Capital Controls and International Financial Stability

A Dynamic General Equilibrium Analysis in Incomplete Markets

Adrian Buss∗

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Abstract

In this paper, we conduct an analysis of the implications of capital controls for financial stability. We study a financial transaction (Tobin) tax applicable to cross-border capital flows in a multi-good, multi-country dynamic equilibrium model with incomplete financial markets and heterogeneous agents. The results derived from the model suggest that the impact of capital controls may vary considerably across market segments. In currency markets, capital controls reduce the volatility. However, in international stock markets, their introduction amplifies price movements, thus, increases the volatility; but it reduces a country’s vulnerability to external shocks, thereby limiting spillover effects.

Keywords: capital controls, financial transaction (Tobin) tax, financial stability, incomplete financial markets, general equilibrium

JEL classification: F21, F31, G12, G15

∗INSEAD, Boulevard de Constance, 77305 Fontainebleau Cedex, France. E-mail: adrian.buss@insead.edu.
1 Introduction

“Currency exchanges transmit disturbances originating in international financial markets ... and national governments are not capable of adjusting to massive movements of funds. I therefore regretfully recommend ... to throw some sand in the wheels of our excessively efficient international markets.” James Tobin, “A Proposal for International Monetary Reform,” 1978.

Whenever we observe extreme movements in international financial markets, e.g., huge capital flows or volatile stock markets, the cries for the introduction of regulatory measures to limit these fluctuations get louder. For instance, as a reaction to massive capital inflows that often trigger an increase in exchange rate volatility that might spill over to the stock market, emerging countries often follow Tobin’s advice to “throw some sand in the wheels” of the international financial markets and introduce capital controls on inflows.\(^1\) However, any form of financial regulation, including capital inflow controls, creates distortions in capital allocation and hinders financial integration, thereby eliminating the benefits arising from the free flow of capital. Hence, it is a priori not clear whether financial regulation, e.g., capital controls, will actually ease fluctuations in financial markets.

Consequently, there is currently an intense debate about financial regulation in general and capital controls in specific—in politics and academia. For example, while the ECB Financial Stability Review (2010) argues that the “evidence on the effectiveness of capital controls is inconclusive” and expects “no clear outcome of a cost-benefit analysis,” the International Monetary Fund nowadays\(^2\) accepts capital controls as part of the policy toolkit, and leading economists advocate their implementation.\(^3\)

The objective of this paper is to assess quantitatively the implications of a financial transaction tax applicable to cross-border capital flows—a common form of capital inflow controls—for international financial stability. Specifically, we want to understand if regulating international capital flows can strengthen the stability of international financial markets, e.g., reduce the volatility in the markets and limit the cross-border propagation of shocks.

We therefore study a model of a full-information, multi-good, multi-country, dynamic general equilibrium Lucas exchange economy with uncertainty generated by output as well as demand shocks,\(^4\)

\(^{1}\)Recently, Brazil introduced a transaction tax applicable to capital flowing into the country. Other recent examples include Chile, Colombia, South Korea, and Thailand.

\(^{2}\)Working on the establishment of the IMF, Keynes (1941) suggested that the “control of capital movements, inward and outward, should be a permanent feature of the post-war system.” However, the IMF later advised nations to steer clear of capital controls and wanted to change its articles of agreement to outlaw capital controls (Chwieroth (2009)).

\(^{3}\)In a letter to Treasury Secretary Timothy Geithner, Secretary of State Hillary Clinton, and Trade Representative Ron Kirk, on January 31, 2011, more than 250 economists urged the U.S. to rethink its opinion on capital controls. The authors argue limiting “the inflow of short-term capital into emerging countries can stem the development of dangerous asset bubbles and currency appreciations.”
and the following three central features. The first is the presence of *multiple goods*, which allows us to analyze the effects of capital controls on real exchange rates, and, hence, to study currency markets and international stock markets in a unified framework. The second key features are *heterogeneous agents* with a rich preference structure—Epstein and Zin (1989) and Weil (1990) preferences with a constant elasticity of substitution consumption basket. In particular, the agents can be heterogeneous with respect to all preference parameters. The third central feature of our model is *financial market incompleteness*. Typically, dynamic general equilibrium models in financial economics assume complete markets, as this substantially simplifies the task of identifying the equilibrium. However, in the presence of capital controls, financial markets are inherently incomplete.

Specifically, the introduction of a transaction tax gives rise to a “no-trade region” and agents will only trade if their holdings lie outside this (agent-specific) region. This considerably complicates the identification of the equilibrium, especially so, as we relax the small-open economy assumption typically used in the literature on capital controls, i.e., in our economy all prices are determined endogenously through market clearing. Thus, we have to solve *simultaneously* for asset prices, the individual agent’s boundaries of her no-trade region, and her optimal trading strategy—with all three components influencing each other. For instance, while an agent’s trading strategy depends on the boundaries of the no-trade region, the boundaries itself are driven by current asset prices that in turn influence and are influenced by the trading strategies of the agents.

In this paper, we provide a widely applicable numerical algorithm identifying the equilibrium in the described economy *recursively* on an event tree—in the presence of financial market incompleteness arising from the introduction of a financial transaction tax. The algorithm can solve large scale international finance problems. It provides the optimal consumption as well as the optimal trading strategy of each agent, market clearing asset prices and real exchange rates.

We calibrate a two-country version of our model to the financial markets of the United States and Brazil and assume that *only* Brazil, mimicking an emerging country, introduces the transaction tax on capital inflows. Our quantitative results show that the introduction of the tax effectively reduces the level and the volatility of cross-border capital flows. Importantly, a very small tax, e.g., in the range of 0.1–0.2%, already dramatically reduces the level and the volatility of the flows—by up to 50%. This reduction in cross-border capital flows leads to portfolio positions of the agents that are strongly tilted toward their domestic assets, with the investment home bias strengthening over time.

While the effects on the capital flows are unambiguous, the implications of the transaction tax
for international financial markets are more complex, and heterogeneous effects arise on different market segments. In currency markets, we cannot find any evidence that a financial transaction tax limits currency appreciations, but the transaction tax reduces the volatility of real exchange rate fluctuations for the duration of the capital controls period. That is, the tax stabilizes currency markets, which Tobin (1978) identified as the transmission mechanism for disturbances.

However, we find that the stabilizing effects of the transaction tax do not spill over from the currency market to international stock markets. In contrast, we find that the transaction tax amplifies stock price movements, thereby even increasing the stock market volatility in both countries. On a positive side, the transaction tax reduces a country’s vulnerability to external shocks, i.e., limits spillover effects, by weakening the integration of the international stock markets. That is, as expected by Tobin (1978), in the presence of capital controls, we “move toward greater financial segmentation between nations or currency areas.” In addition, the introduction of the transaction tax causes a reduction in the stock prices of the two countries and raises the market price of risk in the country implementing the tax, i.e., increases the cost of capital.

Finally, we show that in the presence of the tax, the developed country’s agent welfare drops, as she is constrained in her investment decisions. In contrast, the emerging country’s agent welfare is essentially unaffected, causing a shift in relative welfare—in favor of the emerging country’s agent.

The results are driven by two components that may work in opposite directions, thereby easing fluctuations in financial markets, but they may also push in the same direction, hence, amplifying movements in financial markets. The first is a demand effect, as the introduction of capital controls discourages international capital flows, thereby reducing the agents’ demand for foreign assets and increasing their demand for domestic assets. The second component is a Keynes (1929) wealth transfer effect. Recall that in the presence of the transaction tax, the portfolio positions of the agents are tilted toward their domestic assets. That is, in case of a positive shock to a country’s assets, the country’s representative agent profits disproportionately, triggering a wealth transfer to the agent. The additional wealth is used for consumption, mainly in her domestic consumption good, and investment, mainly in her domestic assets, thereby influencing exchange rates and asset prices.

These results have strong policy implications: A financial transaction tax applicable to capital inflows does not unambiguously strengthen financial stability. It eases the fluctuations in currency markets and limits a country’s vulnerability to external shocks, though at the cost of higher stock market volatility. The introduction of a financial transaction tax should therefore be preceded by a
clear outline of its purposes and an extensive cost-benefit analysis.

Our paper relates to the theoretical literature on capital controls, pioneered by Black (1974), Stapleton and Subrahmanyam (1977), and Stulz (1981), who show in a CAPM-type setup that a tax on foreign asset holdings leads to an investment home bias. Later, Errunza and Losq (1989) and Bergström, Rydqvist, and Sellin (1993), still in a static setup, show that investment barriers imply a premium for foreign assets and are welfare-deteriorating due to diversification losses. Recently, Reinhart and Smith (1998, 2002), in a small-open economy model with a tax on net foreign interest payments, and Magud, Reinhart, and Rogoff (2011), also in dynamic small-open economy model but with a reduced form approach for a transaction tax modeled indirectly through returns, show that capital controls in principle limit capital flows and currency appreciations. Mendoza (2002, 2010), Korinek (2010, 2011), and Bianchi (2011) show, in small-open economies with a central planner able to internalize the decisions of the agents, that capital controls can control the impact of sudden stops.

Our contribution to this literature is manifold. First, instead of using a tax applicable to interest rate payments or a reduced form approach, we consider a transaction tax applicable directly to capital inflows. This complicates the identification of the equilibrium because it gives rise to a “no-trade region”—we present a numerical algorithm for this task. Second, we relax the small-open economy restriction that assumes exogenously specified asset returns. Thus, we can study the effects of a transaction tax on prices and returns. Third, instead of assuming the existence of a single agent or of a central planner able to internalize the agents’ decisions, as in the case of small-open economies, we solve the optimization problem for several agents—a necessary condition to be able to study wealth transfer effects. Fourth, we provide a quantitative analysis of the implications of capital controls for international financial stability, especially on volatilities and correlations. Fifth, we consider a richer preference structure as well as an augmented asset menu.

The paper is complementary to the large body of literature on the macroeconomic role of capital controls, reviewed by Kose, Prasad, Rogoff, and Wei (2010) and Frankel (2011). While some studies argue that openness to capital inflows is essential for long-term growth in emerging countries (among others, Dornbusch (1998), Fischer (1998), and Summers (2000)), others regard unregulated capital flows as a serious threat for global financial stability (e.g., Bhagwati (1998), Rodrik (1998), Stiglitz (2002)), calling for the introduction of capital controls. Recently, Ostry, Ghosh, Habermeier, Chamon, Qureshi, and Reinhardt (2010) have concluded that capital controls are justified under certain conditions, but emphasize that controls imposed by one country may lead other countries to
adopt them, creating a chilling impact on financial integration.\footnote{This paper suggested the latest shift in opinion of the IMF. For example, referencing this paper, The Economist on February 18, 2010, states that “the IMF changed its mind on controls on capital inflow” and, similarly, on March 1, 2010, The Guardian notes that the IMF “realized Keynes’s capital controls are a good thing.”} Our contribution to this literature is threefold. First, by studying a theoretical model, we can show through which channels the controls work. Second, we provide explanations for the results discussed in the aforementioned papers. Third, we provide an analysis of the implications for stock markets, often not considered in this literature.

There is currently a growing body of literature that studies optimal policy and optimal regulation of global financial markets, following the historic debate about the (de)stabilizing effects of speculation (Alchian (1950), Friedman (1953)). For example, Fostel and Geanakoplos (2008), and Mendoza (2010) study the effects of collateral restrictions, McCulloch and Pacillo (2010) review the literature on the Tobin tax, Acharya, Pedersen, Philippon, and Richardson (2010) investigate a tax on systemic risk, and Anufriev and Tuinstra (2010) analyze short-sale constraints. We contribute to this literature by analyzing capital controls, another form of financial regulation, in an international setup, showing that regulation may come at high costs and not always strengthens financial stability.

From a modeling perspective, our article relates to the international macro-finance literature, as surveyed by Pavlova and Rigobon (2010b) and pioneered in Helpman and Razin (1978), and Cole and Obstfeld (1991), as well as Zapatero (1995). Specifically, our model extends the log-linear setup in Pavlova and Rigobon (2007, 2008, 2010a) to richer preference structures, i.e., recursive preferences with a constant elasticity of substitution consumption basket. Methodologically, we contribute to this literature by presenting an algorithm that identifies the equilibrium in an economy with these preferences and incomplete financial markets. In their recent survey paper, Pavlova and Rigobon (2010b) note that such a numerical algorithm for international economies is missing.

Finally, the paper complements the literature on transactions costs in general equilibrium, e.g., Heaton and Lucas (1996), Vayanos (1998), Buss and Dumas (2011), and Buss, Uppal, and Vilkov (2011), by studying the impact of a transaction tax—an asymmetric form of transactions costs—on international financial markets. Specifically, in addition to stock markets our model contains currency markets, typically not considered in the single-good economies of the transactions cost literature.

The remainder of the paper is organized as follows: Section 2 presents the analytical framework. Section 3 characterizes the equilibrium and briefly describes the numerical algorithm. Section 4 presents the calibration and discusses the impact of capital controls on financial stability, asset prices, and exchange rates. Finally, Section 5 concludes. The Appendix contains analytical derivations.
2 Basic Model

In this section, we describe the main ingredients of our model. Namely, we consider a general-equilibrium pure-exchange, multi-agent, multi-country, multi-good “world economy” in the spirit of Lucas (1982), with heterogeneous agents, uncertainty due to output and demand shocks and incomplete financial markets. Time is assumed to be discrete, indexed by \( t \), and finite, \( 0 \leq t \leq T \).

Uncertainty in the economy is represented by a \( \sigma \)-algebra \( \mathcal{F} \) on the set of states \( \Omega \) with filtration \( \mathcal{F} \) and probability measure \( P \). We assume a finite number of states in our economy, such that the filtration can be represented by an event tree, each node capturing a particular state \( \omega(t) \).

2.1 Countries

There exist \( N + 1 \) countries in the world economy, indexed by \( j \in \{1, \ldots, N + 1\} \). Each country specializes in producing its own perishable good via a strictly positive Lucas fruit tree output process \( Y_j \). We assume that log output \( \log Y_j(t) = y_j(t) \) has dynamics:

\[
y_j(t) = y_j(t-1) + \bar{g}_j + \epsilon_j(t),
\]

where \( \epsilon_j(t) \) denotes an i.i.d. normal random variable which has a mean of zero, a variance of \( \sigma^2_j \), and may be correlated with \( \epsilon_i(t) \), \( i \neq j \). The deterministic component \( \bar{g}_j \) of expression (1) reflects exogenous output growth, and the random part reflects economic fluctuations.

2.2 Agents

The economy is populated by \( N + 1 \) agents (or classes of agents), indexed by \( \ell \in \{1, \ldots, N + 1\} \), where agent \( \ell \) represents the representative agent of country \( \ell \). Each agent derives utility from lifetime consumption of the different goods, albeit with a preference toward her home good.

Specifically, utility of the agents is assumed to be of the Epstein and Zin (1989) and Weil (1990) type, allowing for the separation of risk aversion and elasticity of intertemporal substitution:

\[
V_\ell(t) = \left[ (1 - \beta_\ell) \cdot C_\ell(t)^{1 - \frac{1}{\psi_\ell}} + \beta_\ell \cdot E_t \left[ V_\ell(t+1)^{1 - \gamma_\ell} \right]^{\frac{1}{1 - \gamma_\ell}} \right]^{\frac{1 - \frac{1}{\psi_\ell}}{1 - \gamma_\ell}},
\]

where \( E_t \) denotes the conditional expectation operator, \( \beta_\ell \) denotes the subjective time-preference rate, \( \gamma_\ell \) is the degree of relative risk aversion, \( \psi_\ell \) denotes the elasticity of intertemporal substitution, and \( C_\ell(t) \) denotes the consumption basket, aggregating consumption in the different goods using a constant elasticity of substitution function:
\[ C_\ell(t) = \left[ \sum_{j=1}^{N+1} \alpha_{\ell,j}(t) \frac{1}{\varphi} \cdot c_{\ell,j}(t) \right] \frac{1}{s_\ell} \]  

(3)

where \( c_{\ell,j}(t) \) denotes the consumption of agent \( \ell \) in the good of country \( j \), and \( s_\ell \) denotes the degree of substitution between the available goods. For \( s_\ell \) approaching zero the goods are perfect complements, and for \( s_\ell \) approaching infinity the goods are perfect substitutes.

The parameter \( \alpha_{\ell,j}(t) > 0 \) reflects agent’s \( \ell \) preference for consumption in good \( j \). We assume that \( \alpha_{\ell,\ell}(t) > \alpha_{\ell,j}(t) \forall j \neq \ell \), implying a “home bias” in consumption.\(^5\) Following Pavlova and Rigobon (2008), we model \( \alpha_{\ell,\ell}(t) \) for \( \ell > 1 \) as stochastic,\(^6\) allowing for demand shifts modeled along the lines of Dornbusch, Fischer, and Samuelson (1977), that influence the extent of the agents’ preferences for their domestic good. Demand shifts reduce the excessively high stock market correlation in the absence of demand uncertainty\(^7\) and Pavlova and Rigobon (2007) have empirically shown their importance. Specifically, we assume that each \( \alpha_{\ell,\ell}(t), \ell > 1 \) is a martingale with dynamics:

\[ \alpha_{\ell,\ell}(t) = \alpha_{\ell,\ell}(t-1) + \nu_\ell(t), \]  

(4)

where \( \nu_\ell(t) \) denotes an i.i.d. normal random variable with mean zero and time-varying variance \( \sigma^2_{\alpha_\ell}(t) \) such that our restriction of a consumption home bias is satisfied.

This preference structure nests several preferences used in the literature. For a one-good economy, the preferences reduce to power utility \( (\psi_\ell = 1/\gamma_\ell) \) or standard Epstein and Zin (1989) and Weil (1990) preferences. Moreover, for two goods, \( s_\ell = \psi_\ell = 1/\gamma_\ell \) and a deterministic consumption home bias \( \alpha_{\ell,j} \), the preferences are similar to Hollifield and Uppal (1997). In the presence of demand shocks and log utility, we arrive at the log-linear specification, used by Pavlova and Rigobon (2007, 2008) for analytical tractability. In addition, for power utility, a deterministic consumption home bias \( \alpha_{\ell,j} \), and a constant elasticity of substitution consumption basket the model reduces to Coeurdacier (2009).

### 2.3 Goods Markets

We assume that there exist no market frictions in the goods markets or transportation (shipping) costs,\(^8\) i.e., goods markets are perfectly integrated, and trading is unrestricted as well as cost-free. In single-good economies one typically normalizes the price of the consumption good to be equal to one. Such a simplification is not possible in multi-good economies—the prices of the goods matter.

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\(^{5}\)This is a standard assumption and captures, in reduced form, the existence of non-tradeable goods.

\(^{6}\)\( \alpha_{1,1} \) is assumed to be constant. The demand shocks for agents \( \ell > 1 \) can then be regarded as relative shifts.


\(^{8}\)For shipping costs models, see Dumas (1992), Sercu, Uppal, and Van Hulle (1995), and Sercu and Uppal (2000).
Hence, we denote the world price of the good of country \( j \) with \( p_j(t) \). However, as prices are not pinned down in absolute terms in a real model such as ours, we also need to specify a numeraire. We therefore fix the price of a world numeraire basket containing \( \lambda_j > 0 \) units of the good produced by each country \( j \) to be equal to unity. This pins down the prices of all goods.

### 2.4 Financial Markets

Each country’s financial market consists of a short-lived, in terms of the local consumption good risk-free country bond\(^9\) and a risky stock. The locally risk-free bond represents a claim to one unit of the country’s good in the next period and is in zero net supply. The stock is a claim to the country’s output stream and available in unit supply. We denote the bond and stock prices of country \( j \) as \( B_j(t) \) and \( S_j(t) \), respectively. Risk-free rates, expected returns, and volatilities for all assets are determined endogenously in equilibrium.

The agents can invest in all \( 2 \times (N + 1) \) assets, i.e., all available stocks and bonds—home as well as foreign. In our economy there exist \( N + 1 \) output shocks and \( N \) demand shocks, such that, in the absence of market frictions international financial markets are complete.

We denote the number of shares of the bond and the stock of country \( j \) in the hands of investor \( \ell \) after all transactions at date \( t \) by \( \theta_{B,\ell,j}^j(t) \) and \( \theta_{S,\ell,j}^j(t) \), respectively. Initially, at date 0, agent \( \ell \) is endowed with the total stock market of her domestic country, such that her wealth is given by \( S_\ell(0) \).

### 2.5 Capital Controls

We assume that only one of the countries introduces capital controls, which we assume, without loss of generality, is country \( N + 1 \). For the implementation of the capital controls we consider a financial transaction tax on capital inflows, similar to the Tobin (1978) tax, but applicable to capital inflows only. That is, while the tax discourages capital inflows, it poses no restrictions on capital outflows. Specifically, we assume that on each unit of capital that an agent \( \ell \in \{1, \ldots, N\} \) transfers into country \( N + 1 \), she has to pay transaction taxes equal to the percentage tax level \( \kappa \).\(^{10}\) Changing the composition of the investment within country \( N + 1 \) is not costly. For example, selling bonds of country \( N + 1 \) and using this capital to buy stocks of the same country is cost-free. Transaction taxes are treated as deadweight costs.

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\(^9\)While the country bonds are locally risk-free, the truly (globally) riskless bond, postponing one unit of consumption of the numeraire, consists of a bond portfolio with weight \( \lambda_j \) for the bond of country \( j \).

\(^{10}\)This type of capital inflow controls resembles very much the ones imposed by Brazil: Since October 19, 2009, capital flowing into Brazil has been subject to transaction taxes, a so-called Tax on Financial Operations.
Capital inflow is measured by the amount that the investment at date \( t \) in country \( N + 1 \), given by \( \theta_{S,\ell,N+1}(t)S_{N+1}(t) + \theta_{B,\ell,N+1}(t)B_{N+1}(t) \), exceeds the date \( t \) value of shares purchased at date \( t - 1 \), given by \( \theta_{S,\ell,N+1}(t-1)S_{N+1}(t) + \theta_{B,\ell,N+1}(t-1)p_{N+1}(t) \). Technically, the transaction taxes paid by agent \( \ell \in \{1, \ldots, N\} \) for transferring capital into country \( N + 1 \) are thus given by:

\[
TT_\ell(t) = \left[ (\theta_{S,\ell,N+1}(t) - \theta_{S,\ell,N+1}(t-1)) S_{N+1}(t) + \theta_{B,\ell,N+1}(t)B_{N+1}(t) - \theta_{B,\ell,N+1}(t-1)p_{N+1}(t) \right] + \kappa, \tag{5}
\]

and are zero for \( \ell = N + 1 \), i.e., for an investor transferring capital into her own country.

The transaction tax is imposed exogenously, e.g., by the government or the central bank, and the agents cannot influence their introduction or removal. Specifically, the capital controls are introduced at \( t_1 > 0 \) and lifted at \( t_2 < T \). As illustrated in Panel (A) of Figure 1, this implies that, at first, from date 0 up to \( t_1 - 1 \), investment is unrestricted; then for periods \( t_1 \) to \( t_2 - 1 \) (shaded area), capital controls are in place; and, finally, for \( t \geq t_2 \), the economy is again free of capital controls.

In this setup, the process of the introduction as well as the removal of the capital controls is crucial. For example, in the extreme case where the agents know exactly when capital controls are imposed and lifted, they can easily hedge against these changes in the investment environment. We assume that the introduction/removal of the controls follows a two-state Markov chain, illustrated in Panel (B) of Figure 1. Specifically, the economy can be in one of two states: (1) without capital controls, and (2) with capital controls. At each point in time \( t \), the probability to move from the state without controls to the same state in the next period \( t + 1 \) is \( p_1 \), and with probability \( 1 - p_1 \) capital controls will be introduced. Similarly, if capital controls are currently imposed, with probability \( p_2 \) they will also be in place in the next period and will be removed with probability \( 1 - p_2 \). The agents know the process of the implementation of the controls, i.e., have rational expectations and behave accordingly.

![Figure 1: Capital Controls](image)
2.6 Financial Market Incompleteness

In the presence of the financial transaction tax, international financial markets are inherently incomplete. Specifically, the introduction of the transaction tax gives rise to a so-called no-trade region.

In detail, if a foreign agent’s total investment in country $N + 1$ is between the lower and the upper boundary of the no-trade region, the agent abstains from transferring additional capital into the country. Within this region the trading costs induced by the financial transaction tax dominate the additional benefits arising from a better diversification that could be achieved if the agent transfers capital into the country. Only if the agent’s total investment in country $N + 1$ is below (above) the lower (upper) boundary she will trade—to the nearest boundary. For example, if the total investment is below the lower boundary, the agent will transfer exactly such an amount of capital into the country, that, after trading, her total investment is equal to the level of the lower boundary. Importantly, the boundaries of the no-trade region are agent-specific.

In incomplete financial markets the identification of the equilibrium is substantially more complex. That is, in complete markets we can solve the problem sequentially. First, identify the optimal consumption plans of the agents, and, second, obtain the portfolio positions of the agents that finance these consumption plans. In contrast to this, in incomplete markets we have to solve simultaneously for the consumption and portfolio strategies of the agents, because for a specific consumption plan there may not exist an admissible trading strategy to finance it.

The existence of the no-trade region further complicates the solution. Specifically, we have to solve simultaneously for the each agent’s boundaries of her no-trade region, each agent’s optimal trading strategy and the market clearing asset prices—all three components influencing each other. For instance, while an agent’s trading strategy depends on the boundaries of the no-trade region, the boundaries, in terms of total investment in the country, depend on the current asset prices that are at the same time influencing and influenced by the trading strategies of all agents in the economy.

3 Equilibrium

In this section we present the characterization of the equilibrium in our economy. In Section 3.1 we describe the individual agent’s optimization problem. In Section 3.2 we introduce the notion of equilibrium and impose market clearing to obtain the characterization of the equilibrium. Finally, in Section 3.3, we briefly describe the numerical algorithm used to identify the equilibrium.
3.1 Individual Agent’s Optimization Problem

The objective of each agent $\ell \in \{1, \ldots, N + 1\}$ is to maximize her lifetime expected utility:

$$V_\ell(t) = \max_{\{c_{\ell,j}(t), \theta^B_{\ell,j}(t), \theta^S_{\ell,j}(t)\}} \left( (1 - \beta_\ell) \cdot C_\ell(t)^{1 - \frac{1}{\psi_\ell}} + \beta_\ell \cdot E_\ell \left[ V_\ell(t + 1)^{1 - \gamma_\ell} \right]^{1 - \frac{1}{\psi_\ell}} \right)^{1 - \frac{1}{\psi_\ell}}, \tag{6}$$

by choosing consumption $c_{\ell,j}(t), \forall \ell$ in the available goods, which are aggregated into $C_\ell(t)$ by the constant elasticity of substitution function (3), by choosing investment $\theta^B_{\ell,j}(t)$ in the countries’ bond markets and by choosing investment $\theta^S_{\ell,j}(t)$ in the countries’ stock markets. The optimization is subject to the agent’s dynamic flow budget equation:

$$\sum_{j=1}^{N+1} \theta^S_{\ell,j}(t-1) \cdot (S_j(t) + p_j(t) \cdot Y_j(t)) + \sum_{j=1}^{N+1} \theta^B_{\ell,j}(t-1) \cdot p_j(t) \tag{7}$$

$$= \sum_{j=1}^{N+1} p_j(t) \cdot c_{\ell,j}(t) + \sum_{j=1}^{N+1} \theta^S_{\ell,j}(t) \cdot S_j(t) + \sum_{j=1}^{N+1} \theta^B_{\ell,j}(t) \cdot B_j(t) + TT_\ell(t),$$

where the left-hand side of the equation captures the agent’s wealth:

$$W_\ell(t) = \sum_{j=1}^{N+1} \theta^S_{\ell,j}(t-1) \cdot (S_j(t) + p_j(t) \cdot Y_j(t)) + \sum_{j=1}^{N+1} \theta^B_{\ell,j}(t-1) \cdot p_j(t), \tag{8}$$

given by the date $t$ value of the assets purchased at date $t - 1$ and the dividends received from the assets. The right-hand side of equation (7) captures the consumption expenditures of the agent, the investment expenditures, i.e., the amount of wealth allocated to the purchase of the available assets, and the tax payments.

The optimization problem of the $N+1$th agent is standard, as he faces complete markets. We thus focus on the problem for an agent $\ell \in \{1, \ldots, N\}$.\footnote{One can nest the problem of agent $N + 1$ into this setup by using agent-specific transaction taxes $\kappa_{N+1} = 0$.} This problem is non-standard and quite challenging due to two reasons. First, in the presence of the transaction tax we need to treat the “total investment in country $N + 1$” as an additional state variable. However, it is a priori not clear what values this variable will take on, i.e., we do not know its range. Second, the maximum operator in the transaction tax formulation entails the emergence of a step function in the first-order conditions of the agent, i.e., a discontinuity.

To overcome this problem, we use the insights in Buss and Dumas (2011) and introduce two additional decision variables, first, the capital inflow $x_\ell(t)$ to and, second, the capital outflow $y_\ell(t)$ from the $N + 1$th country’s securities market triggered by agent $\ell$. Note that transaction taxes can
then be written as \( TT_{i}(t) = x_{i}(t) \cdot \kappa \). That way, we can later use a “dual variable” that has a clear defined range and eliminates the step function from the first-order conditions, instead of the primal variable “total investment in country \( N + 1 \).”

With these definitions, we can rewrite the dynamic budget equation (8) as:

\[
\sum_{j=1}^{N+1} x_{i,j}(t) \cdot \kappa = \sum_{j=1}^{N+1} p_{j}(t) \cdot \kappa.
\]

However, we now have to make sure that the evolution of the agent’s investment positions in country \( N + 1 \) is consistent with the capital inflow and outflow decisions of the agent. We therefore impose the following conditions:

\[
(\theta_{S_{N+1}}(t) - \theta_{S_{N+1}}(t-1)) S_{N+1}(t) + \theta_{B_{N+1}}(t-1) B_{N+1}(t) - \theta_{B_{N+1}}(t) p_{N+1}(t) + x_{i}(t) - y_{i}(t) = 0; \quad (10)
\]

\[
x_{i}(t) \geq 0; \quad y_{i}(t) \geq 0,
\]

that ensure that the agent’s investment in country \( N + 1 \), after trading at date \( t \), is equal to the date \( t \) value of assets purchased at date \( t - 1 \) plus capital inflows minus capital outflows.

In Appendix A we derive the first-order conditions for the problem in terms of the dual variable.

### 3.2 Characterization of the Equilibrium

The notion of equilibrium in our economy is an extension of the single-agent, single-country equilibrium in Lucas (1978). All agents maximize their expected utility and all markets clear. Accordingly, equilibrium is defined as price processes \( \{S_{j}(t); B_{j}(t); p_{j}(t)\} \) and consumption as well as trading strategies \( \{c_{i,j}(t); \theta_{B_{i,j}}(t); \theta_{S_{i,j}}(t); x_{i}(t); y_{i}(t)\} \) such that \( \forall t, 0 \leq t \leq T \):

1. the consumption plan \( \{c_{i,j}(t)\} \) maximizes objective (6), given initial asset allocations;

2. the consumption plan \( \{c_{i,j}(t)\} \) is financed by the trading strategy \( \{\theta_{B_{i,j}}(t); \theta_{S_{i,j}}(t); x_{i}(t); y_{i}(t)\} \);

3. financial markets clear; and

4. consumption good markets clear.

Imposing market clearing on the agents’ first-order conditions presented in Appendix A, the equilibrium is characterized by the following system of equations.
First, the dynamic flow budget equation for each agent $\ell \in \{1, \ldots, N+1\}$:

$$
\sum_{j=1}^{N+1} \theta_{\ell,j}^S(t-1) \cdot (S_j(t) + p_j(t) \cdot Y_j(t)) + \sum_{j=1}^{N+1} \theta_{\ell,j}^B(t-1) \cdot p_j(t) = \sum_{j=1}^{N+1} p_j(t) \cdot c_{\ell,j}(t) + \sum_{j=1}^{N+1} \theta_{\ell,j}^S(t) \cdot S_j(t) + \sum_{j=1}^{N+1} \theta_{\ell,j}^B(t) \cdot B_j(t) + x_\ell(t) \cdot \kappa,
$$

that, as described above, equates the wealth of an agent, with her consumption and investment expenditures plus tax payments.

Second, the kernel conditions for countries $j \in \{1, \ldots, N\}, \forall \ell \in \{1, \ldots, N\}$:

$$
E_t \left[ \frac{M_{N+1}(t+1)}{M_{N+1}(t)} \cdot p_{N+1}(t+1) \right] = \frac{1}{R_\ell(t)} E_t \left[ \frac{M_{\ell}(t+1)}{M_{\ell}(t)} \cdot R_\ell(t+1) \cdot p_{N+1}(t+1) \right], 
$$

$$
E_t \left[ \frac{M_{N+1}(t+1)}{M_{N+1}(t)} \cdot (Y_{N+1}(t+1) \cdot p_{N+1}(t+1) + S_{N+1}(t+1)) \right] = \frac{1}{R_\ell(t)} E_t \left[ \frac{M_{\ell}(t+1)}{M_{\ell}(t)} \cdot (Y_{N+1}(t+1) \cdot p_{N+1}(t+1) + R_\ell(t+1) \cdot S_{N+1}(t+1)) \right],
$$

as well as the kernel conditions for the bond and stock from country $N+1, \forall \ell \in \{1, \ldots, N\}$:

$$
E_t \left[ \frac{M_{N+1}(t+1)}{M_{N+1}(t)} \cdot p_{N+1}(t+1) \right] = \frac{1}{R_\ell(t)} E_t \left[ \frac{M_{\ell}(t+1)}{M_{\ell}(t)} \cdot R_\ell(t+1) \cdot p_{N+1}(t+1) \right],
$$

$$
E_t \left[ \frac{M_{N+1}(t+1)}{M_{N+1}(t)} \cdot (Y_{N+1}(t+1) \cdot p_{N+1}(t+1) + S_{N+1}(t+1)) \right] = \frac{1}{R_\ell(t)} E_t \left[ \frac{M_{\ell}(t+1)}{M_{\ell}(t)} \cdot (Y_{N+1}(t+1) \cdot p_{N+1}(t+1) + R_\ell(t+1) \cdot S_{N+1}(t+1)) \right],
$$

with the pricing kernel

$$
\frac{M_\ell(t+1)}{M_\ell(t)} = \frac{\beta_\ell \cdot p_\ell(t+1)}{p_{\ell+1}(t+1)} \cdot \left( \frac{\alpha_\ell,t_{N+1}(t+1)}{\alpha_{\ell,N+1}(t)} \right)^{\frac{1}{\gamma_\ell}} \cdot \left( \frac{t_{\ell,t}(t+1)}{t_{\ell,N+1}(t)} \right)^{-\frac{1}{\gamma_\ell}} \cdot \left( \frac{V_\ell(t+1)^{1-\gamma_\ell}}{E_t[V_\ell(t+1)^{1-\gamma_\ell}]} \right)^{1-\frac{1}{1-\gamma_\ell}},
$$

ensuring that the agents agree on the prices of the traded assets (bonds and stocks). For the assets of country $N+1$ this accounts for possible transaction payments.

The dual variable $R_\ell(t)$, capturing the shadow costs of the transaction tax, is thereby defined, according to equation (A.23), as:

$$
\phi_1(t) \cdot (R_\ell(t) - 1) \equiv \phi_1(t) \cdot \kappa - \mu_{\ell,1}(t) = \mu_{\ell,2}(t),
$$
and lie in the range $[1, 1 + \kappa]$. The variable merges the two Lagrange multiplier $\mu_{\ell,1}(t)$ and $\mu_{\ell,2}(t)$, associated with the inequality conditions (11), into one multiplier $R_\ell(t)$.

Third, consumption good pricing conditions, $\forall \ell \in \{1, \ldots, N\}$, $\forall j \in \{2, \ldots, N+1\}$:

$$
\left( \frac{\alpha_{N+1,j}(t)}{\alpha_{N+1,1}(t)} \right)^{-\frac{1}{\tau_{N+1}}} \cdot \left( \frac{c_{N+1,j}(t)}{c_{N+1,1}(t)} \right)^{-\frac{1}{\tau_{N+1}}} = \left( \frac{\alpha_{N+1,j}(t)}{\alpha_{\ell,1}(t)} \right)^{-\frac{1}{\tau_\ell}} \cdot \left( \frac{c_{\ell,j}(t)}{c_{\ell,1}(t)} \right)^{-\frac{1}{\tau_\ell}},
$$

(19)
equating the relative good prices across agents, i.e., the agents agree on the prices of the goods.

Fourth, market clearing conditions, ensuring that financial markets clear, $\forall j \in \{1, \ldots, N+1\}$:

$$
\sum_{\ell=1}^{N+1} \theta_{\ell,j}^B(t) = 0; \quad \sum_{\ell=1}^{N+1} \theta_{\ell,j}^S(t) = 1.
$$

(20)
Fifth, aggregate resource constraints, equating endowment with consumption plus taxes:

$$
\sum_{\ell=1}^{N+1} c_{\ell,N+1}(t) = Y_{N+1}(t); \quad \sum_{\ell=1}^{N+1} c_{\ell,j}(t) + \sum_{\ell=1}^{N} \frac{x_{\ell}(t) \cdot \kappa}{p_j(t)} = Y_j(t), \quad \forall j \in \{1, \ldots, N\}.
$$

(21)
Sixth, buying/selling definitions, $\forall \ell \in \{1, \ldots, N\}$:

$$
(\theta_{\ell,N+1}^S(t) - \theta_{\ell,N+1}^S(t - 1)) S_{N+1}(t) + \theta_{\ell,N+1}^B(t) B_{N+1}(t) - \theta_{\ell,N+1}^B(t - 1) p_{N+1}(t) + x_{\ell}(t) - y_{\ell}(t) = 0,
$$

(22)

ensuring that the agents’ investment in country $N+1$ after trading at date $t$ is equal to the date $t$ value of assets purchased at date $t - 1$ plus capital inflows minus capital outflows.

Finally, complementary slackness conditions, $\forall \ell \in \{1, \ldots, N\}$:

$$
(1 + \kappa - R_\ell(t)) \cdot x_{\ell}(t) = 0; \quad (1 - R_\ell(t)) \cdot y_{\ell}(t) = 0,
$$

(23)
with accompanying inequality conditions, $\forall \ell \in \{1, \ldots, N\}$:

$$
1 \leq R_\ell(t) \leq (1 + \kappa); \quad x_{\ell}(t) \geq 0; \quad y_{\ell}(t) \geq 0,
$$

(24)
ensuring that the inequality conditions (11) are fulfilled for all agents – now expressed in terms of the dual variables $R_\ell(t)$.

After solving this system of equations, one can compute the market clearing asset prices at date $t$, given by the discounted future value of the asset and its dividend payments – discounted with the pricing kernel:

$$
B_j(t) = E_t \left[ \frac{M_{N+1}(t + 1)}{M_{N+1}(t)} \cdot p_j(t + 1) \right],
$$

(25)

$$
S_j(t) = E_t \left[ \frac{M_{N+1}(t + 1)}{M_{N+1}(t)} \cdot (S_{N+1}(t + 1) + Y_{N+1}(t + 1) \cdot p_{N+1}(t + 1)) \right].
$$

(26)
3.3 Numerical Solution Technique

One way to identify the equilibrium in our economy would be solve the system of equations at all nodes of the event tree simultaneously—the so-called “global method.” However, in the presence of the financial transaction tax the problem is path-dependent, because the optimal consumption and trading strategies of an agent depend on the preceding period’s total investment in country $N + 1$. The number of equations to be solved simultaneously would therefore easily be in the billions. A recursive solution would therefore be desirable.

Note that in the system of equations presented in the preceding section the current, date $t$, asset prices, given by (25) and (26), depend on future consumption through the pricing kernel. That is, if we would solve this specific system of equations recursively, consumption at date $t + 1$ and, accordingly, asset prices at date $t$ would already be determined when we start the computations at date $t$. However, when we solve for the current, date $t$, optimal consumption and investment decisions of the agents in general equilibrium, the prices must be able to adjust such that financial markets can clear. Consequently, to obtain a solution for these equations we would need to iterate backwards and forwards—essentially over all nodes in the tree—until the future prices are such that the financial markets at date $t$ clear.

To overcome this problem, we rely on the algorithm proposed by Dumas and Lyasoff (2010). Dumas and Lyasoff show how to identify the equilibrium in economies with incomplete financial markets recursively on an event tree. They therefore propose a “time-shift,” i.e., shifting some of the equations one period ahead. This renders the system of equations at each node backward only, such that a recursive approach is possible. The recursive scheme has the advantage that one only has to solve a small number of equations at each node. However, this standard form of the algorithm cannot capture the form of market incompleteness arising in the presence of a transaction tax. Accordingly, we base our algorithm on an extension of the Dumas and Lyasoff (2010) scheme. Specifically, we use the insights in Buss and Dumas (2011) who show how to identify the equilibrium in an economy with proportional transaction costs and how to use dual variables instead of the primal variables “total holdings in country $N + 1$” as state variables. Here, we extend this algorithm further such that it can handle Epstein and Zin (1989) and Weil (1990) type of utility functions with a constant elasticity of substitution consumption basket over multiple goods.

The system of equations that we need to solve recursively on the event tree is presented in Appendix B together with some auxiliary computations and a brief description of the implementation.
4 Quantitative Analysis of Capital Controls

In this section, we analyze the quantitative implications of capital inflow controls in a calibrated version of our model. In Section 4.1, we present the calibration of our world economy. In Section 4.2, we discuss the impact of the controls on international capital flows and portfolio holdings. Next, in Sections 4.3 and 4.4, we study their effect on international financial markets—focusing on stability and prices, respectively. Finally, in Section 4.5, we briefly discuss welfare effects.

4.1 Calibration

We set one period in our model to be one year and, accordingly, calibrate our model, having in mind the financial markets of the United States, representing an developed country in our model, and Brazil, representing an emerging country in our model. Given the drastic changes in the Brazilian economy around 2000, we rely on financial market data for the years 2003 to 2010 in the calibration.\footnote{In January 1999 the central bank announced that the Brazilian Real would no longer be pegged to the U.S. dollar, and in mid-2002 Brazil needed a $30.4-billion rescue package from the IMF to avoid default.}

First, we set the output growth rate in our model equal to the real GDP growth rates of the United States and Brazil, obtained from the Bureau of Economic Analysis and the Banco Central do Brasil, the Brazilian central bank. Specifically, the real GDP growth rates for the years 2003–2010 were 1.74\% and 4.01\%, respectively. In addition, we assume that the random variables $\epsilon_j(t), j = 1, 2$, driving the output shocks, are uncorrelated with the random variable $\nu_2(t)$, driving the emerging country’s demand shocks.

Second, we assume that the world numeraire basket, used to pin down the prices of the goods, consists of 92\% of the developed country’s good and 8\% of the emerging country’s good, based on the relative size, measured by real GDP, of the U.S. and the Brazilian economy.

Third, to simplify numerical computations, we set the elasticity of intertemporal substitution equal to the coefficient of consumption substitution. Specifically, for all agents we assign a value of 2.35—a number consistent with both variables. For example, Barro (2009) and Drechsler and Yaron (2011) choose an elasticity of intertemporal substitution of 2, and estimates for consumption substitution from RBC-type macro models lie in the range between 1 and 3 (see Backus, Kehoe, and Kydland (1994)).

Fourth, we set the coefficient of relative risk-aversion for the developed country’s representative agent to 1.25 and for the emerging country’s representative agent to 3, both values within the range
between 1 and 10, which the literature typically views as reasonable. We chose a higher risk aversion for the emerging country’s representative agent to accommodate the higher equity risk premium in emerging countries. Finally, we assume that the initial, i.e., date 0, consumption home bias of the emerging country’s representative agent is equal to the constant consumption home bias of the developed country’s agent, and we normalize the initial output of the developed country to 1.

We then use the remaining eight parameters, namely, the two countries’ output growth volatilities, the correlation between the countries’ output growth, the two subjective time-preference parameters of the agents, the (constant) consumption home bias of the developed country’s representative agent, the volatility of the emerging country’s agent demand shocks, and the initial output level of the emerging country, to match asset pricing moments of the United States and Brazil. Specifically, we select a total of eight moments: the domestic risk-free rate, the domestic equity premium, and the domestic stock market volatility—always for both countries—as well as the cross-country stock market correlation and the real exchange rate volatility. Following Barro (2009), we assume that stock returns reflect leverage and associate real world stock returns with a leveraged claim on output in the model, using a leverage factor of 1.5.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_1 )</td>
<td>output growth rate in developed country</td>
<td>0.0174</td>
</tr>
<tr>
<td>( g_2 )</td>
<td>output growth rate in emerging country</td>
<td>0.0401</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>output growth volatility in developed country</td>
<td>0.1090</td>
</tr>
<tr>
<td>( \sigma_2 )</td>
<td>output growth volatility in emerging country</td>
<td>0.1835</td>
</tr>
<tr>
<td>( \rho_{1,2} )</td>
<td>output growth correlation developed / emerging country</td>
<td>0.2340</td>
</tr>
<tr>
<td>( Y_1(0) )</td>
<td>initial output level developed country</td>
<td>1.00</td>
</tr>
<tr>
<td>( Y_2(0) )</td>
<td>initial output level emerging country</td>
<td>0.95</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demand Process Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1(0), \alpha_2(0) )</td>
<td>initial consumption home bias</td>
</tr>
<tr>
<td>( \sigma_{\alpha_2} )</td>
<td>demand shock volatility emerging country</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Preference Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_1 )</td>
<td>coefficient of relative risk aversion of representative agent developed country</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>coefficient of relative risk aversion of representative agent emerging country</td>
</tr>
<tr>
<td>( \psi_1 \equiv s_1 )</td>
<td>coefficient of EIS and consumption elasticity of representative agent developed country</td>
</tr>
<tr>
<td>( \psi_2 \equiv s_2 )</td>
<td>coefficient of EIS and consumption elasticity of representative agent emerging country</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>rate of subjective time-preference representative agent of developed country</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>rate of subjective time-preference representative agent of emerging country</td>
</tr>
<tr>
<td>( \lambda_1 )</td>
<td>share of developed country in consumption basket</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Capital Controls</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa )</td>
<td>level of the transaction tax</td>
</tr>
<tr>
<td>( t_1 )</td>
<td>date of introduction of capital controls</td>
</tr>
<tr>
<td>( t_2 )</td>
<td>date of removal of capital controls</td>
</tr>
<tr>
<td>( p_1 )</td>
<td>probability of staying in state “no controls”</td>
</tr>
<tr>
<td>( p_2 )</td>
<td>probability of staying in state “capital controls”</td>
</tr>
</tbody>
</table>

Table 1: Model Parameters. This table lists the parameter values used for the quantitative analysis, including the parameter values for the output processes, the demand process, the preference parameters of the agents, and the values for the capital controls.
### Table 2: Asset Pricing Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S.</td>
<td>Brazil</td>
</tr>
<tr>
<td>risk-free rate (local)</td>
<td>-0.23%</td>
<td>9.08%</td>
</tr>
<tr>
<td>expected stock return (local)</td>
<td>6.51%</td>
<td>27.04%</td>
</tr>
<tr>
<td>stock market volatility (local)</td>
<td>20.21%</td>
<td>43.57%</td>
</tr>
<tr>
<td>stock market correlation (U.S. dollar)</td>
<td>81.41%</td>
<td>81.66%</td>
</tr>
<tr>
<td>exchange rate volatility</td>
<td>16.08%</td>
<td>15.83%</td>
</tr>
<tr>
<td></td>
<td>Developed Country</td>
<td>Emerging Country</td>
</tr>
<tr>
<td>risk-free rate (local)</td>
<td>1.55%</td>
<td>8.18%</td>
</tr>
<tr>
<td>expected stock return (local)</td>
<td>5.71%</td>
<td>27.81%</td>
</tr>
<tr>
<td>stock market volatility (local)</td>
<td>21.39%</td>
<td>42.64%</td>
</tr>
</tbody>
</table>

This table presents the risk-free interest rate, the expected stock return, the stock market volatility, stock market correlation as well as exchange rate volatility in real terms for our model and the empirical data for the years 2003–2010. The developed country in the model is calibrated to match the U.S. data and the emerging country in the model is calibrated to match the Brazilian data.

We use the S&P500 and the Bovespa, as representative stock markets for the U.S. and Brazil, and obtain total return data, including dividends, through Datastream and the Brazilian Central Bank, respectively. As proxy for the riskless bond, we use three-month zero coupon government bonds for each country, obtained from the Federal Reserve Board and the Banco Central do Brasil. As our model is in real terms, we use inflation data, specifically Consumer Price Indices from the Bureau of Labor Statistics, as well as the Brazilian Central Bank, to deflate returns and risk-free rates. Finally, we obtain real exchange rate data for the U.S. dollar and the Brazilian Real from the International Financial Statistics dataset of the International Monetary Fund.

The values assigned to all parameters are listed in Table 1. Note that the eight parameters used to match the empirical moments, are assigned economically reasonable values though some slightly differ from the values typically encountered in the literature. For example, while the subjective time-preference rate for the Brazilian representative agent is 0.868—reasonable, given the fact that Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe (2011) use values between 0.785 and 0.919 for Latin American countries, the a bit too high subjective time-preference rate for the U.S. representative agent (0.999) is necessary in our exchange economy to calibrate the low U.S. interest rate.

A notable point is that our calibration assigns volatilities for output growth that are higher than the empirically observable real GDP growth volatilities. This was to be expected, as we face the well-known dilemma that with standard preferences one cannot match the equity risk premium and the stock market volatility using the volatility of GDP growth.\textsuperscript{13} To match the equity premium and stock market volatility with GDP data, we would need more complex models, like long-run risk or

\textsuperscript{13}See, among others, the seminal work of Mehra and Prescott (1985).

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habit formation.\footnote{See Colacito and Croce (2011) and Heyerdahl-Larsen (2010) for the use of these models in international settings.} Though possible, this would further complicate the model.

Table 2 shows the empirical and the model-implied moments. Note that due to the finite nature of our economy, we disregard the last three model periods in the moment computations, as the uncertainty in the economy diminishes as we reach the terminal date \( T \). The calibration comes close to matching the empirical moments. The average absolute deviation between the empirical and the model-implied moments is 0.85%.

We assume that the financial transaction tax on capital inflows is introduced for the emerging country’s financial market at date \( t_1 = 2 \) and lifted at time \( t_2 = 8 \), so that they are imposed in total for seven years. A seven-year horizon is consistent with the empirical evidence on the use of capital inflow controls. For example, Brazil implemented capital controls from 1993 to 1997 and Chile from 1991 to 1998. We vary the level of the transaction tax between 0.25% and 1%, well in line with currently discussed levels for a financial transaction tax.\footnote{For example, in 2001, James Tobin suggested the rate as “let’s say 0.5%.”}

To compute the quantities reported in the subsequent sections, we simulate for each level of the transaction tax 10,000 paths of two economies—one economy with and one economy without capital controls. We then average at each point in time over all 10,000 paths for the two economies. Finally, we compute either the relative (percentage) deviation, i.e., the ratio of the average value in an economy with capital controls in place and the average value in an economy without capital controls, or, in the case that the quantity may become zero, we compute the absolute deviation, i.e., the difference of the average value in an economy with capital controls in place and the average value in an economy without capital controls. Consequently, values below (above) 0 indicate that the quantity is lower (higher) in the case that capital controls are imposed.

### 4.2 International Capital Flows, Portfolio Holdings, and Wealth Transfers

The introduction of the transaction tax renders capital flows into the emerging country costly, and, accordingly, discourages the developed country’s agent from investing into the country. Specifically, in the presence of capital controls the developed country’s agent faces a tradeoff between better diversification and low tax payments. Below a certain threshold of tax payments, and, accompanying, capital flows, the benefits of better diversification will dominate. However, above the threshold the costs due to the transaction tax will prevail, limiting the investment in the emerging country. Intuition, therefore, strongly suggests that the introduction of capital controls should lead to lower
Figure 2: International Capital Flows, Portfolio Holdings and Wealth Transfers. This figure presents the average relative deviation in the level and the volatility of capital flows into the emerging country (Panel (A)), the average relative deviation in the developed country’s representative agent portfolio share of domestic assets (Panel (C)), and the average relative deviation in the change of the wealth share of the developed country’s agent (Panel (D)) over the duration of the capital controls period for different levels of the transaction tax. Panel (B) shows the absolute deviation in the level of capital inflows in the time-dimension for a transaction tax of 0.75%. Numerical values for the model parameters are as in Table 1.

Panel (A) of Figure 2 presents the relative reduction in capital flows into the emerging country, averaged over the duration of the capital controls period. As expected, the introduction of the transaction tax strongly reduces the level of capital inflows. Interestingly, a rather small level of the tax already leads to a significant reduction in capital inflows, while further reductions due to higher levels of the transaction tax are comparably small. For example, for a transaction tax of only 0.15%, capital flows into the emerging country are reduced by more than 50%. That is, even for a low level of the transaction tax, the direct costs of the tax dominate the losses due to insufficient diversification.

Panel (A) of Figure 2 also shows the volatility of capital flows to the emerging country, again, averaged over the duration of the capital controls period. We can observe that the volatility is also significantly reduced. The intuition behind this result is as follows: First, the costs that the developed
country’s agent incurs when transferring capital into the emerging country reduce capital inflows. Second, by reducing capital inflows, the investment of the developed country’s agent in the emerging country is automatically reduced, implying that possible future capital outflows must also be lower. In summary, in the presence of a transaction tax on capital inflows, capital flows into as well as capital flows out of the emerging country are smaller, and, accordingly, more clustered around the mean. This reduces the volatility of the capital flows into and out of the emerging country. While being substantial, the reduction in the volatility of capital flows is weaker than for the level of the capital flows. For example, a reduction of about 50% can be achieved with a transaction tax of 0.35%—still a rather small tax level.

The result that already a small transaction tax causes strong reductions in the level as well as the volatility of capital flows into the emerging country confirms the Tobin’s (1978) suspicion that a small tax would “throw some sand in the wheels of our excessively efficient international markets, ... slowing the flow of capital across borders.”

The results presented up to now constituted averages over the full capital controls period. In Panel (B) of Figure 2 we present the implications of capital controls on the absolute level of capital inflows in the time dimension—for a transaction tax of 0.75%. We can observe that the reduction in capital inflows is most prevalent at the time of their introduction. For the last couple of years the reduction is not that strong, which can be explained as follows. For the year of their introduction, the costs incurred due to the transaction tax dominate the losses due to insufficient diversification. However, the deviation in the developed country’s agent portfolio holdings from her preferred holdings, and, consequently, the losses due to insufficient diversification increase with each unit of capital not transferred into the emerging country. Therefore, as time proceeds, the losses due to insufficient diversification amplify, and, accordingly, the developed country’s agent starts to transfer more capital into the emerging country, accepting the costs induced by the transaction tax.

In addition, we observe a strong “backlash effect” when capital controls are removed. After their removal, transferring capital into the emerging country has returned to being cost-free, and, consequently, the agent makes up for the reduced capital flows of the preceding periods, leading to sudden and huge capital inflows. This observation strongly suggests a gradual relaxation of the controls—common sense in the financial integration literature.

The reduction in capital flows into the emerging country in the presence of capital controls implies that the developed country’s agent portfolio holdings deviate from her preferred holdings.
Specifically, the lower level of cross-border capital flows automatically leads to a higher portfolio share of her domestic assets—a stronger investment home bias. For example, Panel (C) of Figure 2 shows the relative deviation between the developed country’s agent portfolio share of her domestic assets in economies with and without capital controls, averaged over the capital controls period. Her portfolio share of domestic assets grows monotonically with the level of the transaction tax. For instance, for a transaction tax of 0.5%, the average increase in the share of the domestic assets, in relative terms, is about 40% during the duration of the capital controls period. This implies that, say, a 60% share of domestic assets in an economy without capital controls increases to more than 84% in an economy with capital controls—a rather drastic shift in portfolio positions.

The increase in the portfolio share of domestic assets observable in the presence of capital controls triggers a *wealth transfer*, similar to the classic “Transfer Problem.” For example, in the presence of capital controls, a positive output shock, say, to the developed country, leads to a disproportionately positive wealth shock for the developed country’s representative agent, i.e., a wealth transfer to the agent, as she outweighs her domestic assets. Similarly, a negative output shock to the developed country implies a wealth transfer away from the country’s representative agent. Hence, an agent’s wealth is more sensitive to domestic output shocks and less susceptible to foreign output shocks, and, accordingly, wealth shocks are less correlated in an economy with capital controls.

To illustrate this mechanism, define the developed country’s agent wealth share as:

\[
x(t) = \frac{W_1(t)}{W_1(t) + W_2(t)}.
\]

where \( W_\ell(t) \) denotes the wealth of agent \( \ell \), as defined in expression (8). We can now analyze how output shocks to the developed country affect the share of total wealth of the developed country’s representative agent—comparing economies with and without controls.

In Panel (D) of Figure 2 we report how the change in the developed country’s wealth share \( \Delta x(t)/x(t) \) differs in economies with and without capital controls as a reaction to output shocks, averaged over the capital controls period. The results confirm our expectations discussed above. A higher output shock for the developed country than for the emerging country leads to an excessive increase in the wealth share of the developed country’s agent, compared to an economy without capital controls, i.e., a wealth transfer to this agent. Similarly, a lower output shock for the developed country compared to the emerging country causes an excessive reduction in the wealth share, inducing a wealth transfer.

\[16\] The Transfer Problem originates from Keynes (1929), reasoning that in the presence of a consumption home bias, transferring income from one country to another leads to an appreciation in the exchange rate of the transfer’s recipient.
away from the developed country’s representative agent.

4.3 International Financial Markets: Stability and Integration

We will now study the impact of capital controls on the stability and integration of international financial markets. Specifically, we focus on currency and stock markets.

4.3.1 Currency Markets

Keeping in mind the implications of the transaction tax on the agents’ portfolio holdings, we can now address the question whether capital controls can reduce the volatility of exchange rate movements, i.e., stabilize foreign exchange markets—as intended. The real exchange rate $X_{2,1}(t)$ between the emerging country and the developed country is given by the ratio of the price indices $P_j(t)$ for the two countries:

$$X_{2,1}(t) = \frac{P_2(t)}{P_1(t)}. \quad (28)$$

Following Obstfeld and Rogoff (1996), we define the price index of country $j$ as the lowest cost for the agent inhabiting country $j$ to achieve one unit of aggregate consumption. One can show that this definition implies a price index for country $j$ of:$^{17}$

$$P_j(t) = \left[ \sum_{k=1}^{N+1} \alpha_{j,k}(t) \cdot p_k(t)^{1-s_j} \right]^{1/s_j}. \quad (29)$$

$^{17}$See, among others, Obstfeld and Rogoff (1996).
The changes $r_{X_{2,1}}(t)$ in the real exchange rate between the emerging and the developed country can then easily be computed as:

$$
r_{X_{2,1}}(t) = \frac{X_{2,1}(t) - X_{2,1}(t-1)}{X_{2,1}(t-1)},
$$

(30)

and can be used to compute, at each date, the realized volatility of the exchange rate movements.

We now contrast the volatilities in economies with and without capital controls in place.

As Panel (A) of Figure 3 reveals, a transaction tax on capital inflows causes a reduction in the volatility of real exchange rate movements for the duration of the capital controls period. For instance, for a transaction tax of 1%, the relative deviation from an economy without capital controls is more than 1%. This confirms the Tobin (1978) suspicion, that a tax on cross-border transactions can “cushion exchange rate fluctuations.”

The underlying economic force driving this result is the aforementioned wealth transfer. Namely, in a pure exchange economy—with and without capital controls in place—a higher output shock, say, for the emerging country, all else being equal, directly leads to a lower price $p_2(t)$ relative to the other country’s good price as the supply of the country’s good increases more strongly. This is the so-called Ricardian effect, as described in Ricardo (1817). A lower price of the domestic good relative to the foreign good implies a lower domestic price index $P_2(t)$ compared to the foreign index $P_1(t)$, and, accordingly, a reduction in the level of the real exchange rate.

In the presence of capital controls, the higher output shock for the emerging country triggers a wealth transfer to the country’s representative agent. The agent profits more from the shock than the developed country’s agent as she outweighs her domestic assets. Due to the consumption home bias, the emerging country’s agent utilizes this excess wealth to disproportionately demand her home good. This leads to an increase in the price of the emerging country’s good, which pushes the real exchange rate up.

Comparing economies with and without capital controls, we can see that in the economy without capital controls, only the Ricardian effect takes place. In contrast to this, in the presence of the transaction tax, we observe, in addition to the Ricardian effect, a wealth transfer effect. Importantly, as the discussion above highlights, the wealth effect always works in the opposite direction of the Ricardian effect. Thus, shocks to the real exchange rate are smoothed, leading to the observed reduction in the volatility of the exchange rate movements in the presence of the transaction tax.

To understand the implications of the transaction tax on the volatility of the real exchange rate
fluctuations at different points in time, Panel (B) of Figure 3 depicts the volatility in the time dimension for a transaction tax of 0.75%. The tax consistently reduces the volatility over the horizon of the capital controls period. However, the figure also illustrates that the volatility of the real exchange rate movements spikes up when the controls are lifted. This effect is due to the huge capital flows at this date and is probably a bit exaggerated in our model due to the abrupt removal of the controls, but would most probably, though in a weaker fashion, be observable if one lifts the controls gradually.

4.3.2 International Stock Markets

Given that currency markets and international stock markets are closely intertwined, intuition might suggest that a reduction in real exchange rate movements would also induce lower stock market volatilities. Hence, we now study the effects of the transaction tax on the stock market volatilities of the two countries.

We therefore first briefly review the components that drive stock market returns in an economy absent of capital controls. Recall that the stock is modeled as a claim to the future output stream of a country, and, accordingly, we define the return $r_j^S(t)$ of stock $j$ as:

$$r_j^S(t) = \frac{S_j(t) + p_j(t)Y_j(t)}{S_j(t-1)}, \quad (31)$$

where the first part of the numerator denotes the current stock price, i.e., the price of a claim to future dividends, and the second part denotes the value of the current dividend payment. Accordingly, the
stock return is driven by two components: (i) the level of current and future dividends, and (ii) the current and future prices of the consumption good.

For illustration purposes, let us now consider the case of a positive output shock, say, for the developed country. First, a positive output shock implies a high level of current dividend payments and a higher expectation of future dividends, boosting the price of the stock. Second, in a pure-exchange economy the positive output shock implies a higher current and future supply of the developed country’s good, lowering the price of the good—for today and the future. Lower prices for the good of the developed country lead to a decline in the value of current and future dividends, and, consequently, to downward pressure on the stock. In the model, the first effect dominates, leading to a higher stock price, i.e., a positive stock return.

Now, consider the economy with a transaction tax on capital inflows in place. Recall that the developed country’s agent invests more in her domestic stock in the presence of capital controls. This implies that, compared to an economy without capital controls, the developed country’s agent profits more from the positive stock return. The agent now uses her additional wealth to consume and invest, tilting her additional investment strongly toward her domestic stock, thereby increasing the demand for this stock in the presence of capital controls. For financial markets to clear, the stock price will raise above the level in an economy without capital controls. Consequently, the return of the stock is higher in the presence of capital controls. Similar lines of reasoning show that a negative output shock for the developed country leads to a stock return that is more negative than in an economy without capital controls.

To summarize, in the presence of capital controls, we should observe an amplification of positive and negative stock returns, increasing the dispersion in the stock return distribution and, accordingly, the stock market volatility. The results for our model are presented in Panel (A) of Figure 4 and confirm these expectations. In both countries—developed and emerging—the stock market volatility is higher in the presence of capital controls. Hence, the stabilizing effect of the capital controls in currency markets does not spill over to the international stock markets.

Not only volatilities are important for financial stability, but also the cross-border propagation of shocks, the so called “spillover effects.” To understand how a transaction tax applicable to capital inflows affects the cross-border propagation of output shocks, let us abstract for now from the existence of demand shocks and analyze the comovement of the countries’ stock returns in a model similar to ours but without demand shocks.
Ceteris paribus, a positive output shock for the developed country would have two effects. First, as discussed above, the stock price of the developed country would increase. Second, the higher supply in the good of the developed country would lead to a decrease in the country’s good price relative to the price of the emerging country’s good (Ricardian effect). The higher relative price of the emerging country’s good would then boost the value of future dividends of the country’s stock, raising its price. Specifically, one can show that the prices of the two stocks will always move in the same direction, implying a perfect correlation between the stock returns in the two countries.\footnote{See, for instance, Helpman and Razin (1978), and Cole and Obstfeld (1991), as well as Zapatero (1995).}

In the presence of the transaction tax, the positive output shock for the developed country triggers a wealth transfer to the country’s representative agent, as she outweighs the domestic stock. The higher wealth increases her demand for the consumption goods, but due to the consumption home bias, her demand for her local good increases disproportionately, raising the price of her domestic good relative to the foreign one. In the case that this wealth transfer effect dominates the Ricardian supply effect, the price of the emerging country’s stock will drop, i.e., stock prices will move inversely. That is, while stock prices can still move in the same direction, it is now possible that they move in opposite directions, reducing the correlation of the countries’ stock returns.

The introduction of demand shocks, as in our model, reduces the comovement of the stock returns in the economy without capital controls, i.e., it leads to a correlation below 1. But, the basic mechanisms in the presence of capital controls are still the same, leading to the same conclusion: the stock return correlation should drop if capital controls are introduced. In Panel (C) of Figure 4 we show the deviation in the correlation for economies with and without capital controls, again, averaged over the full capital controls period. The results confirm our suspicion. The correlation drops, as we increase the level of the transaction tax. For example, for a transaction tax of 1%, the correlation between the two countries’ stock returns drops, in relative terms, by about 1.5%. Economically, this means that shocks to one country’s stock market influence the other country’s stock less in the presence of capital inflow taxes. That is, the emerging country’s vulnerability to external shocks is reduced.

4.4 International Financial Markets: Prices

We will now in turn discuss the impact of the transaction tax on the level of the real exchange rate and on stock prices as well as on the market price of equity risk in the country implementing the
4.4.1 Currency Markets

One of the purposes for introducing capital controls is to reduce the upward pressure on a country’s exchange rate. We therefore now analyze the ability of a transaction tax applicable to capital inflows to reduce the real exchange rate between the emerging country and the developed country.

As already discussed, the introduction of capital inflow controls leads to a reduction in the developed country’s agent demand for the emerging country’s assets. Both assets, the emerging country’s bond and stock, are claims to the emerging country’s good. Specifically, the bond is a claim to one unit of the emerging country’s good in the next period, and the stock is a claim to the output of the emerging country. The lower demand for the emerging country’s assets, therefore, automatically leads to a lower demand for the emerging country’s good. The lower demand for the emerging country’s good would then lead to a depreciation in the real exchange rate. Intuition, therefore, suggests that the introduction of capital controls should lead to a depreciation in the real exchange rate between the emerging country and the developed country.

However, Panel (A) of Figure 5 reveals that there is no impact of the capital controls on the average level of the real exchange rate over the duration of the capital controls. One of the economic forces driving this result is, again, the wealth transfer. Specifically, consider, all else equal, a positive output shock for the emerging country in the economy with capital controls. The positive output shock triggers a shift in the wealth distribution—a wealth transfer to the emerging country’s representative agent. As the wealth of the agent goes up, her total demand increases, but due to the consumption
home bias the agent’s demand for her domestic good increases disproportionately, driving up the emerging country’s good price relative to the developed country’s good price. If this wealth effect dominates the demand effect described above, the use of capital controls leads to a higher level of the real exchange rate compared to an economy without capital controls. Instead of reducing the upward pressure on the real exchange rate, capital controls can even increase the upward pressure.

Note that the wealth transfer effect cannot be solely responsible for the weak effects of the capital inflow controls on the real exchange rate. Specifically, the wealth transfer only leads to an increase in the real exchange rate if (i) the emerging country exhibits a higher output shock than the developed country, and (ii) the effect induced by the wealth transfer dominates the demand effect.

To illustrate the other economic forces driving the results presented in Panel (A), we present in Panel (B) of Figure 5 the evolution of the real exchange rate in the time dimension for a 0.75% transaction tax. Note that for the first couple of years of the implementation of the controls, their use effectively depreciates the real exchange rate. However, the figure also shows that the impact of the controls is only temporary, i.e., they lose their effectiveness rather quickly. Namely, in the last couple of years while capital controls are still in place, the average level of the exchange rate is about the same in the economy with capital controls than in the economy without capital controls. To grasp this effect, let us briefly concentrate on the behavior of the real exchange rate at the time of the removal of the controls. We have already established that at this point in time the demand of the developed country’s agent for the assets of the emerging country sharply spikes up, and, accordingly, we observe a strong appreciation in the real exchange rate when the capital controls are lifted. Although the agents do not know when the controls will be lifted, they anticipate this appreciation in case the controls are removed in the next period. Thus, if either the probability of moving from the state “capital controls” to the state “no controls” or the expected appreciation is strong enough to dominate the demand effect, the real exchange rate will appreciate. Ceteris paribus, the expected appreciation is stronger the longer the capital controls are already in place, because the capital inflows at the time of the removal increase with the length of the imposition of the capital controls. Consequently, the real exchange rate increases at the end of the capital controls period.

To summarize, capital controls can only lead to a reduction in the real exchange rate for a very short time, and they may be even counterproductive if kept too long in place. Moreover, the temporary nature of the controls implies future revaluation pressure on the exchange rate.\footnote{This may explain Brazil’s intervention in the futures market on January 7, 2011—one and a half years after imposing capital controls—unexpectedly announcing a new measure to abate the short-selling of the dollar against the Real.}

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Figure 6: Stock Markets. This figure presents the average relative deviation in the countries’ stock prices over the duration of the capital controls period for different levels of the transaction tax (Panel (A)), the relative deviation in the emerging country’s stock price in the time-dimension for a transaction tax of 0.75% (Panel (B)), and the average relative deviation in the emerging country’s market price of risk over the duration of the capital controls period for different levels of the transaction tax (Panel (C)). Numerical values for the model parameters are as in Table 1.

4.4.2 International Stock Markets

The shifts in the agents’ portfolio positions induced by the introduction of the transaction tax on capital inflows obviously also affect the international stock markets. On the level of stock prices, there are two major economic forces. First, in the presence of capital controls, the agents’ portfolio holdings deviate from their preferred holdings, and, accordingly, the agents are less able to smooth consumption across states. These deviations in consumption are transmitted through the pricing kernel, and, consequently, affect stock prices. Specifically, we would expect that a higher consumption volatility induces lower stock prices. Second, the introduction of the transaction tax entails a demand shift between the stocks of the two countries. On the one hand, the costs of foreign investment, in the presence of capital controls, force the developed country’s agent to reduce her demand for the foreign stock, and, hence, to increase her demand for the domestic stock. On the other hand, the demand of the emerging country’s agent for the two stocks is not affected by the transaction tax, as
she is not subject to the tax. That is, for financial stock markets to clear, the excess demand for the
developed country’s stock should lead to upward pressure on its stock price and the lower aggregate
demand for the emerging country’s stock should lead to downward pressure on its stock price.

One can thus summarize the two effects as follows: While for the emerging country’s stock both
components induce downward pressure on its stock price, the two components work in opposite
directions for the developed country’s stock price. The net effect for the developed country’s stock
price thus depends on the magnitude of the two components, and it is not easily predictable.

The effects of the transaction tax on the stock prices in our economy are presented in Panel (A)
of Figure 6, averaged over the full capital controls period. For the emerging country we observe,
as predicted, a monotone decreasing stock price in the level of the transaction tax. For example,
for a transaction tax of 1%, the average reduction in the emerging country’s stock price amounts to
about 0.35%. For the developed country we find an even higher impact on the stock price for low
levels of the tax. However, after this initial decrease, higher tax levels do not have much of a further
impact on the country’s stock price. These results can be explained as follows: A small transaction
tax already substantially reduces capital flows to the emerging country, and, accordingly, affects the
portfolio holdings of the developed country’s agent. Consequently, the agent’s consumption volatility
increases, dominating the demand effect and reducing the price of the stock. For higher levels of the
tax, the additional impact on capital streams and, hence, consumption volatility, is small, such that
the demand effect and the pricing kernel effect almost cancel out, leaving the stock price unaffected.

Supplementary to these aggregate effects over the duration of the capital controls period, we show
in Panel (B) of Figure 6 the evolution of the emerging country’s stock price in the time-dimension
for a 0.75% transaction tax. We can see that the reduction in the stock price is strongest for the
first couple of years of the introduction of the controls. For instance, we can observe a reduction
in the stock price of up to 0.5%—about half of the size of the transaction tax. During these years
the reduction in the developed country’s agent demand for the emerging country’s stock is strongest,
putting downward pressure on the stock. However, as discussed above, the reduction in capital
inflows and, accordingly, the reduction in demand weakens as time proceeds, such that the reduction
in the stock price is smaller for the later years. Moreover, at the time of the removal of the controls,
the agents’ demand for the emerging country’s stock spikes up, creating upward pressure on the stock
price. Namely, the stock price increases by about 1%, i.e., the same order of magnitude as the tax,
and only slowly converges back to the level in the economy without capital controls.
In addition to affecting the stock prices, we would also expect that the introduction of a transaction tax on capital flows to the emerging country affects the price of equity risk, defined as the ratio of the excess return of a country’s stock and the volatility of the stock returns:

\[
MPR_j(t) = \frac{E_t \left[ r_j^S(t+1) - r_j^B(t+1) \right]}{Vol_t \left( r_j^S(t+1) \right)},
\]

where \( E_t \left[ r_j^S(t+1) - r_j^B(t+1) \right] \) denotes the conditional expectation of the stock’s return \( r_j^S(t+1) \) in excess of the one-period bond return \( r_j^B(t+1) \), and \( Vol_t \left( r_j^S(t+1) \right) \) denotes the conditional volatility of the stock returns—for country \( j \).

Specifically, both—the shift in the demand for the stocks and the fact that trading in the emerging country’s assets induces trading costs—may influence the market price of risk. For example, a lower demand for a specific stock should lead in equilibrium to an increase in the agent’s required compensation for holding the stock and, accordingly, increase the price of equity risk. Similarly, the developed country’s agent will require compensation for the costs incurred in buying the foreign stock, which should also increase the price of risk in the emerging country.

Panel (C) of Figure 6 depicts the quantitative implications of the transaction tax on the market price of risk in the emerging country, averaged over the capital controls period. We can observe that the introduction of a transaction tax on capital inflows leads to an increase in the market price of risk, increasing the cost of financing for firms in the country—by more than 1%.

4.5 Welfare

Unilateral capital controls on inflows, as in our model, exhibit two forms of asymmetry. The first is the fact that the controls discourage capital inflows but do not pose any restrictions on capital outflows. The second asymmetry relates to the investment opportunity sets of the agents.

In particular, in the presence of the transaction tax, the developed country’s agent is constrained in her investment in the emerging country’s bond and stock market, leading to strong deviations in her portfolio holdings. Accordingly, the portfolio of the agent is less diversified. In contrast, the emerging country’s agent is not constrained in her investment decision—neither in her domestic nor in the foreign asset market. She is only indirectly affected through the changes in portfolio holdings that the developed country’s agent conducts.

To cleanly analyze the effects of the transaction tax on the welfare of the agents, we take only the
Figure 7: Welfare Emerging Country. This figure presents the average relative deviation in the developed and the emerging country’s representative agent welfare over the duration of the capital controls period for different levels of the transaction tax. Numerical values for the model parameters are as in Table 1.

consumption decisions within the capital controls period into account. That is, we compute for all dates within the capital controls period the effect of the transaction tax on the period utility function \( u_\ell(t) \), defined as:

\[
\begin{align*}
    u_\ell(t) &= C_\ell(t)^{1-\psi_\ell} \\
    &= \left[ \sum_{j=1}^{N+1} \alpha_{\ell,j}(t)^{1/S_\ell} \cdot c_{\ell,j}(t)^{S_{\ell,j}-1/S_{\ell,j}} \right]^{S_{\ell,j}/(S_{\ell,j}-1) \times (1-\psi_\ell)}.
\end{align*}
\]  

(33)

In Figure 7 we present the impact of the transaction tax on the level of the felicity function \( u_\ell(t) \) of the two representative agents, averaged over the capital controls period. We observe that the introduction of the transaction tax leads to a reduction in the welfare of the developed country’s agent. That is, her insufficiently diversified portfolio causes welfare losses. In the case of the emerging country’s agent the introduction of the tax has no effect on welfare. Taken together, this implies that the emerging country’s agent welfare relative to the developed country’s agent welfare improves.

5 Conclusion and Avenues for Future Research

Capital controls have been adopted by several emerging countries in the last few years as a reaction to large capital inflows, with the goal of reducing the volatility in financial markets, specifically in currency and stock markets. However, \textit{a priori} it is not clear whether the introduction of capital controls actually strengthens financial stability, and what the possible adverse consequences are.

In this paper, we develop an international asset pricing model in the presence of capital inflow controls and analyze their implications for international financial markets, with a major focus on financial stability. Our results show that a transaction tax on capital inflows has ambiguous implications for financial stability. Specifically, while it reduces the exchange rate volatility and a country’s
vulnerability to external shocks, the stock market volatility increases. The underlying economic force driving these results is the Keynes (1929) wealth transfer.

Our model yields several empirical predictions about the behavior of financial markets in the presence of capital controls, that might be tested empirically. For example, the prediction of the model that capital inflow controls have essentially no effect on the level of the real exchange rate is a result that has been often discussed in the empirical literature. In cross-country studies, Montiel and Reinhart (1999) and Magud and Reinhart (2007) find only weak evidence for a reduction in exchange rate pressure by capital controls. Similarly, the prediction that capital controls increase the cost of capital is confirmed by Forbes (2007). However, several other predictions, especially the ones addressing financial stability, have not been tested yet.

A natural extension of the model would be to move from a pure-exchange economy to a production economy, such that we can also study the effects on the real side of the economy, e.g., output or labor supply. Another extension would be to include “money” into the model. We would then be able to make predictions about nominal quantities. Moreover, the controls in this paper are imposed exogenously, while, ideally, one would like to know the properties of optimal capital controls.

Methodologically, the numerical algorithm proposed in this study allows us to study several other interesting international finance and financial regulation research questions. Specifically, the algorithm can handle various forms of financial market incompleteness, e.g., transaction costs, short-sale or borrowing constraints, but also leverage or value-at-risk constraints.

For example, one of the major puzzles in international finance is the “Equity Home Bias,” i.e., the empirical observation that investors’ portfolio positions are strongly tilted toward domestic stocks. In this paper, we show that a small transaction tax already has a strong impact on the agents’ portfolio positions. It may, therefore, be worthwhile to study an international economy in which stock trading entails asymmetric transactions costs—i.e., a model where transaction costs are slightly higher for foreign investors than for domestic ones, e.g., due to information acquisition costs (like in van Nieuwerburgh and Veldkamp (2010)). If small differences between the transaction costs for foreign and domestic investors, force them to substantially overweight their domestic stock, we might be able to resolve the puzzle—at least partially.

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Appendix A  Equilibrium Derivations

The Lagrangian $\mathcal{L}_{\ell,t}(c_{\ell,j}(t); \theta_{e,j}^B(t); \theta_{e,j}^S(t); x_{\ell}(t); y_{\ell}(t); \phi_{\ell}(t); \varphi_{\ell}(t); \mu_{\ell,1}(t); \mu_{\ell,2}(t); \forall j)$ for the optimization problem stated in (6), (9), (10) and (11) can written as:

$$
\mathcal{L}_{\ell,t} = \left[(1 - \beta_{\ell}) \cdot C_{\ell}(t)^{1 - \frac{1}{\theta_{\ell}}} + \beta_{\ell} \cdot E_t \left[V_{\ell}(t + 1)^{1 - \gamma_{\ell}}\right]^{1 - \frac{1}{\theta_{\ell}}} \right]^{\frac{1}{1 - \gamma_{\ell}}}
+ \phi_{\ell}(t) \cdot \left\{ \sum_{j=1}^{N+1} \theta_{e,j}^S(t-1) \cdot (S_j(t) + p_j(t) \cdot Y_j(t)) + \sum_{j=1}^{N+1} \theta_{e,j}^B(t-1) \cdot p_j(t) + x_{\ell}(t) \cdot \kappa \right. \\
- \sum_{j=1}^{N+1} p_j(t) \cdot c_{\ell,j}(t) - \sum_{j=1}^{N+1} \theta_{e,j}^S(t) \cdot S_j(t) - \sum_{j=1}^{N+1} \theta_{e,j}^B(t) \cdot B_j(t) \right\} \\
+ \varphi_{\ell}(t) \cdot \left\{ \theta_{e,N+1}(t-1) \cdot S_{N+1}(t) + \theta_{e,N+1}(t-1) \cdot p_{N+1}(t) + x_{\ell}(t) \\
- \theta_{e,N+1}(t) \cdot S_{N+1}(t) - \theta_{e,N+1}(t) \cdot B_{N+1}(t) - y_{\ell}(t) \right\} \\
+ \mu_{\ell,1}(t) \cdot x_{\ell}(t) + \mu_{\ell,2}(t) \cdot y_{\ell}(t),
$$

where $\phi_{\ell}(t)$ denotes the Lagrange multiplier attached to budget equation (9), $\varphi_{\ell}(t)$ denotes the Lagrange multiplier attached to definition (10), $\mu_{\ell,1}(t)$ as well $\mu_{\ell,2}(t)$ denote the Lagrange multipliers attached to inequality conditions (11) and the consumption index $C_{\ell}(t)$ is defined in expression (3).

Define

$$
\eta_{\ell}(t) = \beta_{\ell} \cdot V(t)^{\frac{1}{\theta_{\ell}}} \cdot E_t \left[V_{\ell}(t + 1)^{1 - \gamma_{\ell}}\right]^{\frac{\gamma_{\ell}}{1 - \gamma_{\ell}}},
$$

then the Karush-Kuhn-Tucker first-order conditions for the Langragian (A.1) can be written as:

$$
\frac{\partial \mathcal{L}_{\ell,t}^{FC}}{\partial c_{\ell,j}(t)} = V(t)^{\frac{1}{\theta_{\ell}}} \cdot (1 - \beta_{\ell}) \cdot C_{\ell}(t)^{\frac{1}{\theta_{\ell}}} \cdot \alpha_{\ell,j}(t)^{\frac{1}{\theta_{\ell}}} \cdot c_{\ell,j}(t)^{\frac{1}{\theta_{\ell}}} - \phi_{\ell}(t) \cdot B_j(t) = 0 \ \forall j; \\
$$

$$
\frac{\partial \mathcal{L}_{\ell,t}^{FC}}{\partial \theta_{e,j}^B(t)} = \eta_{\ell}(t) \cdot E_t \left[V_{\ell}(t + 1)^{1 - \gamma_{\ell}} \cdot \frac{\partial V_{\ell}(t + 1)(\cdot)}{\partial \theta_{e,j}^B(t)}\right] - \phi_{\ell}(t) \cdot S_j(t) = 0 \ \forall j \in \{1, \ldots, N\};
$$

$$
\frac{\partial \mathcal{L}_{\ell,t}^{FC}}{\partial \theta_{e,j}^S(t)} = \eta_{\ell}(t) \cdot E_t \left[V_{\ell}(t + 1)^{1 - \gamma_{\ell}} \cdot \frac{\partial V_{\ell}(t + 1)(\cdot)}{\partial \theta_{e,j}^S(t)}\right] - \phi_{\ell}(t) \cdot B_{N+1}(t) - \varphi_{\ell}(t) \cdot B_{N+1}(t) = 0;
$$

$$
\frac{\partial \mathcal{L}_{\ell,t}^{FC}}{\partial \theta_{e,N+1}(t)} = \eta_{\ell}(t) \cdot E_t \left[V_{\ell}(t + 1)^{1 - \gamma_{\ell}} \cdot \frac{\partial V_{\ell}(t + 1)(\cdot)}{\partial \theta_{e,N+1}(t)}\right] - \phi_{\ell}(t) \cdot S_{N+1}(t) - \varphi_{\ell}(t) \cdot S_{N+1}(t) = 0;
$$

$$
\frac{\partial \mathcal{L}_{\ell,t}^{FC}}{\partial \theta_{e,N+1}(t)} = \eta_{\ell}(t) \cdot E_t \left[V_{\ell}(t + 1)^{1 - \gamma_{\ell}} \cdot \frac{\partial V_{\ell}(t + 1)(\cdot)}{\partial \theta_{e,N+1}(t)}\right] - \phi_{\ell}(t) \cdot S_{N+1}(t) - \varphi_{\ell}(t) \cdot S_{N+1}(t) = 0;
$$

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with complementary slackness and inequality conditions:

\[ \mu_{\ell,1}(t) \cdot x_{\ell}(t) \equiv 0; \quad \mu_{\ell,2}(t) \cdot y_{\ell}(t) \equiv 0; \quad x_{\ell}(t) \geq 0; \quad y_{\ell}(t) \geq 0; \quad \mu_{\ell,1}(t) \geq 0; \quad \mu_{\ell,2}(t) \geq 0. \]  

Applying the envelope theorem we get:

\[ \frac{\partial L_{\ell,t}^{\text{CFC}}}{\partial \theta_{\ell,j}^p(t-1)} = \frac{\partial L_{\ell,t}^{\text{CFC}}}{\partial \theta_{\ell,j}^S(t-1)} = \frac{\partial L_{\ell,t}^{\text{CFC}}}{\partial \theta_{\ell,N+1}^B(t-1)} = \frac{\partial L_{\ell,t}^{\text{CFC}}}{\partial \theta_{\ell,N+1}^S(t-1)} = \frac{\partial L_{\ell,t}^{\text{CFC}}}{\partial \phi_{\ell}(t)} = 0. \]  

Equations (A.8) and (A.9), together with (A.19) and (A.20), imply:

\[ \varphi_{\ell}(t) = \phi_{\ell}(t) \cdot \kappa - \mu_{\ell,1}(t); \]  

\[ \varphi_{\ell}(t) = \mu_{\ell,2}(t). \]
such that we can merge two Lagrange multiplier $\mu_{\ell,1}(t)$, $\mu_{\ell,2}(t)$ into one multiplier $R_\ell(t)$, defined as:

$$\phi_\ell(t) (R_\ell(t) - 1) \triangleq \phi_\ell(t) \cdot \kappa - \mu_{\ell,1}(t) = \mu_{\ell,2}(t) \quad (= \varphi_\ell(t)).$$  \hfill (A.23)

With this definition we can rewrite:

$$\phi_\ell(t) + \varphi_\ell(t) = \phi_\ell(t) + \phi_\ell(t) \cdot (R_\ell(t) - 1) = \phi_\ell(t) \cdot R_\ell(t),$$  \hfill (A.24)

and use this in the first-order conditions (A.6) and (A.7) to get:

$$\eta_\ell(t) \cdot E_t \left[ V_\ell(t + 1)^{-\gamma_\ell} \cdot \frac{\partial V_\ell(t + 1)}{\partial \theta_{\ell,N+1}(t)} \right] - \phi_\ell(t) \cdot R_\ell(t) \cdot B_{N+1}(t) = 0; \quad (A.25)$$

$$\eta_\ell(t) \cdot E_t \left[ V_\ell(t + 1)^{-\gamma_\ell} \cdot \frac{\partial V_\ell(t + 1)}{\partial \theta_{\ell,N+1}(t)} \right] - \phi_\ell(t) \cdot R_\ell(t) \cdot S_{N+1}(t) = 0; \quad (A.26)$$

Moreover, from definition (A.23) it follows:

$$\mu_{\ell,1}(t) = \phi_\ell(t) \cdot (1 + \kappa - R_\ell(t)); \quad (A.27)$$

$$\mu_{\ell,2}(t) = \phi_\ell(t) \cdot (R_\ell(t) - 1); \quad (A.28)$$

$$R_\ell(t) = (1 + \kappa) - \frac{\mu_{\ell,1}(t)}{\phi_\ell(t)}; \quad (A.29)$$

$$R_\ell(t) = 1 + \frac{\mu_{\ell,2}(t)}{\phi_\ell(t)}; \quad (A.30)$$

such that we can rewrite the complementary slackness and inequality conditions (A.13) and (A.14) as:

$$(1 + \kappa - R_\ell(t)) \cdot x_\ell(t) = 0; \quad (1 - R_\ell(t)) \cdot y_\ell(t) = 0; \quad (A.31)$$

$$1 \leq R_\ell(t) \leq (1 + \kappa); \quad x_\ell(t) \geq 0; \quad y_\ell(t) \geq 0. \quad (A.32)$$

Defining the pricing kernel

$$\frac{M_\ell(t+1)}{M_\ell(t)} = \beta_\ell \cdot \frac{p_{N+1}(t+1)}{p_{N+1}(t)} \cdot \left( \frac{\alpha_{\ell,N+1}(t+1)}{\alpha_{\ell,N+1}(t)} \right)^{\frac{1}{\gamma_\ell}} \cdot \left( \frac{c_{\ell,N+1}(t+1)}{c_{\ell,N+1}(t)} \right)^{-\frac{1}{\gamma_\ell}} \cdot \left( \frac{C_\ell(t+1)}{C_\ell(t)} \right)^{\frac{1}{\gamma_\ell}} \cdot \left( \frac{V_\ell(t+1)^{1-\gamma_\ell}}{E_t [V_\ell(t+1)^{1-\gamma_\ell}]} \right)^{\frac{\gamma_\ell - \frac{1}{\gamma_\ell}}{1-\gamma_\ell}}, \quad (A.33)$$

the final set of first-order conditions are given by:

$$\frac{p_j(t)}{p_1(t)} = \left( \frac{\alpha_{\ell,j}(t)}{\alpha_{\ell,1}(t)} \right)^{\frac{1}{\gamma_\ell}} \cdot \left( \frac{c_{\ell,j}(t)}{c_{\ell,1}(t)} \right)^{-\frac{1}{\gamma_\ell}} \cdot \left( \frac{V_\ell(t+1)^{1-\gamma_\ell}}{E_t [V_\ell(t+1)^{1-\gamma_\ell}]} \right)^{\frac{\gamma_\ell - \frac{1}{\gamma_\ell}}{1-\gamma_\ell}} \cdot \left( \frac{C_\ell(t+1)}{C_\ell(t)} \right)^{\frac{1}{\gamma_\ell}} \cdot \forall j \in \{2, ..., N+1\} \quad (A.34)$$

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\begin{align*}
B_j(t) &= E_t \left[ \frac{M_t(t+1)}{M_t(t)} \cdot p_j(t+1) \right]; \ \forall j \in \{1, \ldots, N\} \tag{A.35} \\
S_j(t) &= E_t \left[ \frac{M_t(t+1)}{M_t(t)} \cdot (S_j(t+1) + Y_j(t+1) \cdot p_j(t+1)) \right]; \ \forall j \in \{1, \ldots, N\} \tag{A.36} \\
B_{N+1}(t) &= \frac{1}{R_\ell(t)} E_t \left[ \frac{M_t(t+1)}{M_t(t)} \cdot R_\ell(t+1) \cdot p_j(t+1) \right]; \tag{A.37} \\
S_{N+1}(t) &= \frac{1}{R_\ell(t)} E_t \left[ \frac{M_t(t+1)}{M_t(t)} \cdot (R_\ell(t+1)S_{N+1}(t+1) + Y_{N+1}(t+1)p_{N+1}(t+1)) \right]; \tag{A.38} \\
\sum_{j=1}^{N+1} \theta^S_{\ell,j}(t-1) \cdot (S_j(t) + p_j(t) \cdot Y_j(t)) + \sum_{j=1}^{N+1} \theta^B_{\ell,j}(t-1) \cdot p_j(t) \\
&= \sum_{j=1}^{N+1} p_j(t) \cdot c_{\ell,j}(t) + \sum_{j=1}^{N+1} \theta^S_{\ell,j}(t) \cdot S_j(t) + \sum_{j=1}^{N+1} \theta^B_{\ell,j}(t) \cdot B_j(t) + x_\ell(t) \cdot \kappa; \tag{A.39} \\
\theta^S_{\ell,N+1}(t-1) \cdot S_{N+1}(t) + \theta^B_{\ell,N+1}(t-1) \cdot p_{N+1}(t) + x_\ell(t) \\
&= \theta^S_{\ell,N+1}(t) \cdot S_{N+1}(t) + \theta^B_{\ell,N+1}(t) \cdot B_{N+1}(t) + y_\ell(t); \tag{A.40} \\
(1 + \kappa - R_\ell(t)) \cdot x_\ell(t) = 0; \ (1 - R_\ell(t)) \cdot y_\ell(t) = 0; \tag{A.41} \\
1 \leq R_\ell(t) \leq (1 + \kappa); \ x_\ell(t) \geq 0; \ y_\ell(t) \geq 0. \tag{A.42}
\end{align*}

We can now aggregate the individual agent’s optimality conditions to characterize the equilibrium in our economy, i.e., impose the market-clearing and aggregate resource constraints. The full set of equation characterizing the equilibrium is presented in expressions (12) to (24) in the main part of the paper.
Appendix B  Numerical Solution Technique

B.1 System of Equations

To render the system of equations backward only and to be able to solve recursively for the equilibrium on an event tree, Dumas and Lyasoff (2010) suggest to shift all equations, except the kernel conditions and market clearing conditions one period ahead. Applying this transformation to equation system (12) to (24), we get the following “shifted system” of equations, where for a specific time $t$ node in the tree we index the $K_{t+1}$ subsequent nodes by $i = 1, \ldots, K_{t+1}$ and the transition probabilities by $\pi_t(t+1,i)$.

First, the dynamic flow budget equations for agents $\ell \in \{1, \ldots, N\}$, $\forall i$:  

$$
\sum_{j=1}^{N+1} p_j(t+1,i) c_{\ell,j}(t+1,i) + \sum_{j=1}^{N+1} \theta_{\ell,j}^S(t+1,i) S_j(t+1,i) + \sum_{j=1}^{N+1} \theta_{\ell,j}^B(t+1,i) B_j(t+1,i) = \sum_{j=1}^{N+1} \theta_{\ell,j}^S(t) (S_j(t+1,i) + p_j(t+1,i) Y_j(t+1,i)) + \sum_{j=1}^{N+1} \theta_{\ell,j}^B(t) p_j(t+1,i) - TT_{\ell}(t+1,i) \tag{B.1}
$$

where $TT_{\ell}(t+1)$ is defined in expression (5).

Second, the kernel conditions for bonds and stocks from countries $j \in \{1, \ldots, N\}$, $\forall \ell \in \{1, \ldots, N\}$:

$$
\sum_{i=1}^{K_{t+1}} \left[ \pi_t(t+1,i) \cdot \frac{M_{N+1}(t+1,i)}{M_{N+1}(t)} \cdot p_j(t+1,i) \right] = \sum_{i=1}^{K_{t+1}} \left[ \pi_t(t+1,i) \cdot \frac{M_\ell(t+1,i)}{M_\ell(t)} \cdot p_j(t+1,i) \right], \tag{B.2}
$$

$$
\sum_{i=1}^{K_{t+1}} \left[ \pi_t(t+1,i) \cdot \frac{M_{N+1}(t+1,i)}{M_{N+1}(t)} \cdot (S_j(t+1,i) + Y_j(t+1,i) \cdot p_j(t+1,i)) \right] = \sum_{i=1}^{K_{t+1}} \left[ \pi_t(t+1,i) \cdot \frac{M_\ell(t+1,i)}{M_\ell(t)} \cdot (S_j(t+1,i) + Y_j(t+1,i) \cdot p_j(t+1,i)) \right], \tag{B.3}
$$

as well as the kernel conditions for the bond and stock from country $j = N+1$, $\forall \ell \in \{1, \ldots, N\}$:

$$
\sum_{i=1}^{K_{t+1}} \left[ \pi_t(t+1,i) \cdot \frac{M_{N+1}(t+1,i)}{M_{N+1}(t)} \cdot p_{N+1}(t+1,i) \right] = \frac{1}{R_\ell(t)} \sum_{i=1}^{K_{t+1}} \left[ \pi_t(t+1,i) \cdot \frac{M_\ell(t+1,i)}{M_\ell(t)} \cdot R_\ell(t+1,i) \cdot p_{N+1}(t+1,i) \right]. \tag{B.4}
$$
\[
\sum_{i=1}^{K_{t+1}} \left[ \pi_t(t+1,i) \cdot \frac{M_{N+1}(t+1,i)}{M_{N+1}(t)} \cdot (S_{N+1}(t+1,i) + Y_{N+1}(t+1,i) \cdot p_{N+1}(t+1,i)) \right] = \frac{1}{R_\ell(t)} \cdot \sum_{i=1}^{K_{t+1}} \left[ \pi_t(t+1,i) \cdot \frac{M_\ell(t+1,i)}{M_\ell(t)} \cdot (R_\ell(t+1,i) \cdot S_{N+1}(t+1,i) + Y_{N+1}(t+1,i) \cdot p_{N+1}(t+1,i)) \right].
\]  

(B.5)

Third, the consumption \textit{good pricing conditions}, \(\forall \ell \in \{1, \ldots, N\}, \forall j \in \{2, \ldots, N+1\}, \forall i:\)

\[
\left( \frac{\alpha_{N+1,j}(t+1,i)}{\alpha_{N+1,1}(t+1,i)} \right)^{-\frac{1}{\pi_{N+1}}} \cdot \left( \frac{c_{N+1,j}(t+1,i)}{c_{N+1,1}(t+1,i)} \right)^{-\frac{1}{\pi_{N+1}}} = \left( \frac{\alpha_{\ell,j}(t+1,i)}{\alpha_{\ell,1}(t+1,i)} \right)^{-\frac{1}{\pi_\ell}} \cdot \left( \frac{c_{\ell,j}(t+1,i)}{c_{\ell,1}(t+1,i)} \right)^{-\frac{1}{\pi_\ell}}.
\]  

(B.6)

Fourth, the \textit{market clearing conditions}, \(\forall j \in \{1, \ldots, N+1\}:\)

\[
\sum_{\ell=1}^{N+1} \theta_{\ell,j}^B(t) = 0; \quad \sum_{\ell=1}^{N+1} \theta_{\ell,j}^S(t) = 1.
\]  

(B.7)

Fifth, the \textit{aggregate resource constraints}, \(\forall i:\)

\[
\sum_{\ell=1}^{N+1} c_{\ell,j}(t+1,i) + \sum_{t=1}^{N} x_{j}(t+1,i) \cdot \kappa \cdot p_{j}(t+1,i) = Y_j(t+1,i), \quad j \in \{1, \ldots, N\},
\]  

(B.8)

\[
\sum_{\ell=1}^{N+1} c_{\ell,N+1}(t+1,i) = Y_{N+1}(t+1,i).
\]  

(B.9)

Finally, the \textit{complementary slackness conditions}, \(\forall \ell \in \{1, \ldots, N\}, \forall i:\)

\[
(-R_\ell(t+1,i) + (1 + \kappa)) \cdot x_\ell(t+1,i) = 0;
\]  

(B.11)

\[
(-R_\ell(t+1,i) + 1) \cdot y_\ell(t+1,i) = 0,
\]  

(B.12)

with accompanying \textit{inequality conditions}, \(\forall \ell \in \{1, \ldots, N\}, \forall i:\)

\[
1 \leq R_\ell(t+1,i) \leq (1 + \kappa); \quad x_\ell(t+1,i) \geq 0; \quad y_\ell(t+1,i) \geq 0.
\]  

(B.13)

In total, we have \(2(N+1) + K_{t+1}(N^2 + 6N + 2)\) equations (not counting the inequalities) with variables \(\{c_{\ell,j}(t+1,i); \theta_{\ell,j}^B(t); \theta_{\ell,j}^S(t); \forall \ell \forall i\}\) and \(\{x_\ell(t+1,i); y_\ell(t+1,i); R_\ell(t+1,i); \forall \ell \in \{1, \ldots, N\} \forall i\}\), i.e., in total \(2(N+1) + K_{t+1}(N^2 + 6N + 1)\) variables.
B.2 Redundancy Equations

However, one can show that \( K_{t+1} \) budget equations are redundant. Summing up the budget equations (B.1) of all agents \( \forall \ell \in \{1, \ldots, N + 1\} \), we get:\(^{21}\)

\[
\begin{align*}
\sum_{\ell=1}^{N+1} \left[ \sum_{j=1}^{N+1} p_{j}(t+1) c_{\ell,j}(t+1) + \sum_{j=1}^{N+1} \left( \theta_{\ell,j}^{S}(t+1) - \theta_{\ell,j}^{S}(t) \right) \cdot S_{j}(t+1) + \sum_{j=1}^{N+1} \theta_{\ell,j}^{B}(t+1) B_{j}(t+1) \right] \\
= \sum_{j=1}^{N+1} \left[ \sum_{\ell=1}^{N+1} \theta_{\ell,j}^{S}(t) \cdot p_{j}(t+1) \cdot Y_{j}(t+1) + \sum_{j=1}^{N+1} \theta_{\ell,j}^{B}(t) \cdot p_{j}(t+1) + c_{\ell,j} \cdot x_{\ell}(t+1) \cdot \kappa \right] + \sum_{\ell=1}^{N+1} p_{N+1}(t+1) \cdot c_{\ell,N+1}(t+1)
\end{align*}
\]

\[
\Leftrightarrow \sum_{j=1}^{N+1} \sum_{\ell=1}^{N+1} p_{j}(t+1) \cdot Y_{j}(t+1) + \sum_{j=1}^{N+1} \sum_{\ell=1}^{N+1} \theta_{\ell,j}^{S}(t) \cdot p_{j}(t+1) + \sum_{j=1}^{N+1} \theta_{\ell,j}^{B}(t) \cdot p_{j}(t+1)
\]

\[
\Leftrightarrow \sum_{j=1}^{N+1} \sum_{\ell=1}^{N+1} \theta_{\ell,j}^{S}(t) \cdot p_{j}(t+1) \cdot Y_{j}(t+1) + \sum_{j=1}^{N+1} \sum_{\ell=1}^{N+1} \theta_{\ell,j}^{B}(t)
\]

\[
\Leftrightarrow \sum_{\ell=1}^{N+1} c_{\ell,N+1}(t+1) = Y_{N+1}(t+1),
\]

where we used in the second-to-last step the resource constraints (B.8) as well as the market clearing conditions (B.7). The final equation (B.17) is equivalent to the aggregate resource constraint (B.9) for country \( N + 1 \), implying that one can eliminate \( K_{t+1} \) equations from the system of equations.

B.3 Prices

After solving the system of equations at time \( t \), we can compute asset and relative good prices as:

\[
B_{j}(t) = \sum_{i=1}^{K_{t+1}} \left[ \pi_{t}(t+1, i) \cdot \frac{M_{N+1}(t+1, i)}{M_{N+1}(t)} \cdot p_{j}(t+1, i) \right], \forall j;
\]

\[
S_{j}(t) = \sum_{i=1}^{K_{t+1}} \left[ \pi_{t}(t+1, i) \cdot \frac{M_{N+1}(t+1, i)}{M_{N+1}(t)} \cdot \left( S_{j}(t+1, i) + Y_{j}(t+1, i) \cdot p_{j}(t+1, i) \right) \right], \forall j
\]

\[
p_{j}(t) = p_{1}(t) \left( \frac{\alpha_{N+1,j}(t,i) \cdot \frac{1}{c_{N+1,1}(t)}}{\alpha_{N+1,1,j}(t)} \right)^{\frac{1}{s_{N+1}}} \cdot \left( \frac{c_{N+1,j}(t,i)}{c_{N+1,1}(t)} \right)^{\frac{1}{s_{N+1}}}, \forall j > 1.
\]

\(^{21}\)For ease of exposition we omit the the indexation \( i \) for future nodes.
B.4 Time 0 (Initial Node) Equation System

By solving the shifted system of equations recursively on the event tree, one solves all equations from the corresponding “global system,” except the initial budget equations, the initial pricing equation for the consumption goods, the initial aggregate resource constraints, the initial “buying/selling” definitions and the initial complementary slackness conditions with accompanying inequality conditions. That is, after the backward steps we have to solve at the initial node the following, remaining equations:

\[
\begin{align*}
    & \sum_{j=1}^{N+1} p_j(0) \cdot c_{\ell,j}(0) + \sum_{j=1}^{N+1} \theta_{\ell,j}^S(0) \cdot S_j(0) + \sum_{j=1}^{N+1} \theta_{\ell,j}^B(1) \cdot B_j(0) + TT_\ell(0) \\
    = & \sum_{j=1}^{N+1} \theta_{\ell,j}^S(-1) \cdot (S_j(0) + p_j(0) \cdot Y_j(0)) + \sum_{j=1}^{N+1} \theta_{\ell,j}^B(-1) \cdot p_j(0); \quad \forall \ell \in \{1, \ldots, N + 1\} \quad (B.21)
\end{align*}
\]

where \( TT_\ell(0) \) is defined in equation (5).

\[
\left( \frac{\alpha_{N+1,j}(0)}{\alpha_{N+1,1}(0)} \right)^{\frac{1}{N+1}} \left( \frac{c_{N+1,j}(0)}{c_{N+1,1}(0)} \right)^{-\frac{1}{N+1}} = \left( \frac{\alpha_{\ell,j}(0)}{\alpha_{\ell,1}(0)} \right)^{\frac{1}{\ell}} \left( \frac{c_{\ell,j}(0)}{c_{\ell,1}(0)} \right)^{-\frac{1}{\ell}} ; \quad \forall \ell \in \{1, \ldots, N\} \quad (B.22)
\]

\[
\sum_{\ell=1}^{N+1} c_{\ell,j}(0) + \sum_{\ell=1}^{N} \frac{x_\ell(0) \cdot \kappa}{p_j(0)} = Y_j(0); \quad \forall j \in \{1, \ldots, N\}; \quad (B.23)
\]

\[
\sum_{\ell=1}^{N+1} c_{\ell,N+1}(0) = Y_{N+1}(0); \quad (B.24)
\]

\[
\theta_{\ell,N+1}^S(-1) \cdot S_{N+1}(0) + \theta_{\ell,N+1}^B(-1) \cdot p_{N+1}(0) + x_\ell(0) = 0; \quad \forall j \in \{1, \ldots, N\}; \quad (B.25)
\]

\[
(-R_\ell(0) + (1 + \kappa)) \cdot x_\ell(0) = 0; \quad (-R_\ell(0) + 1) \cdot y_\ell(0) = 0; \quad \forall \ell \in \{1, \ldots, N\}; \quad (B.26)
\]

\[
1 \leq R_\ell(0) \leq (1 + \kappa); \quad x_\ell(0) \geq 0; \quad y_\ell(0) \geq 0, \quad \forall \ell \in \{1, \ldots, N\}. \quad (B.27)
\]

In total we have \((N + 1)^2 + 3N + 1\) equations (not counting inequalities) with \((N + 1)^2 + 3N\) unknowns: \(\{c_{\ell,j}(0); \forall \ell, j \in \{1, \ldots, N + 1\}\} \) and \(\{x_\ell(0); y_\ell(0); R_\ell(0); \forall \ell \in \{1, \ldots, N\}\}\). One can show, using the same lines of reasoning as in Appendix B.2 that one equation is redundant.
B.5 Implementation

To create the recombining event tree, capturing the evolution of the exogenous variables output shocks and demand shocks, given in (1) and (4), we utilize the approach outlined in He (1990).

As endogenous variables, we use the consumption of the agents in the $N+1st$ consumption good: $c_{\ell,N+1}(t)$ and the current consumption basket $C_\ell(t)$ of the agents. In addition to the endogenous and exogenous state variables, future stock prices $S_j(t + 1)$, future exiting wealth $X_\ell(t + 1)$ as well as future good prices $p_j(t + 1)$ are needed to solve the system of equations. While the exiting wealth and stock prices are obtained by backward interpolation, subject to terminal conditions $X_\ell(T) = 0$, as well as $S_j(T) = 0$, good prices follow from (A.34).

The implementation itself is done in MATLAB, using the NAG Toolbox for MATLAB for interpolation as well as solving the system of equations.
References


