The Marginal Productivity Theory of Capital in Intermediate Microeconomics Textbooks: A Critique

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The production function has been a powerful instrument for miseducation.

Joan Robinson (1953-54, p. 81)

When it comes to the neo-classical theory of income distribution, you are either in or out with regard to the concept of marginal product... However, if marginal product is an incoherent or unusable concept, most of neo-classical economics (including its approach to income distribution) disintegrates; hence, the unwillingness to question marginal product analysis.

Thomas Palley (2007; emphasis added)

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The most important question in a theory of capitalism is the question of profit – where does profit come from, and what determines its magnitude? Profit is the main motive and overriding purpose of capitalist production, and the main determinant of the dynamics of capital accumulation. When profits are high, then the rate of capital accumulation will be strong, unemployment will fall, and the overall condition of the economy will be good (a period of prosperity). On the other hand, when profits are low or insufficient, then the rate of capital accumulation will be weak, unemployment will rise, and the overall condition of the economy will be poor (a period of crisis and recession or depression).

In addition, the question of the origin of profit is also important for reasons of economic justice. If the source of profit is the work and effort of capitalists or the machines they own, then the receipt of profit by capitalists is just and fair and equitable (if we ignore the prior unequal distribution of ownership of productive resources). On the other hand, if the source of profit is the unpaid labor of workers, then the appropriation of profit by capitalists is exploitation, and capitalism is inherently an unjust and unfair economic system.

The question of profit is the main question in Marx’s theory of capitalism. Most of Volume 1 of Capital is about this all-important question, and most of the main conclusions of Marx’s theory follow directly from the basic theory of profit: conflicts between capitalists and workers over wages and over the length of the working day and the intensity of labor, inherent technological change, trend in the rate of profit, endogenous causes of crises, etc. The question of profit was also emphasized by the classical economists, especially Ricardo.

By contrast, the question of profit and the return to capital has been given much less attention in neoclassical economics. The main neoclassical theory to explain the return to capital over the last century has been marginal productivity theory. As we shall see below, the return to capital is redefined
in marginal productivity theory as the “price of capital”, or the “rental rate of capital”. Capital and the return to capital have always received much less attention in marginal productivity theory than labor and wages, and in recent decades the theory of capital has been almost entirely ignored. Nonetheless, marginal productivity theory remains today the most widely accepted theory of the return to capital by neoclassical economists and is widely used in empirical work.

The marginal productivity theory of distribution was developed in the late 19th century by J.B. Clark (US) and Philip Wicksteed (UK) and others. The variables determined in the modern versions of this theory are the prices of the factors of production – the price of labor (wages), the price of capital (rental rate), and the price of land (I will ignore land in what follows). According to marginal productivity theory, the prices of the factors of production are determined by the supply and demand of these factors.

As is well known, the demand functions for labor and capital are derived in essentially the same way, from given production functions \( Q = f(K, L) \) and the firm’s profit-maximizing condition that the price of each factor should be equal to the marginal product of each factor.\(^1\) The marginal product of each factor is the extra output that is produced if that factor is increased by one unit, and all other factors remain constant. In terms of calculus, the marginal product of each factor is the partial derivative of output with respect to that factor; e.g. the marginal product of capital is the partial derivative with respect to capital: \( \text{MPK} = \partial Q / \partial K \).

The supply functions of labor and capital are derived in different ways. The supply of labor is derived from the utility function of individuals and the assumption that each individual chooses the number of hours she wishes to work on the basis of a “labor-leisure” trade-off in order to maximize her

\(^1\) More precisely, the firm’s profit-maximizing condition is that the price of each factor should be equal to its marginal revenue product, which is its marginal physical product multiplied times the unit price of the output. This complication will be mostly ignored in what follows, because the key issue is the existence or non-existence of the physical marginal products.
utility.\(^2\) The supply of capital is more problematic and has not yet received a definitive treatment. One common interpretation of the supply of capital (that will be discussed below) is that capital goods are supplied by rental firms whose business is to rent capital goods to other firms.

Marginal productivity theory comes to the harmonious conclusion that in equilibrium the price of each factor is equal to its marginal revenue product, which is widely interpreted to mean that each factor is paid according to its productivity, i.e. what it specifically contributes to the production of the output. Thus, workers in general are not exploited in capitalism but instead receive their “just deserts”, i.e. their marginal products.

This paper focuses on the marginal productivity theory of capital and the price of capital, and will discuss some fundamental problems in this theory and how these problems have been dealt with in four leading undergraduate intermediate microeconomics textbooks by Varian, Frank, Nicholson-Snyder, and Pindyck-Rubinfeld.\(^3\) The three main problems to be discussed are: (1) it is impossible to incorporate material inputs (and intermediate inputs in general) into marginal productivity theory because the existence of material inputs in production processes invalidates the basic concept of the marginal product of capital (or labor), which is the foundation of the theory; (2) because the concept of the marginal product of capital (or labor) is invalid, so is the derivation of the demand for capital (or labor), which is based on the marginal product of capital; and (3) ultimately the marginal productivity theory of capital takes as given what it is supposed to explain – the return to capital.

The following additional criticisms of the marginal productivity of capital will not be discussed in this paper (I have discussed these criticisms in Moseley 2012a and 2012b): (1) it assumes perfect competition; (2) the “aggregation problem”, according to which it is not possible to add together heterogeneous capital goods to obtain a single quantity of capital in production functions, even for

\(^2\) L\(^s\) is in units of hours, whereas L\(^d\) is in units of workers, so there is a fundamental logical inconsistency between the units of \(L^s\) and \(L^d\), which makes the usual labor market analysis incoherent.

\(^3\) Actually, five textbooks will be discussed because Nicholson and Snyder have two textbooks, one with calculus and one without.
individual firms, and especially for aggregate production functions;\(^4\) (3) the “disentanglement problem”, according to which it is not possible to isolate the “marginal products” of capital goods and labor separately (i.e. all products are joint products of capital goods and labor);\(^5\) (4) fixed proportions between capital and labor in many production technologies, which makes marginal products non-existent; (5) the “reswitching critique”, according to which in some technologies the relation between the price of capital and the demand for capital switches from an inverse relation to a direct relation, thus contradicting marginal productivity theory; (6) the “exhaustion problem”, according to which the sum of the factor incomes is equal to the value of the output only if the technology is characterized by constant returns to scale (also called the “adding-up problem”, which is different from the “aggregation problem”); and (7) the theory is static and provides no explanation of the dynamic trends in the distribution of income over time in capitalist economies.

1.1 Materials inputs in the production function

The first problem with marginal productivity theory to be discussed is that it is not possible to incorporate material inputs (and intermediate goods in general) into the theory in a reasonably coherent way, because materials inputs cannot be held constant as output increases. In order to increase output in goods-producing industries, the quantity of materials used to produce these goods must also be increased (e.g. more wood to produce another chair, more cloth to produce another shirt, more tires to produce another car, etc.). However, the concept of the marginal physical product of capital (i.e. the partial derivative of output with respect to capital) logically requires that the input of capital goods be increased by one unit, and all other inputs must be held constant. But it is not possible

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\(^4\) The absence of a satisfactory method of aggregating capital means that the “aggregate production function” in macroeconomic growth theory is not a valid theoretical concept, despite its widespread use.

\(^5\) Pullen 2010 is an excellent book-length survey of the history of marginal productivity theory, and especially criticisms of the theory, which emphasizes the “disentanglement problem”.
in general to hold the material inputs constant and produce more goods. Therefore, the existence of material inputs in the production process renders the concept of the marginal productivity of capital impossible in reality (this fundamental problem also applies to the marginal product of labor).

In terms of calculus, this means that production functions with material inputs are not differentiable. The reason that production functions with material inputs are not differentiable is not because the functions are kinked or discontinuous, but because the basic assumption of partial differentiation – that the independent variables must themselves be mutually independent – is violated. Material inputs and capital goods or labor are not mutually independent; if capital goods or labor increases in order to produce more output, then materials would also have to increase.

Similarly, materials inputs cannot be substituted with capital or labor in order to produce the same quantity of output (e.g. a tire cannot be substituted for a worker in order to produce the same car). Therefore, isoquants for materials and capital or labor do not exist, and neither does the “marginal rate of substitution” between materials and capital or labor, because there is no substitution between them.

The impossibility of substitution between capital goods and labor in fixed coefficients production functions is often cited as an important criticism of marginal productivity theory (which assumes “perfect substitution”). The attempted “solution” to this problem is to assume that there are many possible fixed proportion technologies between capital and labor (not just one fixed proportion) to produce a given output, which results in a kinked isoquant, but which approaches a smooth isoquant (perfect substitution) as the number of fixed proportions technologies increases substantially. This attempt hardly seems like a realistic solution in most cases of fixed proportions between capital and labor, but it is definitely not a solution for the fixed proportions of material inputs and outputs. In the materials case, there are not many possible fixed proportions, but only one fixed proportion between the quantity of material inputs and the quantity of each output (e.g. a car will always require four tires,
unless we are talking about a different kind of car). So the “convergence in the limit” argument doesn’t work in the case of material inputs.

Furthermore, if material inputs are included as an input in production functions, as they should be, then the price of materials would presumably also be determined by the same theory as the other factors, i.e. by the equating price of materials with their marginal revenue product (equal to the marginal physical product of materials times the price of the output). But what is the meaning of the “marginal physical product of materials”? The concept of the marginal physical product of materials requires that output could be increased by increasing materials by one unit and holding all other inputs constant. But how is it possible to increase output if both labor and capital are held constant – by magic? Therefore, the concept of the marginal product of materials is also invalid, and for all these reasons material inputs cannot be reasonably incorporated into marginal productivity theory.

One way that the problem of material inputs has been dealt with – especially in empirical work – has been to assume material inputs away, i.e. to assume that the production functions are “value added production functions”, without material inputs. However, this attempted solution does not work, because a production function is a physical concept – a technical relation between physical quantities of inputs and outputs, like an engineering plan – and value added is a nominal price concept – the difference between the price of the output and the prices of intermediate inputs. Nominal value added and prices do not exist in engineering production plans. One can subtract the price of material inputs from the price of the output to calculate value added, because both prices are in nominal terms which are commensurable. However, one cannot subtract the physical quantity of material inputs from the physical quantity of output, because materials and output in a given firm are different kinds of physical goods which are not commensurable (e.g. what is the “value added product” of a car? a car without wheels?). There is no common unit of measure in terms of which this subtraction of physical quantities of inputs and outputs could be made. Therefore, a “value added production function” is an oxymoron.
In empirical work, the physical quantities in production functions are usually estimated by means of the prices of the physical quantities. This estimation procedure has problems of its own (see Felipe and McCombie). But in a theory of factor prices, it is not admissible to take factor prices as given, and use these factor prices to determine a value added production function. To take factor prices as given in a theory of factor prices is circular reasoning, as Joan Robinson pointed out many years ago.

It is sometimes argued that the problem of material inputs can be solved by assuming that the production function is “separable”, in such a way that capital and labor are separable from material inputs, which allows for sequential optimization – first holding materials constant and optimizing quantities of K and L to produce “net output” (sometimes called “real value added”), and then optimizing quantities of materials and “net output” to produce gross output (Bernedt and Christiansen 1973, Arrow 1985, Frondel and Schmidt 2004). However, the condition for separability is that the marginal rate of substitution between capital and labor must be independent of the quantity of materials (first articulated by Leontief 1947 and often called the “Leontief condition”), which in turn requires that the production functions must be twice differentiable (because the marginal rate of substitution is a ratio of partial derivatives of capital and labor). But production functions with material inputs are not even differentiable once. Therefore the condition for separability is not satisfied in production functions with material inputs, and separability does not solve the problem of incorporating material inputs into marginal productivity theory. Furthermore, as discussed on the previous page, “net output” in a given firm in physical terms has no meaning, and does not belong in production functions. There can be no optimization and substitution between “net output” and materials because “net output” does not exist and materials cannot be substituted.6

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6 Arrow 1985 has said that real value added is a “latent concept” which cannot be directly observed. I would go further and say that real value added in physical terms is a non-existent concept. Arrow continued: “Without the separability assumption, however, it is hard to assign any definite meaning to real value added, and probably the best thing to say is that the concept should not be used when capital and labor are not separable from materials in production.” (p. 458; emphasis added) However, Arrow
1.2 Materials inputs and the demand for capital goods

The demand for capital goods is derived from the fundamental concept of the marginal product of capital goods and the profit maximization condition for firms – that the price of capital goods (or the “rental rate”) should equal the marginal revenue product of capital goods (i.e. the marginal physical product of capital goods times the unit price of the output). However, since the marginal physical product of capital goods is not a valid concept in production processes with material inputs, the derivation of the demand of capital goods is itself invalid (the same point also applies to the derivation of the demand for labor).

Furthermore, if material inputs are not included in production functions, then they would also not be included in the cost functions and in the profit maximization conditions for firms, and firms would make the wrong decision concerning whether or not to increase the demand for capital goods (or labor) by one unit in order to increase output. Firms would compare the marginal revenue of the additional output with the additional cost of an additional unit of capital goods only, and would not include the additional cost of the necessary increase of raw materials. Thus firms would demand a greater than to all goods-producing industries.

seems to forget that capital and labor are generally not separable from materials in production functions because production functions with material inputs are not twice differentiable. Thus, following Arrow’s advice, we should “not use the concept” of real value added or net output. Value added cannot be deflated because it is not the product of a price and an existing quantity of output.

7 Keynes 1936, p. 272
8 Blaug 1995, p. 408.
9 Mandler 1999 stated that “it would be foolhardy to deny the manifest implausibility ... of at least the crude versions of marginal productivity theory”. (p. 6; emphasis added) A crude version of marginal productivity theory is usually what is presented in textbooks.
10 The unrealistic assumption that firms rent their capital goods results in the following bizarre implication: if the producing firms actually paid the average return to capital to the rental firms as rent, then in the long-run the producing firms would make no profit. They would rent capital goods and hire labor, but they would make no profit in the long-run. But why would capitalist firms continue to rent capital goods and produce output, if they make no profit in the long-run?
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2.2 Material inputs and the demand for capital goods

In Chapter 19 (“Profit Maximization”), Varian derives the demand for factors in generic terms – the factors are referred to as $x_1$ and $x_2$. Nothing is said explicitly about capital and labor and materials inputs. The usual profit maximization condition for firms is stated as the marginal revenue product of each factor equal to the given price of the factor, with the marginal revenue product equal to the marginal product of the factor times the price of the output. However, as discussed above, this profit maximization condition does not apply to goods-producing industries because the concept of marginal
product does not apply to goods-producing industries, because of the existence of material inputs in production processes, which cannot be held constant as output increases. Thus, the derivation of the demand for factors based on this condition also does not apply to goods-producing industries. If firms decided their demand for capital or labor on the basis of this condition, they would lose money because they would not have taken into account the extra cost of additional material inputs. The generic presentation of the inputs obscures this crucial difference between materials inputs and capital or labor.

2.3 The rental rate of capital goods

As mentioned above, Varian does not mention at all the supply of capital goods and does not explain the determination of the rental rate of capital goods. In Chapter 19, it is emphasized that the prices of the factors of production, including the rental rate of capital goods, should be valued at their opportunity cost to the firm. The rental rate of capital goods is defined simply as “the rate at which you can rent a machine for a given period.” (p. 346) But it is not further specified that the opportunity cost that is included in the rental rate of capital is the prevailing rate of interest and that the rate of interest (the return to capital) is taken as an exogenous variable in marginal productivity theory, not explained by this theory. Varian’s vague and incomplete explanation of the rental rate of capital obscures this fundamental weakness of the marginal productivity theory of capital.

3. Frank

Robert Frank’s *Microeconomics and Behavior* (8th edition) presents a much more complete discussion of marginal productivity theory than Varian. Part 4 is devoted to “Factor Markets”, with a separate chapter on “Labor” (Chapter 13) and “Capital” (Chapter 14). In the chapter on capital, Frank presents a theory of the supply of capital, and it turns out that the rental rate of capital goods is
determined solely by the supply of capital, and does not depend on the demand for capital and is not
determined by the supply and demand in the capital goods market.

3.1 Materials inputs, value added, and the production function

In Chapter 9 ("Production"), the inputs to production that are listed are land, labor, capital, and
"the more elusive category of entrepreneurship". (p. 264) Entrepreneurship is not mentioned again in
the book. Materials inputs are not explicitly mentioned in this list, but two pages later, in a discussion of
an example of a production function for meals, Frank acknowledges that of course meals can be
produced only with raw foodstuffs as inputs, and that in general "intermediate products" are necessary
for the production of other goods. (p. 266)

In the sixth edition, Frank proposed to deal with intermediate products in the production
function as follows:

Strictly speaking, the output of this process is not the meals themselves, but the value
added to the raw foodstuffs... For the sake of simplicity, we will ignore the complication of
intermediate goods in the examples we discuss in this chapter. But this feature could be
built into all these examples without changing any of our essential conclusions. (2006, p.
290)

However, as discussed above in Section 1.1 above, a "value added production function" is an
oxymoron. The production function is a purely technical physical relation between quantities of inputs
and outputs and value added is a monetary concept in terms of the prices of the inputs and outputs, and
prices are not included in physical production functions. It makes no sense to say that the "output" of a
production function is the "value added" in price terms. Therefore, this value added solution to the
problem of raw materials in production functions is no solution at all.

In the eighth edition, Frank changed his treatment of intermediate goods somewhat. (p. 266)
He no longer referred to value added, but he continued to reassure students that intermediate goods
could be incorporated into this theory, even though he does not explain how intermediate goods can be
held constant while increasing output nor how intermediate goods can be substituted with capital or labor. There is no further discussion of intermediate goods in the rest of the book, including especially no discussion in Part 4 on “Factor Markets”. Are students supposed to take Frank’s word for it on this crucial point, without an explanation or at least a suggestion as to how intermediate goods could be reasonably incorporated into a neoclassical production function and marginal productivity theory?

3.2 Demand for capital goods and material inputs

Frank derives the demand for capital in the usual way, from the firm’s profit maximization condition that the marginal revenue of capital should be equal to the rental rate of capital. However, again, this derivation of the demand for capital is not valid for goods-producing industries because it is based on the concept of the marginal product of capital, and the marginal product of capital is not a valid concept in goods-producing industries because of the existence of raw materials in production processes, which cannot be held constant as output increases. If firms followed this “profit maximization rule”, they would lose money.

In a discussion of the demand for labor and the decision of a firm whether or not to hire an additional unit of labor, Frank stated:

If the manager of a firm is thinking like an economist, she will reason as follows: “The benefit of hiring an extra unit of labor will be the amount for which I can sell the extra output I will get. The cost will be the wage rate. Thus I should hire an extra unit of labor as long as the former exceeds the latter...” (p. 460)

However, if the firm produces physical goods, extra output will also in general require extra material inputs as well as extra labor. So I hope that the business manager is not “thinking like an economist”, but is instead thinking like a reality based manager who would also include the extra material costs in her calculations.

Frank also discusses the case in which firms have the option to purchase capital goods rather than rent them. (p. 508) It is assumed that a new machine would increase the firm’s total revenue by R
per year (for N years) and it will cost M per year to operate the machine, so that its expected annual stream of net revenue is R – M. In this case, the firm should calculate the present value of this expected future stream of net revenue, which varies inversely with the rate of interest; and the firm should purchase the machine only if this present value is greater than or equal to the price of the machine.

However, important costs are missing in this calculation. If the new machine increases output, then additional materials would have to be purchased (as usual, nothing is said about material inputs). If these additional costs of materials are not subtracted from the total revenue, then the net revenue would be overestimated and the wrong investment decisions would be made. In order to accurately calculate the net revenue, the cost of additional materials must also be subtracted from the expected total revenue.

Furthermore, the demand for capital goods derived in this way is not put together with the supply of capital goods in order to determine the price of capital goods. Instead the price of capital goods is taken as given; and in fact the price of capital goods that is taken as given in this comparison with present value is the actual price of capital goods, which is different from the price of capital goods as usually defined in marginal productivity theory, which is the actual price of capital goods plus an imputed opportunity cost.

3.3 The supply of capital goods and the rental rate of capital goods

Chapter 15 (“Capital”) presents a discussion of the supply of capital goods and the determination of the rental rate of capital goods, in favorable comparison to Varian who has no such discussion. It is assumed to begin with that firms rent their capital goods from rental firms who provide the supply of capital goods, and the rental rate of capital goods is analyzed from the point of view of the rental firms; i.e. from the point of view of the supplier of capital goods, not the demander of capital goods.
The rental rate of capital goods is assumed to be determined by the sum of three types of costs of the rental firms: depreciation cost, maintenance cost, and “opportunity cost”, similar to the equation for the rental rate in Section 1.3 above, with a third component for maintenance cost. All these costs are taken as given in the determination of the rental rate of capital goods. The marginal product of capital goods and the demand for capital goods play no role in the determination of their rental rates, and thus marginal productivity theory plays no role in the determination of the rental rate of capital goods. The rental rates are determined by these costs, and these costs alone, which are taken as given. The “opportunity cost” of capital goods that is taken as given in the determination of the rental rate is in turn determined by Frank in the loanable funds market, by supply and demand in the loanable funds market, not by supply and demand in the capital goods market.

4. Nicholson and Snyder (11th editions)

Walter Nicholson and Christopher Snyder have two intermediate microeconomics textbooks, both in their 11th editions, one more advanced that includes calculus (Microeconomic Theory: Basic Principles and Extensions) and the other one does not include calculus (Intermediate Microeconomics and its Applications). My discussion will be mainly in terms of the more advanced textbook, but a few differences in the presentations in the two textbooks will be noted. Nicholson was the sole author of the first nine editions.

Nicholson and Snyder present a fairly complete and integrated discussion of marginal productivity theory. Part 7 is entitled “Pricing in Input Markets” and consists of two chapters, “Labor Markets” (Chapter 16) and “Capital in Time” (Chapter 17) (there is a similar structure in the other textbook). The chapter on capital presents a similar theory of the supply of capital goods and the rental rate of capital goods as Frank’s, with one important exception to be discussed below.
4.1 Material inputs in the production function

In Chapter 9 ("Production Functions"), the inputs to production that are listed are machines, labor, and raw materials. (p. 303) It is noted in a footnote that:

In empirical work, raw materials are often disregarded and output is measured in terms of value added. (p. 303)

The suggestion seems to be that raw materials can also be disregarded in theory; but this is not true. In the theory of production, raw materials are necessary inputs to the production of outputs. It cannot be disregarded that raw materials cannot be held constant while output is increasing, which renders invalid the fundamental concept of the marginal products of other factors. And it also cannot be disregarded that raw materials cannot be substituted for capital or labor in order to produce the same amount of output, which renders invalid the isoquants and the marginal rate of substitution between raw materials and capital or labor.

The non-calculus textbook also has a footnote on value added:

Sometimes the output for a firm is defined to include only its ‘value added’; that is, the value of raw materials used by the firm is subtracted to arrive at a net value of output for the firm. This procedure is also used in adding up gross domestic product to avoid double counting of inputs. (p. 216)

Here the suggestion seems to be that the dependent variable in the production function could also be defined as the “value added” of the output; but again this is not true. The production function is a purely technical physical relationship between quantities of inputs and outputs. Value added is not a physical quantity, but is instead a monetary quantity, which has no place in a physical production function.

And the analogy with GDP measurement doesn’t work. GDP is a macro concept and a firm’s production function is a micro concept. GDP is the net output produced in the economy as a whole during a given period, which makes some sense at the macro level, because materials distributed as inputs to other industries could be subtracted from the total output of materials in order to arrive at the
net output for the economy as a whole (although aggregating the different kinds of output into a single macro measure for the economy as a whole poses further formidable problems). However, in a firm’s production function, the inputs and outputs are different physical goods and thus the physical inputs cannot be subtracted from the physical outputs to obtain a “net output of the firm”.

It is also stated in the beginning of this chapter that it is “more convenient” to use a simplified production function with only capital and labor as inputs, but that *the same theory applies to all inputs*. (pp. 303-04) But again this is not true; material inputs are different from capital goods and labor. It is possible in some production processes that capital goods or labor could be held constant while the other one is increasing, but it is not possible in general to hold material inputs constant while capital or labor is increasing in order to produce additional output.

In the section on input substitution and isoquants and the marginal rate of substitution, it is stated:

Again, we study a production function of the form \( q = f(k,l) \), with the understanding that “capital” and “labor” are simply convenient examples of *any two inputs* that happen to be of interest. (p. 306; emphasis added).

But this assertion is false. Materials cannot be substituted for capital or labor and thus there are no isoquants for materials and these other inputs, and materials do not have a marginal rate of substitution with other inputs.

In the non-calculus textbook, an example is presented of the production of hamburgers (by a firm called Hamburger Heaven). Tables 6.1, 6.2, and 6.3 present different combination of the inputs of grills and workers to produce different quantities of output of hamburgers (pp. 233-36). But this example leaves out the most important input in the hamburger production function – the hamburger meat! One cannot produce hamburgers without hamburger meat, and one cannot produce more hamburgers without more hamburger meat. (One is reminded of the old Wendy’s commercial that

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12 Although even here many production processes have relatively fixed proportions between capital and labor and variable proportions is not possible or very limited.
complained about McDonald’s hamburgers: “Where’s the beef?”). It is acknowledged that this example is “obviously unrealistic”; but it is argued that “it does reflect the way production is analyzed in the real world.” I would say that this unrealistic assumption might reflect the way that neoclassical microeconomics analyzes production, but I doubt very much that real world hamburger firms analyze production in this unrealistic way, which ignores essential material inputs and the cost of these inputs.

Also in their non-calculus textbook, there is a brief subsection entitled “Appraising the Marginal Product Concept”, in which it is candidly acknowledged that:

The concept of marginal product itself may sometimes be difficult to apply because of the ceteris paribus assumption used in its definition. Both the levels of other inputs and the firm’s technical knowledge are assumed to be held constant when we perform the conceptual experiment of say, adding one more worker to an orange grove. But in the real world, that is not the way new hiring would likely occur. Rather, additional hiring would probably also necessitate adding additional equipment (ladders, crates, tractors, and so forth)... In such cases, the ceteris paribus assumptions incorporated in the definition of marginal productivity would be violated ... (p. 220; emphasis added, except ceteris paribus)

Here Nicholson and Snyder are talking about fixed proportions between labor and capital inputs, not about fixed proportions between material inputs and outputs. The latter are even more widespread than the former and thus so are the violations of the ceteris paribus assumptions incorporated in the definition of marginal products.

Nicholson and Snyder conclude this appraisal as follows:

For this reason, it is more common to study the entire production function for a good, using the marginal product concept to help understand the overall function. (p. 220)

However, in the rest of the book, the ceteris paribus assumption in the definition of marginal product is maintained throughout in spite of its widespread violation in the real world, and the “entire production function” in the real world, including fixed proportions and material inputs, is hardly discussed at all. Nine pages later, there is a brief discussion of fixed proportions, and it is acknowledged that “fixed proportion production functions have a wide range of applications.” (p. 229) However, it is not mentioned that, with fixed proportions, the ceteris paribus assumption in the definition of marginal
product is violated and thus the concept of marginal product does not apply to this wide variety of applications.

Unfortunately, there is no similar appraisal of the concept of marginal product in their higher level textbook.

4.2 Material inputs and the demand for capital

The demand for capital is derived in the usual way (similar to Varian and Frank), based on the concept of the marginal product of capital and the profit maximization condition that the marginal revenue product of capital is equated with the rental rate of capital. Again, nothing is said about material inputs, which invalidates this derivation of the demand for capital, because materials cannot be held constant while output is increasing, which makes the marginal product of capital (or labor) impossible in goods-producing industries.

Nicholson and Snyder also discuss the case in which firms have the option to purchase capital goods rather than rent them (pp. 618-21), and their analysis is essentially the same as Frank’s analysis discussed above, and thus is subject to the same criticisms.

4.3 The supply of capital goods and the rental rate of capital goods

Nicholson and Snyder present a good discussion of the supply of capital goods and the determination of the rental rate of capital goods, which is similar to Frank’s. It is assumed to begin with that firms rent their capital goods from rental firms who provide the supply of capital goods. The rental rate of capital goods is analyzed from the point of view of these rental firms, and the long-run equilibrium rental rate is determined entirely by the costs of the rental firm (depreciation cost plus opportunity cost), and does depend on the demand for capital goods, and thus the rental rate of capital
goods is not determined by the marginal product of capital (even if the MPK existed, which in general it does not).

Nicholson and Snyder explain the determination of the rental rate of capital by the costs of the rental firms as follows:

Consider a firm in the business of renting machines to other firms. Suppose the firm owns a machine (say, a car or a backhoe) that has a current market price of \( p \). How much will the firm charge its clients for the use of the machine? The owner of the machine faces two kinds of costs: depreciation on the machine and the opportunity cost ... If we assume that the machine rental market is perfectly competitive, then no long-run profits can be earned by renting machines. The workings of the market will ensure that the rental rate per period is exactly equal to the costs of the machine owner. (p. 617; emphasis added)

Thus the rental rate of capital goods is “exactly equal to the cost” of the rental firms and the demand for capital goods and the marginal product of capital goods play no role in the determination of the rental rate of capital goods (see Figure 1 in Section 1.3 above).

The opportunity cost of the rental firm is the prevailing rate of interest. Nicholson and Snyder present two different theories of the determination of the rate of interest in their two textbooks. In the higher level book, the rate of interest is determined by supply and demand in a one-commodity “future goods” market, as inversely related to the price of the future good (is a simplified version of Fisher’s theory of the rate of interest). In their non-calculus book, the rate of interest is determined by supply and demand in the “loan” market, similar to Frank’s loanable funds market. But it still remains true in both theories that the rental rate of capital goods is determined solely by the costs of the rental firm, one of which is the interest rate, and thus the rental rate does not depend on the demand for capital and the marginal product of capital.13

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13 In the non-calculus book, much more attention is given to the determination of the rate of interest (the main subject of Chapter 14) than to the determination of the rental rate of capital goods, which is discussed in only a few paragraphs (pp. 492-94); and the main point of this brief discussion is that the exogenous rate of interest is one component of the rental rate, so that e.g. an increase in the rate of interest causes an increase in the rental rate (which in turn reduces the demand for capital goods and also the demand for loans).
Nicholson and Snyder also argue that if firms own their machines, rather than rent them (which is of course the normal case), this does not change the essentials of the supply of capital goods and the determination of the rental rate of capital goods. (pp. 617-18) Supply is still determined by costs and the same costs must be incurred in the case of ownership, including the imputed opportunity cost of owning capital goods. It is argued that, in this case, the firm that owns its capital goods would charge itself the opportunity cost, along with the depreciation cost, and the long-run equilibrium rental rate would still be determined solely by the sum of these two costs. Thus the demand for capital goods and the marginal product of capital goods would still play no role in this determination of the rental rate.

In their non-calculus textbook, Nicholson and Snyder briefly present (one paragraph) another assumption concerning the supply of capital goods – that capital goods are supplied by firms that produce capital goods rather than by rental firms or by firms that own their own capital goods (p. 460). In this case, it is argued, the derivation of the supply of capital goods is essentially the same as the derivation of the supply of output in the theory of the firm presented in previous chapters, and thus no further derivation or discussion of the supply of capital goods specifically is needed.

Hence, we can safely assume that firms that produce inputs to be sold to other firms have upward-sloping supply curves. (p. 460)

However, there is an important difference between the price of capital goods as outputs and the price of capital goods as inputs. The price of capital goods as outputs in the usual theory of supply is their actual market price, whereas the price of capital goods as inputs in marginal productivity theory is the rental rate of capital goods, which is the actual market price plus an imputed opportunity cost of investing in capital goods rather than alternative investments, as all these authors emphasize. This important difference – different price variables – should be acknowledged and an explanation given of how the usual theory of the supply of output can be modified in marginal productivity theory to be a function of the rental rate of capital goods rather than their actual market price. I don’t see how this can be done coherently.
If the supply of capital goods is assumed to be a function of the rental rate, which includes the rate of interest as the opportunity cost, then an exogenous increase of the interest rate from outside the capital goods market would increase the rental rate, which in turn is supposed to increase the supply of capital goods. But why would a firm produce more machines if the interest rate rises? The imputed interest rate is not actual revenue received by firms that produce capital goods and thus there is no real incentive for these firms to increase their quantity supplied. Indeed the opposite is true: in the theory of the firm, the interest rate is one of the costs of the firm (not revenue), and thus an increase in the interest rate would increase the firm’s cost and would likely reduce the firm’s quantity supplied (not increase it) (i.e. would shift the firm’s supply curve to the left). Therefore, we cannot “safely assume” that when the supply of capital goods function is modified to be a function of the rental rate that the supply curve will be upward sloping.

In the next section (“Equilibrium Input Price Determination”), Nicholson and Snyder put together the supply of inputs and the demand for inputs in order to determine the equilibrium price of inputs. A graph of the labor market and the determination of the equilibrium wage is presented with the familiar supply and demand curves, and they assert:

For this figure we have chosen to diagram equilibrium wage determination in the general labor market, but the graph would serve equally well ... for any other input market. (p. 462; emphasis added)

However, there is again an important difference between the equilibrium price of capital goods and the equilibrium wage (price of labor). The equilibrium wage that is determined in the labor market is the actual market wage, whereas the equilibrium price of capital goods that is determined in the capital goods market is in general not the actual price of capital goods, but is instead a hypothetical rental rate of capital goods, which also includes an imputed opportunity cost. Again, this important difference between the labor market and the capital market – different kinds of price variables – should be acknowledged and an explanation given of how the capital market functions with this different kind
of price variable. A starting point would be to explain the derivation of the supply of capital goods as a function of the hypothetical rental rate of capital goods, as discussed in the previous paragraph.

5. Pindyck and Rubinfeld (8th edition)

Pindyck and Rubinfeld's *Microeconomics* (8th edition) is currently the #1 best-selling intermediate microeconomics textbook. Chapter 14 (“Markets for Factors of Production”) is about “labor, raw materials, and other inputs” (p. 529), but (surprisingly) capital is not discussed in this chapter (so the general title is somewhat misleading). This chapter presents the standard derivation of the demand for labor, based on the marginal product of labor, and the standard derivation of the supply of labor, based on the labor-leisure trade-off (with an unusual emphasis on the backward-bending labor supply curve).

Chapter 14 also derives the demand for *material inputs*, using *fabric* as an example to make clothing as an example. The demand for fabric is derived in the usual way – by equating the “marginal revenue product of fabric” (the “marginal physical product of fabric” times the unit price of clothing) with the price of the fabric. But what is the meaning of the “marginal physical product of fabric” – the extra clothing produced from an extra unit of fabric, holding both labor and capital constant? But how is it possible to produce more clothing if both labor and capital goods are held constant? No explanation is given. This is the only textbook I have seen that derives the demand for material inputs in this way and it clearly makes no sense. Fabric cannot make more clothing by itself.

Chapter 14 also has a featured example about “The Demand for Jet Fuel”. (Example 14.1, pp. 536-37) It is explained that when the price of jet fuel increased substantially in the 1970s, airlines responded by adopting more efficient flying practices, like carrying less excess fuel and flying at slower speeds, which reduced their demand for fuel. Hence the demand for jet fuel varies inversely with its price (although estimates show that it is fairly inelastic in the short-run). In the last paragraph of this
feature, the concept of the “marginal revenue product of jet fuel” is mentioned for the first time as a synonym for the “demand for jet fuel”. Figure 14.6 (“The Short-Run and Long-Run Demand for Jet Fuel”) shows two downward-sloping demand curves that are labeled MRP_{SR} and MRP_{LR}. However, no explanation is given of what is meant by the “marginal revenue product of jet fuel”. The airlines’ response to rising prices had nothing to do with the “marginal revenue product of jet fuel” (whatever that is), but consisted of changing flying practices. The concept of the “marginal revenue product of jet fuel” requires that all other inputs remain the same. But in this case, other inputs were not remaining the same, but were changing, and these other changes (not the “marginal revenue product of jet fuel”) were responsible for the reduced demand for jet fuel.\(^\text{14}\)

Chapter 15 (“Investment, Time, and Capital Markets”) derives the demand for the purchase of capital goods (not rentals), similar to that of Frank and Nicholson-Snyder discussed above, except that Pindyck and Rubinfeld do take into account the cost of materials in the calculation of net profit. Therefore, their calculation is more correct, but the extra output that results is not the marginal product of the machine alone, because the materials also increase.

Furthermore, as in the other textbooks, the demand for the purchase of capital goods derived in this way is not put together with the supply of capital goods in order to determine the price of capital goods. Nothing is said in this chapter about the supply of capital goods. As with Varian, a blade is missing from the scissors of the marginal productivity theory of the price of capital. Instead the price of capital goods is taken as given; and the price of capital goods that is taken as given in this comparison

\(^{14}\) Cigliano 1982 is the original research article on which this example is based, and it does not mention the “marginal revenue product of jet fuel” at all. In fact, a production function derivation of the demand for jet fuel is explicitly rejected on the grounds that output in the airlines industry is not a single product, but many different routes.

Nicholson and Snyder (2010b, p. 455) also discuss the adjustment of the airline industry to changing fuel prices since the 1970s, and emphasize new fuel-economy engines (i.e. a change of capital goods). Their analysis does not mention the “marginal product of jet fuel”.
with present value is the actual price of capital goods, which is different from the price of capital goods as usually defined in marginal productivity theory, which also includes an imputed opportunity cost.

**Conclusion**

All these textbooks acknowledge the existence of material inputs in many production processes, and they all assert that material inputs can be incorporated into marginal productivity theory, but none of them explain precisely how this necessary incorporation could be accomplished. No explanation is given of how material inputs could be held constant while output is increasing, which is a logical prerequisite for the concept of the marginal products of capital and labor (i.e. for partial derivatives). Also, no explanation is given of how materials could be substituted for capital or labor in order to produce the same quantity of output. In earlier editions, Frank assumed a “value added production function” in an attempt to solve the materials inputs problem, but he abandoned that attempt in the most recent edition. Nicholson and Snyder present footnotes which seem to suggest that a value added definition of production functions is acceptable, but they do not present one in the text. Pindyck and Rubinfeld include materials in the production function, using fabric as an example, but it is not explained what is meant by the “marginal product of fabric”.

All these textbooks present the same standard derivation of the demand for capital (and labor), and thus all make the same logical error – ignoring the extra materials that are required in order to produce extra output, and thus ignoring the extra costs of materials, which results in making the wrong decision about the profit-maximizing demand for capital goods or labor. The derivation of the demand for inputs from their marginal products is the main point of marginal productivity theory, but this derivation is logically flawed because of the existence of material inputs (and also because of fixed
proportions between capital and labor in many production processes). Marginal productivity theory is sometimes described as a theory of the demand for factors only, but it is not even that.

These textbooks make different assumptions about the supply of capital goods and the determination of the rental rate of capital goods. Varian and Pindyck-Rubinfeld do not discuss the supply of capital goods at all and thus do not actually present a theory of the determination of the price of capital goods by supply and demand in the capital goods market. Frank and Nicholson-Snyder present an interpretation of the supply of capital goods based on the assumption that capital goods are supplied by rental firms, and they argue that the long-run equilibrium rental rate of capital goods is determined solely by the costs of these rental firms (depreciation cost plus opportunity cost). Therefore, the marginal product of capital and the demand for capital play no role in this determination of the rental rate (see horizontal rental rate line in Figure 1 in Section 1.3).

The “opportunity cost” of investing in a machine rather some alternative investment is the prevailing rate of interest times the price of capital goods (iP_c). The rate of interest is taken as given in marginal productivity theory and not explained. Thus marginal productivity theory takes as given the main variable that it is supposed to explain – the return to capital. Instead, the rate of interest is determined by some other theory besides marginal productivity theory. The main other theory presented in these textbooks to determine the rate of interest is the loanable funds theory, a macroeconomic theory that has been around a long time, and long before marginal productivity theory.

These textbooks (and other textbooks) should acknowledge these logical problems and criticisms of marginal productivity theory (and also the “additional criticisms” mentioned in the introduction), and should discuss how these problems could or might be resolved within marginal productivity theory, instead of ignoring or denying these problems and attempting to finesse them or

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15 All these textbooks mention fixed proportions in their discussion of production functions, but none of them mention that marginal productivity theory of the demand for factors does not apply in the case of fixed proportion production functions.
cover them up. Textbooks (and professors) should encourage critical and independent thinking about marginal productivity theory (and all economics theories), rather than memorization and acceptance of authority. Textbooks should also discuss alternative theories of profit and income distribution and compare these alternative theories with marginal productivity theory – e.g. Marx’s theory, Post-Keynesian theory, etc. “Compare and contrast” is a well-known and highly recommended pedagogical approach, which is usually an effective way to learn a theory and its strengths and weaknesses in comparison to other theories and also to learn how to think critically about all theories, which seems especially appropriate in this case of alternative theories of distribution.

A good example of the kind of critical and open-minded presentation of marginal productivity theory that I am suggesting is the 1st edition of Nicholson’s advanced textbook (Microeconomic Theory) published in 1972. Essentially the same theory is presented as in later editions, but the main difference is that the 1st edition explicitly acknowledged criticisms of marginal productivity theory and even devoted an appendix to the chapter on capital to these criticisms and even a brief discussion of Marx’s alternative theory of profit (the appendix is entitled “Alternative Views on the Theory of the Factoral Distribution of Income”).

In the Introduction to Part V on factor prices, Nicholson remarked that:

the theory presented in the three chapters of this part is not as widely accepted by economists as are the theories of prices in the goods market ... far more objections have been raised about the application of supply-demand analysis to factor markets ... particularly true of the capital market. (pp. 326-27; emphasis added)

And he concluded the Introduction with the expressed intention to present the issues in marginal productivity theory in a balanced way that “permits the reader to carry forward a discussion of this question on his own.” (p. 327). I strongly endorse this pedagogical goal.

In the appendix to the chapter on capital, Nicholson remarked that it would be a mistake to think that all economists are “happy” with the marginal productivity theory of capital. The two main

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16 I am grateful to Blythe Keller for discovering this 1st edition of Nicholson’s textbook.
logical problems in marginal productivity theory that Nicholson discussed in this appendix are the “adding up problem” (i.e. the “exhaustion problem”) and the “aggregation problem” (both of these are included in my list of “additional criticisms” in the introduction) and he acknowledged that these problems have not been completely solved. I would of course go further and discuss all the other logical problems in marginal productivity theory, and I would also include a more substantial discussion of Marx’s theory. But Nicholson’s presentation was a significant step in the right direction of openness and pluralism and critical thinking in the teaching of economics and marginal productivity theory in particular.

Unfortunately, Nicholson’s critical presentation was largely abandoned in the 2nd edition (1978) and in all editions after that. The introduction to the part on factor prices does not mention that marginal productivity theory is not as widely accepted by economists, nor the objections to the theory, especially to the theory of capital. There is no appendix to the chapter on capital which discusses some of these objections and presents Marx’s alternative theory (the earlier “alternative views” appendix is replaced by mathematical appendixes on compound interest and optimal resource allocation over time).

I hope that Nicholson and Snyder will revive the more critical approach to marginal productivity theory in future editions, and that other textbook authors will do the same.

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