Experience, Skill Composition, and the Persistence of Unemployment Fluctuations∗

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Abstract

Standard models are unable to generate the persistence in unemployment fluctuations found in the data. This paper constructs a job search model consistent with age patterns of unemployment outcomes to quantitatively assess potential explanations for persistence. Changes in the composition of workers across experience groups with different unemployment rates generate persistent unemployment fluctuations. This paper also assesses the role a thin market externality in generating persistence as in Pissarides (1992). While the externality adds to the level of persistence, it cannot generate the levels of persistence observed in the data without compositional changes in the distribution of workers across groups.

Keywords: Persistence, unemployment, experience, skill composition.

JEL Codes: E24, J64.

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1 Introduction

Fluctuations in monthly unemployment rates are highly persistent. The autocorrelation of
monthly unemployment rates in the U.S. exceeds 0.95 and can be as high as 0.99 for prime
age workers. This implies a half-life of shocks to unemployment ranging between 13 and
69 months.\footnote{The half-life of fluctuations in unemployment can be calculated from the empirical employment data
by estimating an AR(1) process on monthly unemployment rates. Autocorrelations of U.S.
monthly unemployment rates generate coefficients that exceed 0.95. Given coefficient \(\rho\), the half-life of unemployment
fluctuation is given by:

\[ t_{\text{hl}} = \frac{\log .5}{\log \rho} \]

resulting in a lower bound estimate for the half-life of unemployment fluctuations from the data is 13.5
months. When \(\rho = 0.99\), the half-life is 69.0 months.} Moreover, the level of persistence has become even higher in recent recessions.
Therefore, figuring out the cause is critical as explaining the mechanisms that generate these
persistent fluctuations is important for understanding the propagation of shocks over the
business cycle. While models of the business cycle have developed to account for many
patterns in the data, the ability to provide an internal propagation mechanism for shocks
remains a major challenge for many models.

Neither standard real business cycle models nor the canonical Mortensen and Pissarides
(1994) model successfully propagate shocks to unemployment. Standard real business cycle
models generate time series of aggregate variables that closely follow the shock process.\footnote{The ability of labor market frictions to provide a propagation mechanism in standard dynamic stochastic
general equilibrium models has been explored by Merz (1995), Andolfatto (1996), and den Haan et al. (2000).
See Pries (2004) for a discussion.} While search frictions embodied in search and matching models provide an intuitive expla-
nation for persistence, the ability of these models to explain the persistence of unemployment
fluctuations depends on both the speed at which workers find jobs when unemployed and
separate from their job when employed. Observed levels of labor market flows imply a
half-life of only one to two months. Even with a sizable decline in job finding rates after
2000, Tasci (2012) shows that the trend rate of convergence is consistent with a half-life of
unemployment fluctuations of less than two months.\footnote{To understand this point consider a standard search model where unemployed workers find jobs at rate 
\(f\) and are separated from their jobs at rate \(s\). Using the continuous time formulation as discussed in Shimer}
tent with observed job finding and separation probabilities are unable to generate persistent unemployment fluctuations.

The goal of this paper is to understand how shocks to the level of unemployment are propagated to generate persistent fluctuations. We propose a model where changes in the composition of worker types away from their steady state distribution can generate persistence while maintaining the observed high levels of inflows and outflows of unemployment for all groups of workers. To show this, we extend a standard search and matching model to include two types of workers with different steady state unemployment rates. Having two groups is a deliberate simplification that keeps the model tractable and allows the quantitative implications of the mechanism to be easily assessed. While short-run unemployment dynamics are governed by flows into and out of unemployment within each group, the model generates new long-run dynamics that are governed by how long it takes workers to transition between groups. In particular, the two groups can represent experienced and inexperienced workers. They can differ in their productivity when employed, probability of finding a productive match, and their exogenous job separation probability so that they have different steady state rates of unemployment. Even with rapid within group worker flows, the model can generate persistent unemployment fluctuations from compositional changes in the pool of workers across groups when the transition rate between groups is slow.

Next, we quantitatively assess the importance of these compositional changes. Since different groups of workers have different baseline unemployment rates, the model is calibrated to match life cycle patterns of employment outcomes. By targeting employment outcomes (2012) and Elsby et al. (2009), if all workers start off unemployed then unemployment at time $t$ is given by:

$$u(t) = u^* + (1 - u^*)e^{-(s+f)t}$$

where $u^* = \frac{s}{s+f}$ is the steady state level of unemployment. In this case the rate of convergence of the system is governed by $s + f$ since the half-life of any difference in unemployment from steady state is given by:

$$t_{hl} = -\log\frac{5}{s+f}$$

Observed worker flows in the U.S. imply that $s + f \approx 0.5$. Therefore, the half-life is just over one month. Even with lower transitions rates of about 0.1 found in many European countries the half life is only about 6 months.
for experienced and inexperienced workers in the model to match those of young and old workers in the data, steady state outcomes replicate empirical age patterns of unemployment rates, job finding probabilities, job separation probabilities, and wages. Inexperienced workers have a higher baseline unemployment rate, which drives persistent unemployment fluctuations in the model when there is a compositional change in workers from experienced to inexperienced. This occurs even though inexperienced workers have higher within job finding probabilities.

The calibrated model is simulated for different initial compositions of workers across types to assess the persistence of unemployment fluctuations. Increases in unemployment without changes in the composition of workers across type have the same rapid dynamics as the Mortensen and Pissarides (1994) model. When changes in unemployment include changes in the composition of workers the model generates long-run persistence of unemployment of similar magnitudes documented in the data. Compositional changes generate non-linear rates of convergence to the steady state. The model reproduces the same short-run dynamics as a regular search and matching model, but now has new long-run dynamics. The time to close half of the gap of the initial shock is rapid at 1.7 months. However, when closing the last 10% of the increase in unemployment the half-life rises to above 11 months and above 77 months when closing the final 5%. The non-linearity occurs because when workers are displaced and need to reacquire skills it takes them a long time to learn a new skill or regain skills in order to return to their previous lower average rate of unemployment. The calibration strategy of targeting employment outcomes of young and old workers may also understate the amount of persistence generated as workers who lose skills with job loss may have lower job finding probabilities.

4Elsby et al. (2010) document that there are sizable differences in labor market flows by gender, age, race, and education. While a number of these characteristics are fixed, different outcomes by age and education could proxy for differences in skill or experience. Hence, one interpretation matching age patterns of employment outcomes for high school educated workers is that changes in the composition of workers after a shock could reflect the loss of skills during unemployment as in Ljungqvist and Sargent (1998). Moreover, parameterizing the model to match unemployment dynamics by age is appealing as Jaimovich and Siu (2009) show that accounting for the employment experiences of young workers is crucial to understand aggregate employment dynamics.

5Inexperienced workers in the model have short unemployment durations corresponding to young workers
An alternate explanation for the persistence of unemployment is the existence of a thin market externality as first proposed by Pissarides (1992). Such an externality arises in models with skill loss when the fraction of unskilled workers in the unemployment pool increases causing firms to post fewer vacancies and hence reduce a worker’s probability of finding a job. While Pissarides (1992) develops a simple theoretical model to highlight the possibility that these externalities can generate persistent unemployment, their quantitative importance has not been studied. After showing that compositional changes generate persistent unemployment fluctuations, we assess whether a thin market externality can generate similar levels of persistence. To study the role of such an externality, our baseline model where experienced and inexperienced workers have separate matching functions is modified so that both groups search for jobs in the same market. In this environment, fluctuations in labor market tightness arising from the thin market externality quantitatively generate only a small amount of persistence. The intuition for this result is that the aggregate match rate is bounded between the match probability of each type of worker when they have separate matching functions. Since each group of workers has high a job finding probability, a shock that increases the number of inexperienced workers in the unemployment pool has only a modest impact on persistence.

Finally, to assess the business cycle implications of the model, simulations are run where all experienced workers who lose their jobs unexpectedly become inexperienced for 18 months. While there are no new shocks added to the model, this exercise is consistent with interpret-

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who find jobs rapidly. In contrast, Valletta (1991) shows that high tenure workers have longer spells of unemployment following job displacement and Kletzer (1998) finds that displaced workers have an average unemployment duration of 17 weeks compared to just 7.2 for workers who are just laid-off. These high durations could arise due to hope by workers that they will regain their lost jobs, a buffer stock of assets, learning about future job quality as in Gorry (2012), or the combination of skill loss and unemployment insurance as in Ljungqvist and Sargent (1998). More generally, a large literature on displaced workers shows that such workers have worse outcomes than other unemployed workers. See for instance, Jacobson et al. (1993) who find long term wage losses for displaced workers and Stevens (1997) who shows that more frequent job loss explains an important part of the average wage loss experienced by displaced workers.

6Thin market externalities have been studied in Wasmer (2004). Such externalities can also give rise to multiple equilibria as noted by Diamond (1982), Howitt (1985), and Mortensen (1989). Despite the potential for multiple equilibria, there exists a unique steady state equilibrium for reasonable parameterizations of the model developed in this paper. This is consistent with estimates of the matching function that do not exhibit sufficient increasing returns to generate multiple equilibria as noted by Pissarides (1986), Blanchard and Diamond (1989), and Petrongolo and Pissarides (2001).
ing business cycles as a time when job loss leads to skill loss among workers. The baseline model with separate matching functions and the model with a thin market externality generates both an increase in unemployment and a slight increase in job separation probabilities that decline slowly after the 18 month period. While the baseline model has the counterfactual implication that job finding probabilities increase as unemployment rises because inexperienced workers find jobs more rapidly, the model with a thin market externality generates lower job finding probabilities. These simulations suggest that while the thin market externality does not generate substantial persistence on its own, it may be important in explaining observed cyclical patterns in worker flows.\footnote{Another way to reconcile cyclical patterns in worker flows is to have workers who lose their jobs experience lower job finding probabilities, rather than the higher ones assumed. The model could easily accommodate this by adding a third group of workers who have longer unemployment durations.}

In related work, Pries (2004) shows that persistence can be generated by workers learning about the quality of a new job match. Learning implies that unemployed workers have rapid turnover on new jobs because with some probability they learn that they are unproductive soon after starting a new job. Additionally, explanations of persistent unemployment from heterogeneity have previously been discussed in Pries (2008) and Ravenna and Walsh (2012). This paper compliments previous explanations as the model generates both an increase in job separation probabilities and predictions about the cyclicality of job finding probabilities.

While persistence has always been a feature of unemployment fluctuations, it has increased during the great recession. See Elsby et al. (2010) and Elsby et al. (2011) for a summary of labor market outcomes during the great recession. The aim of this paper is to understand the mechanism that generates these persistent fluctuations rather than understanding how persistence has changed over time.

Section 2 of the paper presents the model. Section 3 describes the parameterization of the model. Section 4 presents the results on persistence, the effect of a thin market externality, and business cycle implications of the model. Section 5 concludes with a discussion of the results and their relation to explanations for unemployment during the great recession and recent jobless recoveries.
2 Model

This section presents the baseline model of heterogeneous workers who have different steady state unemployment rates. Experienced workers and inexperienced workers search for jobs in separate markets. The model is designed to match life-cycle patterns of unemployment as in Gorry (2013). The description of the model does not include any shocks. Changes in the initial composition of workers across groups will be considered to measure the time to converge back to the steady state. Heterogeneity in worker types is the key feature that allows the model to generate persistent unemployment fluctuations through changes in the composition of workers. In the results section, the model will be modified to have a single matching function to understand the effects of a thin market externality on the persistence of unemployment fluctuations.

2.1 Setup and Worker Flows

Time is discrete. In any period there is a unit mass of workers who maximize the present discounted value of their consumption stream and discount the future at rate $\beta$. Workers can be either employed or unemployed and experienced or inexperienced. Inexperienced workers become experienced while employed with probability $\alpha$ and remain experienced until they exit the labor force. Workers leave the labor market at rate $\delta$ and are replaced by a new cohort of inexperienced, unemployed workers. This assumption prevents all workers from becoming experienced in the steady state model.

There is a continuum of infinitely lived firms that can search for workers by posting vacancies for either an experienced or an inexperienced worker at flow cost $k$ per vacancy. Production occurs when a worker is paired with a firm. Workers of each type search for firms in a separate market characterized by a constant returns to scale matching function $m(v, u) = u^\eta v^{1-\eta}$ where $i \in \{e, n\}$. Let $e$ denote experienced and $n$ denote inexperienced workers. $\theta_i = \frac{u}{v}$ denotes the tightness of the labor market for workers of type $i$. With this matching function, an unemployed worker meets a job in a given period with probability
\[\lambda(\theta_i) = \frac{m(v_i, u_i)}{u_i} = \theta_i^{1-\eta} \] and open vacancies are matched with a worker with probability \[q(\theta_i) = \frac{m(v_i, u_i)}{v_i} = \theta_i^{-\eta}.\]

When a worker and a firm meet they realize a shock to determine if the match is productive. Experienced matches are productive with probability \(p_e\) and inexperienced matches are productive with probability \(p_n\). These probabilities enable job finding probabilities to differ for experienced and inexperienced workers.\(^8\) When a worker and firm of type \(i \in \{e,n\}\) form a productive match they produce \(y_i\) units of output. In general we assume that \(y_e > y_n\). Nash bargaining determines wages for both types of workers.

With this setup, workers of type \(i \in \{e,n\}\) find jobs with probability \((1-\delta)\lambda(\theta_i)p_i\). Worker separations arise from labor force exit and exogenous employment separation shocks. Experienced workers separate from their jobs with probability \(\delta + (1-\delta)s_e\) and inexperienced workers separate with probability \(\delta + (1-\delta)(1-\alpha)s_n\). Also, with probability \((1-\delta)\alpha\) inexperienced workers become experienced, remaining employed.

### 2.2 Value Functions and Equilibrium

Value functions for unemployed and employed workers of each type are as follows:

\[U_n = b + \beta(1-\delta)\left[\lambda(\theta_n)(p_nE_n + (1-p_n)U_n) + (1-\lambda(\theta_n))U_n\right]\]  (1)

\[U_e = b + \beta(1-\delta)\left[\lambda(\theta_e)(p_eE_e + (1-p_e)U_e) + (1-\lambda(\theta_e))U_e\right]\]  (2)

\[E_n = w_n + \beta(1-\delta)\left[\alpha E_e + (1-\alpha)(s_nU_n + (1-s_n)E_n)\right]\]  (3)

\[E_e = w_e + \beta(1-\delta)\left[s_eU_e + (1-s_e)E_e\right]\]  (4)

Unemployed workers get flow value \(b\) and move to employment with probability \(\lambda(\theta_i)p_i\) if they do not exit the labor market. \(b\) can be interpreted as some combination of unemployment

\(^8\)Alternately, differences in job finding rates could be generated by differences in the cost of posting vacancies \(k\) across different types of workers. While the results are identical for the baseline model, this setup simplifies the analysis when considering the model with a single matching function to understand the quantitative relevance of thin market externalities.
benefits, the value of leisure, and the value of home production. When they become employed they get their employment value $E_i$.

Inexperienced employed workers receive their wage $w_n$ and with probability $\alpha$ become experienced employed in the next period. When they do not become experienced they are separated from their job with probability $s_n$ becoming unemployed inexperienced. Experienced employed workers receive wage $w_e$ and are separated from their jobs with probability $s_e$ when they do not exit the labor market.

Next, firms can choose to open vacancies to meet workers and search directly for inexperienced or experienced workers. Their value functions are as follows:

$$V_n = -k + \beta q(\theta_n)p_nJ_n$$  \hspace{1cm} (5)

$$V_e = -k + \beta q(\theta_e)p_eJ_e$$  \hspace{1cm} (6)

$$J_n = y_n - w_n + \beta(1 - \delta) [\alpha J_e + (1 - \alpha)(s_nV_n + (1 - s_n)J_n)]$$  \hspace{1cm} (7)

$$J_e = y_e - w_e + \beta(1 - \delta) [s_eV_e + (1 - s_e)J_e]$$  \hspace{1cm} (8)

Firms post vacancies at period flow cost $k$. Jobs are then created if the workers and firms form a productive match. Inexperienced matches produce output $y_n$ and the firm pays the worker wage $w_n$. In each period an inexperienced worker becomes experienced with probability $\alpha$ and when she does not become experienced the worker and firm separate with probability $s_n$. Likewise, experienced matches produce output $y_e$ and earn $w_e$ and workers are separated with probability $s_e$ each period.

In this economy, a steady state equilibrium is defined as follows:

**Definition 1.** A steady state equilibrium consists of the value functions for the worker, $U_n$, $U_e$, $E_n$, and $E_e$, the value functions of the firm, $V_n$, $V_e$, $J_n$, and $J_e$, the aggregate state variables, $u_n$, $u_e$, $e_n$, $e_e$, $\theta_n$, and $\theta_e$:

1. Value functions are satisfied: Given $w_n$, $w_e$, $u_n$, $u_e$, and $\theta$, then $U_n$, $U_e$, $E_n$, $E_e$, $V_n$, $V_e$, $J_n$, and $J_e$ satisfy equations (1)–(8).
2. Match Formation: Given \( w_n, w_e, u_n, u_e, \theta_n \) and \( \theta_e \), it is optimal for workers to form productive matches.

3. Free Entry: The value of posting a vacancy for each type of worker is given by \( V_n = V_e = 0 \).

4. Bargaining: \( w_n \) and \( w_e \) are determined by Nash bargaining equations with weight \( \gamma \) given to workers:

\[
E_n - U_n = \gamma [J_n + E_n - U_n] \\
E_e - U_e = \gamma [J_e + E_e - U_e]
\]

5. Steady State: The following four worker flow equations hold:

\[
\delta + (1 - \delta)(1 - \alpha)s_n e_n = (\delta + (1 - \delta)\lambda(\theta_n)p_n)u_n \\
(1 - \delta)\lambda(\theta_n)p_n u_n = (\delta + (1 - \delta)\alpha + (1 - \delta)(1 - \alpha)s_n)e_n \\
(1 - \delta)s_e e_e = (\delta + (1 - \delta)\lambda(\theta_e)p_e)u_e \\
(1 - \delta)\lambda(\theta_e)p_e u_e + (1 - \delta)\alpha e_n = (\delta + (1 - \delta)s_e)e_e
\]

The steady state equilibrium can easily be solved. For details see the appendix.

3 Paramaterization

This section parameterizes the model to match key features of life-cycle patterns of unemployment rates in the United States. Matching life cycle employment outcomes provides discipline on model parameters. The approach is similar to the one used in Gorry (2013). The model period is assumed to be one month. Therefore, \( \delta = \frac{1}{480} \) so that the expected length of time in the labor market for each worker is 40 years. The discount rate is set using \( \beta(1 - \delta) = 0.9967 \) to match an annual interest rate of 4%. As normalizations, \( y_n = 1 \) and
\( p_n = 1 \) so that \( y_e \) is interpreted as the relative productivity of experienced workers and \( p_e \) is the relative probability that is match is productive for experienced workers compared to inexperienced.

The matching function takes the standard Cobb-Douglas form, \( m(u, v) = u^\eta v^{1-\eta} \). \( \eta \) is set to 0.5. This value is at the lower end of the range of estimates found in Petrongolo and Pissarides (2001). The choice of \( \gamma = \eta \) insures that the Hosios (1990) condition applies.

Next, observed job separation probabilities are used to set \( s_e \). Micro-data from the Current Population Survey (CPS) is used to construct job finding and job separation probabilities for high school educated workers. Targeting only high school educated workers insures that the age patterns observed are due to experience rather than changes in composition as workers of different skills enter the labor force. The separation probability for experienced workers can be set directly from the measured job separation probability of 50-54 year old workers. The separation probability solves: \( 0.011 = \delta + (1 - \delta)s_e \). This gives \( s_e = 0.009 \).

The remaining parameters of the model are the productivity of experienced workers \( y_e \), the separation probability for inexperienced workers \( s_n \), the probability that a match is productive for experienced workers \( p_e \), the probability with which experienced workers gain experience \( \alpha \), the value of unemployment \( b \), and the cost of posting a vacancy \( k \). These parameters are calibrated jointly to match targets about individual wage growth, job finding and job separation probabilities, and unemployment benefits.

The following targets are used. First, \( y_e \) is set to match the amount of wage growth observed in the data. Using data from the Merged Outgoing Rotation Groups (MORG) from the CPS the mean hourly wage for 18-year-old workers from 2002-2007 is $8.44 and the mean hourly wage for 50-54 year-old workers is $16.18 (both values are in 2009 dollars). Therefore,
Table 1: Baseline values for model parameters along with targets.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>$1/480$</td>
<td>40 year working life</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$0.999$</td>
<td>Annual Interest rate of 4%</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$0.5$</td>
<td>Petrongolo &amp; Pissarides (2001)</td>
</tr>
<tr>
<td>$p_n$</td>
<td>$1$</td>
<td>Normalization</td>
</tr>
<tr>
<td>$p_e$</td>
<td>$0.766$</td>
<td>Ratio of job finding probabilities</td>
</tr>
<tr>
<td>$y_n$</td>
<td>$1$</td>
<td>Normalization</td>
</tr>
<tr>
<td>$y_e$</td>
<td>$1.76$</td>
<td>Wage growth from MORG</td>
</tr>
<tr>
<td>$s_n$</td>
<td>$0.037$</td>
<td>20-24 year-old separation probability</td>
</tr>
<tr>
<td>$s_e$</td>
<td>$0.009$</td>
<td>50-54 year-old separation probability</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$0.0076$</td>
<td>Share of experienced workers is 0.78</td>
</tr>
<tr>
<td>$b$</td>
<td>$0.45$</td>
<td>$b = 0.5w_n$</td>
</tr>
<tr>
<td>$k$</td>
<td>$9.42$</td>
<td>50-54 finding probability of 0.32</td>
</tr>
</tbody>
</table>

the wage for experienced workers is targeted to be 1.92 times the wage of inexperienced workers. The monthly job separation probability for 20-24 year old individuals is 3.9\%. Using the flow equation for separations the following target is used: $0.039 = \delta + (1 - \delta)(1 - \alpha)s_n$. Third, $(1 - \delta)\theta_e^{1-\eta}p_e = 0.316$ is targeted to match the job finding probability for 20-24 year old workers of 31.6\%. Next, since the mean hourly wage from 2002-2007 in the MORG for 18-64 year-old workers is $14.46$ this implies a target of the fraction of experienced workers in the population to be $\frac{e}{e_n + e_e} = 0.78$ so that the average wage in the model matches the average wage in the data. The flow value of unemployment is targeted to be half of the wage of inexperienced workers. Finally, $(1 - \delta)\theta_e^{1-\eta}p_e = 0.274$ is targeted to match the job finding probability for 50-54 year old workers.

These targets imply parameter values of $y_e = 1.76$, $s_n = 0.037$, $\alpha = 0.0076$, $b = 0.45$, and $k = 9.42$. Steady state wages generated by these parameters are $w_e = 1.71$ and $w_n = 0.89$. The parameters are summarized along with their calibration targets in Table 1.
4 Numerical Results

This section reports the numerical results from the steady state model. First, the life-cycle outcomes of the model are simulated to demonstrate that the parameterized model matches observed patterns of unemployment and worker flows. Next, the ability of compositional changes in the distribution of workers across states to generate persistent unemployment is assessed. To do so, the dynamics of the model are solved for the case where 1% of workers employed in the steady state begin unemployed and inexperienced. With this formulation, we compute how long it takes the model to converge back to steady state. After an initial period of quick convergence of workers finding new jobs, the model generates substantial persistence in unemployment rates. Because the baseline unemployment rates are higher for inexperienced compared to experienced workers, increases in the share of inexperienced workers in the economy leads to a persistent increase in unemployment. Next, the model is modified to have only one matching function to assess the ability of a thin market externality to generate persistence. Finally, to interpret the cyclical dynamics of worker flows, both models are simulated for an 18 month period where all workers who lose jobs unexpectedly become inexperienced. While the thin market externality alone does not generate persistent unemployment, it helps the model generate the lower job finding probabilities during recessions.

4.1 Unemployment, Wages, and Worker Flows

With the steady state values of \( \theta_e \) and \( \theta_n \) the flow equations can be solved for the steady state number of workers in each state \( \{u_n, u_e, e_n, e_e\} \). Table 2 summarizes the number of workers in each state. Using these figures, the total steady state unemployment rate in the model is 5.6%, while the unemployment rates for inexperienced and experienced workers are 14.8 and 3.4% respectively. With these baseline unemployment rates the model matches average levels of unemployment among high school educated workers in the United States between 2002 and 2007.
<table>
<thead>
<tr>
<th>State</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_n$</td>
<td>0.031</td>
</tr>
<tr>
<td>$u_e$</td>
<td>0.025</td>
</tr>
<tr>
<td>$e_n$</td>
<td>0.209</td>
</tr>
<tr>
<td>$e_e$</td>
<td>0.735</td>
</tr>
</tbody>
</table>

Table 2: Steady state results for share of population in each state in the economy.

Figure 1: Monthly wages simulated from the model and compared to the data in five year age bins.

The most important parameter in the model to determine the persistence of unemployment fluctuations is the rate at which workers become experienced, $\alpha$. The calibration chooses $\alpha$ to match the average wages for high school workers by targeting the share of experienced workers in the economy. Figure 1 shows the simulated pattern of wage growth by age compared with mean wages for each 5 year age group from CPS MORG data. The parameterized model generates much of the observed wage growth in the data.

In order to assess the quality of this target, $\alpha$ also determines how quickly job finding
Figure 2: Monthly job finding probabilities (left panel) and job separation probabilities (right panel) for high school educated workers in the United States by five year age group and simulated from the model.

and separation probabilities decline over the life-cycle. A strength of the model is that it is consistent with age patterns of job finding and job separation probabilities. The left panel of Figure 2 shows average unemployment to employment transition probabilities for high school educated workers in the United States between 2002 and 2007 for each five year age group from 20-24 through 50-54. The figure also shows average job finding probabilities for each age for 10,000 worker outcomes simulated from the model where each worker enters the labor force unemployed and inexperienced at age 18. The model captures much of the observed decline in the job finding probability by age. Observed job separation probabilities by age are shown in the right panel of Figure 2. The simulated model closely replicates the job separation probabilities by age observed in the data, providing additional evidence that the calibrated value of $\alpha$ is reasonable.

While the endpoints of both job finding and separation rates are targeted through other parameters of the model, $\alpha$ determines the life cycle patterns of job finding and separation probabilities between the endpoints.
4.2 Persistence

This section assesses the ability of compositional changes from the steady state distribution of workers across states to generate persistent unemployment fluctuations. As discussed in the introduction, the half-life of convergence can be computed separately for inexperienced and experienced workers given their steady state worker flows. For inexperienced workers, the baseline calibration implies that \( s = 0.04 \) and \( f = 0.38 \).\(^{11}\) This implies that the half-life for changes in the unemployment rate is 1.65 months. For experienced workers, \( s = 0.011 \) and \( f = 0.32 \) gives a half-life of 2.09 months. The short duration of deviations generated by each group is similar to the result in standard search models that do not generate persistence when calibrated to match the observed levels of worker flows. Individually, neither group of workers exhibits persistent deviations in their unemployment rates. Even though the rates of employment transitions have declined during the great recession lower job finding and separations rates do not account for observed levels of persistence.

To understand how much persistence in unemployment is generated by compositional changes, the model is simulated for monthly employment outcomes after 1% of employed workers (both experienced and inexperienced) from the steady state distribution start off unemployed. Two scenarios are considered. In the first scenario, workers remain in their original experience group. That is 1% of experienced employed workers start experienced unemployed and 1% of inexperienced employed workers begin inexperienced unemployed. This scenarios is denoted as no skill loss. In the second scenario, the 1% of experienced workers who start off unemployed also begin inexperienced. This scenario is referred to as skill loss in the results and documents the main mechanism for persistence in the model.

The 1% of employed workers who begin unemployed increases the unemployment rate by nearly one percentage point from the steady state rate of 5.59% to 6.53%. After computing the share of workers in each state, the unemployment dynamics of the model can be easily computed using the following first order difference equations that give the number of workers

\[ s = -\log(1 - S) \]

where \( S \) is the monthly probability. An analogous equation holds for \( f \).

\(^{11}\) This comes from converting the targeted monthly job finding and job separation probabilities into rates. The formula for the rate (given in lower case letters) is given by: \( s = -\log(1 - S) \). Here \( S \) is the monthly probability. An analogous equation holds for \( f \).
in each state in the next period:

\[
\begin{align*}
    u'_{n} &= u_n + \delta + (1 - \delta)(1 - \alpha)s_ne_n - (\delta + (1 - \delta)\lambda(\theta_n)p_n)u_n \\
    e'_{n} &= e_n + (1 - \delta)\lambda(\theta_n)p_nu_n - (\delta + (1 - \delta)(1 - \alpha)s_n + (1 - \delta)\alpha)e_n \\
    u'_{e} &= u_e + (1 - \delta)s_ee_e - (\delta + (1 - \delta)\lambda(\theta_e)p_e)u_e \\
    e'_{e} &= e_e + (1 - \delta)\lambda(\theta_e)p_eu_e + (1 - \delta)\alpha e_n - (\delta + (1 - \delta)s_e)e_e
\end{align*}
\]

In the above equations, \(u'_n\), \(e'_n\), \(u'_e\), and \(e'_e\) denote the values for the number of workers in each state in the next period. Because there are separate matching functions for each group, \(\theta_e\) and \(\theta_n\) do not depend on the composition of workers across states in the economy so the simulation is simple to execute.

Figure 3 plots the monthly unemployment rate for five years after the increase in unemployment for each scenario. While the simulations do not specify the shock that generates the increase in unemployment, the graphs can be interpreted as impulse response functions to changes in the composition of workers across states from their steady state distribution. The gray line depicts the steady state unemployment rate of 5.6%. In both simulations, the initial unemployment rate is 6.5%. The dashed line shows that when there is no skill loss the unemployment rate converges rapidly back to the steady state level of unemployment. The dotted line shows the monthly unemployment rates for the scenario with skill loss. The figure demonstrates two results. First, compositional shocks generate substantial persistence in unemployment, as the unemployment rate does not fully converge back to the steady state level after five years. Second, the convergence generated by the model is highly non-linear. In the first few months unemployment declines rapidly in both scenarios (in fact, it declines even more rapidly in the skill loss scenario as inexperienced workers have high job finding rates), but the rate of convergence slows dramatically in the scenario with skill loss as it takes workers a long time to become experienced. The larger fraction of inexperienced workers can generate a persistent increase in unemployment.
To get a better sense of how unemployment converges after compositional changes, Table 3 reports a number of measures of the speed of convergence for each scenario. Because of the non-linearity in convergence, the half-life is no longer a sufficient statistic for the speed of convergence in the case of skill loss. Therefore, the number of months it takes for the unemployment to close 50%, 75%, 90%, 95% and 97.5% of the initial shock are reported. In the case of no skill loss, the speed of convergence is rapid with a half-life of approximately 2 months. This rate of change remains nearly constant as the time to go from 90 to 95% and 95 to 97.5% are each 2 months (each of these differences represents closing half of the remaining distance to the steady state). In contrast, the results for the case with skill loss are highly non-linear. For the half-life, there is less persistence than in the case with no skill loss as it takes 1.7 months to close half of the initial shock. This occurs as newly unemployed workers quickly converge to the baseline unemployment rate for inexperienced workers. This quick convergence continues through closing 75% of the gap, then slows down dramatically
Table 3: Time to return to steady state unemployment rate from 1% of employed workers starting unemployed compared to the steady state distribution of workers for the cases with and without skill loss.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>No Skill Loss</th>
<th>Skill Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half-life</td>
<td>1.9</td>
<td>1.7</td>
</tr>
<tr>
<td>Converge 75%</td>
<td>3.9</td>
<td>3.6</td>
</tr>
<tr>
<td>Converge 90%</td>
<td>6.5</td>
<td>7.0</td>
</tr>
<tr>
<td>Converge 95%</td>
<td>8.5</td>
<td>18.5</td>
</tr>
<tr>
<td>Converge 97.5%</td>
<td>10.5</td>
<td>95.9</td>
</tr>
</tbody>
</table>

after closing 90%. It takes 11.5 months to go from 90-95% and over 77 months to go from 95-97.5%. A portion of the initial shock to unemployment remains highly persistent as it takes workers a long time to gain experience.

4.3 Thin Market Externalities

An alternate explanation for persistent unemployment fluctuation is the presence of a thin market externality as proposed by Pissarides (1992). The idea is that when the composition of the pool of unemployed workers is worse it gives firms a lower incentive to post vacancies. Therefore, when unemployment pool has more low quality workers job finding probabilities are low and unemployment can remain higher than it otherwise would. Such externalities do not arise in the baseline model as experienced and inexperienced workers search for jobs in separate labor markets. While Pissarides (1992) develops the possibility of such an externality generating persistent unemployment fluctuations, the quantitative relevance of this channel has never been assessed.

To assess the role of a thin market externality, the model is modified so that there is a single matching function for both types of workers. The thin market externality arises as both experienced and inexperienced workers search for jobs in the same labor market. Since workers become experienced through employment, low rates of unemployment lead to a higher fraction of experienced workers in the unemployment pool. Experienced workers have higher work productivity and are more valuable to firms. Therefore, firms post more
vacancies when the composition of the unemployment pool is better.

Specifically, it is assumed that both workers now match using the same constant returns to scale matching function \( m(v, u) = u^\eta v^{1-\eta} \). Let \( u = u_e + u_n \) be the aggregate number of unemployed workers where \( u_i \) is the number of unemployed workers of type \( i \in \{e,n\} \). \( \theta = \frac{u}{u} \) denotes the tightness of the labor market. The fraction of experienced workers in the unemployment pool is denoted by \( \mu = \frac{u_e}{u_n + u_e} \). Given that firms post vacancies of a single type, their value of posting vacancies is given by:

\[
V = -k + \beta q(\theta)[(1 - \mu)p_n J_n + \mu p_e J_e]
\]

With probability \( q(\theta) \) an open vacancy meets a worker who with probability \( \mu \) is experienced and with probability \( 1 - \mu \) is inexperienced. For the model to generate an externality it is assumed that experienced workers are more productive than inexperienced ones so that \( y_e > y_n \). Moreover, it must be the case that \( J_e > J_n \) so that experience workers are more valuable to firms. A sufficient condition for this to be the case is that \( s_n \geq s_e \) and \( y_e - w_e \geq y_n - w_n \) with one strict inequality. The second inequality holds in a standard Nash bargaining solution.

By assumption firms cannot search separately for experienced workers. This assumption overstates the potential of the thin market externality to account for persistence as any ability to sort workers reduces the externality from changes in the quality of the pool of unemployed workers. To the extent that labor markets are able to sort workers, these externalities would be less important even though there is a fair amount of segmentation by education and experience.

In addition to having full segmentation, we assume that wages for each type of worker, \( w_e \) and \( w_n \), are fixed at the steady state level of the baseline model. If wages were allowed to adjust, they would lessen the impact of the thin market externality as workers in bad labor markets are willing to accept lower wages, partially offsetting the lower incentive for firms to post vacancies. Both of these assumptions give the model the best shot for the externality to generate persistence.
With this setup, the model is reparameterized to match the same targets. Since there is only one matching function, the ratio of job finding rates for younger and older workers implies that $p_e = 0.866$. All of the remaining parameters are identical except for the cost of posting vacancies $k$. Since $k$ must now account for the possibility of firms meeting different types of workers, targeting the job finding probability for young workers implies a value of $k = 10.6$.

To assess the role of the thin market externality, the model is simulated as follows. First, the proportion of experienced workers in the pool of unemployed workers, $\mu = \frac{u_e}{u_n + u_e}$ is computed. Second, the zero profit condition is used to find the labor market tightness $\theta$ associated with the current value of $\mu$ by finding the value of $\theta$ that solves:

$$0 = -k + q(\theta)\beta(1 - \delta)(\mu p_e J_e + (1 - \mu)p_n J_n)$$

Finally, the following first order difference equations are used to find the next periods number of workers in each state:

$$u_n' = u_n + \delta + (1 - \delta)(1 - \alpha)s_n e_n - (\delta + (1 - \delta)\lambda(\theta)p_n)u_n$$

$$e_n' = e_n + (1 - \delta)\lambda(\theta)p_n u_n - (\delta + (1 - \delta)(1 - \alpha)s_n + (1 - \delta)\alpha)e_n$$

$$u_e' = u_e + (1 - \delta)s_e e_e - (\delta + (1 - \delta)\lambda(\theta)p_e)u_e$$

$$e_e' = e_e + (1 - \delta)\lambda(\theta)p_e u_e + (1 - \delta)\alpha e_n - (\delta + (1 - \delta)s_e)e_e$$

In the above equations, $u_n'$, $e_n'$, $u_e'$, and $e_e'$ denote the number of workers in each state in the next period. Using the new values for $u_n$ and $u_e$, the simulation method can be repeated to generate a monthly time series for $\theta$ and unemployment rates.

In order to compare the results of the models with and without an externality, two simulations are conducted. First, the model with an externality is simulated for the same skill loss scenario as presented above. This simulation explains how much more persistence the model with a thin market externality can generate than the baseline model. Second, to
see how much persistence the externality generates on its own, both models are simulated for the case where all of the 1% of workers who begin unemployed come from the pool of inexperienced workers. This scenario has an identical effect on the composition of the pool of unemployed workers as the simulation with skill loss, but does not change the composition of experienced and inexperienced workers in the workforce. Therefore, this scenario assesses how much persistence in unemployment fluctuations a thin market externality generates on its own.

Figure 4 plots the results from the skill loss simulations for the baseline model and the model with a single matching function. The dotted line replicates the results from Figure 3 showing that the baseline model initially has quick convergence followed by persistent unemployment. The model with a single matching function shows a similar pattern. Unemployment begins at 6.5% before quickly dropping below 5.7%. After the initial decline, the model with the externality generates persistent unemployment. While the patterns are very similar, the figure shows that the thin market externality only slightly increases the persistence of unemployment fluctuations over the persistence generated by compositional changes alone.

Next, Figure 5 plots the monthly unemployment rate in response to a shock where all additional unemployment relative to the steady state comes from inexperienced workers for both the baseline model and the model with a single matching function. Even though the fraction of unemployed workers who are experienced, $\mu$, is the same as in the skill loss simulation, the unemployment rate quickly converges back to the steady state level for both models. While this is expected for the baseline model, the fact that the model with a single matching function converges rapidly implies that the externality alone does not generate persistent unemployment fluctuations.

Finally, the rates of convergence from each simulation are reported in Table 4. The first column replicates the rates of convergence from the baseline skill loss scenario reported in Table 3. Next, the table shows that the externality only modestly increases the half-life from 1.7 months to 2.0 months. However, when converging the final 10% the half-life increases
from 11.5 to 31.3 months and even further from 77.4 to 78.8 months for the final 5%. While the model with an externality generates more persistence, it does not substantially alter the amount of persistence generated by the model. The final two columns show the rates of convergence for the case where all of the workers who start out unemployed come from the group of employed inexperienced workers. The baseline model converges very rapidly as inexperienced workers just need to regain their steady state level of unemployment. The half-life for convergence is 1.6 months at all durations. In the case with an externality it takes moderately longer to converge, but convergence is still rapid. Closing half the distance to the steady state occurs in 1.8 months and does not vary much as the model approaches the steady state. The thin market externality alone does not generate the level of persistence observed in the data.

Figure 4: Monthly unemployment rate in response to 1% of employed workers starting out unemployed and inexperienced for the baseline model and the model with a thin market externality.
Figure 5: Monthly unemployment rate from the distribution where all additional workers who start unemployed come from inexperienced employed workers with and without the thin market externality.

4.4 Interpreting Business Cycles

This section uses the model to assess how unemployment recovers after a period of unanticipated skill loss. One way to interpret such an exercise is to understand how unemployment recovers after a recession that generates skill loss among workers. Skill loss is a plausible outcome of recessions as the portion of workers who separate from their jobs due to layoffs increases while the portion of workers who quit declines. Davis et al. (2012) show that during the past recession quits dropped dramatically from their pre-recession high in 2006 of nearly 8 percent of employment to under 5.5 percent of employment by the end of the recession. At the same time, layoffs increased from about 6 percent to over 8 percent of employment.

To explore this feature of recessions in the model, these patterns are interpreted as experienced workers losing their skills when they have a job loss. This section simulates a recessionary episode where experienced workers who are separated from their jobs unexpect-
edly lose their skills by becoming inexperienced. The subsequent recovery is then simulated as in the previous sections. When thinking about the results of this exercise and recent recessions, a number of caveats are in order. First, the simulations assume that all workers lose their skills during the recession and no workers become inexperienced during the recovery. While in practice there are certainly workers who lose their skills and do not in any given period, these extreme assumptions clarify the mechanisms in the model. Second, the exercise does not change the magnitude of the shocks in the model. It is assumed that the probability of job separation remains constant for both types of workers over the business cycle. Hence, it will not attempt to generate the magnitude of fluctuations in unemployment observed during the recession. The benefit of this approach is that the value of filled matches of each type remains constant, which makes the model easier to solve. The simulation does not generate as much unemployment as observed during the past recession. Finally, the simulation assumes that the skill loss that moves experienced workers to inexperienced is unanticipated so that the value functions remain unchanged from those previously described. While modifying expectations modestly changes the dynamics of the system, the purpose of the simulation is to evaluate the persistence in the recovery rather than identify the shock that caused the recession.

We first present the series for unemployment for the baseline model and the model with a single matching function. Figure 6 plots monthly unemployment rates for the period before the recession, an 18 month recession where workers who are separated from their jobs also lose their skills, and the following five years of recovery. The 18-month recessionary period

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Skill Loss</th>
<th>Skill Loss, Externality</th>
<th>n Job Loss</th>
<th>n Job Loss, Externality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half-life</td>
<td>1.7</td>
<td>2.0</td>
<td>1.6</td>
<td>1.8</td>
</tr>
<tr>
<td>Converge 75%</td>
<td>3.6</td>
<td>4.2</td>
<td>3.2</td>
<td>3.6</td>
</tr>
<tr>
<td>Converge 90%</td>
<td>7.0</td>
<td>8.5</td>
<td>5.3</td>
<td>5.9</td>
</tr>
<tr>
<td>Converge 95%</td>
<td>18.5</td>
<td>39.8</td>
<td>6.9</td>
<td>7.8</td>
</tr>
<tr>
<td>Converge 97.5%</td>
<td>95.9</td>
<td>118.6</td>
<td>8.5</td>
<td>9.6</td>
</tr>
</tbody>
</table>

Table 4: Time to return to steady state unemployment rate from 1% of employed workers starting out unemployed for each scenario.
Figure 6: Monthly unemployment rate simulated from 18 months of skill loss with job separation for each model. 18-month recession shaded in gray followed by five year recovery.

is chosen to match the length of the great recession and is shaded in gray. In the baseline model the initial impact of the skill loss is for the unemployment rate to go down. This is the case because inexperienced workers have higher job finding rates than experienced ones, so the separations with skill loss lead to lower average unemployment durations. This effect dominates for a few months until the unemployment rate begins to increase due to the compositional effect of a now higher portion of workers who are inexperienced and hence have higher unemployment rates. In the model with a single matching function where firms reduce the average number of vacancies per unemployed worker, the initial decline in unemployment is almost completely muted. The reduction in job finding probabilities for all workers implies that there is a larger increase in the unemployment rate from the change in the composition of the unemployment pool. In both cases, unemployment continues to rise after the recessionary period of skill loss ends as the job separation rate increases due to the compositional change in the workforce. Moreover, the recovery in each case is eventually
characterized by persistence in that unemployment only slowly returns to its steady state level, as the composition of workers across experience groups is slow to recover.

Finally, we look at the cyclical patterns of job finding and job separation probabilities from each simulation. Aggregate job finding and job separation probabilities are computed using the composition of the pools of unemployed and employed workers in each period multiplied by the probability of each type of workers experiencing a change in their employment status. The left panel of Figure 7 plots the pattern of job finding probabilities for both simulations. Here, the baseline model generates the counterfactual result that job finding probabilities increase during the period of skill loss. This occurs due to the assumption that inexperienced workers have higher job finding probabilities than experienced ones. If there were a third state for displaced workers with lower job finding probabilities this result could be reversed. However, with a thin market externality job finding probabilities move in the opposite direction. This is consistent with the empirical evidence that job finding probabilities are
procyclical. Simulated results for job separations are depicted in the right panel of Figure 7. Both simulations generate nearly identical patterns of job separation probabilities as they slowly increase during the recession as the fraction of workers who are inexperienced (with high job separation probabilities) increases.

5 Discussion

The goal of this paper is to quantitatively assess potential channels to generate persistent unemployment fluctuations in search and matching models. The results suggest that compositional changes among heterogeneous groups of workers with different baseline unemployment rates generate persistent unemployment fluctuations. This explanation is related to the heterogeneity explanation explored in Ravenna and Walsh (2012) and the learning story in Pries (2004). This paper compliments previous explanations as it generates a theory of long-run unemployment fluctuations that also has predictions about the cyclicality of both job finding and job separation probabilities. Learning implies that periods of high unemployment are persistent due to higher than normal job separation probabilities. However, Shimer (2012) shows that variation in job finding rates are an important component of cyclical unemployment fluctuations. The cyclical properties of job finding and job separation probabilities in this paper depend on the separate probabilities for inexperienced and experienced workers. The baseline parameterization of the model where inexperienced workers have higher finding and lower separation probabilities implies changes in the composition of workers generates counterfactually high job finding rates. However, the model can generate higher job finding rates either with a thin market externality that amplifies persistence or by including additional worker types with different job finding probabilities.

While the focus of this paper is to understand the theoretical propagation mechanism that can generate persistent unemployment fluctuations rather than the shocks that cause unemployment to change, it relates to a number of papers that seek to understand changes in unemployment during the great recession. For a summary of the labor market with a
focus on worker flows through the recession see Elsby et al. (2010) and Elsby et al. (2011). An increase in long-duration unemployment is a key feature in the recent US recession and has been a constant feature of higher rates of unemployment in Europe. Another possible explanation for the deterioration of labor market conditions is mismatch as described in Shimer (2007). A large literature has attempted to assess the role of mismatch in increased unemployment after the recession, but has only found modest effects. 12

When assessing the evidence for compositional changes proposed in this paper with respect to the thin market externality proposed by Pissarides (1992) there are a number of pieces to evidence to consider. First, the explanations are not exclusive in that both can play important roles in explaining cyclical patterns of unemployment outcomes. Second, even with generous assumptions about the size of the externality including a single labor market, assuming that all workers become inexperienced when shocks hit the economy, and fixed wages to magnify their effect, the thin market externality alone only generates moderate amounts of persistence. In contrast, compositional changes can generate substantial persistence on their own that can be enhanced through an externality. Mueller (2012) provides further evidence between these mechanisms by showing that during recessions the pool of unemployed workers is composed of more workers who were separated from high wage jobs. While this evidence makes a thin market externality less likely as the composition of the unemployment pool is improving, such separations could still generate persistent unemployment fluctuations if they lose skills when they separate.

The mechanism of compositional changes in workers across skills is potentially related to a recent literature on job polarization. 13 In particular, Jaimovich and Siu (2012) show that the disappearance of jobs in occupations in the middle of the skill distribution has been concentrated during recessions. They argue that this factor contributes to jobless recoveries, but could also contribute to compositional changes where workers previously employed in middle skill occupations can no longer find jobs in that area. Therefore, job polarization could contribute to the compositional story proposed to account for the persistence of

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12 See papers by Barlevy (2011), Herz and van Rens (2011), and Sahin et al. (2012) among others.
13 For a discussion see Acemoglu (1999), Autor et al. (2006) among many others.
unemployment fluctuations proposed in this paper.

Finally, in attempting to understand how high observed labor market flows can be reconciled with persistence in the unemployment rate, this paper has abstracted away from the influence of policies on labor market outcomes. Ljungqvist and Sargent (1998) and Pries and Rogerson (2005) show that policies can have important effects on the level of worker turnover. In relation to explanations that focus on the role of policy, this paper provides a complimentary explanation that emphasizes the compositional role of skill differences for unemployment outcomes in the absence of policy differences. Exploring how policy interacts with heterogeneity and labor market shocks is an intriguing avenue for future study.
References


A Appendix: Steady State Model Solution

This section shows the steps taken to solve for the steady state equilibrium of the model. Using the free entry condition for experienced worker firms, $J_e$ can be solved for using (8):

$$J_e = \frac{y_e - w_e}{1 - \beta(1 - \delta)(1 - s_e)}$$

Subtracting (2) from (4) yields:

$$E_e - U_e = \frac{w_e - b}{1 - \beta(1 - \delta)(1 - s_e - \lambda(\theta_e)p_e)}$$

Substituting this into the Nash Bargaining solution yields:

$$w_e = \frac{\gamma y_e (1 - \beta(1 - \delta)(1 - s_e - \lambda(\theta_e)p_e))}{1 - \beta(1 - \delta)(1 - s_e - \gamma \lambda(\theta_e)p_e)} + \frac{(1 - \gamma)b(1 - \beta(1 - \delta)(1 - s_e))}{1 - \beta(1 - \delta)(1 - s_e - \gamma \lambda(\theta_e)p_e)}$$

$E_e - U_e$ can be substituted into (4) to solve for $E_e$:

$$E_e = \frac{1}{1 - \beta(1 - \delta)} \left[ w_e - s_e \beta(1 - \delta) \frac{w_e - b}{1 - \beta(1 - \delta)(1 - \theta e^{-\eta} p_e - s_e)} \right]$$

Following the same approach, subtracting (1) from (3) gives:

$$(1 - \beta(1 - \delta)(1 - (1 - \alpha)s_n - \lambda(\theta_n)p_n)(E_n - U_n) = w_n - b + \alpha \beta(1 - \delta)(E_e - E_n)$$

Solving for $E_e - E_n$ and substituting into the above equation yields:

$$E_n - U_n = \frac{w_n - b + \alpha \beta(1 - \delta)\frac{(1 - \beta(1 - \delta))E_e - w_n}{1 - \beta(1 - \delta)(1 - \alpha)} A}{A}$$

where:

$$A = 1 - \beta(1 - \delta)(1 - \lambda(\theta_n)p_n - (1 - \alpha)s_n) - \alpha(1 - \alpha)(\beta(1 - \delta))^2 s_n$$

$$1 - \beta(1 - \delta)(1 - \alpha)$$
Equation (7) and the zero profit condition combined with the solution for $J_e$ implies that $J_n$ is given by:

$$J_n = \frac{1}{1 - \beta(1 - \delta)(1 - \alpha)(1 - s_n)} \left( y_n - w_n + \frac{\alpha \beta(1 - \delta)}{1 - \beta(1 - \delta)(1 - s_e)}(y_e - w_e) \right)$$

Finally, plugging these into the Nash bargaining equation and solving for $w_n$ gives:

$$w_n = \frac{AC\gamma \left( y_n + \frac{\alpha \beta(1 - \delta)(y_n - w_n)}{1 - \beta(1 - \delta)(1 - s_e)} \right) + B(1 - \gamma)(Cb - \alpha \beta(1 - \delta)(1 - \beta(1 - \delta))E_e)}{B(1 - \gamma)(C - \alpha \beta(1 - \delta)) + AC\gamma}$$

where:

$$B = 1 - \beta(1 - \delta)(1 - \alpha)(1 - s_n)$$

and

$$C = 1 - \beta(1 - \delta)(1 - \alpha)$$

To solve for the steady state of the model, the above equations for $J_n$ and $J_e$ can be substituted into the value functions for vacancies with the zero profit condition imposed:

$$J_n = \frac{k}{\beta q(\theta_n)p_n}$$  \hspace{1cm} (9)$$

$$J_e = \frac{k}{\beta q(\theta_e)p_e}$$  \hspace{1cm} (10)$$

Solving the steady state flow equations as a function of $\theta_i$ provides an expression that can be substituted into the zero profit conditions. For any given set of parameters, these conditions determine the equilibrium number of workers in each state.