

Diversification Benefits of REIT Preferred and Common Stock: New Evidence from a Utility based Framework

Walter I. Boudry, Jan A. deRoos, and Andrey D. Ukhov¹

December 20, 2013

Abstract

We study the diversification benefits of REIT preferred and common stock using a utility based framework. Taking the view of a long run investor, we conduct our analysis using data from 1992 to 2012. We examine optimal mean-variance portfolios of investors with different levels of risk aversion given access to different classes of assets and establish two main results. First, REIT preferred and common stock provides significant diversification benefits to investors. REIT common stock helps low risk aversion investors attain portfolios with higher returns, while REIT preferred stock helps high risk aversion investors by providing a venue for risk reduction. Both asset classes receive material allocations over plausible levels of risk aversion. Second, while REIT preferred stock appears to behave somewhat like a hybrid debt/equity asset, its risk/return profile appears to not easily be replicated by those asset classes. When given the opportunity, investors will reduce allocations to REIT common stock and investment grade bonds and invest in REIT preferred stock.

¹ Corresponding author: Walter Boudry, School of Hotel Administration, Cornell University, wb242@cornell.edu; Jan A. deRoos, School of Hotel Administration, Cornell University, jad10@cornell.edu; Andrey Ukhov, School of Hotel Administration, Cornell University, au53@cornell.edu. We would like to thank Cornell's Center for Real Estate and Finance for support, Crocker Liu and Bob Jarrow for helpful comments, Dmitry E. Zeliakh for superb assistance with computations, and MSCI for providing the REIT preferred stock index used in our analysis. All errors are our own.

1 Introduction

We employ a utility based approach to examine the diversification benefits of REITs. In the standard Markowitz (1952) setting where investors are unconstrained in their portfolio formation and can long and short the risk free asset, all investors hold portfolios that are a combination of the risk free asset and the market portfolio. In this setting maximizing the Sharpe Ratio of the market portfolio is equivalent to maximizing utility. Any asset that increases the Sharpe Ratio of the market portfolio is valuable to investors. However, we know investors do face constraints and cannot borrow at the risk free rate. Importantly, when investors are constrained, the way we think about diversification benefits also changes. Investors no longer hold portfolios comprised of just the risk free asset and the market portfolio. Their portfolios differ based on their risk preferences. In this setting, the question is not whether an asset provides diversification benefits? But, to whom does the asset provide diversification benefits?

Our motivation for employing a utility based approach to examine diversification is twofold. First, we know investors face constraints. They can't short all assets or borrow at the risk free rate. And as we show, this has material implications for how we should judge the diversification benefits of an asset. Second, using the standard Markowitz framework typically employed in the prior literature,² it is unclear that REITs provide any diversification benefits to investors when they are given anything but a very restricted set of assets to invest in.³ It is possible that REITs are important to investors, we have just been examining the issue in a framework that fails to detect these benefits.

To examine the benefits that the REIT industry may provide to investors, we include both REIT common stock and REIT preferred stock in our analysis. To our knowledge we are the first study to examine the benefits of REIT preferred stock to investors. While preferred stock has received some attention in the academic literature, these papers tend to focus on either why firms choose to issue preferred stock over other classes of securities,⁴ the role of preferred stock in the capital structure,⁵ or on the pricing of preferred stock.⁶ The issue we examine in this paper is why investors might choose to hold these securities. This is important because preferred stock forms a significant portion of REIT capital structure. In equilibrium, for REITs to be able to raise capital through preferred issuance, this asset class needs to offer attractive risk-return characteristics for investor portfolio formation and diversification.

To make our results as general as possible, we give our investor access to global and US large cap stock indices, US small and mid-cap equity portfolios, US small and mid-cap growth and value portfolios, investment grade and high yield bond indices, an equity REIT index, and a REIT preferred stock index. We conduct our analysis using data from the largest time series available, with our data spanning the period November 1992 to November 2012.⁷

² Sa-Aadu, Shilling, and Tiwari (2010) is an exception.

³ See Chiang and Lee (2007) for a discussion.

⁴ See, Boudry, Kallberg, and Liu (2010).

⁵ See, for example, Ott, Riddiough and Yi (2005), and Heinkel and Zechner (1990).

⁶ See, for example, Bildersee (1973), Sorensen and Hawkins (1981), Emanuel (1983), and Ferreira, Spivey, and Edwards (1992).

⁷ This sample period covers the modern REIT era. While REITs have existed since 1960s, our sample period is the most relevant for a current REIT investor. Given that our methodology relies on return dynamics and there are large structural differences between the old and modern REIT eras, it is sensible to confine our interest to the latter time period.

We find two main results. First, both REIT common and preferred stock provide diversification benefits, but they provide these benefits to different sets of investors. In our utility based framework under realistic constraints, investors separate based on risk aversion. Investors with different levels of risk aversion will hold materially different portfolios. REIT common stock allows investors to form portfolios with higher total return. This is beneficial to low risk aversion investors who seek to form such portfolios. On the other hand, REIT preferred stock is a venue for risk reduction. This is prized by high risk aversion investors who allocate heavily to preferred stock. This view of REIT preferred and common stock is quite different to the one that is obtained using the classical unconstrained Markowitz framework. In that setting REITs provide little benefit. Adding realistic constraints to the investor's optimization problem materially changes how we should judge the benefits of the REIT sector to investors.

Second, REIT preferred stock play an important role in portfolio formation. While the common perception is that preferred stock is some kind of hybrid between common stock and investment grade bonds, our analysis indicates that preferred stock is actually preferred to a combination of those asset classes. That is, when an investor is given access to REIT preferred stock, they will reduce their allocation to REIT common stock and investment grade bonds. This helps explain why REITs choose to issue these securities: there is a set of investors out there who prefer REIT preferred stock over holding a combination of REIT common stock and investment grade bonds.

Overall our results are relevant to both academics and practitioners. First, they suggest that we need to get away from judging diversification benefits simply using Sharpe Ratios and correlations. Investors face constraints and these constraints materially change how we should judge the benefits of asset classes. This is especially true for the REIT sector, because based on Sharpe Ratios, the REIT sector provides no real benefits to investors. Once we impose those constraints we observe the benefits that these securities provide. Second, they also suggest that investors, especially those with high risk aversion who focus on equity portfolios, would be well served by investing in REIT preferred stocks. Doing so would be beneficial to the risk adjusted performance of their portfolios.

The remainder of this study is organized as follows. Section 2 provides a review of the literature, while Section 3 describes our empirical methodology and data. Section 4 presents our results, and Section 5 provides our conclusions.

2 Literature Review

The literature that this paper is most closely related to is the literature examining the role of real estate in mixed asset portfolios. This literature applies the Modern Portfolio Theory framework of Markowitz (1952) and Merton (1972) to examine the benefit of holding real estate (both direct and securitized in the form of REITs) in mixed asset portfolios.

Beginning in the early 1980's a series of papers demonstrates that positive allocations to real estate produce superior risk-adjusted returns to portfolios that omit real estate, especially for risk sensitive investors. These studies rely on the NCREIF index of real estate returns, a database of private real estate returns or an index of returns from comingled real estate funds. Using this data, Miles and McCue (1984) find that real estate acts as an inflation hedge and that there are other intriguing properties of real estate returns, notably the low correlation with stocks and bonds; Ross and Zissler (1991) estimate allocations in the range of 9 to 13 percent while Folger (1984) makes the case for a 20% allocation using these data. Ibbotson and Siegel (1984) confirm the findings of Miles and McCue (1984) and Folger (1984), but also show that the pricing of real estate reacts

to very different stimuli than stocks and bonds; real estate prices are shown to be sensitive to residual risk and factors such as taxes, marketability and information costs that are not captured in the traditional measure of risk as represented by market or beta risk.

An early critique of these studies is that they rely on appraisal based returns indexes, which are known to understate the volatility of real estate returns, called appraisal smoothing. Corgel and deRoos (1999) correct for the understated volatility and find significant allocations to real estate in the ranges found by previous authors. While the correction for appraisal smoothing adds volatility to real estate returns, thus reducing the allocation, the correction also makes real estate less correlated with stocks and bonds, increasing the allocation. Kallberg, Liu, and Greig (1996) find support for allocations of approximately 9% to real estate, but in addition, find a “size effect,” that is, small properties tend to bring much larger diversification benefits to mixed asset portfolios than large properties due to their lower correlation with other financial assets while maintaining a similar risk/return profile as large properties.

Starting in the mid-1990’s, real estate researchers added REITs to the NCREIF return series, allowing an exploration of both public market real estate returns in addition to the private market returns. Gyourko and Nelling (1996) explore the impact of different property types and location in real estate portfolio allocation and find that diversification by location and property type is independent to market based diversification measures. Kallberg, Liu, and Trzcinka (2000) find that REIT mutual funds have persistent positive net alphas, a significant finding. Combining the conventional wisdom that real estate provides diversification and an inflation hedge, Lin and Yung (2006) find that REIT returns affect REIT equity market flows, not vice versa, meaning that the market demand for REIT equities is closer to horizontal than downward sloping.

Some papers have questioned the benefits of REITs in mixed asset portfolios. Most notably, Chiang and Lee (2007) demonstrate using spanning tests that the diversification benefits of REIT common stock is very sensitive to the set of benchmark assets. In many cases REIT common stock is spanned by the test assets and as such provides no diversification benefits.

More recently, Sa-Aadu, Shilling, and Tiwari (2010) examine the performance of real estate in mixed asset portfolios in a conditional asset pricing framework and find that real estate is one of two asset classes that deliver portfolio gains during bad times; that is, when investors really care about returns. They argue that investors may underweight real estate due to this misunderstanding of real estate return behavior in bad times.

3 Data and Methodology

3.1 Methodology

In the standard Markowitz (1952) setting where investors are unconstrained in their portfolio formation and can long and short the risk free asset, all investors hold portfolios that are a combination of the risk free asset and the market portfolio (two fund separation). The composition of the market portfolio (relative weights of different risky securities) is independent of investor preferences. In this setting diversification benefits provided by a given risky asset can be measured by a Sharpe ratio and the improvement in the Sharpe ratio is independent of investor risk preferences. However, when investors face constraints and cannot borrow at the risk free rate there are important changes. Two fund separation no longer holds and the way we think about diversification benefits provided by a given risky asset also changes. Investors no longer hold risky assets in

the same proportion, their portfolio holdings depend on their risk preferences. We now develop this case in more detail.

To illustrate the role of constraints, consider a simple constrained portfolio problem. Let there be two risky assets, with returns jointly normally distributed with expected returns μ_1, μ_2 , standard deviations σ_1, σ_2 , and correlation ρ .⁹ Without loss of generality assume that $\mu_1 < \mu_2$ and $\sigma_1 < \sigma_2$. Consider investors with negative exponential utility of wealth, $U(W) = -e^{-a_i W}$, where $a_i > 0$ is risk aversion for investor i . An investor is endowed with initial wealth, W_0 , and maximizes expected value of terminal wealth. When there is no risk-free asset and no short sales of risky assets, an investor chooses investments in the two risky assets, w_1 and w_2 , to maximize expected utility,

$$\max_{\substack{w_1, w_2 \\ w_1 \geq 0, w_2 \geq 0 \\ w_1 + w_2 = W_0}} E[U(\tilde{W})] = \max_{\substack{w_1, w_2 \\ w_1 \geq 0, w_2 \geq 0 \\ w_1 + w_2 = W_0}} -e^{-a_i(\mu_p - 0.5 a_i \sigma_p^2)}$$

where μ_p and σ_p are the expected return and standard deviation of the portfolio with w_1 invested in asset one and w_2 invested in asset two. The equality obtains because a linear combination of normally distributed variables (assets) is normally distributed (portfolio), and by taking the integral of negative exponential distribution with respect to the probability density function of a Normal distribution. To solve this maximization problem, we need to solve

$$\max_{\substack{w_1, w_2 \\ w_1 \geq 0, w_2 \geq 0 \\ w_1 + w_2 = W_0}} (\mu_p - 0.5 a_i \sigma_p^2)$$

Let λ_1 and λ_2 be the Lagrange multipliers for the two inequality constraints, and φ the Lagrange multiplier associated with the budget constraint. Forming the Lagrangian and differentiating it with respect to w_1, w_2 , the solution to the problem is:

1. Corner solution, with all wealth in the first risky asset: $w_1 = W_0, w_2 = 0, \lambda_1 = 0, \lambda_2 = \mu_1 - \mu_2 - a_i W_0(\sigma_1^2 - \rho\sigma_1\sigma_2), \varphi = 1 + \mu_1 - a_i W_0 \sigma_1^2$.

2. Corner solution, with all wealth invested in the second risky asset (symmetric to the first case above): $w_1 = 0, w_2 = W_0, \lambda_1 = -\mu_1 + \mu_2 - a_i W_0(\sigma_2^2 - \rho\sigma_1\sigma_2), \lambda_2 = 0, \varphi = 1 + \mu_2 - a_i W_0 \sigma_2^2$.

3. Interior solution (both risky assets held in positive quantity): $\lambda_1 = 0, \lambda_2 = 0$,

$$w_1 = \frac{\mu_1 - \mu_2 + a_i W_0(\sigma_2^2 - \rho\sigma_1\sigma_2)}{a_i(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)}$$

$$w_2 = \frac{-\mu_1 + \mu_2 + a_i W_0(\sigma_1^2 - \rho\sigma_1\sigma_2)}{a_i(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)}$$

⁹ The results in this section hold for the general case of n risky assets with expected returns vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. The two asset case is described for tractability and is without loss of generality. Dybvig (1984) studies the shape of efficient frontiers under constraints but does not include investor preferences.

$$\varphi = \frac{-(2 + \mu_1 + \mu_2)\rho\sigma_1\sigma_2 + (1 + \mu_1)\sigma_2^2 + (1 + \mu_2)\sigma_1^2 - a_i W_0 \sigma_1^2 \sigma_2^2 (1 - \rho^2)}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}$$

Consider the interior solution. What types of investors invest more in the first asset than in the second asset? That is, for what values of risk aversion a_i , is $w_1 > w_2$? Note that the denominator in the expressions for w_1 and w_2 is positive, because risk aversion is positive ($a_i > 0$) and $(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2) > 0$ because this is the variance of the portfolio that invests one in the first asset and minus one in the second asset. Then,

$$\begin{aligned} w_1 > w_2 &\Leftrightarrow \mu_1 - \mu_2 + a_i W_0 (\sigma_2^2 - \rho\sigma_1\sigma_2) > -\mu_1 + \mu_2 + a_i W_0 (\sigma_1^2 - \rho\sigma_1\sigma_2) \\ &\Leftrightarrow a_i W_0 (\sigma_2^2 - \rho\sigma_1\sigma_2) - a_i W_0 (\sigma_1^2 - \rho\sigma_1\sigma_2) > 2(\mu_2 - \mu_1) \\ &\Leftrightarrow a_i W_0 (\sigma_2^2 - \rho\sigma_1\sigma_2 - \sigma_1^2 + \rho\sigma_1\sigma_2) > 2(\mu_2 - \mu_1) \\ &\Leftrightarrow a_i > \frac{2(\mu_2 - \mu_1)}{W_0(\sigma_2^2 - \sigma_1^2)} \end{aligned}$$

Recall that the two assets are ordered so that $\mu_1 < \mu_2$ and $\sigma_1 < \sigma_2$. Therefore, there exists sufficiently high level of risk aversion, so that investors with risk aversion above this level of risk aversion hold more wealth in the less risky asset, than in the more risky asset.

This case shows that in the presence of constraints (and no risk free asset) investors no longer hold risky securities in the same proportion. This case also shows that investor preferences (risk aversion) affects the composition of the portfolio. Because different investors hold risky assets in different proportions, diversification benefits of a given risky asset will be different for different investors. Below we show that this intuition holds when a risk-free asset is a part of the investment opportunity set but investors cannot borrow at the risk free rate.

3.1.1 Constrained investment in the riskless asset

Now consider a case where risk free asset paying the rate of interest r is a part of the investment opportunity set, but investors cannot borrow at a risk-free rate ($w_{rf} \geq 0$). The utility maximization problem is now

$$\max_{\substack{w_1, w_2, w_{rf} \\ w_1 \geq 0, w_2 \geq 0, w_{rf} \geq 0 \\ w_1 + w_2 + w_{rf} = W_0}} E[U(\tilde{W})] = \max_{\substack{w_1, w_2, w_{rf} \\ w_1 \geq 0, w_2 \geq 0, w_{rf} \geq 0 \\ w_1 + w_2 + w_{rf} = W_0}} -e^{-a_i(\mu_p - 0.5 a_i \sigma_p^2)}$$

Form the Lagrangian,

$$\begin{aligned} L(w_1, w_2, w_{rf}, \lambda_1, \lambda_2, \lambda_3, \varphi) \\ = (1 + \mu_1)w_1 + (1 + \mu_2)w_2 + (1 + r)w_{rf} - \frac{1}{2} a_i (w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho \sigma_1 \sigma_2) \\ - \lambda_1 (-w_1 - 0) - \lambda_2 (-w_2 - 0) - \lambda_3 (-w_{rf} - 0) - \varphi (w_1 + w_2 + w_{rf} - W_0) \end{aligned}$$

Differentiating the Lagrangian and equating the derivatives to zero, we obtain characterization of the solution (detailed characterization of the solution is given in the Appendix). The solution consists of seven cases: (1) Interior solution, positive investment in the risk-free asset and both risky assets. (2) Corner solution: All money is invested in the risk-free asset. (3) No investment in the first risky asset, positive investment in the

second risky asset and risk-free asset. (4) Corner solution: all funds invested in the second risky asset. (5) No investment in the second risky asset, positive investment in the first risky asset and risk-free asset. (6) Corner solution, all funds invested in the first risky asset. (7) Portfolios with zero investment in the risk free asset and non-negative investment in the risky assets.

It is shown in the Appendix that cases (2), (3), (5), and (6) are dominated by case (1). The important cases are: (1) portfolios with investment in the risk free asset and both risky assets; (7) portfolios with no investment in the risk free asset and positive investment in the two risky assets; and (4) all funds invested in the second risky asset. We now discuss these cases.

Case 1. Interior Solution: Positive investment in the risk-free rate and both risky assets:

$$w_{rf} = \frac{\mu_2\sigma_1(\sigma_1 - \rho\sigma_2) - r(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2) + a_i W_0(\rho^2 - 1)\sigma_1^2\sigma_2^2 + \mu_1\sigma_2(\sigma_2 - \rho\sigma_1)}{a(\rho^2 - 1)\sigma_1^2\sigma_2^2}$$

$$w_1 = \frac{(\mu_1 - r)\sigma_2 - (\mu_2 - r)\rho\sigma_1}{a_i \sigma_1^2\sigma_2(1 - \rho^2)}, \quad w_2 = \frac{(\mu_2 - r)\sigma_1 - (\mu_1 - r)\rho\sigma_2}{a_i \sigma_1^2\sigma_2(1 - \rho^2)}$$

Tangency portfolio (the tangency of the straight line with the origin at the risk-free rate and the efficient frontier curve) is characterized by zero investment in the risk free asset. The weights of the two risky assets in the tangency portfolio are:

$$w_{1,Tangency} = W_0 \frac{(\mu_2\rho\sigma_1 - \mu_1\sigma_2 + r(\sigma_2 - \rho\sigma_1))\sigma_2}{(r - \mu_2)\sigma_1^2 + (\mu_1 + \mu_2 - 2r)\rho\sigma_1\sigma_2 + (r - \mu_1)\sigma_2^2}$$

$$w_{2,Tangency} = W_0 \frac{(-\mu_2\sigma_1 + \rho\mu_1\sigma_2 + r(\sigma_1 - \rho\sigma_2))\sigma_1}{(r - \mu_2)\sigma_1^2 + (\mu_1 + \mu_2 - 2r)\rho\sigma_1\sigma_2 + (r - \mu_1)\sigma_2^2}$$

As investor risk aversion increases, the investor holds more in the risk free asset. As risk aversion decreases, investment in the tangency portfolio increases. This case is the closest to the canonical unconstrained portfolio problem where two fund separation obtains. An important distinction, however, exists. In the canonical unconstrained problem the two fund separation covers *all* optimal portfolios and therefore applies to *all* investors. In the constrained case we consider, this is only part of the efficient frontier, and it applies only to investors with sufficiently high risk aversion.

Case 7. Zero investment in the risk free asset and positive investment in the risky assets,

$$w_1 = \frac{\mu_1 - \mu_2 + a_i W_0(\sigma_2^2 - \rho\sigma_1\sigma_2)}{a_i(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)}$$

$$w_2 = \frac{-\mu_1 + \mu_2 + a_i W_0(\sigma_1^2 - \rho\sigma_1\sigma_2)}{a_i(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)}$$

The composition (relative investment in the risky assets) of the portfolios in this segment depends on investor risk aversion, a_i . As risk aversion decreases, a higher proportion of funds are invested in the riskier asset.

Case 4. This is simply a corner solution with all funds invested in the risky asset with the highest expected return (the second risky asset).

The efficient frontier in the standard deviation (x-axis) - return (y-axis) space is characterized as follows (the derivation is in the Appendix):

Segment 1 - Straight line with the origin at the risk-free rate, ending at the tangency portfolio. Portfolios on this segment are characterized by the case 1 and include investment in the risk-free asset and in the tangency portfolio. Investors with sufficiently high risk aversion hold portfolios on this segment,

$$a_i > \frac{(r - \mu_2)\sigma_1^2 + (\mu_1 + \mu_2 - 2r)\rho\sigma_1\sigma_2 + (r - \mu_1)\sigma_2^2}{W_0(-1 + \rho^2)\sigma_1^2\sigma_2^2}$$

Segment 2 - A curve beginning at the tangency portfolio and ending at the risky asset with the highest expected return (the second risky asset). Investors whose risk aversion satisfies the inequality below hold portfolios located in this segment,

$$\frac{\mu_2 - \mu_1}{W_0(\sigma_2 - \rho\sigma_1)\sigma_2} < a_i \leq \frac{(r - \mu_2)\sigma_1^2 + (\mu_1 + \mu_2 - 2r)\rho\sigma_1\sigma_2 + (r - \mu_1)\sigma_2^2}{W_0(-1 + \rho^2)\sigma_1^2\sigma_2^2}.$$

Segment 3 - Corner solution, with all wealth invested in the second risky asset. Investors with sufficiently low risk aversion hold this allocation,

$$0 < a_i \leq \frac{\mu_2 - \mu_1}{W_0(\sigma_2 - \rho\sigma_1)\sigma_2}.$$

What is crucial to observe is that the composition of the optimal portfolio along the overall frontier (the relative amount invested in the risky assets) depends on investor risk preferences. Investors with sufficiently high risk aversion will hold portfolios on segment 1; investors with lower risk aversion will hold portfolios on segment 2, with all portfolios on this segment having different composition of the risky assets; and finally investors with sufficiently low risk aversion will hold a corner solution (segment 3).

In this setting, asset characteristics of different assets, or in other words, diversification benefits of different assets, affect different segments of the frontier. Because risk aversion separates investors across segments of the frontier, diversification benefits will be different for investors with different risk preferences. This is our main conclusion. It implies that investor characteristics must be taken into account when measuring and discussing diversification benefits of various asset classes in the investment opportunities set.

3.1.2 A Utility-Based Framework for Measuring Diversification Benefits: Compensation Ratios

When investors face constraints the relative compositions of their optimal portfolios depend on risk preferences. Diversification benefits of a risky asset, then, need to be measured taking into account investor preferences. In other words, a utility-based framework for measuring diversification benefits is needed. To assess diversification benefits of different assets to different investors we employ compensation ratios.

Consider an investor with initial wealth W_0 solving an expected utility maximization problem with a risk free asset (no borrowing), and n risky securities (the unrestricted set). The investor achieves derived utility of wealth,

$$V_n(W_0) := E[U(\tilde{W})],$$

where the subscript n denotes that investor had access to unrestricted asset set with n risky assets. Now, restrict the investor from investing in one of the risky assets.¹⁰ Namely, the asset whose diversification benefits are being assessed. The investment opportunity set now contains $(n - 1)$ assets and the investor achieves derived utility of wealth under the constrained set (denoted by the subscript $(n - 1)$),

$$V_{n-1}(W_0) := E[U(\tilde{W})].$$

Wealth compensation, ΔW_k is the additional wealth required when asset k is removed from the investment opportunity set to restore utility to the level achieved under the unrestricted risky asset set, and is the solution to the equation,

$$V_{n-1}(W_0 + \Delta W_k) = V_n(W_0).$$

Wealth compensation is measured in terms of time zero (initial) wealth. To evaluate the impact of different assets in the investment opportunity set, consider *compensation ratio* -- a ratio of compensation required for removal of asset k relative to compensation required for removal of asset j ,

$$\text{Compensation Ratio} = \frac{\Delta W_k}{\Delta W_j}.$$

This approach can be applied to individual assets, as well as to collections of assets. For example, we examine compensation ratios (as a function of risk aversion) of the wealth required to compensate an investor for losing access to REIT preferred stock (or REIT common) dividend by the compensation required to compensate them for losing access to both REIT common and preferred stock. In this sense the ratios measure the relative importance of REIT preferred stock and REIT common to an investor over different levels of risk aversion.

3.2 Data

We collect monthly returns from November 1992 to November 2012 from four sources. Datastream provides returns on the Barclays Investment Grade Corporate Bond Index, the Barclays High Yield Corporate Bond Index, the MSCI World Ex-US index, the Russell 2000 index, the Russell 2000 growth index, the Russell 2000 value index, the Russell Mid Cap index, the Russell Mid Cap growth index, and the Russell Mid Cap value index. From SNL we collect the SNL US Equity REIT index. MSCI provides the MSCI REIT Preferred Index. From CRSP we obtain returns on the 30-day T-Bill and the returns on the S&P500.¹¹

All of the data we use in the analysis is quite standard apart from the MSCI REIT Preferred Index, so some discussion of that series is warranted. REITs are one of the few industries that issue large quantities of preferred stock. According to NAREIT, from 1992 through 2012 equity REITs raised \$52.4 billion through preferred issuance, a number larger than the \$41.3 billion raised in REIT IPOs, and approximately one fifth

¹⁰ Ukhov (2006) studies changes in the frontier caused by addition and removal of assets from the opportunity set.

¹¹ The one asset class we do exclude from the analysis is private real estate. The reason for this is that it isn't apparent that an investor can easily obtain these returns. NCREIF appraisal based returns have well known statistical issues that would be particularly problematic in a mean-variance setting and Boudry, Coulson, Kallberg and Liu (2013) show that transactions-based returns can't easily be replicated by investors.

of all the public equity issued by REITs.¹² The typical REIT preferred stock has a fixed dividend rate, is issued at par, and is callable after 5 years. In terms of dividend priority, preferred stock dividends take priority in payment to common dividends, and also typically count towards the dividend payout requirements of the REIT.¹³ The MSCI REIT Preferred Index consists of non-convertible preferred stock issued by US equity and hybrid REITs. The preferred stock must be traded on the NYSE, AMEX, or NASDAQ.¹⁴

Table 1 reports descriptive statistics for the indices used in our analysis. Panel A reports means and standard deviations, while Panel B reports correlations. The average annualized return for REIT preferred stock over the sample period was 10.3% with a standard deviation of 11.4%. This compares to an average return of 12.9% for REIT common stock, which was the asset class with the highest average return in our sample. The S&P 500 also performed well over the sample period with a mean return of 9.2%. As would be expected over a long sample, investment grade bonds had the lowest average return of 7.1%.

Turning to the correlations in Panel B, we observe some noticeable characteristics. First, preferred stock is most highly correlated with high yield bonds with a 68% correlation, but also shares significant co-movement with REITs. Second, an examination of the correlation between REIT common stock and the other equity indices shows that REITs are more highly correlated with small and mid-cap stocks than large cap, but also that REITs appear to have a significant value component to them.¹⁵ For both the mid-cap and Russell 2000 universes, REITs are more highly correlated with value stocks than growth stocks.¹⁶ Finally, the asset class that appears to be least correlated with the others is investment grade bonds. It has high correlations with only the high yield bond index and REIT preferred stock index.

Figure 1 reports the current value of a dollar invested in each asset class in 1992. Some clear patterns emerge regarding the time series behavior of the asset classes. First, consistent with the average returns observed in Table 1, REIT common stock was the best performing asset class by the end of the sample period. Notice however, that this was not always the case. Prior to 2000 REIT common shares were nearly always the worst performing equity index and up to that time had a total return nearly identical to high yield corporate bonds. In fact prior to 2002, REIT common stock and preferred stock had remarkably similar performance. Second,

¹² Lee and Johnson (2009) report that 20% of NYSE firms, 15% of AMEX firms and 175 of NASDAQ firms have preferred stock in their capital structure; Yaman (2011) reports that that slightly more than one-third of all non-IPO equity raised by U.S. firms was preferred equity over the 1985-1999 timeframe, with the U.S. public market for preferred stock reaching \$193 billion in 2005. Kallberg, Liu and Villupuran (2013) report that U.S. firms used preferred equity to raise over 47% of all equity capital from 1999 – 2005, including IPOs; further preferred equities were almost 60% of all non-IPO equity raised during this period.

¹³ See Boudry (2011) for a discussion of the dividend payout requirements of REITs.

¹⁴ We are able to obtain the constituents of the MSCI REIT Preferred Index from 2005 on. An examination of these constituents shows that the index is extremely broad based. There are over 100 constituents in the index at any time and no individual constituent is more than 2% of the index. We are also able to obtain daily REIT preferred stock prices and trading volume from 2000 on and observe that most REIT preferred stock trades at a daily frequency. This suggests that although they are less liquid than their common share counterparts, they are liquid enough for an investor to be able to form portfolios.

¹⁵ See Ambrose, Lee, and Peek (2007) for a discussion of REIT co-movement after index inclusions.

¹⁶ Notice that this is unlikely to be driven by REITs being components of midcap and small cap indices for two reasons. First, these are very broad based indices. Second, the Russell 2000 value index has a 97% correlation with the small/value Fama French portfolio that by construction does not include REITs. The REIT common index also has a correlation of 73% with the small/value portfolio. In this sense it appears that REITs are highly correlated with small and midcap value indices not because they are part of those indices, but because they tend to be small and midcap value stocks. We use the Russell indices in our analysis instead of Fama French portfolios simply because an investor is far more likely to hold a Russell index than a portfolio of stocks they created with size and book to market sorts.

from 2002 to 2007 we observe a marked increase in the performance of midcap, small and midcap value, and REIT common shares. The close relationship between REIT common shares and midcap value stocks become quite apparent in Figure 2 when we plot just the REIT common and preferred shares, the S&P 500, and the mid cap value index. Finally, the effects of the global financial crisis, subsequent recession, and recovery are evident in the latter part of the sample. We observe all risky asset classes declining significantly during the crisis and rebounding during the recovery. This V-shaped pattern being quite pronounced in both the REIT common and preferred shares.

4 Results

4.1 Classical Analysis

We begin our minimum-variance analysis using the standard mean-variance framework of Markowitz (1952) where the investors are unconstrained in their portfolio formation and can freely borrow and lend at the risk free rate. In such a framework, the diversification benefits of any given asset can be measured by its improvement in the Sharpe Ratio of the tangency portfolio. Table 2 shows the characteristics of the market (tangency) portfolios created with different mixes of equity and debt asset classes. The left columns are for the case where bonds are excluded from the analysis and the right columns refer to when bonds are included in the investor's opportunity set. Panel A is the most inclusive opportunity set and includes all equity classes. Panel B removes the value and growth portfolios but leaves the investor access to size portfolios. Panel C gives the investor access to size and value/growth portfolios. Panel D is the most restrictive set and give the investor access only to the S&P 500 and the World-Ex-US portfolios. In each case we use excluding both REIT common and preferred stock as the base case and then compare improvements in the Sharpe Ratio when either REIT preferred stock, or REIT common, or both is added to the opportunity set.

A few results are apparent from Table 2. REIT common stock is really only valuable when the investor is given a very limited opportunity set. This is evident in the left columns of Panel D where the Sharpe Ratio improves by 28.62% (from 0.1202 to 0.1546, 0.4164 to 0.5418 annualized)¹⁷ when the investor is given access to REIT common stock. However, notice that this effect is mitigated if the investor is given access to bonds (Panel D right columns) or value/growth portfolios (Panel C). In this sense REIT common appears to be spanned by a value/growth factor and a fixed income factor, at least in this standard setting. This is important, because numerous studies do not give investors access to debt or value/growth portfolios. It is also supportive of the results of Chiang and Lee (2007) who find that the diversification benefits of REIT common stock is sensitive to benchmark portfolios.

The second result from Table 2 is that REIT preferred stock appears to be very much like a bond substitute. Investors who are constrained to invest only in equity asset classes always find access to preferred stock beneficial.¹⁸ The magnitude of the improvement ranges from 8.13% when all other equity asset classes are included (Panel A left columns), to 63.39% when the most restrictive set of equity classes is considered (Panel D left columns). In fact nearly all the benefit to equity investors from the REIT market comes from access to their preferred stock, not their common stock (compare the first and third row in each panel). Once

¹⁷ To obtain annualized returns, monthly returns should be multiplied by 12; to obtain annualized standard deviation, monthly standard deviation should be multiplied by square root of 12. Sharpe ratio is defined as $(r - r_f)/\sigma$. Therefore, the annualized Sharper Ratio is obtained from Sharpe Ratio based on monthly numbers by multiplying it by $\sqrt{12}$.

¹⁸ The economic significance is similar in magnitude to the benefits from diversifying internationally by UK equity investors as reported in Goetzmann and Ukhov (2006).

investors have access to bond indices, the benefits of preferred stock are reduced quite dramatically. This is evident from the reduced increases in Sharpe Ratios reported in the right columns of Table 3. In the most inclusive case, REIT preferred stock improves the Sharpe Ratio by only 0.28%.

Taken together the results of Table 2 suggest that REITs potentially provide little diversification benefit to investors. If an investor is given access to easily tradable portfolios of value stocks or bonds, then Sharpe Ratios are not improved greatly by also giving the investor access to REIT common or preferred stock. This leads one to question why investors should hold REITs in their portfolios.

4.2 Constrained Analysis

We start our constrained analysis by considering the simplest case where there is no risk free asset and the investor cannot short the risky assets. While this scenario may be extreme, it provides a base to understand how constraints affect investors and is easily extendable to more realistic cases.

4.2.1 No Risk Free Asset and No Bonds

Figure 3 reports efficient frontiers and indifference curves for the case of constrained risky asset portfolio (no shorts) and no risk free asset. In Panel A, the dashed lines represent the case where preferred stock is excluded from the opportunity set and the solid lines represent the case where they are included. Black lines are the mean-variance frontier, while red lines are indifference curves for the high risk aversion investor and blue lines are indifference curves for the low risk aversion investor. In Panel B we repeat the exercise, but dashed lines represent the case where REIT common stock is excluded from the opportunity set and the solid lines represent the case where they are included. In both cases the investment opportunity set includes all the other non-REIT equity classes.

Examining Panel A, we observe that the addition of preferred stock has a dramatic effect on the mean-variance frontier. Preferred stock provides a venue for significant risk reduction. This is evident in the shift to the left of the frontier from the dashed curve to the solid curve. This risk reduction is most beneficial to the high risk aversion investor. The high risk aversion investor's tangency portfolio changes quite dramatically and they are able to obtain a higher indifference curve (solid red line lies above the dashed red line). However, notice that the addition of preferred stock does little for the low risk aversion investor. Their tangency portfolio remains essentially identical (solid blue and dashed light blue lines lie on top of each other.) This is the sense in which one needs to be specific about which set of investors benefit from access to a given asset. Preferred stock is highly beneficial to high risk aversion investors, but of little consequence to low risk aversion investors.

Turning to Panel B, we observe that the addition of REIT common stock has a different effect on the mean-variance frontier. Including REIT common stock allows investors to form portfolios with higher total return than they otherwise would have been able to attain. That is, the solid efficient frontier (including REIT common) lies above the dashed efficient frontier curve (excluding REIT common) with this difference being most pronounced for high return portfolios. In this case it is the low risk aversion investor who benefits. Their optimal portfolio changes and they are able to obtain a higher indifference curve when given access to REIT common stock (solid blue indifference curve is above the dashed light blue indifference curve.) Notice that this is true even though the investor already has access to both size and value/growth tilted portfolios. Examining the high risk aversion investor, we see that they are largely indifferent to access to REIT common stock (the solid red and dashed red indifference curves lie on top of each other.) This once again

demonstrates how an asset class can have different benefits to different investors. REIT common stock benefits low risk aversion investors, but is of little consequence to high risk aversion investors.

To further examine the benefit of REIT common and preferred stock to investors with different risk aversions, in Figure 4 we plot compensation ratios for investors with different levels of risk aversion. The compensation ratio is measured as the wealth required to compensate an investor for losing access to preferred stock (red line) or REIT common (blue line) divided by the compensation required to compensate them for losing access to both REIT common and preferred stock. In this sense they measure the relative importance preferred stock (red line) and REIT common (blue line) to an investor over different levels of risk aversion.

Both the blue and red lines confirm what we observed in Figure 3 for only two levels of risk aversion. The benefit of access to REITs both in the form of common and preferred stock is dominated by preferred stock for high risk aversion investors (blue line declines with risk aversion.) Examining the red line (that excludes preferred stock), for high levels of risk aversion the compensation ratio moves towards one. This indicates that for high risk aversion investors, nearly all the compensation they require for losing access to the REIT market is because of the lack of access to REIT preferred stock. If we examine the blue line (no REIT common) we see the opposite is true for low risk aversion investors. Most of the compensation for losing access to the REIT assets is due to losing access to REIT common stock.

To provide some economic significance to our analysis, in Figure 5 we plot the weights in the optimal portfolio of REIT preferred stock (blue), REIT common stock (red), and all other equities (green) for investors of different risk aversion. In both Panel A and Panel B, the heavy solid line represents the base case for the analysis where the investor has access to all the equity asset classes including REIT common and preferred stock. For extremely low levels of risk aversion, we observe that the investor's optimal portfolio is dominated by REIT common stock. This allocation declines as risk aversion increases, although is non-zero for many levels of risk aversion that would be considered plausible in the literature.¹⁹ Turning our attention to preferred stock, we observe that low risk aversion investors put very little weight in preferred stock, but this allocation grows quite markedly as risk aversion increases. For high levels of risk aversion we observe that the investor's portfolio is dominated by preferred stock. While such an allocation is economically unreasonable for what we observe in practice, in this scenario the investor doesn't have access to either risky or riskless fixed income securities.

The dashed lines in Panel A represent the case where the investor is not given access to REIT common stock and we examine how the investor allocates their portfolio over different levels of risk aversion. At low levels of risk aversion, the investor's allocation to preferred stock is slightly increased compared to the case with all equity asset classes. The gap left by not having access to REIT common stock is filled by increasing the allocation to the other non-preferred equity classes. This suggests that REIT preferred stock is not a great substitute for REIT common stock.

In Panel B we repeat the analysis but in this case the dashed line represents the case where the investor doesn't have access to REIT preferred stock. In this case we observe that while the allocation to other equity classes increases for all levels of risk aversion, so does the allocation to REIT common, especially for higher risk aversion investors. This suggests that REIT common may provide some bond like characteristics that the risk averse investor seeks, but can't obtain in other equity classes.

¹⁹ See Bliss and Panigirtzoglou (2004) and Blackburn, Goetzmann, and Ukhov (2009) for a discussion.

4.2.2 Long Only Risk Free Asset and No Bonds

We now extend our analysis by giving the investor access to a risk free security. Importantly, we allow the investor to freely invest, but not borrow at the risk free rate. This matches the realistic constraint that investors' face in practice. In Figure 6 we plot the mean-variance frontier and indifference curves as in Figure 3. In Panel A the solid line refers to the case with REIT preferred stock and the dashed the case without REIT preferred stock. In Panel B the solid line refers to the case with REIT common stock and the dashed the case without REIT common stock. Red indifference curves relate to the high risk aversion investor, while blue indifference curves are for the low risk aversion investor. Because we have introduced the risk free asset, the investor is no longer constrained to portfolios that sit on the mean-variance frontier. The mean-variance efficient portfolios (black lines) are made up of the line starting from the risk free rate and extending to the green tangency portfolio. The efficient frontier then traces along the upper portion of the mean-variance curve.

Examining Figure 6 Panel A, we can determine the usefulness of preferred stock to investors. Once again the inclusion of preferred stock shifts the mean-variance frontier to the left. This allows high risk aversion investors to attain a higher indifference curve (solid red indifference curve lies above the dashed red indifference curve.) As was the case without the risk free rate, access to the preferred stock has little effect on the low risk aversion investor. Their portfolio and level of utility remain unchanged (solid blue and dashed blue lie on top of each other.)

Turning to Figure 6 Panel B, we examine the effect of giving the investor access to REIT common stock. As was the case without the risk free asset, including REIT common stock allows investors to form portfolios with higher total returns (the solid efficient frontier lies above the dashed frontier curve.) This is once again beneficial to the low risk aversion investor. They are able to attain a higher indifference curve when they have access to REIT common stock (solid blue indifference curve is above the dashed light blue indifference curve.) The high risk aversion investor is unaffected by the inclusion of REIT common stock. They hold the same portfolio and attain the same level of utility they did when they did not have access to REIT common stock.

Taken together, the results of Figure 6 show that giving an equity investor access to invest in the risk free rate has not changed their preferences towards REIT common and preferred stock. High risk aversion investors still benefit from access to REIT preferred stock, and low risk aversion investors still benefit from access to REIT common stock.

Unsurprisingly, when we examine the compensation ratios in Figure 7, we observe a similar relationship to the one seen in Figure 4. High risk aversion investors still require the majority of their compensation for losing access to REIT preferred stock, while low risk aversion investors require relatively more compensation for losing access to REIT common stock.

Examining the optimal portfolio weights in Figure 8, we observe that both REIT common and preferred stock still form an economically significant portion of investor portfolios. In Figure 8, the red line represents the weight in REIT common stock, the blue line is the weight in REIT preferred stock, the green line is the weight in the other equity asset classes, and the black line is the allocation to the risk free asset. Panel A is for the case where the investor has access to all equity asset classes, while Panel B is the case where the investor no longer has access to REIT common stock, and Panel C is the case where the investor no longer has access to REIT preferred stock.

Contrasting Figure 8 Panel A to Figure 5, we observe the main effect of introducing the risk free rate is to dampen the allocation to preferred stock for high levels of risk aversion. Highly risk averse investors substitute out of REIT preferred stock and into the risk free asset, while allocations to REIT common and the other equity classes remain similar to before. This makes intuitive sense because as the investor's risk aversion increases they are slowly holding a portfolio that is dominated by the risk free asset.

If we remove the investor's ability to invest in REIT common (Panel B) we observe, like before, that the investor allocates slightly more to REIT preferred stock at lower levels of risk aversion and substantially more to the other risky asset classes. If we remove the investor's access to REIT preferred stock (Panel C) we observe that the investor allocates more heavily towards REIT common than they did before, but now with access to the risk free asset, they also start investing in the risk free asset at lower levels of risk aversion. They also invest substantially more in the risk free asset at extremely high levels of risk aversion. In this sense the investor appears to be attempting to replicate REIT preferred stock as a combination of REIT common stock and the risk free asset.

4.2.3 Long Only Risk Free Asset Plus Bonds

The final case we consider is when the investor has access to all the equity asset classes, can invest in the risk free asset, and also has access to investment grade and high yield debt. This represents the most realistic investment opportunity set that an investor might have. And remember from Table 2, it is also the case where at least in the classical Markowitz setting, that REIT common and preferred stock appeared to provide little if any benefit to investors.

Figure 9 reports mean-variance frontiers and indifference curves for the case with investment grade and high yield bonds, investment in the risk free asset, and access to all the equity securities. The high risk aversion investor's indifference curves are in red, while low risk aversion investor's indifference curves are in blue. In Panel A, the solid lines are for the case where the investor has access to all securities, while the dashed case is when the investor does not have access to REIT preferred stock. In Panel B, the dashed lines refer to the case where the investor does not have access to REIT common stock. As was the case in Figure 6, efficient portfolios are those that extend from the risk free rate to the green tangency portfolios, and then trace the upper portion of the mean-variance frontiers.

Examining Panel A first, the most notable effect of including investment grade and high yield debt into the investment set is that REIT preferred stock no longer dramatically shifts the mean variance frontier to the left. In fact, inclusion of REIT preferred stock only marginally expands the frontier. While the expansion isn't as dramatic as before, it is still beneficial to highly risk averse investors who are able to attain a higher indifference curve. Low risk aversion investors are unaffected by access to REIT preferred stock, with their portfolios and utility remaining unchanged.

In Panel B, we observe that the inclusion of investment grade and high yield bonds has not changed the effect of REIT common stock on the frontier. Once again, REIT common stock allows the investor to create portfolios with higher returns than they otherwise would have been able to form. This benefits low risk aversion investors, allowing them to attain higher utility levels than before. Notice that REIT common stock providing investors some benefit when the investor has access to both bonds and value portfolios is quite different to the conclusion from the standard case. When we introduce realistic constraints into the portfolio maximization problem, we find that REIT common and preferred stock do play a beneficial role in investor

portfolios. It is just that they benefit different investors. Low risk aversion investors benefit from REIT common stock, while high risk aversion investors benefit from REIT preferred stock.

Consistent with what we observed for the case with no bonds, high risk aversion investors still require the majority of their compensation for losing access to REIT preferred stock, and low risk aversion investors require the majority of the compensation for losing access to REIT common stock. This is evident in the compensation ratios reported in Figure 10.

In Figure 11 we report optimal portfolio weights over different levels of risk aversion. The assets are REIT preferred stock (blue), REIT common stock (red), other equity classes (green), investment grade bonds (brown), high yield bonds (orange), and the risk free asset (black.) Panel A reports the case where the investor has access to all asset classes, while Panel B removes REIT common stock, and Panel C removes REIT preferred stock.

In Panel A we observe that for low risk aversion investors, REIT common stock plays a dominant role in their portfolio. As risk aversion increases, REIT preferred stock starts increasing in importance, and for very high levels of risk aversion the optimal portfolio consists of investment grade bonds, the risk free asset, and 10% allocations each to REIT preferred stock and other equity asset classes. Interestingly, in this scenario, the investor makes no use of high yield bonds. Overall, for plausible levels of risk aversion we observe investors holding significant amounts of both REIT common and preferred stock.

When we remove access to REIT common stock in Panel B, the investor allocated slightly more to REIT preferred stock at low levels of risk aversion, and considerably more to the other equity asset classes. Allocations to investment grade bonds and the risk free rate remain similar.

In Panel C, where we remove access to REIT preferred stock, we observe that investors allocate more heavily towards REIT common equity and start to invest in investment grade bonds at lower levels of risk aversion. Notice that if we compare Panel C to Panel A, we can observe an interesting relationship between REIT preferred stock, REIT common stock, and investment grade bonds. While access to REIT preferred stock only expanded the frontier modestly in Figure 9 when the investor also had access to investment grade bonds, the allocations to preferred stock in the optimal portfolio suggests that at least for some investors preferred stock and investment grade bonds are not substitutes. If an investor does not have access to preferred stock they will invest more heavily in investment grade bonds and REIT common stock (especially at moderate levels of risk aversion.) Suggesting that REIT preferred is some combination of REIT common stock and investment grade bonds. However, if you give the investor access to REIT preferred stock, they will use them instead of the combination of REIT common stock and investment grade bonds. In this sense REIT preferred stock provides the investor with some unique risk return profile that they couldn't otherwise replicate in their portfolio. It also explains in part why REITs issue preferred stock – there is at least some subset of investors to whom they provide benefits that can't be easily replicated using other securities.

5 Conclusion

We study diversification benefits of REIT preferred and common stocks using a utility based framework. Employing a utility based framework allows us to examine the diversification benefits of different asset classes, while taking into consideration realistic constraints that investors' face. Being unable to short risky assets or borrow at the risk free rate has a material impact on how investors' should gauge the benefit of different asset classes. In our utility based framework, investors separate based on risk aversion. That is,

investors no longer hold the market portfolio and a long or short position in the risk free asset. This means that standard metrics of diversification benefits, such as Sharpe Ratios, are no longer valid measures of utility maximization.

Taking the view of a long run investor, we conduct our analysis using data from the largest time series available, with our data spanning the period November 1992 to November 2012. We allow the investor to choose from a wide variety of asset classes: the World Ex-US stock portfolio, the S&P 500, midcap and small cap indices, midcap and small cap value and growth portfolios, investment grade bonds, high yield bonds, and the risk free asset. We establish two main results. First, REIT common and preferred stock do provide diversification benefits, they just provide them to different investors. REIT common stock allows investors to form higher return portfolios. This is beneficial to low risk aversion investors who seek to form such portfolios. REIT preferred stock is beneficial to high risk aversion investors. REIT preferred stock allows investors to form portfolios with lower risk. This is particularly true if the investor doesn't have access to investment grade bonds. Over realistic levels of risk aversion, we find both REIT common and preferred stock forming substantial portions of an investor's optimal investment portfolio.

Our second main result sheds some light on why firms issue preferred stock. Conventional wisdom is that preferred stock is either a bond substitute or a hybrid security of debt and equity. If this is true, then investors shouldn't change their portfolio allocations when given access preferred stock, if they already have access to these other asset classes. We find that this is not the case. When given access to REIT preferred stock, investors reduce their allocation to investment grade bonds and REIT common stock quite substantially. They are also able to attain higher levels of utility when they have access to preferred stock. In this sense, while the conventional wisdom is true, preferred stock is a hybrid asset, the reality is that the exact risk return characteristics of preferred stock can't easily be replicated by the two other asset classes. Investors would still rather have the ability to invest in preferred stock than be constrained to just common stock and investment grade debt. This is especially true for high risk aversion investors.

APPENDIX

To derive the efficient frontier in the standard deviation (x-axis)-return (y-axis) space trace the frontier by comparing expected utility under different cases. Case 2 (corner solution with all money invested in the riskless asset) is dominated by Case 1 (with positive investment in the risk-free asset and both risky assets).

Case 1. Interior Solution: Positive investment in the risk-free rate and both risky assets: $\lambda_1 = \lambda_2 = \lambda_3 = 0$,

$$w_{rf} = \frac{\mu_2 \sigma_1 (\sigma_1 - \rho \sigma_2) - r(\sigma_1^2 - 2\rho \sigma_1 \sigma_2 + \sigma_2^2) + a_i W_0 (\rho^2 - 1) \sigma_1^2 \sigma_2^2 + \mu_1 \sigma_2 (\sigma_2 - \rho \sigma_1)}{a(\rho^2 - 1) \sigma_1^2 \sigma_2^2}$$

$$w_1 = \frac{(\mu_1 - r) \sigma_2 - (\mu_2 - r) \rho \sigma_1}{a_i \sigma_1^2 \sigma_2 (1 - \rho^2)}, \quad w_2 = \frac{(\mu_2 - r) \sigma_1 - (\mu_1 - r) \rho \sigma_2}{a_i \sigma_1^2 \sigma_2 (1 - \rho^2)}$$

The Lagrange multiplier associated with the budget constraint: $\varphi = 1 + r$.

The investor who hold all his wealth in the tangency portfolio has zero investment in the risk free asset. Risk aversion of this investor can be found by solving $w_{rf} = 0$ and equals,

$$a_{Tangency} = \frac{(r - \mu_2) \sigma_1^2 + (\mu_1 + \mu_2 - 2r) \rho \sigma_1 \sigma_2 + (r - \mu_1) \sigma_2^2}{W_0 (-1 + \rho^2) \sigma_1^2 \sigma_2^2}$$

The allocation to the risk free asset is increasing in risk aversion. To show this, consider the derivative of the allocation to the risk free asset with respect to risk aversion,

$$\frac{\partial w_{rf}}{\partial a_i} = \frac{(r - \mu_2) \sigma_1^2 + (\mu_1 + \mu_2 - 2r) \rho \sigma_1 \sigma_2 + (r - \mu_1) \sigma_2^2}{a_i^2 (-1 + \rho^2) \sigma_1^2 \sigma_2^2}$$

Comparing the above expression for $\frac{\partial w_{rf}}{\partial a_i}$ with the expression for $a_{Tangency} > 0$, it follows that $\frac{\partial w_{rf}}{\partial a_i} > 0$. Therefore, allocation to the risk-free asset is increasing in risk aversion.

Case 2. Corner Solution: All money invested in the riskless asset.

$$w_{rf} = W_0, \lambda_1 = r - \mu_1, \lambda_2 = r - \mu_2, \varphi = 1 + r, \lambda_3 = 0, w_1 = 0, w_2 = 0.$$

Under the maintained assumption $r < \mu_1 < \mu_2$, for all positive levels of wealth, $W_0 > 0$ and for all positive levels of risk aversion $a_i > 0$, the expected utility under this solution is lower than the expected utility under case 1 (holding of the risk free asset and both risky assets). This case is therefore dominated by case 1.

Case 3. No investment in the first risky asset, $w_1 = 0$, positive investment in the second risky asset and risk-free asset ($\lambda_2 = 0, \lambda_3 = 0$),

$$w_{rf} = W_0 + \frac{r - \mu_2}{a_i \sigma_2^2}, \lambda_1 = r - \mu_1 + \frac{(\mu_2 - r) \rho \sigma_1}{\sigma_2}, w_2 = \frac{\mu_2 - r}{a_i \sigma_2^2}, \varphi = 1 + r$$

To determine when this case yields higher expected utility than case 1, consider the following inequality,

$$\text{Expected Utility Under Case 1} > \text{Expected Utility Under Case 3}$$

$$\frac{(r - \mu_2)^2 \sigma_1^2 - 2(r - \mu_1)(r - \mu_2) \rho \sigma_1 \sigma_2 + ((r - \mu_1)^2 - 2a_i(1 + r)W_0(-1 + \rho^2) \sigma_1^2) \sigma_2^2}{2a_i(-1 + \rho^2) \sigma_1^2 \sigma_2^2} > (1 + r)W_0 + \frac{(r - \mu_2)^2}{2a_i \sigma_2^2}$$

Under the maintained assumptions $0 < r < \mu_1 < \mu_2$, $a > 0$, $W_0 > 0$, $0 < \sigma_1 < \sigma_2$, $-1 < \rho < 1$, the expected utility under case 3 is always smaller than the expected utility under case 1 (the above inequality holds) and case 3 is dominated by case 1.

In particular, for non-positive correlation, $-1 < \rho \leq 0$ the above inequality always holds under the maintained assumptions $0 < r < \mu_1 < \mu_2$, $a > 0$, $W_0 > 0$, $0 < \sigma_1 < \sigma_2$. The above inequality also always holds for the case of all correlations, $-1 < \rho < 1$ when the Sharpe ratio of asset two is lower than the Sharpe ratio of asset 1, $\frac{\mu_2 - r}{\sigma_2} < \frac{\mu_1 - r}{\sigma_1}$. Other cases can be considered similarly with the conclusion that case 3 is strictly dominated by case 1 in terms of expected utility for all cases, except the unique case when $\mu_2 = r + (\mu_1 - r)\sigma_2/\rho\sigma_1$ (or, equivalently, $\rho = (\mu_1 - r)\sigma_2/[(\mu_2 - r)\sigma_1]$ when case 3 coincides with case 1.

Case 4. All wealth is invested in the second risky asset, $w_1 = 0$, $w_2 = W_0$, $w_{rf} = 0$. The Lagrange multipliers are: $\lambda_1 = -\mu_1 + \mu_2 + a_i W_0(\rho\sigma_1 - \sigma_2)\sigma_2$, $\lambda_2 = 0$, $\lambda_3 = \mu_2 - r - a_i W_0 \sigma_2^2$, and the Lagrange multiplier for the budget constraint is $\varphi = 1 + \mu_2 - a_i W_0 \sigma_2^2$.

The second case is preferred to case 7 only for investors with sufficiently low risk aversion,

$$a_i \leq \frac{\mu_2 - \mu_1}{W_0(\sigma_2 - \rho\sigma_1)\sigma_2}.$$

Case 5. No investment in the second risky asset, $w_2 = 0$, $\lambda_1 = 0$, $\lambda_3 = 0$, $\varphi = 1 + r$,

$$w_{rf} = W_0 + \frac{r - \mu_1}{a_i \sigma_1^2}, \quad \lambda_2 = r - \mu_2 + \frac{(-r + \mu_1)\rho\sigma_2}{\sigma_1}, \quad w_1 = \frac{\mu_1 - r}{a_i \sigma_1^2}$$

To determine when this case yields higher expected utility than case 1, consider the following inequality,

$$\textit{Expected Utility Under Case 1} > \textit{Expected Utility Under Case 5}$$

Under the maintained assumptions $0 < r < \mu_1 < \mu_2$, $a > 0$, $W_0 > 0$, $0 < \sigma_1 < \sigma_2$, $-1 < \rho < 1$, the above inequality always holds and case 5 is dominated by case 1.

Case 6. Corner solution, all funds invested in the first risky asset, $w_1 = W_0$, $\lambda_1 = 0$, $w_2 = 0$, $w_{rf} = 0$, $\lambda_2 = \mu_1 - \mu_2 + a_i W_0 \sigma_1(\rho\sigma_2 - \sigma_1)$, $\lambda_3 = \mu_1 - r - a_i W_0 \sigma_1^2$, $\varphi = 1 + \mu_1 - a_i W_0 \sigma_1^2$.

Under the maintained assumption $r < \mu_1 < \mu_2$, when is this solution dominated by case 1 (holding of the risk-free asset and both risky assets)?

(Actually, it is possible to have a risky asset with expected return less than the risk-free rate, $\mu_1 < r$, and can consider this case separately).

Case 1 dominates this case when

$$\begin{aligned} \mu_{p1} - 0.5 a_i \sigma_{p1}^2 &> \mu_{p6} - 0.5 a_i \sigma_{p6}^2 \\ (1 + \mu_1)w_1 + (1 + \mu_2)w_2 + (1 + r)w_{rf} - \frac{1}{2}a_i(w_1^2\sigma_1^2 + 2w_1w_2\rho\sigma_1\sigma_2 + w_2^2\sigma_2^2) \\ &> (1 + \mu_1)W_0 - 0.5a_iW_0^2\sigma_1^2 \end{aligned}$$

Under the maintained assumptions $0 < r < \mu_1 < \mu_2$, $a > 0$, $W_0 > 0$, $0 < \sigma_1 < \sigma_2$, $-1 < \rho < 1$, the above inequality always holds and case 6 is dominated by case 1.

Case 7. No investment in the risk free asset, $w_{rf} = 0$, positive amounts of the two risky assets held ($\lambda_1 = \lambda_2 = 0$):

$$\begin{aligned} w_1 &= \frac{\mu_1 - \mu_2 + a_i W_0 (\sigma_2^2 - \rho \sigma_1 \sigma_2)}{a_i (\sigma_1^2 - 2\rho \sigma_1 \sigma_2 + \sigma_2^2)} \\ w_2 &= \frac{-\mu_1 + \mu_2 + a_i W_0 (\sigma_1^2 - \rho \sigma_1 \sigma_2)}{a_i (\sigma_1^2 - 2\rho \sigma_1 \sigma_2 + \sigma_2^2)} \\ \lambda_3 &= \frac{\mu_1 (\sigma_2^2 - \rho \sigma_1 \sigma_2) + \mu_2 (\sigma_1^2 - \rho \sigma_1 \sigma_2) - a_i W_0 \sigma_1^2 \sigma_2^2 (1 - \rho^2)}{\sigma_1^2 - 2\rho \sigma_1 \sigma_2 + \sigma_2^2} - r \\ \varphi &= \frac{-(2 + \mu_1 + \mu_2)\rho \sigma_1 \sigma_2 + (1 + \mu_1)\sigma_2^2 + (1 + \mu_2)\sigma_1^2 - a_i W_0 \sigma_1^2 \sigma_2^2 (1 - \rho^2)}{\sigma_1^2 - 2\rho \sigma_1 \sigma_2 + \sigma_2^2} \end{aligned}$$

Comparing expected utility under case 1 with expected utility under case 7, we conclude that for risk aversion that satisfies

$$a_i > \frac{(r - \mu_2)\sigma_1^2 + (\mu_1 + \mu_2 - 2r)\rho \sigma_1 \sigma_2 + (r - \mu_1)\sigma_2^2}{W_0(-1 + \rho^2)\sigma_1^2 \sigma_2^2}$$

case 1 is preferred to case 7. That is, investors with sufficiently higher risk aversion hold portfolios described by case 1 and investors with sufficiently low risk aversion hold portfolios described by case 7.

Comparing case 7 (investment in both risky assets) with case 4 (investment in the second risky asset only), shows that investors with sufficiently low risk aversion,

$$a_i \leq \frac{\mu_2 - \mu_1}{W_0(\sigma_2 - \rho \sigma_1)\sigma_2},$$

achieve higher expected utility under case 4.

The expressions for portfolio weights w_1 and w_2 are the same as in the case when risk-free asset is not a part of the investment opportunity set. This case is analogous to the case of no risk-free asset. Therefore, there exists a sufficiently high risk aversion level, $a_i > \frac{2(\mu_2 - \mu_1)}{W_0(\sigma_2^2 - \sigma_1^2)}$, so that investment in the first (less risk) asset is larger than the investment in the second (more risky) asset, $w_1 > w_2$.

References

- Ambrose, Brent W., D.W. Lee, and Joe Peek, "Industry Comovement After Joining an Index: Spillovers of Nonfundamental Effects", *Real Estate Economics*, January 2007, 35, 57-90.
- Bildersee, J.S. 1973. Some Aspects of the Performance of Non-Convertible Preferred Stocks. *Journal of Finance*. 28, 1187-1201.
- Blackburn D., W.N. Goetzmann and A.D. Ukhov. 2009. Risk Aversion and Clientele Effects, NBER Working Paper 15333.
- Bliss, R.R., and N. Panigirtzoglou. 2004. Option-implied Risk Aversion Estimates. *Journal of Finance* 59, 407-446.
- Boudry. W.I. 2011. An Examination of REIT Dividend Payout Policy. *Real Estate Economics* 39(4) 601-634.
- Boudry W.I., Coulson, N.E, Kallberg J., & Liu.C. 2013. On Indexing Commercial Real Estate Properties and Portfolios, *Journal of Real Estate Finance and Economics* 47, 617-639.
- Boudry W., Kallberg J., & Liu.C. 2010. An Analysis of REIT Security Issuance Decisions. *Real Estate Economics* 38, 91-120.
- Chiang, K.C.H, and M.L. Lee. 2007. Spanning tests on public and private real estate, *Journal of Real Estate Portfolio Management* 13, 7-15.
- Corgel, J. B. and deRoos, J.A. 1999. Recovery of Real Estate Returns for Portfolio Allocation, *Journal of Real Estate Finance and Economics*. 18(3), 279-296.
- Dybvig, P. H. (1984) Short Sales Restrictions and Kinks of the Mean Variance Frontier, *Journal of Finance* 39, 239-244.
- Emanuel, David. 1983. A Theoretical Model for Valuing Preferred Stock. *Journal of Finance*. 38, 1133-1155.
- Ferreira, Eurico. J., Spivey, Michael. F., and Charles E. Edwards (1992) Pricing New-Issue and Seasoned Preferred Stocks: A Comparison of Valuation Models. *Financial Management*. 21, 52-62.
- Folger, H.R. 1984. Twenty Percent in Real Estate: Can Theory Justify It? *Journal of Portfolio Management* 10(2), 6-13.
- Goetzmann, W., A. Ukhov. (2006) British Investment Overseas 1870--1913: A Modern Portfolio Theory Approach, *Review of Finance* 10, 261--300.
- Graham, Benjamin, and David Dodd. 1934. *Security Analysis*. McGraw-Hill.
- Gyourko, Joseph, & Nelling, Edward F. 1996. Systematic Risk and Diversification in the Equity REIT Market. *Journal of Real Estate Economics*. 24(4), 493-515.
- Heinkel R., & Zechner J. 1990. The Role of Debt and Preferred Stock as a Solution to Adverse Investment Incentives. *Journal of Financial and Quantitative Analysis* 25, 1-24.

- Ibbotson, Roger G., & Siegel, Laurence B. 1984. Real Estate Returns: A Comparison with Other Investments. *Journal of Real Estate Economics* (AREUEA Journal) 12:3, 219-242
- Kallberg, Jarl G., Liu, Crocker H., & Greig, Wylie. 1996. The Role of Real Estate in the Portfolio Allocation Process. *Journal of Real Estate Economics*. 24:3, 359-377.
- Kallberg, Jarl G., Liu, Crocker L., & Trzcinka, C. 2000. The Value Added from Investment Managers: An Examination of Funds of REITs. *Journal of Financial and Quantitative Analysis*, 35(3), 387-408.
- Kallberg, Jarl G., Liu, C., & Villupuram, S. 2013. Preferred Stock: Some Insights into Capital Structure. *Journal of Corporate Finance* 21, 77-86.
- Lee, H., & D. T. Johnson. 2009. The Operating Performance of Preferred Stock Issuers. *Applied Financial Economics*, 19(5), 397.
- Lin, Crystal Y., and Yung, K. 2006. Equity Capital Flows and Demand for REITs. *Journal of Real Estate Finance and Economics* 33:3, 275-291.
- Markowitz, H. (1952) Portfolio Selection, *Journal of Finance* 7, 77-91.
- Merton, R. C. (1972) An Analytical Derivation of the Efficient Portfolio Frontier, *Journal of Financial and Quantitative Analysis* 7, 1851—72.
- Miles, Mike E.; & McCue, Thomas E. 1984. Commercial Real Estate Returns. *Journal of Real Estate Economics*. (AREUEA Journal) 12:3, 355-377
- Ott S. H., Riddiough T.J., & Ha-Chin Yi. 2005. Finance, Investment and Investment Performance: Evidence from the REIT Sector. *Real Estate Economics*. 33(1), 203-235.
- Ross. S. and R. Zisler. 1991. Risk and Return in Real Estate. *Journal of Real Estate Finance and Economics* 4(2), 175-190.
- Sa-Aadu, J., Shilling, J., & Tiwari, A. 2010. On the Portfolio Properties of Real Estate in Good Times and Bad Times. *Real Estate Economics*, 38(3), 529-565.
- Sorensen E.H., and C.A. Hawkins. 1981. On the Pricing of Preferred Stock. *Journal of Financial and Quantitative Analysis*. 515-528
- Ukhov, A.D. (2006) Expanding the frontier one asset at a time, *Finance Research Letters* 3, 194—206.
- Yaman, D. 2011. Long-Run Operating Performance of Preferred Stock Issuers. *The International Journal of Business and Finance Research*, 5(2), 61-73.

Figure 1.
Performance of Asset Classes

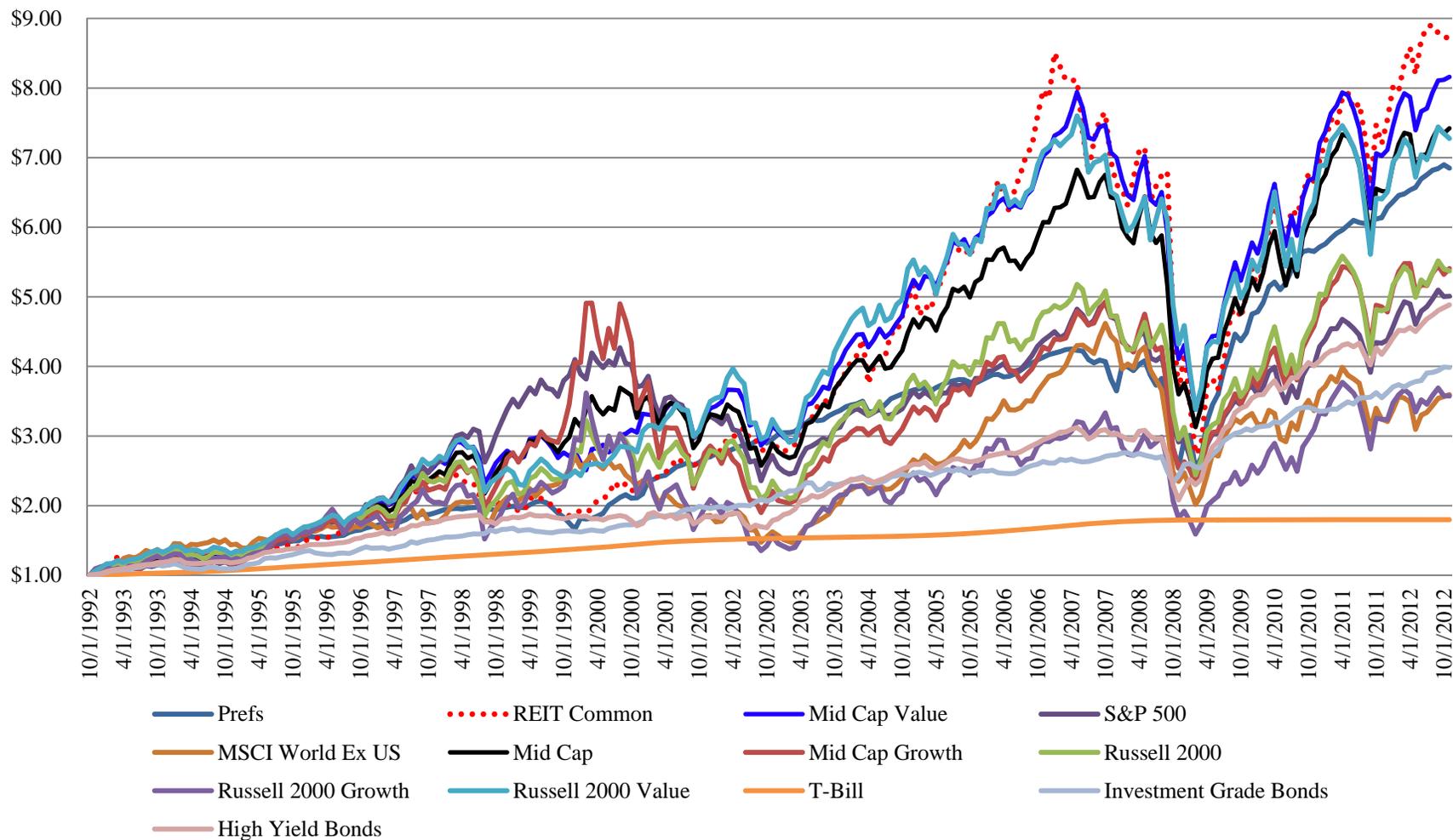


Figure 2.
Performance of Select Equity Asset Classes

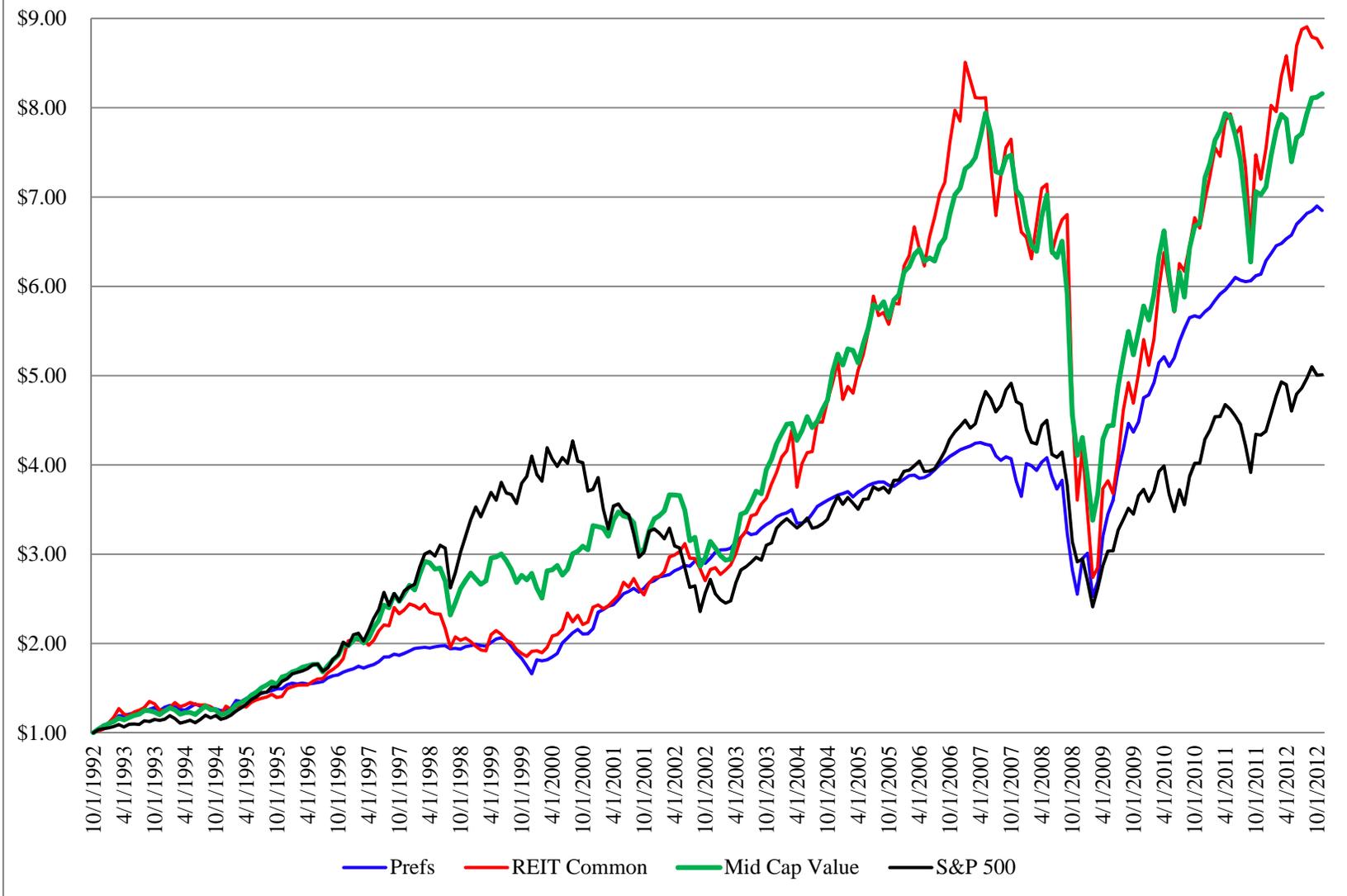
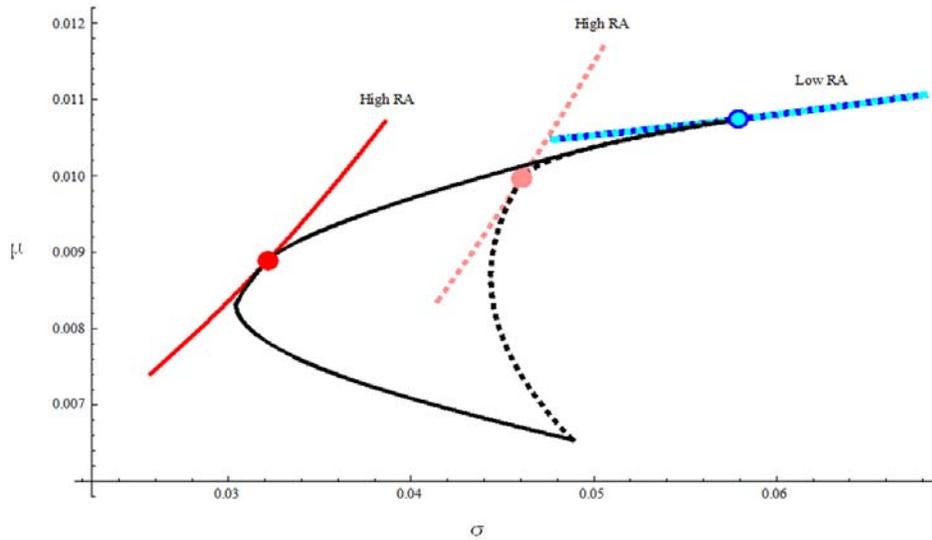


Figure 3: Equity Only and No Risk Free Asset

Figure displays mean-variance frontiers (black) and indifference curves for high (red) and low (blue) risk aversion investors. In Panel A, the dashed line represents the case where the investor has access to all equity asset classes but excludes REIT preferred stock, while the solid line presents the case where REIT preferred stock is also included. In Panel B, the dashed line represents the case where the investor has access to all equity asset classes but excludes REIT common stock, while the solid line presents the case where REIT common stock is also included. Data are monthly and span the period November 1992 to November 2012.

Panel A



Panel B

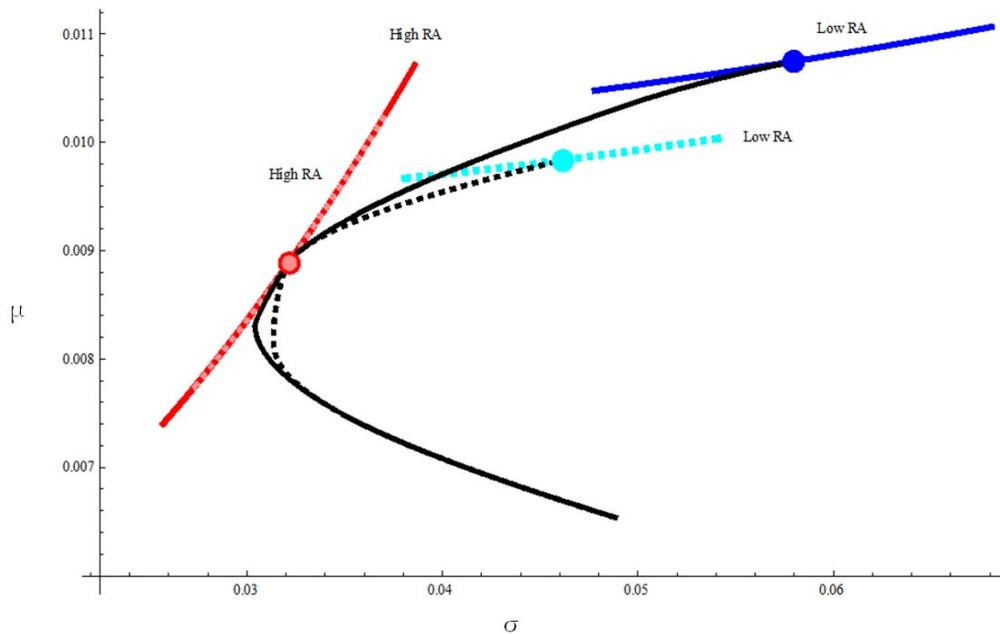


Figure 4: Compensation Ratios

Figure reports compensation ratios over different levels of risk aversion. Compensation ratios are calculated as the dollar compensation required to compensate the investor for losing access to REIT common stock (blue) and REIT preferred stock (red) divided by the compensation required to compensate the investor for losing access to both REIT common and preferred stock. The investment opportunity set is the same as in Figure 3.

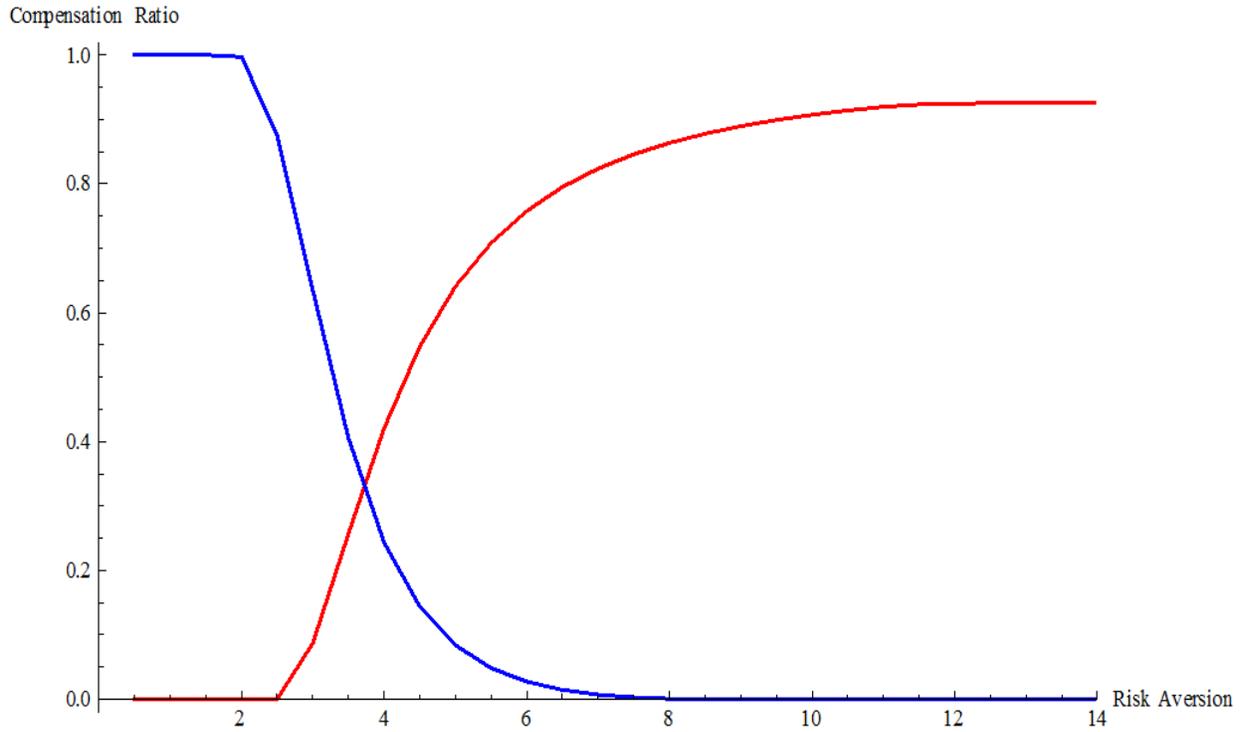
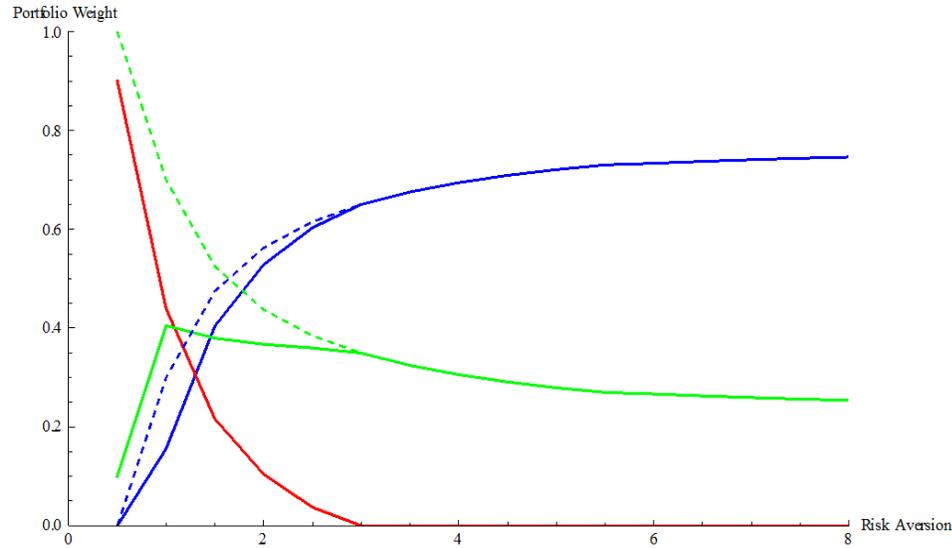


Figure 5: Optimal Portfolio Weights

Figure reports optimal portfolio weights over different levels of risk aversion. Red is the weight to REIT common stock, blue is the weight to REIT preferred stock, and green is the weight to all other equity asset classes. Solid lines present the weights when the investor has access to all equity classes plus REIT common and preferred stock. In Panel A, the dashed line presents weights when the investor loses access to REIT common stock, and in Panel B the dashed lines presents weights when the investor loses access to REIT preferred stock.

Panel A



Panel B

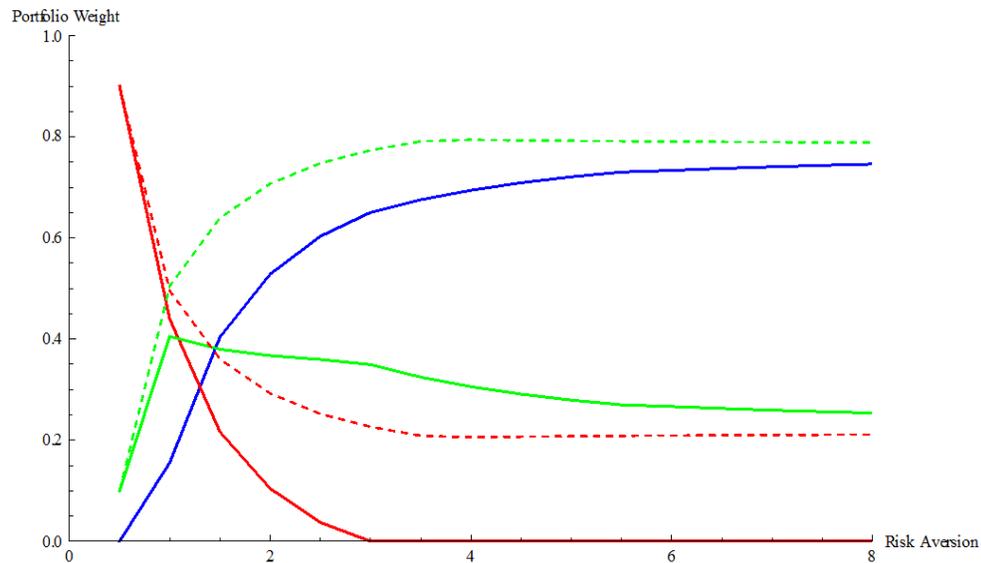
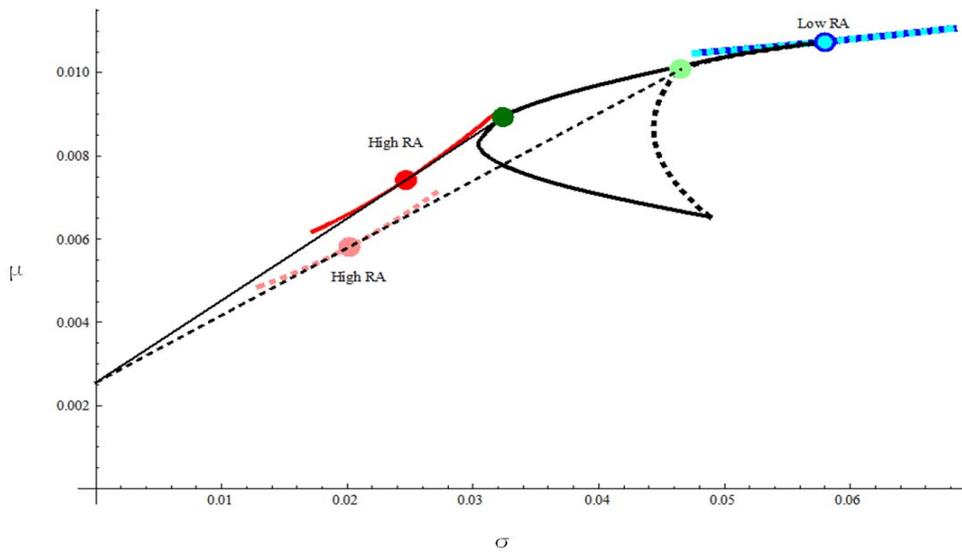


Figure 6: Long Only Risk Free Asset and No Bonds

Figure displays mean-variance frontiers (black) and indifference curves for high (red) and low (blue) risk aversion investors. In Panel A, the dashed line represents the case where the investor has access to all equity asset classes but excludes REIT preferred stock, while the solid line presents the case where REIT preferred stock is also included. In Panel B, the dashed line represents the case where the investor has access to all equity asset classes but excludes REIT common stock, while the solid line presents the case where REIT common stock is also included. The investor has access to the risk free asset and tangency portfolios are given by green circles. Data are monthly and span the period November 1992 to November 2012.

Panel A



Panel B

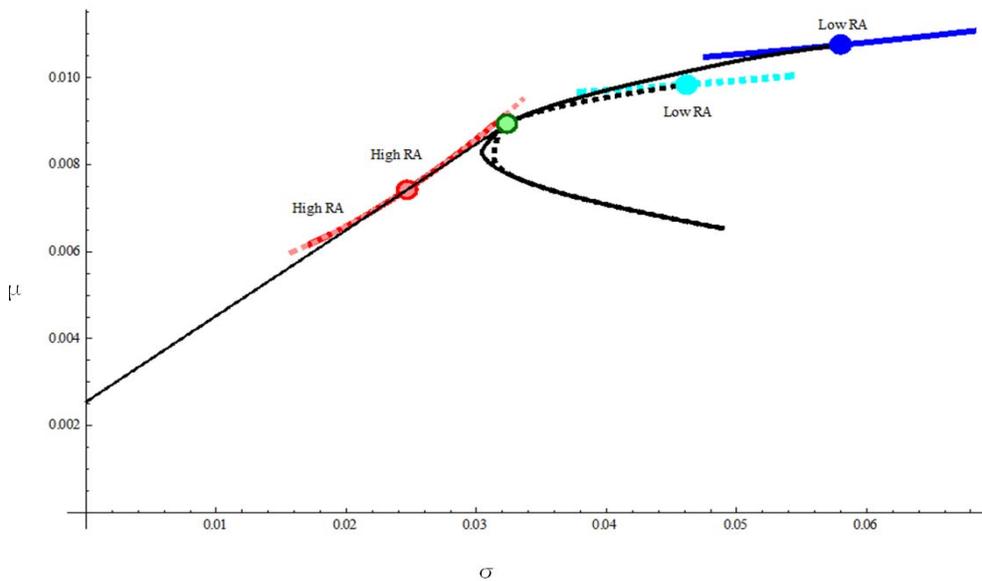


Figure 7: Compensation Ratios

Figure reports compensation ratios over different levels of risk aversion. Compensation ratios are calculated as the dollar compensation required to compensate the investor for losing access to REIT common stock (blue) and REIT preferred stock (red) divided by the compensation required to compensate the investor for losing access to both REIT common and preferred stock. The investment opportunity set is the same as in Figure 6.

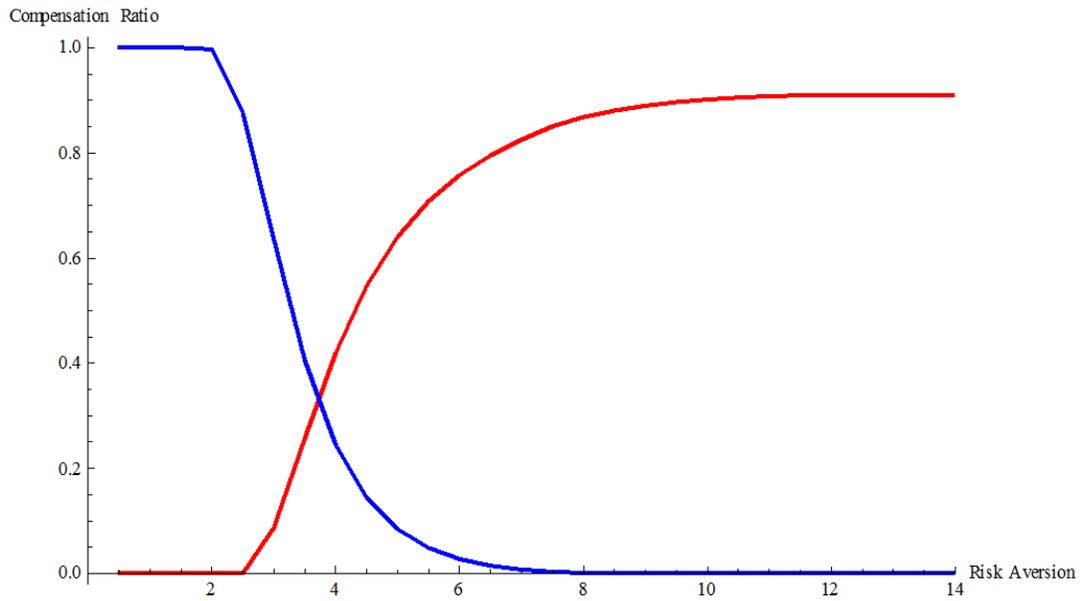
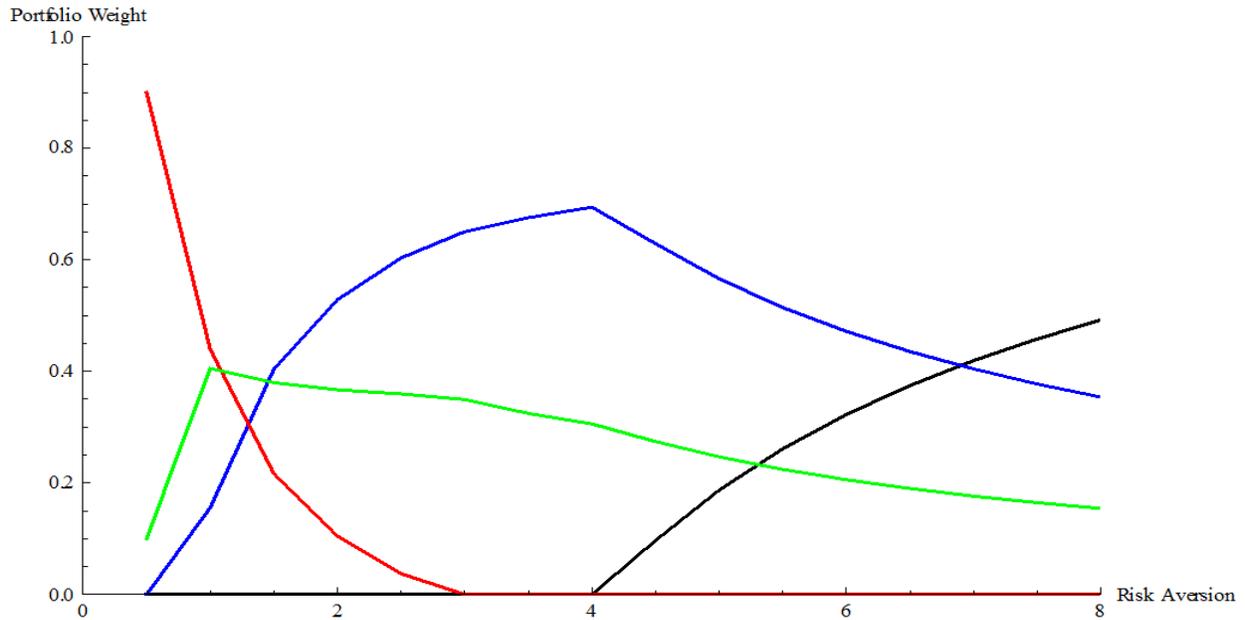


Figure 8: Optimal Portfolio Weights

Figure reports optimal portfolio weights over different levels of risk aversion. Red is the weight to REIT common stock, blue is the weight to REIT preferred stock, green is the weight to all other equity asset classes, and black is the allocation to the risk free asset. Panel A presents weights when the investor has access to all assets. Panel B, reports weights when the investor loses access to REIT common stock, and Panel C reports weights when the investor loses access to REIT preferred stock.

Panel A



Panel B

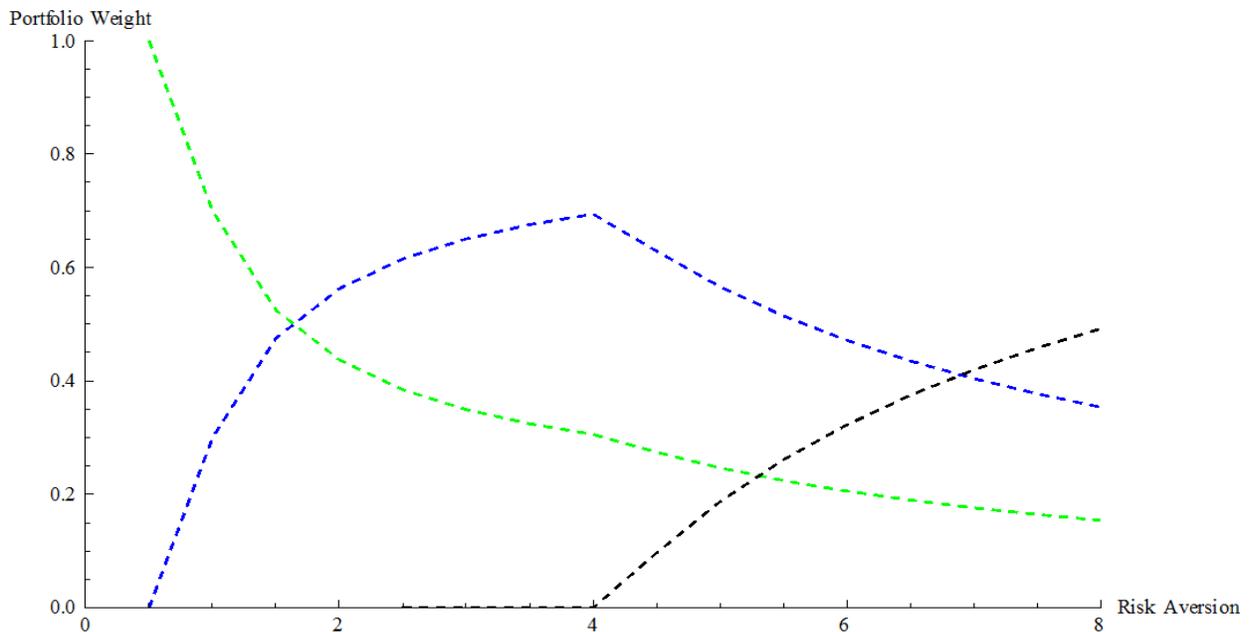


Figure 8 ctd...

Figure reports optimal portfolio weights over different levels of risk aversion. Red is the weight to REIT common stock, blue is the weight to REIT preferred stock, green is the weight to all other equity asset classes, and black is the allocation to the risk free asset. Panel A presents weights when the investor has access to all assets. Panel B, reports weights when the investor loses access to REIT common stock, and Panel C reports weights when the investor loses access to REIT preferred stock.

Panel C

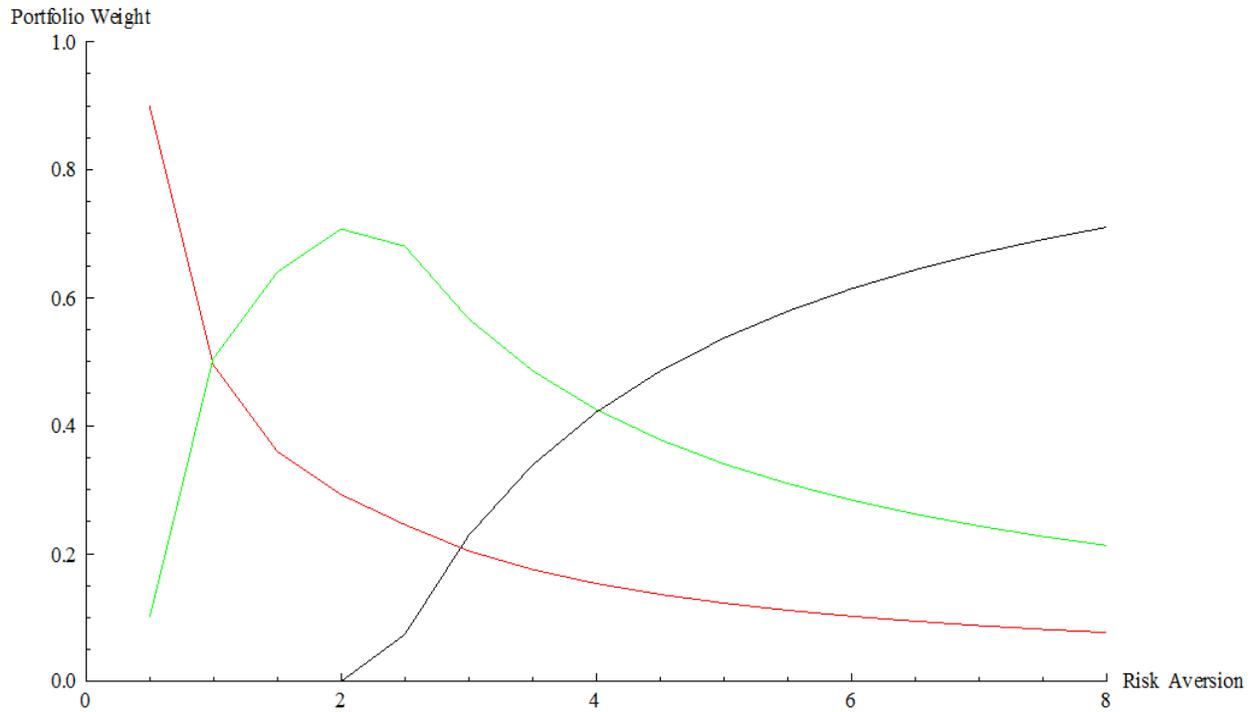
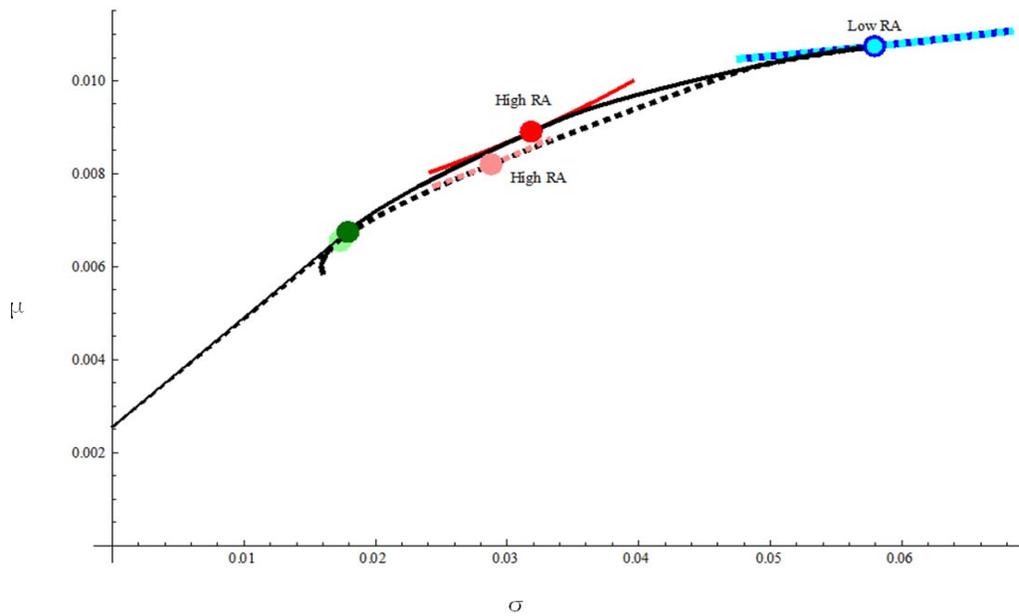


Figure 9: Long Only Risk Free Asset Plus Bonds

Figure displays mean-variance frontiers (black) and indifference curves for high (red) and low (blue) risk aversion investors. In Panel A, the dashed line represents the case where the investor has access to all equity asset classes and investment grade and high yield bonds, but excludes REIT preferred stock, while the solid line presents the case where REIT preferred stock is also included. In Panel B, the dashed line represents the case where the investor has access to all equity asset classes and investment grade and high yield bonds, but excludes REIT common stock, while the solid line presents the case where REIT common stock is also included. The investor has access to the risk free asset and tangency portfolios are given by green circles. Data are monthly and span the period November 1992 to November 2012.

Panel A



Panel B

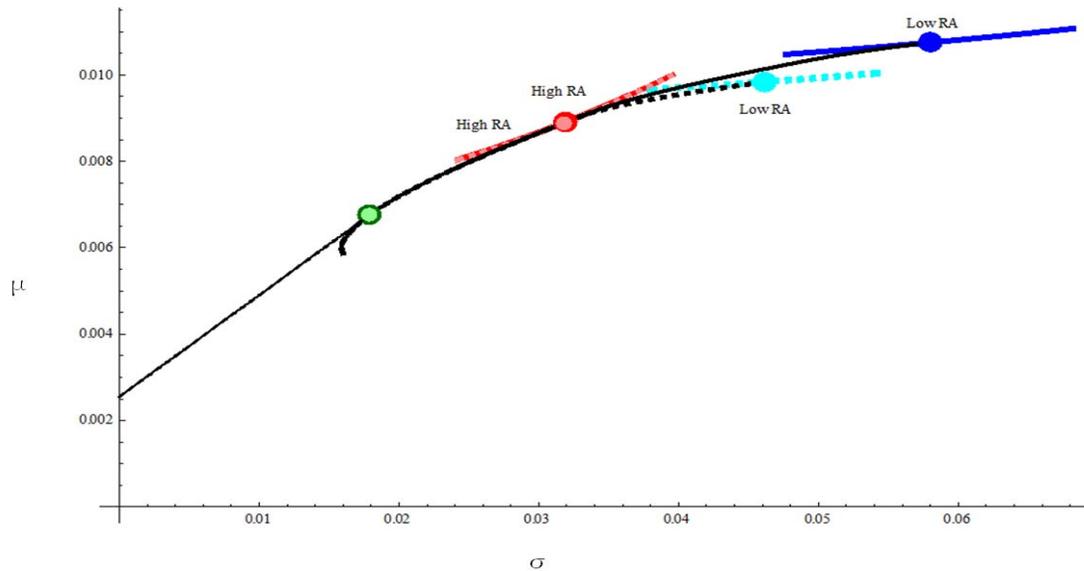


Figure 10: Compensation Ratios

Figure reports compensation ratios over different levels of risk aversion. Compensation ratios are calculated as the dollar compensation required to compensate the investor for losing access to REIT common stock (blue) and REIT preferred stock (red) divided by the compensation required to compensate the investor for losing access to both REIT common and preferred stock. The investment opportunity set is the same as in Figure 9.

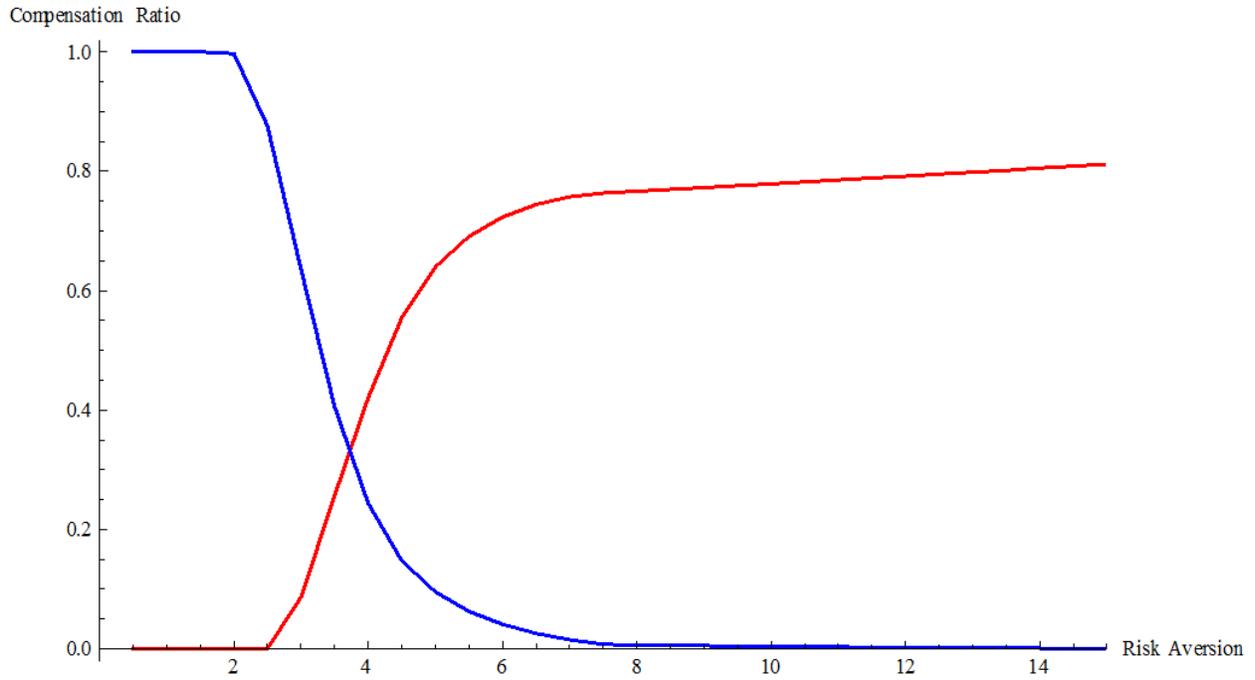
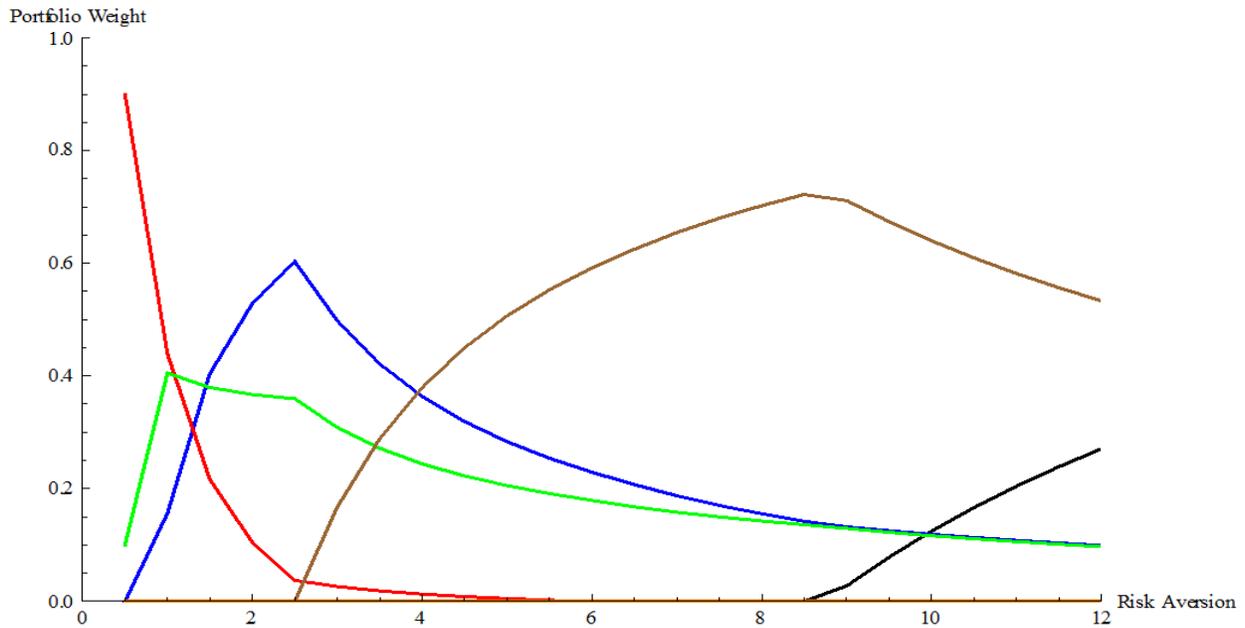


Figure 11: Optimal Portfolio Weights

Figure reports optimal portfolio weights over different levels of risk aversion. Red is the weight to REIT common stock, blue is the weight to REIT preferred stock, green is the weight to all other equity asset classes, brown is the weight in investment grade bonds, orange is the weight in high yield bonds, and black is the allocation to the risk free asset. Panel A presents weights when the investor has access to all assets. Panel B, reports weights when the investor loses access to REIT common stock, and Panel C reports weights when the investor loses access to REIT preferred stock.

Panel A



Panel B

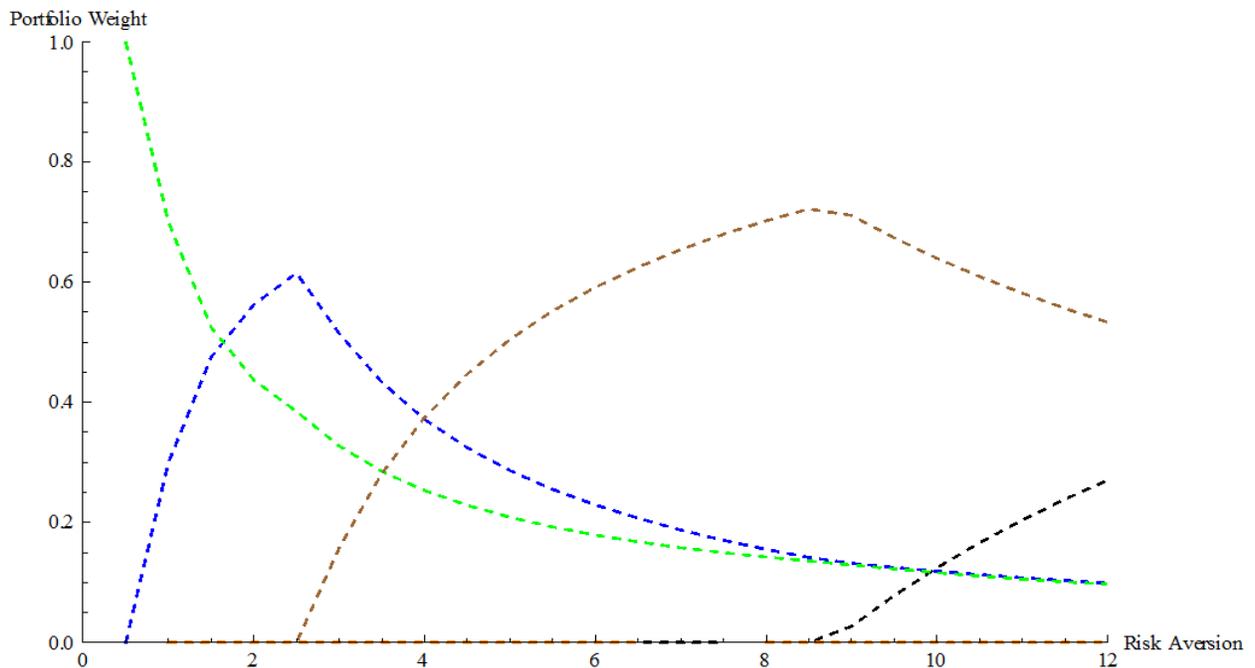


Figure 11 ctd...

Figure reports optimal portfolio weights over different levels of risk aversion. Red is the weight to REIT common stock, blue is the weight to REIT preferred stock, green is the weight to all other equity asset classes, brown is the weight in investment grade bonds, orange is the weight in high yield bonds, and black is the allocation to the risk free asset. Panel A presents weights when the investor has access to all assets. Panel B, reports weights when the investor loses access to REIT common stock, and Panel C reports weights when the investor loses access to REIT preferred stock.

Panel C

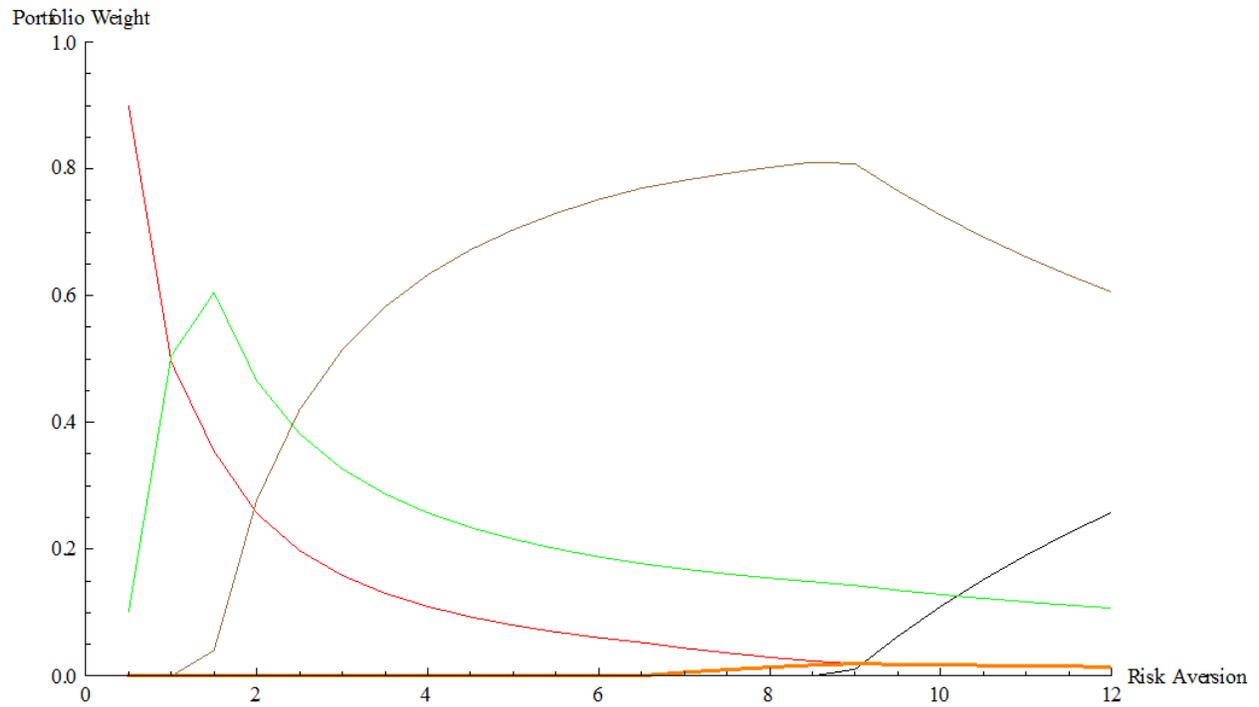


Table 1: Returns and Correlations

Table reports descriptive statistics of monthly returns on 13 indices between November 1992 and November 2012. Panel A reports means and standard deviations, while Panel B reports correlations. Pref is the MSCI REIT Preferred index, IG Corp is the Barclays Investment Grade Corporate Bond index, HY Corp is the Barclays High Yield Corporate Bond index, World Ex-US is the MSCI World Ex-US index, REIT is the SNL Equity REIT index, MidCap is the Russell MidCap index, MidCap Growth is the Russell MidCap Growth index, MidCap Value is the Russell MidCap Value index, Rus2000 is the Russell 2000 index, Rus2000 Growth is the Russell 2000 Growth index, Rus2000 Value is the Russell 2000 value index, and SP500 is the S&P 500 index.

Panel A												
	Pref	IG Corp	HY Corp	World Ex-US	REIT	MidCap	MidCap Growth	MidCap Value	Rus2000	Rus2000 Growth	Rus2000 Value	SP500
<i>Monthly</i>												
Mean	0.009	0.006	0.007	0.007	0.011	0.010	0.009	0.010	0.009	0.008	0.010	0.008
Std	0.033	0.016	0.026	0.049	0.058	0.049	0.062	0.046	0.057	0.067	0.050	0.044
<i>Annualized</i>												
Mean	0.103	0.071	0.083	0.078	0.129	0.115	0.107	0.118	0.103	0.091	0.115	0.092
Std	0.114	0.055	0.090	0.170	0.201	0.168	0.213	0.160	0.196	0.233	0.174	0.151
Panel B: Correlations												
Pref	1.00	0.54	0.68	0.40	0.62	0.47	0.34	0.52	0.42	0.34	0.49	0.38
IG Corp	0.54	1.00	0.54	0.29	0.31	0.28	0.21	0.31	0.19	0.16	0.22	0.27
HY Corp	0.68	0.54	1.00	0.63	0.61	0.68	0.60	0.66	0.63	0.59	0.63	0.62
World Ex-US	0.40	0.29	0.63	1.00	0.54	0.82	0.75	0.77	0.75	0.72	0.72	0.82
REIT	0.62	0.31	0.61	0.54	1.00	0.65	0.47	0.75	0.65	0.52	0.77	0.56
MidCap	0.47	0.28	0.68	0.82	0.65	1.00	0.93	0.93	0.93	0.89	0.90	0.93
MidCap Growth	0.34	0.21	0.60	0.75	0.47	0.93	1.00	0.74	0.90	0.95	0.76	0.85
MidCap Value	0.52	0.31	0.66	0.77	0.75	0.93	0.74	1.00	0.82	0.72	0.91	0.88
Rus2000	0.42	0.19	0.63	0.75	0.65	0.93	0.90	0.82	1.00	0.97	0.94	0.81
Rus2000g	0.34	0.16	0.59	0.72	0.52	0.89	0.95	0.72	0.97	1.00	0.83	0.78
Rus2000v	0.49	0.22	0.63	0.72	0.77	0.90	0.76	0.91	0.94	0.83	1.00	0.79
SP500	0.38	0.27	0.62	0.82	0.56	0.93	0.85	0.88	0.81	0.78	0.79	1.00

Table 2: Optimal Portfolios: Unconstrained Case

Optimal portfolios are constructed using different investment opportunity sets. For each opportunity set four cases are considered: (a) Both preferred index and REIT Common index are included; (b) REIT Common index is included, but preferred index is not; (c) Preferred index is included, but REIT Common is not; (d) Neither preferred index nor REIT Common are included (no real estate). The table reports expected return, standard deviation, the Sharpe ratio of the market (tangency) portfolio, and the percent improvement in the Sharpe ratio relative to the benchmark case (d) when no REIT common or preferred stock is included. The data is monthly for the period November 1992 through November 2012. The average monthly risk-free rate (1 month T-Bill) is 0.00255. Equity indices are (1) MSCI REIT Preferred Index; (2) MSCI World Ex-US Index; (3) SNL REIT Index; (4) S&P 500 Index; (5) Russell Mid Cap Index; (6) Russell Mid Cap Growth Index; (7) Russell Mid Cap Value Index; (8) Russell 2000; (9) Russell 2000 Growth; (10) Russell 2000 Value. Bond indices are the Barclays Investment Grade Bond Index and the Barclay's High Yield Bond Index.

	Market Expected Return	Market St.Dev.	Sharpe Ratio	Increase In Sharpe (%)	Market Expected Return	Market St.Dev.	Sharpe Ratio	Increase In Sharpe (%)
	Equity Only				Equity and Bonds			
Panel A: All Equity Indices								
Including Prefs & REITS	0.01315	0.0404	0.2622	8.26%	0.0089	0.02203	0.2881	0.38%
Including REITs, No Prefs	0.01715	0.05988	0.2438	0.66%	0.0088	0.02176	0.2872	0.07%
Including Prefs, No REITs	0.01327	0.04093	0.2619	8.13%	0.00894	0.02221	0.2877	0.24%
No REITs, No Prefs	0.01729	0.06085	0.2422	Base	0.00884	0.02189	0.287	Base
Panel B: S&P 500, World Ex-US, MidCap, Russell 2000 (No Midcap V/G, No Russell 2000 V/G)								
Including Prefs & REITS	0.01179	0.04158	0.2223	19.58%	0.00789	0.02096	0.2546	1.56%
Including REITs, No Prefs	0.01651	0.07029	0.1986	6.83%	0.00776	0.0206	0.2528	0.84%
Including Prefs, No REITs	0.01162	0.04087	0.2219	19.37%	0.00783	0.02077	0.254	1.32%
No REITs, No Prefs	0.01714	0.07847	0.1859	Base	0.00758	0.02007	0.2507	Base
Panel C: S&P 500, World Ex-US, MidCap V/G, Russell 2000 V/G (No MidCap, No Russell 2000)								
Including Prefs & REITS	0.01255	0.04139	0.242	10.50%	0.00831	0.0208	0.2766	0.44%
Including REITs, No Prefs	0.0172	0.06656	0.22	0.46%	0.00824	0.02059	0.2759	0.18%
Including Prefs, No REITs	0.01275	0.04235	0.241	10.05%	0.00835	0.02103	0.2757	0.11%
No REITs, No Prefs	0.01729	0.06728	0.219	Base	0.00829	0.02083	0.2754	Base
Panel D (Most Restricted Set): S&P 500, World Ex-US (No MidCap, MidCap V or G, Russell 2000, Russell 2000 V or G)								
Including Prefs & REITS	0.00894	0.03238	0.1973	64.14%	0.00682	0.01794	0.2376	3.85%
Including REITs, No Prefs	0.01026	0.04987	0.1546	28.62%	0.00663	0.01741	0.2343	2.40%
Including Prefs, No REITs	0.00872	0.03138	0.1964	63.39%	0.00671	0.01763	0.2358	3.06%
No REITs, No Prefs	0.00804	0.04568	0.1202	Base	0.00634	0.01654	0.2288	Base