual economies while holding world prices and interest rates constant so that no wealth effects arise. As a result, the global planner’s capital control policies constitutes a global Pareto improvement at a first-order approximation.
The following lemma demonstrates how a global planner can manipulate world prices by simultaneously adjusting the capital controls in all countries worldwide; then we show how this mechanism can be used to hold world prices fixed so as to avoid redistributive effects when correcting for externalities in a given country.

**Lemma 3** Consider a global competitive equilibrium with an allocation \( \{M^j\}_j \), capital account interventions \( \{\xi^j\}_j \) and world prices \( Q \). A global planner can change world prices by \( dQ \) while keeping the bond allocations for all countries constant by moving the capital account interventions in each country \( j = 1 \ldots N \) by

\[
dx^j = - (M^j_\xi)^{-1} M^j_Q dQ \tag{17}
\]

**Proof.** We set the total differential of net imports of a given country \( j \) with respect to world prices and capital account interventions to zero,

\[
dM^j = M^j_Q dQ + M^j_\xi d\xi^j = 0
\]

and rearrange to obtain equation (17). □

**Corollary 1** (Pareto-Improving Capital Account Intervention, No Transfers) Assume an exogenous marginal increase in the domestic externalities of country \( i \) that raises the optimal unilateral capital control by \( d\xi^* > 0 \) in period \( t \). A global planner can correct for this while keeping world prices constant \( dQ = 0 \) to avoid income and wealth effects by adjusting

\[
dx^j = -\omega^i (M^j_\xi)^{-1} M^j_Q (M_Q)^{-1} M^j_\xi d\xi^* \quad \text{and} \quad d\xi^*_{t+1} = \left[I - \omega^i (M^j_\xi)^{-1} M^j_Q (M_Q)^{-1} M^j_\xi\right] d\xi^*
\]

In the resulting equilibrium, net imports \( \{M^j\}_j \) are altered but world prices are unchanged. By the envelope theorem, welfare is unchanged at a first-order approximation.

**Proof.** If the planner implemented the unilaterally optimal increase \( d\xi^*_i > 0 \) in capital account interventions of country \( i \), then world prices would move by

\[
dQ = -\omega^i (M_Q)^{-1} M^j_\xi d\xi^*. \quad \text{According to Lemma 3, the move in the interest rate can be undone if the capital controls of all countries } j = 1 \ldots N \text{ are simultaneously adjusted by } - (M^j_\xi)^{-1} M^j_Q dQ, \text{ which delivers the first equation of the proposition. The second equation is obtained by adding the optimal unilateral change in intervention } d\xi^* \text{ plus the adjustment given by the first equation with } j = i. \text{ In the resulting equilibrium, the increase in the externality } d\xi^* \text{ is corrected but the world interest rate is unchanged. Furthermore, by the envelope theorem, for constant world prices, the change in welfare that results from a marginal change in } M^j \text{ is}
\]

\[
dV^j |_{dQ=0} = (V^i_M + V^i_m) dM^j = 0
\]

\[\footnote{For non-infinitesimal changes in } \xi^*, \text{ changes in net imports } \Delta M^j \text{ have second-order effects on welfare (i.e. effects that are negligible for infinitesimal changes but growing in the square of } \Delta M^j \text{) even if world prices are held constant. Under certain conditions, e.g. if there are only two types of countries in the world economy, a global planner can undo these second-order effects via further adjustments in the world prices } Q.\]
Numerical Illustration  Let us illustrate the mechanics of Pareto-improving capital controls by returning to the numerical illustration in section 2.4. Assuming a world economy in the steady state described there, an increase in the externality in country $i$ that would call for an optimal unilateral response $d\tau_{i,t+1}^i$ in the country’s level of capital controls can also be corrected by setting

$$\frac{d\tau_{i,t+1}^i}{d\tau_{i,t+1}^i} = 1 - \omega^i$$

$$\frac{d\tau_{j,t+1}^j}{d\tau_{i,t+1}^i} = -\omega^i$$

In short, the country that experiences the externality corrects only a fraction $(1 - \omega^i)$ of it and the rest of the world imposes a capital control to correct the remaining fraction $\omega^j$ corresponding to the country’s weight in the world economy. For example, small open economies would meet the burden of adjustment by themselves since $\omega^j = 0$ and they do not affect the world interest rate. For large economies, we refer to the country weights implied by Table 2 on page 11. For example, if China experienced a positive externality from current account surpluses that calls for a 1% unilateral subsidy to capital outflows, then a global planner who follows the described scheme would impose a 0.90% subsidy on outflows in China and a 0.10% subsidy to inflows in the rest of the world to keep the world interest rate unchanged. Similarly, if Brazil experienced a $-1\%$ externality from capital inflows, the global planner would impose a 0.97% tax on inflows to Brazil and a 0.03% tax on outflows in the rest of the world in order to keep the world interest rate stable.

In Figure 1 on page 9, a national planner corrects for a negative externality to borrowing $\tau_{i,t}^i$ in country $i$. A global planner could achieve a Pareto improvement by splitting the burden of regulating capital flows between the two countries. He would tax outflows in country $j$ such that $1 - \tau^j = R_{NP}/R_{LF}$ and tax inflows for the remaining part of the externality such that $1 - \tau^i = \frac{1 - \tau^j}{1 - \tau^j}$ in country $i$. As a result, the interest rate would be unchanged at $R_{LF}$ and the welfare loss by country $i$, indicated by the shaded area in the figure, would be limited to the Harberger triangle between $b_{NP}^i, b_{LF}^i$ and $R_{LF}$.\footnote{Since there are only two countries in this example, country $i$ could be compensated for this second order loss by raising the interest rate on the remainder of its bond holdings, as described in Lemma 3. achieving an unambiguous Pareto improvement.} Such a policy response shares certain characteristics with voluntary export restraints (VERs) in trade policy: If a borrowing country imposes controls on capital inflows, the world interest rate will decline and all lending countries experience negative wealth effects. However, if lenders restrict outflows by imposing controls of their
own, they can keep the surplus. A global planner would share the burden of adjustment between borrower and lender in proportion to their elasticities of demand and supply so as to keep the world interest rate constant.

4.2 Monopolistic Behavior

Assume next that there is a monopolistic planner in a given country \( i \) with positive mass \( \omega_i > 0 \) that maximizes the utility of the representative consumer \( U^i \) and internalizes that she has market power over world prices \( Q \). We solve the problem of the monopolistic planner under the assumption that the remaining countries \( j \neq i \) behave according to the competitive planning setup in section 3.3. (Our findings can easily generalized to the case where other countries engage in monopolistic behavior or operate under laissez-faire.

Global market clearing requires that

\[
\omega^i M^i + M^{-i}(Q) = 0 \quad \text{with} \quad M^{-i}(Q) = \sum_{j \neq i} \omega^j M^j(Q)
\]

where \( M^{-i}(Q) \) denotes the excess demand of the rest-of-the-world excluding country \( i \) as described in section 3.3, which satisfies \( \partial M^{-i}_t / \partial Q_t < 0 \) for each element \( t \), i.e. that each good is an ordinary good, reflecting that its price needs to decline for other countries to absorb more of it. Furthermore, we assume that the function can be inverted to obtain an inverse rest-of-the-world excess demand function \( Q^{-i}(M^{-i}) \).

A monopolistic planner solves the optimization problem

\[
\max V^i(M^i, M^i) \quad \text{s.t.} \quad Q^{-i}(-\omega^i M^i) \cdot M^i \leq B^i_0
\]

leading to the optimality condition

\[
V^i_m + V^i_M = \lambda^i Q \left( 1 - \mathcal{E}^i_{Q,M} \right) \quad \text{with} \quad \mathcal{E}^i_{Q,M} = \omega^i Q^{-i}_M M^i / Q \quad (18)
\]

where \( \mathcal{E}^i_{Q,M} \) represents the elasticity of the world price \( Q \) with respect to imports in country \( i \) and consists of four elements: the country weight \( \omega^i \) reflects the country’s market power in the world market; the square matrix \( Q^{-i}_M = \partial Q^{-i} / \partial M^{-i} \) which captures how much the world market price has to respond to absorb an additional unit of exports from country \( i \); this is multiplied by the vector \( M^i \) to obtain the marginal revenue accruing to country \( i \) as a result of monopolistic distortions; finally the expression is divided element-by-element by the vector \( Q \) in order to normalize it and obtain a vector of elasticities.

\(^{10}\)The only important assumptions for our problem are that each country \( j \) has a well-defined and continuous demand function. This rules out, for example, discontinuous trigger strategies.
Lemma 4 (Monopolistic Capital Account Intervention) The allocation of the monopolistic planner who internalizes her country’s market power over world prices can be implemented by setting the vector of external policy instruments to

$$1 - \xi_{i,m}^{t} = \frac{1 + V_{m}^{i}/V_{m}^{i}}{1 - \mathcal{E}_{Q,M}^{i}}$$

where all divisions are performed element-by-element.

Proof. The tax $\xi_{i,m}^{t+1}$ ensures that the private optimality condition of consumers (13) replicates the planner’s Euler equation (18).

To provide some intuition, assume that the matrix $Q_{M}^{-i}$ in a country with $\omega^{i} > 0$ is a diagonal matrix and that there are no domestic externalities so $V_{M}^{i} = 0$.

Then all diagonal entries will satisfy $\partial Q_{M}^{-i}/\partial M_{i}^{i} < 0$, reflecting that greater imports $M_{i}^{i}$ of good $t$ require a lower world price $Q_{i}$.

If the country is a net importer $M_{i}^{i} > 0$ of good $t$ then the elasticity $\mathcal{E}_{Q,M}^{i}$ will be negative and the optimal monopolistic tax on imports $\xi_{i,m}^{t} > 0$ is positive. Similarly, for goods that are net exports $M_{i}^{i} < 0$ the planner will reduce the quantity exported by a tax $\xi_{i,m}^{t} < 0$.

Proposition 4 (Inefficiency of Exerting Market Power) An equilibrium in which domestic planners impose capital controls to exert market power is Pareto-inefficient.

Proof. The result is a straightforward application of proposition 2 that optimality requires competitive behavior.

Returning to equation (18), a monopolistic planner equates the social marginal benefit of imports $V_{m}^{i} + V_{M}^{i}$ to the marginal expenditure $\lambda^{i} Q (1 - \mathcal{E}_{Q,M}^{i})$ rather than to the world price $Q$. She intervenes up to the point where the marginal benefit of manipulating world prices – captured by the elasticity term $\lambda^{i} Q \mathcal{E}_{Q,M}^{i}$ – equals the marginal cost of giving up profitable consumption opportunities $\lambda^{i} Q - V_{m}^{i} - V_{M}^{i}$. However, giving up profitable consumption opportunities creates a deadweight loss – the planner introduces a distortion to extract monopoly rents from the rest of the global economy. The intervention therefore constitutes a classic beggar-thy-neighbor policy.

Distinguishing Competitive and Monopolistic Behavior

The spillover effects of capital account intervention are the same, no matter what the motive for intervention. However, whether capital account interventions that are

\[\text{11}^\text{This would be the case, for example, if the reduced-form utility } V (m^{i}, M^{i}) \text{ is Cobb-Douglas in } m^{i}. \text{ Otherwise, the off-diagonal elements of } Q_{M}^{-i} \text{ reflect how changes in exports in one good affect the price of other goods and are positive for goods that are complements and negative for goods that are substitutes.}\]
unilaterally imposed lead to Pareto efficient outcomes depends crucially on whether they correct for domestic distortions (as described in lemma 2) or whether they serve to exert market power (as described in lemma 4). Distinguishing between the two motives for intervention is therefore an essential step in determining whether global allocations are Pareto efficient or whether there is scope for global cooperation to achieve Pareto improvements.

Unfortunately it is impossible to answer this question in general. It is often easy for policymakers to claim various market imperfections, domestic objectives or different political preferences to justify an arbitrary set of policy interventions in the name of domestic efficiency, and it is close to impossible for the international community to disprove them. Specifically, for any reduced-form utility function $V^i(m^i, M^i)$ and monopolistic interventions $\xi^{i,m}$ as described by lemma 4, there exists an alternative reduced-form utility function $\tilde{V}^i(m^i, M^i)$ such that the described interventions implement the optimal competitive allocation defined by

$$\tilde{V}^i(m^i, M^i) = V^i(m^i, M^i) - \left(\xi^{i,m} V^{i}^i(\cdot)\right) \cdot M^i$$

With some extra work, the reduced-form utility function $\tilde{V}^i(\cdot)$ can be translated into a fundamental utility function $U^i(x^i)$ and a set of constraints $f^i(\cdot)$ that would justify it.

Nonetheless, the direction of optimal monopolistic policy interventions is often instructive to determine whether it is plausible that a given intervention is for domestic or monopolistic reasons. In the following, we describe optimal monopolistic capital account interventions along a number of dimensions. If the observed interventions of a policymaker are inconsistent with these observations, then they are likely not for monopolistic reasons. For the following discussion, we repeat the definition of the elasticity of the world price with respect to imports in country $i$ from above,

$$\mathcal{E}_{Q,M}^i = \omega^i Q_M^{-i} M^i / Q$$

and discuss the implications of the various parameters:

**Country Size** The optimal monopolistic intervention is directly proportional to the country’s weight $\omega^i$ in the world economy. Larger countries have a greater impact on the rest of the world since market clearing requires $M^{-i} = \omega^i M^i$. For example, if a small open economy with $\omega^i \approx 0$ and undifferentiated exports regulates capital in- or out-flows, the reason cannot be monopolistic.

**Responsiveness of Price** The second factor determining the optimal monopolistic intervention is the responsiveness of the world market price to changes in consumption. Observe that $Q_M^{-i} = (\partial Q_t / \partial M_k^{-i})$ is a square matrix of how changing imports $M_k$ affects the world price $Q_t$. This formulation allows for the possibility that distorting the consumption of one type of flow affects the world price of other flows. Along the diagonal $\partial Q_t / \partial M_t^{-i}$ is negative for ordinary goods since
the price needs to fall for the rest-of-the-world to absorb more of a good. Off the
diagonal the derivative is negative for substitutes and positive for complements.

**Direction and Magnitude of Flows** The intervention to manipulate a given price
$Q_t$ is directly proportional to the magnitude of a country’s net imports $M^i_t$ in
that time period/good/state of nature and has the opposite sign of $M^i_t$ as it
aims to reduce the value of imports $M^i_t > 0$ and increase the value of exports
$M^i_t < 0$. The larger $M^i_t$ in absolute value, the greater the benefits from distorting
the price $Q_t$ and the more advantageous to engage in monopoly behavior. By
contrast, if $M^i_t \approx 0$, the optimal monopolistic intervention is zero.

The direction and magnitude of flows has the following implications for monop-
olistic behavior:

- **Intertemporal trade:** If the different elements of $M^i$ capture the trade
  balance $M^i_t = B^i_{t+1}/R - B^i_t$ and there is a single good per time period, then
  a zero trade balance $M^i_t \approx 0$ makes it impossible to engage in monopolistic
  behavior. By contrast, monopolistic reasons may be involved if a country
  with a large current account deficit $M^i_t > 0$ taxes inflows $\xi^i_t > 0$ to keep
  world interest rates lower or a country with a large current account surplus
  $M^i_t < 0$ restricts outflows $\xi^i_t < 0$ to push up interest rates.

- **Risk-sharing:** In a model of uncertainty in which $M^i_t(s_t)$ denotes different
  states of nature, each country has – by definition – monopoly power over
  its own idiosyncratic risk. Optimal risk-sharing implies greater imports
  in bad states and greater exports in good states of nature. A planner
  who exerts monopoly power would restrict risk-sharing so as to reduce the
  price of insurance or keep the price of participating in the country’s shock
  process elevated. By contrast, if a country encourages insurance (e.g by
  forbidding foreign currency debt and instead encouraging FDI), the motive
  is unlikely to be monopolistic.

- **Intratemporal trade:** Exercising monopoly power in intratemporal trade
  consists of tariffs $\xi^i_{t,k} > 0$ on imported goods $k$ with $M^i_{t,k} > 0$ and taxes on
  exports $\xi^i_{t,k} < 0$ for $M^i_{t,k} < 0$, as is well known from a long literature on
  trade policy (see e.g. ?).

In all these cases, observe that the optimal monopolistic intervention reduces
the magnitude of capital or goods flows but does not change their direction.

It is straightforward that any price intervention $\xi^i_t$ can also be implemented by
an equivalent quantity restriction $M^i_t$, for example by imposing a ceiling on capital
inflows rather than a tax on inflows. Furthermore, under closed capital accounts,
onimal monopolistic capital controls are isomorphic to reduced reserve accumu-
lation/decumulation.\footnote{For example, when a country that accumulates reserves is concerned that it is not earning...}
<table>
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<tr>
<th>Country</th>
<th>( GDP^i )</th>
<th>( NW^i )</th>
<th>( NW^i/Y^{-i} )</th>
<th>( \tau^{i,M} )</th>
</tr>
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<td>World</td>
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<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
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</tr>
<tr>
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<td>0.42%</td>
</tr>
<tr>
<td>South Korea</td>
<td>$1,014bn</td>
<td>$30bn</td>
<td>0.05%</td>
<td>-0.2%</td>
</tr>
</tbody>
</table>

Table 3: Monopolistically optimal capital controls (Source: IMF IFS and author’s calculations) [needs updating]

Numerical Illustration  In the following we determine the monopolistically optimal level of capital controls for a variety of countries numerically based on equation (??). From our earlier analysis, we observe that the steady-state response of the world interest rate to additional saving is \( \partial R/\partial B^{-i} = \frac{1+\beta}{\sigma \beta Y^{-i}} \) in a steady state. We identify \( m^b/R \) in the data as the net external wealth \( NW^i \) of different countries and express the monopolistic capital controls of country \( i \) as

\[
\tau^{i,M}_{t+1} = \frac{1 + \beta}{\sigma \beta} \cdot \frac{NW^i}{Y^{-i}}
\]

For our earlier value of the intertemporal elasticity of substitution \( \sigma = 2 \), the first term of this expression is approximately 4. In short, the monopolistically optimal capital control of a country is roughly four times its current account relative to the GDP of the rest of the world.

We report the resulting calculations for a number of countries in Table 3. Countries for which the external wealth represents a significant fraction of rest-of-the-world GDP have a strong motive for imposing monopolistic capital controls. The United States, for example, would optimally impose a 4% tax on capital inflows so as to exert monopoly power over the availability of global savings instruments and benefit from a lower world interest rate. By contrast, China would optimally impose a 2% tax on capital outflows (or subsidy on capital inflows) so as to exert monopoly power over its supply of worldwide savings and raise the interest rate. Countries that make up a smaller share of the world capital market have less market power and choose accordingly smaller capital controls.

The table highlights that it is difficult to reconcile the capital controls observed in the real world with the monopolistic motive for imposing capital controls. This suggests that some of the other motives for imposing controls that we studied in

“sufficient” interest on its reserves because its accumulation is pushing down the world interest rate, this is non-competitive behavior and is equivalent to distortive capital controls.
earlier sections were more relevant for most countries that imposed capital controls in recent years.

Figure 2 illustrates our results in a framework of two countries $i$ and $j$ of equal mass for a given time period. The line $R^i(b^i)$ represents the (inverse) supply of bonds, the two lines $R^j(-b^j)$ and $R^j(-b^j) - b^j R_b$ represent the demand for bonds as well as the ‘marginal revenue’ curve for country $i$ that takes into account the decline in the interest rate from supplying additional bonds. The laissez faire equilibrium is characterized by an interest rate $R_{LF}$ and bond positions $b^i = b^j = b^{LF}$. A monopolistic planner in country $i$ would reduce the quantity of bonds supplied to $j$ such that her marginal valuation $R^i(b^i)$ equals the marginal revenue derived from country $j$. This monopolistic equilibrium is indicated by the quantity of bonds sold $b^{MP}$ and interest rate $R^{MP}$. The described policy shifts the surplus between $R^{MP}$ and $R^*$, marked by the dotted area in the figure, from country $j$ to country $i$. It also introduces a deadweight loss indicated by the triangular vertically-shaded area. Monopolistic capital controls constitute a classic beggar-thy-neighbor policy and are always inefficient: they introduce a distortion into the Euler equation of domestic agents, which reduces global welfare, in order to shift welfare from foreigners to domestic agents – the policy represents a “negative-sum” game overall.

[update] A monopolistic planner in country $i$ would reduce the quantity of bonds supplied to $j$ such that her marginal valuation $R^i(b^i)$ equals the marginal revenue derived from country $j$. This monopolistic equilibrium is indicated by the quantity of bonds sold $b^{MP}$ and interest rate $R^{MP}$. The described policy shifts the surplus between $R^{MP}$ and $R^*$, marked by the dotted area in the figure, from country $j$ to country $i$. It also introduces a deadweight loss indicated by the triangular vertically-
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5 Examples and Applications

This section investigates several examples of externalities that have been used in the literature and in policy circles to motivate capital account intervention and that are relevant for our analysis of spillover effects and efficiency of equilibrium. We start with two simple examples of learning externalities that are triggered either by exporting or by producing and in which capital account interventions represent first-best and second-best policy instruments, respectively. Then we analyze aggregate demand externalities that may occur if a country experiences a liquidity trap.

Even if one is skeptical of the existence of some of the described externalities, these are important question to analyze since policymakers have explicitly invoked such externalities when they engaged in capital account interventions, exemplified by numerous comments of the Brazilian finance minister Guido Mantega (see Wheatley and Garnham, 2010).

5.1 Learning-by-Exporting Externalities

Our first and simplest example are learning-by-exporting externalities. Assume a representative agent in an economy $i$ that behaves as in our baseline model, except that the endowment income $y_{i,t+1}$ is a function $\varphi_t(\cdot)$ of the economy’s aggregate net imports $M_{i,t}$ that satisfies $\varphi_t(0) = 0$ and that is continuous and decreasing $\varphi_t'(M_t) \leq 0$ to capture that higher exports increase growth,

$$y_{i,t+1} = y_{i,t} + \varphi_t(M_{i,t})$$  \hspace{1cm} (20)

The reduced-form utility of a representative agent in country $i$ is

$$V^i(m^i, M^i) = \sum_t \beta^t u \left( y_0 + \sum_{s=0}^{t-1} \varphi_s(M^i_s) + m^i_t \right)$$

with marginal utility of private and aggregate capital inflows of $V^i_{m,t} = \beta^t u'(C^i_t)$ and $V^i_{M,t} = \varphi_t'(M^i_t) \beta^t v_{t+1}$ where $v^i_{t+1} = \sum_{s=t+1}^{\infty} \beta^{s-t} u'(C^i_s)$ is the PDV of one unit of output growth at time $t+1$, capturing the growth externalities from exporting. Following

\footnote{There is a considerable theoretical literature that postulates that such effects are important for developing countries in the phase of industrialization. See for example Rodrik (2008) and Korinek and Servén (2010). In the empirical literature there have been some studies that document the existence of learning externalities, whereas others are more skeptical. For a survey see e.g. Giles and Williams (2000).}
lemma 2, a planner can implement the socially efficient allocation in economy \( i \) by imposing capital controls

\[
\xi_t^i = -V_{M,t}^i/V_{m,t}^i = -\frac{\varphi_t^i(M_t^i)}{w_t^i(C_t^i)} \geq 0
\] (21)

The planner subsidizes exports (taxes imports) of capital in periods in which net exports generate positive externalities.

Observe that capital control are the first-best policy tool to internalize learning-by-exporting externalities in this framework, since they directly target net saving and hence the trade balance of the economy.

Since our model of learning-by-exporting externalities nests into the general model of section 3, it is a straightforward application of proposition 2 that the intervention of a competitive planner \( CP^i \) to internalize such externalities leads to a Pareto efficient outcome from a global perspective.

5.2 Learning-by-Doing Externalities

Capital account intervention may also serve as a second-best instrument in an economy where it would be desirable to use domestic policy measures to correct a distortion but such measures are not available.

The following example show how capital controls may serve to internalize learning-by-doing externalities in a production economy in which productivity growth is an increasing function of aggregate employment. The first-best policy instrument in such a setting is a subsidy to employment. However, if such an instrument is not available (for example, because of a lack of fiscal resources, a large informal sector, or the risk of corruption), it may be optimal to resort to capital controls as a second-best instrument to improve welfare.

Assume that the output of a representative worker in economy \( i \) is given by \( y_t^i = A_t^i \ell_t^i \), where labor \( \ell_t^i \) imposes a convex disutility \( d(\ell_t^i) \) on workers. We capture learning-by-doing externalities by assuming that productivity growth \( A_t^i \) in the economy is a continuous and increasing function of aggregate employment \( \psi_t(L_t^i) \) that satisfies \( \psi_t^i(\cdot) \geq 0 \) so that

\[
A_{t+1}^i = A_t^i + \psi_t(L_t^i) = A_0^i + \sum_{s=0}^{t} \psi_s(L_s^i)
\] (22)

In the described economy, the first-best policy instrument to internalize such learning effects would be a subsidy \( s_t^i \) to wage earnings in the amount of \( s_t^i = \psi_t^i(L_t^i) v_{A,t+1}/[u^i(c_t^i) A_t^i] \) where \( v_{A,t+1} = \sum_{s=t+1}^{\infty} \beta^{s-t} u^i(c_s^i) L_s^i \) is the PDV of one unit of productivity growth starting period \( t+1 \).

In the absence of a policy instrument to target the labor wedge, a planner faces the implementability constraint

\[
A_t^i u^i (A_t^i L_t^i + M_t^i) = d^i(L_t^i)
\] (23)

which reflects the optimal labor supply condition of individual workers. Observe that reducing $M^i_t$ in this constraint is akin to a negative wealth effect and increases the marginal utility of consumption, which in turn serves as a second-best instrument to raise $L^i_t$ and trigger learning-by-doing externalities.

Accounting for this implementability constraint and imposing the consistency condition $\ell^i_t = L^i_t$, a constrained planner recognizes that the reduced-form utility of the economy is

$$V(m^i, M^i) = \max_{L^i_t} \sum_t \beta^t \left\{ u \left( A^i_t L^i_t + m^i_t \right) - d \left( L^i_t \right) \right\} \quad \text{s.t.} \quad (22), (23)$$

with marginal utility of private and aggregate capital inflows of $V^i_{m,t} = \beta^t u^\prime (C^i_t)$ and $V^i_{M,t} = -\lambda^i_t \beta^t A^i_t u^{\prime\prime} (C^i_t) < 0$ where $\lambda^i_t$ is the shadow price on the implementability constraint (23) and is given by

$$\lambda^i_t = \frac{\psi^i_t(L^i_t) u_{A,t+1}}{d''(L^i_t) - (A^i_t)^2 u''(C^i_t)} > 0$$

In this expression, the positive learning externalities (in the numerator) are scaled by a term that reflects how strongly labor supply responds to changes in consumption (in the denominator).

Following lemma 2, the planner can implement this second-best solution by imposing capital controls of

$$\xi^i_t = -\frac{\lambda_t A^i_t u''(C^i_t)}{u'(C^i_t)} = -\frac{\psi^i_t(L^i_t) u_{A,t+1}}{u'(C^i_t)} \cdot \frac{A^i_t u''(C^i_t)}{d''(L^i_t) - (A^i_t)^2 u''(C^i_t)}$$

This control reduces capital inflows and stimulates demand production, which in turn triggers learning-by-doing externalities. Observe that the first term in the expression is analogous to the optimal capital control (21) under learning-by-exporting. Using the optimality condition on labor (23), the second term can be approximated by $-\frac{1}{A^i_t \sigma_C \eta_L}$ for small $M^i_t$, where $\eta_L$ and $\sigma_C$ are the Frisch elasticity of labor supply and the intertemporal elasticity of substitution: the second-best intervention is more desirable the more responsive the marginal utility of consumption (low $\sigma_C$) and the less responsive the marginal disutility of labor (high $\eta_L$).

Given that there are no first-best policy instruments available, the PDV of one unit of productivity growth also includes the effects of higher productivity on future labor supply decisions: on the one hand, higher productivity increases incentives to work, on the other hand it makes the agent richer and reduces the incentive to work via a wealth effect. The two effects are captured by the two expressions in square brackets,

$$v_{A,t+1} = \sum_{s=t+1}^{\infty} \beta^{s-t} \left\{ u^\prime (C^i_s) L^i_s + \lambda^i_s \left[ u^\prime (C^i_s) + A^i_s u''(C^i_s) L^i_s \right] \right\}$$

If the economy has outgrown its learning externalities, the term drops to zero.
Proposition 2 implies, as in the previous case, that the application of second-best capital controls lead to a globally Pareto efficient outcome. Intuitively, even though capital controls (24) are just second-best instruments, they are chosen so as to equate the marginal social benefit from indirectly triggering the LBD-externality to their marginal social cost. Reducing domestic consumption by running a trade surplus is the only way available to induce domestic agents to work harder, given the restrictions on policy instruments. Since a global planner does not have superior instruments, he cannot do better than this and chooses an identical allocation.

Remark There has been a lively debate on the multilateral desirability of capital account intervention (see e.g. IMF, 2012; Forbes et al., 2012; Ostry et al., 2012). This debate has sometimes suggested that interventions that are of a second-best nature, such as those to internalize LBD externalities in our example above, should be viewed with particular skepticism. However, our general model demonstrates that the global efficiency implications of second-best capital controls to internalize LBD-externalities are no different from other reasons to implement capital controls, given the restrictions on the set of policy instruments. A global planner who faces the same constraints on his policy instruments would implement an identical allocation.\textsuperscript{15}

5.3 Aggregate Demand Externalities at the ZLB

Next we study the multilateral implications of capital controls to counter aggregate demand externalities at the zero lower bound (ZLB) on nominal interest rates. We develop a stylized framework that captures the essential nature of such externalities in the spirit of Krugman (1998) and Eggertsson and Woodford (2003), adapted to an open economy framework as in Jeanne (2009).

Assume that a representative consumer in country $i$ derives utility from consuming $c^i_t$ units of a composite final good and experiences disutility from providing $\ell^i_t$ units of labor. Collecting the two time series in the vectors $c^i$ and $\ell^i$, we denote

$$U^i(c^i, \ell^i) = \sum \beta^i \left[ u(c^i_t) - d(\ell^i_t) \right]$$

As is common in the New Keynesian literature, we assume that there is a continuum $z \in [0, 1]$ of monopolistic intermediate goods producers who are collectively owned by consumers and who each hire labor to produce an intermediate good of variety $z$ according to the linear function $y^i_{t}z = \ell^i_{t}z$, where labor market clearing requires $\int \ell^i_{t}z dz = \ell^i_t$. All the varieties are combined in a CES production function to produce final output

$$y_t^i = \left( \int_0^1 \left( y^i_{t}z \right)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{1}{\sigma-1}}$$

\textsuperscript{15}The second-best nature of capital account interventions is only relevant in the debate on global coordination if a global planner has access to a superior set of (first-best) policy instruments than national planners.
where the elasticity of substitution is \( \varepsilon > 1 \). We assume that the monopoly wedge arising from monopolistic competition is corrected by a proportional subsidy \( \frac{1}{\varepsilon-1} \) that is financed by a lump-sum tax on producers. This implies that the wage income and profits of the representative agent equal final output, which in turn equals labor supply \( w_i \ell^i_t + \pi^i_t = y^i_t = \ell^i_t \). In real terms and vector notation, the period budget constraints of a representative agent and the external budget constraint are given by

\[
Q \frac{1}{1 - \varepsilon^i} \cdot m^i_t - T^i_t \leq b^i_t.
\]

The condition for the optimal labor supply of the representative agent is

\[
d^i_t (\ell^i_t) = w^i_t.
\]

We assume that the nominal price of one unit of consumption good follows an exogenous path \( P^i = (1, P^i_2, P^i_3, \ldots) \) that is credibly enforced by a central bank (see Korinek and Simsek, 2013, for further motivation). This assumption precludes the central bank from committing to a future monetary expansion or future inflation in order to stimulate output in the present period.\(^{16}\) The corresponding gross rate of inflation is given by \( \Pi^i_{t+1} = P^i_{t+1}/P^i_t \) or by \( \Pi^i = P/L(P) \) in vector notation with lag operator \( L(\cdot) \). One example is a fixed inflation target \( \Pi^i_{t+1} = \hat{\Pi}^i \forall t \).

Combining the ZLB constraint on the nominal interest rate \( i^i_{t+1} = R_{t+1} - \Pi^i_{t+1} - 1 \geq 0 \) with the aggregate Euler equation to substitute for \( R_{t+1} \), the ZLB in period \( t \) imposes a ceiling on aggregate period \( t \) consumption,

\[
u^i (C^i_t) \geq \frac{\beta}{\Pi^i_{t+1}} u^i (C^i_{t+1})
\]

Intuitively, a binding ZLB implies that consumption in period \( t \) compared to consumption in the following period, limiting aggregate demand in period \( t \) to the level indicated by the constraint.

In the laissez-faire equilibrium, this constraint is satisfied with strict inequality if world aggregate demand for bonds and by extension the world interest rate is sufficiently high, i.e. if \( R_{t+1} \geq 1/\Pi^i_{t+1} \). Then the market-clearing wage \( W^i_t = 1 \) will prevail and output \( Y^i_t \) is at its efficient level determined by the optimality condition \( u^i (C^i_t) = d^i (L^i_t) \). We call this output level potential output \( Y^{\text{pot}}_t \).

If worldwide aggregate demand declines and the world real interest rate hits the threshold \( R_{t+1} = 1/\Pi^i_{t+1} \), then the ZLB on the nominal interest rate is reached and the domestic interest rate cannot fall any further. Instead, any increase in the world supply of bonds will flow to economy \( i \), which pays a real return of \( 1/\Pi^i_{t+1} \) by the feature of offering liabilities with zero nominal interest rate. Given the high return on

\(^{16}\)It is well known in the New Keynesian literature that the problems associated with the zero lower bound could be avoided if the monetary authority was able to commit to a higher inflation rate. See e.g. Eggertsson and Woodford (2003).
nominal bonds, consumers in economy $i$ find that today’s consumption goods are too expensive compared to tomorrow’s consumption goods and consumers reduce their aggregate demand for today’s consumption goods. Output is demand-determined, $Y^i_t$ falls below potential output $Y^i_{t^*}$ in order to satisfy equation (25) and the wage falls below $W^i_t < 1$. This situation captures the essential characteristic of a liquidity trap: at the prevailing nominal interest rate of zero, consumers do not have sufficient demand to absorb both domestic output and the capital inflow $M^i_t$. Intermediate producers cannot reduce their prices but let domestic output adjust so that demand equals supply. If the domestic real interest rate could fall, consumers would have incentive to consume more in period $t$ and the problem would be solved. We will show in the following that a planner in such a situation finds it optimal to erect barriers against capital inflows or encourage capital outflows in order to stimulate domestic aggregate demand.

We substitute the domestic period budget constraint and the consistency condition $x^i = X^i$ and denote the reduced-form utility maximization problem of a planner in country $i$ as defined in section 3.2 by

$$V (m^i, M^i) = \max_{L^i} \sum \beta^t{(C^i_t) - d(L^i_t)} s.t. \quad u' (L^i + M^i) \geq \frac{\beta}{\Pi^t_{t+1}} u' (L^i_{t+1} + M^i_{t+1}) \forall t$$

$$d'(L^i_t) \leq u' (L^i_t + M^i_t) \forall t$$

where the last constraint ensures that the central bank follows a time-consistent policy and cannot commit to future monetary expansion to induce workers to produce more, as we assumed earlier.

Assigning the shadow prices $\beta^t \mu_t$ and $\beta^t \gamma_t$ to the two constraints, the associated optimality conditions are

$$FOC (L^i_t) : u' (C^i_t) - d'(L^i_t) + [\mu_t - \mu_{t-1}/\Pi^i_t] u'' (C^i_t) - \gamma_t [d'' (L^i_t) - u'' (C^i_t)] = 0$$

When the ZLB constraint is loose, the shadow prices $\mu_t$ and $\gamma_t$ are zero. If the ZLB is binding in period $t$, then $\mu_t = \frac{u'(C^i_t)-d'(L^i_t)}{-u''(C^i_t)} > 0$ reflects the labor wedge in the economy created by the lack of demand and the second constraint is trivially satisfied so $\gamma_t = 0$. If the ZLB is loose in the ensuing period $t + 1$, then the planner would like to commit to stimulate output in that period as captured by the term $-\mu_t/\Pi^i_{t+1} u'' (C^i_{t+1})$ so as to relax the ZLB constraint at date $t$, but we imposed the second constraint to reflect that the planner cannot commit to do this. Therefore $u' (C^i_{t+1}) = d' (L^i_{t+1})$ in that period and the shadow price $\gamma_{t+1}$ adjusts so that the optimality condition is satisfied $\gamma_{t+1} = \frac{-\mu_t u'' (C^i_t) + \mu_{t+1} \gamma_t}{d'(L^i_{t+1}) - u'' (C^i_{t+1})} > 0$.

The externalities of capital inflows in periods $t$ and $t + 1$ in such an economy are given by the partial derivatives

$$V_{M,t} (\cdot) = [\mu_t - \mu_{t-1}/\Pi^i_t + \gamma_t] u'' (C^i_t)$$

35
If the economy experiences a liquidity trap in period $t$ but has left the trap in period $t + 1$, then $V_{M,t} = \mu_t u''(C_t^i) = -u'(C_t^i) - d'(L_t) < 0$ - the externality from a unit capital inflow is to reduce aggregate demand by one unit, which wastes valuable production opportunities as captured by the positive labor wedge $u'(C_t^i) - d'(L_t)$. It is optimal to set the policy instrument $\xi_t^i = 1 - \frac{d(L_t)}{w(L_t)} > 0$ precisely such as to reflect this social cost, thereby restricting capital inflows or encouraging capital outflows.

In the following period, it is beneficial to commit to setting $\xi_{t+1}^i < 0$ so as to restrict capital outflows or subsidize capital inflows since

$$V_{M,t+1} = -\mu_t / \Pi_{t+1} \left[ \frac{d''(L_{t+1})}{d''(L_{t+1}) - u''(C_{t+1}^i)} \right] u''(C_{t+1}^i) > 0$$

This has the effect of raising future consumption, which stimulates consumption during the liquidity trap by relaxing the ZLB constraint (25).\(^{17}\)

Note that the capital account interventions of a planner in this setting are second-best policies since the first-best policy would be to restore price flexibility to abolish the ZLB constraint.

### 6 Financial Constraints

This section analyzes how capital controls can be employed to deal with financial constraints in international capital markets. We delineate circumstances under which a global planner can fully circumvent financial constraints. Even though the conditions necessary for this may not always be met in practice, they are instructive for how globally coordinated capital controls may contribute to mitigating financial constraints. Next we study prudential capital controls that are imposed to alleviate domestic pecuniary externalities as in Korinek (2010).

#### 6.1 Financial Constraints and Welfare Effects

Assume that consumers in country $i$ are subject to a commitment problem that limits how much they can borrow from international lenders.\(^{18}\) For now, we assume that consumers may threaten to abscond and renegotiate their debts after obtaining loans. If they do so, international lenders can take them to court and seize at most $-\phi^i > 0$ from them, which is a country-specific constant that reflects the quality of creditor

\(^{17}\)In a time-consistent setting for capital account interventions, the planner would not be able to commit to future policy actions. The intervention during a liquidity trap would still be given by the same expression $V_{M,t} = d'(L_t) - u'(C_t^i)$, but after the liquidity trap has passed the planner would find $V_{M,t+1} = 0$ and no further intervention would occur.

\(^{18}\)Since all agents within a given economy are identical, there is no domestic bond market.
protections in country $i$. To avoid absconding, lenders impose a constraint on new borrowing of\footnote{Our main findings are unaffected if we impose the constraint on repayments $b_{t+1}^i \geq \phi^i$ instead of new borrowing.}

\[
\frac{b_{t+1}^i}{R_{t+1}} \geq \phi^i
\]  

(26)

When this constraint (26) is binding, equilibrium borrowing is determined by $b_{t+1}^i/R_{t+1} = \phi^i$, and there is a wedge in the Euler equation of constrained consumers that corresponds to the shadow price of the constraint $\lambda^i_{t+1}$,

\[
(1 - \tau_{t+1}^i) u''(c_t^i) = \beta R_{t+1} u''(c_{t+1}^i) + \lambda_{t+1}^i
\]  

(27)

The welfare effects of marginal changes in $\phi^i$ depend on both the constraint itself and on the resulting general equilibrium effect on the world interest rate. The marginal welfare cost of tightening the constraint $d\phi^i > 0$ for a given interest rate is is $\lambda_{t+1}/R_{t+1}$. The tighter borrowing limit reduces the effective global demand for bonds and reduces the world interest rate by

\[
\frac{dR_{t+1}}{d\phi^i} = -m^i \frac{\partial R_{t+1}}{\partial B_{t+1}^i} < 0
\]

which is always beneficial for country $i$ since a constrained country is by definition a borrower. The total welfare effect is the sum of the two,

\[
\frac{dW_t^i}{d\phi^i} = u_{T,t}^i m^i \eta_{RB-i} - \frac{\lambda_{t+1}^i}{R_{t+1}} \geq 0
\]  

(28)

For relatively lax borrowing constraints in large economies with $m^i > 0$, the interest effect is larger and the constrained country benefits from a tightening of the constraint. This may seem counter-intuitive, but recall that a tighter constraint moves the country closer to the level of borrowing that would be chosen by a monopolistic planner who internalizes the country’s market power. For relatively tight constraints, the welfare cost of the constraint outweighs any positive terms of trade effects on the world interest rate. The cutoff at which $dW_t^i/d\phi^i = 0$ corresponds to the monopolistic level of borrowing that is described in more detail in section 4.2.

The interest rate effects of one country’s tightening borrowing limit on other countries are identical to those described in lemma 1 – they improve the welfare of other borrowing countries (who compete for funds) and reduce the welfare of lending countries (who experience a decline in the effective demand for their lending).

6.2 Restoring the First-Best Allocation

Binding financial constraints impede optimal consumption smoothing and therefore pose a challenge to a planner who wants to equate the marginal rates of substitution
of different agents. Here we delineate circumstances under which a global planner can in fact employ capital controls to fully undo the effects of financial constraints $\phi^i < 0$ in a two-country framework.

The planner can do so by taking advantage of the indeterminacy in the setting of capital controls and the world interest rate that we identified in proposition 2. What matters for the decisions of decentralized agents is the fraction $\frac{R_{t+1}}{1-\tau_{t+1}}$ not the levels of the interest rate and the capital controls.

Although the conditions necessary for restoring the first-best are unlikely to be met in practice, they are instructive for how globally coordinated capital controls may contribute to mitigating financial constraints. Hence, a global planner can restore the first-best equilibrium in our setup.

**Proposition 5 (Restoring the First-Best)** In a world with two countries $i, j$ that are subject to the financial constraint (26), a global planner who can determine the capital controls $\tau^i_t, \tau^j_t$ of both countries can implement the first-best equilibrium.

**Proof.** Denote variables in the first-best allocation by $\{c^i_t\}, \{b^i_t\}$ and $\{R^*_t\}$ and focus on a period $t$ in which the first-best level of new borrowing is below what the financial constraint permits, i.e. $b^i_{t+1}/R^*_t < \phi^i$. The global planner implements the first-best allocation by reducing both the repayment and the new borrowing in period $t$ by the excess over the borrowing limit $\Delta = \phi^i - b^i_{t+1}/R^*_t$, i.e. by setting $b^i_t = b^i_t + \Delta$ and $b^i_{t+1}/R^*_t = b^i_{t+1}/R^*_t + \Delta = \phi^i$. This leaves period $t$ consumption unchanged. At the same time, the planner uses his control over the interest rates $R_t$ and $R_{t+1}$ to keep borrowing in the previous period $b^i_t/R_t = b^i_t/R_t$ and the repayment next period $b^i_{t+1}/R^*_t$ constant at the first-best levels, which guarantees that consumption in all time periods is unchanged. Substituting the latter two equations into the former two, we find

$$R_t = R^*_t \frac{b^i_t + \Delta}{b^i_t} < 1 \quad \text{and} \quad R_{t+1} = R^*_t \frac{b^i_{t+1}/R^*_t + \Delta}{b^i_{t+1}/R^*_t} > 1.$$

In other words, the planner reduces the world interest rate for repayments and increases it proportionately for new borrowing in period $t$. To achieve this, he imposes capital controls

$$\tau^i_t = -\frac{\Delta}{b^i_t} > 0 \quad \text{and} \quad \tau^i_{t+1} = \frac{\Delta}{b^i_{t+1}/R^*_t + \Delta} < 0.$$

By engaging in this manipulation in a given period $t$, the planner can circumvent any level of the borrowing constraint that satisfies $\bar{b}^i_{t+1} < 0$. The intervention can be repeated for arbitrarily many periods. ■

Intuitively, the global planner circumvents the financial constraint by reducing both the repayment and the (constrained) level of new borrowing in period $t$ by
identical amounts while manipulating interest rates such that nothing changes in adjacent time periods. In period $t-1$, both countries agree to impose capital controls $\tau_t > 0$ (i.e. controls on inflows in the borrowing country and subsidies to outflows in the lending country) to push down the world interest rate and “help” country $i$, which would otherwise be constrained in the following period, to reduce its repayment $b_i^t$ for a given level of borrowing $b_i^t/R_t$. In period $t$, both countries agree to impose capital controls in the opposite direction (i.e. subsidies on inflows in the borrowing country and taxes on outflows in the lending country) to push up the world interest rate. This implies that the borrowing country obtains less $b_i^{t+1}/R_{t+1}$ for a given face value of debt $b_i^{t+1}$, which makes up for the loss in interest payments that the lending country would otherwise have suffered.

**Remark 1:** The results of proposition 5 are robust to alternative specifications of the financial constraint. For example, the same argument could be applied to period $t+1$ if the interest rate was omitted in the denominator of constraint (26). What matters is that the planner can change the amount borrowed $b_t/R_t$ and repaid $b_t$ independently in two consecutive periods because he can determine the level of the interest rate $R_t$.

**Remark 2:** Our results can easily be generalized to a world with multiple states of nature in which two countries trade contingent securities $b_{i+1}^t$ in a complete market. Following the recipe of proposition 5, a global planner would reduce the payoffs of contingent liabilities of the borrowing country $i$ that pay out in states of nature when the constraint is binding by imposing capital controls $\tau_i^* > 0$ and reduce new borrowing once such a state is reached. In practice, securities that pay out in constrained states of nature can be interpreted as “hard claims” such as dollar debt. The planner would impose inflow controls in the recipient country and subsidies to outflows in the source country. This reduces the need for new financing in country $i$ if one of those states of nature materializes. In that event, the planner would subsequently impose capital controls in the opposite direction on all securities (i.e. subsidies on inflows in the recipient country and on outflows in source country) to push up the world interest rate and compensate the source country for the lower returns in the prior period. If a different state of nature materializes in which there is no risk of binding constraints, the planner would take no further action in period $t$. Again, the resulting real allocations replicate the first-best.

**Remark 3:** There are also a number of limitations to restoring the first best. In particular, the implementation of proposition 5 requires that unconstrained countries set capital controls in favor of the the constrained country in period $t-1$ and constrained countries return the favor in period $t$. This requires a significant extent of cooperation and commitment. The first-best may therefore be difficult to implement through capital controls in practice. If a global planner has a superior enforcement technology, similar mechanisms such as crisis lending to provide a constrained country with additional borrowing capacity may restore the first best.
7 Imperfect Capital Controls

This section analyzes capital controls that are imperfect policy tools and investigates under what circumstances such imperfections lead to a case for global coordination of capital control policies. In the previous section, we emphasized that the international spillover effects of perfectly targeted capital controls constitute pecuniary externalities that are mediated through a well-functioning market and therefore lead to Pareto-efficient outcomes, as long as domestic policymakers act competitively and impose such controls to internalize domestic externalities. This result follows from the first welfare theorem if we view the domestic policymakers in each country as competitive agents who optimize domestic welfare. By implication, we found that there is no need for global coordination to achieve Pareto-efficient outcomes. Our result relies on the assumption that domestic policymakers have the instruments to perfectly and costlessly control the amount of capital flows to the country.

In practice capital controls sometimes differ from the perfect policy instruments that we have depicted in our earlier analysis in that they create ancillary distortions (see e.g. Carvalho and Marcio, 2006). In the following two subsections, we analyze two types of such distortions: implementation costs of capital controls and imperfect targeting of capital controls. We formalize both examples and analyze whether a global planner could achieve a Pareto improvement by coordinating the capital control policies of different countries in the presence of such ancillary distortions.

7.1 Costly Capital Controls

The simplest specification of such a setup is to assume that capital controls impose a resource cost $C^i(\tau)$ on the economy that represents enforcement costs or distortions arising from attempts at circumvention. Assume that the function $C^i(\cdot)$ is twice continuously differentiable and satisfies $C^i(0) = C''^i(0) = 0$ and $C''^i(\tau) > 0 \forall \tau$, i.e. it is convex.

The optimization problem of a national policymaker, where we use the summary notation $V^i(b^i) = V^i(b^i; b^i)$, is then

$$\max_{b^i, c^i, \tau^i} u(c^i) + \beta W^i(b^i) - \lambda^i \left[ c^i - y^i + \frac{b^i}{R} + C^i(\tau^i) \right] - \mu^i \left[ (1 - \tau^i) u'(c^i) - \beta RV''^i(b^i) \right]$$

The first-order conditions are

$$FOC(b^i) : \quad \beta W''^i(b^i) = \lambda^i / R - \mu^i \beta RV''^i(b^i)$$
$$FOC(c^i) : \quad u'(c^i) = \lambda^i + \mu^i (1 - \tau^i) u''(c^i)$$
$$FOC(\tau^i) : \quad \lambda^i C''^i(\tau^i) = \mu^i u'(c^i)$$

20 Analogous results can be derived if the cost of capital controls is proportional to the amount of bond holdings, e.g. $c(\tau, b) = C(\tau b)$, which may specifically capture the costs associated with attempts at circumvention.
and can be combined to the optimality condition
\[
\frac{u'(c^i)}{u'(c^i)} = \beta RW''(b) \cdot \frac{1 + \frac{\beta RV'u''}{(u')^2} C'^i}{1 - \frac{\beta RV''}{u'} C'^i}
\] (29)

We find:

**Proposition 6 (Costly Capital Controls)** If capital controls impose a resource cost \( C^i(\tau^i) \) as defined above and if \( \xi^i \neq 0 \), then a national planner imposes an optimal level of capital controls of the same sign as \( \xi^i \) but of smaller absolute magnitude, i.e. \( \tau^i \) satisfies \( 0 < |\tau^i| < |\xi^i| \).

**Proof.** The planner implements the optimality condition (29) by setting the capital control in the decentralized optimality condition (3) to
\[
\tau^i = \frac{\beta R\xi^i}{u'(c^i)} + \beta R C'^i \cdot \frac{V''u'' + u'V''}{(u')^2 + \beta RV'u''C'}
\]
The first additive term corresponds to the optimal costless capital controls \( \hat{\tau}^i \). If this term is positive because the country experiences a negative externality \( \xi^i > 0 \) from capital inflows, then \( C'^i > 0 \) and the second additive term is negative, which mitigates the optimal magnitude of the capital control to \( \tau^i < \hat{\tau}^i \). (This holds as long as the denominator is positive, i.e. \( (u')^2 + \beta RV'u''C' > 0 \), which is satisfied as long as the marginal cost of the capital control \( C' \) is not too large.) For \( \xi^i > 0 \), the second term never flips the sign of the control \( \tau^i \) to make it negative. If it did, then \( C'^i \) would switch sign as well and the second term would become positive, leading to a contradiction. The argument for \( \xi^i < \tau^i < 0 \) follows along the same lines. ■

### 7.2 Global Coordination of Costly Capital Controls

We next determine under what conditions the equilibrium in which each national planner imposes capital controls according to equation (29) is globally Pareto efficient. In other words, if national planners follow the described rule, can a global planner achieve a Pareto improvement on the resulting equilibrium?

We analyze a global planner who maximizes global welfare in the described environment who has access to lump-sum transfers between countries. This implies that he is not bound by the period 1 budget constraints of individual countries and can undo the redistributions that stem from changes in the world interest rate.

Formally, a global planner maximizes the sum of the surplus of all nations for some set of welfare weights \( \{\phi^i\} \). He internalizes that the world interest rate \( R \) is a choice variable and that the optimality conditions of individual agents (with shadow price \( \mu^i \)) as well as global market clearing must hold, i.e. \( \Sigma_i m^i b^i = 0 \) (with shadow price \( \nu \)). In addition, we include a transfer \( T^i \) in our optimization problem, which needs
to satisfy global market clearing \( \Sigma_i m^i T^i = 0 \) (with shadow price \( \gamma \)). The associated Lagrangian is

\[
\mathcal{L} = \sum_i \phi^i \left\{ u^i (c^i) + \beta W^i (b^i) - \lambda^i \left[ c^i - y^i + b^i / R + C^i (\tau^i) - T^i \right] - 
- \mu^i \left[ (1 - \tau^i) u^i (c^i) - \beta RV^i (b^i) \right] \right\} - \nu \sum_i m^i b^i - \gamma \sum_i m^i T^i
\]

The first-order conditions of the global planner are

\[
FOC (b^i) : \beta W^{i^u} (b^i) = \lambda^i / R - \mu^i \beta RV^{i^u} (b^i) + m^i \nu / \phi^i \\
FOC (c^i) : u^i (c^i) = \lambda^i + \mu^i (1 - \tau^i) u^i (c^i) \\
FOC (\tau^i) : \lambda^i C^{i^u} (\tau^i) = \mu^i u^i (c^i) \\
FOC (R) : \sum_i \phi^i \left\{ \frac{\lambda^i b^i}{R} + \mu^i (1 - \tau^i) u^i (c^i) \right\} = 0 \\
FOC (T^i) : \phi^i \lambda^i = \gamma m^i
\]

The uncoordinated Nash equilibrium among national planners is constrained Pareto efficient under the given set of instruments if and only if we can find a set of welfare weights \( \{ \phi^i \} \) such that the allocations of national planners satisfy the maximization problem of the global planner. If we substitute the allocations from the Nash equilibrium, we find that the second and third optimality conditions are unchanged compared to the national planner’s equilibrium and can be solved for \( \lambda^i \) and \( \mu^i \) that are identical to the shadow prices in the Nash equilibrium between national planners. Substituting these in the optimality condition \( FOC (b^i) \), we find that this condition is satisfied for all countries if we set \( \nu = 0 \). The fifth optimality condition is satisfied if we set \( \phi^i = \gamma m^i / \lambda^i \forall i \). The Nash equilibrium among planners is therefore efficient if the described variables also satisfy the fourth optimality condition \( FOC (R) \).

**Proposition 7 (Coordination of Costly Controls with Transfers)** If capital controls to correct national externalities are costly, then the uncoordinated Nash equilibrium between national planners is Pareto efficient with respect to a global planner who can engage in transfers if and only if the resulting allocation satisfies

\[
\sum_i m^i (1 - \tau^i) C'' (\tau^i) = 0 \tag{30}
\]

**Proof.** The optimality condition (30) can be obtained by substituting \( FOC (T^i) \) into the condition \( FOC (R) \) and accounting for market clearing \( \Sigma_i m^i b^i = 0 \) as well as for \( FOC (\tau^i) \).

In a Pareto-optimal allocation, the weighted average marginal distortion imposed by capital controls must be zero. If there are no externalities, this can be achieved
by having zero controls in all countries. Otherwise, the planner combines controls in capital inflow and outflow countries in a way that their weighted average marginal distortion is zero.

The planner’s country weights $\phi^i$ do not show up in condition (30) since the condition is purely about efficiency, i.e. about minimizing the overall resource cost of imposing capital controls. Since the planner has access to lump-sum transfers, she can undo any redistributions created by movements in the interest rate according to her welfare weights.

8 Conclusions

This paper has studied the effects of capital controls in a general equilibrium model of the world economy and has delineated under what conditions such controls may be desirable from a global welfare perspective. In our positive analysis, we found that capital controls in one country push down the world interest rate and induce other countries to borrow and spend more. We then analyzed three motives for imposing capital controls. If capital controls are imposed to combat national externalities, then controls are Pareto efficient from a global welfare perspective. As long as national policymakers can impose such controls optimally, there is no need for global coordination of such controls as the Nash equilibrium between national planners is socially efficient. Under fairly mild conditions, capital controls that combat national externalities can make everybody in the world economy better off.

On the other hand, if national planners impose capital controls to exert market power and manipulate a country’s terms of trade, then they have beggar-thy-neighbor effects and reduce global welfare.

If we deviate from the assumption that national policymakers can optimally address externalities, for example, if imposing capital controls has distortionary side-effects or if they cannot perfectly target different types of capital flows, then global policy coordination is also desirable. The goal of such coordination is to minimize the aggregate distortions created from capital controls.

Finally, if prudential capital controls are imposed that are designed to mitigate the risk of systemic crises after a surge in capital inflows, we have shown that our insights on technological externalities carry through. In particular, capital controls are Pareto efficient from a global perspective. Under certain circumstances, they may even lead to a global Pareto improvement since they reduce financial instability and create the potential for larger gains from trade in the future.

References


A Mathematical Appendix

A.1 Decentralized demand for bonds

We denote the optimization problem of an individual consumer in recursive form as

$$V^i_{t+1}(b^i_{t+1}) = \max_{b^i_{t+1}} u(y^i_{t+1} + b^i_{t+1} - (1 - \tau^i_{t+1}) b^i_{t+1}/R_{t+1} - T^i_{t+1}) + \beta V^i_t(b^i_{t+1})$$

and observe that $$V^i_{t+1}(b^i_{t+1}) > 0 > V^i_{t+1}(b^i_{t+1})$$. For given $$b^i_{t+1}$$ and after substituting the government budget constraint, the Euler equation that results from this problem defines an implicit function

$$F(b^i_{t+1}; R_{t+1}, \tau^i_{t+1}) = (1 - \tau^i_{t+1}) u'(y^i_{t+1} + b^i_{t+1}/R_{t+1} - \beta R_{t+1} V^i_{t+1}(b^i_{t+1})) = 0$$

which satisfies

$$\frac{\partial F}{\partial b^i_{t+1}} = - (1 - \tau^i_{t+1}) u''(c^i_{t+1})/R_{t+1} - \beta R_{t+1} V^i_{t+1}(b^i_{t+1}) > 0$$

$$\frac{\partial F}{\partial R_{t+1}} = (1 - \tau^i_{t+1}) u''(c^i_{t+1}) b^i_{t+1}/(R_{t+1})^2 - \beta V''(b^i_{t+1}) \geq 0$$

$$\frac{\partial F}{\partial \tau^i_{t+1}} = -u'(c^i_{t+1}) < 0$$

The first partial derivative is always positive, allowing us to implicitly define a demand function $$b^i_{t+1}(R_{t+1}; \tau_{t+1})$$.

The second partial derivative is negative as long as saving $$b^i_{t+1}$$ is sufficiently high. Specifically, we write the condition as

$$(1 - \tau^i_{t+1}) u''(c^i_{t+1}) b^i_{t+1}/R_{t+1} - \beta R_{t+1} V''(c^i_{t+1}) < 0$$

We employ the Euler equation to substitute for the second term and rearrange to

$$or \quad \frac{b^i_{t+1}/R_{t+1}}{c^i_{t+1}} > \frac{u'(c^i_{t+1})}{c^i_{t+1} u''(c^i_{t+1})} = -\sigma(c^i_{t+1})$$

i.e. the savings/consumption ratio is greater than the negative of the elasticity of intertemporal substitution $$\sigma(c^i)$$, as we stated in Assumption 1. If this inequality is satisfied then for given $$b^i_{t+1}$$, the demand function $$b^i_{t+1}(R_{t+1}; \tau_{t+1})$$ is strictly increasing in $$R_{t+1}$$, which allows us to invert it into a strictly increasing inverse demand function $$R_{t+1}(b^i_{t+1}; \tau_{t+1})$$.

The third partial derivative is always negative – this is because we assumed that the revenue from capital controls is rebated so that there are only substitution effects and no income effects from capital controls.