Measuring the Unequal Gains from Trade

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First Draft: September 2013
This Draft: May 2014

Abstract

Individuals that consume different baskets of goods are differentially affected by relative price changes caused by international trade. We develop a methodology to measure the unequal gains from trade across consumers within countries that is applicable across countries and time. The approach uses data on aggregate expenditures across goods with different income elasticities and parameters estimated from a non-homothetic gravity equation. We find considerable variation in the pro-poor bias of trade depending on the income elasticity of each country’s exports. Non-homotheticities across sectors imply that the gains from trade typically favor the poor, who concentrate spending in more traded sectors.

Key words: International Trade, Inequality, Non-Homothetic Preferences
JEL Classification: D63, F10, F60

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We thank Andrew Atkeson, Joaquin Blaum, Ariel Burstein, Arnaud Costinot, Juan Carlos Hallak, Esteban Rossi-Hansberg, Jonathan Vogel and seminar participants at Brown, Chicago, Harvard, Princeton, Stanford, UCLA, USC, and International Growth Centre meeting at Berkeley for helpful comments. We acknowledge funding from the Jerome A. Chazen Institute of International Business at Columbia Business School.

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1 Introduction

Understanding the distributional impact of international trade is one of the central tasks pursued by international economists. A vast body of research has examined this question through the effect of trade on the distribution of earnings across workers (e.g., Stolper and Samuelson (1941)). A second channel operates through the cost of living. It is well known that the consumption baskets of high- and low-income consumers look very different (e.g., Deaton and Muellbauer (1980b)). Therefore, international trade has a distributional impact whenever it affects the relative price of goods that are consumed at different intensities by rich and poor consumers. For example, a trade-induced increase in the price of food has a stronger negative effect on low-income consumers, who typically have larger food expenditure shares than richer consumers. How important are the distributional effects of international trade through this expenditure channel? How do they vary across countries? Do they typically favor high- or low-income consumers?

In this paper we develop a methodology to answer these questions. The approach is based on aggregate statistics and model parameters that can be estimated from readily available bilateral trade and production data. It can therefore be implemented across many countries and over time. In designing this methodology, we are influenced by a recent literature in international trade, including Arkolakis et al. (2012), Melitz and Redding (2014) and Feenstra and Weinstein (2010), which studies the aggregate welfare gains from trade. These approaches estimate model parameters from a gravity equation (typically, the trade-cost elasticity of imports) and then combine these parameters with aggregate statistics to calculate the impact of trade on aggregate real income. We estimate model parameters from a non-homothetic gravity equation (both the trade-cost elasticity and the income elasticity of imports) to calculate the impact of trade on the real income of consumers with different patterns of expenditures within the economy.

The premise of our analysis is that consumers at different income levels within an economy dedicate different expenditure shares on imported goods (or vary in their share of expenditures across import countries and sectors). Studying the distributional implications of trade in this context requires a non-homothetic demand structure with good-specific Engel curves. That is, the elasticity of the expenditure share with respect to individuals’ total expenditures is allowed to vary across goods. The Almost-Ideal Demand System (AIDS), introduced by Deaton and Muellbauer (1980a), is a natural choice. It is a first-order approximation to any demand system and is widely used in applied work because it generally provides a good fit of individual expenditure data. Importantly for our purposes, it is flexible enough to satisfy the key requirement of good-specific income elasticities and has convenient aggregation properties.

We start by showing a demand-side result: in the AIDS, the first-order approximation to the

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1 Costinot and Rodriguez-Clare (2013) summarize this literature.
2 Only a few studies to our knowledge have used the AIDS in an international trade context. Feenstra and Reinsdorf (2000) shows how prices and aggregate expenditures relate to the Divisia index in the AIDS, and suggests that this demand system could be useful for welfare evaluation in a trade context. See also Feenstra (2010). Chaudhuri et al. (2006) use the AIDS to determine the welfare consequences in India of enforcing the Agreement on Trade-Related Intellectual Property Rights.
welfare change through the expenditure channel experienced by consumers at each expenditure level as a result of changes in prices, can be recovered from demand parameters and aggregate statistics. These aggregate statistics include the initial levels and changes in aggregate expenditure shares across commodities, and moments from the distribution of expenditure levels across consumers. The intuition for this result is that, conditioning on moments of the expenditure distribution, changes in aggregate expenditure shares across goods can be mapped to changes in the relative prices of high- versus low-income elastic goods by inverting aggregate demand. These relative price changes and demand parameters, in turn, suffice to measure the variation in real income of consumers at each expenditure level through changes in the cost of living.

To study the distributional effects of trade through the expenditure channel, we embed this demand structure in a benchmark model of international trade, the Armington model, which is also convenient as an empirical framework. There are multiple sectors, cross-country differences in sectoral productivity, and bilateral trade costs. Within each sector, goods are differentiated by country of origin. We extend this supply-side structure with two features. First, the endowment of the single factor of production varies across consumers, which generates within-country inequality. Second, consumer preferences are given by the Almost-Ideal Demand System, allowing goods from each sector and country of origin to enter with different income elasticity into the demand of individual consumers. Aggregate trade patterns are driven by standard Ricardian forces (differences in productivities and trade costs across countries and sectors) and by demand forces (cross-country differences in income distribution and differences in income elasticity across sectors and source countries).

We first pursue the theoretical and empirical analysis in a single-sector version of the model, in which each country produces a differentiated good that may vary by income elasticity of demand. This benchmark is useful to see how the income elasticity of each country’s exports shapes the gains from trade of poor relative to rich consumers within each country. In countries that export low income-elastic goods, or that are proximate to exporters of high income-elastic goods, the gains from trade are relatively biased to richer consumers, because opening to trade decreases the relative price of high-income elastic goods. Then, we proceed to the multi-sector analysis. Because sectors vary in how tradable they are (e.g., food versus services), demand non-homotheticity across sectors is an additional force that shapes unequal gains from trade across consumers.

In either context, the application of our starting demand-side result implies that demand-side parameters and aggregate import shares can be used to measure the welfare change experienced by consumers at any income level through the expenditure channel in response to foreign shocks such as changes in trade costs or foreign productivities. For example, a tilt in the aggregate import basket towards goods consumed mostly by the rich may reveal a fall in the import prices of these goods, and a relative welfare improvement for high-income consumers.

To quantify these results, we need estimates of the elasticity of individual expenditure shares by sector and country of origin with respect to both prices and income. A salient feature of the

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3Since there is a single factor of production, we abstract from changes in nominal income inequality.
model is that it delivers a sectoral non-homothetic gravity equation to estimate these key parameters from readily-available data on production and trade flows across countries.⁴ The estimation identifies which countries produce high or low income-elastic goods by projecting importer budget shares within each sector on standard gravity forces (e.g., distance) and a summary statistic of the importer’s income distribution whose elasticity can vary across exporters. Consistent with the existing empirical literature, such as Hallak and Schott (2011) or Feenstra and Romalis (2014), we find that richer countries export goods with higher income elasticities. The estimation also identifies the sectors whose goods are relatively more valued by rich consumers by projecting sectoral expenditure shares on a summary statistic of the importer’s income distribution. Consistent with Hallak (2010), our results also suggest non-homotheticities not only across origin countries but also across sectors.

Using the estimated parameters, we apply the results from the theory to ask: who are the winners and losers of trade within countries, how large are the distributional effects, and what country characteristics are important to shape these effects? To answer these questions we perform the counterfactual exercise of increasing trade costs so that each country is brought from its current trade shares to autarky, and compute the gains from trade corresponding to each percentile of the income distribution in each country (i.e., the real income that would be lost by each percentile because of a shut down of trade).

The single-sector analysis reveals that, in most countries, the gains from trade are U-shaped with consumer income. This reflects that imported goods are more likely to suit the tastes of very rich and poor consumers. For example, rich consumers devote relatively large shares to goods originated from exporters of high-income elastic goods such the U.S. or Japan, while the poor spend heavily in imports from India or Indonesia. While this U-shaped pattern arises within most countries, there is also large cross-country heterogeneity in the difference between the gains from trade of poor and rich consumers. In countries that specialize in low income-elastic goods, such as India, or that are proximate to exporters of high income-elastic goods, such as Mexico, the gains from trade are relatively less favorable to poor consumers because opening to trade increases the relative price of low-income elastic exported goods.

The multi-sector estimation further reveals an important role for non-homotheticities across sectors. While the single-sector model implies that in some countries the rich (e.g. 90th percentile of the income distribution) gain more from trade than the poor (e.g., 10th percentile), the multi-sector model implies that poor consumers gain more from trade than rich consumers in every country. On average over the countries in our sample, the real income loss from closing off trade are 70 percent for the 10th percentile of the income distribution and 27 percent for the 90th percentile.⁵ This strong

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⁴In principle, one could use micro data to recover the parameters of the model, and then apply aggregate information to simulate counterfactuals. The reason we do not pursue this approach is that, although micro data measure expenditures across products by individuals with different incomes, these data rarely ask households about the origins of goods. (For example, it is not possible to compute import shares at the household level using standard consumer survey data). As such, it is not feasible to estimate income elasticities of sector and origin countries using typical micro data. This data challenge is a motivating feature of our methodology that permits aggregation across consumers within a country and estimation of the demand parameters using aggregate expenditures shares.

⁵This difference between the average gains from trade of poor and rich consumers is considerably larger than in
bias in the gains from trade toward poor consumers hinges on the fact that these consumers are predicted to spend relatively more on sectors that are more traded, while high-income individuals consume relatively more services, which are the least traded sector. Additionally, low-income consumers happen to concentrate spending on sectors with a lower elasticity of substitution across source countries. As a result, the multi-sector setting implies larger expenditures in more tradable sectors and a lower rate of substitution between imports and domestic goods for poor consumers; these two features lead to larger gains from trade for the poor than the rich.

The movement-to-autarky counterfactuals give a sense of the distributional implications of reducing exposure to trade. In a second set of exercises, we use data on changes in expenditure shares from the mid-1990’s to the mid-2000’s, a period characterized in most countries by an increase in the share of imports in total spending. As domestic shocks to each country may have also caused changes in expenditure shares, this experiment provides an upper bound for the distributional impact of trade over time through the expenditure channel. For almost all countries, these changes in trade costs lead to welfare changes favorable to the poor due to increased imports in low-income elastic sectors.

As we mentioned, our approach to measure welfare gains from trade using aggregate statistics is close to a recent literature that studies the aggregate welfare gains from trade summarized by Costinot and Rodriguez-Clare (2013). This literature confronts the challenge that price changes induced by trade costs are not commonly available by inferring them through the model structure from changes in trade shares. However, these approaches are designed to measure only aggregate gains rather than distributional consequences. In our setting, we exploit properties of a non-homothetic demand system that also allows us to infer changes in prices from trade shares, and in addition the non-homothetic structure enables us to trace out the welfare consequences of these price changes across different consumers within countries. We are motivated by the belief that an approach that is able to quantify the (potentially) unequal gains from trade through the expenditure channel for many countries is useful in assessing the implications of trade, particularly because much of the public opposition towards increased openness stems from the notion that welfare changes are unevenly distributed.

Of course, we are not the first to allow for differences in income elasticities across goods in an international trade framework. Theoretical contributions to this literature such as Markusen (1986), Flam and Helpman (1987) and Matsuyama (2000) develop models where richer countries specialize in high-income elastic goods through supply-side forces, while Fajgelbaum et al. (2011) study cross-country patterns of specialization that result from home market effects in vertically

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6 We compute the distributional effects associated with hypothetical changes in trade costs that lead, in each country, to changes in expenditure shares that actually occurred.

7 For example, autarky prices are rarely observed in data but under standard assumptions on preferences the autarky expenditure shares are generally known. The difference between autarky and observed trade shares can then be used to back out the price changes caused by a counterfactual movement to autarky.
differentiated products. Recent papers by Hallak (2006), Fieler (2011), Caron et al. (2012) and Feenstra and Romalis (2014) find that richer countries export goods with higher income elasticity.\(^8\) This role of non-homothetic demand and cross-country differences in the income elasticity of exports in explaining trade data is an important motivation for our focus on explaining the unequal gains from trade through the expenditure channel.\(^9\)

Porto (2006) and Faber (2013) analyze the effects of trade on consumer welfare through heterogeneity of tastes. Porto (2006) studies the effect of price changes implied by a tariff reform on the distribution of welfare using consumer survey data from Argentina. In a related paper, Faber (2013) exploits Mexico’s entry into NAFTA to study the effect of input tariff reductions on the price changes of final goods of different quality. While these papers utilize detailed micro data for specific countries in the context of major reforms, our approach provides a framework to quantify the unequal gains from trade across consumers over a large set of countries using aggregate trade and production data. Within our framework we are able to show theoretically how changes in trade costs map to the welfare changes of individuals in each point of the expenditure distribution, how to compute these effects using model parameters and aggregate statistics, and how to estimate the parameters from cross-country trade and production data.

There is of course a large literature that examines trade and inequality through the earnings channel. A dominant theme in this literature, as summarized by Goldberg and Pavcnik (2007), has been the poor performance of Stolper-Samuelson effects, which predict that trade increases the relative wages of low-skill workers in countries where these workers are relatively abundant, in rationalizing patterns from low-income countries.\(^{10}\) We complement these and other studies that focus on the earnings channel by examining the implications of trade through the expenditure channel. While in the multi-sector estimation we find that the magnitude of the gains from trade is typically larger for relatively poor than rich consumers, we also find, in contrast to textbook Stolper-Samuelson effects, that the gains from trade are relatively less favorable to the poor in lower-income countries because these countries tend to specialize in goods that low-income consumers are more likely to purchase.

\(^8\)See also Schott (2004), Hallak and Schott (2011) and Khandelwal (2010) who provide evidence that richer countries export higher-quality goods, which typically have high income elasticity of demand. In this paper we abstract from quality differentiation within sectors, but note that our methodology could be implemented using disaggregated trade data where differences in the income elasticity of demand may be driven by differences in quality.

\(^9\)These theoretical and empirical studies use a varied set of demand structures. A challenge in our case is to accommodate within-country inequality for which the AIDS is naturally suited. The AIDS is also a first-order approximation to any demand and has the property that distinct parameters identify income and substitution elasticities. If good-specific income elasticities are neutralized, the AIDS collapses to the homothetic translog demand system studied in an international trade context by Feenstra and Weinstein (2010), Arkolakis et al. (2010) and Novy (2012). Neither the AIDS nor the translog has closed-form direct-utility representations but appropriate parameter restrictions ensures that both are well-defined demand systems. See Deaton and Muellbauer (1980b).

\(^{10}\)Several recent studies, such as Feenstra and Hanson (1996), Helpman et al. (2012), Brambilla et al. (2012), Frias et al. (2012), and Burstein et al. (2013) study different channels through which trade affects the distribution of earnings such as outsourcing, labor market frictions, quality upgrading, or capital-skill complementarity. Costinot and Vogel (2010) and Burstein et al. (2013) study the role of trade and other forces in shaping the distribution of earnings in high-dimensional environments that include multiple sectors and factors. We study an economy with multiple goods (which may vary in their income elasticity) and heterogeneous consumers (who vary in their preferences for these goods).
The remaining of the paper is divided into five sections. Section 2 uses standard consumer theory to derive generic expressions for the distribution of welfare changes across consumers, and applies these expressions to the AIDS. Section 3 embeds these results in a standard trade framework. Section 4 estimates the parameters and quantifies the unequal gains from trade. Section 5 extends the theory and empirics to multiple sectors. Section 6 concludes.

2 Consumers

We start by deriving generic expressions for the distribution of welfare changes across consumers that vary in their total expenditures. We only use properties of demand implied by standard demand theory. The results from this section correspond to changes in prices and expenditures exogenously taken as given by consumers. In Section 3, we link these results to a standard model of trade in general equilibrium.

2.1 Definition of the Expenditure Channel

We study an economy with $J$ goods for final consumption with price vector $p = \{p_j\}_{j=1}^J$ taken as given by $h = 1, ..., H$ consumers. Consumer $h$ has indirect utility $v_h$ and total expenditures $x_h$. We denote by $x(v_h, p)$ the expenditure function with associated indirect utility function $v(x_h, p)$. We also let $s_{j,h} = s_j(x_h, p)$ be the share of good $j$ in the total expenditures of individual $h$, and $S_j$ be the share of good $j$ in aggregate expenditures. We introduce supply-side assumptions in Section 3.

Consider the change in the indirect utility of consumer $h$ due to infinitesimal changes in log-prices $\{\hat{p}_j\}_{j=1}^J$ and in the log of the expenditure level $\hat{x}_h$:

$$\hat{v}_h = \sum_{j=1}^J \frac{\partial \ln v(x_h, p)}{\partial \ln p_j} \hat{p}_j + \frac{\partial \ln v(x_h, p)}{\partial \ln x_h} \hat{x}_h. \tag{1}$$

The equivalent variation of consumer $h$ associated with the price and expenditure changes $\{\hat{p}_j\}_{j=1}^J, \hat{x}_h$ is defined as the increase in individual expenditures, $\hat{w}_h$, that leads to the indirect-utility change $\hat{v}_h$ at constant prices:

$$\hat{w}_h = \frac{\partial \ln v(x_h, p)}{\partial \ln x_h} \hat{w}_h. \tag{2}$$

Combining (1) and (2) and applying Roy’s identity gives a well-known formula for the equivalent variation:

$$\hat{w}_h = \sum_{j=1}^I (-\hat{p}_j) s_{j,h} + \hat{x}_h. \tag{3}$$

The first term on the right-hand side of (3) is an expenditure-share weighted average of price changes and represents what we refer to as the expenditure effect. It is the increase in the cost of living of

\[\text{We use } \hat{z} \equiv d\ln (z) \text{ to denote the infinitesimal change in the log of variable } z.\]

\[\text{See Theil (1975).}\]
a consumer caused by a change in prices at the pre-shock expenditure basket. Henceforth, we refer to \( \hat{w}_h \) as the welfare change of individual \( h \), acknowledging that by this we mean the equivalent variation, expressed as share of the initial level of expenditures, associated with a change in prices or in the expenditure level of individual \( h \).

To organize our discussion it is useful to rewrite (3) as follows:

\[
\hat{w}_h = \hat{W} + \hat{\psi}_h + \hat{x}_h, \tag{4}
\]

where

\[
\hat{W} \equiv \sum_{j=1}^{I} (-\hat{p}_j) S_j, \tag{5}
\]

is the aggregate expenditure effect, and

\[
\hat{\psi}_h \equiv \sum_{j=1}^{I} (-\hat{p}_j) (s_{j,h} - S_j) \tag{6}
\]

is the individual expenditure effect of consumer \( h \).

The term \( \hat{W} \) is the welfare change through the expenditure channel in the absence of within-country inequality or when all consumers feature the same distribution of expenditures (e.g., if consumers only vary by income and demand is homothetic). It also corresponds to the welfare change through the cost of expenditures for a hypothetical representative consumer.\(^{13}\) In turn, the individual welfare change \( \hat{\psi}_h \) captures that consumers may be differentially affected by the same price changes due to differences in the composition of their expenditure basket. It is different from zero for some consumers only if there is variation across consumers in how they allocate expenditure shares across goods. The focus of this paper is to study how international trade impacts the distribution \( \{ \hat{\psi}_h \}_{h=1}^{H} \) in (6).

### 2.2 Almost-Ideal Demand

The Almost-Ideal Demand System (AIDS) introduced by Deaton and Muellbauer (1980a) belongs to the family of Log Price-Independent Generalized Preferences defined by Muellbauer (1975). The latter are defined by the indirect utility function

\[
v (x_h, p) = F \left[ \left( \frac{x_h}{a(p)} \right)^{1/b(p)} \right], \tag{7}
\]

\(^{13}\)For example, suppose that consumers have preferences with constant elasticity of substitution (CES). In that case, for any pair of goods \( j \) and \( k \) we can write \(-\hat{p}_j = \frac{\hat{S}_j - \hat{S}_k}{\hat{p}_k} \), where \( \sigma \) is the elasticity of substitution between goods. Since we have not yet chosen a numeraire, we can set \( \hat{p}_k = 0 \). Replacing into (5) yields \( \hat{W} \equiv \frac{1}{1-\sigma} \hat{S}_k \). Therefore, with CES demand the change in the aggregate share of just one (arbitrarily chosen) good \( k \) suffices to capture the aggregate expenditure effect (e.g., see Arkolakis et al. (2012)).
where \( a(p) \) and \( b(p) \) are price aggregators and \( F(\cdot) \) is a well-behaved increasing function. The AIDS is the special case that satisfies

\[
\begin{align*}
    a(p) &= \exp \left( \alpha + \sum_{j=1}^{J} \alpha_j \ln p_j + \frac{1}{2} \sum_{j=1}^{J} \sum_{k=1}^{J} \gamma_{jk} \ln p_j \ln p_k \right), \\
    b(p) &= \exp \left( \sum_{j=1}^{J} \beta_j \ln p_j \right),
\end{align*}
\]

where the parameters satisfy the restrictions \( \sum_{j=1}^{J} \alpha_j = 1, \sum_{j=1}^{J} \beta_j = \sum_{j=1}^{J} \gamma_{jk} = 0, \) and \( \gamma_{jk} = \gamma_{kj} \) for all \( j, k \).

The first price aggregator, \( a(p) \), has the translog functional form. It is independent from non-homotheticities and can be interpreted as the cost of a subsistence basket of goods. The second price aggregator, \( b(p) \), captures the relative price of high-income elastic goods. For our purposes, a key feature of these preferences is that the larger is the consumer’s expenditure level \( x_h \) relative to \( a(p) \), the larger is the welfare gain from a reduction in the cost of high income-elastic goods, as captured by a reduction in \( b(p) \).

Applying Shephard’s Lemma to the indirect utility function defined by equations (7) to (9) generates an expenditure share \( s_{j,h} = s_j(p,x_h) \) in good \( j \) for individual \( h \), where

\[
s_j(p,x_h) = \alpha_j + \sum_{k=1}^{I} \gamma_{jk} \ln p_k + \beta_j \ln \left( \frac{x_h}{a(p)} \right)
\]

for \( j = 1, \ldots, I \). These expenditure shares have two features that suit our purposes. First, the elasticity with respect to the expenditure level is allowed to be good-specific. Goods for which \( \beta_j > 0 \) have positive income elasticity, while goods for which \( \beta_j < 0 \) have negative income elasticity.

Second, they admit aggregation, in the sense that market-level behavior is represented by the behavior of a single consumer. The aggregate market share of good \( j \) is

\[
S_j = s_j(p,\bar{x}),
\]

where \( \bar{x} \) is an inequality-adjusted mean of the distribution of expenditures across consumers,

\[
\bar{x} = \bar{e}^{\Sigma},
\]

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14These parameter restrictions correspond to the adding up, homogeneity, and symmetry constraints implied by individual rationality, and ensure that the AIDS is a well-defined demand system. No direct-utility representation of the AIDS exists, but this poses no restriction for our purposes. See Deaton and Muellbauer (1980b).

15Expenditure shares must be restricted to be non-negative for all goods. We assume that (10) predicts non-negative expenditure shares for all goods and consumers, so that the non-negativity restriction is not binding. Since expenditure shares add up to one, this guarantees that expenditure shares are also smaller than 1. We discuss how to incorporate this restriction in the empirical analysis of Section 4.

16Even though we define \( x_h \) as the individual expenditure level, we follow standard terminology and refer to \( \beta_j \) as the income elasticity of the expenditure share in good \( j \).
where \( \bar{x} = \mathbb{E}[x_h] \) is the mean and \( \Sigma = \mathbb{E}\left[\frac{x_h}{\bar{x}} \ln \left(\frac{x_h}{\bar{x}}\right)\right] \) is the Theil index of the expenditure distribution.\(^\text{17}\) We identify \( \bar{x} \) as the expenditure level of the representative consumer, so that the distribution of budget shares for the aggregate economy is the same as the distribution of budget shares for an individual with expenditure level \( \bar{x} \).

To shorten notation, we let \( \hat{\mathbf{p}} \) be a column vector with the proportional price changes \( \hat{p}_j \) and \( \{S, \hat{S}\} \) be vectors with the levels and changes in aggregate expenditure shares, \( S_j \) and \( \hat{S}_j \). We also collect the parameters \( \alpha_j \) and \( \beta_j \) in the vectors \( \{\alpha, \beta\} \) and define \( \Gamma \) as the matrix with element \( \gamma_{jk} \) in row \( j \), column \( k \). With this notation, the AIDS is characterized by the parameters \( \Theta = \{\alpha, \alpha, \beta, \Gamma\} \), and the aggregate expenditure shares in (11) are represented by

\[
S = \alpha + \Gamma \ln \mathbf{p} + \beta y. \tag{12}
\]

The term \( y = \ln \left(\frac{\bar{x}}{a} (\mathbf{p})\right) \) denotes the ratio between the adjusted mean of the expenditure distribution and the homothetic price index. Henceforth, we follow Deaton and Muellbauer (1980a) and refer to \( y \) as adjusted “real” income.\(^\text{18}\)

### 2.3 The Individual Expenditure Effect with Almost-Ideal Demand

From (10) and (11), the difference in the budget shares of good \( j \) between a consumer with expenditure level \( x_h \) and the representative consumer is

\[
s_{j,h} - S_j = \beta_j \ln \left(\frac{x_h}{\bar{x}}\right). \tag{13}
\]

Consumers who are richer than the representative consumer have larger expenditure shares than the representative consumer in positive-\( \beta_i \) goods and lower shares in negative-\( \beta_i \) goods. Combining (13) with the individual expenditure effect defined in (6) we obtain

\[
\hat{\psi}_h = \underbrace{-\text{COV}\left[\{\beta_j\}_{j=1}^J; \{\hat{p}_j\}_{j=1}^J\right]}_{\hat{b}} \times \ln \left(\frac{x_h}{\bar{x}}\right), \tag{14}
\]

where \( \hat{b} \) is the proportional change in the non-homothetic price index \( b(\mathbf{p}) \).\(^\text{19}\) This expression says that the covariance between the income elasticities \( \{\beta_j\} \) and the price changes \( \{\hat{p}_j\} \) summarizes unequal welfare changes through the expenditure channel. A positive (negative) value of \( \hat{b} \) reflects a relative price increase of high (low) income elastic goods, leading to a relative welfare loss for rich (poor) consumers.

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\(^{17}\)The Theil index is a measure of inequality which takes the minimum \( \Sigma = 0 \) if the distribution is concentrated at a single point. In the case of a lognormal expenditure distribution with variance \( \sigma^2 \), it is \( \Sigma = \frac{1}{2} \sigma^2 \).

\(^{18}\)Clearly, \( y \) does not represent the actual real income of any specific consumer. In the absence of within-country inequality, \( y \) can be interpreted as the total expenditure of a representative consumer after paying for a subsistence-level consumption basket with price equal to \( a \).

\(^{19}\)Because the elasticities \( \{\beta_j\} \) add up to zero, \( \hat{b} \) is the covariance between the good-specific income elasticities and the price changes.
Collecting terms, the welfare change of consumer $h$ is

$$\hat{w}_h = \hat{W} + \hat{b} \times \ln \left( \frac{x_h}{x} \right) + \hat{x}_h.$$  \hfill (15)

Given a distribution of expenditure levels $x_h$ across consumers, this expression generates the distribution of welfare changes in the economy through the expenditure channel.

A useful property of this structure is that the coefficients $\{\hat{W}, \hat{b}\}$ can be expressed as function of demand parameters and aggregate statistics. From (5) and (14), these terms are simply weighted averages of price changes, $\hat{W} = S' \hat{p}$ and $\hat{b} = \beta' \hat{p}$. In turn, assuming that $\Gamma$ has a well defined inverse, $\hat{p}$ can be expressed as function of aggregate expenditure shares inverting the demand system (12),

$$\hat{p} = \Gamma^{-1} (dS - \beta dy).$$

This leads us to the following result.

**Proposition 1.** The aggregate and the individual expenditure effects $\{\hat{W}, \hat{b}\}$ corresponding to arbitrary infinitesimal price changes are

$$\hat{W} = S' \Gamma^{-1} (dS - \beta dy),$$  \hfill (16)

$$\hat{b} = \beta' \Gamma^{-1} (dS - \beta dy).$$  \hfill (17)

A direct corollary is that computing $\hat{W}$ and $\hat{b}$ only demands knowledge of the parameters $\{\Gamma, \beta\}$, the levels and changes in aggregate expenditure shares $\{S, dS\}$, and the change in adjusted real income, $dy$. Therefore, as long as the substitution and income-elasticity parameters $\{\Gamma, \beta\}$ are known, a researcher armed with a sequence of the aggregate statistics $\{S_j\}_{j=1}^T$ and $y$ over time can account not only for the aggregate expenditure effect, $\hat{W}$, but also for the deviation from that aggregate effect corresponding to consumers at each level of expenditures, $\hat{\psi}_h$.

### 3 International Trade Framework

We have used properties of demand to express the distribution of welfare changes across consumers as function of aggregate expenditure shares and demand parameters. Now, we embed these results in a standard model of trade and use it to measure the importance of trade as a driver of inequality through the expenditure channel. A natural benchmark that is useful as empirical framework is the canonical Armington model, in which products are differentiated by country of origin.\(^{20}\) In this section and the next we proceed in the context of a single-sector model, which is useful to see how the income elasticity of each country’s exports shapes the gains from trade of poor relative

\[^{20}\text{Anderson and Van Wincoop (2003) pioneered the use of the Armington model as a quantitative tool in international trade. Extra margins, such as product differentiation as in Krugman (1980) or input-output linkages as in Caliendo and Parro (2012) could be naturally incorporated to the model and empirical analysis along the lines of Costinot and Rodríguez-Clare (2013). However, a full analysis of firm heterogeneity as in Melitz (2003) or competitive effects as in Feenstra and Weinstein (2010) or Arkolakis et al. (2012) would need to confront that heterogeneous firms plausibly sell goods that are valued distinctively by rich or poor consumers and is outside the scope of this paper.}\]
to rich consumers. We introduce multiple sectors in Section 5 to study the role of sectoral non-homotheticities.

3.1 Single-Sector Model

The world economy consists of $N$ countries, each of them specialized in the production of a different good. From the perspective of an individual consumer, these goods can be demanded with different income elasticities. For example, expenditure shares on Indian goods may decrease with total individual expenditures. We let $p_{in}$ be the price of goods from country $n$ in country $i$ and $\mathbf{p}_i$ be the price vector in country $i$. We denote the local price in country $i$ of domestically produced goods by $p_i \equiv p_{ii}$. Bilateral iceberg trade costs $\tau_{in}$ and perfect competition imply that $p_{in} = \tau_{in} p_n$.

Labor is the only factor of production. We let $z_h$ be the effective units of labor of individual $h$ and $Z_i$ be the productivity of each unit of labor in country $i$. Therefore, the wage rate per unit of labor is $p_i Z_i$, and individual $h$ in country $i$ receives income of $x_h = z_h p_i Z_i$. Individual income equals expenditure. Each country is characterized by a mean $\bar{z}_i$ and a Theil index $\Sigma_i$ of its distribution of effective units of labor across the workforce, leading to a mean $\bar{x}_i = \bar{z}_i p_i Z_i$ and a Theil index $\Sigma_i$ of the income distribution.

The demand side is given by the AIDS. Let $X_{in}$ be the value of exports from exporter $n$ to importer $i$ and let $Y_i$ be the total income of importer $i$. Using (11), the aggregate expenditure share in country $i$ for goods originated in country $n$ is

$$S_{in} = \frac{X_{in}}{Y_i} = \alpha_n + \sum_{n' = 1}^N \gamma_{nn'} \ln (p_{in'}) + \beta_n y_i,$$

(18)

where, letting $a_i = a(p_i)$ be the homothetic price index in country $i$, the term $y_i = \ln (\bar{x}_i / a_i)$ denotes as before the ratio of the adjusted mean of the expenditure distribution to the homothetic price index, $\bar{x}_i = \bar{z}_i e^{\Sigma_i}$. The richer is the importing country (higher $\bar{x}_i$) or the more unequal it is (higher $\Sigma_i$), the larger is its expenditure share from countries that produce goods with positive income elasticity, $\beta_n > 0$. The parameters $\alpha_n$ captures the overall taste for the goods exported by country $n$ independently from prices or income in the importer. The expenditure share in goods originated from $n$ for an individual consumer $h$ in country $i$ is

$$s_{in,h} = \alpha_n + \sum_{n' = 1}^N \gamma_{nn'} \ln (p_{in'}) + \beta_n y_{i,h},$$

(19)

where $y_{i,h} = y_i + \ln (x_h / \bar{x}_i)$.

For cleaner analytic expressions we assume that cross-elasticities are symmetric,

$$\gamma_{nn'} = \begin{cases} \frac{\gamma}{N} & \text{if } n \neq n' \\ - \left(1 - \frac{1}{N}\right) \gamma & \text{if } n = n' \end{cases}.$$
While this assumption simplifies the algebra, it is not necessary to reach analytic results.\footnote{In the empirical analysis of Section 4, this assumption simplifies the estimation because it restricts the number of parameters to be estimated, but it can be relaxed if there is enough variation to estimate asymmetric elasticities across, for example, groups of countries. In the multi-sector estimation of Section 5, we allow $\gamma$ to vary by sector. The normalization by $N$ in (20) only serves the purpose of shortening notation.}

Before we proceed it is useful to define the following measure of dispersion among the $\beta_n$’s,

$$\sigma_\beta^2 = \sum_{n=1}^{N} \beta_n^2,$$

as well as the “aggregate beta” of economy $i$,

$$\bar{\beta}_i = \sum_{n=1}^{N} \beta_n S_{in}.$$  

The parameter $\sigma_\beta^2$ is proportional to the variance of the $\beta_n$’s and captures the strength of non-homotheticities across goods from different origins. If preferences are homothetic then $\sigma_\beta^2 = 0$. The aggregate beta $\bar{\beta}_i$ measures the bias in the composition of aggregate expenditure shares of country $i$ towards goods from high-$\beta$ exporters. The larger is $\bar{\beta}_i$, the relatively more economy $i$ spends in goods that are preferred by high-income consumers.

### 3.2 Distributional Impact of a Foreign-Trade Shock

Without loss of generality we normalize the wage in country $i$ to 1, $p_i Z_i = 1$. Consider a foreign shock to this country consisting of an infinitesimal change in foreign productivities, foreign endowments or trade costs between any country pair. From the perspective of an individual consumer $h$ in country $i$, this shock affects welfare through the ensuing changes in prices $\{\hat{p}_n\}_{n=1}^{N}$ and income $\hat{x}_h$. Because only foreign shocks are present, the change in income $\hat{x}_h$ is the same for all consumers and equal to change in the price of the domestic commodity, $\hat{x}_h = \hat{p}_i = 0$.

Applying Proposition 1 to this context gives an aggregate expenditure effect in country $i$ of

$$\hat{W}_i = \hat{W}_{H,i} + \hat{W}_{NH,i},$$

where

$$\hat{W}_{H,i} = \frac{1}{\gamma} \left( \sum_{n=1}^{N} S_{in} dS_{in} - dS_{ii} \right),$$

$$\hat{W}_{NH,i} = \frac{1}{\gamma} \left( \beta_i - \bar{\beta}_i \right) dy_i,$$

and a change in the relative price of high-income elastic goods of

$$\hat{b}_i = \frac{1}{\gamma} \left( \sigma_\beta^2 dy_i - d\bar{\beta}_i \right).$$
The application of Proposition 1 to this trade environment generates expressions that link the aggregate and individual expenditure effects, \( \hat{W}_i \) and \( \hat{b}_i \), to the demand parameters \( \gamma \) and \( \{ \beta_n \} \), the level and changes in aggregate expenditure shares \( \{ S_{in}, dS_{in} \}_{n=1}^N \), and the level and change in adjusted real income of country \( i \), \( \{ y_i, dy_i \} \). Additionally, the supply-side structure from the Armington environment allows to express the change in adjusted real income \( dy_i \) caused by foreign shocks as function of \( \{ S_{in}, dS_{in} \}_{n} \) and the demand parameters:\(^{22}\)

\[
dy_i = \frac{\sum_{n=1}^{N} S_{in} dS_{in} - dS_{ii} - y_i \beta_i}{\beta_i - \beta_i + \gamma - \sigma y_i}.
\]

Expressions (23) to (27) provide a closed-form characterization of the first-order approximation to the welfare effects of a foreign-trade shock that includes three novel margins.\(^{23}\) First, preferences are non-homothetic with good-specific income elasticities. Second, the formulas accommodate within-country inequality through the Theil index of expenditure distribution \( \Sigma_i \), which enters through the level of \( y_i \). Third, and key for our purposes, they characterize the welfare change experienced by individuals at each income level, so that the entire distribution of welfare changes through the expenditure channel can be computed using (15).

The aggregate expenditure effect, \( \hat{W}_i \), includes a homothetic part \( \hat{W}_{H,i} \) that is independent from the \( \beta'_i \)s and a non-homothetic part, \( \hat{W}_{NH,i} \), which adjusts for the country’s pattern of specialization in high- or low-income elastic goods and the change in adjusted real income. Assuming that \( \gamma > 0 \), the richer or the more unequal country \( i \) becomes (the higher \( dy_i \) is), the larger the aggregate non-homothetic term is when the country is specialized in high income elastic goods (\( \beta_i > \beta_i \)). When non-homotheticities are shut down, the aggregate welfare effect \( \hat{W}_i \) collapses to \( \hat{W}_{H,i} \), which corresponds to the aggregate gains under translog demand.\(^{24}\)

The key term for measuring unequal welfare effects is \( \hat{b}_i \). As we have established, \( \hat{b}_i < 0 \) implies a decrease in the relative price of high income-elastic goods which favors high-income consumers. To understand expression (26), we note that changes in import shares reflect both changes in relative prices and in the aggregate real income of the importing economy. Suppose that we observe \( d\beta_i > 0 \), which means that aggregate trade shares have moved towards high-\( \beta \) exporters; for example, this would occur if the U.S. exports goods that are mostly consumed by rich consumers and the importing country increases its imports from the U.S. In this circumstance,

\(^{22}\)To derive (27), we use that \( \hat{a}_i = \sum_{n=1}^{J} \frac{\partial \ln \sigma_n}{\sigma_n} \hat{p}_{in} = \sum_{n=1}^{J} (S_{in} - \beta_n y_i) \hat{p}_{in} \), where the second line follows by Shephard’s Lemma. Also, totally differentiating \( S_{ii} \) and \( S_{in} \) from (18), we reach \( \hat{p}_{in} = \hat{p}_i - \frac{1}{\beta_i} [dS_{in} - dS_{ii} - (\beta_i - \beta_n) dy_i] \). Combining the last two expressions and using that by definition \( dy_i = \hat{p}_i - \hat{a}_i \) gives the solution.

\(^{23}\)Below we also show expressions for discrete, rather than infinitesimal change in parameters. Note that the empirical results rely on exact welfare changes rather than first-order approximations.

\(^{24}\)The homothetic part, \( \hat{W}_{H,i} \), includes the entire distribution of levels and changes in expenditure shares, \( \{ S_{in}, dS_{in} \} \). With CES preferences the equivalent term is \( \frac{1}{\sigma} S_{ii} \) where \( \sigma \) is the elasticity of substitution, so it depends on just the own trade share. These results hold under perfect competition. Feenstra and Weinstein (2010) measures the aggregate gains from trade in the U.S. under translog preferences stemming from competitive effects, and Arkolakis et al. (2010) study the aggregate gains from trade with competitive effects under homothetic translog demand and Pareto distribution of productivity. The AIDS nests the demand system studied in these papers in the case that \( \beta_n = 0 \) for all \( n \), but we abstract from competitive effects.
if \( \gamma > 0 \) and the aggregate real income of the economy stayed constant \( (dy_i = 0) \), observing \( d\beta_i > 0 \) implies a reduction in the relative price of imports from the U.S. and a positive welfare impact on sufficiently rich consumers. However, the increase in imports from the U.S. captured by \( d\beta_i > 0 \) may also reflect an increase in aggregate real income of the importer, \( dy_i > 0 \). Hence, the change in the aggregate beta is adjusted by the change in real income to infer the bias in relative price changes.

Equations (23) to (27) express changes in individual welfare as the equivalent variation of a consumer (relative to initial income) that corresponds to an infinitesimal change in prices caused by foreign shocks. We use these results to measure exact welfare changes that correspond to discrete changes in the vector of prices. Consider two scenarios, A and B, with associated distributions of prices \( \{p_i^A, p_i^B\} \) and aggregate expenditure shares \( \{S_i^A, S_i^B\} \). As in the previous subsection, we assume that the endowments and productivity in country \( i \) are the same in the two scenarios, while foreign trade costs, endowments or technology may vary. Integrating (15) we obtain the exact change in real income experienced by an individual with expenditure level \( x_h \) in country \( i \) when conditions change from A to B:

\[
\frac{w_B}{w_A} = \frac{W_B}{W_A} \left( \frac{x_h}{\bar{x}_i} \right) - \ln \left( \frac{W_B}{W_A} \right).
\] (28)

If \( w_B < w_A \), individual \( h \) is willing to pay a fraction \( 1 - \frac{w_B}{w_A} \) of her income in scenario A to avoid the movement to scenario B. Measuring this exact individual-level change in real income requires the ratio between the non-homothetic price index in the two scenarios, \( b_i^B / b_i^A \), as well as the change in aggregate welfare, \( W_i^B / W_i^A \). By construction, the latter equals the welfare change of the representative consumer in country \( i \), which from (23) can be expressed as

\[
\frac{W_i^B}{W_i^A} = \frac{W_{H,i}^B}{W_{H,i}^A} \frac{W_{NH,i}^B}{W_{NH,i}^A}.
\] (29)

Integration of equations (23) to (27) between the expenditure shares \( \{S_{in}^A\} \) and \( \{S_{in}^B\} \) yields the following result.\(^{25}\)

**Proposition 2.** Consider two scenarios, A and B, with different foreign conditions (trade costs or productivities) to country \( i \) and associated aggregate expenditure shares \( \{S_i^A, S_i^B\} \). The total welfare change to consumer \( h \) in country \( i \) is given by (28), where the homothetic component of the

\(^{25}\)Reaching (30) to (32) is straightforward. As for (33), rearranging terms in (27) and using (24) gives

\[
d (y_i \beta_i) + (\gamma - \beta_i) dy_i - (\sigma^2 / 2) d (y_i^2) - \gamma \hat{W}_{H,i} = 0.
\]

Integrating between \( \{S_{in}^A\} \) and \( \{S_{in}^B\} \) gives

\[
(\sigma^2 / 2) \left( y_i^B \right)^2 - (\gamma - \beta_i + \beta^B_i) y_i^B + \left[ (\gamma - \beta_i + \beta^A_i) y_i^A + \gamma \ln \left( W_i^B / W_i^A \right) \right] = 0,
\]

with roots for \( y_i^B \) reported in (33). Evaluating the roots at \( \{S_{in}^B\} = \{S_{in}^A\} \) determines which root must be preserved.
aggregate effect in (29) is
\[
\frac{W^B_{H,i}}{W^A_{H,i}} = e^{\frac{1}{\gamma} \left( \sum_{n=1}^{N} (s^B_{in})^2 - \sum_{n=1}^{I} (s^A_{in})^2 \right) - \frac{1}{\gamma} (\bar{s}^B_{ii} - \bar{s}^A_{ii})},
\]
(30)

the non-homothetic component of the aggregate effect is
\[
\frac{W^B_{NH,i}}{W^A_{NH,i}} = e^{\frac{1}{\gamma} \beta_i (y^B_i - y^A_i) - \frac{1}{\gamma} f^B \beta_i dy_i},
\]
(31)

the change in the non-homothetic price index is
\[
\frac{b^B_i}{b^A_i} = e^{\frac{1}{\gamma} [ (y^B_i - y^A_i) \sigma^2 - (\bar{\beta}^B_i - \bar{\beta}^A_i) ]},
\]
(32)

and the adjusted real income in the final scenario is
\[
y^B_i = \frac{1}{\sigma^2} \left\{ \gamma + \bar{\beta}^B_i - \beta_i \pm \sqrt{ (\gamma + \bar{\beta}^B_i - \beta_i)^2 - 2\sigma^2 \left[ (\gamma + \bar{\beta}^A_i - \beta_i) y^A_i - \frac{\sigma^2}{2} (y^A_i)^2 + \gamma \ln \left( \frac{W^B_{H,i}}{W^A_{H,i}} \right) \right] } \right\}
\]
(33)
in which the larger root is chosen if \( \gamma + \bar{\beta}^A_i - \beta_i - \sigma^2 y^A_i < 0 \) and the smaller root is chosen otherwise.

Equations (28) to (33) can be used to make either \textit{ex post} evaluations of the distribution of welfare changes (for an observed change in trade shares corresponding to foreign shocks) or \textit{ex ante} evaluations (for a counterfactual change in trade shares such as moving to autarky).\textsuperscript{26}

### 3.3 Non-Homothetic Gravity Equation

The model yields a non-homothetic gravity equation that we will take to the data to estimate the key parameters needed for welfare assessment. These parameters are the elasticity of substitution \( \gamma \) across exporters and the income elasticity of the goods supplied by each exporter, \( \{\beta_n\} \). Combining (18) and the definition of \( y_i \) gives
\[
\frac{X_{in}}{Y_i} = \left[ \alpha_n - \gamma \ln \left( \frac{\tau_{in}p_n}{\bar{\tau}_i p} \right) \right] + \beta_n \left[ \ln \left( \frac{\bar{x}_i}{a(P_i)} \right) + \Sigma_i \right],
\]
(34)

where
\[
\bar{\tau}_i = \exp \left( \frac{1}{N} \sum_{n=1}^{N} \ln (\tau_{in}) \right)
\]

\textsuperscript{26}If we set the \( \beta_n \)'s to zero, the exact aggregate welfare change \( W^B_i/W^A_i \) collapses to \( W^B_{H,i}/W^A_{H,i} \). The solution for this term in (30) is the same as the aggregate welfare change with translog demand in equation (10) of Feenstra and Weinstein (2010) fixing the number of varieties over time in their case (proof of this equivalence is available upon request). We also note that, as in Feenstra and Weinstein (2010) and Feenstra (2014), the homothetic part of the aggregate effect in (30) includes the Herfindahl of the distribution of aggregate expenditure shares.
and
\[ \bar{p} = \exp \left( \frac{1}{N} \sum_{n=1}^{N} \ln (p_n) \right). \]

Total income of each exporter \( n \) equals the sum of sales to every country, \( Y_n = \sum_{i=1}^{N} X_{in} \). Using this condition we can solve for the first term in square brackets in (34) and express import shares in country \( i \) in standard gravity form,
\[ \frac{X_{in}}{Y_i} = \frac{Y_n}{Y_W} - \gamma T_{in} + \beta_n \Omega_i, \tag{35} \]
where \( Y_W = \sum_{i=1}^{I} Y_i \) stands for world income, and where
\[ T_{in} = \ln \left( \frac{\tau_{in}}{\bar{\tau}_i} \right) - \sum_{n'=1}^{N} \left( \frac{Y_{n'}}{Y_W} \right) \ln \left( \frac{\tau_{n'n}}{\bar{\tau}_{n'}} \right), \tag{36} \]
\[ \Omega_i = \left[ \ln \left( \frac{\bar{x}_i}{a_i} \right) + \Sigma_i \right] - \sum_{n'=1}^{N} \left( \frac{Y_{n'}}{Y_W} \right) \left[ \ln \left( \frac{\bar{x}_{n'}}{a_{n'}} \right) + \Sigma_{n'} \right]. \tag{37} \]

The first two terms in the right-hand side of (35) are standard gravity terms. They capture relative market size of the exporter, bilateral trade costs, and multilateral resistance through trade costs relative to third countries. The last term, \( \beta_n \Omega_i \), is the non-homothetic component of the gravity equation, which includes the good-specific Engel curves that are needed to measure the unequal gains from trade across consumers. This term captures resistance to trade through mismatch between the income elasticity of the exporter and the income distribution of the importer. The larger \( \Omega_i \) is, either because average income or inequality in the importing country \( i \) is high relative to the rest of the world, the higher is the share of expenditures devoted to goods from country \( n \) when \( n \) is specialized in high income elastic goods (\( \beta_n > 0 \)).

4 Empirical Results for Single-Sector Economy

4.1 Data and Empirical Implementation

To estimate the gravity equation we use the World Input-Output Database (WIOD) which contains bilateral trade flows and production data for 40 countries (27 European countries and 13 other large countries) to compute expenditure shares. These are the same data used by Costinot and Rodriguez-Clare (2013), and cover food, manufacturing and service sectors.\(^{27}\) The data also delineates expenditures by sector and country of origin by final consumption or intermediate in-

\(^{27}\)We take an average of flows between 2005-2007 to smooth out annual shocks. Following Costinot and Rodriguez-Clare (2013), we aggregate the service sectors into a single sector. The choice of WIOD is a natural benchmark because it covers all sectors of the economy, can distinguish country of origin by final consumption or intermediate use, and has been used in previous work. In principle, one could apply the methodology to any dataset that contains production and trade flows across sectors or products (e.g., the Food and Agriculture Organization’s FAOSTAT dataset which records this information by food item).
put use. We use total expenditures as the benchmark and report robustness checks that restrict attention to final consumption.

In this section, we aggregate sector-level flows to a single sector; in Section 5 we examine the sector-level flows allowing for non-homothetic preferences across both sectors and countries. We merge the bilateral and production information with CEPII’s Gravity database to obtain bilateral distance and other gravity measures. Price levels, income per capita and population are from the Penn World Tables, and we obtain gini coefficients from the World Income Inequality Database (Version 2.0c, 2008) published by the World Institute for Development Research.\(^{28}\)

The term \(T_{in}\) in (36) captures bilateral trade costs between exporter \(n\) and importer \(i\) relative to the world. Direct measures of bilateral trade costs across countries are not available, so we proxy them with weighted-average distance \((d_{in})\) from the CEPII Gravity database. This variable calculates bilateral distances between the largest cities in each country, with the distances weighted by the share of the city in the overall country’s population.\(^{29}\)

We start by assuming that \(\tau_{in} = d_{in} \tilde{\epsilon}_{in}\) where \(\rho\) reflects the elasticity between distance and trade costs and \(\tilde{\epsilon}_{in}\) is an unobserved component of the trade cost between \(i\) and \(n\), and then include other gravity terms besides distance.\(^{30}\) This modifies the gravity specification in (35) to the estimated equation

\[
\frac{X_{in}}{Y_{i}} - \frac{Y_{n}}{Y_{W}} = -(\gamma\rho)D_{in} + \beta_{n}\Omega_{i} + \epsilon_{in},
\]

(38)

where, letting \(\bar{d}_{i} = \frac{1}{N} \sum_{n=1}^{N} \ln (d_{in})\),

\[
D_{in} = \ln \left( \frac{d_{in}}{\bar{d}_{i}} \right) - \sum_{j=1}^{I} \left( \frac{Y_{j}}{Y_{W}} \right) \ln \left( \frac{d_{jn}}{d_{j}} \right),
\]

(39)

and where we assume that the error term is iid.\(^{31}\) In the data, we measure \(\frac{X_{in}}{Y_{i}}\) from exporter \(n\)’s share in country \(i\)’s expenditures. Similarly, we use country \(n\)’s share in worldwide expenditure to construct \(\frac{Y_{n}}{Y_{W}}\). Since we do not directly observe trade costs, we cannot separately identify \(\gamma\) and \(\rho\). Following the literature we set \(\rho = 0.177\).\(^{32}\)

The term \(\Omega_{i}\), defined in (37), captures importer \(i\)’s inequality-adjusted real income relative to the world. To construct this variable we assume that the distribution of efficiency units in each country \(i\) is log-normal, \(\ln z_{h} \sim \mathcal{N}(\mu_{i}, \sigma_{i}^{2})\), leading to a log-normal distribution of expenditures

\(^{28}\)The World Income Inequality Database provides gini coefficients from both expenditure and income data. Ideally, we would use ginis from only the expenditure data, but this is not always available for some countries during certain time periods. We construct a country’s average gini using the available data between 2001-2006. Data for Iran is not contained in the database, so we obtain its gini from the World Development Indicators database.

\(^{29}\)Waugh (2010) includes exporter effects in the trade-cost specification. In our context, his assumption amounts to formulating \(\tau_{in} = c_{ex} d_{in} \tilde{\epsilon}_{in}\), where \(c_{ex}\) is the exporter fixed effect. The gravity equation (38) would be unchanged in this case because the exporter effect \(c_{ex}\) washes out from \(T_{in}\) in (36).

\(^{30}\)From the structure of trade costs it follows that the error term is \(\epsilon_{in} = -\gamma \left( \ln \left( \frac{\tilde{\epsilon}_{in}}{\tilde{\epsilon}_{i}} \right) - \sum_{n'} \left( \frac{Y_{n'}}{Y_{W}} \right) \ln \left( \frac{\tilde{\epsilon}_{i n'}}{\tilde{\epsilon}_{n'}} \right) \right)\)

where \(\tilde{\epsilon}_{i} = \exp \left( \frac{1}{N} \sum_{n=1}^{N} \ln (\tilde{\epsilon}_{in}) \right)\).

\(^{31}\)This is the same value used by Novy (2012).
with Theil index equal to \( \sigma_i^2/2 \) where \( \sigma_i^2 = 2\left[ \Phi^{-1}\left( \text{gini}_i + 1 \right) \right]^2 \). We construct \( \bar{x}_i \) from total expenditure and total population of country \( i \).

To build \( \Omega_i \) we also need to construct the homothetic price index \( a_i \). Following Deaton and Muellbauer (1980a) and more recently Atkin (2013), we replace the price index in \( i \) with a Stone index, 
\[
a_i = \sum_n S_{in} \ln (\tau_{in} p_n),
\]
where \( p_n \) comes from the price levels reported in the Penn World Tables. The obvious advantage of this approach is that it sidesteps estimation of the \( \alpha_n \), which enter the gravity specification non-linearly and are not required for our welfare calculations. The measure of real spending per capita divided by the Stone price index, \( \bar{x}_i / a_i \), is strongly correlated with countries’ real income per capita; this suggests that \( \Omega_i \) indeed captures the relative difference in real income across countries. Below, we show that the parameters look very similar in a robustness check that replaces \( \ln \left( \bar{x}_i / a_i \right) \) in (37) with income per capita.

We cluster errors at the importer level. Since the market shares sum to one for each importer, it is guaranteed that \( \sum_n \beta_n = 0 \) in the estimation, as the theory requires. We estimate the gravity equation on the 40 countries in the WIOD.

The results of the baseline estimation are reported in columns 1A and 1B of Table 1. As is well known in the literature, bilateral distance reduces trade flows between countries, which is captured by the statistically significant coefficient on \( D_{in} \). This implies \( \gamma = 0.533 \) (=.0943/.177). The 40 \( \beta \) parameters are reported in the subsequent rows. The exporters with the highest \( \beta \)'s are the U.S. and Japan, while Indonesia and India have the lowest \( \beta \)'s. This means that U.S. and Japan export goods that are preferred by richer consumers, while the latter export goods that are preferred by poorer consumers.

It is perhaps easiest to visualize the \( \beta \)'s by plotting them against the exporter’s per capita income in Figure 1. The relationship is strongly positive and statistically significant. We emphasize that this relationship is not imposed by the estimation. Rather, these exporter-specific coefficients reflect that richer countries are more likely to spend on products from richer countries, conditional on trade costs. We also note that the \( \beta \)'s are fully flexible, which is why the coefficients are often not statistically significant from zero. The finding that a subset are statistically significant is sufficient to reject homothetic preferences in the data and is consistent with the existing literature who finds that richer countries export goods with higher income elasticities.\(^{33}\)

The remaining columns of Table 1 check the sensitivity of the parameters to alternative specifications. In columns 2A and 2B, we re-estimate (38) using income per capita rather than our constructed measure \( \ln (\bar{x}_i / a_i) \) to build the term \( \Omega_i \). The \( \gamma \) parameter is stable and the correlation of the beta coefficients with the baseline column 1 is 0.94. This gives us confidence that the approximation of real income with the Stone index captures meaningful differences in income levels across countries. In column 3, we introduce additional trade costs: common language and a contiguous border term.\(^{34}\) As expected, the coefficient on \( D_{in} \) falls but remains statistically significant.


\(^{34}\)For this we define trade costs as \( \tau_{in} = d_{in,\rho}^{\rho} b_{in}^{\delta} \epsilon_{in} \), where \( l_{in} \) is an indicator of common language between \( i \) and \( n \) and \( b_{in} \) is an indicator for a contiguous border. As a result, the right-hand side of the gravity equation (38) becomes
\[-(\gamma \rho) D_{in} + (\gamma \delta) L_{in} + (\gamma \delta) B_{in} + \beta_i \Omega_i + \epsilon_{in}, \]
where \( L_{in} = \ln(l_{in}/\bar{l}_i) - \sum_{j=1}^m \left( Y_j / Y_{Wj} \right) \ln(l_{jn}/\bar{l}_j) \) and \( B_{in} \) is defined.
implied parameter $\gamma$, noted in the last row, falls to 0.239 in this specification. Importantly, the correlation with the baseline $\beta'$s remains high (correlation is 0.86). In the results below, we use the gravity estimates from column 3 as our baseline parameter values.

4.2 Measuring The Unequal Gains from Trade in a Single-Sector Economy

We now have the parameters to measure the unequal distribution of the gains from trade across consumers. For that, we perform the counterfactual experiment of bringing each country to autarky. In the model, this can be accomplished by sufficiently increasing trade costs for all countries. Because we know the changes in expenditure shares that take place between the observed trade shares and autarky, we can apply the results from Section 3.2 to measure the exact changes in real income between trade and autarky for consumers at each income level. Since we study an endowment economy, every consumer must be better off with trade, but the magnitude of the consumer-specific gains from trade (i.e., the real income lost by a consumer when trade is prohibited) vary across consumers within a country depending on their income level.

We begin by computing the gains from trade for the representative agent in country $i$ defined in (29). The exact expression for the first term is given in (30) and the second term is the non-homothetic component defined in (31). This non-homothetic term includes the real income term under autarky, which we solve for using (33). Moving the economy to autarky amounts to setting $A = \text{trade}$ and $B = \text{autarky}$ in those expressions, so that $\{S^A_{in}\}$ equals the observed trade shares in country $i$, and $\{S^B_{in}\}$ equals the autarky trade shares, $S^B_{ii} = 1$ and $S^B_{in} = 0$ for all $n$ different from $i$.

The first column of Table 2 reports the real income loss for the representative consumer in each country, $1 - W^\text{autarky}_i/W^\text{trade}_i$. To compare the results with and without non-homothetic preferences, we re-estimate (38) imposing $\beta_n = 0$ for all $n$; this corresponds to a translog specification. This analogously.

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35Our estimate of $\gamma$ in column 3 of Table 1 is very similar to Novy (2012), who reports $\gamma = 0.167$. That paper does not include domestic budget shares in the estimation; that is, it estimates an import translog demand system. When we include controls for common language and border, the variation in our budget shares resembles more closely the setting in that paper. Note that Feenstra and Weinstein (2010) report a median $\gamma$ of 0.19 using different data, a different level of aggregation and a different estimation procedure, so our estimate is in line with the few papers that have run gravity regressions with the translog specification.

36Changes in the level of earnings (common to all consumers) are captured by the real income change of the representative consumer. Our counterfactuals measure the real-income effects of changes in trade costs through consumption effects keeping fixed the distribution of relative earnings across consumers. Including Stolper-Samuelson effects would generate distributional effects of trade through changes in the earnings distribution, but we do not pursue this channel here.

37Throughout the analysis we take as given the specialization pattern of countries across goods with different income elasticity of demand. This pattern could change as they moved to autarky, but we note that the direction of that change will depend on what forces determine specialization. If specialization is driven by home-market effects, as in Fajgelbaum et al. (2011), poor countries would specialize less in low-income elastic goods as they move to autarky. However, if specialization is driven by relative factor endowments, as in Mitra and Trindade (2005) or Caron et al. (2012), poor countries would specialize more in low-income elastic goods as they move to autarky. To our knowledge, no study has established the relative importance of these forces for international specialization patterns in goods with different income elasticity.

38Since the term $\frac{1}{2} \int_A^B \beta_i dy_i$ in (31) does not have a closed form solution we numerically integrate it.
estimation yields a $\gamma^{\text{translog}} = 0.240$ which we then feed into (30) to compute loss of moving to autarky. These outcomes are reported in the second column. Since the estimated $\gamma$ hardly changes when we impose homothetic preferences and the non-homothetic component of aggregate welfare $W_{NH,i}^{\text{autarky}} / W_{NH,i}^{\text{trade}}$ is small, the differences between the translog or AIDS estimates are negligible. This suggests that non-homotheticities, in our context, do not fundamentally change the estimates of the aggregate gains from trade (but will have a strong impact on their distribution across consumers).\textsuperscript{39}

The main advantage of our approach, and the focus of this paper, lies in measuring how trade affects individuals across the expenditure distribution. To compute real income changes by income level, we must account for the restriction of non-negative expenditure shares at the individual level, which commonly binds under autarky.\textsuperscript{40} As discussed in Appendix A.1, we follow the approach in Feenstra (2010) and treat these goods as not consumed. The welfare and the expenditure shares of consumer $h$ under autarky can then be calculated using the consumer-$h$ specific choke prices that bring that consumer exactly to autarky, so that $s_{ii}^h = 1$ and $s_{in}^h = 0$ for $n \neq i$ in (19). Even though the actual prices under autarky are common to all consumers, the effective price changes used to measure welfare vary by consumer.\textsuperscript{41}

Following these steps, we find the effective change in the non-homothetic price index experienced by each consumer when the country moves from autarky to the observed level of trade,

$$\ln \left( \frac{b_{i,h}^{\text{trade}}}{b_{i,h}^{\text{autarky}}} \right) = \frac{1}{\gamma} \left[ (y_{i,h}^{\text{trade}} - y_{i,h}^{\text{autarky}}) \sigma_\beta^2 - (\bar{\beta}_{i,h}^{\text{trade}} - \beta_i) \right].$$

(40)

A positive value for $(\bar{\beta}_{i,h}^{\text{trade}} - \beta_i)$ in the right hand side means that, as the country opens up to trade, it shifts its expenditure basket towards higher income elastic goods. Assuming a $\gamma > 0$, this implies a reduction in the relative price of these goods as the country moves from autarky to trade.

Armed these relative price changes, we can determine the real income lost in a movement to autarky by each percentile of a country’s expenditure distribution using equation (28). We evaluate that equation at the shares defined in (A.1), letting $A$ and $B$ in those formulas correspond, respectively, to the actual scenario under trade and the counterfactual scenario under autarky. We assume, as we do throughout the empirical analysis, that the expenditure distribution in country $i$ is lognormal with variance $\sigma_i^2$. From now on, we index consumers by their percentile in the income distribution, so that $h \in (0,1)$.

Then, the gains from trade for a consumer at percentile $h$ of the income distribution of country

\textsuperscript{39}The aggregate gains from trade tend to be larger under translog compared to a CES Armington model, as reported in column 1 of Table 1 of Costinot and Rodriguez-Clare (2013), although the correlation is quite high (equal to 0.96).

\textsuperscript{40}For example, if (19) predicts that consumer $h$ has expenditure share $s_{in,h} = 0$ in a good with $\beta_n < 0$, then the non-negative constraint binds in good $n$ for every consumer richer than $h$.

\textsuperscript{41}The restriction that individual shares must be non-negative binds for some percentile-importer-exporter combinations at the estimated parameters. As discussed in Appendix A.3, in these cases we also compute the individual expenditure shares by assigning consumer-specific choke prices. For this reason, $y_{i,h}^{\text{trade}}$ and $\bar{\beta}_{i,h}^{\text{trade}}$ in (40) have an index $h$. This adjustment does not affect the aggregate predictions of the model, as the observed aggregate expenditure shares have a correlation of 0.99 with the aggregate expenditure shares generated by adding up the adjusted individual shares across all percentiles and countries.
\[ 1 - \frac{W_{i,h}^{autarky}}{W_{i,h}^{trade}} = 1 - \frac{W_{i,h}^{autarky}}{W_{i,h}^{trade}} \left( \frac{W_{i,h}^{form}}{W_{i,h}^{form}} \right) \sigma_i (1 - z_h). \] (41)

Figure 2 plots the gains from trade by percentile of the income distribution for all the countries in our data. To facilitate the comparison, we express them as difference from the gains from trade of the 50th percentile in each country, and report the gains from trade for the 50th percentile in column 3 of Table 2. The solid red line in the figure shows the average gains from trade for each percentile across the 40 countries in our sample. The typical U-shape relationship between the gains from trade and the relative position in the income distribution implies that poor and rich consumers within each country tend to reap larger benefits from trade compared to middle-income consumers.

### 4.3 Determinants of the Unequal Gains From Trade

The average U-shape relationship between gains from trade and relative income masks large heterogeneity across countries in the difference between the gains from trade of relatively poor and rich consumers. In 19 out of the 40 countries trade is unambiguously “pro-poor”, in the sense that gains from trade are larger for any consumers above the median than for any consumer below the median, while in one country, India, using that same metric, trade is unambiguously “pro-rich”.

What determines the strength in the overall pro-poor bias of trade? The answer naturally lies in the income elasticity of each country’s products vis-à-vis its natural trade partners. In countries that export relatively low income-elastic goods, such as India, or that trade considerably with the U.S., such as Mexico or Canada, the gains from trade are relatively less biased to poor consumers. In these countries, opening to trade increases the relative price of low-income elastic goods (which are exported), or decreases that of high-income elastic goods (which are imported).

To examine these patterns more systematically we consider the cross-country variation in the change of the non-homothetic price index from autarky to trade expressed in (40). The more positive this term is, the larger the increase in the relative price of high-income elastic goods as the country opens up, and the more strongly pro-poor the bias of the gains from trade.

As implied by (40), one of the determinants in the change in the non-homothetic price index is the income elasticity of each country’s exports, captured by \( \beta_i \). This relationship is plotted in Figure 3. The positive slope from the figure means that opening to trade is associated with a larger increase (or a less negative decrease) in the relative price of high income elastic goods goods in countries specialized in the production of these goods, a reflection of standard terms-of-trade

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\(^{42}\)Under the log-normal distribution, the expenditure level of a consumer at percentile \( h \) in country \( i \) is \( e^{x_i \sigma_i + \mu_i} \), where \( x_h \) denotes the value from a standard normal \( z \)-table at percentile \( h \), and \( \tilde{x}_i = e^{\mu_i} \). Applying this to 28 yields (41). The lognormal distribution leads to \( Pr[\tilde{x}_i < \tilde{x}_j] = 0.841 \), which implies that the representative agent corresponds to the 84.1th percentile in each country.

\(^{43}\)We report the effective price change for the individual that corresponds to the representative agent in each country. Similar cross-country patterns emerge for the effective price change of any specific percentile.
effects. Therefore, trade is relatively more favorable to poor consumers in countries that export high income elastic goods. Because, as implied in Figure 1, these countries tend to be richer, this implies that trade tends to be relatively more pro-poor in richer countries.

As is also implied by (40), another determinant of the non-homothetic price index is the income elasticity of the goods exported by each country’s trade partners, as captured by $\bar{\beta}_{i,\text{trade}}$. A higher value for $\bar{\beta}_{i,\text{trade}}$ means that country $i$ imports more from exporters of higher income elastic goods. This depends not only of importer $i$’s income level, but also on standard gravity forces that determine the prices of these goods. Figure 4 plots $\ln\left(\frac{b_{i,\text{trade}}}{b_{i,\text{autarky}}}\right)$ against distance to the U.S., an exporter of very high income elastic goods. Countries that are closer to the U.S., specifically Mexico and Canada, face lower relative prices for U.S. imports, and a reduction in the non-homothetic price index due to trade. In these countries, trade tends to favor the rich (as manifested in Figure 2, which shows that Mexico and Canada exhibit higher gains at top percentiles than at bottom percentiles).

Appendix Figure A.1 shows the average gains by percentile from Figure 2 when the service sector is excluded from the data (and all the parameters are re-estimated). Since trade in services is small, dropping that sector increases import shares and accentuates the differences in the gains from trade across percentiles.

Appendix Figure A.2 replicates Figure 2 using final consumption values reported in WIOD (and again all the parameters are re-estimated). The $\beta$ coefficients are slightly different from column 3 of Table 1, and these changes are intuitive. For example, China becomes a negative $\beta$ exporter when examining final consumption, which is consistent with large differences between China’s gross vs. value-added exports. We also find that Germany’s $\beta$ increases relative to the baseline setting. Nevertheless, the overall U-shaped gains from trade pattern is consistent with the baseline figure.

Overall, the results demonstrate the importance of both specialization and geography in shaping the unequalizing effects of trade. In countries that specialize in high-income elastic goods or that are far, in a gravity sense, from exporters of this type of goods, the changes in relative prices caused by opening up to trade tends to benefit poor consumers relatively more.

5 Multiple Sectors

So far we have characterized unequal gains from trade through the expenditure channel in a single-sector Armington model where goods are only differentiated by country of origin. Costinot and Rodriguez-Clare (2013) and Ossa (2012) show that sectoral heterogeneity is an important driver of the aggregate gains from trade, and Hallak (2010) argues that non-homotheticities across sectors are important. In this section, we extend the analysis to allow for multiple sectors, so that now each good is defined by a sector-origin dyad. This approach allows for differences in income elasticity across both sectors and source countries, as well as international specialization across sectors, to be drivers of the unequal gains from trade.

First, we describe the key expressions corresponding to the extended environment. The multi-
sector model produces a non-homothetic sectoral gravity equation which aggregates exactly to the gravity equation of the single-sector model, as well as formulas for the unequal gains from trade that nest our previous expressions. Then, we estimate the parameters from the multi-sector model and contrast the results with the single-sector estimates to isolate the effect non-homotheticities across sectors, and sectoral heterogeneity more generally, in shaping the results.

5.1 Multi-Sector Environment

Goods are indexed by country of origin, $i = 1, \ldots, N$ and sector $s = 1, \ldots, S$, so that in total there are $N \times S$ goods. Country $i$ has labor productivity $Z_i^s$ in sector $s$ and an exogenous distribution of effective units of labor as in the single-sector case. Assuming that every country has positive production in every sector, the wage rate per effective unit of labor in country $i$ is $p_i^s Z_i^s$ for all $s = 1, \ldots, S$, where $p_i^s = p_{ii}^s$ is the domestic price of sector-$s$ goods and $p_{in}^s$ is the price of goods from exporter $n$ to importer $i$ in sector $s$. Trade costs $\tau_{in}^s$ may vary by sector, so that $p_{in}^s = \tau_{in}^s p_n^s$.

Assuming Almost-Ideal Demand, the share of total expenditures in country $i$ devoted to goods from country $n$ in sector $s$ is

$$S_{in}^s = \alpha_{in}^s + \sum_{s' = 1}^S \sum_{n' = 1}^N \gamma_{nn'}^{ss'} \ln p_{in'}^{s'} + \beta_n^s y_i, \quad (42)$$

where, as in the single-sector model, $y_i = \ln \left( \bar{x}_i / a(p_i) \right) + \Sigma_i$ is the adjusted real income of the economy. The income elasticity $\beta_n^s$ is allowed to vary across both sectors and exporters. In turn, $\alpha_{in}^s$ may vary across exporters, sectors, and importers. These coefficients must satisfy $\sum_{n} \sum_{s} \alpha_{in}^s = 1$ for all $i = 1, \ldots, N$ and $\sum_{n} \sum_{s} \beta_n^s = 0$.

The coefficient $\gamma_{nn'}^{ss'}$ is the elasticity of the expenditure share in good $(n, s)$ with respect to the price of good $(n', s')$. We assume no cross-substitution between goods in different sectors ($\gamma_{nn'}^{ss'} = 0$ if $s \neq s'$). Within each sector $s$, and similarly to the single-sector case, we assume the same elasticity between goods from different sources within a sector ($\gamma_{nn'}^{ss}$ is the same for all $n' \neq n$ for each $s$), but this elasticity may vary across sectors:

$$\gamma_{nn'}^{ss'} = \begin{cases} \frac{\gamma^s}{N} & \text{if } s = s' \text{ and } n' = n, \\ -(1 - \frac{1}{N}) \gamma^s & \text{if } s = s' \text{ and } n' \neq n, \\ 0 & \text{if } s \neq s'. \end{cases} \quad (43)$$

This structure on the elasticities is convenient because it allows us to cast the demand structure as a two-tier system, across sectors in the upper tier and across source countries within each sector in the lower tier, and to relate it to existing homothetic multi-sector gravity models in the literature.\textsuperscript{44}

\textsuperscript{44}This nesting is a standard approach to the demand structure in multi-sector trade models. For example, see Broda and Weinstein (2006), Feenstra and Romalis (2014), or Costinot and Rodriguez-Clare (2013).
In the upper tier, the share of sector $s$ in the total expenditures of country $i$ is:

$$S_i^s = \sum_n S_{in}^s$$

$$= \alpha_i^s + \beta_i^s y_i,$$  \hspace{1cm} (44)

where $\alpha_i^s = \sum_n \alpha_{in}^s$ and

$$\beta^s = \sum_{n=1}^N \beta_n^s.$$

Equation (44) shows that the expenditure shares across sectors have an “extended Cobb-Douglas” form, which allows for non-homotheticities across sectors through $\beta_i^s$ on top of the fixed expenditure share $\alpha_i^s$. This expression is useful because, given the value for $y_i$ under autarky, it pins down the distribution of autarky expenditure shares across sectors, which are needed to compute the gains from trade by percentile of the income distribution. We refer to the $\beta_i^s$ in (44) as sectoral betas, and we explore how results change depending on whether non-homotheticities across sectors are allowed (in which case $\beta_i^s$ may be different from zero) or not (in which case all the $\beta_i^s = 0$).

Using (43), the expenditure share in goods from source country $n$ in sector $s$ adopts the symmetric-AIDS form,

$$S_{in}^s = \alpha_{in}^s - \gamma^s \ln \left( \frac{p_{in}^s}{p_i^s} \right) + \beta_n^s y_i,$$  \hspace{1cm} (45)

where $P_i^s \equiv \exp \left( \frac{1}{N} \sum_{n'=1}^N \ln p_{i'n'}^s \right)$. The corresponding expenditure share for consumer $h$ is

$$s_{in,h}^s = S_{in}^s + \beta_n^s \ln \left( \frac{x_h}{x_i} \right).$$  \hspace{1cm} (46)

### 5.2 Expressions for The Unequal Gains From Trade with Multiple Sectors

As in Section 3.2, we consider the distribution of welfare changes associated with a foreign trade shock to country $i$, choosing the wage in country $i$ as numeraire. Applying Proposition 1 to the multi-sector model, the aggregate expenditure effect in country $i$ is

$$\widehat{W}_i \equiv \sum_{s=1}^S \widehat{W}_{H,i}^s + \sum_{s=1}^S \widehat{W}_{NH,i}^s,$$  \hspace{1cm} (47)
where
\[
\hat{W}^s_{H,i} = \frac{1}{\gamma^s} \left( \sum_{n=1}^{N} S^s_{in} dS^s_{in} - dS^s_{ii} \right),
\]
(48)

\[
\hat{W}^s_{NH,i} = \frac{1}{\gamma^s} \sum_{n=1}^{N} S^s_{in} (\beta^s_i - \beta^s_n) dy_i.
\]
(49)

In turn, the change in the non-homothetic price index in country \(i\) is
\[
\hat{b}_i = \sum_{s=1}^{S} \hat{b}^s_i,
\]
(50)

where
\[
\hat{b}^s_i = \sum_{n=1}^{N} \frac{\beta^{s_n}}{\gamma^s} \left[ dS^s_{ii} - dS^s_{in} + (\beta^s_n - \beta^s_i) y_i \right].
\]
(51)

Finally, following similar steps to the single-sector case,\(^{46}\) the change in adjusted real income is
\[
dy_i = \frac{\sum_{s=1}^{S} \hat{W}^s_{H,i} + y_i \sum_{s=1}^{S} \sum_{n=1}^{N} \frac{\beta^{s_n}}{\gamma^s} (dS^s_{ii} - dS^s_{in})}{1 - \sum_{s=1}^{S} \sum_{n=1}^{N} \frac{1}{\gamma^s} (S^s_{in} - \beta^s_n y_i) (\beta^s_i - \beta^s_n)}.\]
(52)

These expressions collapse to (23) to (27) in the single-sector case (\(S = 1\)). They characterize welfare changes to a first-order approximation. Integrating them between two sets of aggregate expenditure shares gives the exact changes in real income by level of income caused by a discrete change in trade costs or foreign productivities.

5.3 Estimation of the Sectoral Non-Homothetic Gravity Equation

The model delivers a sector-level non-homothetic gravity equation. Letting \(X^s_{in}\) denote exports from \(n\) to \(i\) in sector \(s\) and \(Y^s_n = \sum_i X^s_{in}\) be the total sales of country \(n\) in sector \(s\) we reach
\[
\frac{X^s_{in}}{Y^s_i} = \frac{Y^s_n}{Y^s_W} + A^s_{in} - \gamma^s T^s_{in} + \beta^s_n \Omega_i,
\]
(53)

where \(Y_i\) and \(Y_W\) are respectively country-\(i\) and world income, and where \(T^s_{in}\) is defined analogously to \(T_{in}\) in (36),
\[
T^s_{in} = \ln \left( \frac{\tau^s_{in}}{\tau^s_i} \right) - \sum_{n'=1}^{N} \left( \frac{Y_{n'}}{Y^s_W} \right) \ln \left( \frac{\tau^s_{n'\infty}}{\tau^s_{n'}} \right),
\]

\(^{46}\)See footnote 22.
for $\tau^s_i = \exp \left( \sum_{n=1}^N \ln(\tau^s_{in}) / N \right)$. Compared to the single-sector gravity equation (35), the sector-level gravity equation (53) includes the extra term

$$A^s_{in} = \alpha^s_{in} - \sum_{n'=1}^N \left( \frac{Y_{n'}}{Y_W} \right) \alpha^s_{n'n},$$

(54)

which captures cross-country differences in tastes across sectors or exporters. This term vanishes if $\alpha^s_{in}$ is constant across importers $i$.

We estimate the sectoral gravity equation using the sector-level trade flows in the WIOD. To reach a sectoral gravity equation that is amenable to estimation, it is useful decompose $\alpha^s_{in}$ into an exporter effect $\alpha_n$ (which has the same interpretation as in the single-sector case), a sector-specific effect $\alpha^s$ (which captures that some sectors command higher shares in every country) and an imposing country-specific taste for each sector $\varepsilon^s_i$,

$$\alpha^s_{in} = \alpha_n (\alpha^s + \varepsilon^s_i).$$

(55)

We assume, as in the single-sector case, that $\sum_{n=1}^N \alpha_n = 1$. Under this assumption, the sectoral expenditure shares from the upper-tier equation (44) become

$$S^s_i = \alpha^s + \bar{\beta}^s y_i + \varepsilon^s_i.$$ 

(56)

This equation is an Engel curve. It is a projection of sector shares on sector dummies and a measure of importer income interacted with sector dummies; the interacted coefficients have the structural interpretation as the sectoral betas $\bar{\beta}^s$.\footnote{The term $\varepsilon^s_i$ captures cross-country differences in expenditure shares across sectors that are not explained by differences in income or inequality levels. As in Costinot et al. (2012) or Caliendo and Parro (2012), among other papers that estimate multi-sector trade models with Cobb-Douglas demand across sectors, this flexibility is needed for the model to match sectoral shares by importer. This approach to measuring taste differences is also in the spirit of Atkin (2013), who attributes regional differences in tastes to variation in demand that is not captured by observables.}

Similarly to the single-sector case we assume that trade costs are $\tau^s_{in} = d^s_{in} \tilde{\varepsilon}^s_{in}$. Using (56) together with (53) and (54), the sectoral gravity equation that we take to the data is

$$\frac{X^s_{in}}{Y_i} - \frac{Y^s_{n}}{Y_W} = \alpha_n (S^s_i - S^s_W) - (\gamma^s \rho) D_{in} + \tilde{\beta}^s_n \Omega_i + \varepsilon^s_{in},$$

(57)

where $S^s_W = \sum_n \sum_i X^s_{in}/Y_W$ is the share of sector $s$ in world expenditures, $D_{in}$ is defined in (39), and

$$\tilde{\beta}^s_n = \beta^s_n - \alpha_n \bar{\beta}^s.$$ 

(58)

We note that the gravity equation identifies $\tilde{\beta}^s_n$, the difference between $\beta^s_n$ and the level $\alpha_n \bar{\beta}^s$. However, to compute welfare changes we need the underlying structural parameters, $\beta^s_n$. We identify these by first estimating $\tilde{\beta}^s$ from the upper tier regression in (56) and then using these estimates together with those for $\alpha_n$ from (57) to infer $\beta^s_n$ from (58).
We also highlight that the sectoral gravity equation aggregates to the single-sector case. As discussed below, this facilitates identifying the role of sectoral heterogeneity in shaping the results. Summing (57) across sectors gives the total expenditure share dedicated to goods from \( n \) in the importing country \( i \),

\[
\frac{X_{in}}{Y_i} - \frac{Y_n}{Y_W} = -\left( \sum_s \gamma_s \right) \rho D_{in} + \left( \sum_s \beta_{n}^s \right) \Omega_i + \sum_s \epsilon_{in}^s.
\]

This coincides exactly with the single-sector gravity equation (38) if we define \( \gamma = \sum_s \gamma_s \), \( \beta_n = \sum_s \beta_{n}^s \) and \( \epsilon_{in} = \sum_s \epsilon_{in}^s \). Thus, summing our estimates of \( \{\gamma_s\} \) from (57) matches the parameter \( \gamma \) estimated from (38) in Section 4. Likewise, the sum of the sector-specific income elasticities by exporter \( \sum_s \beta_{n}^s \) estimated from (57) matches the income elasticity of exports from \( n \) estimated from (38), \( \beta_n \).

The gravity results are reported in Table 3. As in column 3 of Table 1, the sectoral gravity equation also includes language and border terms, but these coefficients are suppressed. Columns 1A and 1B of Table 3 report the 17 sector-specific distance coefficients multiplied by the elasticity of trade costs with respect to distance, \( \rho \gamma^s \) (where \( \rho = 0.177 \) as before). As mentioned in the previous paragraph, these distance coefficients sum exactly to the \( \rho \gamma \) reported in the single-sector case in column 3 of Table 1. (Likewise, the unreported sector-specific language and border coefficients sum exactly to the corresponding coefficients in the single-sector estimation.)

Columns 2A and 2B report the sectoral betas \( \bar{\beta}^s = \sum_n \beta_{n}^s \). These coefficients are obtained from the Engel curve in (56) which projects importers’ sectoral expenditure shares on the importer’s income per capita. To get a visual sense of these sectoral betas, in Figure 5 we plot the relationship for three broad sector groups (Food, Manufacturing and Services). Services is a strongly high-income elastic sector. However, there is large heterogeneity across exporters (i.e., across \( \{\beta_{n}^s\}_{n=1}^{N} \) given \( s \)). Some countries export negative-beta Services, while others export positive-beta Food or Manufacturing. This heterogeneity in \( \beta_{n}^s \) across exporters within sectors is represented in Figure 6, which plots the betas of each exporter aggregated within Food, Manufacturing and Services against exporter per capita income. (Again note that the sum of the betas of each exporter across sectors matches exactly the coefficients reported in column 3 of Table 1.) The pattern from the single-sector model in Figure 1 still holds within sectors, as richer countries tend to export goods with higher income elasticities within broad sectors.

\[\text{In total, there are 680 (=17 sectors times 40 countries) } \beta_{n}^s \text{ and 17 distance, language and border coefficients. We suppress the } \beta_{n}^s \text{ for readability purposes and are available upon request. For each exporter the sum of these coefficients across sectors, } \sum_s \beta_{n}^s, \text{ equals the exporter-specific income elasticities } \beta_n \text{ reported in Table 1.}\]

\[\text{Food includes “Agriculture” and “Food, Beverages and Tobacco”, services is a single sector, and manufacturing includes the remaining 14 sectors listed in Table 1. Though the import share within services is often low, no country is completely autarkic in services and 2.7% of world service expenditure is traded internationally. This provides the variation in expenditure shares across exporters in services needed to identify the } \beta_{n}^s \text{’s for services.}\]
5.4 Measuring The Unequal Gains From Trade with Multiple Sectors

With the parameters in hand, we can now compute the gains from trade for each percentile in each country. As before, the gains from trade at percentile $h$ are given by equation (41). However, the measurement of $W^\text{autarky}_{i,h}/W^\text{trade}_{i,h}$ and $b^\text{autarky}_{i,h}/b^\text{trade}_{i,h}$ is now done using (47)-(52). We numerically integrate these expressions between observed trade shares and counterfactual autarky shares which are calculated using the procedure described in Appendix (A.2).\footnote{In the single-sector model, the autarky shares trivially equal 1 for the (only) domestically produced good and 0 for all the imported goods. In the multi-sector model, there is a distribution of autarky shares across sectors. Due to non-homotheticities across sectors ($\beta^s \neq 0$ in (56)), this distribution depends on the adjusted real income under autarky, which is also computed in the numerical integration of (47)-(52).}

Figure 7 reports the unequal gains from trade with multiple sectors. As in Figure 2 from the single-sector analysis, it shows the gains from trade for each percentile in each country as difference from the median percentile of each country. Table 4 reports the gains from trade at the 10th, median, and 90th percentile, as well as for the representative consumer of each country.

There are two important differences between the results under the single- and under the multi-sector framework. First, the relative effects across percentiles are considerably larger. In the single-sector case from Figure 2, the gains from trade (relative to the median) lie within the -5 percent to 10 percent band across most countries and percentiles, while in the multi-sector case the range increases to -40 percent to 60 percent.\footnote{Osca (2012) and Costinot and Rodriguez-Clare (2013) show that allowing for sectoral heterogeneity leads to larger measurement of the aggregate gains from trade in CES environments. In our context, sectoral heterogeneity also leads to larger measurement of the aggregate gains from trade, but we concentrate on studying the asymmetric effects on rich and poor consumers which are specific to our framework.}

Second, poor consumers are now predicted to gain more from trade than rich consumers in every country, in the sense that every consumer below the median income gains more from trade than every consumer above the median (recall that in the single-sector model, this occurred in 19 out of 40 countries). Across the countries in our sample, the gains from trade are 70 percent for the 10th percentile of the income distribution and 27 percent for the 90th percentile.

Why does the multi-sector estimation imply such pro-poor bias in the gains from trade? The multi-sector model allows for two key additional margins: heterogeneity in the elasticity of substitution \{\gamma_s\} and in the sectoral betas \{\beta^s\}.$^{52}$ To gauge the importance of each of these margins in shaping the unequal gains, we re-estimate the gravity equation in (57) allowing for heterogeneity in $\gamma_s$ but imposing $\beta^s = 0$, which shuts down the sectoral non-homotheticities. Figure 8 shows the average gains from trade by percentile across all countries from the single-sector model, the multi-sector model with homothetic sectors, and the more flexible multi-sector model with non-homothetic sectors. We find that non-homotheticities across sectors are crucial for the strongly pro-poor bias of trade, as only allowing for heterogeneity in $\gamma_s$ across sectors (i.e., comparing the single-sector with the multi-sector homothetic estimation) slightly bias the gains from trade towards rich consumers.

\footnote{By construction, if we restricted the \{\gamma_s\} and \{\beta^s_n\} to be constant across sectors in the multi-sector estimation, we would recover the same unequal gains from trade as in the single-sector estimation, and Figure 7 would look identical to Figure 2.}
There are two key reasons why allowing for non-homotheticities across sectors so starkly affects the measurement of the bias of trade. First, low-income consumers are predicted to spend relatively more on sectors that are more traded whereas high-income consumers spend relatively more on services, which is the least internationally traded sector (recall the Engel curves plotted in Figure 5). Second, low-income consumers concentrate spending on sectors with a lower substitution parameter $\gamma_s$. To visualize this, we construct, for each percentile in each country, an expenditure-share weighted average of the sectoral gammas. Then, we average across all countries and report the results in Figure 9. As we see, higher percentiles concentrate spending in sectors where exporters sell more substitutable goods. So, even though allowing for heterogeneity in $\{\gamma_s\}$ does not by itself affect much the predictions for the unequal gains from trade relative to the single-sector model (as already established by the comparison between the single-sector and the multi-sector homothetic estimations of Figure 8), the interaction between heterogeneity in $\gamma_s$ and $\beta^s$ does.\

5.5 Ex Post Measurement, 1996-2006

We illustrate an alternative use of our methodology by performing an ex post welfare evaluation using actual changes in aggregate expenditure shares over time from the data rather than simulating a counterfactual movement to autarky. In this case, we interpret the results as indicating the distribution of welfare changes across consumers that would be associated with hypothetical foreign-trade shocks to each country, such as changes in trade costs, that cause changes in expenditure shares for each country of equal magnitude as those that are observed over time. Since domestic shocks may have also caused observed changes in expenditure shares, we see this type of experiment as providing an upper bound for the distributional impact of trade over time through the expenditure channel.

We use changes trade shares between 1996 and 2006 and report the distributions of welfare changes in Figure 10. The figure shows welfare changes biased to the poor in all countries. To understand the forces driving these results, we again note that they are generated using exclusively data on aggregate expenditure shares and model parameters. Under the assumption that these changes in aggregate expenditure shares only result from foreign shocks, we use them to infer changes in the relative price of high income elastic goods using the equations that result from integrating (50) and (51) between the initial and the final year,

$$
\Delta^{96-06} \ln b_i = \sum_{s=1}^{S} \sum_{n=1}^{N} \frac{\beta_n^s}{\gamma^s} \left[ \Delta^{96-06} S_{i i}^s - \Delta^{96-06} S_{i n}^s \right] + (\beta_n^s - \beta_i^s) \Delta^{96-06} y_i,
$$

More precisely, the figure reports $\gamma_{i,v}^h = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{S} s_{i,h}^{j,v} \gamma_j^s$, where $s_{i,h}^{j,v}$ is the expenditure share of percentile $h$ in country $i$ on goods in sector $j$ defined in equation (A.9). By construction, both the single-sector and the multi-sector homothetic models predict that $\gamma_{i,v}^h$ is flat across percentiles.

The positive relationship between percentiles and average $\gamma$ from Figure 9 is partly explained by the high elasticity of the service sector. If we exclude services from the sample and re-estimate all the parameters, that relationship is still positive and the gains from trade remain biased to low-income consumers. These results are available upon request.

We use averages of values between 1994-1996 for the initial period and average values between 2005-2007 for the final period.
where $\Delta^{96-06}$ denotes the difference between 1996 and 2006, and where $\Delta^{96-06} y_i$ is computed from (52).\footnote{To construct the welfare change for an individual at percentile $h$ in country $i$ we feed these relative price changes into the expression for the change in real income in (41), which applies to both the single-sector and multi-sector model.}

The pro-poor effects that we find must be driven by a reduction in the relative price of high-income elastic goods ($\Delta^{96-06} \ln b_i > 0$), which must be driven by increases in import shares in negative income-elastic sectors (sectors with $\beta^s < 0$ in Table 3) or from negative income-elastic exporters (imports from countries where $\beta_n < 0$ in Table 1\footnote{Recall that $\beta_n$ from the single-sector estimation equals $\sum_s \beta^s_n$ in the multi-sector estimation that we use now.}). To examine the importance of country and sector effects, we project the difference between the change in welfare of the the 90th and 10th percentiles on the changes in import shares from negative-beta sectors or negative-beta countries. The results in Table 5 indicate that increases in import shares from negative-income elastic sectors are correlated with more pro-poor effects. This suggests that the pro-poor welfare changes between 1996 and 2006 are driven primarily by importing countries increasing their import shares of low-income elastic sectors. Of course, this relationship is not a causal statement but simply reinforces the relative importance of sector heterogeneity in understanding the unequal gains from trade.

6 Conclusion

This paper develops a methodology to measure the distribution of welfare changes across individuals through the expenditure channel for many countries over time. The broad applicability of the approach results from using a demand system that is a first-order approximation to any demand system—the Almost-Ideal Demand System—which allows us to characterize the exact changes in welfare experienced by individuals in each point of the expenditure distribution as a function aggregate statistics, such as expenditure shares across goods, and model parameters that can be estimated from bilateral trade data.

We estimate a non-homothetic gravity equation generated by the model to obtain the key parameters required by the approach, and identify the effect of trade on the distribution of welfare changes through counterfactual changes in trade costs. Although all consumer lose when moving to autarky, the estimated parameters suggest stark differences in how trade affects individuals along the income distribution in different countries. A stronger specialization in high-income elastic goods relative to a country’s trade partners biases the gains from trade towards low-income consumers. Using a single-sector model we find the gains from trade to be typically U-shaped with individual income, while the multi-sector analysis leads to the conclusion that the gains from trade are typically biased towards the poor, who concentrate expenditures in sectors that are more traded. Overall, the results demonstrate the importance of specialization across goods with different elasticity, geography and non-homotheticities across sectors in shaping the unequalizing effects of trade through the expenditure channel.
Additional margins, such as product differentiation as in Krugman (1980), input-output linkages as in Caliendo and Parro (2012), or different worker types as in Heckscher-Ohlin, could be naturally incorporated to the model and empirical analysis along the lines of Costinot and Rodriguez-Clare (2013). However, a full analysis of firm heterogeneity as in Melitz (2003), or competitive effects as in Feenstra and Weinstein (2010) or Arkolakis et al. (2012), would need to confront, among other things, that heterogeneous firms plausibly sell goods that are valued distinctively by rich or poor consumers. This would entail a different modeling and empirical strategy, as well considerably different data requirements. A unified approach that measures unequal gains from trade through the expenditure channel incorporating these margins would be a promising area for future work.

Finally, while our goal in this paper is to demonstrate the potential importance of accounting for demand heterogeneity across consumers in understanding the distributional effects of trade, we believe an another promising avenue for future works lies in developing a unified approach that measures the impact of trade on inequality through both expenditure and income channels.

References


### A Zeros in the Individual Shares

#### A.1 Autarky in the Single-Sector Model

In Section 4.2, we simulate a movement from the observed trade shares in the economy, \( \{S^{\text{trade}}_n\} \) to the autarky shares \( s^{\text{autarky}}_i = 1 \) and \( s^{\text{autarky}}_n = 0 \) for \( n \neq i \). The restriction that the individual expenditure shares \( s_{in,h} \) defined in (19) are greater than zero binds in some goods for all consumers. We follow the approach in Feenstra (2010) set the price of these goods equal to the reservation value at which the expenditure share predicted by (19) equals zero. Feeding these choke prices into the demand system delivers a set \( p^{\text{autarky}}_{i,h} \) of consumer-specific autarky prices which ensures that \( s^{\text{autarky}}_{in,h} = 1 \) and \( s^{\text{autarky}}_{in,n} = 0 \) for all \( n \neq i \) for all goods and all consumers. Even though under autarky all consumer face the same vector of actual prices, the welfare and the expenditure shares of consumer \( h \) under autarky can be calculated as if that consumer faced the consumer-\( h \) effective prices \( p^{\text{autarky}}_{i,h} \). To measure the welfare change of consumer \( h \) from the actual trade scenario to the counterfactual consumer-\( h \) specific prices \( p^{\text{autarky}}_{i,h} \), it suffices to set the counterfactual aggregate shares under autarky to

\[
S^{\text{autarky}}_{in,h} = s^{\text{autarky}}_{in,h} - \beta_n \ln \left( \frac{x_h}{x_i} \right),
\]

where \( s^{\text{autarky}}_{ii,h} = 1 \) and \( s^{\text{autarky}}_{in,n} = 0 \) for all \( n \neq i \). By construction, the aggregate shares \( S^{\text{autarky}}_{in,h} \) correspond to the set of effective prices \( p^{\text{autarky}}_{i,h} \) at which consumer \( h \) chooses the autarky distribution of expenditure shares.\(^{58}\) Applying the aggregate shares defined in (A.1) in place of the share \( \{S^{B}_n\} \) in the equations (28) to (33) leads to the consumer-\( h \) effective change in the non-homothetic price index shown in (40), where \( y^{\text{autarky}}_{i,h} \) results from evaluating (33) at the shares defined in (A.1) in place of the shares \( \{S^{B}_n\} \).

#### A.2 Autarky in the Multi-Sector Model

The steps from the single-sector model in A.1 generalize the multi-sector model. From (44), the expenditure share of consumer \( h \) in sector \( s \) is

\[
s^s_{i,h} = \sum_n s^s_{in,h} = \alpha^s_i + \beta^s \left( y_i + \ln \left( \frac{x_h}{x_i} \right) \right),
\]

To compute the real income loss of consumer \( h \) from moving to autarky, we find a set of sectoral aggregate expenditure shares \( \{S^{\text{autarky}}_{in,s,h}\} \) corresponding to the consumer-\( h \) effective autarky prices. At these autarky prices, the individual expenditure shares must satisfy

\(^{58}\)Note that the \( s^{\text{autarky}}_{in,h} \) are not restricted to lie in \([0, 1]\). They formally correspond to the shares predicted by the equation (18) when individual \( h \) chooses the expenditure shares \( \{s^{\text{autarky}}_{in,h}\} \).
\[ S_{in,h}^{s,\text{autarky}} = \begin{cases} 0 & \text{if } n \neq i \\ s_{i,h}^{s,\text{autarky}} & \text{if } n = i \end{cases} \quad (A.3) \]

Combining (46) in the main text with (A.2) and (A.3) leads to

\[ S_{in,h}^{s,\text{autarky}} = \begin{cases} -\beta_{in}^{s} \ln \left( \frac{x_{i}}{x_{n}} \right) & \text{if } n \neq i \\ \alpha_{i,h}^{s} + \beta_{in}^{s} y_{ih}^{\text{autarky}} - \left( \beta_{i} - \beta_{n}^{s} \right) \ln \left( \frac{x_{n}}{x_{i}} \right) & \text{if } n = i \end{cases} \quad (A.4) \]

for \( s = 1, \ldots, S \). A change in trade costs that moves the representative consumer to the shares \( \{ S_{in,h}^{s,\text{autarky}} \} \) and the adjusted real income \( y_{ih}^{\text{autarky}} \) brings consumer \( h \) to autarky, causing to this consumer the same welfare change as increasing trade costs to infinity. We measure welfare changes integrating equations (48) to (51) from the aggregate trade shares \( \{ S_{in,h}^{s,\text{trade}} \} \) and adjusted real income \( y_{ih}^{\text{trade}} \) to the autarky shares \( \{ S_{in,h}^{s,\text{autarky}} \} \) and real income \( y_{ih}^{\text{autarky}} \). To compute \( y_{ih}^{\text{autarky}} \) we integrate \( dy_{i} \) in (52) between observed and autarky aggregate shares,

\[ y_{ih}^{\text{autarky}} = \int_{S_{i,1}^{\text{autarky}}}^{S_{i,N}^{\text{autarky}}} \int_{S_{1,1}^{\text{autarky}}}^{S_{1,N}^{\text{autarky}}} \cdots \int_{S_{i,1}^{\text{autarky}}}^{S_{i,N}^{\text{autarky}}} \int_{S_{1,1}^{\text{autarky}}}^{S_{1,N}^{\text{autarky}}} \cdots \int_{S_{N_{i,h},1}^{\text{autarky}}}^{S_{N_{i,h},N}^{\text{autarky}}} dy_{i} \left( \{ S_{in}^{s} \}_{i,n} \right), \quad (A.5) \]

with initial value \( y_{i} \left( \{ S_{in}^{s,\text{trade}} \} \right) = y_{ih}^{\text{trade}} \), where \( dy_{i} \left( \{ S_{in}^{s} \}_{i,n} \right) \) is shown in (52). This integral can be solved numerically or, following steps similar to the single-sector model described in footnote (25), we can reach a partial analytic solution for \( y_{ih}^{\text{autarky}} \) (available upon request). Note that the endpoints \( S_{in,h}^{s,\text{autarky}} \) are function of \( y_{ih}^{\text{autarky}} \) through (A.4). For each of the 100 percentiles of each importer \( i \) we numerically find the value of \( y_{ih}^{\text{autarky}} \). We iterate on its value starting from the initial condition \( y_{ih}^{\text{trade}} \), then using (A.4) to compute the shares \( S_{in,h}^{s,\text{autarky}} \), then generating a new value using (A.5) that is used as guess for the following iteration.

### A.3 Initial Scenario of the Counterfactuals

At the estimated parameters there are percentiles \( h \) in importer \( i \) for which the restriction to non-negative shares binds from some exporters \( n \). For these percentile-importer-exporter combinations we follow Feenstra (2010) and choke prices \( p_{in,h}^\text{adj} \). Let \( I_{i,h} \) be the set and \( N_{i,h} \) be the number of source countries for which the constraint on the expenditure share of percentile \( h \) located in country \( i \) binds. Then, the choke prices must be such that, for all \( n \in I_{i,h} \), the adjusted shares satisfy \( s_{in,h}^{\text{adj}} = 0 \). Equation (19) then implies

\[ 0 = \alpha_{n} - \gamma \ln p_{in,h}^{\text{adj}} + \frac{\gamma}{N} \left( \sum_{n' \notin I_{i,h}} \ln (p_{in'}) + \sum_{n' \in I_{i,h}} \ln (p_{in',h}) \right) + \beta_{n} \left( \ln y_{i} + \ln \left( \frac{x_{h}}{x_{i}} \right) \right). \quad (A.6) \]

Adding up the last equation across \( n \in I_{i,h} \) gives

\[ \sum_{n \in I_{i,h}} \gamma \ln p_{in,h}^{\text{adj}} = \frac{N}{N - N_{i,h}} \sum_{n' \in I_{i,h}} \left( \alpha_{n'} + \beta_{n'} y_{i} + \beta_{n'} \ln \left( \frac{x_{h}}{x_{i}} \right) \right) + \frac{N_{i,h}}{N} \sum_{n' \notin I_{i,h}} \gamma \ln (p_{in'}). \]
and using (18) we reach
\[
\frac{\gamma}{N} \sum_{n' \in I_{i,h}} \left( \ln p_{in',h}^{\text{adj}} - \ln p_{in'} \right) = \frac{1}{N - N_{i,h}} \left( \sum_{n' \in I_{i,h}} S_{in'} + \beta_n \ln \left( \frac{x_h}{\bar{x}_i} \right) \right).
\]

Feeding in these adjusted prices into (19) we compute the adjusted shares \( s_{in,h}^{\text{adj}} \) for goods \( n \notin I_{i,h} \). Using (18) we reach
\[
s_{in,h}^{\text{adj}} - S_{in} = \frac{\gamma}{N} \sum_{n' \in I_{i,h}} \left( \ln p_{in',h}^{\text{adj}} - \ln p_{in'} \right) + \beta_n \ln \left( \frac{x_h}{\bar{x}_i} \right),
\]
which as established in Feenstra (2010), defines an adjusted AIDS for these goods. Combining the last two equations we can write
\[
s_{in,h}^{\text{adj}} = \left( S_{in} + \frac{1}{N - N_{i,h}} \sum_{n' \in I_{i,h}} S_{in'} \right) + \left( \beta_n + \frac{1}{N - N_{i,h}} \sum_{n' \in I_{i,h}} \beta_n' \right) \ln \left( \frac{x_h}{\bar{x}_i} \right).
\]
This last equation can also be written as
\[
s_{in,h}^{\text{adj}} = S_{in} + \beta_n \ln \left( \frac{x_h}{\bar{x}_i} \right) + \frac{1}{N - N_{i,h}} \sum_{n' \in I_{i,h}} s_{in'}, \tag{A.7}
\]
where the \( s_{in} \) in the last line refers to shares where equation (19) gives a negative number. Equation (A.8) implies that assigning choke prices to a subset of goods \( I_{i,h} \) is equivalent to a uniform redistribution of the shares resulting from the demand system in (19), \( \sum_{n' \in I_{i,h}} s_{in'} \), to the remaining \( N - N_{i,h} \) spending categories.

We follow similar steps in the multi-sector model. We set choke prices \( p_{in,h}^{s,\text{adj}} \) to adjust the expenditure shares by percentile-importer-exporter-sector in which (46) gives a negative number to \( s_{in,h}^{s,\text{adj}} = 0 \). Letting \( N_{i,h}^s \) be the number of exporters in sector \( s \) for which \( s_{in,h}^{s,\text{adj}} = 0 \) for percentile \( h \) in country \( i \), and letting \( I_{i,h}^s \) be the set of such goods, for all \( n \in I_{i,h}^s \), we have
\[
s_{in,h}^{s,\text{adj}} = \left( \frac{1}{N - N_{i,h}^s} \sum_{n' \in I_{i,h}^s} S_{in'}^s + S_{in}^s \right) + \left( \frac{1}{N - N_{i,h}^s} \sum_{n' \in I_{i,h}^s} \beta_n' + \beta_n^s \right) \ln \left( \frac{x_h}{\bar{x}_i} \right). \tag{A.9}
\]
We apply these steps to the percentile-specific expenditure shares in the initial trade scenario before running each of our counterfactuals (single-sector analysis of Section 4.2, multi-sector analysis of Section 5.4, and ex post analysis of Section 5.5). Specifically, we proceed as follows: i) for all \( n \in I_{i,h}^s \) we set \( s_{in,h}^{s,\text{adj}} = 0 \); ii) for all \( n \notin I_{i,h}^s \) we set \( s_{in,h}^{s,\text{adj}} \) using (A.7); and iii) for all \( \left\{ s_{in,h}^{s,\text{adj}} \right\}_{i,n,s,h} \) we follow the same steps as in Appendix (A.2) to express the adjusted percentile-specific prices as function of aggregate shares: \( S_{in,h}^{s,\text{adj}} = s_{in,h}^{s,\text{adj}} - \beta_n^s \ln \left( \frac{x_h}{\bar{x}_i} \right) \).

We verify that this adjustment to the individual shares does not affect the aggregate predictions of the model by checking that, at the estimated parameters, the observed aggregate expenditure shares used to estimate the representative-agent model (constructed under the assumption that the constraint to non-negative shares does not bind) almost exactly match the numerical aggregation of the adjusted shares across all percentiles (some of which have a binding non-negative constraint in some shares). More precisely, the aggregate shares \( \left\{ \hat{S}_{in} \right\}_{i,n,s} \) generated by adding up the adjusted percentile-specific shares \( \left\{ s_{in,h}^{s,\text{adj}} \right\}_{i,n,s,h} \)
in each importer-exporter-sector ($\hat{S}_{in} = \sum_h \left( \frac{x_h}{\sum_{n',x_{i,n'}}} \right) s_{i,n,h}^{s,adj}$) have a correlation of 0.99 with the observed shares $\{S_{in}\}_{i,n,s}$ (in both single- and multi-sector models).
Tables and Figures

Table 1: Gravity Equation Estimates

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<td>$\beta$-LTU</td>
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<td>(0.012)</td>
<td>(0.014)</td>
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<td>(0.012)</td>
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<td>0.005</td>
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<td>(0.011)</td>
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<td>-0.030 **</td>
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<td>(0.007)</td>
<td>(0.011)</td>
<td>(0.008)</td>
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<tr>
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<td>$\beta$-CYP</td>
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<td>(0.007)</td>
<td>(0.005)</td>
<td>(0.006)</td>
<td>(0.012)</td>
</tr>
</tbody>
</table>

Notes: Table reports the estimates of the single-sector gravity equation. The results are split into two columns. Columns 1A and 1B report the baseline specification run on aggregate bilateral trade shares. Columns 2A and 2B use $\Omega$ constructed from GDP per capita (see text for details). Columns 3A and 3B add language and border terms. We assume that $\rho=0.177$. The implied $\gamma=\text{coefficient on D}/\rho$ is noted at the bottom of the table. Standard errors are clustered by importer. Significance * .10; ** .05; *** .01.
<table>
<thead>
<tr>
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<td>1.4%</td>
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</tr>
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<td>12.5%</td>
<td>13.5%</td>
<td>ITA</td>
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<td>2.5%</td>
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<td>16.6%</td>
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<td>0.5%</td>
<td>1.6%</td>
</tr>
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<td>14.7%</td>
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<td>0.4%</td>
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<td>24.2%</td>
</tr>
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<td>6.2%</td>
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</tr>
<tr>
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<td>PRT</td>
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<td>7.8%</td>
</tr>
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<td>8.0%</td>
<td>9.2%</td>
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<td>1.9%</td>
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<td>3.0%</td>
<td>3.1%</td>
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<td>16.4%</td>
<td>17.3%</td>
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<td>5.3%</td>
<td>7.5%</td>
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<td>8.1%</td>
<td>9.1%</td>
</tr>
<tr>
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<td>20.4%</td>
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<td>2.1%</td>
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<td>1.2%</td>
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<td>9.5%</td>
<td>11.5%</td>
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<tr>
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<td>0.8%</td>
<td>0.3%</td>
<td>USA</td>
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<td>5.9%</td>
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<tr>
<td><strong>Average</strong></td>
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<td>10.2%</td>
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<td></td>
</tr>
</tbody>
</table>

Notes: Table reports gains from trade. The first column uses the parameters of the AIDS from column 3 of Table 1. The second column computes welfare changes using a translog demand system; the parameters are obtained from re-running the gravity equation imposing $\beta=0$ (we obtain $\gamma=0.240$). The third column reports gains from trade for the median individual.
### Table 3: Sectoral Gravity Estimates

<table>
<thead>
<tr>
<th>Variables</th>
<th>Sector (ρ*γ) Coefficients</th>
<th>Sector β's from Engel Curve Regression</th>
<th>Sector (ρ*γ) Coefficients</th>
<th>Sector β’s from Engel Curve Regression</th>
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<td>(1A)</td>
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<td>(1B)</td>
<td>(2B)</td>
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<tr>
<td><strong>D_i X</strong></td>
<td>Ω_i X</td>
<td>-B_i X</td>
<td>Ω_i X</td>
<td>-B_i X</td>
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<tr>
<td>Agriculture</td>
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<td>-0.0288 ***</td>
<td>0.0005 ***</td>
<td>-0.0017 *</td>
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<td>(0.000)</td>
<td>(0.003)</td>
<td>(0.000)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Mining</td>
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<td>-0.0105 ***</td>
<td>0.0005 ***</td>
<td>-0.0033 **</td>
</tr>
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<td>(0.000)</td>
<td>(0.003)</td>
<td>(0.000)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Food, Beverages and Tobacco</td>
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<td>-0.0171 ***</td>
<td>0.0018 ***</td>
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<td>(0.000)</td>
<td>(0.004)</td>
<td>(0.000)</td>
<td>(0.005)</td>
</tr>
<tr>
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<td>0.0009 ***</td>
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<td>(0.000)</td>
<td>(0.002)</td>
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<tr>
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<td>0.0015 ***</td>
<td>-0.0013</td>
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<td>(0.000)</td>
<td>(0.000)</td>
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<td>0.0011 ***</td>
<td>-0.0046 *</td>
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<td>(0.000)</td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Printing and Publishing</td>
<td>0.0007 ***</td>
<td>0.0016</td>
<td>0.0003 ***</td>
<td>-0.0007</td>
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<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.001)</td>
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<tr>
<td>Coke, Refined Petroleum and Nuclear Fuel</td>
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<td>-0.0065 **</td>
<td>0.0290 ***</td>
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<td>(0.004)</td>
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<td>Chemicals and Chemical Products</td>
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<td>(0.000)</td>
<td>(0.002)</td>
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</table>

**Notes:** Table reports the estimates of the multi-sector gravity equation. The results report sector-specific (ρ*γ)'s, the coefficients on distance variable in columns 1A and 1B. The sum of these coefficients exactly sums to the distance coefficient in column 3 of Table 1. The table supresses the 17 sector-specific border, 17 sector-specific language coefficients, and the 680 (=17 sectors*40 exporters) sector-exporter dummies to save space. Instead, columns 2A and 2B report the sector betas from the first stage engel curve; these betas are equal to the sum of the sector-exporter dummies for each sector across exporters. The sum of the sector-exporter dummies for each exporter across sectors exactly equals the betas reported in column 3 of Table 1. The sample is constructed from total expenditures from the WIOD database. Standard errors are clustered by importer. Significance *.10; ** .05; *** .01.
Table 4: Unequal Gains From Trade, Multi-Sector Case

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<tr>
<th>Country</th>
<th>10th Percentile</th>
<th>50th Percentile</th>
<th>Aggregate Change</th>
<th>90th Percentile</th>
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<td>74%</td>
<td>58%</td>
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<td>37%</td>
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<tr>
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<td>81%</td>
<td>68%</td>
<td>51%</td>
<td>46%</td>
</tr>
<tr>
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<td>81%</td>
<td>65%</td>
<td>50%</td>
<td>45%</td>
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<td>2%</td>
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<td>5%</td>
</tr>
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<td>51%</td>
<td>47%</td>
</tr>
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<td>46%</td>
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<td>41%</td>
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<td>Average</td>
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<td>50%</td>
<td>31%</td>
<td>27%</td>
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</table>

Notes: Table reports gains from trade for the multi-sector case and uses the parameters reported in Table 3. The columns report welfare changes associated at the 10th, 50th, the representative consumer, and the 90th percentiles.

Table 5: Ex Post 90th-10th Welfare Difference and Sources of Import Growth, 1996-2006

<table>
<thead>
<tr>
<th>Difference in 90th-10th welfare change, 1996-2006</th>
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<td>ΔImport Shares (β&lt;0)</td>
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<td>ΔImport Shares (β&lt;0)</td>
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<tr>
<td>R-squared</td>
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<td>Observations</td>
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</tbody>
</table>

Notes: Table regresses the difference between welfare changes at the 90th and 10th percentile on changes in import shares between 1996 and 2006. The import shares are partitioned into two groups: changes in import shares from negative-beta sectors; and, changes in import shares from negative-beta exporters. Significance * .10; ** .05; *** .01.
Figure 1: $\beta$ of Exporters and GDPPC

Figure 2: The Unequal Gains from Trade, Single-Sector

The deviations are relative to the median individual.
The red line is the average across countries.
Figure 3: Change in Non-Homothetic Price Index from Autarky to Trade vs Own $\beta$

Figure 4: Change in Non-Homothetic Price Index from Autarky to Trade vs Distance to USA

Figure generated for the representative agent

Figure excludes USA
Figure 5: Engel Curves, by Broad Sector Groups

Figure 6: $\beta$ by Exporter and Broad Sector Group vs GDPPC

Figure sums betas across broad sectors for each country.
Figure 7: Distribution of Unequal Gains, Multi-Sector Case

Figure 8: Comparison of Distribution of Unequal Gains, Means across Countries
Figure 9: Average $\gamma$, by Percentile

The deviations are relative to the median individual.
The red line is the average across countries.

Figure 10: *Ex Post* Unequal Gains from Trade, Multi-Sector 1996-2006

The deviations are relative to the median individual.
The red line is the average across countries.
Appendix Figures

Figure A.1: Comparison of All Expenditures vs Manufacturing Expenditures, Average by Percentile

The deviations are relative to the median individual.
Figure shows averages across countries, by percentile

Figure A.2: Unequal Gains from Trade, Final Expenditures

The deviations are relative to the median individual.
The red line is the average across countries