Building up Financial Flexibility

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December 2013

Abstract

We ask how firms should build up financial flexibility by initially choosing high or low target leverage. This depends on whether additional financing is raised at competitive terms, as then there will be a problem of either overinvestment or underinvestment. The role of the initial (or target) capital structure is that it affects the "outside options" of both insiders and outside investors. Our novel insights stem from characterizing how this creates countervailing incentives (to those typically analyzed) when firms face a problem of asymmetric information in new financing rounds. Our theory also entails implications for start-up and venture capital financing.

Keywords: Financial Contracting, Optimal Security Design, Capital Structure
JEL Classification: G32

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1 Introduction

Exploring the determinants of financial contracting has been in the forefront of both the theoretical and empirical literature in the last decades. However, existing evidence does not give clear support to any of the more influential theories. In particular, while the pecking order theory seems to hold for very large firms (Shyam-Sunder and Myers, 1999), other considerations (also besides trade-off theory arguments) seem to be more important as the focus shifts towards smaller firms (Frank and Goyal, 2003; Leary and Roberts, 2010). Some of these considerations are the firms’ ability to tap the capital markets at competitive terms and their desire to build up financial flexibility when they are not under duress (Lemmon and Zender, 2007).

In this paper, we explicitly take into account the access to competitive financing to develop a theory of how firms should build up financial flexibility by initially choosing a low or a high target leverage. Thereby, we obtain a rich set of implications from a single agency problem. We show that in the absence of a competitive market for capital, the "pecking order" of how firms raise outside financing under asymmetric information does not hold, and is actually inverted. Moreover, initial financing contracts generate countervailing incentives to those typically analyzed in models with asymmetric information. This gives firms a powerful tool to manage their financial flexibility, leading to contrasting implications depending on whether firms are able to raise financing at competitive terms when asymmetric information is an issue. Since small and large firms may differ in their access to competitive financing, the resulting implications of our model could help to shed some light on the existing contradictory evidence. Our optimal security design approach allows us to interpret our result also more broadly. For example, one open question that we address is why venture capital financing contracts in countries with weaker enforcement of investor rights are systematically different than those in the US (Cumming, 2008; Kaplan et al., 2007). We also interpret our model in the context of target capital structure and short-term deviations from it.

We consider the following financing problem. After raising initial capital, a firm may have to raise additional financing in the future, possibly under asymmetric information because the information advantage of the firm’s insiders has increased over time, and financing must be secured at short notice. Notably, the firm’s initial capital structure
matters as it shapes the "outside options" both of the insider (owner-manager) and of the providers of outside funding—i.e., the profits that both parties would realize if the firm failed to raise additional finance. Based on a single financial imperfection, namely information asymmetry at a later stage, our model generates a simple theory of how firms raise finance and adjust leverage over time. Firms that expect to be locked-in with investors who exert substantial bargaining power at later financing rounds benefit from initially using up their debt capacity. This makes it easier for these firms to raise equity financing, helping to reduce an underinvestment problem in the future. Instead, firms that expect to receive future financing at competitive terms are better off initially avoiding debt. Interestingly, this limits (rather than expands) their access to debt financing in the future, helping to reduce an overinvestment problem.

The important innovation at the heart of our results is that the initial financing structure generates countervailing incentives that could make asymmetric information irrelevant, and first-best financing could be achieved. Intuitively, countervailing incentives are generated by the fact that the existing financing claims on a firm with good investment opportunities are worth more regardless of whether it raises a new round of financing or not. Thus, claiming that the firm’s prospects are better or worse also has implications for how expensive it is to potentially modify existing contracts and to get initial investors on board for new financing rounds.

Only if the countervailing incentives created by the initial financing structure are not sufficiently strong when firms have access to competitive financing do we obtain the overinvestment problem emphasized by the extant literature (e.g., Myers and Majluf, 1984; Nachman and Noe, 1994). For this case we show that avoiding debt helps build up financial flexibility by maximizing countervailing incentives and, thus, efficiency.\footnote{The need to build up debt capacity has also been motivated with Myers’ (1977) debt overhang problem. However, absent asymmetric information, this problem can be resolved through renegotiations.}

More important, our prescription for preserving financial flexibility is reversed when firms cannot raise financing at competitive terms. This case seems more relevant for small firms, private firms, and also firms that are "locked-in" with existing (relationship) investors. One reason why incumbent investors in such firms may have bargaining power is that their refusal to (co-)finance new investment may make it impossible for these firms to raise new financing elsewhere. Such a refusal sends a negative signal about the
firms’ overall prospects ("lemons"). An important novel insight for this case is that (if the countervailing incentives are not sufficiently strong) underinvestment can result as investors seek to extract a larger share of the gains from a new investment. This problem is mitigated when refinancing leads to a decrease in the firms’ leverage. Thus, these firms should initially use up their debt capacity and then decrease leverage when raising financing under asymmetric information.

An important empirical implication, arising from these results, is that firms that don’t have access to competitive financing should pursue a higher target leverage, and they should deviate down from this target when raising capital at short notice. The opposite should hold for firms that can expect to raise financing at competitive terms even when information asymmetry is an issue. Indeed, large and public firms, which arguably have an easier access to competitive financing, issue debt to finance their internal financial deficit (Shyam-Sunder and Myers, 1999), and they issue equity when they are not under duress (Fama and French, 2005), possibly to stock-pile debt capacity (Lemmon and Zender, 2009). However, smaller firms fill their financial deficit by issuing equity (Frank and Goyal, 2003), and they issue debt when asymmetric information is not a factor, thereby using up their debt capacity (Leary and Roberts, 2010; Gomes and Phillips, 2012). Thus, taking into account that the lack of access to competitive financing is a distinguishing feature of small and private firms, we contribute to the better understanding of the financing strategies of such firms.

We derive also additional predictions for start-up and young firm financing. Then the distinction between facing a strong or a weak investor could be interpreted as whether the firm faces a specialist or nonspecialist investor. Taking this perspective, we predict that nonspecialist investors in start-up firms will initially demand less protection on the downside. However, financing will change over time as entrepreneurs gain an informational advantage (vis-a-vis investors) about the firm’s prospects, and they will issue more senior securities at later stages. This prediction corresponds to financing patterns of firms initially raising financing from nonspecialist investors, such as friends and family and business angels, turning to various type of debt financing only at later stages (Berger and Udell, 1998; Wong, 2009). Thereby, observe that issuing pure common equity is by far not an obvious choice for a young firm even if it lacks a stable cash flow stream. Indeed, specialized investors, such as venture capitalists in the U.S., additionally demand a liquidation
preference, as predicted by our "strong investor" case. These contracts then convert to common equity only at later stages as the venture capitalist takes the firm to the equity markets (Kaplan and Strömberg, 2003).²

Interestingly, another implication of our model is that such U.S.-style venture capital contracts are not optimal if a weak protection of investor rights puts even specialist investors in a weak-investor position. There is, indeed, evidence for this prediction (Lerner and Schoar, 2005; Cumming 2005, 2008; Kaplan et al., 2007). Venture capitalists are more likely to take common equity in first rounds in countries with weak enforcement of investor rights. More successful firms then issue more senior securities in later rounds of financing (Kaplan et al., 2007). Thus, considering the access to competitive financing and the effect of countervailing incentives could shed new light on patterns in start-up financing.³

Our contribution to the corporate finance theory literature is to solve for a security design problem under asymmetric information where both the privately informed owner-manager and the original investors already have a stake in the company. Their existing claims create "outside options", whose value depends on the firm’s profitability when no new financing is raised. Technically, we solve for a game of screening (when the investor has bargaining power) and a game of signaling (when the owner-manager has bargaining power) with so-called type-dependent reservation values (e.g., Jullien, 2000).⁴ By affecting the "outside options" at the refinancing stage, the firm’s initial financing structure becomes relevant even though it is chosen under symmetric information. We obtain conditions when despite private information at the refinancing stage, the outcome is efficient—i.e., there is no under- or overinvestment. This is made possible as also the value of the stakeholders’ "outside options" depends on the firm’s profitability. Since in practice (fresh) financing is frequently raised when the firm already has outstanding securities, our three-period model of financing under asymmetric information should add realism and clarity how firms should

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²We show that the sequence of financing contracts that we derive can also be interpreted as a single convertible contract. For a more detailed analogy to venture capital financing, see Sections 3.3 and 5.

³The bulk of the existing venture capital literature, which has focused on incomplete contracts and (two-sided) moral hazard, motivates only the financing contracts used in countries with strong law enforcement (e.g., Schmidt, 2003).

⁴The countervailing incentives starkly differentiate our paper from the literature on incomplete contracts, which also discusses the role of existing contracts as outside options (Hart and Moore, 1988; Noldeke and Schmidt, 1995). What is interesting is that in our setting only a single security instead of a menu is always optimal at the refinancing stage (see, instead, the literature on screening with type-dependent reservation values, e.g., Jullien 2000).
build up their financial flexibility.\(^5\)

In a related paper, Axelson et al. (2009) also take a security design approach in a model with interim private information. Similarly to our case of a "weak" investor at the refinancing stage, they derive debt as the unique interim security. This confirms the predictions of Myers and Majluf’s (1984) pecking order theory (cf., also Nachman and Noe, 1994). In our model, however, (re)financing is not always optimal for the owner-manager, as the existing capital structure serves as an outside option.\(^6\) The overinvestment problem is then less severe. In fact, first-best investment may be achieved, and the pecking order is reversed if the bargaining power shifts to the uninformed investor.\(^7\)

There is a growing body of research on the dynamics of a firm’s optimal capital structure focusing on dynamic trade-off explanations (cf., Hennesy and Whited, 2005; Miao, 2005), but also on agency problems (DeMarzo and Sannikov, 2006; DeMarzo et al., 2012; for an overview see Sannikov, 2012). The common theme in the latter papers is optimally containing moral hazard with dynamic incentive contracts. Similarly to this literature, we also focus on a single agency problem, but in contrast to it we analyze an adverse selection problem arising over time. While our dynamics are simpler, as they are captured with a stylized three period model, this framework is sufficient to derive our main results. Thereby, our contribution is to show how existing contracts create countervailing incentives in new rounds of financing, and to show that the pecking order in which firms raise financing crucially depends on their ability to raise financing at competitive terms.\(^8\)

Finally, the separation of initial financing and possible refinancing relates our paper also to the literature on stage financing, which is a well documented fact in start-up finance (e.g., Gompers, 1995; Kaplan and Strömberg, 2003). A number of authors have

\(^5\)Other recent papers that explore the role of (type-dependent) outside options, albeit in different contexts, are Tirole (2011) and Burkart and Lee (2011).

\(^6\)The optimal ex ante financing in our model, which is chosen under symmetric information, is also very different from Axelson et al. (2009). They propose a mixture of debt and levered equity: Debt is used to deter the entry of fraudulent entrepreneurs, while levered equity mitigates the risk shifting incentives caused by debt financing. In one of our cases, we also obtain that levered equity may be optimal, because it reduces overinvestment in later stages. What drives this result, however, is the role of levered equity for shaping the outside options of the refinancing stage.

\(^7\)Though DeMarzo and Duffie (1999) and Biais and Mariotti (2005) also consider a two-stage game, the security in their models is designed before private information is revealed, and ultimately only a single security is issued.

\(^8\)These insights also differentiate our paper from dynamic papers on debt capacity, analyzing the role of collateral and the trade-off between current investment and risk management (Rampini and Viswanathan, 2010; 2013).
used staging in a security design context when there is no commitment to refinancing (e.g., Cornelli and Yosha, 2003). Further, following Aghion and Bolton (1992), several papers have motivated the use of contingent securities in venture capital financing in the context of incomplete contracting (e.g., Berglöf, 1994). Our contribution lies, especially, in showing how variations in bargaining power, as arising from different legal environments or different stages of firm development, significantly changes the shape of optimal securities.

The rest of this paper is organized as follows. Section 2 describes the model. Section 3 solves for the optimal initial and interim financing when the investor has bargaining power at the refinancing stage. Section 4 characterizes security design when the owner-manager has bargaining power at the interim stage. Empirical implications of our results are presented in Section 5, and Section 6 concludes. All proofs are in the Appendix.

2 The Model

We envisage a firm that raises financing in an initial period, \( t = 1 \), and that raises additional financing in a subsequent period, \( t = 2 \). Precisely, suppose that at \( t = 1 \) a penniless owner-manager ("she") needs to raise initial financing \( K_1 > 0 \). At \( t = 2 \), another investment \( K_2 > 0 \) can be made. For simplicity only, we assume that the investment opportunity in \( t = 2 \) arises with probability one. The investment opportunity will be profitable only sometimes, though. Cash flows are realized in the final period, \( t = 3 \). Both the owner-manager and investors are risk neutral, and we abstract from discounting.

Financing and Contracting The firm’s verifiable cash flow at \( t = 3 \) is either low or high: \( x_l \geq 0 \) or \( x_h > x_l \). The likelihood of realizing high cash flow depends both on whether additional capital was injected at \( t = 2 \) and on the firm’s underlying profitability (its "type"). The restriction to two cash flow states is for simplicity only. At the end of this section, we explain how our results can be generalized to a continuum of cash flows.

To raise \( K_1 \) at \( t = 1 \), the owner-manager issues a security \( R^1(x) \) that conditions the repayment on the final cash flow. The firm can initially raise capital competitively, so that the resulting security design problem will be to maximize the ex-ante value of the owner-manager’s claims. For this reason, we stipulate that at \( t = 1 \) the owner-manager can offer \( R^1(x) \) to investors. This assumption is inconsequential for our qualitative results and we relax it later.
Raising financing at $t = 2$, provided this is successful, involves a fresh injection of $K_2$ by the investor. It is convenient to suppose initially that all outside claims are held by one investor. Then, the initial security $R^1(x)$ that is held by the outside investor is replaced by a new security $R^2(x)$. In Section 3.3, we argue why it is not optimal to raise all financing ex ante. We also discuss financing from new investors.

We make the standard assumptions that $0 \leq R^t(x) \leq x$ and that both $R^t(x)$ and $x - R^t(x)$ are nondecreasing. According to the first assumption, the security can only distribute the cash flows that are realized by the firm. As the owner-manager is assumed to be penniless, the relevant restriction is that $R^t(x) \geq 0$: The security cannot specify a "wage" that is paid to the owner-manager over and above the firm’s cash flow. This assumption is common in the literature. Such a payment could lure "non-serious" operators into the market. Also, it is common to assume that both $R^t(x)$ and $x - R^t(x)$, i.e., the payouts to outside investors and the owner-manager, are nondecreasing. Otherwise, either party could have an incentive to "destroy" cash flow by obstructing the operations of the firm.

**Information**  The firm’s profitability can take on two values. We refer to these as a good or bad "state", $\theta = \{B, G\}$. A priori, the likelihood of $\theta = G$ is given by $0 < \hat{q} < 1$. At $t = 1$, this is common knowledge between the owner-manager and potential investors. At $t = 2$, when the refinancing decision must be made, the owner-manager has already gained an informational advantage regarding $\theta$, which she cannot credibly share with the investor, at least not at such short notice. Based on this private information, the posterior belief of the owner-manager regarding $\Pr (\theta = G)$ is $q$. We refer to the owner-manager’s private information $q$ as her "type" at $t = 2$. It is a priori distributed according to the CDF $F(q)$ over $q \in [0, 1]$, satisfying $\hat{q} = \int q dF(q)$.\(^9\)

The probability of achieving the high cash flow state, $p_{dG}$, depends on the firm’s profitability state $\theta = \{B, G\}$ and the decision whether to refinance at $t = 2$, $d \in D := \{Y, N\}$ ("Yes" and "No"). Assuming that $p_{dG} > p_{dB}$ holds for all $d \in D$, state $G$ can be unambiguously referred to as the "good" state. In what follows, we are interested in a setting in which refinancing in the good state is at least as efficient as in the bad state—i.e., if

\(^9\)Alternatively, we may, instead, stipulate that the owner-manager privately observes some signal $\vartheta$, which is generated by the CDFs $\Psi_{\vartheta}(\theta)$. We can then generate $q$ as well as $F(q)$ by using Bayes’ rule.
refinancing increases the success probability by a factor of $\beta_\theta$, we have

$$\beta_G \geq \beta_B,$$

where $\beta_\theta$ can be stated equivalently as $\beta_\theta := p_{Y\theta}/p_{N\theta}$. Observe that this implies that $p_{YG} - p_{NG} > p_{YB} - p_{NB}$.

**Discussion of Contracting**  Given symmetric information at $t = 1$, one can solve the financing and refinancing problem that we describe above with a single contract signed at $t = 1$. By committing the owner-manager and the investor to a refinancing contract at $t = 2$, which replaces the initial security if the owner-manager chooses to refinance, such a contract could ensure efficient refinancing. However, in some settings committing both parties to a refinancing contract may not be feasible or renegotiation-proof. For now we make this assumption. In Sections 3.3 and 4.3 we show several ways to endogenize it, which will then also resonate in our empirical implications. In these sections, we also motivate why not all of the required funds, $K_1 + K_2$, can be raised initially.

The main objective in what follows is to analyze a sequence of simple contractual games: raising initial financing in $t = 1$ through $R_1(x)$ and refinancing in $t = 2$ through an exchange for $R^2(x)$. As we discuss below, offering a menu of refinancing contracts is always suboptimal. For our analysis, we distinguish two cases. We capture the case in which the firm is "locked-in" with the initial investor by granting the investor all bargaining power: It is the investor who makes a take-it-or-leave-it offer. When there is no such "lock-in", the owner-manager has all bargaining power. Then, it is the owner-manager who makes a take-it-or-leave-it offer. While these two settings are certainly extreme, they allow to capture the fundamental sources of inefficiency that arise from asymmetric information. After deriving our first characterization, we will be more explicit about endogenizing a possible "lock-in", e.g., through an additional layer of information asymmetry between initial investors and new investors at $t = 2$. In Sections 3.3 and 4.3, we also discuss the possibility that fresh capital is raised from a new investor.

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10Assuming instead that $\beta_G < \beta_B$ could (if $\beta_B$ is sufficiently larger) cause the incentives we discuss below to invert—i.e., instead of being better off in the good state, the owner-manager is better off in the bad state)—leading respectively to inverse results. However, a setting with such "inverse" incentives seems to be a less relevant assumption for healthy (and growing) firms.

11Note that a solution in which the investor *optimally* retains $R^1$ and receives in addition a new security in exchange for $K_2$ can be modelled as giving the investor a single aggregate security $R^2$. 
To ease exposition, we use the following short-hand notation: $R^t_i := R^t(x_i)$ denotes the repayment for low cash flow, $\Delta R^t := R^t(x_h) - R^t(x_l)$ denotes the outside investor’s upside, and $\Delta x := x_h - x_l$ denotes the upside of the whole firm. One can then represent the investor’s payoff when there is refinancing at $t = 2$ as

$$v_Y(R^2, q) := R^2_l + (p_{YB} + q(p_{YG} - p_{YB})) \Delta R^2 - K_2,$$

depending on the refinancing security contract $R^2$ and the owner-manager’s private information (his "type") $q$. Note that the payoff is gross of the initial outlay $K_1$, but net of $K_2$. Likewise, denote the investor’s expected payoff without refinancing by

$$v_N(R^1, q) := R^1_l + (p_{NB} + q(p_{NG} - p_{NB})) \Delta R^1.$$

Total expected cash flow in either case is given by the joint surplus

$$s_Y(q) = x_l + (p_{YB} + q(p_{YG} - p_{YB})) \Delta x - K_2,$$

$$s_N(q) = x_l + (p_{NB} + q(p_{NG} - p_{NB})) \Delta x.$$

With this at hands, the owner-manager’s expected payoff can be written as

$$u_N(R^1, q) = s_N(q) - v_N(R^1, q),$$

$$u_Y(R^2, q) = s_Y(q) - v_Y(R^2, q).$$

To limit trivial case distinctions, we suppose that refinancing is efficient only in $\theta = G$:

$$(p_{YB} - p_{NB})\Delta x < K_2 < (p_{YG} - p_{NG})\Delta x. \tag{2}$$

This implies that there exists a cutoff $0 < q_{FB} < 1$, so that refinancing increases the joint surplus only if the owner-manager’s type (the probability of being in $G$) is above $q_{FB}$. Similarly, it is convenient to stipulate that

$$K_1 > x_l,$$

as this allows to rule out the use of safe debt. Throughout the analysis we assume that the firm is financially viable at $t = 1$ under the respective optimal contracts.

Finally, note that our results also generalize to a setting with a continuum of cash flows. Such a setting requires slightly more structure, similar to that in the related security design
literature (e.g., Nachman and Noe, 1994), without leading to material new insights. Thus, our restriction to only two positive cash-flow realizations allows us to focus the analysis on the novel aspect, which is the interaction of the security design problems in the two periods.

3 The Case with a Strong Investor

We proceed backwards and take, first, the refinancing stage at \( t = 2 \). The equilibrium outcome is then plugged into the problem that arises at the initial financing stage, \( t = 1 \). Recall that we presently consider a game in which the initial investor has bargaining power vis-à-vis the (locked-in) firm when it needs refinancing at \( t = 2 \), and can, thus, make a take-it-or-leave-it offer at this stage.

3.1 Refinancing (Strong Investor)

To obtain refinancing \( K_2 \), the owner-manager must agree to a new contract, \( R^2 \), proposed by the investor, which replaces her existing security \( R^1 \).\(^{13}\) Otherwise, no new capital is injected, and the original security \( R^1 \) stays in place. Denote the set of all types \( q \) for whom it is profitable to accept the offer with \( A \subseteq [0,1] \)-i.e., \( u_Y(R^2,q) \geq u_N(R^1,q) \) for \( q \in A \). Then, the investor’s expected payoff at \( t = 2 \) is given by

\[
\int_A v_Y(R^2,q) dF(q) + \int_{[0,1]/A} v_N(R^1,q) dF(q),
\]

and his objective is to maximize this payoff, subject to the imposed "feasibility" constraints that \( 0 \leq R^2(x) \leq x \) and that both \( R^2(x) \) and \( x - R^2(x) \) are non-decreasing. This program is solved next.

The best that the investor can do to maximize his profit is to refinance all owner-manager types for whom this is efficient, and then extract all surplus created from refinancing.\(^{12}\)

\(^{12}\)Denoting with \( H_d(x|\theta) \) the distribution function over cash flows for all combinations \( d = \{Y,N\} \) and \( \theta = \{G,B\} \), we can first generalize \( p_{dB}(x) := 1 - H_d(x|\theta) \). Following Nachman and Noe (1994), assume that the distribution for \( G \) dominates that for \( B \) in terms of conditional stochastic dominance (CSD): \( p_{dG}(x'|z) \geq p_{dB}(x'|z) \) for \( x', z \in X \), where \( p_{d}(x|z) \) is the conditional probability \( 1 - \text{Pr}(x' \leq x \leq x' + z) \), which implies that high cash flows are increasingly more likely in state \( G \) compared to state \( B \): \( \frac{\partial}{\partial z} \left( \frac{p_{dG}(x)}{p_{dB}(x)} \right) \geq 0 \). More efficient refinancing means again shifting more probability mass to the high cash flow states--i.e., \( \beta_G(x) \geq \beta_B(x) \), where as before \( \beta_{\theta}(x) := \frac{p_{\theta}(x)}{p_{\theta}(x')} \). We have shown in a working-paper version how these assumptions jointly ensure that our subsequent results and predictions still hold.

\(^{13}\)Clearly, offering the owner-manager to keep his initial contract is just a special case.
financing. This requires making an offer for which each owner-manager type \( q \in A \) is exactly indifferent between accepting and rejecting:

\[
 u_Y(R^2, q) = u_N(R^1, q) \quad \forall q \in [0, 1],
\]

implying that the investor’s incremental payoff from refinancing is

\[
 v_Y(R^2, q) - v_N(R^1, q) = s_Y(q) - s_N(q).
\]

The one-shot analogy of our refinancing stage, in which the owner-manager has a constant reservation utility \( u_N \), is a special case of our problem, and it helps highlight one of our main novel insights. In particular, note that if \( u_N > x_l \), any security that the owner-manager retains in the firm must give her a share on the upside, \( \Delta x \). Thus, it will depend on the probability of achieving this state, which is determined by her type. Since high types are more likely to achieve higher cash flows, the investor can offer such types a smaller claim on the firm’s cash flows that still leaves them better off participating. In a setting with asymmetric information, this creates incentives for the owner-manager to understate her type.

The conceptual innovation when the owner-manager already has a claim on the firm’s cash flows is that her type determines not only the profitability of refinancing, but also the value of her outside option of not receiving refinancing. This creates the following countervailing incentives. On the one hand, for any given outside option, a high-type owner-manager has an incentive to understate her type (for the same reasons as above). On the other hand, a high-type owner-manager needs to be compensated for having a higher outside option. This creates a countervailing incentive for the owner-manager to overstate her type.

The investor should, therefore, aim at offering a security that optimally balances these countervailing forces. Suppose that security \( R^2 = \hat{R} \) that satisfies (4), is feasible. Then, \( \hat{R} \) implements first best refinancing, as then no owner-manager has an incentive to under- or overstate the firm’s success probability. To analyze the properties of this security, we can use (4) to obtain the "upside" \( \Delta \hat{R} \) and the respective safe repayment, \( \hat{R}_l \)

\[
\Delta \hat{R} = \Delta x - \frac{p_{NG} - p_{NB}}{p_{YG} - p_{YB}} (\Delta x - \Delta R^1),
\]

(5)

\[
\hat{R}_l = R^1_l - p_{NB} (\Delta x - \Delta R^1) + p_{YB} \left( \Delta x - \Delta \hat{R} \right).
\]

(6)
These expressions reveal that, if \( \hat{R} \) is feasible, it gives the investor a higher participation on the upside, i.e., \( \Delta \hat{R} > \Delta R^1 \), while providing him with less protection on the downside—i.e., \( \hat{R}_l < R^1_l \) (after plugging in for (5) in (6)). In what follows, we refer to such securities as being "steeper" than the initial security \( R^1 \) from the investor’s perspective. Note that by making the investor’s payoff more dependent on achieving the high cash flow state, steeper securities are more dependent on the owner-manager’s true type.

Intuitively, to keep the owner-manager indifferent between refinancing and not refinancing regardless of her type, as required by (4), the owner-manager must be made less sensitive to achieving (her share of) the upside, \( \Delta x - \Delta R^2 \). The reason is that this upside has a higher weight in the owner-manager’s expected payoff due to the higher success probability after refinancing. In exchange, the owner-manager should be compensated with more of the cash flows in the low cash flow state. Thus, giving the investor a steeper security can balance the owner-manager’s countervailing incentives. By making the owner-manager’s choice between refinancing and not refinancing independent of her type, \( \hat{R} \) fully neutralizes the effect of asymmetric information, and extracts all surplus from refinancing.

The "first-best" security, \( \hat{R} \), may not be feasible, though. This is the case when a new security that extracts all surplus from refinancing cannot be made sufficiently steep, as it would demand a "negative repayment" to the investor in the low state: \( \hat{R}_l < 0 \).\(^{14}\) As a result, there is no single security that allows the investor to extract all of the surplus from refinancing for each type—i.e., the difference in the owner-manager’s expected payoff with and without refinancing, \( u_Y(R^2,q) - u_N(R^1,q) \), is no longer zero everywhere, as it is when \( R^2 = \hat{R} \), but it is strictly increasing in \( q \). For this case, denote the unique point of intersection of \( u_Y(R^2,q) \) and \( u_N(R^1,q) \) by \( q^* : \)

\[
\tag{7}
\left. u_Y(R^2,q) \right|_{q^*} = u_N(R^1,q) .
\]

The set of owner-manager types who accept a refinancing offer with \( R^2 \) in \( t = 2 \) becomes, thus, \( A = [q^*, 1] \): The owner-manager prefers to accept \( R^2 \) if and only if \( q \geq q^* \) and strictly so if \( q > q^* \). All types \( q > q^* \) who accept \( R^2 \) now receive an \textit{information rent} of size

\[
\tag{8}
\left. u_Y(R^2,q) \right|_{q^*} - u_N(R^1,q) .
\]

\(^{14}\)As noted above, this could prescribe a wage that is paid to the owner-manager regardless of the firm’s performance, which—following standard restrictions—we excluded.

\(^{15}\)While we could simply set \( q^* = 0 \) for the case where there is no intersection as also \( u_Y(R^2,q = 0) > u_N(R^1,q = 0) \), this case will not arise by optimality for the investor. In fact, we show that always \( q^* \geq q_{FB} \).
The investor’s expected payoff is maximized when the rent left to the owner-manager is minimized. While the investor can no longer offer a steep enough security that brings each owner-manager type to her reservation value, he can offer a contract (or a menu of contracts) that is as steep as possible through maximizing $\Delta R^2$, while ensuring that $R^2$ remains feasible. Intuitively, such a contract maximally reduces the owner-manager’s claim on the upside from refinancing, $\Delta x - \Delta R^2$. Therefore, it brings each type who accepts refinancing as close as possible to her outside option of forgoing refinancing $u_N(R^1, q)$, in which case the upside potential carries only little weight due to the lower success probability.

**Proposition 1** With a strong investor at the refinancing stage, the investor offers a security $R^2$ that is steeper than the initial security $R^1$: $R^2_t \leq R^1_t$ and $\Delta R^2 \geq \Delta R^1$ (the inequalities being strict if initially $R^1_t > 0$ or $\Delta R^1 < \Delta x$). There is refinancing if and only if $q \geq q^*$. Furthermore:

(i) If

$$R^1_t \geq \left( \frac{PYGPNB - PYBPNB}{PYG - PYB} \right) (\Delta x - \Delta R^1),$$

the "first-best" security $R^2 = \hat{R}$, as characterized in (5)-(6), is feasible and uniquely optimal, in which case the refinancing decision is always efficient: $q^* = q_{FB}$.

(ii) Otherwise, if (9) does not hold, the new security is levered equity with $R^2_t = 0$, and there is underinvestment as $q_{FB} < q^* < 1$.

**Proof.** See Appendix.

If condition (9) does not hold, Proposition 1 pins down levered equity with $R^2_t = 0$ as the uniquely optimal shape of the single optimal security at the refinancing stage.\(^{16}\) Such a security maximally shifts the claim of the owner-manager to the low cash-flow realization and that of the investor to the high cash-flow realization. As desired, this minimizes the owner-manager’s information rent. Note that we presently take a general security design perspective and, thus, consider the full replacement of the initial security $R^1$ by a new security $R^2$. After deriving the optimal initial security $R^1$ in the following section, we offer more interpretation for Proposition 1—e.g., in terms of a change in leverage or in terms of converting an existing security.

\(^{16}\)We discuss condition (9) after characterizing also the initial security $R^1$. 

Furthermore, observe that the investor would not gain from offering a menu of securities. Intuitively, any non-degenerate menu of contracts would have to include also a flatter security than levered equity (the flatter security will be offered to lower types). However, any such incentive compatible menu is altogether flatter than a single levered equity security $R^2$. Thus, the investor has to share more of the upside from refinancing with the owner-manager, implying that he extracts less, rather than more, surplus on every type who receives refinancing.

The underinvestment problem in the second part of Proposition 1 is another important novel insight from our model. More specifically, if $\tilde{R}$ is not feasible, the investor must face a trade-off between maximizing surplus and reducing the owner-manager’s information rent. While the countervailing incentives of the owner-manager help ameliorate this problem, they are not strong enough to help the investor extract the full surplus from refinancing for every type, resulting in inefficient refinancing: $q^* > q_{FB}$.

It remains to pin down the resulting cutoff $q^*$. For this we can now substitute the cutoff rule, $A = [q^*, 1]$, into the investor’s objective function (3) and use that the uniquely optimal refinancing security, when refinancing is inefficient, is levered equity with $R^2_l = 0$. The upside $\Delta R^2$ is pinned down by the owner-manager’s indifference condition for $q^*$ (cf. condition (7)) as a monotonic function of $q^*$. Intuitively, giving the investor a higher upside $\Delta R^2$, leaves the owner-manager with a smaller share of the benefit from refinancing. Hence, only higher owner-manager types (i.e., higher $q^*$) still find it optimal to accept the refinancing offer. Differentiating the investor’s expected profit (3) with respect to $q^*$, gives us, therefore, the following first-order condition

$$- [s_Y(q^*) - s_N(q^*)] f(q^*) + \frac{d\Delta R^2}{dq^*} \int_{q^*}^{1} \frac{dY(R^2, q^*)}{d\Delta R^2} dF(q) = 0. \quad (10)$$

The second term in (10) captures the benefits from "implementing" a higher $q^*$ and, thereby, extracting a higher payoff from all types $q > q^*$. The loss in surplus, given that the underinvestment problem becomes more severe, is captured by the first term in (10). Expression (10) implies immediately that $q^* > q$.

It is important to note that by allowing the investor to extract more surplus, levered equity financing also induces him to implement more efficient refinancing. Intuitively, by internalizing a higher portion of the social surplus, the investor has a higher incentive to implement a more efficient decision. Observe that, while underinvestment will also result
when \( v_N \) is constant, the crucial difference is that the countervailing incentives created by the initial contracts can help ameliorate, and even eliminate, this problem. In what follows, we explore how the efficiency of the refinancing decision depends on the shape of the initial security \( R_1 \).

### 3.2 Raising Initial Finance (Strong Investor)

Initially, at \( t = 1 \), there is no private information, and financing can be raised at competitive terms. As noted previously, we relax this assumption below. Using that \( u_d(R^t, q) = s_d(q) - v_d(R^t, q) \), the owner-manager maximizes her expected payoff

\[
\int_0^{q^*} (s_N(q) - v_N(R^1, q)) dF(q) + \int_{q^*}^1 (s_Y(q) - v_Y(R^2, q)) dF(q)
\]

subject to the participation constraint of the investor at \( t = 1 \)

\[
\int_0^{q^*} v_N(R^1, q) dF(q) + \int_{q^*}^1 v_Y(R^2, q) dF(q) \geq K_1,
\]

where, importantly, \( q^* \) and \( R^2 \) are determined at the interim stage.\(^{17}\) In what follows, we start with the case in which there is inefficiency at the interim stage regardless of the security contract that is offered initially.

Recall that the source of inefficiency at \( t = 2 \) is that information asymmetry forces the investor to trade off rent extraction from the owner-manager with efficiency. We showed that the investor internalizes a higher portion of social surplus (by extracting more information rent from the owner-manager) for levered equity financing, and such financing, thus, induces him to make a more efficient refinancing offer.

The role of the initial financing contract should be, therefore, to create countervailing incentives for the owner-manager at the interim state, which would allow the investor to internalize even more of the social surplus (by extracting even more rent). Thus, the initial security should give the owner-manager as much of the upside of the non-refinanced firm as possible. Intuitively, the best way to make the owner-manager’s claim on a project with a low success probability (non-refinanced firm) similarly dependent on achieving the upside \( \Delta x - \Delta R \) as a project with a high success probability (refinanced firm) is to offer the

\(^{17}\)Note that, to simplify the exposition, we have presently assumed that, for given \( R^1 \), the investor chooses a pure strategy in \( t = 2 \), so that \( R^2 \) and \( q^* \) are pinned down uniquely. As we show in the proof of Proposition 2, this must indeed hold in equilibrium, even though the investor’s program at \( t = 2 \) may not be strictly quasiconcave.
owner-manager initially the highest possible participation on the upside. This is ensured by initially signing a debt contract with the investor, since such financing minimizes the investor’s claim on the upside.

It remains to argue that offering debt is also what the owner-manager prefers at \( t = 1 \). Under symmetric information at \( t = 1 \) both contracting parties can benefit from more efficiency at the interim stage, as this would give them claims on a larger "pie". Thus, allowing the investor to extract more surplus at the interim stage is optimal for the owner-manager, as it implies that the owner-manager can keep initially a larger claim and, thus, must be offered a larger claim on the refinanced firm also at the refinancing stage.

**Proposition 2** Take the case with a strong investor who determines the refinancing terms at \( t = 2 \). Then, if there is underinvestment at \( t = 2 \) (\( q^* > q_{FB} \)), it is uniquely optimal for the firm to raise initial financing at \( t = 1 \) through a debt contract, \( R^1_l = x_l \).

**Proof.** See Appendix.

Raising the initial amount \( K_1 \) through debt expands the firm’s ability to raise financing at the latter refinancing stage. Somewhat loosely speaking, it improves its "equity capacity". The potential to replace the initial security by a relatively steeper ("levered equity") security reduces underinvestment and maximizes ex-ante firm value. What matters is, thus, how steep the new security in case of refinancing, \( R^2 \), can become relative to the initial security \( R^1 \). It is notable that, though initial financing is chosen under symmetric information (and absent any other agency problem, for that matter), Proposition 2 still pins down a unique security: Debt.

We derive now a simple condition showing when it is feasible to construct an initial security that leads the investor to implement the efficient cutoff \( q^* = q_{FB} \). Denote the maximum feasible joint surplus, after subtraction of the initial outlay \( K_1 \), by

\[
S_{FB} := \int_0^{q_{FB}} s_N (q) dF (q) + \int_{q_{FB}}^1 s_Y (q) dF (q) - K_1
\]

and, to ease notation for the rest of the paper, let

\[
p_d (q) := p_{dG} + q (p_{dG} - p_{dB}) \text{ for } d = \{Y, N\}.
\]

Using that from condition (9) an efficient outcome in \( t = 2 \) is feasible only if \( R^1_l \) is sufficiently high, we obtain:
Proposition 3 If a strong investor determines the refinancing conditions in \( t = 2 \), the first-best investment outcome \( (q^* = q_{FB}) \) is obtained if

\[
x_l \geq \frac{p_{NB}py - p_{YB}p_{NG}}{(p_{YG} - p_{YB})p_N(q)} S_{FB} + \frac{(p_{NG} - p_{NB})p_Y(q)}{(p_{YG} - p_{YB})p_N(q)} \max (0, S_{FB} - p_N(q)\Delta x),
\]

while there is, otherwise, underinvestment with \( q^* > q_{FB} \). In both cases, the security that is held by the investor after refinancing is unambiguously steeper from the investor’s perspective than the initial security \( (\Delta R^2 > \Delta R^1 \text{ and } R^2_l < R^1_l) \).

Proof. See Appendix.

The intuition for condition (14) is simple. If \( x_l \) is large enough, the owner-manager can ensure that the investor just breaks even with a security that leaves most of the upside from the non-refinanced firm to the owner-manager. As explained above, this makes it possible to offer the owner-manager a claim that gives her a low participation on the upside of the refinanced firm, such that (despite the higher success probability after refinancing) her expected payoff is equally sensitive to her true type as with the initial contract.

Taken together, Propositions 2 and 3 entail the following predictions. If refinancing is obtained from a strong investor—e.g., an investor with whom the firm is presently "locked-in"—then the firm should initially raise financing through debt. When the firm obtains refinancing, it should reduce its leverage. There is scope for underinvestment, especially for firms that repay little in case of failure, relative to total profits (if condition (14) does not hold). Hence, such a problem of underinvestment at the refinancing stage should be more likely for firms with a more severe downside risk. In Section 5, we offer various interpretations for these results and relate them to existing evidence on firms’ choice of financing.

Finally, it should be noted that the owner-manager could offer an initial security that would induce first-best refinancing with \( q^* = q_{FB} \) even if (14) does not hold by promising him more than is required to make him break even. (Trivially, this would always be the case if the owner-manager no longer had a stake in the firm as \( R^1_l = R^2_l = x \).) This is, however, never optimal. Intuitively, at \( q^* = q_{FB} \) a marginal distortion has a zero first-order effect on total surplus, but a large effect on the owner-manager’s payoff.\(^\text{18}\) Related, observe

\(^{18}\text{Note that our arguments do not depend on assuming that under the optimal initial security, } R^1, \text{ the investor just breaks even. Debt is uniquely optimal in Proposition 2 as it allows to reduce underinvestment in } t = 2 \text{ for any given level of the investor’s ex-ante payoff. The efficiency gains obtained thereby accrue to the owner-manager.}\)
that our qualitative results remain unchanged if the capital market is not competitive also at \( t = 1 \). Given that information is symmetric at this stage, we can equivalently model this by assuming that the investor requires that his ex ante expected payoff be \( \tilde{K}_1 > K_1 \). The only additional insight is that the investor internalizes a higher proportion of social surplus at the interim stage, inducing him to implement a more efficient refinancing decision \( (q^* \) is lower).

### 3.3 Assumptions and Interpretation of Contracting I

**Staging of Financing** An important feature of our model, which is shared with many of the financial contracting models that we reviewed in the Introduction, is that the owner-manager does not raise \( K_1 + K_2 \) ex ante. We can apply standard arguments for why this would not be optimal. For instance, this can be endogenized by appealing to the existence of an unlimited supply of fraudulent entrepreneurs who realize zero cash flows regardless of how much capital is sunk. Precisely, suppose that at the end of \( t = 1 \) there is a publicly observable, but unverifiable signal whether the entrepreneur is such a "fly-by-night operator" (Rajan, 1992). Then, if the owner-manager had the unconditional right to decide on investing \( K_2 \), a fraudulent owner-manager could "blackmail" the initial investor by demanding a sufficiently large transfer in return for paying back \( K_2 \). Conferring the right to stop refinancing to the investor, which is equivalent to the stipulated staging of financing, renders entry for such fraudulent entrepreneurs unprofitable.\(^ {19} \) As we argue below, this assumption also implies that a single contract signed in \( t = 1 \), fixing the refinancing terms in \( t = 2 \), will not be renegotiation-proof unless it coincides with the contracts we have derived above.

The above motivation relates our paper to the strand of literature on incomplete contracting, which emphasizes the role of existing contracts as outside options (Hart and Moore, 1988; Noldeke and Schmidt, 1995). While our initial contract also shapes the outside options to interim negotiations, our paper differs in a crucial way. The contracting parties in the incomplete contracts literature renegotiate existing contracts if they commonly see a scope for improving efficiency. In contrast, we analyze a setting in which only the owner-manager privately learns her type. Hence, she has an informational advantage

\(^ {19} \)Note that allowing the investor to claw back \( K_1 \) before it is sunk also stops fly-by-night operators from demanding a payment for returning \( K_1 \).
over the true value of both negotiating parties’ outside option (i.e., value of existing contracts if no agreement is reached). This channel creates countervailing incentives at the refinancing stage, relating our paper more closely to the respective literature on countervailing incentives (e.g., Lewis and Sappington, 1989; Jullien 2000) than to that on incomplete contracting.

Next, we motivate giving all the bargaining power to the investor at the interim stage. As noted in the Introduction, the present case where the firm is locked-in may capture various forms of relationship financing. We relate our results to the respective empirical evidence in Section 5. Formally, such a "lock-in" can be obtained when there is an information asymmetry between the original investor and new investors. In fact, our model could be readily extended by introducing an additional layer of information asymmetry between the owner-manager and the original investor, on one side, and new investors in \( t = 2 \), on the other side. New investors would then face a "lemons problem" when being asked to fund all or a part of \( K_2 \). This may make access to new investors very costly or even impossible for the firm at \( t = 2 \) (cf. Sharpe, 1990; Rajan, 1992).\(^{20}\) As we argue in Section 5, we expect this case to be relevant not only in the context of relationship banking, but also in the context of venture capital financing when the initial investor enjoys strong investor protection.

**Other Interpretations and Extensions** We can also interpret our results in the context of a single renegotiation-proof contract signed at \( t = 0 \), which takes into account the option to withhold refinancing together with the distribution of bargaining power at \( t = 1 \). To see this, note that the ability to claim that the owner-manager is a "flight-by-night" operator allows the investor to threaten to withhold refinancing. Since \( R_1 \) will be then renegotiated in the investor’s interest alone, the only renegotiation proof contract that can be signed at \( t = 1 \) would stipulate that \( R_1 \) converts into \( R_2 \) as described in Proposition 1. Thus, a renegotiation-proof contract \((R_1, R_2)\) could be interpreted also as a (single) convertible security, which exchanges senior financing \( R_1 \) for junior financing \( R_2 \) when additional capital, \( K_2 \), is injected. We defer the empirical implications of this insight to

\(^{20}\)To incorporate this into our model, we suppose that the stipulated cash flow realizations only apply for some "type" \( \phi = h \), while with positive probability the firm may be of some "type" \( \phi = l \) with substantially worse cash flow realizations. The type \( \phi \) is observed before the refinancing decision, but only by "insiders" (cf. also Inderst, Münich, and Mueller (2007) for a formalization along these lines).
Section 5.

Finally, consider the following extension to the baseline setting, which is relevant for the discussion of venture capital financing in Section 5. Suppose that the initial investor does not provide fresh financing, but that he remains "strong" even if financing is raised from a new investor. One reason could be that his certification is crucial for a firm to obtain financing even in a competitive market. In this case, the coalition of the initial (strong) and new investors will jointly hold the steepest security (Proposition 1). Thereby, the new investors will be compensated with a "share" of this security for which they break even for offering $K_2$.

**Refinancing from a New (Strong) Investor**  Another way to endogenize when fixing the refinancing terms already at $t=1$ may not be feasible is to assume that there is some probability that the initial investor cannot provide the necessary refinancing at $t=2$. Then, if the owner-manager succeeds in finding a new investor, this investor would enjoy an exclusive bargaining position, allowing him to make a take-it-or-leave-it refinancing offer to the owner-manager (we discuss the case with competition among investors in the next section). This setting could be relevant for firms, financed by small or specialized investors, who could be cash-constrained themselves or are restricted from investing a large proportion of their funds in the same firm. In such a case, non-specialist outside investors may be uninformed about the real reason for the refusal of refinancing, and (faced with a lemons problem) may refuse refinancing. In this case, it would take another specialist financier--i.e., who has the same information as the initial investor--to provide fresh capital.

Solving such a setting is straightforward and leads to similar qualitative results as before. The new investor obtains a levered equity contract, and the new claim jointly held by the owner-manager and the old investor has the same shape as the security originally held by the initial investor (i.e., debt). If the initial investor cannot provide new financing, the owner-manager and the initial investor cannot commit to sticking to the initial contract. Since refinancing increases the firm’s value, a refinancing offer (replacing the initial contracts), can make them better off than their outside option of no refinancing.

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$^{21}$If the initial investor cannot provide new financing, the owner-manager and the initial investor cannot commit to sticking to the initial contract. Since refinancing increases the firm’s value, a refinancing offer (replacing the initial contracts), can make them better off than their outside option of no refinancing.
4 The Case with a Weak Investor

We now consider the case in which the owner-manager has all the bargaining power at the refinancing stage. We capture this, in analogy to the previous section, by stipulating that she makes a take-it-or-leave-it offer $R^2$. Similarly to above, we discuss further motivations of our contracting assumptions in Section 4.3. The present setting gives rise to a game of signaling, as at $t = 2$ the owner-manager is privately informed about the probability of the good state, $q$. (Recall that one way to motivate this is that the firm needs to raise fresh financing relatively quickly, so that an information asymmetry between insiders and outside investors cannot be resolved in time.) Again, we solve first for the equilibrium in the interim period before turning to the optimal contract to raise initial financing at $t = 1$.

4.1 Refinancing (Weak Investor)

Game of Signaling A candidate for an equilibrium of the signaling game where each "type" $q$ plays a pure strategy is a triple of functions $(R^2(q), \mu^*, \pi)$: $R^2(q)$ is the security issued by the owner-manager of type $q$, where we allow for $R^2(q) = \emptyset$ to capture the case where no new security is offered; $\mu^*$ is the investor’s posterior belief, which maps the proposed security contract into the set of probability distributions over the type set $q \in [0, 1]$; and $\pi$ represents the investor’s decision to refinance the project, where $\pi : R^2 \rightarrow [0, 1]$ (with $\pi = 1$ corresponding to $d = Y$ and $\pi = 0$ corresponding to $d = N$). Our equilibrium concept is that of a Perfect Bayesian Equilibrium.

Efficient Refinancing The key feature of our model is that also with a strong investor the initially issued security $R^1$ generates countervailing incentives. On the one hand, the owner-manager has an incentive to claim that her type is higher. Since high types are more likely to achieve high cash flows, they need to promise the investor a lower share of these cash flows in return for refinancing. On the other hand the owner-manager also has an incentive to claim that her type is lower. Then the investor’s existing claim on the non-refinanced firm is worth less and he needs to be promised less in exchange for taking the new contract $R^2$ and providing $K_2$. The latter effect is crucial, as it makes our paper different from previous security design papers such as Nachman and Noe (1994), in which only the first (standard) incentive to claim that one’s type is higher is present.

These countervailing incentives make it possible to construct a refinancing security,
which is "beliefs-free" for the investor. Let \( R^2 = \hat{R}^M \) be defined such that the investor is indifferent between refinancing and not refinancing for all \( q \)

\[
v_Y(\hat{R}^M, q) = v_N(R^1, q) \quad \text{for all } q,
\]

implying that it allows the owner-manager to extract all of the surplus obtained from refinancing. Thus, regardless of the firm’s profitability type, the investor is indifferent between retaining his old claim \( R^1 \) without refinancing and exchanging it for \( R^2 = \hat{R}^M \) after additionally investing \( K_2 \). The decision to accept \( \hat{R}^M \) does not depend on the firm’s type and, thus, on the owner-manager’s private information.\(^{22}\) When feasible, this security is characterized by

\[
\Delta \hat{R}^M_i = \frac{(p_{NG} - p_{NB})}{(p_{YG} - p_{YB})}\Delta R^1, \quad (15)
\]

\[
\hat{R}^M_i = R^1_i + p_{NB}\Delta R^1 - p_{YB}\Delta \hat{R}^M + K_2. \quad (16)
\]

Observe that \( \hat{R}^M \) must be flatter than \( R^1 \). Refinancing increases the upside probability in the good state relative to that in the bad state. Thus, to make the investor indifferent between refinancing and his outside option for all \( q \) (ensuring that the owner-manager extracts the full surplus), the new contract must give the investor less from the upside and more when the low cash flow is realized.

It is straightforward that if \( \hat{R}^M \) is feasible, the owner-manager obtains refinancing if and only if it is efficient—i.e., if and only if \( q > q_{FB} \)—by issuing \( R^2 = \hat{R}^M \). As a first step in the argument, note that the investor will accept \( \hat{R}^M \) whenever it is offered.\(^{23}\) This implies that any type \( q > q_{FB} \) can ensure himself the full surplus from refinancing, so that there can be no "cross-subsidization" among types. This also implies that there is no refinancing for all \( q < q_{FB} \). Second, observe that a different contract will not be offered, even though for any given type \( q \) there is more than one refinancing contract \( R^2 \) satisfying \( v_Y(R^2, q) = v_N(R^1, q) \)—so that for this type \( q \) the investor would be indifferent between refinancing and not refinancing. Intuitively, if such a contract were offered and it were not equal to \( \hat{R}^M \), there would be types \( q' > q \) or types \( q < q' \) that would gain from

\(^{22}\)For comparison, observe that in the one-shot analogy of our setting (as in Nachman and Noe, 1994) the only such security would be riskless debt (if feasible).

\(^{23}\)To be precise, observe that by marginally increasing either \( R^1_i \) or \( \Delta \hat{R}^M \), the investor’s preference for accepting can be made strict, provided that \( R^2 = \hat{R}^M \) is feasible.
cross-subsidization. This is why in equilibrium refinancing must be obtained with \( R^2 = \hat{R}^M \).

**Proposition 4** Security \( \hat{R}^M \) is feasible at the refinancing stage \( t = 2 \) if

\[
x_t \geq R_1^l + \frac{p_Y p_{NB} - p_{NG} p_Y}{p_Y - p_Y^B} \Delta R^1 + K_2.
\]

Then, with a weak investor at \( t = 2 \), refinancing is obtained if and only if \( q > q_{FB} \). Types \( q_{FB} < q < 1 \) uniquely offer \( \hat{R}^M \), where \( \hat{R}^M \) is flatter from the investor’s perspective than the initial security \( R^1 \): \( \hat{R}_t^M \geq R_1^l \) and \( \Delta \hat{R}^M \leq \Delta R^1 \) (the inequalities are strict if \( \Delta R^1 > 0 \)).

**Proof.** See Appendix.

**Equilibrium Security When Refinancing is Inefficient** Offering security \( \hat{R}^M \) is not feasible, however, if condition (17) is not satisfied. This happens when the investor’s initial security \( R^1 \) on the non-refinanced firm gives him a low participation on the cash flows in case of success. Then, since the investor needs to be compensated for additionally providing \( K_2 \), it may not be possible to offer him a sufficiently low participation on the upside of the refinanced firm (which has a higher success probability) to make him indifferent between refinancing and not refinancing for every type. In this case, there will be cross-subsidization in case refinancing is obtained. Specifically, if there is an owner-manager type who extracts the full surplus (or more) from refinancing by offering some security \( R^2 \), there must also be higher types offering this security, extracting less than the full surplus.

The intuition behind this claim is as follows. Suppose that type \( q = q^H \) extracts the full surplus from refinancing—i.e., \( v_Y(R^2, q^H) = v_N(R^1, q^H) \) and that \( q^H \) is the highest type issuing \( R^2 \). Since (17) is not satisfied, it is not possible to offer a security for which the investor is indifferent about the owner-manager’s type. Precisely, the investor will be worse off refinancing the firm than under his outside option of not doing so for all types \( q < q^H \)—i.e., \( v_Y(R^2, q) \) intersects \( v_N(R^1, q) \) from below at \( q^H \). This implies that, if \( R^2 \) is issued in equilibrium, all types \( q < q^H \) must extract more than the full surplus (i.e., the investor is better off with his outside option). Thus, since \( q^H \) is the highest type issuing

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\( ^{24} \text{Strictly speaking, this argument does not apply at the boundaries } q = q_{FB} \text{ and } q = 1, \text{ where incentive compatibility can be ensured with a flatter or steeper contract, respectively, provided that condition (17) is slack so that the construction is feasible. The realizations of } q = 0 \text{ and } q = q_{FB} \text{ are, however, zero-probability events.} \)
$R^2$, the investor will reject this security. It is now straightforward that this implies that there will be cross-subsidization and pooling in equilibrium.\footnote{Note there will always be a deviation from an equilibrium candidate in which no single type extracts the full surplus from refinancing. Thus, there must be a type who extracts the full surplus who is pooled with a type who extracts less than that.}

A key insight that is shared with much of the literature on security design with adverse selection is that the degree of the resulting "cross-subsidization" is lowest when, in a given pool, the respective security is debt (Nachman and Noe, 1994). Intuitively, the difference $v_Y(R^2, q) - v_N(R^1, q)$, which is strictly increasing in $q$, is smaller for debt, as debt is least "information-sensitive" to the private information $q$. Following this literature, we apply criterion D1 to refine the out-of-equilibrium beliefs (e.g., Nachman and Noe, 1994; DeMarzo and Duffie, 1999; or DeMarzo et al., 2005).\footnote{D1 was introduced and extended to a continuum of types by Cho and Kreps (1987) and Ramey (1996), respectively.} Roughly speaking, if type $q'$ has a weak incentive to deviate to some security $R^{2'}$, while type $q$ has a strict incentive, D1 requires that the investor should put probability zero on type $q'$ making this deviation.

Using this criterion, it is straightforward to show that, in equilibrium, all types must offer a (pooling) debt contract. In any other case, the highest type (in a pool) can credibly deviate by offering a debt contract to the investor. Since debt maximally protects the investor in low cash flow states, it is less sensitive to the firm’s probability of success. Thus, the highest type can construct a deviation that is expensive for lower types to mimic (compared to alternative cross-subsidized contracts), and which (given a lower degree of cross-subsidization) makes both the highest type and the investor better off.

**Proposition 5** If condition (17) does not hold, there is pooling and cross-subsidization among the types who receive refinancing. In the pooling equilibrium that survives D1, refinancing is obtained by issuing debt (with $R^2_l = x_l$) if and only if $q \geq q^*_M$. If the investor breaks even, the debt contract is unique and there is overinvestment: $q^*_M < q^{FB}$.

**Proof.** See Appendix.

Proposition 4 and 5 jointly imply that the refinancing security is always flatter than the original security $R^1$, regardless of whether there is cross-subsidization or not.

Define, next, for given initial security $R^1$ a pooling debt security $R^2 = R^P$ for which
the investor is just indifferent to refinancing: \( R^P = (x_t, \Delta R^P) \) and \( q_M^* \) jointly satisfy

\[
\int_{q_M^*}^{1} [v_Y (R^P, q) - v_N (R^1, q)] \frac{dF(q)}{1 - F(q)} = 0, \tag{18}
\]

\[
u_Y (R^P, q_M^*) - u_N (R^1, q_M^*) = 0.
\]

Note that \( q_M^* < q_{FB} \), as there is cross-subsidization of lower types under refinancing. Hence, there is overinvestment at the refinancing stage. The outcome where all \( q \geq q_M^* \) pool at this particular (break-even) debt contract \( R^P \) can be supported by beliefs that satisfy the imposed refinement D1. However, D1 does not eliminate other pooling equilibria with debt where the investor is left with a strictly positive "rent" under refinancing: D1 uniquely pins down the shape of the refinancing security, but not the level. In what follows, we impose the common restriction that the investor just breaks even, so that the equilibrium at \( t = 2 \) is uniquely pinned down by (18), provided that condition (17) does not hold.\(^{27}\)

### 4.2 Raising Initial Finance (Weak Investor)

Recall that at \( t = 1 \) there is no private information and financing can be raised at competitive terms. The owner-manager maximizes

\[
\int_{q_M^*}^{1} (s_N (q) - v_N (R^1, q)) dF(q) + \int_{q_M^*}^{1} (s_Y (q) - v_Y (R^2, q)) dF(q),
\]

subject to the ex ante participation constraint of the investor

\[
\int_{0}^{q_M^*} v_N (R^1, q) dF(q) + \int_{q_M^*}^{1} v_Y (R^2, q) dF(q) \geq K_1, \tag{19}
\]

where \( q_M^* \) and \( R^2 = R^P \) are determined either from (18), if (17) does not hold, or from \( q_M^* = q_{FB} \) and \( R^2 = \hat{R}^M \), if (17) holds. Suppose, first, that there is inefficiency at the refinancing stage. We present below a simple condition when this is the case in equilibrium and show that then (19) binds. The owner-manager is the residual claimant in \( t = 1 \). Hence, his aim is to maximize the expected surplus at \( t = 1 \). That is, the security she

\(^{27}\)In fact, it can be shown that this is also the unique equilibrium outcome if there is competition by outside investors at \( t = 2 \). Precisely, suppose there are at least two new outside investors who could offer to refinance \( K^* \) and, at the same time, to buy out the incumbent investor. To preserve the bargaining power of the owner-manager, assume that the owner-manager must agree first to such a proposal, before it is passed on to the incumbent investor. Then, \( R^P \) is the unique outcome in this competition game.
offers in the initial period should be designed so that the gap between the cutoffs \( q^*_M \) and \( q_{FB} \) is minimal.

**Proposition 6** Suppose there is overinvestment at the refinancing stage. Then, it is uniquely optimal for the firm to raise initial financing through a levered equity contract, \( R^1_l = 0 \).

**Proof.** See Appendix.

Levered equity maximizes the financier’s participation on the cash flows in the high cash flow states, while minimizing his participation in the low cash flow states. In other words, it maximizes the investor’s sensitivity to the owner-manager’s type. This helps in making the investor’s payoff from the non-refinanced firm (which has a low success probability) similarly sensitive to achieving the upside as his payoff from a security on the refinanced firm (which has a higher success probability). As argued above, the latter is crucial for minimizing cross-subsidization and, thus, overinvestment.

An interesting aspect of our result of how levered equity financing affects the firm’s "debt capacity" is that it appears to run counter to conventional wisdom. Indeed, issuing such a security limits, rather than expands (as stressed by the previous literature), the scope for projects that can be refinanced at the interim stage. This is welfare increasing, as the owner-manager faces a problem of overinvestment at this stage, the cost of which she must bear at the initial financing stage.

Finally, we derive a condition when it is feasible to offer an initial contract \( R^1 \), which can lead to efficient refinancing at the interim stage. Using (17), we obtain:

**Proposition 7** With a weak investor at \( t = 2 \), the first-best refinancing outcome \( (q^*_M = q_{FB}) \) is obtained if

\[
x_l \geq K_2 + \frac{p_{NB}p_{YG} - p_{YB}p_{NG}}{(p_{YG} - p_{YB})p_{N}(\hat{q})} K_1 + \frac{(p_{NG} - p_{NB})p_{Y}(\hat{q})}{(p_{YG} - p_{YB})p_{N}(\hat{q})} \max \left( 0, K_1 - p_{N}(\hat{q}) \Delta x \right),
\]

(20)

There is overinvestment with \( q^*_M < q_{FB} \) if (20) is not satisfied. In either case, the security that is held by outside investors after refinancing at \( t = 2 \) is unambiguously flatter from the investor’s perspective than the initial security (\( \Delta R^2 < \Delta R^1 \) and \( R^2_l > R^1_l \)).

**Proof.** See Appendix.
The intuition for condition (20) is straightforward. If $x_t$ is large enough, the owner-manager can offer the investor a security at the refinancing stage that compensates him both for his initial claim and $K_2$, without having to promise him a large participation on the cash flows in the high cash flow state. This makes it possible to limit the dependence of the refinancing security on achieving the upside $\Delta R$. Thus, it also makes it possible to offer an initial security, which—despite the lower success probability of the not-refinanced firm—can be just as sensitive to the firm’s probability of success, so that the investor can be made indifferent between refinancing and not refinancing for every type.

4.3 Assumptions and Interpretations of Contracting II

An alternative way to motivate the assumption of a weak investor and the inability to commit both parties to a refinancing contract at $t = 1$ is by assuming that the owner-manager can threaten to withhold essential human capital that is needed to grow the business (cf. Hart and Moore, 1994). Indeed, such a threat shifts the bargaining power to the owner-manager at the interim stage, and it can be shown that it also necessitates staging of investment (Neher, 1999). Hence, with the weak-investor case we can address financing of firms that depend heavily on human capital provided by insiders. When we take this interpretation, we expect that investors are more likely to suffer from the threat of a hold-up in countries with weak protection of investor rights or with weak legal enforcement of these rights.\(^{28}\) We explore the respective interpretations in more detail in Section 5.

**Refinancing from a New (Weak) Investor.** Equally important is the interpretation that we offered initially, according to which the owner-manager has access to a competitive market for capital at the refinancing stage. Then, if the owner-manager cannot commit to raising refinancing from the initial investor (e.g., for the reasons above or because the initial investor is cash-constrained as in Section 3.3), new investors will compete to refinance the firm. In this case, when the owner-manager cannot be committed to raising financing from the initial investor, she will always go for the best refinancing terms. We have already characterized these terms in Propositions 4 and 5, implying that the financial

\(^{28}\)Note that countries with weaker investor protection could also be faced by the problem that the pledgeability of the assets is lower. This only gives an additional reason for debt financing when information asymmetry is an issue (e.g., Bolton and Scharfstein, 1996).
claims held by the owner-manager after refinancing will be the same. We relegate again a more detailed discussion to Appendix B.

5 Implications and Evidence

5.1 Start-Up Financing and Young Firm Financing

A suitable environment to test the contrasting predictions of our model is the financing of young firms with growth potential. The discussion of fly-by-night operators and the inalienability of human capital, which we use as one way of motivating our model assumptions in Sections 3.3 and 4.3 are then especially relevant (e.g., Rajan, 1992). In this context, we can take the two-stage nature of our model literally. Initially, when marketing her business plan to investors, we stipulate that the owner-manager can bridge any information asymmetry vis-à-vis outsiders.29 At a later stage, however, when fresh financing has to be raised, the information gap with outside investors may have widened as the owner-manager is more involved in the firm’s day-to-day operations, providing her with a better insights about the potential profitability of the firm’s investment opportunities. If capital needs to be raised at short notice, there may then be insufficient time to credibly divulge all relevant information to investors.

Taking this perspective, the contrasting implications of our model should provide for a better understanding of how firms’ financing strategies depend on whether they are facing a specialist or a nonspecialist investor. As argued above, a firm that needs to raise financing from specialist investors is less likely to face a competitive market for capital when it needs fresh financing at short notice.

Implication 1: (i) When raising financing from specialist investors, firms should use up their debt capacity early and then decrease their leverage whenever raising capital for investments marked by asymmetric information. (ii) Firms should follow the opposite strategy if they face nonspecialist investors: They should initially avoid debt and then increase leverage whenever raising capital for projects marked by asymmetric information.

Implication 1 is consistent with financing patterns in start-up firms. One of the main source of financing for such firms, which is financing from friends and family, takes the

29In fact, it could also be argued that at this stage the investor, due to his industry knowledge as a "long-run" player, may be better able to gauge the prospects of a business plan (e.g., Inderst and Müller 2006).
form of equity financing (Berger and Udell, 1998). Also business angels who, while also not specialist, are more sophisticated investors use predominantly equity financing (Wong et al., 2009). Firms take then on debt financing only in later stages of their development. While one could attribute the reluctance to use debt to the lack of a stable cash flow stream, this does not explain why equity investors do not take preferential liquidation rights, as we predict would be the case for specialist investors.

Indeed, preferential liquidation rights make equity financing more debt-like, and such rights are used predominantly by specialist investors, such as venture capitalists (Gompers, 1995; Kaplan and Strömberg, 2003). Thus, on the one hand, we relate to extant papers that try to explain why venture capitalists [VC] provide funds in exchange for senior securities with the option of converting them into junior ones as venture capitalists take the firm to the equity markets (recall our previous interpretations in terms of convertible securities and financing from new investors in Section 3.3).\textsuperscript{30} However, on the other hand, our contribution to this literature is to show that the exact opposite financing patterns are optimal for non-specialist investors. Furthermore, we predict:

**Implication 2:** (i) If the enforcement of investor rights is strong, investors should initially demand more liquidation rights in initial financing stages, and they should take a higher participation on the upside in later financing stages. (ii) If the enforcement of investor rights is weak, investors should follow the opposite strategy, initially demanding a higher participation on the upside and increasing their protection on the downside in later financing rounds.

Our intuition for this prediction is borrowed again from Sections 3.3 and 4.3. The enforcement of investor rights can help predict whether entrepreneurs are more or less likely to hold up initial investors (which is one of our motivations why both parties cannot pre-commit to refinancing terms at \( t = 1 \)) and whether initial investors would be able to maintain a strong bargaining position in such cases. Derived from this prediction, we expect:

**Implication 3.** U.S.-style VC contracts are not optimal in countries with weak enforcement of investor rights when entrepreneurs are likely to develop an information advantage over time.

\textsuperscript{30}See, e.g., Berglöf (1994), Cornelli and Yosha (1997), Hellmann (2006). Contrary to our focus on asymmetric information, this literature has focused on effort incentives and allocation of control rights.
Recent empirical work shows that the typical U.S.-style VC contracts are less common outside the U.S. (e.g., Cumming 2008). In particular, Lerner and Schoar (2005) find that, while convertible preferred equity is common in strong enforcement countries, common stock is the favored instrument in weak-enforcement countries. This is consistent with our results, as we predict a switch from senior to junior financing when the initial investor is strong in $t = 2$, but the opposite when the investor is weak. The latter case is also consistent with Kaplan et al. (2007). They find that equity is more frequently issued in first rounds in countries with weaker creditor rights, and that the more successful firms that have issued equity in earlier rounds require more protection on the downside in later rounds.\(^{31}\)

Our model also points to a previously neglected aspect in the, admittedly multifaceted, discussion whether banks should be allowed to buy equity in start-up firms.

**Implication 4.** *Banks expecting to finance a firm also at a later stage in its development, can reduce future credit rationing by making equity investments in earlier financing rounds.*

Hellmann et al. (2008) analyze banks’ private equity investments in start-up firms. They find that banks direct their *equity* investments towards later stages of the development of start-ups. The same banks are then significantly more likely to subsequently grant a loan (i.e. debt financing) to these firms. This finding, which is also supported by Fang et al. (2010), is in line with our predictions when we apply the following reasoning. Banks are more likely to lack specific industry knowledge and management skills, implying that firms are less likely to end up locked-in compared to when they raise financing from specialist investors, such as venture capitalists. This translates to our weak-investor case, for which we predict that it is optimal to shift from equity to debt financing—i.e., initially avoid debt financing.

### 5.2 Long-Term vs. Short-Term Financial Strategy

Interpreting our model more broadly, allows us to address the recent conflicting evidence on financial contracting. In particular, while Shyam-Sunder and Myers (1999) find strong support for the pecking order theory in a sample of large firms, their findings seem to be reversed as the focus shifts towards smaller firms (Frank and Goyal, 2003; Leary and

\(^{31}\)Kaplan et al. (2007) interpret this as a sign of learning that US-style contracts are more efficient.
The evidence suggests that a firm’s debt capacity and its ability to tap the capital markets at competitive terms are additional important factors determining the type of new issues (Lemmon and Zender, 2007).

By focusing explicitly on these factors, we derive predictions, which seem in line with the above evidence. In particular, we can interpret our results also in terms of a firm’s long-term and short-term financial strategy. Our motivation for this is as follows. Even though we only look at a stylized three period model, this setting is sufficient for deriving our main results on how initial contracting creates countervailing incentives for new rounds of financing. We stipulate that information asymmetry should be more relevant when financing is needed at short notice, e.g., to realize an investment opportunity that is open only for a short time. Instead, when a firm chooses its long-term (or target) financial structure, it would have more time to narrow an informational gap vis-à-vis outside investors. These two choices correspond to the security design problems in $t = 2$ and $t = 1$ of our model, where the security held by outside investors after refinancing in $t = 2$ may represent a firm’s temporary deviation from its long-term financial strategy. Importantly, observe that if refinancing comes from a new investor in our model, the owner-manager and the initial investor jointly hold a claim, which has the same shape as the initial investor’s security. Thus, our model is consistent with old investors optimally retaining the same (type of) claim even if new investors, joining at a later stage, take different types of securities.

Implication 5. *Firms that have access to competitive financing pursue a lower target leverage and deviate upwards from this target when they need to raise capital at short notice. The opposite holds for firms that don’t have access to competitive financing.*

When we take this perspective, an application of our weak-investor case yields the following implications. Large firms with access to capital markets may choose "armslength" financing as a long-term capital strategy, when even in $t = 2$ of our model they will face a competitive financial market (albeit one plagued by information asymmetry). Indeed, Fama and French (2005) find that equity issuance is often observed in companies that are not under duress. They interpret this as a violation of Myers and Majluf’s (1984) pecking order theory (cf. also Leary and Roberts, 2010). Our model shows that this allows firms

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32 In a multiperiod setting in which a firm repeatedly undertakes finite-horizon projects, the firm will revert to our $t = 1$-type of financing whenever it has to raise fresh financing (after repaying old investors) and information asymmetry is not an issue.
to build up financial flexibility, so that they can use up their "debt capacity" if financing is needed at short notice (cf. Lemmon and Zender, 2009).

More important is our novel insight that these predictions reverse for firms that raise financing from (relationship) investors who hold bargaining power, when additional financing is needed at a relatively short notice. These firms should prefer debt financing when information asymmetries are not an issue, which is in line with the survey evidence in Graham and Harvey (2001). These predictions are also in line with the evidence that debt is the predominant choice for small firms that have close ties to a single bank (Petersen and Rajan, 1994; Detragiache et al. 2000; see also our discussion in Section 3.3). Our results also find support in Leary and Roberts (2010) and Gomes and Phillips (2012) who document that smaller growth firms indeed prefer debt when asymmetric information is not a factor and, thus, use up their debt capacity. In summary, by taking into account that the lack of access to competitive financing is a distinguishing feature when comparing small and private with large and public firm, our model could help shed some light on why these firms differ in their choices of how to build up financial flexibility and raise new financing.

Implication 6. (i) Small, young, and private firms are more likely to issue equity when raising financing at short notice. They issue debt when information asymmetry is not an issue. (ii) Large firms are more likely to follow the pecking order by avoiding debt when not under duress and using up their debt capacity when seeking financing at short notice.

Finally, based on Propositions 3 and 7, we expect that the countervailing incentives created by initial financing might not be sufficiently strong when the ability to repay investors dramatically depends on new successful financing rounds. In such cases, investment inefficiencies, such as under- and overinvestment, are more likely.

Implication 7. Firms for which success is more dependent on new rounds of outside financing, such as growth firms in earlier stages of their life-cycle, are more likely to suffer from investment inefficiencies.

6 Conclusion

We develop a theory of how firms build up financial flexibility and their optimal capital structure. The key linkage between the firm’s choice of initial financing, which is raised under symmetric information "for the long-term", and its subsequent financing under
asymmetric information is that the former affects the "outside options" for both insiders and outside investors when new financing must be raised. The model’s implications for the optimal financial structure and its change over time differ sharply depending on whether the bargaining power at the refinancing stage lies (more) with the firm, as it faces a competitive capital market, or with initial investors.

If incumbent investors have bargaining power at the refinancing stage, there can be underinvestment, as they attempt to extract higher "rents" from better informed insiders (the "owner-manager" in our model). This underinvestment can be minimized, and even eliminated, by adequately designing the initial financing contracts. These contracts create countervailing incentives when fresh financing needs to be raised under asymmetric information. Specifically, when investors have bargaining power in such cases, leverage decreases when the firm raises additional financing. Then, the firm’s long-term (target) capital structure should preserve the firm’s "equity capacity", as this mitigates underinvestment in the future. Instead, a problem of overinvestment is likely if bargaining power lies with the better informed insiders. Then the initial (or long-term) leverage decision serves to reduce an overinvestment problem: The firm should avoid issuing debt, which limits its incentives to overinvest when issuing debt under asymmetric information.

Our implications are richer than those derived from most standard theories of security design under asymmetric information, and they are largely in line with the sometimes contrasting recent evidence in the literature. Our polar cases with strong or weak investors may also shed light on cross-country differences in start-up and young firm financing, on differences in financing patterns between specialist and nonspecialist investors, as well as on differences between early- and later-stage financing. We also derive implications for firms’ choice of optimal target financial structure and the direction of temporary deviations when financing has to be raised at relatively short notice. Though we show how our restriction on the amount of financing that is raised initially (or, for that matter, for the long term) can be endogenized, a firm may hold free cash as part of its optimal financial strategy when the agency problems that this engenders are not too large. This possibly represents an avenue for future research.
References


Appendix A

Proof of Proposition 1. The proof follows from a sequence of auxiliary results.

Claim 1. The first-best security $\hat{R}$ is feasible if and only if condition (9) holds.

Proof. Note first that if the initial security $R^1$ is feasible, then from $\Delta x - \Delta R^1 \geq 0$ and from the construction of $\Delta \hat{R}$ in (5) we also have that $\Delta x - \Delta \hat{R} \geq 0$. Further, as condition (1) implies that $p_Y G - p_Y B > p_N G - p_N B$, we have from (5) that $\Delta \hat{R} \geq 0$. To see next that $\hat{R}_l \leq x_l$ holds, we substitute (5) into (6) and obtain

$$\hat{R}_l = R^1_l - \left( \frac{p_Y G p_{NB} - p_Y B p_{NG}}{p_Y G - p_Y B} \right) (\Delta x - \Delta R^1) . 
(21)$$

This implies from (1) that $\hat{R}_l < R^1_l$ and thus also $\hat{R}_l < x_l$, given that $R^1$ was feasible. The remaining condition is thus that $\hat{R}_l \geq 0$, which from (21) is just condition (9). From this it also follows that (9) is necessary for $\hat{R}$ to be feasible. Q.E.D.

The next claim establishes that by optimality of $R^2$, the set of owner-manager types that accepts, $q \in A$, is always characterized by a cutoff $q^*$. We argue to a contradiction, showing that if there existed a security $R^2$ so that the owner-manager would prefer acceptance for low but not for high $q$, then the first-best contract $\hat{R}$ would be feasible, instead. Then, as argued in the main text, it is clearly optimal to offer $\hat{R}$. Q.E.D.

Claim 2. If a security $R^2$ satisfying $u_Y(R^2, 0) > u_N(R^1, 0)$ together with $u_Y(R^2, 1) < u_N(R^1, 1)$ is feasible, then also the first-best security $\hat{R}$ is feasible.

Proof. Note first that from the assumed inequalities $u_Y(R^2, 0) > u_N(R^1, 0)$ (owner-manager prefers refinancing for $q = 0$) and $u_N(R^1, 1) > u_Y(\hat{R}^2, 1)$ (owner-manager prefers no-refinancing for $q = 1$), $\Delta \hat{R} < \Delta R^2$ must hold to ensure that the slope of $u_Y(R^2, q)$ is strictly smaller than that of $u_Y(\hat{R}, q)$. But then $u_Y(R^2, 0) > u_Y(\hat{R}, 0)$ implies that $R^2_l < \hat{R}_l$. By the assumed feasibility of $R^2$, we have from this that $\hat{R}_l > 0$, so that (9) holds strictly. Q.E.D.

From Claims 1-2 refinancing takes place whenever $q \geq q^*$ (with $q^* = q_{FB}$ if $\hat{R}$ is feasible). It is straightforward to rule out optimality of the case $q^* = 1$ (zero probability of
If \( q^* < 1 \), then the cutoff is pinned down by the requirement that \( u_Y(R^2, q^*) = u_N(R^1, q^*) \) (cf. also (7)).

**Claim 3.** Levered-equity with \( R_i^2 = 0 \) is the uniquely optimal security for the investor if the first-best security \( \hat{R} \) is not feasible.

**Proof.** We argue to a contradiction. Suppose that, so as to implement some \( q^* \in [0, 1] \), another security \( R^2 \) with \( R_i^2 > 0 \) were optimal. Choose now \( \hat{R}^2 = (0, \Delta \hat{R}^2) \) so that \( u_Y(\hat{R}^2, q^*) = u_N(R^1, q^*) \), which implies that the owner-manager’s conditional expected acceptance set, \([q^*, 1]\), remains unchanged, while at \( q^* \) the investor’s conditional expected payoff does not change: \( v_Y(\hat{R}^2, q^*) = v_Y(R^2, q^*) \). However, as \( u_Y(\hat{R}^2, q^*) = u_Y(R^2, q^*) \) together with \( \hat{R}^2_i = 0 < R_i^2 \) must imply that \( \Delta \hat{R}^2 > \Delta R^2 \), we have that \( v_Y(\hat{R}^2, q) - v_Y(R^2, q) > 0 \) holds for all \( q > q^* \). Thus, provided it is feasible, the investor is indeed strictly better off under the newly constructed contract \( \hat{R}^2 \).

It remains to show that \( \hat{R}^2 \) is indeed feasible. By the assumed feasibility of \( R^2 \) and construction of \( \hat{R}^2 \), this is the case if \( \Delta \hat{R}^2 \leq \Delta x \). (The other feasibility restrictions on \( \hat{R}^2 \) are satisfied by feasibility of \( R^2 \).) From \( u_Y(\hat{R}^2, q^*) = u_Y(R^2, q^*) \) and \( \hat{R}^2_i = 0 \), we can obtain

\[
\Delta \hat{R}^2 = \frac{R_i^2}{p_Y B + q^* (p_Y G - p_Y B)} + \Delta R^2,
\]

so that \( \Delta \hat{R}^2 \leq \Delta x \) holds whenever

\[
0 \leq -R_i^2 + (p_Y B + q^* (p_Y G - p_Y B)) (\Delta x - \Delta R^2).
\]  

(22)

However, (22) is implied by the assumption that the first-best security is not feasible, i.e., that (9) does not hold. To see this, note first that from the definition of \( q^* \), i.e. \( u_Y(R^2, q^*) = u_N(R^1, q^*) \), condition (22) is equivalent to

\[
0 \leq -R_i^1 + (p_{NB} + q^* (p_{NG} - p_{NB})) (\Delta x - \Delta R^1).
\]  

(23)

As, by assumption, \( \hat{R} \) is not feasible, it holds from transforming the "first-best condition" (9) that

\[
0 < -R_i^1 + \left(\frac{p_Y G p_{NB} - p_Y B p_{NG}}{p_Y G - p_Y B}\right) (\Delta x - \Delta R^1) < -R_i^1 + (p_{NB} + q^* (p_{NG} - p_{NB})) (\Delta x - \Delta R^1),
\]

(29)
where the last inequality holds for any $q^*$. But this is just what we needed to show (condition (23)). Q.E.D.

To conclude the proof of Proposition 1, we solve the investor’s program when $\hat{R}$ is not feasible. For this observe that from the indifference condition of the owner-manager at $q^*$, (7), we have that

$$\Delta R^2 = \Delta x - \frac{R^2_t - R^1_t + [p_{NB} + q^* (p_{NG} - p_{NB})] (\Delta x - \Delta R^1)}{p_{YB} + q^* (p_{YG} - p_{YB})},$$

(24)

from which we obtain explicitly

$$\frac{d\Delta R^2}{dq^*} = \frac{(R^2_t - R^1_t) (p_{YG} - p_{YB}) + (p_{NB} p_{YG} - p_{YB} p_{NG}) (\Delta x - \Delta R^1)}{[p_{YB} + q^* (p_{YG} - p_{YB})]^2} > 0,$$

(25)

where the inequality follows as $R^2_t = 0$ when (9) does not hold.

We can next substitute for the acceptance set $A = [q^*, 1]$ into the investor’s objective function (3), where $q^*$ is given by the indifference condition for the owner-manager (cf. condition (7)). Differentiating with respect to $q^*$, we have the first-order condition (cf. also (10))

$$-[s_Y(q^*) - s_N(q^*)] f(q^*) + \frac{d\Delta R^2}{dq^*} \int_{q^*}^1 \frac{dv_Y(R^2, q)}{d\Delta R^2} dF(q) = 0,$$

where the first term follows from $s_d(q) = u_d(R^t, q) + v_d(R^t, q)$ and (7). As $\frac{d\Delta R^2}{dq^*} > 0$,

$$\frac{dv_Y(R^2, q)}{d\Delta R^2} = p_{YB} + q(p_{YG} - p_{YB}),$$

while $s_Y(q^*) - s_N(q^*)$ is strictly increasing and equal to zero when $q^* = q_{FB}$, we have that $q^* > q_{FB}$.

Finally, we show that levered equity not only maximizes the investor’s ability to extract rent from the owner-manager, but it also induces him to implement a more efficient $q^*$. To see this, suppose that $R^2_t = \varepsilon > 0$. The cross-partial of the investor’s expected payoff with respect to $q^*$ and $\varepsilon$ shows that it is supermodular in these variables

$$\frac{(p_{YG} - p_{YB})}{[p_{YB} + q^* (p_{YG} - p_{YB})]^2} \int_{q^*}^1 \frac{dv_Y(R^2, q)}{d\Delta R^2} dF(q) > 0.$$ 

Therefore, by monotonic selection arguments, $q^*$ increases in $\varepsilon$. Thus, reducing $\varepsilon$ leads to a lower $q^*$. Q.E.D.

Proof of Proposition 2. The proof is by contradiction. Suppose that $R^1$ with $R^1_t < x_t$
were optimal and that there is inefficiency at $t = 2$. By Proposition 1 the investor chooses a security $R^2 = (0, \Delta R^2)$ that implements a cutoff $q^*_{old} > q_{FB}$. Note that we relegate to the end of the proof the argument why, in the equilibrium of the whole game, the investor must always choose the most efficient cutoff from his optimal correspondence and thus plays a pure strategy. We proceed in several steps.

**Step 1.** We start by constructing $\tilde{R}^1 = (x_i, \Delta \tilde{R}^1)$ together with $\tilde{R}^2 = (0, \Delta \tilde{R}^2)$ so that two conditions are satisfied: The owner-manager is still indifferent at his old cutoff $q^*_{old}$ and, holding this cutoff fixed, the ex ante payoff for both parties stays the same. By construction, it then holds that

$$0 = \int_0^{q^*_{old}} [v_N(\tilde{R}^1, q) - v_N(R^1, q)] dF(q) + \int_{q^*_{old}}^1 [v_Y(\tilde{R}^2, q) - v_Y(R^2, q)] dF(q),$$

(26)

together with $u_Y(R^2, q^*_{old}) = u_N(R^1, q^*_{old})$ and $u_Y(\tilde{R}^2, q^*_{old}) = u_N(\tilde{R}^1, q^*_{old})$. To ease exposition, let

$$\tilde{p}_N := p_{NB} + (p_{NG} - p_{NB}) \int_0^{q^*_{old}} q \frac{dF(q)}{F(q^*_{old})},$$

$$\tilde{p}_Y := p_{YB} + (p_{YG} - p_{YB}) \int_{q^*_{old}}^1 q \frac{dF(q)}{1 - F(q^*_{old})}.$$ 

Further, let $p_d(q) := p_{dB} + q (p_{NG} - p_{NB})$ be defined as in (13) in the main text. Recall also that, for given $q^*$ and $R^1$, $\Delta R^2$ is given in (24). Plugging into (26) we have

$$0 = \left( x_i - R^1_i + \tilde{p}_N \left( \Delta \tilde{R}^1 - \Delta R^1 \right) \right) F(q^*_{old}) + \frac{\tilde{p}_Y}{p_Y(q^*_{old})} \left( x_i - R^1_i + p_N(q^*_{old}) \left( \Delta \tilde{R}^1 - \Delta R^1 \right) \right) (1 - F(q^*_{old})), $$

from which we can express $\Delta \tilde{R}^1$ as

$$\Delta \tilde{R}^1 = \Delta R^1 - \left( \frac{x_i - R^1_i}{\tilde{p}_N} \right) \left( \frac{p_Y(q^*_{old}) F(q^*_{old}) + \tilde{p}_Y (1 - F(q^*_{old}))}{p_Y(q^*_{old}) F(q^*_{old}) + \frac{p_N(q^*_{old})}{\tilde{p}_N} \tilde{p}_Y (1 - F(q^*_{old}))} \right).$$

(27)

**Step 2.** We now show that, if offered $\tilde{R}^1$ in the initial period, the investor will actually offer a different security $\tilde{R}^2 \neq \tilde{R}^2$ at $t = 2$ that implements a strictly lower cutoff. For this purpose we look at the expected payoff of the investor at $t = 2$ when he is faced with $R^1$ or $\tilde{R}^1$, respectively, and then apply monotone comparative statics.
As the second security is levered equity with \( R^2_l = \tilde{R}^2_l = 0 \), the indifference condition of the owner-manager at a cutoff \( q^* \) gives the respective value \( \Delta R^2 \) as a unique function of \( R^1 \) and \( q^* \) only (cf. (24)). We use \( \Delta R^2 (q^*, R^1) \) and \( \Delta R^2 (q^*, \tilde{R}^1) \), making thereby explicit that \( \Delta R^2(\cdot) \) presently denotes a function. Next, we define the investor’s expected payoff at \( t = 2 \) for some \( q^* \) and an initial contract \( R^1 \) by

\[
V (q^*, R^1) := \int_0^{q^*} v_N (R^1, q) dF (q) + \int_{q^*}^{1} v_Y (R^2, q) dF (q) .
\]

Defining \( V(q^*, \tilde{R}^1) \) accordingly, we now show that the difference \( V(q^*, \tilde{R}^1) - V(q^*, R^1) \) is decreasing in \( q^* \). (Importantly, note that \( q^* \) is not an optimal selection from the investor’s optimization problem at this point.) After some transformations we have

\[
\frac{d}{dq^*} \left[ V(q^*, \tilde{R}^1) - V(q^*, R^1) \right] = \int_{q^*}^{1} p_Y (q) \left( \frac{d\Delta R^2 (q^*, \tilde{R}^1)}{dq^*} - \frac{d\Delta R^2 (q^*, R^1)}{dq^*} \right) dF (q).
\]

Next, using (25) and (27), we obtain an explicit expression for the second term under the integral in (29). Importantly, observe that \( \tilde{R}^1 \) is defined as a function of \( q^* \) and not \( q^* \). We have

\[
\frac{d\Delta R^2 (q^*, \tilde{R}^1)}{dq^*} - \frac{d\Delta R^2 (q^*, R^1)}{dq^*} = \frac{(x_l - R^1_l) (p_{YG} - p_{YB}) + (p_{NB} p_{YG} - p_{YBP_N}) (\Delta \tilde{R}^1 - \Delta R^1)}{p_Y(q^*)^2} \]

\[
= \frac{-(x_l - R^1_l) (p_{YG} - p_{YB})}{p_Y(q^*)^2} \times \left( 1 - \frac{(p_{NB} p_{YG} - p_{YBP_N})}{(p_{YG} - p_{YB}) \hat{p}_N} \frac{p_Y(q^*_{old}) F(q^*_{old}) + \hat{p}_Y (1 - F(q^*_{old}))}{p_Y(q^*_{old}) F(q^*_{old}) + \hat{p}_Y (1 - F(q^*_{old}))} \right) \]

\[
< \frac{-(x_l - R^1_l) (p_{YG} - p_{YB})}{p_Y(q^*)^2} \left( 1 - \frac{(p_{NB} p_{YG} - p_{YBP_N})}{(p_{YG} - p_{YB}) \hat{p}_N} \right) < 0,
\]

where for the first inequality we use that \( p_N (q^*_{old}) / \hat{p}_N > 1 \), and for the second inequality we use that \( \hat{p}_N > p_{NB} \). From (29), it follows, therefore, that

\[
\frac{dV(q^*, \tilde{R}^1)}{dq^*} < \frac{dV(q^*, R^1)}{dq^*} .
\]

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Thus, the difference \( V(q^*, \tilde{R}^1) - V(q^*, R^1) \) decreases in \( q^* \). By standard monotone selection arguments, strictly decreasing differences imply the following: Any optimal cutoff \( q_{new}^* \) that the investor chooses given \( \tilde{R}^1 \) is lower than any optimal cutoff \( q_{old}^* \) that he selects given \( R^1 \), so that \( q_{new}^* < q_{old}^* \).

**Step 3.** In this step we show that the owner-manager is indeed better off with the considered deviation. Observe first that by construction both the owner-manager and the investor are ex ante indifferent between \((R^1, R^2)\) and \((\tilde{R}^1, \tilde{R}^2)\), when holding \( q^* = q_{old}^* \) constant. But as \( q_{new}^* < q_{old}^* \), it follows from (25) \( d\Delta R^2/dq^* > 0 \) that for the new optimal second-period contract, which implements some \( q_{new}^* \), we have that \( \Delta R^2(q_{new}^*, \tilde{R}^1) < \Delta R^2(q_{old}^*, \tilde{R}^1) \). Denote this contract by \( \tilde{R}^2 \). Hence, \( u_Y(\tilde{R}^2, q) > u_Y(\tilde{R}^2, q) \) holds for all \( q \), and the ex ante expected payoff of the owner-manager with \((\tilde{R}^1, \tilde{R}^2)\) is strictly higher than with either \((\tilde{R}^1, \tilde{R}^2)\) or \((R^1, R^2)\), respectively. To finish this step, note that by optimality of \( \tilde{R}^2 \) the investor is also at least weakly better off with \((\tilde{R}^1, \tilde{R}^2)\) than with \((\tilde{R}^1, \tilde{R}^2)\), so that \((\tilde{R}^1, \tilde{R}^2)\) satisfies the investor’s break-even condition. Taken together, this contradicts the optimality of \( R^1 \).

To conclude the proof, we can make use of the preceding results to show that, as asserted in the main text, in equilibrium the investor chooses a pure strategy and, thereby, implements the most efficient (i.e., lowest) \( q^* \) in case his optimal contractual choice at \( t = 2 \) is not uniquely determined. Given a debt security at \( t = 1 \), one can use the indifference condition (7) to express the second-stage levered equity security \( R^2 \) as a function of \( \Delta R^1 \) and \( q^* \). We can thus write \( V(q^*, \Delta R^1) \) instead of \( V(q^*, R^1) \) (cf. expression (28)). Further, we use \( Q^* = \arg \max V(q^*, \Delta R^1) \) to denote the optimal choice correspondence subject to (12). Observe now that given \( R^1 \), \( V(q^*, \Delta R^1) \) is strictly submodular in \( q^* \) and \( \Delta R^1 \):

\[
\frac{\partial^2 V(q^*, \Delta R^1)}{\partial q^* \partial \Delta R^1} = - \frac{(\rho_{NP} \rho_{YG} - \rho_{YP} \rho_{NG})}{\rho_Y(q^*)^2} \int_{q^*}^{1} p_Y(q) \, dF(q) < 0.
\]

Therefore, again by monotonic selection arguments, relaxing the investor’s ex ante participation constraint by increasing \( \Delta R^1 \) results in a lower set \( Q^* \). Since \( Q^* \) is monotonic, it must be almost everywhere a singleton and continuous. Then, while the investor’s payoff is continuous in \( \Delta R^1 \) everywhere, the owner-manager’s expected payoff is continuous a.e. and, where \( Q^* \) is not a singleton, the owner-manager strictly prefers the lowest (most efficient) value \( q^* = \min Q^* \). Consequently, analogously to a tie-breaking condition, by
optimality for the owner-manager the investor must choose $q^* = \min Q^*$ with probability one in equilibrium. Q.E.D.

**Proof of Proposition 3.** Recall from Proposition 1 that if the investor implements $q_{FB}$, then $u_N(R^1, q) = u_Y(\hat{R}, q)$ holds for all $q \in [0, 1]$. Using this and the identity $s_d(q) = v_d(R^t, q) + u_d(R^t, q)$ to plug into (12), if the investor just breaks even at $t = 1$, one can express $\Delta R^1$ as

$$\Delta R^1 = \Delta x - \frac{S_{FB} - (x_l - R^1_l)}{p_N(q)}.$$  

A first-period security that satisfies (30) is feasible if

$$x_l \geq R^1_l \geq 0,$$

$$\Delta x \geq \Delta R^1 = \Delta x - \frac{S_{FB} - (x_l - R^1_l)}{p_N(q)} \geq 0,$$

$$R^1_l \geq \left(\frac{p_{NB}p_{YG} - p_{YB}p_{NG}}{p_{YG} - p_{YB}}\right) \frac{S_{FB} - (x_l - R^1_l)}{p_N(q)},$$

where the last inequality is just condition (9) from Proposition 1. These three conditions can be rewritten as follows:

$$\min (x_l, x_l + p_N(q)\Delta x - S_{FB}) \geq R^1_l \geq \max \left( x_l - S_{FB}, \frac{p_{NB}p_{YG} - p_{YB}p_{NG}}{(p_{NG} - p_{NB})p_Y(q)} (S_{FB} - x_l) \right).$$

Since the left-hand side must be greater than the right-hand side, it must be that

$$x_l \geq \max \left( x_l - S_{FB}, \frac{p_{NB}p_{YG} - p_{YB}p_{NG}}{(p_{NG} - p_{NB})p_Y(q)} (S_{FB} - x_l) \right) + \max (0, S_{FB} - p_N(q)\Delta x).$$

Simple transformations yield condition (14). If (14) holds, by optimality for the owner-manager we then have that $q^* = q_{FB}$: The optimal security $R^1$ then maximizes joint surplus and, by making the investor just break even, achieves the maximum feasible payoff for the owner-manager.

We finally formalize the argument from the main text that in equilibrium $q^* > q_{FB}$ if (14) does not hold. That is, though we noted in the text that first-best efficiency could be achieved by granting the investor a sufficiently large payoff, this is not optimal. Using the optimality of debt, consider the owner-manager’s optimal choice of $\Delta R^1$. Differentiating
her expected profits with respect to $\Delta R^1$ yields at points of differentiability of $q^*(\Delta R^1)$
\[
(s_N(q^*) - s_Y(q^*)) f(q^*) \frac{dq^*}{d\Delta R^1} \\
- \frac{d}{d\Delta R^1} \left( \int_0^{q^*(\Delta R^1)} v_N(R^1, q) dF(q) + \int_{q^*(\Delta R^1)}^1 v_Y(R^2, q) dF(q) \right) \\
- \frac{d}{dq^*} \left( \int_0^{q^*(\Delta R^1)} v_N(R^1, q) dF(q) + \int_{q^*(\Delta R^1)}^1 v_Y(R^2, q) dF(q) \right) \frac{dq^*}{d\Delta R^1} \\
= - (s_Y(q^*) - s_N(q^*)) f(q^*) \frac{dq^*}{d\Delta R^1} - \left( \bar{p}_N F(q^*) + \frac{p_N(q^*)}{p_Y(q^*)} \bar{p}_Y (1 - F(q^*)) \right),
\]
where we used that the third line is zero by the investor’s FOC at $t = 2$. Similarly, $dq^*/d\Delta R^1$ is computed from the solution to the investor’s optimization problem at $t = 2$ and, from (7), we use that $\Delta R^2$ is a function of $\Delta R^1$ in the last line. This expression is strictly negative at $q^* = q_{FB}$, since then the first term is zero. Q.E.D.

**Proof of Proposition 4.** To obtain condition (17), note first that by construction $\Delta x \geq \Delta \hat{R}^M \geq 0$ is always satisfied from feasibility of $R^1$ and from condition (1). Further, $\hat{R}^M_i \geq 0$ never binds as after substitution
\[
\hat{R}^M_i = R^1_i + \frac{p_{NB} p_{YG} - p_{YB} p_{NG}}{p_{YG} - p_{YB}} \Delta R^1_i + K_2.
\]
The remaining condition $x_i \geq \hat{R}^M_i$ transforms to (17). Having derived $\hat{R}^M$ this way, (1) implies that $\Delta \hat{R}^M \leq \Delta R^1$ and $\hat{R}^M_i > R^1_i$, where the inequalities are strict if $\Delta R^1_i > 0$.

By the arguments in the main text, in equilibrium refinancing is obtained by types $q > q_{FB}$ but not by types $q < q_{FB}$, and types $q_{FB} < q < 1$ must obtain refinancing by issuing $\hat{R}^M$. Further, the offer is accepted with probability one. It is straightforward to support this outcome as a Perfect Bayesian Equilibrium by adequately choosing out-of-equilibrium beliefs. Q.E.D.

**Proof of Proposition 5.** The proof follows from a series of results. Arguing that $v_Y(R^2, q)$ can only cross $v_N(R^1, q)$ from below, we first show that there is no equilibrium in which the highest type that issues a certain security extracts the whole surplus from refinancing. For this result (Claim 2) we make use of the following auxiliary result.

**Claim 1.** If a security $R^2$ satisfying $v_N(R^1, 0) < v_Y(R^2, 0)$ and $v_N(R^1, 1) > v_Y(R^2, 1)$
is feasible, so that \( v_Y(R^1, q) \) crosses \( v_N(R^1, q) \) from above, then also the first-best security \( \hat{R}^M \) is feasible.

**Proof.** From the definition of \( \hat{R}^M \) we have \( v_Y(\hat{R}^M, 0) = v_N(R^1, 0) < v_Y(R^2, 0) \) and \( v_Y(\hat{R}^M, 1) = v_N(R^1, 1) > v_Y(R^2, 1) \), so it follows that \( \Delta R^2 < \Delta \hat{R}^M \) to make sure that the slope of \( v_Y(R^2, q) \) is strictly smaller than that of \( v_Y(\hat{R}^M, q) \). But then \( v_Y(\hat{R}^M, 0) < v_Y(R^2, 0) \) implies that \( R^2 > \hat{R}^M \). By assumed feasibility of \( R^2 \), we therefore have that \( x_1 > \hat{R}^M \), so that (17) holds strictly. Hence, if (17) does not hold, \( v_Y(R^2, q) \) can only cross \( v_N(R^1, q) \) from below. **Q.E.D.**

**Claim 2.** For any security issued in equilibrium, the highest type that issues this security extracts strictly less than the full surplus from refinancing.

**Proof.** Observe first that if some type \( q^H \) has no incentive to mimic a higher type, the same holds strictly for all types \( q < q^H \). Next, if some type \( q \in [0, q_{FB}] \) extracts more than the full surplus, she must be pooling with some type \( q > q_{FB} \). Suppose therefore that \( q^H \in (q_{FB}, 1] \) is the highest type that issues some security \( R^2(q^H) \) and that \( q^H \) extracts (weakly) more than the full surplus from refinancing, i.e. \( v_N(R^1, q^H) \geq v_Y(R^2(q^H), q^H) \). From Claim 1 we know that for any feasible security \( R^2(q^H), v_Y(R^2(q^H), q) \) can only cross \( v_N(R^1, q) \) from below. Hence, it must be that \( v_N(R^1, q) > v_Y(R^2(q^H), q) \geq v_Y(R^2(q), q) \) for \( q \in [q_{FB}, q^H] \), where the last inequality follows as by incentive compatibility: \( u_Y(R^2(q^H), q) \leq u_Y(R^2(q), q) \) and where \( R^2(q) \) is the equilibrium security issued by type \( q \). Hence, in this candidate equilibrium all types \( q \in [q_{FB}, q^H] \) would extract more than the full surplus. We have thus obtained a contradiction, as the investor is then always better off with his outside option rather than refinancing these types. Hence, it must be that \( v_Y(R^2(q^H), q^H) > v_N(R^1, q^H) \). **Q.E.D.**

We now define more formally the refinement \( D1 \).\(^{33}\) Let \( U(\hat{R}^2, q, \pi) \) be the expected payoff of the owner-manager when offering a security \( \hat{R}^2 \)

\[
U(\hat{R}^2, q, \pi) := \pi u_Y(\hat{R}^2, q) + (1 - \pi) u_N(R^1, q).
\]

\(^{33}\)Originally, as discussed in Cho and Kreps (1987), \( D1 \) was defined for discrete type spaces. The extension to continuous types follows, e.g., Ramey (1996) or DeMarzo et al. (2005).
For each type $q$, determine the minimum probability of acceptance, $\Pi(q|R^2)$, that would make offering $R^2$ weakly attractive

$$\Pi(q|R^2) = \min\{\pi : U(R^2, q, \pi) \geq U^*(q)\},$$

where $U^*(q)$ denotes the equilibrium payoff of type $q$. Then, provided that this leads to a non-empty set, D1 restricts the support of the investor’s beliefs to those types that would find $R^2$ attractive for the lowest probability of acceptance

$$Q^{dev}(R^2) = \left\{ q \in [0, 1] | \Pi(q|R^2) = \min_{q'}\Pi(q'|R^2) \right\}.$$

**Claim 3.** In an equilibrium satisfying D1, all types that obtain refinancing offer the same debt security.

**Proof.** This follows standard arguments (cf. Nachman and Noe, 1994), so we omit the formal details of the proof for the sake of brevity. We showed that when the first best is not feasible, as (17) does not hold, then the highest type issuing a certain security, i.e., the highest type in the respective "pool", never extracts the full surplus. This type would thus strictly benefit from "separating away" from the pool. Given that higher types strictly prefer to share cash flow for the (less likely) low realization, this is possible under D1, provided that the initial security was not debt. Clearly, it is not incentive compatible to have more than one debt security in equilibrium. Finally, pooling with debt for all types who receive refinancing can be supported by beliefs that satisfy D1. **Q.E.D.**

**Proof of Proposition 6.** Suppose first that the investor just breaks even ex-ante, so that

$$\Delta R^1 = \frac{K_i - R^1_l}{p_N(\hat{q})},$$

$$\Delta R^2 = \Delta x - \frac{R^2_l - R^1_l + p_N(q^*_M)(\Delta x - \Delta R^1)}{p_Y(q^*_M)}.$$

(Recall that $\hat{q}$ is the unconditional expectation of $q$.) Note that $R^2_l = x_l$, so that we can represent the equilibrium security $R^2$ as a function of $R^1$ and $q^*_M$ only. By plugging (31) into (19), one can express the binding ex ante participation constraint of the investor
entirely as a function of \( R^1_l \) and \( q^*_M \)

\[
K_1 = \int_0^{q^*_M} \left( R^1_l + p_N(q) \frac{K_1 - R^1_l}{p_N(q)} \right) dF(q) + \int_{q^*_M}^1 \left( R^2_l + p_Y(q) \left( \Delta x - \frac{x_l - R^1_l + p_N(q^*_M) \left( \Delta x - \frac{K_1 - R^1_l}{p_N(q)} \right)}{p_Y(q^*_M)} \right) - K_2 \right) dF(q). \tag{32}
\]

Taking the total derivative of (32) allows us, therefore, to examine how a change in \( R^1_l \) affects the equilibrium cutoff \( q^*_M \) at the interim stage, given that \( R^1 \) and \( R^2 \) adjust so that the investor has the same ex ante expected payoff under the old and the new equilibrium.

From total differentiation we obtain

\[
0 = \left[ (R^1_l + p_N(q^*_M) \Delta R^1_l - x_l - p_Y(q^*_M) \Delta R^2_l) f(q^*_M) + \int_{q^*_M}^{q^*_M} p_Y(q) \frac{d \Delta R^2_l}{dq^*_M} dF(q) \right] dq^*_M
\]

\[
+ \left[ \int_0^{q^*_M} \left( 1 - \frac{p_N(q)}{p_N(q)} \right) dF(q) + \int_{q^*_M}^1 \frac{p_Y(q)}{p_Y(q^*_M)} \left( 1 - \frac{p_N(q^*_M)}{p_N(q)} \right) dF(q) \right] dR^1_l,
\]

where for ease of exposition only we have plugged back in for \( R^t_l \) in the first line. With overinvestment, \( q^*_M < q_{FB} \), the first term in the first line is positive. Also the second term is positive, as \( d \Delta R^2_l / dq^*_M > 0 \). Finally, the second line is also positive. To see this, note that differentiating the terms in front of \( dR^1_l \) with respect to \( q^*_M \), we have

\[
\int_{q^*_M}^1 \left[ \frac{p_Y(q)}{p_N(q)} \left( \frac{p_Y p_{NB} - p_{NG} p_Y B}{p_Y(q^*_M)^2} - \frac{(p_Y G - p_Y B) p_N(q)}{p_Y(q^*_M)} \right) \right] dF(q) < 0.
\]

Further, these terms are zero at \( q^*_M = 1 \), while \( q^*_M \leq q_{FB} < 1 \). Taken together, from the preceding observations on (33) we obtain \( dq^*_M / dR^1_l < 0 \). As the owner-manager is the residual claimant and as \( q^*_M < q_{FB} \), we thus have that \( R^1_l \) is optimally chosen as small as possible: \( R^1_l = 0 \).

It remains to show that it is optimal for the owner-manager to offer the investor a contract for which he just breaks even at \( t = 1 \). For this it is sufficient to show that \( q^*_M \) decreases (i.e. becomes more inefficient) as the investor’s ex-ante payoff increases. To avoid new notation, note that we can likewise analyze a change in \( K_1 \), while still assuming

\[34\text{See (31) and (25) and recall that } R^2_l = x_l.\]
that the investor just breaks even. Total differentiation yields then

\[
0 = \left[ (p_N(q_M^*) \Delta R^1 - x_l - p_Y(q_M^*) \Delta R^2) f(q_M^*) + \int_{q_M^*}^1 p_Y(q) \frac{d \Delta R^2}{dq_M^*} dF(q) \right] dq_M^* \\
+ \left[ \int_0^{q_M^*} \frac{p_N(q)}{p_N(q)} dF(q) + \int_{q_M^*}^1 \frac{p_Y(q)}{p_Y(q_M^*)} \frac{p_N(q_M^*)}{p_N(q)} dF(q) - 1 \right] dK_1.
\]

Since the terms in the second line are positive, it must be that \( dq_M^* / dK_1 < 0 \).  

Proof of Proposition 7. We only have to check the feasibility requirements for \( R^1 \) and \( \hat{R}^M \):

\[
x_l \geq R^1_l \geq 0, \\
\Delta x \geq \Delta R^1 = \frac{K_1 - R^1_l}{p_N(\hat{q})} \geq 0, \\
x_l \geq \hat{R}^M_l = R^1_l + \frac{p_{NB}p_Y - p_{YB}p_{NG}}{p_Y - p_{YB}} \frac{K_1 - R^1_l}{p_N(\hat{q})} + K_2,
\]

where the last condition is just (17) from Proposition 4. These conditions can be rewritten as

\[
\begin{align*}
R^1_l \leq \min \left( \frac{(p_{YG} - p_{YB}) p_N(\hat{q})}{(p_{NG} - p_{NB}) p_Y(\hat{q})} \left( x_l - K_2 - \frac{p_{NB}p_{YG} - p_{YB}p_{NG}}{(p_{YG} - p_{YB}) p_N(\hat{q})} K_1 \right), x_l \right), \\
R^1_l \geq \max \left( 0, K_1 - p_N(\hat{q}) \Delta x \right),
\end{align*}
\]

where we have already used that \( x_l < K_1 \). One can construct a feasible security \( R^1 \) only if the right-hand side in the first line is greater than the right-hand side in the second line.

Note now that from \( x_l < K_1 \) we have that:

\[
\frac{(p_{YG} - p_{YB}) p_N(\hat{q})}{(p_{NG} - p_{NB}) p_Y(\hat{q})} \left( x_l - K_2 - \frac{p_{NB}p_{YG} - p_{YB}p_{NG}}{(p_{YG} - p_{YB}) p_N(\hat{q})} K_1 \right) - x_l \\
= \frac{p_{NB}p_{YG} - p_{YB}p_{NG}}{(p_{NG} - p_{NB}) p_Y(\hat{q})} (x_l - K_1) - \frac{(p_{YG} - p_{YB}) p_N(\hat{q})}{(p_{NG} - p_{NB}) p_Y(\hat{q})} K_2 < 0.
\]

As thus the first term on the right-hand side of (34) is the smallest, we obtain after some transformations immediately condition (20) in the main text.  

\[\text{Q.E.D.}\]

\[\text{Q.E.D.}\]

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\[\text{Q.E.D.}\]
Appendix B: For Online Publication

In this appendix we discuss in more detail financing from a new investor. The additional question that we need to address in such a setting is what happens with the old investor upon refinancing. First, the new investor or the owner-manager could offer the initial investor a security contract on the refinanced firm that offers him at least the same expected payoff as without refinancing. The case in which the old investor retains his old contract is a special case of this alternative. Second, the new investor could provide the owner-manager with the necessary capital to repay in cash the old investor or buy the initial investor’s contract in cash himself. These alternatives lead to the same qualitative results.

Proposition 8 If the owner-manager raises $K_2$ from a new strong investor, optimal financing contracts are analogous to Propositions 1-3.

Proof of Proposition 8. We show the case in which the new investor offers the initial investor and the owner-manager a new claim on the refinanced firm. The case in which the new investor buys out the initial investor’s claim is simpler, as then the setting is the same as before.\footnote{However, note that the new investor will not buy the old investor’s security if it commits him to offering refinancing at (from his view) suboptimal terms.} Furthermore, the case in which the owner-manager raises in addition cash to buy out the initial investor herself or, instead, offers herself the initial investor a new financing contract (next to accepting a contract from the new strong investor), leads to the same qualitative results. To see this, observe that the owner-manager will never make an offer that signals her true type, as she would then never be able to obtain more than $u_N(R^1, q)$ upon refinancing.

Consider the offer made by the new investor to the old investor. The new contract $R^{2, ini}$ should be such that the initial investor receives at least the expected value of the old contract $E[R^1|q \geq q^*]$. Importantly, the owner-manager and the old investor cannot commit to sticking to $R^1$ when the old investor cannot provide refinancing. The reason is that by being able to provide refinancing and increase everyone’s expected payoff, the new strong investor renders the old contract not renegotiation proof. (Note that for our arguments it does not matter whether the initial contract stipulates that $R^1$ converts to a different security upon refinancing from a new investor.)
Define security $R^2 = R^{2,\text{ini}} + R^{2,\text{new}}$ as the sum of the new securities issued to the new and the old investor. The new investor’s gross expected payoff is just

$$E[R^2 - R^{2,\text{ini}}|q \geq q^*] = E[R^2|q \geq q^*] - (K_1 - E[R^1|q < q^*])$$

$$= E[R^2|q \geq q^*] + E[R^1|q < q^*] - K_1$$

where we use that the initial investor breaks even at $t = 1$.\(^{37}\) This is qualitatively the same problem faced by the initial investor in Proposition 2 up to a constant. It is, thus, immediate to see that all of our arguments from Proposition 2 apply. (The only qualitative difference is that the expected payoff from the initial security $R^1$ must be higher to compensate the initial investor for investing $K_1$ in the first place. Facing now an owner-manager who has a lower claim on the cash flows of the non-refinanced firm, the new investor effectively internalizes a higher portion of the social surplus created by refinancing, inducing him to make a more efficient refinancing decision.\(^{38}\) Q.E.D.

Suppose now that the owner-manager faces a "weak" new investor. In analogy to above, there are several ways to deal with the initial investor that all lead to the same qualitative results. First, the owner-manager could offer the initial investor a new contract on the refinanced firm that gives him at least the same expected payoff.\(^{39}\) Second, the owner-manager could raise cash from the new investor allowing her to buy out the initial investor. These alternatives lead again to the same qualitative results.

**Proposition 9** Refinancing from a new investor leads to the same qualitative predictions on financial contracting as in Propositions 4-7.

**Proof of Proposition 9.** Consider offering the initial investor a new security on the

\[^{37}\text{We do not explicitly model the bargaining game with the initial investor. However, all we need to}\]
\[^{38}\text{This effect becomes weaker if the initial investor is also able to demand part of the surplus from}\]
\[^{39}\text{Offering the initial investor to keep his initial contract is just a special case of this alternative. Fur-}\]
refinanced firm. Let $R^2$ be the sum of the securities offered to the initial investor and the new investor $R^2 = R^{2,\text{ini}} + R^{2,\text{new}}$ (If the initial investor is bought out in cash, $R^{2,\text{ini}} = \emptyset$). Since the owner-manager’s expected payoff is determined by $R^2$ regardless of how it is split between the new and the old investor, the same arguments as in Propositions 4-7 apply, and the aggregate security $R^2$ must be debt. Clearly, $R^{2,\text{ini}}$ is such that the initial investor is not worse off than without refinancing—i.e., $E[R^{2,\text{ini}}|q > q^*] = E[R^1|q > q^*]$—regardless of whether $R^1$ stipulates conversion conditional on new financing or not. As noted in Proposition 8, the owner-manager and the old investor cannot commit to sticking to $R^1$ when the old investor cannot provide refinancing. The reason is that by being able to provide refinancing and increase everyone’s expected payoff, financing from a new investor renders the old contract not renegotiation proof.$^{40}$ Q.E.D.

$^{40}$Note that stipulating that the owner-manager sticks to the initial contract (as opposed to offering the initial investor a new contract for which he is equally well off) is not renegotiation proof when the initial investor cannot provide $K_2$. 

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