Election Cycle of Real Exchange Rate in Latin America and East Asia

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Among East Asian countries, real exchange rates (RER) tend to be more depreciated before and appreciated after elections, forming an electoral cycle in the opposite direction of the one exhibited by Latin American countries. This paper proposes a theoretical model that explains the opposite RER electoral cycle in these two regions. In a setup where policy-makers differ in their preference bias towards non-tradable and tradable sector citizens, the RER is used a noisy signal of the incumbent’s type in an uncertain economic environment. The mechanism behind the cycle is engendered by the incumbent trying to signal he is median voter’s type, biasing his policy in favor of the majority of the population before elections. The driving forces of the opposite exchange rate populism in these two regions is the RER distributive effects and the difference of the relative size of tradable and non-tradable sectors in these two regions.

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1 Introduction

Empirical studies on the political economy of exchange rate policy in Latin America have identified an electoral cycle of exchange rate: the real exchange rate (RER) is more appreciated than average before elections and more depreciated after elections (Bonomo and Terra [1999], Frieden and Stein [2001] and Pascó-Font and Ghezzi [2000]). In a more recent study, Ryou [2008] identified for Korea the opposite electoral cycle to that in Latin America, that is, more depreciated RERs before elections and appreciated after. Huang and Terra [2012], on their turn, perform a broad comparison between the Latin American and the East Asian experiences, and they find that RERs in these two regions do exhibit opposite election cycles.

There are basically two competing explanations for the RER electoral cycles in Latin America. Stein and Streb [2004] and Stein, Streb and Ghezzi [2005] suggest that exchange rate cycles are generated by politicians who signal their competence by temporarily slowing the rate of currency depreciation below its sustainable level before elections, thus generating the exchange rate electoral cycles observed in the region. Alternatively, Bonomo and Terra [2005] emphasize the distributive impact of RER as the main ingredient leading to exchange rate policy cycles. More specifically, a RER depreciation favors exporters and import competing domestic industries, to the detriment of non-tradable sector workers. Policymakers’ preferences, which are biased towards different groups in society, are concealed from voters with the help of an unstable macroeconomic environment. RER electoral cycles is then the result of the incumbent’s attempt to emulate a preference bias towards the median voter, who is a non-tradable citizen, and thereby increase his re-election probability.

The two alternative explanations for the RER electoral cycles, in a nutshell, competence or preferences signaling, were equally capable of explaining the Latin American experience. The recent empirical findings of RER electoral cycles in opposite direction among Asian economies can help to disentangle the two explanations. While the competence signaling could not generate such cycles, we show in this paper that preferences signaling can encompass both types of cycles.
East Asian countries are relatively more open compared to countries in Latin America. In East Asian export-oriented economies, the majority of the population works in the tradable sector, whereas in Latin America it is the non-tradable sector that attracts the highest share of workers. As a result, while an appreciated currency are more “popular” in Latin America, the majority of East Asian citizens should prefer a more depreciated exchange rate. The exchange rate populism goes then in opposite directions in these two regions: in Latin America, a RER appreciation pleases the median voter, whereas a depreciation is more popular in East Asia.

We generalize the theoretical model in Bonomo and Terra [2005], to develop a dynamic, multidimensional signaling game between incumbent and forward-looking rational voters that generate RER election cycles. Policymakers differ in their preferences bias towards citizens in tradable and in non-tradable sectors, and this difference is concealed from the public with the help of an unstable macroeconomic environment. Government policy affects the level of the RER which, in turn, have a distributive impact: depreciated RER favors tradable sector citizens in detriment to non-tradable sector citizens.

Voters, who would like to elect the politician that attributes more weight to her own welfare, infer the incumbent’s type from the observed RER level. Intuitively, a more depreciated exchange rate has a higher probability to be the result of economic policy from a government that favors the tradable sector. Hence, the incumbent has an incentive tilt economic policy in favor of the median voter to increase his probability of re-election. This behavior generates policy cycles around elections, and a corresponding RER cycle. Moreover, the direction of the RER cycle depends on the median voter’s type. In economies where the median voter is a tradable sector citizen, the RER will be on average more depreciated before and appreciated after elections, as observed in East Asian economies. With a median voter from the non-tradable sector, the opposite election cycle should be observed, as the one in Latin America.

The paper is organized as follows. Section 2 compares Latin American and East Asian economies, with supporting evidence of our hypothesis that generate the opposite exchange rate populism in these two regions. Section 3 describes the model’s setup, whereas the equilibrium is presented in section 4. In section 5 we
show how the model generates the two possible RER election cycles. Section 6 concludes.

2 Opposite Exchange Rate Populism: Latin America and East Asia

Bonomo and Terra [2005] highlight the distributive impact of RER as the center piece of the electoral RER cycles in Latin America. More specifically, a RER appreciation favors the citizens in non-tradable sector, to the detriment of tradable sector citizens. Hence, if the majority of the population works in non-tradable sector, appreciation is more popular, and policy-maker use policies that appreciate the RER to increase their chances of getting reelected. However, if most of the population is in the tradable sector it is depreciation that is more popular, and policies that depreciate the currency are the ones that increase re-election probability. In this section we present evidence that suggests that the median voter in these two regions are in different sectors, which, we argue, is the driving force of the opposite electoral RER cycle.

Table 1: Exports-To-GDP Ratio

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<td>(1)</td>
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<tr>
<td>L.A.</td>
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<td></td>
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<tr>
<td>Brazil</td>
<td>11%</td>
<td>9%</td>
<td>13%</td>
</tr>
<tr>
<td>Chile</td>
<td>33%</td>
<td>29%</td>
<td>37%</td>
</tr>
<tr>
<td>Colombia</td>
<td>17% 21%</td>
<td>17% 18%</td>
<td>17% 23%</td>
</tr>
<tr>
<td>Mexico</td>
<td>25%</td>
<td>22%</td>
<td>28%</td>
</tr>
<tr>
<td>Peru</td>
<td>17%</td>
<td>13%</td>
<td>20%</td>
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<tr>
<td>E.A.</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Indonesia</td>
<td>31%</td>
<td>27%</td>
<td>36%</td>
</tr>
<tr>
<td>Korea</td>
<td>34% 52%</td>
<td>28% 43%</td>
<td>39% 60%</td>
</tr>
<tr>
<td>Malaysia</td>
<td>100%</td>
<td>85%</td>
<td>114%</td>
</tr>
<tr>
<td>Philippine</td>
<td>43%</td>
<td>35%</td>
<td>50%</td>
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Source: World Bank. 2. All value added percentage.

The development strategy adopted by East Asian countries was based on export-
oriented policies, whereas policies in Latin America have been more import-oriented. The ratio of export to GDP, presented in Table 1 for selected countries from these two regions, is relatively higher in East Asia compared to Latin America. The exports-to-GDP ratio ranges from 31% to 100% in East Asia, whereas the highest ratio is only 33% in Latin America. Furthermore, the average ratio of export to GDP in East Asia (52%) is almost 2.5 times higher than in Latin America (21%) over the 1990-2007 period. The gap widens after the 1997 Asian financial crisis.

Table 2: GDP in non-tradable sector (% of GDP)

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<tr>
<td>L.A.</td>
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<td></td>
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<tr>
<td>Brazil</td>
<td>61%</td>
<td>57%</td>
<td>63%</td>
</tr>
<tr>
<td>Chile</td>
<td>48%</td>
<td>47%</td>
<td>47%</td>
</tr>
<tr>
<td>Colombia</td>
<td>51%</td>
<td>56%</td>
<td>48%</td>
</tr>
<tr>
<td>Mexico</td>
<td>62%</td>
<td>62%</td>
<td>62%</td>
</tr>
<tr>
<td>Peru</td>
<td>57%</td>
<td>58%</td>
<td>56%</td>
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<tr>
<td>E.A.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indonesia</td>
<td>40%</td>
<td>41%</td>
<td>39%</td>
</tr>
<tr>
<td>Korea</td>
<td>49%</td>
<td>41%</td>
<td>41%</td>
</tr>
<tr>
<td>Malaysia</td>
<td>41%</td>
<td>41%</td>
<td>41%</td>
</tr>
<tr>
<td>Philippine</td>
<td>35%</td>
<td>34%</td>
<td>37%</td>
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Table 2 compares the ratio of the non-tradable sector production in total GDP between these two regions in order to illustrate the relative importance of non-tradable sector in the national economy. We can see that the non-tradable sector accounts, on average, for 56% of total production in Latin America, compared to 41% in East Asia.

Finally, Table 3 presents the share of workers in the non-tradable sector. In three-sector classification, we take agriculture and industry as the tradable sector and services as the non-tradable one. The column (1) report the average share of workers in the non-tradable sector from 1990 to 2007 for each country, and the average values for each region are in column (2). The last four columns present
Table 3: Percentage of Employees in non-tradable Sectors

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<td>(1)</td>
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<tr>
<td>L.A.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brazil</td>
<td>47%</td>
<td>45%</td>
<td>49%</td>
</tr>
<tr>
<td>Chile</td>
<td>50%</td>
<td>46%</td>
<td>53%</td>
</tr>
<tr>
<td>Colombia</td>
<td>58%</td>
<td>52%</td>
<td>64%</td>
</tr>
<tr>
<td>Mexico</td>
<td>46%</td>
<td>43%</td>
<td>48%</td>
</tr>
<tr>
<td>Peru</td>
<td>59%</td>
<td>66%</td>
<td>53%</td>
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<tr>
<td>E.A.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indonesia</td>
<td>35%</td>
<td>34%</td>
<td>37%</td>
</tr>
<tr>
<td>Korea</td>
<td>53%</td>
<td>43%</td>
<td>48%</td>
</tr>
<tr>
<td>Malaysia</td>
<td>47%</td>
<td>45%</td>
<td>48%</td>
</tr>
<tr>
<td>Philippines</td>
<td>34%</td>
<td>31%</td>
<td>37%</td>
</tr>
<tr>
<td>Panel A: Male</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L.A.</td>
<td>Brazil</td>
<td>71%</td>
<td>69%</td>
</tr>
<tr>
<td>Chile</td>
<td>82%</td>
<td>80%</td>
<td>83%</td>
</tr>
<tr>
<td>Colombia</td>
<td>76%</td>
<td>75%</td>
<td>75%</td>
</tr>
<tr>
<td>Mexico</td>
<td>73%</td>
<td>71%</td>
<td>74%</td>
</tr>
<tr>
<td>Peru</td>
<td>73%</td>
<td>84%</td>
<td>64%</td>
</tr>
<tr>
<td>E.A.</td>
<td>Indonesia</td>
<td>38%</td>
<td>36%</td>
</tr>
<tr>
<td>Korea</td>
<td>65%</td>
<td>55%</td>
<td>58%</td>
</tr>
<tr>
<td>Malaysia</td>
<td>57%</td>
<td>50%</td>
<td>50%</td>
</tr>
<tr>
<td>Philippines</td>
<td>60%</td>
<td>56%</td>
<td>63%</td>
</tr>
<tr>
<td>Panel B: Female</td>
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</tbody>
</table>

those percentages for two sub-periods, before and after the Asian financial crisis from 1997.

Panel A presents the shares for male workers. We can see that in Latin American countries the majority of male workers are in the non-tradable sector, with average shares of 52% over the whole period. By contrast, in East Asia the ratio of male workers in the non-tradable sector to the total male workers is 43% over the period 1990-2007, that is, 10% lower than their Latin America counterparts. The percentages are 39% and 45% for the period 1990-1997 and 1998-2007, respectively, which represent less than half of working men. Hence, the majority of male workers are in in the tradable sectors.

The share of female workers, presented in panel B of the table, is also larger in Latin American countries than in East Asian ones. In Latin American countries, the average share ranges from 71% to 82%, with an average 75% from 1990 to 2007. In East Asian countries, on their turn, it ranges from 38% to 65%, with an average 55%, which is 20% lower than in Latin America on average.

All in all, we have seen that exports and tradables production represent a larger share of GDP in East Asian economies compared to Latin American countries, and that the majority of the working population in non-tradable sector in Latin America, whereas most men are in the tradable sector in East Asia. As a result of these comparisons, the “popularity” of appreciated currency could be a reasonable assumption for Latin America, whereas the majority of East Asian citizens, on their turn, should prefer a more depreciated exchange rate.

3 The model

We propose a theoretical model in which RER election cycles are generated through a signaling game where policy-maker uses exchange rate policy to increase his re-election probability. This model is based on Bonomo and Terra [2005], which explains the RER electoral cycle observed in Latin America with the assumption that the majority of the population prefers an appreciated RER. Therefore, the RER electoral cycle generated by their model is an appreciated RER before elections, and depreciated after elections. We generalize Bonomo and Terra [2005]’s
model by letting the median voter to be either a tradable or non-tradable citizen, so that we can explain the RER election cycle no only in Latin America, but also the opposite cycle observed in East Asia.

3.1 Model Set up

There are two non-storable goods in this model economy, a tradable good ($T$) and a non-tradable one ($N$). We take the non-tradable good as numeraire, and the relative price of tradable $e$ we define as the real exchange rate. Citizens derive utility from the consumption of both types of goods, with Cobb-Douglas preferences,\(^1\) according to which they spend a share $\alpha$, $\alpha \in (0, 1)$, of their income on the consumption of non-tradable goods, and a share $(1-\alpha)$ on tradables. Preferences, both of government and of common citizens, are additively separable in time with discount factor. We assume that there are no financial markets, hence each period’s consumption expenditures must equal disposable income. This assumption will simplify some intertemporal relations by making the consumers’ problem time separable.

We consider an endowment economy, where each citizen receives each period an endowment $y^J$ of the good $J$, for $J = T, N$. There is, however, uncertainty with respect to the amount of good each citizen receives. More specifically, we assume that the endowment of a citizen in sector $J$ is a log-normally distributed random variable with support on $[0, \infty)$, that is, the probability density of $y^J$ is given by:\(^2\)

$$f_J(y^J) = \frac{\exp\left\{-\frac{(\ln y^J - \mu^J)^2}{2\sigma^J}\right\}}{y^J \sigma^J \sqrt{2\pi}}, \text{ for } J = N, T \tag{1}$$

where $\mu^J$ and $\sigma^J$ are parameters representing, respectively, the average and standard deviation of endowment in sector $J$.

\(^1\)We use Cobb-Douglas preferences for simplicity, to have closed form solutions. We would have the same qualitative results with any concave and continuous utility function.

\(^2\)To simplify notation, we omit time subscripts whenever it is not confusing to do so.
Non-tradable sector citizens

A non-tradable citizen receives each period an endowment $y^N$ and pays as taxes a share $\tau$ of her income. With a disposable income of $(1 - \tau)y^N$, she consumes tradable and non-tradable goods, according to the following demand functions:

$$C^N_a(e, y^N) = \alpha(1 - \tau)y^N \quad \text{and} \quad C^T_a(e, y^N) = \frac{(1 - \alpha)}{e}(1 - \tau)y^N,$$

where the subscript $a$ indicates to non-tradable citizens variables, referring to the fact that they prefer an appreciated RER.

Demand functions (2) yield the following indirect utility function for non-tradable sector citizens:

$$V^a(e, y^N) = h y^N e^{-\alpha}$$

where $h \equiv \alpha^\alpha(1 - \alpha)^{1-\alpha}(1 - \tau)$. Note that this is a decreasing function of $e$, that is, non-tradable sector citizens prefer an appreciated RER.

 Tradable sector citizens

Similarly, a tradable sector consumer has a disposable income of $e(1 - \tau)y^T$, and demand function represented by:

$$C^N_d(e, y^T) = \alpha(1 - \tau)e y^T \quad \text{and} \quad C^T_d(e, y^T) = (1 - \alpha)(1 - \tau)y^T$$

which yield the following indirect utility function of tradable sector citizen:

$$V^d(e, y^T) = h y^T e^{\alpha}$$

This indirect utility function is an increasing function of $e$, which means the tradable sector citizens are better off with more depreciated RERs. Subscript $d$ indicates tradable sector citizens, who prefer a depreciated RER.

Policymakers’ preferences

We assume that policy-makers are derive utility not only from citizens welfare, but also from being in office. That is, policy-makers receive additional ego rents
$\chi$, with $\chi = C > 0$ per period in office, and $\chi = 0$ when not elected. Hence, the policy-maker’s per period utility function can be represented by:

$$\tilde{V}^i(e, y^N, y^T) = W(V^a) + \theta^i W(V^d) + \chi, \text{ for } i = a, d \quad (6)$$

Here, we assume that policy-makers are not only concerned about citizens’ utility level, but also about the disparity between two groups’ utility. The concern about the disparity may be motivated by the fact that the inequality between two groups can lead to social unrest. $W(\cdot)$ is thus an increasing and concave function in citizens’ utility: $W'(\cdot) > 0$ and $W''(\cdot) < 0.$

As in Bonomo and Terra [2005], we assume that policy-makers may differ in their preferences. The idea is that tradable sector lobbying may bias policy-maker preferences towards the tradable sector, as proposed by Bonomo and Terra [2010]. As a result, policy-makers may differ in the relative weight they attribute to the welfare of tradable citizens. The result of the lobbying activity is uncertain, and the public cannot observe directly whether the policy-maker has been captured by the tradable sector lobbying.

More specifically, we assume that there are two types of policy-makers: $d$ and $a$. Policymakers of type $d$ give relatively more weight to tradables utility, thus choosing economic policy that generate more depreciated exchange rates on average. Type $a$ policy-makers, on their turn, give relatively less weight to tradables utility, delivering more appreciated RERs. This difference in captured by the parameter $\theta^i$ in the politician’s utility function (6), with $0 < \theta^a < \theta^d$.

We do not model the lobbying activity in this paper, but the mechanism we have in mind is the one proposed by Bonomo and Terra [2010]. As a shortcut, we assume that politicians are randomly assigned a type, $\theta^a$ or $\theta^d$, so that with probability $p^m$ the politician is of the median voter type.

Government finances its expenditures by taxing the endowments of each citizen, and spending it in both tradable and non-tradable goods. Given our assumption that there are no financial markets, the government must have a balanced budget

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3This formulation is similar to that in Rogoff [1990], in which the policy-maker’s per period utility is a concave function of citizen’s consumption and public investment goods, plus ego rents.
at all periods, that is:

\[ G = G^N + G^T = nτy^N + (1 - n)τey^T, \]  

(7)

where \( G \) is total expenditure per capita, \( G^N \) and \( G^T \) represent government spending per capita on non-tradable goods and tradable goods, respectively. \( n \) is the share of the population in non-tradable sector.

We take the tax rate \( τ \) as exogenous. Typically, changing tax rates takes time since it usually has to be approved by congress. Therefore, it cannot be used as short term economic policy, which is the focus of this paper on electoral cycles. Hence, the policy-makers’ policy choice is how to distribute government expenditures between tradable and non-tradable goods. We denote \( s \) as the share of government expenditure used to buy non-tradable goods, so that \( G^N = sG \) and \( G^T = (1 - s)G \).

As we will see in the next session, government spending allocation \( s \) affects the equilibrium RER \( e \), which, in turn, impacts citizens’ utility according to equations (3) and (5). Since we want to focus on the incentives for the government to use economic policy to manipulate the RER, we will abstract from the direct impact of policy choice on citizens’ utility. That is, we assume that expenditure allocation across sectors does not have a direct impact on the utility of individuals. ⁴

**Equilibrium RER**

Since there are no financial markets and there is only one type of tradable good, the market equilibrium conditions for this economy are the same as those of a closed economy. ⁵ Equilibrium RER is the one that assures equilibrium in the markets

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⁴Notice that government expenditures do not appear in the demand functions (2) and (4). Our results would not change if we had included total expenditure per capita \( G \) in the utility function, either multiplicatively (so it would also appear on demand function) or additively. The important assumption is that the two types of citizens are indifferent about whether it is spent on tradables or non-tradables.

⁵Note that the driving force of RER election cycle is the distributive impact of the RER, that is, the relative price of tradable and non-tradable goods, on different citizens’ utility. It is, thus, not related to intertemporal effects.
for tradable and non-tradable goods, that is, the relative price that makes demand equal supply in each sector, as in:

\[
sG + nC^N_a(e, y^N) + (1 - n) C^N_d(e, y^T) = ny^N, \quad \text{and} \quad \quad (8)
\]

\[
(1 - s) \frac{G}{e} + nC^T_a(e, y^N) + (1 - n) C^T_d(e, y^T) = (1 - n)y^T, \quad \quad (9)
\]

where the demand functions \( C^J_i(e, y^J) \), \( J = N, T \), are in equations (2) for non-tradable sector citizens \( i = a \), and in equations (4) for those in the tradable sector \( i = d \).

Solving either one of the market equilibrium equations (8) or (9), and using the government’s budget constraint, we obtain the equilibrium RER:

\[
e(s, y^N, y^T) = \eta H(s) \left( \frac{y^N}{y^T} \right), \quad (10)
\]

where \( \eta = \frac{n}{1 - n} \), and \( H(s) = \frac{1 - s\tau - \alpha(1 - \tau)}{s\tau + \alpha(1 - \tau)} \).

According to equation (10), the equilibrium RER is a function of the allocation of government expenditures on non-tradable goods \( s \). More specifically, since \( H(s) \) is a decreasing function of \( s \), the more the government spends on non-tradable goods, the more appreciated is the equilibrium RER. Equilibrium RER depends also on the relative endowments in the two sectors: a lower relative endowment of non-tradables results in a more appreciated equilibrium RER. Hence, a more appreciated RER may be the result of either more government spending on non-tradable goods or a lower relative endowment in that sector.

### 3.2 Events around elections

Elections are held every other period, with two candidates: the incumbent and the opponent. The citizens in each sector are assumed to be identical, so their voting preferences are the same. Let \( m \) be the sector to which the median voter belongs,
and $\overline{m}$ be the other sector, so that:

$$m = \begin{cases} N & \text{if } n > \frac{1}{2} : \text{ median voter is a notradable sector citizen } a \\ T & \text{if } n < \frac{1}{2} : \text{ median voter is a tradable sector citizen } d \end{cases}$$

$$\overline{m} = \begin{cases} N & \text{if } m = T \\ T & \text{if } m = N \end{cases}$$

The election cycle can be shown in a two-periods setup, with an election between the periods. We first describe the events in the pre-election period $t$. The politicians’ preferences are randomly assigned, and, after observing his own type, the incumbent chooses economic policy, which is the share of government spending on non-tradable goods, $s$. The endowments in two sectors, $y^N, y^T$ are then distributed, determining, with the chosen policy $s$, the equilibrium RER, $e_T = e(s^t, y^N_t, y^T_t)$, as established in equation (10). The median voter does not know neither the politicians’ type $i$, nor the policy chosen $s$, nor the endowment in the other sector $y^{\overline{m}}_t$. She makes her vote decision according to the information she has, which are the endowment in her sector $y^m_t$, and the observed RER $e_T$.

In the post-election period $t + 1$, the election winner sets new policy $s^t_{t+1}$ and the equilibrium RER is determined once the endowments in two sectors are realized, $e^t_{t+1} = e(s^t_{t+1}, y^N_{t+1}, y^T_{t+1})$.

Notice that we assume that there is persistence of the policy-maker’s preferences before and after elections, which is essential for the election cycle to be generated. Voter only care about the policy-maker type if they believe his type will not change completely after election. We use the simplifying assumption that the type does not change at all around elections.

We argue, however, that preferences may change in between elections. In the preparation process for elections, new alliances are made, government’s composition may change, and the result may affect the politicians’ preference bias towards the two sectors in the economy. To capture this change, we assume that preferences are randomly assigned to politicians in the period prior to elections.
4 Equilibrium

Substituting the equilibrium RER from equation (10) into the citizens’ indirect utility functions (3) and (5), the indirect utility function of non-tradable and tradable sector citizens become:

\[ V^a(s, y^N, y^T) = hH(s)^{(1-\alpha)} \] (11)
\[ V^d(s, y^N, y^T) = hH(s)^{\alpha} \] (12)

where \( h \equiv \bar{h}\eta^{\alpha-1}(y^N)^{\alpha}(y^T)^{1-\alpha} \) and \( H(s) \equiv \frac{1 - s\tau - \alpha(1 - \tau)}{s\tau + \alpha(1 - \tau)} \).

Since \( H(s) \) decreases in \( s \) and \( 0 < \alpha < 1 \), the non-tradable citizen’s utility \( V^a \) increases in \( s \), while for the tradable citizen \( V^d \) is a negative function of \( s \). In other words, a higher expenditure share on non-tradable goods favors non-tradable sector citizens’ interests, to the detriment of tradable sector citizens. Substituting equations (11) and (12) into equation (6), the incumbent’s per-period utility function can be written as:

\[ \hat{V}^i(s, y^N, y^T) = W [hH(s)^{(1-\alpha)}] + \theta W [hH(s)^{\alpha}] + \chi \] (13)

The incumbent chooses government expenditure allocation before observing endowments, so he relies on expected value of his utility, which can be obtained from taking expectations of equation (13) with respect to the endowment shocks. We can therefore get the incumbent’s expected utility in a period is a function of \( s \):

\[ F^i(s) \equiv E\left[ \hat{V}^i(s, y^N, y^T) \right] \\
= \int_0^\infty \int_0^\infty W \left(V^a(s, y^N, y^T)\right) f_N \left( y^N \right) f_T \left( y^T \right) dy^N dy^T \\
+ \theta^i \int_0^\infty \int_0^\infty W \left(V^d(s, y^N, y^T)\right) f_N \left( y^N \right) f_T \left( y^T \right) dy^N dy^T + C \\
= E \left[ W \left(V^a(s)\right) \right] + \theta^i E \left[ W \left(V^d(s)\right) \right] + C \] (14)

We now solve the dynamic game between the incumbent and the median voter. With our assumptions of no financial markets, non-storable goods, time separable
utility functions, and the politicians’ preferences independently assigned every two periods, we are able to break our problem into a sequence of identical two-period stage games. This implies that the equilibria strategies of each player are the same in every stage game.

In a stage game, the median voter’s strategy is her vote choice in the pre-election period $\bar{t}$, based on the information she has, which is the observed RER and the endowment in her sector.

The incumbent’s strategy is the expenditure allocation chosen in the pre-election and the post-election periods. The strategy can be represented by $s^* = \{s^a_*, s^a_{+1}, s^d_*, s^d_{+1}\}$, where $s^i_*$ is the expenditure share on non-tradable goods chosen by the incumbent of type $i$ before election, $i = a$ or $d$, and $s^i_{+1}$ is the one chosen by the election winner in the after-election period.

The road-map for finding equilibrium is as follows. Solving by backward induction, we start by finding the optimal policy choice after election. Then, we solve for the optimal strategies of the incumbent and of voters in the pre-election period.

### 4.1 After election policies

In the post-election period, there is no signaling issue in policy choice. New preferences will be assigned to all politicians before next elections, so that the policies chosen in a period following elections have no influence on the re-election probability in the next stage game. Hence, there is no point in trying to signal being of one type or another at that moment. Even though there is still asymmetric information, the incumbent has no strategic considerations in the period following elections, so that he chooses the policy to maximize his expected per-period utility function presented by equation (14). Thus, $s^i_{+1}$ is chosen so as to maximize his expected utility $F^i(s)$ in equation (14), that is:

$$s^{i*}_{+1} = \arg \max F^i(s).$$

As shown in appendix A.1, we have that $s^a_{+1} > s^d_{+1}$, that is, the non-tradable type of policy-maker chooses a relatively higher government expenditure share on non-tradable goods than the tradable type. Since the RER is a decreasing
function of $s$, as established in equation (10), $s_{i+1}^a$ yields more appreciated RERs, preferred by non-tradable citizens $a$, while $s_{i+1}^d$ generates more depreciated RERs, satifying tradable citizens $d$. Hence, citizens always prefer politicians of their own type.

4.2 Before election problem

Let us now analyze the incumbent’s and voters’ strategies in the period preceding elections. The median voter may be either a non-tradable or a tradable citizen. We then define $s_{m}^{i*}$ as expenditure share on non-tradable goods chosen in equilibrium by an incumbent of type $i$, $i = a, d$, in the pre-election period, when the median voter belong to sector $m$, $m = N, T$.

We start by solving the voter’s problem, and then calculating the incumbents re-election probability, which depends on the equilibrium expenditure share chosen by the incumbent, and then we look at the policy-maker’s problem.

The Median Voter’s Problem

We have seen that $s_{i+1}^a > s_{i+1}^d$, that is, after election, the policy-maker that favors the non-tradable type spends relatively more on non-tradables. This generates more appreciated RERs, which are prefered to non-tradable citizens. Hence, they would like to elect a policy-maker of type $a$. Analogously, a tradable citizen would like to elect a type $d$ policy-maker. Hence, the median voter would like to vote for the policy-maker of her own type. However, under asymmetric information, the median voter cannot observe policy-makers’ type. She knows the probability distribution according to which politicians types are assigned, that is, with probability $p^m$ a politician is of median voter’s type.

For the opponent candidate, that is all the information the median voter has. As for the incumbent, she uses the information she has on the economic activity to try to infer his type. In particular she uses the observed RER, which results form the combination of economic policy and the endowment size in both sectors. We denote $\rho^m$ the updated probability the incumbent is of the median voter’s type.
If it is larger than or equal to \( p^m \), it means the incumbent is more likely to be of median voter’s type than the opponent, and the median voter will vote for the incumbent; otherwise she votes for the opponent. Therefore, the vote function can be rewritten as:

\[
\text{vote}_m(\hat{e}, y^m) = \begin{cases} \text{inc} & \text{if } \rho^m(\hat{e}, y^m) \geq p^m \\ \text{opp} & \text{otherwise} \end{cases} \tag{16}
\]

The observed RER is a function of the policy chosen and the endowment level in both sectors: \( \hat{\varepsilon}(s, y^N, y^T) \). Since the voter has information only on the endowment level in her own sector, she is not able to infer precisely the policy chosen from the RER level. She knows, however, the probability distribution for the other sector’s endowment, which she uses to form her belief about the incumbent’s type using Bayes’ rule. The median voter’s updated probability is:

\[
\rho^m = Pr(i = m \mid e = \hat{e}) = \frac{p^m \times g(e = \hat{e} \mid i = m)}{p^m \times g(e = \hat{e} \mid i = m) + (1 - p^m) \times g(e = \hat{e} \mid i = \overline{m})} \tag{17}
\]

where \( i \) denotes the incumbent’s type, \( \hat{\varepsilon} \) is the observed RER, and \( g(\cdot \mid \cdot) \) represents the conditional density function of RER given the incumbent’s type. From equation (17), \( \rho^m \geq p^m \), the condition that the median voter votes for the incumbent in equation (16), is equivalent to:

\[
g(e = \hat{e} \mid i = m) \geq g(e = \hat{e} \mid i = \overline{m}) \tag{18}
\]

According to equation (18), the median voter votes for the incumbent if and only if the observed exchange rate is more likely generated by the incumbent of her type, which is quite intuitive.

**Reelection probability**

The re-election probability, \( \pi \), is the probability that the median voter votes for the incumbent. Referring to the median voter’s voting function equation (16), \( \pi \) equals to the probability of \( \rho^m \geq p^m \), which holds if and only if \( g(e = \hat{e} \mid i = m) \geq g(e = \hat{e} \mid i = \overline{m}) \).
The median voter can observe the endowment in her own sector $y^m$ and the RER $\hat{e}$, but she does not observe the endowment in the other sector $y^\overline{m}$ nor the policy chosen $s$. Since the RER depends on the policy chosen $s$ and the realized endowments in both sectors, $\hat{e} = e(s, y^N, y^T)$, she can compute the endowment level in the other sector $y^\overline{m}$ that would generate the observed RER $\hat{e}$, given the policy chosen in equilibrium by a policy-maker of type $i$, $s^i_m$. Hence, the conditional density function of $\hat{e}$, given the incumbent’s type, $g(e = \hat{e} \mid i)$, is equal to the density function of that endowment $y^\overline{m}$ that would generate $\hat{e}$. That is:

$$g(e = \hat{e} \mid i) = f_{\overline{m}}(y^\overline{m} \mid e(s^i_m, y^N, y^T) = \hat{e}, y^m) \quad (19)$$

The median voter compares the density function of the other sector’s endowment that generates the observed RER for the equilibrium policies from two types of policy-makers: $s^a_m$ and $s^d_m$. Then, given equation (19), the condition for re-election in inequality (18) becomes:

$$f_{\overline{m}}(y^\overline{m} \mid e(s^a_m, y^N, y^T) = \hat{e}, y^m) \geq f_{\overline{m}}(y^\overline{m} \mid e(s^d_m, y^N, y^T) = \hat{e}, y^m) \quad (20)$$

where $e(\cdot)$ is defined by equation (10).

We can show this re-election condition by Figure 1. The horizontal axis shows the observed RER and the vertical axis is the probability density function of the endowment shock non observed by the median voter $y^\overline{m}$, which would yield the observed RER $\hat{e}$ with a given expenditure policy $s$ and the endowment in the median voter’s sector $y^m$, that is, $f_{\overline{m}}(y^\overline{m} \mid e(s, y^N, y^T) = \hat{e}, y^m)$. Since RER decreases in $s$, the curve more to the right corresponds to a lower level of $s$. Hence $s > s'$, that is, the solid curve in Figure1 corresponds the a larger expenditure share on non-tradable goods $s$, and the dotted curve for the smaller one $s'$.

The conditional density of the endowment shock in $\overline{m}$-sector is distinct for different expenditure policies. For instance, as can be seen from the Figure 1, the solid curve is higher than dotted one when $e = \hat{e}$, which means that RER $\hat{e}$ is more likely to be generated by the larger $s$.

By comparing the two conditional density functions at the observed RER level, the median voter makes her vote decision: she votes for the incumbent if the conditional density function with policy chosen by an incumbent of her own type
Figure 1: Re-election Condition

$s_{m}^{s}$ is larger than with policy from the other type of policy-maker $s_{m}^{d}$. She votes for the opponent otherwise. The median voter’s problem can thereby be rewritten as follows:

$$v_{om}(\hat{e}, y^{m}) = \begin{cases} 
  inc & \text{if } f_{m}(y^{m} | e(s_{m}^{s}, y^{N}, y^{T}) = \hat{e}, y^{m}) \geq f_{m}(y^{m} | e(s_{m}^{d}, y^{N}, y^{T}) = \hat{e}, y^{m}) \\
  opp & \text{otherwise}
\end{cases}$$

(21)

It is worth noting that an equilibrium in which both types of policy-makers choose the same policy level cannot exist. If actions chosen by the two types of policy-makers were the same, for every exchange rate level compatible with equilibrium, the median voter would attribute probability $\rho_{m} = p_{m}$ that the incumbent is of her own type. According to the median voter’s voting function in equation (16), the median voter would reelect the incumbent for any observed value of the RER. Since the observed RER would not affect his re-election probability in this case, the incumbent would have an incentive to deviate and choose the policy that maximizes his expected utility (14), that is, the same policy $s_{+1}^{d}$ chosen in the post-election period. We have seen that $s_{+1}^{a} > s_{+1}^{d}$, which means that policy-makers of different types would choose different policies. Therefore, a pooling equilibrium does not exist.

In the equilibria with different policies, there is a cutoff level of RER, $\hat{e}_{m}$ for which
inequality (20) is satisfied with equality, which is the point where the two curves cross in Figure 1.\footnote{Appendix A.2 shows that there will be a cutoff level for the exchange rate to guide the voting rule, with log-normal density distribution for the endowments.} Put in other words, at the RER cutoff level, the probabilities of this RER being generated by either type of incumbent are the same. The RER cutoff levels are determined as follows:

\[
\begin{align*}
\hat{e}_N &= \eta_N \sqrt{H(s^a_N) H(s^d_N)} \frac{y^N}{\exp(\mu^N - (\sigma^N)^2)} \\
\hat{e}_T &= \eta_T \sqrt{H(s^a_T) H(s^d_T)} \frac{y^T}{\exp(\mu^N - (\sigma^N)^2)}
\end{align*}
\]  

(22)

where $\eta_m$ is the share of the population in the non-tradable sector, with $\eta_N > 1$ and $\eta_T < 1$.

The median voter makes her voting decision by comparing the observed RER with cutoff level of RER, as illustrated in Figure 2. The graph (a) represents the voting decision for a median voter from the non-tradable sector and graph (b) when the median voter is a tradable citizen. In both graphs, we have that $s^a_m > s^d_m$, \footnote{In Proposition 4 we show that, indeed, $s^a_m > s^d_m$ in equilibrium.} where the left solid curve corresponds to the conditional density function for the higher value of $s$, $s^a_m$, while the right dotted one for the lower value $s^d_m$. The intersection of the curves determines the RER cutoff level, $\tilde{e}_m$.

A median voter from the non-tradable sector, $m = N$, would like to reelect a policy-maker of type $a$. She will then vote for the incumbent whenever the observed RER is more appreciated, that is, lower that the cutoff level $\tilde{e}_N$, since those RER values are more likely to be generated by the policy set by a type $a$ incumbent, $s^a_N$. Conversely, for more depreciated RERs $\hat{e} > \tilde{e}_N$, the median voter votes for the opponent. Graph (a) in Figure 2 indicates this voting strategy for the median voter, when she is a citizen from the non-tradable sector.

The median voter from the tradable sector, $m = T$, on her turn, would like to reelect a type $d$, policy-maker, who will generate more depreciated RERs on average after election. As shown in graph (b) of Figure 2, an exchange rate $\hat{e}$ more depreciated, that is, higher than $\tilde{e}_T$ is more likely to be generated by policy $s^d_T$, chosen...
Figure 2: Voting Rule of Median Voter
by incumbent of type $d$. Hence, the median voter votes for the incumbent if she observes RER is more depreciated than $\tilde{e}_T$, otherwise votes for the opponent.

Proposition 2 formalizes the median voter’s voting decision.

**Proposition 1 (Median voter’s voting decision)** If the median voter is a non-tradable sector citizen, she votes for the incumbent once she observes a real exchange rate lower (more appreciated) than or equal to the corresponding cutoff real exchange rate, and she votes for the opponent otherwise. When the median voter is a tradable sector citizen, the incumbent is re-elected if the observed real exchange rate is higher (more depreciated) than or equal to corresponding cutoff real exchange rate, and the opponent wins the election otherwise. The median voter’s voting decision is then:

$$
\begin{align*}
\nu^N_\text{inc} (\hat{e}) &= \begin{cases} 
\text{inc} & \text{if } \hat{e} \leq \tilde{e}_N \\
\text{opp} & \text{otherwise}
\end{cases} \\
\nu^T_\text{inc} (\hat{e}) &= \begin{cases} 
\text{inc} & \text{if } \hat{e} \geq \tilde{e}_T \\
\text{opp} & \text{otherwise}
\end{cases}
\end{align*}
$$

(23)

where the cutoff levels $\tilde{e}_N$ and $\tilde{e}_T$ are defined in equation (22).

**Proof.** See Appendix A.2

Now we can compute the re-election probability, which is the probability of having an endowment level for the non-median voter that generates a RER in the range where the median voter votes for the incumbent. From equation (22), the RER cutoff level $\tilde{e}_m$ is determined by the incumbents’ policies at equilibrium and the endowment shocks. Hence, given $\tilde{e}_m$, the incumbent’s policies at equilibrium and the observed endowment shock, the median voter can retrieve a cutoff level for endowment in the other sector $\tilde{y}^m$, which is implicitly defined by $\tilde{e}_m = e(s^a_m, s^d_m, \tilde{y}^m, y_m)$.

Using equations (22) and (10), we get explicit expressions for cutoff levels of endowments $\tilde{y}^T$, for $m = N$, and $\tilde{y}^N$, for $m = T$.

$$
\tilde{y}^T (s^1_N, s^a_N, s^d_N) = \frac{H(s^N_N)}{\sqrt{H(s^a_N) H(s^d_N)}} \exp \left( \frac{\mu T - (\sigma T)^2}{2} \right), \text{ for } m = N
$$

(24)

$$
\tilde{y}^N (s^1_T, s^a_T, s^d_T) = \frac{\sqrt{H(s^a_T) H(s^d_T)}}{H(s^T)} \exp \left( \frac{\mu N - (\sigma N)^2}{2} \right), \text{ for } m = T
$$

(25)
where \( s_i^m \) is the policy chosen by the incumbent of type \( i \), when the median voter is of type \( m \).

Given that exchange rate is a decreasing function of tradable sector endowment \( y^T \) and increasing in non-tradable sector endowment \( y^N \) (see equation (10)), the median voter’s voting decision described in Proposition 2 is equivalent to:

\[
vo_N (\bar{e}, y^N) = \begin{cases} 
inc & \text{if } y^T \geq \tilde{y}^T \\
op & \text{otherwise} \end{cases} \tag{26}
\]

\[
vo_T (\bar{e}, y^T) = \begin{cases} 
inc & \text{if } y^N \geq \tilde{y}^N \\
op & \text{otherwise} \end{cases} \tag{27}
\]

The incumbent is re-elected once the realized endowment is larger than the cutoff level. The re-election probability, which is the probability that the re-election condition occurs, is then \( \pi = Pr(y^m \geq \tilde{y}^m) \). The following proposition formalizes the re-election probability.

**Proposition 2 (The re-election probability)** When the incumbent chooses policy \( s_i \) before election, his re-election probability is:

\[
\pi^i_m (s^i, s^{a*}_m, s^{d*}_m) = \int_{y^m}^{\infty} f_m(y^m) \, dy^m, \text{ for } m = N, T \tag{28}
\]

where:

\[
\tilde{y}^T (s^i, s^{a*}_N, s^{d*}_N) = \frac{H (s^i) \sqrt{H (s^{a*}_N) H (s^{d*}_N)}}{\exp \left( \mu^T - (\sigma^T)^2 \right)} \tag{29}
\]

\[
\tilde{y}^N (s^i, s^{a*}_T, s^{d*}_T) = \frac{\sqrt{H (s^{a*}_T) H (s^{d*}_T)}}{H (s^i)} \exp \left( \mu^N - (\sigma^N)^2 \right) \tag{30}
\]

and \( H(s) \) is defined in equation (10).

**Proof.** See Appendix A.3 

How does the policy choice \( s^i \) affects the re-election probability? Taking the derivative of the re-election probability in equation 28 with respect to \( s^i \), we get:

\[
\frac{\partial \pi^i_m (s^i, s^{a*}_m, s^{d*}_m)}{\partial s^i} = - f_m (y^m) \times \frac{\partial \tilde{y}^m (s^i, s^{a*}_m, s^{d*}_m)}{\partial H (s^i)} \times \frac{\partial H (s^i)}{\partial s^i} \tag{28}
\]
In the formulation above, the first term $f_m \left[ \tilde{y}_m \right]$ is positive and the last term \( \frac{\partial H \left( s^i \right)}{\partial s^i} \) is negative. The sign of the second term \( \frac{\partial \tilde{y}_m \left( s^i, s^a_m, s^d_m \right)}{\partial H \left( s^i \right)} \) depends on the type of the median voter: it is positive when the median voter is a non-tradable sector citizen, and negative if the median voter is a tradable sector citizen. Hence, if the median voter is a non-tradable sector citizen, the re-election probability increases in the government expenditure share on non-tradable goods. Conversely, the re-election probability becomes a negative function of the government expenditure share on non-tradable goods if the majority of the population are tradable sector citizens.

The Incumbent

From the Proposition 3, it is clear that the policy chosen by the incumbent in the pre-election period affects not only his contemporaneous utility, but also the probability of re-election, and hence his next period’s expected gains. Therefore, the policymaker chooses pre-election policy so as to maximize:

\[
\max_{s^i} U^i_m \left( s^i \right) = \begin{cases} 
F^i \left( s^i \right) \\
+ \beta \pi^i_m \left( s^i, s^a_m, s^d_m \right) F^i \left( s^i_x \right) \\
+ \beta \left[ 1 - \pi^i_m \left( s^i, s^a_m, s^d_m \right) \right] p^i F^{i,i} \left( s^i_x \right) \\
+ \beta \left[ 1 - \pi^i_m \left( s^i, s^a_m, s^d_m \right) \right] \left( 1 - p^i \right) F^{i,j} \left( s^j_x \right)
\end{cases} 
\text{ s.t. } 0 \leq s^i \leq 1
\] (31)

where $\beta$ is the incumbent’s intertemporal discount rate. The first term, $F^i(\cdot)$, is the contemporaneous expected utility of the incumbent, defined in equation (14). The sum of the other terms represents the incumbent’s expected utility for the next period: (i) The incumbent will be re-elected with probability $\pi^i_m \left( s^i, s^a_m, s^d_m \right)$, and the corresponding expected utility is $F^i \left( s^i_x \right)$. (ii) With probability $\left[ 1 - \pi^i_m \left( s^i, s^a_m, s^d_m \right) \right] p^i$ an opponent with the same type wins the election and $F^{i,i} \left( s^i_x \right)$ is the corresponding expected utility. Although in both cases the expenditure policy in the next period is $s^i_x$, $F^i \left( s^i_x \right)$ is different from $F^{i,i} \left( s^i_x \right)$ due to the “ego rent”: $F^{i,i} \left( s^i_x \right) \equiv F^i \left( s^i_x \right) - C$. (iii) With probability $\left[ 1 - \pi^i_m \left( s^i, s^a_m, s^d_m \right) \right] \left( 1 - p^i \right)$ an opponent of the other type wins the election, in which the utility of incumbent is denoted as $F^{i,j} \left( s^j_x \right) \equiv F^i \left( s^j_x \right) - C$. 

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Rearranging, problem (31) can be rewritten as:

\[
\max_{s^i} U_i^{s_i} = \begin{cases} 
F^i(s^i) 
+ \beta \pi_m^i(s^i, s^{a*}_m, s^{d*}_m) \left\{ \left( 1 - p^i \right) \left[ F^{i,i}(s^{i*}_{+1}) - F^{i,j}(s^{j*}_{+1}) \right] + C \right\} \\
+ \beta \left( p^i F^{i,i}(s^{i*}_{+1}) + (1 - p^i) F^{i,j}(s^{j*}_{+1}) \right) 
\end{cases}
\]

subject to \(0 \leq s^i \leq 1\). The solution is a fixed point where the solution to the problem above for a type \(a\) incumbent is \(s^{a*}_m\), and \(s^{d*}_m\) for a type \(d\) incumbent.

**Proposition 3** Under asymmetric information about the policy-maker’s type, the policy-maker of non-tradable type chooses in equilibrium a higher expenditure share on non-tradable goods than policy-maker of tradable type in the run-up to the election, that is, \(s^{a*}_m > s^{d*}_m\), for \(m = N, T\).

**Proof.** see Appendix A.4

The following proposition formalizes the equilibrium and its existence.

**Proposition 4 (Equilibrium under asymmetric information)** There is a perfect Bayesian equilibrium in pure strategies. In any perfect Bayesian equilibrium, the incumbents’ strategies prescribe as following: (1) in the pre-election period, an incumbent of type \(i\) will chose an action \(s^i = s^{i*}_m\) such that \(s^{i*}_m \in s^{i*}_m(s^{a*}_m, s^{d*}_m)\), where \(s^{i*}_m(\cdot, \cdot)\) is defined as the solution of problem (32). (2) In the post-election period, an incumbent of type \(i\) will choose an expenditure share \(s^{i*}_{+1}\), defined in equation (15). The non-tradable sector citizen votes for the incumbent if the observed exchange rate is not greater than \(\tilde{e}_N\); while the tradable sector citizen votes for the incumbent if she observes exchange rate is not smaller than \(\tilde{e}_T\), where \(\tilde{e}_m\) is defined by equation (22). (3) The re-election probability of the incumbent of type \(i\), \(i = a, d\), \(\pi_m^i(s^{i*}_m, s^{a*}_m, s^{d*}_m)\), is defined in equation (28). (4) Pre-election policies are biased towards the median voter’s preferences, compared to post-election policies, that is, incumbents of both types spend relatively more on nontradable goods before elections when the median voter is of the non-tradable type, \(s^{i*}_N > s^{i*}_{+1}\), while they spend less on non-tradable goods if the median voter is a tradable sector citizen, \(s^{i*}_T < s^{i*}_{+1}\).

**Proof.** See Appendix A.6.
5 RER Election Cycle

5.1 Conditional electoral cycle

Proposition 5 (Conditional electoral cycle of expenditure policy) In equilibrium, when the election winner is of the same type as the incumbent (including re-election), in the case of median voter being a non-tradable sector citizen, must prescribe for both types of incumbent a pre-election expenditure share on non-tradable goods strictly greater than the one chosen in the post-election period. By contrast, if majority population are tradable sector citizens, both types of incumbent choose a strictly smaller expenditure share on non-tradable goods before than after elections. Hence \( s^*_N > s^*_{i+1} > s^*_T \).

Corollary 6 (Conditional electoral cycle of RER) In equilibrium, when the election winner is of the same type as the incumbent (including re-election), the real exchange rate is on average more appreciated before than after elections if the median voter is a non-tradable sector citizen. Conversely, on average, a more depreciated RER is observed before than after elections if the median voter is a tradable sector citizen. That is:

\[
\begin{align*}
  e^*_i < e^*_{i+1} & \quad \text{if } m = N \\
  e^*_i > e^*_{i+1} & \quad \text{if } m = T
\end{align*}
\]

Proof. See Appendix A.5

5.2 Unconditional electoral cycle of RER

The exchange rate dynamics depend on the policy-maker type before and after election. We use a Markov transition matrix \( P_m \) to describe the probabilities of
those transitions.

\[
P_m = \begin{pmatrix} P_{d,d}^m & P_{d,a}^m \\ P_{a,d}^m & P_{a,a}^m \end{pmatrix} = \begin{pmatrix} \pi^d_m + (1 - \pi^d_m) p^d \\ (1 - \pi^a_m) p^d \end{pmatrix} \begin{pmatrix} 1 - \pi^d_m \\ \pi^a_m + (1 - \pi^a_m) p^a \end{pmatrix}
\]

where \( P_{i,j}^m \) represents the transition probability that the incumbent is of type \( i \) before election and the election winner is of type \( j \), when the median voter is a \( m \) sector citizen. For instance, \( P_{T,a}^d \) corresponds the transition probability that the tradable type incumbent is replaced by a non-tradable type politician when the median voter is a tradable sector citizen.

Let \( \bar{e}^i \) be the average RER before election, and \( \bar{e}^{i+1} \) the post-election average, when the incumbent is of type \( i \). We define \( \Delta E \) as the matrix of the changes in conditional average real exchange rate around elections:

\[
\Delta E_m = \begin{pmatrix} \bar{e}^d_{m+1} - \bar{e}^d_m \\ \bar{e}^a_{m+1} - \bar{e}^a_m \end{pmatrix} = \begin{pmatrix} \Delta E_{m}^d \\ \Delta E_{m}^a \end{pmatrix}
\]

The first row \( \Delta E_{m}^d \) consists the depreciation of average RER around elections when the incumbent is of tradable type \( d \) and the median voter is in \( m \) sector, \( m = N,T \). In the second row \( \Delta E_{m}^a \) is the equivalent vector for a non-tradable incumbent.

When the incumbent is of tradable type, the average RER depreciation is given by:

\[
\Delta \bar{e}^d_m = P_{d}^m \Delta E_{m}^d,
\]

while, if the incumbent is of the non-tradable type, it equals:

\[
\Delta \bar{e}^a_m = P_{a}^m \Delta E_{m}^a.
\]
Thus, the unconditional average RER depreciation after elections is:

\[ \Delta \bar{e}_m = p^d \Delta \bar{e}^d_m + p^a \Delta \bar{e}^a_m \]

In order to illustrate the election cycle of RER, we evaluate the equilibrium with a set of parameter values,\(^8\) using two different values for \(n\): \(n = 0.6\), which correspond to the median voter being from the non-tradable sector, \(m = N\); and \(n = 0.4\), that is, a median voter from the tradable sector \(m = T\). Table 4 presents the simulation results. We first analyze the results for the median voter being of a non-tradable sector citizen, which are reported in the first two columns of the table. The incumbents of both types choose higher expenditure share on non-tradable goods before elections in order to signal he favors the median voter’s interests, and thus, a more appreciated RER is generated in the pre-election period. More specifically, when a non-tradable incumbent is re-elected the real exchange rate will depreciate by 0.091 on average, and depreciate by 0.076 on average when policy-maker of tradable type is re-elected. There will be a larger RER depreciation of 0.591 if the non-tradable incumbent is replaced by the tradable politician.

The re-election probability of the non-tradable type of incumbent (87.6%) is higher than the tradable one (80.0%). There is an expected exchange rate depreciation conditioned to a non-tradable type of incumbent (depreciated by 0.1218), because real exchange rate depreciates when the non-tradable type of incumbent is succeeded by the politician of the same type (0.091), and by a policy-maker of tradable type (0.591). When the incumbent is of the tradable type, there is a RER depreciation when the winner of election is of her own type (0.076), but a RER appreciation when he is replaced by the non-tradable type (−0.424). Nevertheless, RER still depreciates by 0.0255 on average conditioned to a tradable type of incumbent. As a result, the unconditional average RER depreciation equals to 0.0737 after elections.

The opposite election cycle is generated when the median voter is a tradable sector citizen, which can be seen from the last two columns. Each type of incumbent chooses smaller expenditure share on non-tradable goods in the pre-election period.

---

\(^8\)α = 0.5, \(τ = 0.3, \mu^T = 2, \mu^N = 2, \sigma^T = 1, \sigma^N = 1, p_N = 0.5, C = 0.2\). For \(m = N, n = 0.6, \theta^d = 2, \theta^a = 1.5\); for \(m = T, n = 0.4, \theta^d = 1, \theta^a = 0.5\). The detail is specified in Appendix A.7
### Table 4: Numerical Example

<table>
<thead>
<tr>
<th></th>
<th>Non-tradable Median voter</th>
<th>Tradable Median voter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$i = a$</td>
<td>$i = d$</td>
</tr>
<tr>
<td>$s$ before election</td>
<td>$s^a_N$ 0.5520</td>
<td>$s^d_N$ 0.2933</td>
</tr>
<tr>
<td>$s$ after election</td>
<td>$s^a_{+1}$ 0.5</td>
<td>$s^d_{+1}$ 0.2619</td>
</tr>
<tr>
<td>Average pre-election RER</td>
<td>$e^a_N$ 1.409</td>
<td>$e^d_N$ 1.924</td>
</tr>
<tr>
<td></td>
<td>$e^a_{+1}$ 1.500</td>
<td>$e^d_{+1}$ 2.000</td>
</tr>
<tr>
<td>Re-election Probability</td>
<td>$\pi^a_N$ 87.6%</td>
<td>$\pi^d_N$ 80.0%</td>
</tr>
<tr>
<td>Conditional changes in</td>
<td>$\pi^{ad}$ 0.591</td>
<td>$\pi^{dd}$ 0.076</td>
</tr>
<tr>
<td>RER after election $\Delta E$</td>
<td>$e^{ad}_N$ 0.091</td>
<td>$e^{dd}_N$ -0.424</td>
</tr>
<tr>
<td>Transition probability</td>
<td>$p^{ad}$ 6.19%</td>
<td>$p^{dd}$ 90.04%</td>
</tr>
<tr>
<td></td>
<td>$p^{aa}$ 93.81%</td>
<td>$p^{da}$ 9.96%</td>
</tr>
<tr>
<td>Average conditional depreciation</td>
<td>$\Delta e^a_N$ 0.1218</td>
<td>$\Delta e^d_N$ 0.0255</td>
</tr>
<tr>
<td>Average unconditional depreciation $\Delta e_m$</td>
<td>$\Delta e_N$ 0.0737</td>
<td>$\Delta e_T$ -0.0676</td>
</tr>
</tbody>
</table>

To increase his re-election probability. The tradable type of incumbent has a higher re-election probability than the one of non-tradable type (90.2% against 75.9%). When the incumbent is of tradable type, RER exchange rate appreciates by 0.075 in the post-election period when the election winner is of the same type, and appreciates by 0.574 when a non-tradable type of policy-maker wins the election. As a result, there is an expected exchange rate appreciation conditioned to the incumbent of the tradable type (by 0.099 ). When the incumbent is of the non-tradable type, there is a RER appreciation of −0.096 after elections when he is replaced by a policy-maker of her own type, but RER depreciates when the election winner is of the tradable type (0.403). RER still experiences an appreciation of −0.036 on average after election conditioned to a tradable type incumbent. The unconditional average RER appreciation after election is generated, equaling 0.067.
6 Conclusion

Theoretical and empirical literature suggests that politicians in Latin America have a bias towards appreciating their currencies before elections and depreciating after elections. The two alternative explanations for the RER electoral cycles, in a nutshell, competence or preference signaling, were equally capable of explaining the empirical findings in Latin America. This literature paid no attention to the East Asian RER election cycles, which is found to be opposite to the one found in Latin America: in East Asian countries the RER tends to be more depreciated before elections and appreciated after elections. Competence signaling model can not generate such RER electoral cycle in East Asia. This paper shows that preference signaling model could generate both types of cycles based on opposite exchange rate populism in these two regions.

The “popularity” of appreciated currency seems to be the case in Latin America, but not in East Asia. In East Asia, the majority population seems to prefer an depreciated currency, for most a larger share of GDP comes from the tradable sector, and the majority of the population also works in that sector.

We develop a preference signaling model that is able to explain both types of cycles. Cycles occur in a dynamic, multidimensional signaling game between the incumbent and forward-looking rational median voters. Our results show that RER tends to be more appreciated than average in the months preceding elections and more depreciated than average in the months following elections if the median voter prefers a more appreciated RER, and the opposite cycle if the median voter prefers a more depreciated RER.

References


Cukierman, Alex and Allan H Meltzer, A positive theory of discretionary policy, the cost of democratic government and the benefits of a constitution, *Economic Inquiry*, 1986, 24 (3), 367–388.


A APPENDIX

A.1 Proof of Proposition 1

The expected utility function for the policy-makers of type $a$ and $d$ can be written as:

$$F^a(s) = EW(V^a(s)) + \theta^a EW(V^d(s)) + C$$

$$F^d(s) = EW(V^a(s)) + \theta^d EW(V^d(s)) + C$$

Let $s^a_{i+1}$ and $s^d_{i+1}$ be the solutions that maximize $F^a(s)$ and $F^d(s)$, respectively. We then have that:

$$F^d(s^d_{i+1}) = 0 \iff \frac{\partial F^a(s^d_{i+1})}{\partial s} + (\theta^d - \theta^a) \frac{\partial EW[V^d(s^a_{i+1})]}{\partial s} = 0$$

Since we know that $\frac{\partial EW[V^d(s^a_{i+1})]}{\partial s} < 0$, we have that:

$$\frac{\partial F^a(s^d_{i+1})}{\partial s} = - (\theta^d - \theta^a) \frac{\partial EW[V^d(s^a_{i+1})]}{\partial s} > 0$$

Also, since $\frac{\partial F^a(s^a_{i+1})}{\partial s} = 0$, we get:

$$\frac{\partial F^a(s^d_{i+1})}{\partial s} > \frac{\partial F^a(s^a_{i+1})}{\partial s}$$

(33)

Since policy-maker’s expected utility is assumed to be concave in $s$, that is, $\frac{\partial^2 F^a(s)}{\partial s^2} < 0$, inequality (33) is true if, and only if, $s^a_{i+1} > s^d_{i+1}$.

A.2 Proof of Proposition 2: Median voter’s voting decision

A.2.1 Median voter is non-tradable: $m = N$

Using the RER definition in equation (10), we compute the endowment shock in tradable sector $y^T$ that would generate the observed RER for the different
equilibrium policy choices, \( s_N^{a*} \) and \( s_N^{d*} \), given that \( s_N^{a*} > s_N^{d*} \):

\[
w \equiv y^T(\hat{e}, s_N^{a*}, y_N) = \eta H(s_N^{a*}) \frac{y_N}{\hat{e}} < \eta H(s_N^{d*}) \frac{y_N}{\hat{e}} = y^T(\hat{e}, s_N^{d*}, y_N) \equiv v
\]

The density function of endowment shock has a unique maximum point since it log-normally distributed. We will denote the unique maximum point as \( z \).

**Case I:** \( w < v \leq z \Rightarrow f_T(w) < f_T(v) \)

and:

**Case II:** \( z \leq w < v \Rightarrow f_T(w) > f_T(v) \)

The incumbent will not be re-elected in case I, and he will be re-elected in case II. Finally, we investigate the case \( z \) is between \( w \) and \( v \):

**Case III:** \( w < z < v \)

We define a function: \( h_{m=N}(e) \equiv f_T(w) - f_T(v) \). We can find:

\[
\frac{dh_{m=N}(e)}{de} = \frac{df_T}{dw} \frac{dw}{de} - \frac{df_T}{dv} \frac{dv}{de} < 0
\]

We know that the two density functions are equal at the cutoff level of RER, hence \( h_{m=N}(\tilde{e}_N) = 0 \). The condition of re-election could be described as follows:

\[
re - elected \iff \rho^N \geq p^N \implies f_T(w) - f_T(v) \geq 0
\]

\[
\Rightarrow f_T(w) - f_T(v) \geq h_{m=N}(\tilde{e}_N)
\]

\[
\Rightarrow h_{m=N}(\tilde{e}) \geq h_{m=N}(\tilde{e}_N)
\]

\[
\Rightarrow \hat{e} \leq \tilde{e}_N (\text{since } \frac{dh_{m=N}(e)}{de} < 0)
\]

That is, the incumbent is reelected if and only if the observed RER is not greater than the cutoff RER; otherwise, opponent wins the election.
A.2.2 Median voter is tradable: m=T

Using the RER definition in equation 10, with \( s^a_T > s^d_T \), we calculate the endowment shock in non-tradable sector \( y_N^T \) that would generate the observed RER for the different equilibrium policy choices.

\[
\phi \equiv y_N^T (\bar{e}, s^d_T, y^T) = \frac{\hat{e}y^T}{\eta H (s^d_T)} < \frac{\hat{e}y^T}{\eta H (s^a_T)} = y_N^T (\bar{e}, s^a_T, y^T) \equiv \psi
\]

The endowment in non-tradable sector is a log-normal distribution, hence, its density function has a unique maximum point, denoted as \( \varphi \).

Case I: \( \phi < \psi \leq \varphi \Rightarrow f_N(\phi) < f_N(\psi) \)

and:

Case II: \( \varphi \leq \phi < \psi \Rightarrow f_N(\phi) > f_N(\psi) \)

In case I, median voter votes for the opponent; in case II, votes for the incumbent. Finally, we analyze the case that \( \varphi \) is between \( \phi \) and \( \psi \):

Case III: \( \phi < \varphi < \psi \)

We define a function: \( h_{m=T}(e) \equiv f_N(\phi) - f_N(\psi) \). \( h_{m=T}(e) \) is an increasing function of the RER:

\[
\frac{dh_{m=T}(e)}{de} = \frac{df_N d\phi}{d\phi de} - \frac{df_N d\psi}{d\psi de}
\]

\[
= \oplus \times \oplus - \ominus \times \ominus
\]

\[
> 0
\]

Note that \( h_{m=T}(\bar{e}_T) = 0 \). The condition for re-election can be written as:

\[
re - elected \Leftrightarrow f_N(\phi) - f_N(\eta) \geq 0
\]

\[
\Leftrightarrow f_N(\phi) - f_N(\eta) \geq h_{m=T}(\bar{e}_T)
\]

\[
\Leftrightarrow h_{m=T}(\bar{e}) \geq h_{m=T}(\bar{e}_T)
\]

\[
\Leftrightarrow \bar{e} \geq \bar{e}_T (\text{since} \frac{dh_{m=T}(e)}{de} > 0)
\]

Thus, the median voter, who is a non-tradable sector citizen, will vote for the incumbent if and only if she observes RER is not lower than the cutoff RER, vote for the opponent otherwise.
A.3 Proof of Proposition 3: Re-election Probability

A.3.1 Median voter is non-tradable sector citizen: \( m = N \)

According to equation (10), RER is a negative function of the endowment in tradable sector: \( \frac{de}{dy^T} < 0 \).

\[
\Rightarrow Pr(\text{re-election}) = Pr\left[ f_T(w) - f_T(v) \geq 0 \right] \\
= Pr\left[ f_T(w) - f_T(v) \geq h_{m=N}(\bar{e}_N) \right] \\
= Pr\left[ h_{m=N}(\bar{e}) \geq h_{m=N}(\bar{e}_N) \right] \\
= Pr\left[ \bar{e} \leq \bar{e}_N \right] \text{ (since } \frac{dh_{m=N}(e)}{de} < 0) \\
= Pr\left[ y^T \geq \bar{y}^T \right] \text{ (since } \frac{de}{dy^T} < 0)
\]

where we have used \( h_{m=N}(e) = f_T(w) - f_T(v) \), and \( h_{m=N}(\bar{e}_N) = 0 \), from equation (34).

Thus,
\[
\pi_N(s, s^{as}_N, s_{N}^{de}) = \int_{\bar{y}^T}^{\infty} f_T(y^T)dy^T
\]

A.3.2 Median voter is tradable sector citizen: \( m = T \)

From equation (10), the equilibrium RER is a positive function of the endowment in non-tradable sector: \( \frac{de}{dy^N} > 0 \).

\[
\Rightarrow Pr(\text{re-election}) = Pr\left[ f_N(\phi) - f_N(\psi) \geq 0 \right] \\
= Pr\left[ f_N(\phi) - f_N(\psi) \geq h_{m=T}(\bar{e}_T) \right] \\
= Pr\left[ h_{m=T}(\bar{e}) \geq h_{m=T}(\bar{e}_T) \right] \\
= Pr\left[ \bar{e} \geq \bar{e}_T \right] \text{ (since } \frac{dh_{m=T}(e)}{de} > 0) \\
= Pr\left[ y^N \geq \bar{y}^N \right] \text{ (since } \frac{de}{dy^N} > 0)
\]

where we have used that \( h_{m=T}(e) = f_N(\phi) - f_N(\psi) \), and \( h_{m=N}(\bar{e}_T) = 0 \), from equation (35).
As a result, we have that:

$$\pi_T(s, s^a_T, s^d_T) = \int_{y_N}^{\infty} f_N(y_N) \, dy_N$$

### A.4 Proof of Proposition 4

$s^*_m$ is the policy level that maximizes the policy-maker’s utility function (32) before election. Thus, $s^*_m$ is such that $\frac{dU^a_m(s)}{ds} = 0$, and $s^d_m$ is the policy level that satisfies $\frac{dU^d_m(s)}{ds} = 0$. That is:

$$s^*_m : \frac{dU^a_m(s^*_m)}{ds} = \frac{dF^a_m(s^*_m)}{ds} + \beta \left[ (1 - p^a) \left( F^a,a - F^a,d \right) + C \right] \frac{d\pi_m(s^*_m)}{ds} = 0$$

$$s^d_m : \frac{dU^d_m(s^d_m)}{ds} = \frac{dF^d_m(s^d_m)}{ds} + \beta \left[ (1 - p^d) \left( F^d,d - F^d,a \right) + C \right] \frac{d\pi_m(s^d_m)}{ds} = 0$$

We define $B^i \equiv \beta \left[ (1 - p^i) \left( F^{i,i} - F^{i,j} \right) + C \right]$, which corresponds the marginal benefit on re-election probability for the incumbent. With $F^i(s) = E[W(V^a)] + \theta^i E[W(V^d)] + C$, we can get:

$$\frac{dU^a_m(s)}{ds} = \frac{dF^a_m(s)}{ds} + \beta \left[ (1 - p^a) \left( F^a,a - F^a,d \right) + C \right] \frac{d\pi_m(s)}{ds}$$

$$= \frac{dF^a_m(s)}{ds} + \frac{dF^a_m(s)}{ds} + B^d \frac{d\pi_m(s)}{ds}$$

$$= \frac{dF^a_m(s)}{ds} + \frac{dF^a_m(s)}{ds} + \left( \theta^d - \theta^a \right) \frac{\partial EW}{\partial V} \frac{\partial V^d(s)}{\partial s} + \left( B^d - B^a \right) \frac{d\pi_m(s)}{ds}$$

The second term is negative, since $\theta^d - \theta^a > 0, \frac{\partial EW}{\partial V} > 0, \frac{\partial V^d(s)}{\partial s} < 0$. It is reasonable to assume that the marginal benefit on re-election probability does not vary across different types of policy-maker, thus, $(B^d - B^a) \frac{d\pi_m(s)}{ds} = 0$. Therefore:

$$\frac{dU^d_m(s)}{ds} = \frac{dU^a_m(s)}{ds} + \left( \theta^d - \theta^a \right) \frac{\partial EW}{\partial V} \frac{\partial V^d(s)}{\partial s} + \left( B^d - B^a \right) \frac{d\pi_m(s)}{ds}$$

$$= \frac{dU^a_m(s)}{ds} + \text{negative term}$$

38
Hence,
\[
\frac{dU^d_m(s)}{ds} < \frac{dU^a_m(s)}{ds}
\Rightarrow \frac{dU^d_m(s^a_m)}{ds} < \frac{dU^a_m(s^a_m)}{ds}
\Rightarrow \frac{dU^d_m(s^a_m)}{ds} < 0 = \frac{dU^d_m(s^d_m)}{ds}
\]

Given that \(U^d_m(s)\) is a concave function in \(s\), we can deduce that \(s^a_m > s^d_m\).

A.5 Proof of Proposition 5

The incumbent chooses optimal expenditure level for maximizing his utility, which is given by equation (32):

\[
\max_s U^i_m(s) = \left\{ F^i(s) + \beta \pi_m(s, s^a_m, s^d_m) \left[ (1 - p^i) (F^{i,i} - F^{i,j}) + C \right] \right. \\
\left. + \beta \left[ p^i F^{i,i} + (1 - p^i) F^{i,j} \right] \right\}
\]

We denote the optimal expenditure level as \(s^i_m\), such that

\[
\text{FOC}(s) : \frac{dU^i_m(s^i_m)}{ds} = 0
\]

and,
\[
\frac{dU^i_m(s)}{ds} = \frac{dF^i(s)}{ds} + \beta \left[ (1 - p^i) (F^{i,i} - F^{i,j}) + C \right] \frac{d\pi_m(s)}{ds}
\]

We define a function \(h_m(s)\) such that,
\[
h_m(s) \equiv \frac{dF^i(s)}{ds} + \beta \left[ (1 - p^i) (F^{i,i} - F^{i,j}) + C \right] \frac{d\pi_m(s)}{ds}
\]

A.5.1 Median voter is non-tradable sector citizens: \(m = N\)

\[
\beta \left[ (1 - p^i)(F^{i}_i - F^{i}_j) + C \right] > 0, \text{ and the sign of } \frac{d\pi_N(s^i_N)}{ds} \text{ is positive.}
\]

The optimal expenditure level \(s^i_N\) is such that \(h_N(s^i_N) = 0\).

\[
h_N(s^i_N) = \frac{dF^i(s^i_N)}{ds} + \beta [(1 - p^i)(F^{i,j} - F^{i,j}) + C] \frac{d\pi_N(s^i_N)}{ds} = 0
\]
where the second term is strictly greater than 0, hence, the first term should be
smaller than 0: $\frac{dF^i(s^*_N)}{ds} < 0$.

Given that $s^*_f$ is the optimal expenditure level for the incumbent under full
information, that is, $\frac{dF^i(s^*_f)}{ds} = 0$. We know that $F^i(s)$ is concave in $s$, that is,
$F^{ii}(s) < 0$, and it has been verified that:

$$F'''(s^*_N) < 0 = F'''(s^*_f)$$

$$\Rightarrow s^*_N > s^*_f \Rightarrow s^*_N > s^*_{i+1}$$

Since the exchange rate is decreasing in $s$,

$$\Rightarrow e^*_N < e^*_f \Rightarrow e^*_N < e^*_{i+1}$$

A.5.2 Median voter is tradable sector citizens: $m = T$

$\beta \left[ (1 - p^i) (F^{i,i} - F^{i,j}) + C \right] > 0$, and the sign of $\frac{d\pi_T(s^*_T)}{ds}$ is negative.

The optimal expenditure level $s^*_T$ is such that $h_T(s^*_T) = 0$.

$$h_T(s^*_T) \equiv \frac{dF^i(s^*_T)}{ds} + \beta \left[ (1 - p^i)(F^{i,i} - F^{i,j}) + C \right] \frac{d\pi_T(s^*_T)}{ds} = 0$$

where the second term is strictly smaller than 0, hence, the first term is larger
than 0: $\frac{dF^i(s^*_T)}{ds} > 0$.

$s^*_f$ is the optimal expenditure level for the incumbent under full information, that
is, $\frac{dF^i(s^*_f)}{ds} = 0$. $F^i(s)$ is concave in $s$, that is, $F^{ii}(s) < 0$, and it has been verified that:

$$F'''(s^*_N) > 0 = F'''(s^*_f)$$

$$\Rightarrow s^*_N < s^*_f \Rightarrow s^*_T < s^*_N$$

Given the exchange rate is decreasing in $s$,

$$\Rightarrow e^*_N > e^*_f \Rightarrow e^*_N > e^*_{i+1}$$
A.6 Proof of Proposition 5

The set of solutions to Problem (32) is denoted as $s_{m}^{s} (s_{m}^{a}, s_{m}^{d})$, which is an upper hemi-continuous correspondence, since it is the solution set for maximizing of a continuous function over a compact set. The existence of the equilibrium then follows from an application of Kakutani’s fixed-point theorem to the hemi-continuous correspondence vector.

$$s_{m}^{is} = \left( \begin{array}{c} s_{m}^{as} \\ s_{m}^{ds} \end{array} \right)$$

(36)

It is easy to check that $F_{i,i}^{s} (s_{i}^{s}) - F_{i,j}^{s} (s_{j}^{s}) + C$ is positive, so problem (32) makes it clear that a higher re-election probability increases welfare for the incumbent. Since the re-election probability $\pi_{N}(\cdot)$ is strictly increasing in $s$ in the case of the median voter being a non-tradable sector citizen, so any equilibrium strategy for the incumbent in the pre-election period $s_{N}^{i} > s_{N}^{is}$. By contrast, the relationship should be reverse if the median voter is a tradable sector citizen, as the re-election probability $\pi_{T}(\cdot)$ is strictly decreasing in $s$, i.e. $s_{T}^{is} < s_{T}^{is} + 1$.

A.7 Simulation example

According to our assumption, the policy-maker’s utility is an increasing and concave function for both two sector citizens and concave in government expenditure share on non-tradable goods $s$, we use a specified utility function for policy-maker to do the simulation.

$$\tilde{V}^{i} (e_{t}, y_{t}^{N}, y_{t}^{T}) = -\frac{1}{V^{a}} (e_{t}, y_{t}^{N}) - \theta^{i} \frac{1}{V^{d}} (e_{t}, y_{t}^{T}) + \chi, \text{ for } i = a, d$$

(37)

where $\chi = \begin{cases} C & \text{if in office} \\ 0 & \text{otherwise} \end{cases}$, with $C > 0$

Given the indirect utility functions (11) and (12), the utility function of policy-
maker can be written as:
\[
V^i (s_t, y_t^N, y_t^T) = - \left[ \bar{H}^{-1} \eta^{-\alpha} (y_t^N)^{-\alpha} (y_t^T)^{\alpha - 1} \right] \left[ H (s_t)^{1-\alpha} + \frac{\theta^i}{\eta} H (s_t)^{-\alpha} \right] + \chi
\] (38)

If we take expectations of equation (38) with respect to the shocks, which are assumed to be independent, we can get the incumbent’s expected utility in a period:

\[
F^i (s_t) = E \left[ \hat{V}^i (s_t, y_t^N, y_t^T) \right] = \int_0^\infty \int_0^\infty \hat{V}^i (s_t, y_t^N, y_t^T) f_N (y_t^N) f_T (y_t^T) dy_t^N dy_t^T
\]

\[
= - \hat{H} \eta^{-\alpha} E \left[ (y_t^N)^{-\alpha} \right] E \left[ (y_t^T)^{\alpha - 1} \right] \left[ H (s_t)^{1-\alpha} + \frac{\theta^i}{\eta} H (s_t)^{-\alpha} \right] + \chi
\]

\[
= - \hat{H} \eta^{-\alpha} \exp \left\{ - \alpha \mu^N + \frac{1}{2} \sigma^N \right\} E \left[ (y_t^N)^{-\alpha} \right] E \left[ (y_t^T)^{\alpha - 1} \right] \left[ H (s_t)^{1-\alpha} + \frac{\theta^i}{\eta} H (s_t)^{-\alpha} \right] + \chi
\]

for \( i = a, d \).

\( s_{i+1}^* \) maximizes \( F^i (s_t) \), defined in the equation 39, subject to \( s_t^i \in (0, 1) \). In the case of an interior solution, the optimal expenditure level under full information \( s_{i+1}^* \) is given by:

\[
s_{i+1}^* = \frac{\eta(1-\alpha)}{\eta \alpha + \eta(1-\alpha)} - \alpha (1-\tau) \quad \text{for } i = a, d
\] (40)

We will have \( s_{i+1}^{a*} > s_{i+1}^{d*} \) since \( \theta^a < \theta^d \).

**Assumption.** Exogenous variables \( \alpha, \eta, \theta^i \) and \( \tau \) are such that the solution for each type of policy-maker (in the absence of strategic motivations) is interior. \( s_{i+1}^* \in (0, 1) \) will be satisfied if \( \theta^i \in \left( \frac{\eta(1-\alpha) [1 - \alpha (1-\tau) - \tau]}{\alpha} \frac{\eta (1-\alpha) [1 - \alpha (1-\tau)]}{\alpha^2 (1-\tau)} \right) \)

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9If \( X \sim Log - N(\mu, \sigma^2) \) then \( X^a \sim Log - N(a \mu, a^2 \sigma^2) \)

10 \( \text{FOC}(s_i^i) : \frac{\partial F^i(s_i^*)}{\partial s_i^i} = 0 \Leftrightarrow H(s_i^*) = \frac{\theta^i \alpha}{r(1-\alpha)} \). In order to make sure \( s_i^* \) is to maximize \( F^i(s_i^i) \), we check the second-order condition: \( \text{SOC}(s_i^i) : \frac{\partial^2 F^i(s_i^*)}{\partial (s_i^i)^2} < 0 \Leftrightarrow H(s_i^*) < \frac{\theta^i (1+\alpha)}{r(1-\alpha)} \), which always holds; that is, \( s_i^* \) is the solution for maximizing policy maker’s utility function \( F^i(s_i^i) \) under full information.

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