Patterns of Growth in Chinese Cities: Implications of the Land Lease*

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Abstract

A special feature of China’s housing market is land use rights in the form of land leasehold contracts granted by the government. Using a model in which a representative developer may choose to redevelop existing centrally located housing or develop new housing at the periphery of the city, we show that as the city grows, the land leasehold system results in the city center being developed less intensely and more land being used on the outskirts of the city when compared to a “fee simple” environment. Thus, cities in China are likely to be relatively more spread out, with city centers relatively older than would be the case with “fee simple” ownership. Our model suggests that excess residential land use is about 6 percent. In addition, compared with the ownership case, housing supply will grow more quickly in the near future, but more slowly later on during the transition of the Chinese economy. Parallel to the supply growth pattern, equilibrium price grows relative slowly in the near future, but more quickly later on. While we focus on residential uses, we believe our model can be applied to other land uses.

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1 Introduction

Few people dispute that the recent rate of housing construction in China is extraordinary. It has affected the personal wealth, the quality of life and the job mobility of tens of millions of people. The issue of housing has also dominated discussions among policy makers who worry about the price level and, our point of focus, about the quantity, mix and location of housing as the city grows. The institutional framework that facilitates the allocation of land among users and the nature of their rights to develop now and in the future are critical to how cities grow. The allocation is especially important in China where, according to McKinsey projections, the urban population will grow by 350 million people between 2009 and 2025 and over 1 billion people will live in cities by 2030 (Woetzel, Mendonca, Devan, Negri, Hu, Jordan, Li, Maasry, Tsen, and Yu (2009)). We examine the nature of land use rights in China and, depending on the values of certain parameters, argue that the current leasehold system could increase the amount of land used for housing by residents by between 3.7 and 13.7 percent. The system would impose similar pressures on commercial land use which we do not model.

In China, local governments maintain ownership of the land and offer land use rights in the form of ground leases which allow a developer to build on the land. At the end of the lease, 70 years in the case of residential uses, 50 in the case of commercial and 40 in the case of industrial uses, both the land and any buildings become the property of the lessor, the government. The right of the owner of the land use rights (the ground lessee) to redevelop is not well defined. This institutional setting influences the decisions of land users and, as market forces adapt to the institutional setting, the choices available to consumer.

The 70-year lease term for residential projects inevitably alters the decision regarding the development and redevelopment of housing projects. The redevelopment decision is influenced by the length of time during which the investor is able to recover invested capital. Given the same current and future house price, the shorter the investment horizon, the less is redevelopment. Therefore, the overall redevelopment rate is lower under leasehold than under fee simple ownership. The timing of redevelopment also changes. Since developing a new project with a full lease term produces a longer flow of payments than redeveloping an existing property with a shorter remaining lease term, the land lease system frustrates attempts to increase density in the urban core and pushes activity to the periphery.

We construct a general equilibrium model to assess the effects of ground leases on the shape of a city, on land use (and reuse) within a city and on how these effects express themselves dynamically. The model includes consumers and a representative developer who interact in perfectly competitive markets in an infinite period model. The developer takes the house price path as given, leases land from the government,

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1 The McKinsey study indicates significant concern regarding the loss of arable land as China urbanizes. This concern is heightened with more dispersed development scenarios and the McKinsey study recommends policies directed toward building more concentrated cities.

2 More specifically, land in cities and towns are owned by the state, and land in villages are collectively owned. As cities expand, municipal government pays villagers to acquire the collective-owned land. See Chapter 5 of Property Law of the People’s Republic of China.
constructs residences and rents them to households. The developer can redevelop an existing residential structure or develop a new residential structure at the urban periphery or both. Unlike a static model, our model allows the developer to re-develop a location multiple times during the term of the lease. On the demand side, a continuum of infinitely-lived households receives income exogenously, takes rents as given, and allocates their income between consumption and housing. The rental rate in each period is determined endogenously by market clearing conditions. With growing income, households’ demand for housing services increase, which induces more supply through higher rental rates. The city expands gradually because new development occurs only when rental rates in urban core rise enough that the transportation cost is no longer a hurdle. We calibrate the model to Chinese data, and solve it computationally. We also solve a reference model in which land is owned. A comparison of results from the two models illustrates the impact of the leasehold system on patterns of city growth in China. We believe that our model is the first to study the dynamic features of land markets in different land use rights settings, specifically land leasehold and fee simple settings.

Our model is meant to reflect the expected evolution of the Chinese economy. We assume that the growth rate of aggregate income will decline from 8% to 2% over a 30 year horizon as the economy matures. Upon the stabilization of income growth, the government stops contracting new land lease, and the existing urban land reverts to the government when the lease expires. We show that as the growth of income stabilizes, the economy converges to a balanced growth path (BGP). In the BGP, we assume that the government redevelop the structure optimally as a developer would in a fee simple setting. This assumption allows us to focus on the transition period in which the broad use of 70 year leases shapes the supply, price and density of housing in cities in China currently.

We show that, in the end of the transition period, the leasehold system increases the amount of urban land in China by 5.7 percent, which is equivalent to slightly more than 700 sq. km. If land is leased instead of being owned then we suggest that, after 50 years, the rent paid by consumers would be about 7 percent higher and the difference in the age of buildings would represent a loss in consumer value worth about 4 percent.

Land leases have a long history in Hong Kong, dating from the British possession in January 1841 when the land lease was chosen as a means of allocating land. That custom continued after the transfer of sovereignty to the People’s Republic of China. In the current setting, most leases are 50 year leases and are presumed to be renewed unless “circumstances require in the public interest”. The renewal would occur with no premium and an annual lease payment of three percent of ratable value. In China, these institutional arrangements are still evolving but the roots of the Chinese system depend in part on advice from current Hong Kong Chief Executive Leung Chun-ying dating back to 1984.

China and Hong Kong are not alone in their use of ground leases. For various reasons, some jurisdictions prefer to use ground leases rather than fee simple ownership as a means of establishing the use rights for real

http://www.bbc.co.uk/news/world-asia-china-17594452
property. For example, like China, many jurisdictions in transition have chosen to use the land lease as a means of privatizing land use rights because the approach is politically palatable, provides local governments a revenue stream and facilitates the introduction of urban planning in a market setting. See Dale-Johnson and Brzeski (2002).

The remainder of this paper is organized as follows. Section 2 briefly reviews the historic background of housing markets in China. Section 3 discusses some of the previous literature related to our model. Section 4 lays out the model. Section 5 presents a simpler model which illustrates its key features and some of the intuition, at the cost of making some important variables exogenous. Section 6 reports the computational results after a discussion of how we calibrate the model to represent the situation in a Chinese city. The concluding section and Appendix 3 summarize the results and their sensitive to the limitations of the model.

2 Institutional Background

Before the People’s Republic of China (PRC) was founded in 1949, land could be privately owned and legally transferred through mutual agreement. Land use rights were radically transformed after the Communist Revolution. In rural areas, privately-held land was confiscated for the use of peasant farmers who ultimately joined communes by donating their assets including land. In urban areas, the Communist Party moved more slowly. Some property was confiscated from foreign owners and from those who had opposed the Revolution however, private ownership and land transactions continued in some forms. Nevertheless, by the end of the Cultural Revolution and the arrest of the Gang of Four in 1976, virtually all land was either held by the state or by collectives. Meyer (2009) estimates that, between 1949 and 1978, the rate of ownership in Beijing fell from 77 percent to about 10 percent.

On September 9, 1987, the first land leasehold agreement in modern China was signed in Shenzhen, a then rapidly growing city in southern China on the border of Hong Kong’s New Territories. The government signed a fifty year lease on a parcel of residential land of 5321.8 square meters with a local public company following private negotiation. That date preceded a key step in China’s urban land reform, the revision of a constitution that did not permit any land transactions including leasing. In April 1988, the constitution was amended to allow for transactions involving land use. Ground leases were adopted in 1990 as the mechanism for assigning land use rights to urban land users. In 1991, the law was revised again to allow the sale and transfer of ground leases, creating a “secondary land market”. Thus a public market has evolved with brokers facilitating land transactions. Between 1981 and 2005, the size of urban areas in China grew by 340 percent to more than 32,500 square km. (Yan (2008)). The National Bureau of Statistics in China reported that, in 2010, the area of the build up districts was 40058 sq. km.\textsuperscript{4}

Now, in China, the typical lease term is 70 years for residential uses, 50 in the case of commercial and 40 in the case of industrial uses. Leaseholds can be acquired through private negotiation, bidding and public

\textsuperscript{4}Source: \url{http://www.stats.gov.cn:82/tjsj/qtsj/hjtjzl/hjtjzl2010/t20111229_402788833.htm}
auction. It is most common that the acquisition occurs through private negotiation with a local government. However, even if the leasehold is acquired through bidding or auction, the complexity of the issues in the land lease contract for a large project usually will require further private negotiation. The payment for leasing land often consists of three major components: a lump-sum premium, an urban infrastructure fee (city wide levy) and a project related or community infrastructure fee (neighborhood levy). A periodic lease payment is made which reflects a percentage of the value of the property.

According to the Property Law of the People’s Republic of China passed in 2007, Article 149 states “The term of the right to the use of land for building houses shall automatically be renewed upon expiration. The term of the right to the use of land for non-house building purpose shall be renewed according to laws and regulations upon expiration. With regard to ownership of the houses built on the land and other real property related, the relevant agreement (if any) shall be abided by, or, if there is no such agreement, the relevant provisions stipulated by law and administrative regulations shall be observed.” Importantly, the legislation does not address the terms of the renewal. Separate from the mechanism for establishing land use rights for a developer, the allocation of housing units was based in part on where the head of the household was employed. While administratively convenient and consistent with a centralized planning model, consumer welfare was unlikely to have been maximized because some households over-consumed while others under-consumed housing services. Bertaud and Renaud (1997) discuss some of the issues involved with the socialist system of allocating land use and housing. Wang (2011) found that enough Chinese households were under-consuming housing services that privatization of the market unleashed sufficient demand to influence price.

In the short period following its adoption, the new system of land use rights has had a profound impact. While the bundle of property rights in a leasehold system are more restricted than in a freehold system, the introduction of a market for use rights provided Chinese residents and firms with greater economic freedom and signaled a shift in the way of doing business in China. By establishing legal rights of use, China allowed land markets to develop and created a source of revenue for local governments in a socialist setting. The resulting real estate market transformed China’s urban landscape. Meyer (2009) offers a personal account of some of these polices and the positive and negative impacts on the residents.

When the law was initially written, consequences which might occur 40 to 70 years in the future were not well addressed. The property right at the termination date of a ground lease is known as the “residual interest”. Absent defined rights for the lessee, the property and improvements usually revert to the lessor, in this case, the state. Recently, particularly for residential properties, this uncertainty has led to pressure for the government to clarify what will happen when the lease terminates. With respect to Land Use Rights for residences, the Property Law of 2007 revisions clarified that the contracts will be extended but has not clarified the terms and conditions leaving significant uncertainty. For non-residential property, neither issue has been clarified.
3 Background Literature

This paper merges two strands of research. The first focused on urban growth and development and the second focused on land use rights and, in particular, the choice between fee simple and long term land leasehold as a form of land use right. These two strands of literature have not been joined despite their critical importance in one of the largest and most rapidly urbanizing countries in the world. We begin by reviewing the relevant literature in each area and then focus on recent relevant research in Chinese real estate markets.

Anas, Arnott, and Small (1998) summarize the early literature regarding urban spatial structure. Researchers have employed both theoretical and empirical means to explore the dynamics of city growth and structure. Wheaton (1998) develops a model of perfect foresight and shows how congestion raises optimal density at the urban core in a standard urban model. Capozza and Helsley (1990) develops a model incorporating uncertainty which shows that uncertainty delays the conversion of land from agricultural to urban use, imparts an option value to agricultural land, causes land at the boundary to sell for more than its opportunity cost in other uses, and reduces equilibrium city size. Subsequently, Capozza and Sick (1994) explore the role that risk plays in determining the rent gradient and the size of cities. They show that the price of land awaiting conversion increases with the growth rate of urban rents and unsystematic risk but decreases with risk aversion. The impact of systematic risk is uncertain. City size increases with the growth rate of urban rents but decreases with systematic and unsystematic risk. In our work growth of city size is also driven by the growth of rents which is influenced by the form of land use rights.

Other papers have considered the role of durable capital in the redevelopment decision as cities grow. Capozza and Li (1994) model the decision to replace durable capital when intensity is variable. The authors employ an optimal-stopping framework characterizing the value of the project, the timing of investment, and the intensity of development. The authors show that intensity interacts in important ways with timing, taxes, and project values. The ability to vary intensity raises hurdle rents and delays development decisions. We allow our developer to vary intensity but focus on the impact of the nature of land use rights.

Braid (2001) presents a method for deriving perfect-foresight spatial growth paths for an urban area, under the closed-city or open-city assumption, when the housing at any location can be redeveloped many times. In his model of a mono-centric city, which is growing deterministically, building starts from the center. He also shows that, when the city becomes big enough, re-development also starts from the center and expands outward. In theory, the pattern of redevelopment should look like rings on a pond after a rock has been thrown: over time, the peak of each wave expands outward from the center and it will be followed by other waves. In our model, the possible termination of the land use right causes this pattern of redevelopment to be frustrated.

A rich data set of parcel characteristics and real property transactions for the Seattle area is employed by Cunningham (2006) to show that, since the development decision is a real option, there is greater price uncertainty which delays the timing of development and raises land prices. In our work, we consider how
the incentives for redevelopment decisions can change over a long period of time and are influenced by the developer’s land use rights.

Wang and Xie (2011) offer a detailed optimal control perspective on the problem of dynamics in a general equilibrium model with housing, as well as a durable capital which can be used to increase production of a non-housing good, with much attention paid to the question of how the endogenous variables evolve toward a steady state equilibrium. Amongst other results, they note that the transition path is not monotonic; overshooting is expected due to substitution between the different types of durable goods.

In a dynamic model, Turnbull (2004) notes how policies, such as development fees and limitations on the amount of land available for development, can affect both the steady state equilibrium and the transition to an equilibrium. He finds that a market does not allocate land efficiently because green space offers an externality. He shows that this externality can be offset using some familiar policy levers but that, by changing the relative value of different types of land, the policy can also affect the rate of development. Our paper looks at a different aspect of this problem: redevelopment. In our model, the total quantity of land under development can be influenced by the nature of land use rights which, in turn, can impact the quality, mix, age and location of housing within the city.

There is a large literature on urban sprawl which addresses how cities grow and the implications for government policies related to infrastructure development such as public transit (Gordon and Richardson (2011)). Recent work by Glaeser (2012) argues that cities are environmentally efficient because of a kind of returns to scale. Our work complements some of the work written by urban planners who argue that cities become too big because the price of farm land is “too” low (e.g., Bertraud (2010)). We argue that it is the nature of land use rights and related uncertainty that leads to the “over-consumption” of arable land.

The relevant literature in the area of land use rights and land leaseholds includes Grenadier (2005) which offers a comprehensive analysis of lease contracts including a short section on ground leases. Capozza and Sick (1991) examine the implications of the ground lease (right of possession and use for a period of time versus the right of ownership) on the redevelopment decision. They find, using option pricing theory, that the discount in the value of a ground lease relative to the fee is not just the result of the zero terminal value of the lease to the ground tenant, but also the result of the reduced redevelopment opportunity afforded the lessee as a consequence of the foreseeable termination of the lease. A ground lessee will redevelop sooner and at a lesser intensity than a fee owner would in the same economic circumstances. Construction is Cobb-Douglas and rents are stochastic. The fee simple values are determined analytically and the lease values are solved for using numerical methods. Dale-Johnson (2001) examines the nature of that contract and the implications for redevelopment timing and the density of development. Monte Carlo sampling and a genetic algorithm is employed to evaluate alternative well-defined contracts. The author concludes that the nature of the contract and specifically how the residual interest is treated has a significant impact on the timing and intensity of redevelopment in a leasehold scenario.

An alternative may be to suppose that the property rights are defined ambiguously, a problem which is
addressed by Wang and Sun (2011) and Zhu (2012). There are two results from ambiguity or uncertainty regarding public policy. The first is that any increase in the transaction costs establishing the nature of the residual claim necessarily reduces the value of a leasehold interest, with its finite horizon, relative to that of a fee interest. Second, the risk premium associated with any valuation of a leasehold interest increases, with the same result.

Given the rapid growth of Chinese cities, numerous authors have turned their attention to applications of these modeling approaches in a Chinese setting. In a Lincoln Institute volume, Ding and Song (2005) collect several papers focusing on land policy reform and housing policy reform in China. They provide an excellent background to the theoretical and empirical papers cited below.

Zheng and Kahn (2008) and Lichtenberg and Ding (2009) study the rent gradient in Beijing and Shanghai respectively. The former paper finds the classic urban monocentric model’s predictions are upheld while documenting the importance of local infrastructure and services as well as air quality. The latter paper also finds evidence of rent gradients in Shanghai despite administrative restrictions on land conversion. Urban land values exceed agricultural land values by a large margin reflecting the potential of conversion at the periphery. In our work, we find that conversion will occur sooner in an environment where land use rights are allocated through long term ground leases rather than fee simple since developers are pushed to build new projects on the periphery rather than to redevelop existing locations.

Zheng, Fu, and Liu (2006) discuss the Pareto efficient allocation which sorts different types of residents in a Chinese city to their preferred location. In theory, different types of consumers should sort themselves according to their bid rent function at intervals further and further from the city center with higher income residents at the center. Using survey data collected from five large Chinese cities in 2003, they show that this ideal allocation is not achieved in practice. They cite three reasons for the poor allocation: ambiguity in the value of previously state-provided homes, a spatial mismatch between job market and housing market opportunities and, using indirect measures, imperfections in finance markets. We explore the impact that the type of land use right has on the location, mix and quality of housing as the city grows.

Zoning restrictions of various forms can distort market outcomes. This is more probable in rapidly urbanizing areas where constraints are apt to be binding. Fu and Somerville (2001) examine the relationship between the incentives facing local governments and their impact on density constraints in the Shanghai market. Peng and Thibodeau (2012) using data from 2001 to 2004 from seven Chinese cities show that the market for residential land became less efficient after municipal governments gained direct control of the land supply and that land prices became less sensitive to changes in property prices, likely the result of less informed administrators involvement in the market.

Whether local government officials react to common economic incentives when deciding on how much land to convert is considered by Lichtenberg and Ding (2009). A formal model is proposed where officials maximize the present value of their asset (land) where land is more likely to be converted if its value as a farm is lower than its ability to produce revenue for the government. Using data on several fast-growing
coastal provinces for the years 1996 to 2004 and a panel data model, they confirm these predictions. This paper provides insight into the actual experience of agricultural land conversion, which is an outcome in our model.

These papers suggest that, as cities in China have evolved, many of the attributes of cities in developed economies can be identified. In little more than a decade and one half, the existence of many of the phenomena that we observe in western cities can be confirmed in Beijing, Shanghai, and other centers in China. However, we believe there will remain at least one significant difference between western cities, in which fee simple ownership prevails, and Chinese cities, in which land use rights are allocated using a land lease system.

4 Model

4.1 The Economy

The economy is populated by a continuum of consumers of measure one and a representative developer who builds houses either by re-developing existing residential areas or by developing new lands or both. Land is owned by the government, but can be leased to the developer for 70 years.

In period \( t=0 \), there exists a unit measure of housing structures that differ in size (height), house age, and age of land lease. The existing structures and the land they sit on are called the “old town”. We use \( x = 0 \) to denote the land in old town, which amounts to 1 unit in total. Land in the outskirts of the old town that can be converted into residential area is characterized by its distance from the old town. Let \( x \in \{1, 2, \ldots, N\} \) denote N locations, with location \( x = 1 \) nearest to the old town and \( x = N \) farthest. Each location \( x \) is associated with \( 1/N \) unit of land, thus total new land supply is also one unit. In other words, we allow the city to double its size.\(^5\)

If the developer finds it profitable to develop a new residential area, she pays a lump sum amount to acquire the right to use the land for 70 years. Since we assume transportation cost increases with the distance to old town, new development takes place around the existing lots: i.e., it is not optimal to develop land of \( x = 5 \) unless land of \( x = 4 \) has been developed. Development of multiple locations in one period is allowed as long as it is profitable.

The housing service, \( s_t \), provided by any house at any time \( t \) depends on the size (\( h \)), age (\( a \)) and location (\( x \)) of the house. We assume the following functional form

\[
s_t = s(h_t, a, x) = \frac{\psi^a}{f(x)} h_t
\]

where \( \psi \in (0, 1) \) is the discount due to house age. \( f(x) \) is an increasing function of \( x \) so that houses farther away from city center provides less service flow. This captures the idea that houses near the city center

\(^5\)If one thinks of these locations as \( N \) belts around the old town, then the outer ring of belt \( x \) has radius of \( \sqrt{\frac{1+x/N}{\pi}} \), where \( \pi \) is the ratio of a circle’s circumference to its diameter.
provide more conveniences. Hereafter we refer to \( f(x) \) as transportation cost, with the normalization \( f(0)=1 \).

The formulation in equation (1) assumes that consumers care about \( s_t \), but are indifferent to the composition of age, location and size in \( s_t \). Consequently, let \( Q_t \) be the rental price of one unit of housing service with age 1 and in location 0 at time \( t \), then the rental price of one unit of housing service characterized by age \( a \) and in location \( x \) in period \( t \) is

\[
q_{t,x,a} = Q_t \frac{\psi^a}{f(x)}
\]

where \( q_{t,x,a} \) is called the quality-adjusted rent since both age and location are related to utility provided by the house. Thus, our model has the simplified feature that, when solving for the equilibrium, we need only to solve for the price path of houses at \( x = 0 \) with \( a = 1 \): i.e., \( Q_t \). Once \( Q_t \) has been determined, the price at any other location \( x \) of a house aged \( a \) can be inferred from (2).

### 4.2 Developers

The representative developer receives a flow of rental income from the portfolio of locations that she currently leases from the government but, at any point in time, must decide whether to redevelop that location to increase that flow or to build in a new location to attract more tenants, or both or, if insufficiently profitable, neither.

We assume the developer faces no borrowing constraint, so that she can develop/redevelop multiple locations simultaneously, and each development/redevelopment is an independent decision. This is a reasonable assumption since the developer is a representative one in our model.

In our model, we do not make any essential distinction between a developer and an owner since the motives and constraints would be identical. If a consumer occupied a location which they owned then they would pay rent to themselves. A consumer whose income is rising may wish to move and pay rent in the new location while renting out their former residence to a different consumer, or they may sell it for the present value of the future stream of income. The assumption of perfect capital markets implies that nobody is at a relative disadvantage when borrowing or lending. An owner who decides to redevelop a location in which they live could pay rent at another place for as long as the construction occurs and then move back, if it were profitable to do so. We focus on the real dimensions of the problem, relative prices and quantities.

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\(^6\)With borrowing constraint, the developer may have to choose a few projects from the potential developments/redevelopments.

\(^7\)If consumers own their houses then, when the developer finds it profitable to re-develop a residential area, she pays for the houses directly, demolishes them, rebuilds and resells to new owners. Of course, it may also be relevant to consider the rules on appropriate compensation after an expropriation. Our discussion below and in Appendix 3 offers some ideas.

\(^8\)This difference could be important if we were concerned about the implications for the accumulation of wealth or if different types of players in our model used different discount rates or if the differences were created by some combination of government policy and capital market imperfections. An important practical consideration is that banks will be reluctant to lend to the “owners” of a housing unit if the end of the land lease is close. Thus, a lack of clarity concerning whether the lease is renewed, extended or cancelled may have the same effect on the owner as a distortion in the capital market.
4.2.1 Redevelopment of the Existing Structures

For any lot with an existing structure, the developer chooses whether to redevelop the lot or not to maximize the discounted cash flow. Cash flow includes inflow of rental income and outflow of demolition cost and construction cost in case of redevelopment. In the simplest case with no redevelopment option at any point of time, the discounted cash flow is simply the discounted flow of rental income which can be easily calculated. However with the option of redevelopment, the calculation of discounted cash flow involves timing, frequency and intensity of redevelopment. We use the language of dynamic programming to describe the problem.

For a given structure, the developer’s state vector includes house size \((h)\), house location \((x)\), house age \((a)\) and age of land lease \((\tau)\). The developer’s optimal decision depends on the state vector \(\{h, x, a, \tau\}\), as well as \(\{q^j_{x,a}\}_{j=1}^\infty\) and the consumer’s income from time \(t\) on. The income matters because we assume the demolition cost increases with it. We posit that economic growth will be translated into higher incomes, higher rents and higher costs. For simplicity, we omit both the price sequence and the income sequence in the specification of dynamic optimization problem below.

In period \(t\), the value of a structure with state vector \(\{h, x, a, \tau\}\) is

\[
v_t(h, x, a, \tau) = \max\{v_{tNR}(h, x, a, \tau), v_{tR}(h, x, a, \tau)\}
\]

That is, the value of the structure is the greater of the value of not redeveloping it at this time \(v_{tNR}(h, x, a, \tau)\) and the value of redeveloping it \(v_{tR}(h, x, a, \tau)\).

The value of redeveloping the existing structure is

\[
v_{tR}(h, x, a, \tau) = \max_k -M_t - k + q^t_{x,0}h' + \frac{1}{1+r}v_{t+1}(h', x, 1, \tau + 1)
\]

s.t.

\[
h' = Ak^\alpha \\
k > 0
\]

where \(r\) is the developer’s discount rate.

Thus, redevelopment entails two types of cash outflow in the current period – a demolition cost \(M_t\) and a construction cost \(k\) which produces \(h'\) units of housing. The benefit is two-fold. First, it results in the rental income \(q^t_{x,0}h'\) from a house of age 0. Second, it leads to a new stream of future cash flow summarized in the new value function \(v_{t+1}(h', x, 1, \tau + 1)\). We simplify the model by assuming the demolition and construction is instantaneous (at least, relative to the time that the space is used).

Construction of houses has two inputs – land and capital. The housing production function \(h' = Ak^\alpha\) represents the technology of building houses on 1 unit of land, therefore \(\alpha\) is the share of capital (denoted by \(k\)) in the production while \(1 - \alpha\) is the share of land.

The demolition cost, \(M_t\), increases with both the size of the building being demolished, \(h\), and with labor costs (as represented by the income of consumers, \(y_t\)). We assume

\[
M_t = mhy_t^{1-\alpha}
\]
where \( m > 0 \) is a constant. It will be shown that, when the economy reaches a balanced growth path, the demolition cost per unit of \( h \) grows at the same rate as income.

The value of redevelopment decreases with the current size of the structure, because the demolition cost increases with the size of structure. The value of redevelopment increases with the age of the structure and the age of the land lease; the former because older houses rent for less, and the latter because an older lease implies a shorter stream of future cash inflow.

The value of not re-developing the structure during period \( t \), given the current state, is

\[
v_t^{NR}(h, x, a, \tau) = q^t x, h + \frac{1}{1 + r} v_{t+1}(h', x, a + 1, \tau + 1) \tag{6}
\]

which represents the rental income received this period plus the value associated with the future state \((h', x, a + 1, \tau + 1)\).

If the land were never to be redeveloped in the future, then \( v_t^{NR}(h, x, a, \tau) = \sum_{j=0}^{T_0-\tau} \left( \frac{1}{1+r} \right)^j q^t x, h \) and the value would increase with \( h \). Since future redevelopments are possible, and demolition cost increases with \( h \), \( v_t^{NR}(h, x, a, \tau) \) is not always increasing in \( h \).

Intuitively, the timing of redevelopment depends on age of land lease. Redeveloping “too soon” implies that the developer must incur excessive demolition costs. Redeveloping later, when the land is leased, implies that somebody else receives more of the benefit from the investment and that the developer will not adequately recoup her investment.

### 4.2.2 Development of New Locations

At any time \( t \), for the un-developed locations on the periphery, the forward-looking developer decides whether to convert the land to build structures based on the rental rate sequence \( \{q^t x, a\}_{j=t}^{\infty} \) and the time path of consumer income. The value of converting the land and building structures in location \( x \) is

\[
v_t^D(x) = \max_k -\Lambda_t - k + q^t x, 0 h' + \frac{1}{1 + r} v_{t+1}(h', x, 1, 1) \tag{7}
\]

s.t.

\[
\Lambda_t = L + \lambda y_t
\]

\[
h' = A k^{\alpha}
\]

\[k > 0.\]

\( \Lambda \) is the land acquisition cost which has two components. The fixed component, \( L \), may be a function of geography, local regulations or legal details related to contracting. The second component, \( \lambda y_t \), is proportional to income and captures the idea that some costs, such as labor costs in land conversion increase with the level of income in a city. In addition, as the economy grows, the government must charge more for land use rights. This reflects the practice as the value of land use rights is a function of market values of the leased land.
The continuation value in equation (7), \( v_{t+1}(h', x, 1, 1) \), is the developer's value defined in equation (3), with \( a = 1 \) and \( \tau = 1 \).

If \( v_t^D(x) \geq 0 \) then the land is converted and developed. Initially, \( v_t^D(x) < 0 \) for all \( x > 0 \). Over time, the rise in income increases demand, and leads to higher rental rates, which results in positive \( v_t^D(x) \) for locations near the newly developed land. With the fixed component of acquisition costs and increasing rental income, land on the periphery is gradually converted.

Recent policy requires a developer to start construction shortly after acquiring the land. Therefore it seems reasonable to assume that \( k \) is strictly positive. If land prices were rising unusually fast, then a developer might prefer to acquire land without developing it immediately. But this option is ruled out by government policy. The idea that prices would rise “unusually fast” seems more likely to be a hotly-debated transitory feature than to be a persistent feature or a characteristic of a very long run equilibrium.

### 4.2.3 Terminal Conditions

A major concern of the developer is what would happen when the land lease terminates. By affecting the present value of a project, the expected terminal condition impacts the developer’s initial decision and all other decisions during the term of the lease. The terminal condition depends heavily on the interpretation and future execution of the Property Law of the People’s Republic of China, passed in 2007. As mentioned earlier in the discussion of institutional background, the law states that “the term of the right to the use of land for building houses shall automatically be renewed upon expiration”, but the terms and conditions of renewal have never been clarified.

Given this lack of clarity, we model the situation using the following strategy: with probability \( \pi \), the government fully compensates the developer for the value of structures which equals the discounted streams of rent. This assumption is adequate in our model because we do not focus on the terminal condition per se, but on the developer’s decisions especially during a transition to a balanced growth path.

In our programming problem, the terminal condition is summarized in a terminal value (i.e., a value function in the end of the last period), and the parameter \( \pi \) provides enough flexibility for various terminal values. In the computational exercise, we use \( \pi = 0.5 \) as a baseline while the sensitivity analysis consider values of \( \pi = 0 \) and \( \pi = 0.75 \). Qualitatively, our results are robust to these different parameter values.

To solve for an equilibrium price path, one also needs to know how the existing structures would be redeveloped after a land lease terminates. We simply assume the government makes an optimal redevelopment decision as a developer under fee-simple ownership would do. Since we compare results from leasehold to those from ownership, this assumption guarantees that all the differences come from single term of land lease. Intuitively, any differences between the institutions would be larger if multiple terms of land lease are allowed.

\[ k \]

If \( k = 0 \) then, by definition, the new project is not undertaken.
4.3 Consumer and Demand

There exists a continuum of infinitely-lived consumers of measure one indexed by \( i \in [0, 1] \). The consumers receive the same exogenous income \( y_t \), but live in houses of different age and location. Consumer \( i \) solves the following maximization problem.

\[
\max u(c_i^t, s_i^t) \tag{8}
\]

subject to the budget constraint

\[
c_i^t + Q_t s_i^t = y_t
\]

where \( c_i^t \) is non-housing consumption, \( s_i^t \) is the amount of housing services. As mentioned earlier, consumers care about the amount of housing services consumed, but do not care about how age and size are combined to deliver that service. Due to homogenous income, consumers demand the same amount of housing service and non-housing consumption. In this sense the superscript \( i \) is redundant. However we keep it in the notation to remind the readers that consumers live in houses of different ages and locations, essentially trading off age and location.

We assume Cobb-Douglas preferences over two types of consumption.

\[
u(c_i^t, s_i^t) = (c_i^t)^{1-\theta} (s_i^t)^{\theta}
\]

where \( \theta \) is the share of housing in utility. The first order condition of the consumer’s maximization problem yields

\[
Q_t s_i^t = \frac{\theta}{1-\theta} c_i^t \tag{9}
\]

and the consumer’s optimal quantity of housing services is

\[
s_i^t = \frac{\theta y_t}{Q_t} \tag{10}
\]

Consequently we have the following equation

\[
\frac{y_{t+1}}{y_t} = \frac{Q_{t+1}}{Q_t} \times \frac{s_{i+1}^t}{s_i^t} \tag{11}
\]

Thus the exogenous growth in income, the ultimate driving force in our model, translates into growth in price and quantity. Mechanically, the growth in income leads to growth in housing demand, and higher demand raises house price which induces growth in supply. The magnitude of the increase in the market clearing price depends critically on terms of land lease. Therefore, although our modeling of the demand side is simplistic, the equilibrium incorporates a key mechanism through which the terms of a lease affect the endogenous variables.

4.4 Equilibrium

Given the exogenous income path \( \{y_t\}_{t=1}^{\infty} \), an equilibrium in this economy is described by sequences of rental rates \( \{q_{t,a,x}^t\} \), sequences of housing supply \( \{h_{t,a,x}^t\} \), and sequences of housing demand \( \{s_i^t\} \), with \( t = 1, 2, ..., \infty, a = 1, 2, ..., \bar{a}, \tau = 1, 2, ..., 70 \) and \( x = 0, 1, 2, ..., N \), such that
1. Given the price function $Q_t$, $h^t_{\tau,a,x}$ solves the developer’s optimization problem.

2. Given the price function $Q_t$, $s^t_i$ solves a consumer’s optimization problem.

3. Housing market clears, i.e., $\int_x \int_a \int_{\tau} h^t_{\tau,a,x} \frac{\psi}{f(x)} dx d\tau = \int s^t_i di$.

where $\bar{a}$ denotes the upper bound of house age at which it is always optimal to redevelop the existing houses.\footnote{When we compute the equilibrium, we set it to a large number ($\bar{a} = 160$) so that no house in the model can hit that age.}

4.5 Balanced Growth Path

We study how the 70-year ground lease affects the equilibrium price and quantity and pattern of city growth as the Chinese economy matures as reflected by reduced growth assumptions. We assume that, the growth of income declines gradually from the current 8% per year stabilizing at the constant rate $T$ years from now. We show that as income grows at this constant rate, the economy approximately converges to a balanced growth path (BGP) in which each variable in the model grows at its own (constant) rate.

The existence of a BGP enables us to solve the model computationally. Because income grows over time, our model deals with the non-stationary problem. In a BGP, we can transform all the endogenous variables into a stationary equivalent by re-scaling them algebraically. After solving the stationary version of the model computationally, the variables and value functions are transformed back to the actual ones. These value functions serve as the terminal conditions from which we compute the relevant values, decisions and outcomes during the transition period.

**Proposition I** Let the income of each consumer grow at a constant factor $G_Y$, then the economy has a balanced growth path. In the balanced growth path equilibrium

1. Development of new residential area no longer takes place.

2. In the existing residential area, housing supply grows at a constant rate, $G_H = G_Y^\alpha$.

3. Aggregate housing demand grow at a constant rate, $G_H = G_Y^\alpha$.

4. Rental rate grows at a constant rate, $G_Q = G_Y^{1-\alpha}$.

5. Capital investment in housing grows at a constant rate, $G_K = G_Y.$

The proof of Proposition I is given in the Appendix 1. The intuition regarding these properties has two parts. First, on the balanced growth path, both rent and acquisition increase proportionally. Therefore, if it is not optimal to development a new location today, then it is not optimal tomorrow either because the increase in rent is exactly offset by increase in land acquisition cost. Second, in the existing residential area, housing supply grows as redevelopment takes place to accommodate the increased demand. A redeveloped structure will be taller (i.e., have a higher floor area ratio or FAR) as housing prices rise. Proposition I shows how the rate of redevelopment, or height, is related to the growth rate of income.
4.6 The Reference Model: Ownership

To study the effects of leasing versus ownership on the equilibrium price path and quantity of housing and pattern of city growth, we compare the results from the model above to a reference model which is identical except that land is owned by the developer who pays $\lambda^{own} = L^{own} + \lambda^{own}y$ to acquire new land. In the reference model, the developer’s programming problem is relatively simple because the age of the land lease is no longer a state variable. The developer makes decisions based on the state vector $(h,x,a)$ and the projected price path $\{Q_t\}$.

5 A Simple Model

Our dynamic model involves decision making in multiple periods and locations, and it does not have an analytical solution. Before we solve the model computationally, we discuss a simplified version to gain a better understanding of the mechanics of the fully-fledged model.

We assume that a lease lasts only two periods. When the lease terminates, both land and the structure are taken over by the government with probability 1; thus, the developer’s terminal value is zero. We ignore the fixed costs of development, conversion or demolition since the effects of fixed costs on the timing of activities are relatively straightforward. Finally, housing construction function $h' = Ak^\alpha$ is simplified to $h' = k^{1/2}$.

First, we study the redevelopment of an existing structure in location $x$ in period 1. Since we assume a new land lease has only two periods, the lease will terminate at the end of period. Thus no inter-temporal decision is involved. Let the price in period 1 be $Q_1$. Adjusted for distance, the price should be $Q_1/f(x)$. For a structure of size $h$, the profit of the developer if she decides to redevelop is

$$v^R = \max_k -k + \frac{Q_1}{f(x)}h'$$  \hspace{1cm} (12)

subject to $h' = k^{1/2}$. The profit maximizing level of redevelopment is $k = (Q_1/(2f(x)))^2$, or equivalently $h' = Q_1/(2f(x))$. Therefore, a developer would redevelop if the optimized profit $v^R = (Q_1/f(x))h' - (h')^2 = Q_1^2/(4f(x)^2)$ were greater than the profit associated with not redeveloping which is $v^{NR} = h\psi Q_1/f(x)$. That is, a location would be redeveloped if

$$Q_1 > 4\psi h f(x)$$  \hspace{1cm} (13)

Therefore, redevelopment is more likely when the price ($Q_1$) is higher, and is less likely when the existing structure ($h$) is taller. These phenomena are found in the simulation of calibrated model below. In particular,

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11 If the two periods of this presentation correspond to 70 years and if we assume that consumer income doubles in one period, then that is equivalent to an annual income growth rate of less than 2 percent.

12 Ignoring these costs is not an insignificant feature since the restrictions implicit in the simplification may also affect the existence of an equilibrium. Specifically, the assumption that the developer takes the price as given could be questioned in a model with so few locations and so few time periods. Thus, the intent of this discussion is to clarify the intuition which supports the more complex model and its numerical simulation.
when income growth rate is high, price rises quickly and more redevelopment is observed. In addition, smaller
structures are more likely to be redeveloped earlier.

Next, we study the development of new locations. Let $Q_0$ and $Q_1$ be the price in period 0 and 1
respectively. In period 0 the developer considers whether to develop location $x$ from which the flow of
services needs to be discounted by $f(x)$. If the location were developed, then the maximization problem
would be

$$v_0^D = \max_h -h^2 + \frac{Q_0}{f(x)} h + \frac{1}{1 + r} v_1$$

where we have used $h^2$ to substitute for the construction cost $k$. Recall that we assumed $h = k^{1/2}$.

The continuation value in equation (14), $v_1$, depends on whether the structures are to be redeveloped in
period 1. We consider two cases.

**Case (1), the structures are redeveloped in period 1** In this case, $v_1 = v^R = Q_1^2/(4f(x)^2)$ is
independent of $h$, then from first order condition the optimal level of $h$ is

$$h^{(D,R)} = \frac{Q_0}{2f(x)}$$

Here we use superscript (D,R) to remind readers that $h^{(D,R)}$ is the optimal height of structure provided that
the location is developed in period 0 and then redeveloped in period 1.

Substituting $v_1 = Q_1^2/(4f(x)^2)$ and equation (15) into equation (14), we obtain the following value for
the developer.

$$v_0^{(D,R)} = \frac{1}{4f(x)^2} \left( Q_0^2 + \frac{Q_1^2}{1 + r} \right)$$

Again, we use superscript (D,R) to indicate the value of developing in period 0 and then redeveloping in
period 1.

**Case (2), the houses are not redeveloped in period 1** In this case the optimization problem becomes

$$\max_h -h^2 + \frac{Q_0}{f(x)} h + \frac{1}{1 + r} f(x) \psi h$$

From first order condition, the optimal level of housing size is

$$h^{(D,NR)} = \frac{Q_0}{2f(x)} + \frac{Q_1 \psi}{2f(x)(1 + r)}$$

The developer’s value is

$$v_0^{(D,NR)} = \frac{1}{4f(x)^2} \left( Q_0 + \frac{Q_1 \psi}{1 + r} \right)^2$$

Here the superscript (D,NR) denotes the strategy of developing in period 0 and not redeveloping in period
1.
Therefore the value of developing location 1 in period 0 is the maximum of values from the two cases above. That is

$$v_D^0 = \max\{v_{(D,R)}^0, v_{(D,NR)}^0\} = \max\left\{\frac{1}{4f(x)^2} \left(Q_0^2 + \frac{Q_1^2}{1+r}\right), \frac{1}{4f(x)^2} \left(Q_0 + \frac{Q_1\psi}{1+r}\right)^2\right\}$$  \hspace{1cm} (17)

From equations (15)-[17], we gain the following insights. First, development of a new location is more likely if the current and future prices are higher, and less likely if the transportation cost (embedded in \(f(x)\)) is higher. This is easily seen from equation (17). New development entails \(v_D^0 > 0\), while both \(v_{(D,R)}^0\) and \(v_{(D,NR)}^0\) increase with \(Q_0\) and \(Q_1\), and decrease with transportation cost \(f(x)\). Consistent with this property, in the simulation results below, we show that a city expands gradually, as the positive effect of rising price outweighs the increasingly higher transportation cost.

Second, the structure is smaller if the plan is to redevelop it in period 1. Comparing equation (15) with equation (16) shows that \(h_{(D,R)} < h_{(D,NR)}\). Intuitively, if redevelopment is planned for the next period, the developer had better build a relatively small structure now to economize on costs of capital. This mechanism is strengthened if demolition of existing structure entails costs which are proportional to structure size, as in the full model.

Third, whether or not to redevelop in period 1 depends on the growth rate of rental rate. A bit of algebraic manipulation shows that \(v_{(D,R)}^0 > v_{(D,NR)}^0\) is equivalent to

$$\frac{Q_1}{Q_0} > \frac{2(1+r)\psi}{1+r - \psi^2}$$  \hspace{1cm} (18)

Notice that condition (18) is independent of transportation cost \(f(x)\). The condition essentially says that if price grows quickly enough, the developer should construct a smaller structure in period 0, then redevelop it into a bigger one in period 1.

Fourth, whether or not to redevelop in period 1 also depends on \(\psi\), the parameter that determines the housing service discount due to age. This is clear from comparing \(v_{(D,R)}^0\) with \(v_{(D,NR)}^0\). Intuitively, an increase in the depreciation rate (i.e., a decrease in \(\psi\)) would decrease \(v_{(D,NR)}^0\) while \(v_{(D,R)}^0\) is independent of \(\psi\). The quicker reductions in current and future rental revenue from older structures encourages a developer to redevelop a location.

6 Computational Results

A simple model helps to provide insight into the dynamic model. However, the two periods of the simple model do not adequately capture the full implications of the dynamic features which, as we show, are significant. In this section, we calibrate the dynamic model and compute the equilibrium paths of prices and housing supply and city density. These results are compared with results from the reference model to highlight the differences in the market for real estate services and the shape of the city arising from alternative systems of land use rights: long term land leases or fee simple ownership.

Following the standard simulation-based approach, the following tatonnement strategy is used.
1. Guess a sequence of rental rate $Q_t$, for $t = 1, ..., T$

2. Compute the developer’s value at time $T$, $v_T(h, x, a, \tau)$. Since we assume the economy is in the balanced growth path from $T$ on, rental price, housing supply and demand growth at constant rates. We can re-scale the problem and solve for $v_T(h, x, a, \tau)$ as if the economy is in a steady state.

3. For each location, compute the optimal decision rules of consumers and developers from $t=T$ to $t=1$ using backward induction.

4. Based on the optimal decision rules, simulate housing supply by the developer and housing demand by the consumers given $Q_t$ for $t = 1, ..., T$. The simulation requires initial conditions which we specify below.

5. Check for excess demand or excess supply; if $\int_x \int_a \int_\tau h^{\tau,a,x}_{t} \psi_{\tau} f(x) \, dx \, da \, d\tau > \int_i s_i \, di$, then decrease the prices around time $t$. Otherwise increase the prices.

6. Repeat steps (2) and (5) until the market clears.

### 6.1 Initial Conditions

Both the initial income of consumers and initial housing supply are assumed to be 1. The initial house price equals $\theta$. Recall that $\theta$ is the housing share in utility function. This initial price, combined with the initial income of 1, results in initial housing demand that equals 1, hence housing market clears.

Initially all the houses are located in $x = 0$. Lease ages are drawn from uniform distribution between 1 and 20, consistent with the fact that land lease policy has a history of about 20 years. The first lease was negotiated in Shenzhen in 1987 with the market taking hold in the early 1990s. Since little redevelopment has occurred thus far, we further assume the age of existing structure in $t=1$ is the same as the age of lease. Initial height of houses are drawn from uniform distribution between $\mu - 0.5$ and $\mu + 0.5$ with $\mu$ so chosen that initial housing supply is 1.

### 6.2 Calibration

The exogenous force that drives the changes of price and quantity is income growth. We assume the income growth rate is currently 8 percent and declines linearly until it stabilizes at 2 percent. We assume this deceleration takes 30 years and is followed by another 20 years during which the economy converges to the balanced growth path. This assumption is consistent with recent history and would imply that, over 50 years, income would grow by a factor of more than 6 times, which is more than the current difference between per capita GDP of the United Stated and China. Our discussion focuses on this 50 year period. Figure 1

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13 In the computational exercise, we find that the distribution of houses in the state space becomes invariant 20 years after income growth stabilizes.
Figure 1: Income – Growth Rate and Level

Illustrates the assumed growth rate and level of average income. Recall that initial income is normalized to 1.

Table 1 summarizes the other model parameters used in computing the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>discount rate</td>
<td>$r$</td>
<td>0.03</td>
</tr>
<tr>
<td>curvature parameter in housing production function</td>
<td>$\alpha$</td>
<td>0.48</td>
</tr>
<tr>
<td>housing service discount due to age</td>
<td>$\psi$</td>
<td>0.99</td>
</tr>
<tr>
<td>scaling parameter in housing production function</td>
<td>$\Lambda$</td>
<td>0.63</td>
</tr>
<tr>
<td>demolition cost</td>
<td>$m$</td>
<td>1.2</td>
</tr>
<tr>
<td>fixed land acquisition cost under leasehold</td>
<td>$L$</td>
<td>3.79</td>
</tr>
<tr>
<td>fixed acquisition cost under ownership</td>
<td>$L_{own}$</td>
<td>5.23</td>
</tr>
<tr>
<td>proportional land acquisition cost</td>
<td>$\lambda$</td>
<td>0.52</td>
</tr>
<tr>
<td>consumer’s share of income spent on housing</td>
<td>$\theta$</td>
<td>0.24</td>
</tr>
<tr>
<td>probability of full compensation for structure</td>
<td>$\pi$</td>
<td>0.5</td>
</tr>
<tr>
<td>transportation cost in location $x = 0$</td>
<td>$f(0)$</td>
<td>1</td>
</tr>
<tr>
<td>transportation cost in location $x = N$</td>
<td>$f(N)$</td>
<td>2</td>
</tr>
</tbody>
</table>

The developer’s discount rate is assumed to be 0.03. This is the financing cost of the developer. It is set to a relatively low level since the developer faces no uncertainty in our model.

We set $\alpha = 0.48$ so that for a 30-story building, unit construction cost rises by 4% as the number of floors
rise to 31.\textsuperscript{14} Tan (1999) reports that unit construction costs in the US rise by 2\% per floor, which amounts to \( \alpha = 0.612 \). We assume a higher per floor marginal construction cost because the cost of the land use rights (the fee paid by the developer to the government when negotiating the leasehold) also increases with floor area. Specifically, when a piece of land is to be leased, the local Land Resource Bureau specifies the maximum floor area ratio (FAR), the maximum lot coverage ratio, and minimum greening rate (percentage of greenery coverage).\textsuperscript{15} While the land use right is auctioned, the equilibrium lease generally increases with FAR. In the robustness check, we also use \( \alpha = 0.4 \) and \( \alpha = 0.612 \), implying that unit construction cost increase by 5\% and 2\% respectively as the number of floors increases from 30 to 31.

\( \psi = 0.99 \) implies that, after being in use for 50 years, service flow from a unit is about 60\% of that from a new unit.\textsuperscript{16} Another implication of \( \psi = 0.99 \) is that as a house becomes one year older, the service flow (and hence rental rate) declines by 1.01\%.\textsuperscript{17} This is roughly consistent with Rosiers and Theriault (1996).

We choose \( \{A, m, L\} \) jointly to meet three conditions: (i) in period \( t = 1 \), the new structure in location \( x = 1 \) has height 1, consistent with the normalization in initial conditions; (ii) redevelopment of the old town will not take place until 5 years later; (iii) value of new development location \( x = 1 \) in period \( t = 1 \) is zero, i.e., \( v^D_1(x = 1) = 0 \). The reason to target condition (ii) is that little redevelopment is observed currently, but after 5 years some of the leases (and houses) will reach the age of 25, and redevelopment becomes much more likely. Condition (iii) is essentially a zero profit condition. Since new development takes place when the value is zero, the condition guarantees that new development in location \( x = 1 \) occurs in period \( t = 1 \).

For the case of land ownership, the fixed land acquisition cost, \( L_{\text{own}} \), is also chosen so that the value of development in location \( x = 1 \) is also zero in period \( t = 1 \). Notice that if the land acquisition cost was the same, then the value of new development would be higher under ownership than under leasehold due to longer horizon. In our calibration \( L_{\text{own}} > L \) so that the zero profit condition is satisfied under both land use regimes.

The proportional land acquisition cost is \( \lambda = 0.52 \) which equals \( 1 - \alpha \). Recall \( 1 - \alpha \) is the land share in the housing production function. Intuitively, higher housing share leads to more surplus being grabbed by the developer, thus a higher \( \lambda \).

In the baseline, we assume there is a 50\% chance that when the land lease terminates, the government fully compensates for the value of structure which equals the value of the structure under land ownership. In robustness check, we allow \( \pi = 0 \) and \( \pi = 0.75 \) and find similar results as in the baseline.

Following most of the existing literature, we choose \( \theta = 0.25 \). This is roughly the share of total consumer

\textsuperscript{14}Let \( x \) be the per-floor construction cost of a 30-story building, as the number of floors goes from 30 to 31, construction costs increase from 30\( x \) to \((1+4\%)31x \), hence \( \Delta k/k = (1.04\times 31x - 30x)/(30x) = 0.07 \). While the percentage change of housing size is \( \Delta h/h = 1/30 \). Given the house production function \( h = Ak^\alpha \), one can derive \( \alpha = \Delta h/h / \Delta k/k \). Therefore \( \alpha = (1/30)/(0.07) = 0.476 \).

\textsuperscript{15}According to an official of the Land Resource Bureau in Hangzhou, generally lot coverage ratio should be less than 0.4, and greening rate should be above 20\%.

\textsuperscript{16}When interviewed by the media, a number of senior officials from the Ministry of Housing and Urban-Rural Development confirmed that most residential houses in China are designed to be used “normally” for 50 years.

\textsuperscript{17}Recall service flow from a house of height \( h \) and age \( a \) in old town is \( s = h\psi^a \). Taking the derivative of \( s \) with respect to \( a \) yields \( ds/da = h\psi^a \ln(\psi) = s \ln(\psi) \), thus \( \frac{ds}{da} = \ln(\psi) = \ln(0.99) = -0.01 \).
expenditure in China allocated to rent payments (payment for housing services)\(^{18}\).

The transportation cost in old town (location \(x = 0\)) is normalized so that \(f(0)=1\). Recall that we allow a city to double its size. That is, the size of land available for the new town is 1, which should be an upper bound given the strict regulations on conversion of arable land. We divide the land for new town into \(N\) pieces of equal size. Let \(f(x = N)\) be the transportation cost associated with land farthest from old town, we assume \(f(x = N) = 2\). Next, we assume that the transportation cost associated with any houses between \(x = 0\) and \(x = N\) is a linear combination of \(f(x = 0)\) and \(f(x = N)\). We use \(N=60\) so that the discrete representation of \(f(x)\) is fine enough such that new development occurs gradually and the supply function of houses is smooth. In the simulated equilibrium we find that the land used for new town is about 0.8-0.9 when the economy reaches balanced growth path.

### 6.3 Results

We present results from both land leasehold and ownership cases during the economic transition. For readers who are interested comparisons between the two cases in BGP, Appendix 2 provides a discussion.

The upper panel of Figure 2 shows the market-clearing price paths. With leasehold, the price of housing services is lower from \(t= 0\) until 25 to 30 years later then it exceeds the price under a fee simple setting for all later times. The lower panel of Figure 2 shows the quantity adjusted for location and age, i.e., \(\int_{x} \int_{a} \int_{\tau} h_{x,a,\tau}^{\psi} f_{x} dx da d\tau\). In a leasehold setting, when the price is lower in the near future, the quantity supplied is higher. Later, when the price of housing services becomes relatively higher, supply under a leasehold setting is lower compared to the fee simple setting.

As recognized in the Simple Model, this seeming paradox reflects a resolution to an inter-temporal allocation problem: the quantity supplied initially is greater in a leasehold setting because the developer’s initial decision anticipates later decisions concerning the timing and intensity of redevelopment, which in turn affects the age and height of a structure, as shown in Figure 3.

The upper panel plots the average age of houses in location \(x = 0\). Everything starts relatively new since, motivated by recent history, the initial conditions supposes the maximum age of a building to be 20. As time passes, all buildings get older. During the first 5 years, no difference is observed between leasehold and ownership, since the parameter values were chosen so that redevelopment does not occur. Around year 30, the top panel of Figure 3 shows that houses are relatively younger under leasehold because redevelopment occurs earlier. Later, redevelopment rate is clearly lower under leasehold than under ownership, as shown in the bottom panel. Therefore houses tend to be older under leasehold in the later periods.

The middle panel of Figure 3 shows that under leasehold, houses are taller on average from now until about 25 years later. This again is because of earlier redevelopment under leasehold. Redeveloped houses

18 According to the 2010 wave of China Family Panel Studies conducted by the Institute of Social Science Survey of Peking University, on average rental payment takes up 25.37% of the total expenditure of urban renters in China. Notice this is rental payment of renters only, not including the rental equivalence for home owners.
Figure 2: Equilibrium Price and Quantity

The figure plots the price and quantity paths in equilibrium. The initial price is normalized to 0.24 (value of $\theta$, the housing share in consumer’s utility function). The initial quantity is normalized to 1.0.

are taller due to the rising rental price. Later on, lower redevelopment implies less newly-constructed taller buildings, thus the average height is much lower under leasehold than under ownership.

In summary, Figure 3 illustrates how the institutional arrangement affects timing and frequency of redevelopment, which in turn affect average house age and density.

Figure 4 shows new building activity. The top panel shows that the height of new buildings grows over time. New buildings are uniformly less tall under leasehold, because they are redeveloped less often. The bottom panel shows the location of new development, and implicitly, the outer boundary of the city. The vertical axis shows the size of the a defined by the size of land used for residential construction in period $t$. At any period during the transition, the city is more spread out in a leasehold setting and the consumer
The figure shows height, age of buildings in location x=0, and frequency of redevelopment during the transition. Initial average age is set to 15. Initial height is 1.057, so that initially housing supply is 1.0 after adjustment for house age.

experiences a higher cost of living. Of course, the larger footprint of the city implies greater conversion of agricultural land.

Quantitatively, 5.7 percent more land is used to house the same number of residents. The National Bureau of Statistics of China reports that the residential built up area is 12404 sq. km. currently. If we use our benchmark model, these facts would imply an excess land use of slightly more than 700 sq. km. That area is slightly less than the area currently used for residential purposes in the province of Liaoning in north-eastern China or half that used in the province of Guangdong in southern China. \(^\text{19}\) If land leases also have the same effects on industrial and commercial types of land uses, then the analysis implies a current excess land use of about 2300 sq. km., which is more than the built-up areas of Zhejiang province near

\(^\text{19}\)This figure should be interpreted as a modest estimate. To four digits, the difference in land use due if land is owned vs. if land is leased is 1.7585/1.8644- 1 = -0.0568. The data on land use (in 2010) can be found at http://www.stats.gov.cn:82/tjsj/qtsj/hjtsj2010/t20111229_402788833.htm
The figure shows the average height in new town, and city size defined by location of newly developed houses. City size is the sum of residential land in both old town and new town. Residential land in old town equals 1.0.

Shanghai. Given the massive growth in the built-up areas during the last two decades, the size of this excess is certain to grow in the future.

As an additional point of comparison, as shown in Figure 2, when the economy approaches BGP, if land is owned rather than leased, the price would be 7.3 percent lower, supply would be 7.5 percent higher and, as shown in Figure 3, buildings would be 8.3 percent younger. Given $\psi = 0.99$ and an average age of about 50 years at $T = 50$, this difference in age represents a difference in value to consumers of about 4 percent.

The above figures already contain much information about density of the city. Figure 4 further illustrates the situation of both old town and new town, where the white bars stand for average height under ownership and the black bars for leasehold. Location on the horizontal axis is defined by the distance function $f(x)$. Recall that $f(0) = 1$. Clearly, the city expands over time. As the city grows and time passes, the new development that occurs is greater than the new development of the previous period, for both leasehold and ownership. Thus, density is not always greatest at the center of the city.

Comparing the white (ownership) and black bars (leasehold) reveals that, during the later years, the density in the inner core is lower under leasehold. To house the city’s fixed number of people, the density is higher on the periphery and the periphery is more distant, reflecting greater conversion of agricultural land.
Figure 5: Density

The figure illustrates the density of city under different land use policies. The black bars are for heights under a leasehold setting, and white bars are for heights under a fee simple setting. Location is defined by the distance function $f(x)$, with the normalization $f(0)=1.0$.

6.4 Sensitivity and Robustness

One major concern regarding simulation results is the robustness. Any set of parameter values may be considered as a special case. Although we have picked the parameter values carefully so that the model can reflect the reality in China to the extent possible, it is still worthwhile to see if our major results are robust to different parameterizations.

We test the robustness of our results to changes in (i) service discount due to house age ($\psi$); (ii) curvature parameter in housing production function ($\alpha$); and (iii) probability of full compensation for structure ($\pi$). To ensure the model is well-behaved and that the comparison uses the same initial conditions, we re-calibrate four other parameters: the scaling parameter in production function ($A$), the proportional demolition cost ($m$) and the fixed demolition cost ($L_{own}$ and $L_{self}$). As in the baseline case, we choose these four parameters so that the same set of conditions is satisfied. In particular, it is necessary to have zero profit (of the developer)
condition to be satisfied. The new parameter values are reported in Table 2.

<table>
<thead>
<tr>
<th>( \psi =0.98 )</th>
<th>( \psi =0.995 )</th>
<th>( \alpha =0.4 )</th>
<th>( \alpha =0.612 )</th>
<th>( \pi =0 )</th>
<th>( \pi =0.75 )</th>
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<tr>
<td>0.97</td>
<td>0.63</td>
<td>1.02</td>
<td>0.79</td>
<td>1.03</td>
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</tr>
<tr>
<td>0.60</td>
<td>1.20</td>
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<td>4.30</td>
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<td>3.11</td>
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<td>5.98</td>
<td>5.23</td>
<td>6.61</td>
<td>3.38</td>
<td>6.82</td>
<td>5.15</td>
</tr>
</tbody>
</table>

Table 3 reports main results from various parameterizations. We show the averages of price, quantity, age and location for the first 5 years from now (denoted earlier) and the last 5 years before BGP (denoted later). As is evident from the graphic presentation above, differences in these outcomes between two land use regimes can revert over time.

We find that basic results from the baseline model are very robust. Under leasehold, price is initially lower, but becomes higher than under land ownership later. Similarly, housing supply is higher earlier and lower later. As the economy approaches BGP, under leasehold, houses on average are significantly older, and the city is significantly more spread-out.

Regarding density, houses in old town are unanimously less tall under leasehold for various parameterizations. However, houses in new areas are not necessarily less tall. The height of newly constructed houses depends on the marginal benefit of additional height of structure on a given lot. The marginal benefit in turn depends on the expected years of service before either the structure is redeveloped or the lease terminates. Under ownership the “lease” never terminates. From this perspective, new houses should be taller under ownership. However, under leasehold a developer may plan not to have any redevelopment during the lease term. In this case, the expected years of service is longer, and new houses should be taller under leasehold. This point is already made clear in the simple model introduced in Section 5.

The last column of Table 3 reveals a major reason for these difference – the overall redevelopment rate is much lower under leasehold. Had the land use regime been fee simple, the redevelopment rate would have been higher by between 18.7 and 34.3 percent.

7 Conclusion

We study the effects of a government policy which allocates land use by leasing land, rather than permitting ownership of a fee simple interest. The former gives the state a right of reversion when the lease ends while the latter transfers ownership subject only to possible expropriation or other police power held by the state. Where other researchers have considered the problem in a static or single period model, we find that using
<table>
<thead>
<tr>
<th>Table 3: Robustness Check</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Price</strong>, <strong>Supply</strong>, <strong>Age</strong>, <strong>Size</strong>, <strong>Height</strong>, <strong>Redevelop Rate</strong></td>
</tr>
<tr>
<td><strong>earlier</strong>, <strong>later</strong>, <strong>earlier</strong>, <strong>later</strong>, <strong>later</strong>, <strong>later</strong>, <strong>old town</strong>, <strong>new town</strong>, <strong>Rate</strong></td>
</tr>
<tr>
<td>baseline leasehold</td>
</tr>
<tr>
<td>ownership</td>
</tr>
<tr>
<td><strong>3.7%</strong></td>
</tr>
<tr>
<td>( \psi = 0.98 ) leasehold</td>
</tr>
<tr>
<td>ownership</td>
</tr>
<tr>
<td><strong>6.8%</strong></td>
</tr>
<tr>
<td>( \psi = 0.995 ) leasehold</td>
</tr>
<tr>
<td>ownership</td>
</tr>
<tr>
<td><strong>0.3%</strong></td>
</tr>
<tr>
<td>( \alpha = 0.4 ) leasehold</td>
</tr>
<tr>
<td>ownership</td>
</tr>
<tr>
<td><strong>4.1%</strong></td>
</tr>
<tr>
<td>( \alpha = 0.612 ) leasehold</td>
</tr>
<tr>
<td>ownership</td>
</tr>
<tr>
<td><strong>11.4%</strong></td>
</tr>
<tr>
<td>( \pi = 0 ) leasehold</td>
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<td>ownership</td>
</tr>
<tr>
<td><strong>8.5%</strong></td>
</tr>
<tr>
<td>( \pi = 0.75 ) leasehold</td>
</tr>
<tr>
<td>ownership</td>
</tr>
<tr>
<td><strong>5.4%</strong></td>
</tr>
</tbody>
</table>

The table shows the different outcomes under leasehold and ownership, "earlier" represents averages (of price, quantity etc.) for the first 5 years from now, while “later” represents averages in the last 5 years before BGP. "Age" refers to the average age of structures in both old and new town. Size refers to size of land used for residential construction, with land in old town normalized to 1. Height and redevelopment rate are both averages from now until BGP. The bold numbers show the percentage difference between the leasehold and fee simple setting.
a long time horizon and a dynamic model adds an important perspective. Initially, the institution of the
ground lease seems to produce more housing. At a later time when the rate of redevelopment becomes a more
serious concern and when the formerly new buildings have depreciated, the ranking of outcomes is reversed.
We also show that simple aggregate measures, such as price or quantity, obscure dimensions of housing that
are important to consumers, such as location and age. Especially for a country like China which has many
people and many new buildings, the implications of using ground leases on population density and on the
efficient use of land can be significant.

Our numerical analysis indicates that, for the given population and after allowing for the system to evolve
for 50 years, replacing the current system by one which allows land to be owned could shrink the residential
areas of a city by about 5.7 percent (or about 700 sq. km if using the current size of Chinese cities). If the
same effects are evident with non-residential uses of urban land, which are about two to three times larger,
then the impact on rural land use is noteworthy. We also demonstrated that, in equilibrium, consumers
would pay about 7 percent more each year for housing that is worth about 4 percent less.

Our goal was to develop a model simple enough to be manipulated while maintaining enough realism
to offer relevant insight. Appendix 3 discusses some of the possible extensions to our model but the most
important are those which explore the implications of land use rights, either leasehold or fee simple. Our
model suggests that we should distinguish static and dynamic effects. At any point in time, the ability to
extract value from a lease in the future would affect current decisions and those decisions would affect the
market clearing price. We also note that it would affect the timing of future redevelopment. Instead of
developing earlier to extract more value from a lease where there is no reversionary right, lowering the cost
of renewing a lease (or allowing renewal and defining the mechanism through which renewal would occur)
would increase the surplus and encourage developers to allocate their capital more efficiently by redeveloping
a location at the right time with the right product. (See Wong, Chau, and Yiu (2008) for more discussion
of how leases are used in Hong Kong.)

With or without the possibility of renewal, an extension of our model might discuss the optimal term of a
lease from the landowner’s perspective. The current policy imposed different terms for different types of land
use, with residential land use having the longest term of 70 years and the possibility of renewal on uncertain
terms. Our model shows that the costs of a ground lease for residential property varies with environmental
parameters such as the rate of income growth, construction costs and demolition costs. Industrial properties
are very different. Technology changes so tenants come and go and change in size and focus. Natural
turnover in a building’s tenants makes some redevelopment more likely on an on-going basis. Industrial uses
at the periphery often transition to retail, office or residential uses as a city grows. Commercial or industrial
buildings also tend to have a more generic and more flexible design, unlike residential buildings where the
walls between units tend to be fixed. Thus, large scale redevelopments of non-residential buildings would
tend to be driven by the aggregate demand for space instead of the changing demands of individuals.

An interesting aspect of this extension might be that all types of land use would be competing for the
same land. Therefore, if the land leasehold system reduces residential population density then less land is available for commercial uses or such uses may be more dispersed. Even as a numerical exercise, there are complex issues to resolve when allocating land among competing uses. While it would seem valuable to use our numerical model to solve for the allocation which maximizes some measure of social surplus, history and the work of people like Wang (2011) suggest that the practical relevance of this solution is limited. It is useful to remember the simple advice that allowing prices to clear rental markets without a market failure or other distortion often produces an efficient allocation of land between housing and other uses at any point in time and over time.

Appendices

A. Proof of Proposition I

This appendix first proves the existence of and convergence to a balanced growth path (BGP), then it proves certain properties of the BGP. Recall that in the model, we assume no new land lease is contracted upon the stabilization of income growth, and the government makes development/redevelopment decisions as a developer with land ownership would do. So, to understand the very long term characteristics and implications of a balanced growth path, we need only to prove the proposition in the fee simple ownership setting. In what follows, we use the same set of notations as in the main text, but keep in mind that age of land lease (τ) is no longer relevant.

A.1. Existence and Convergence

First of all, recall that land acquisition cost is Λ = L + λy under leasehold. As income increases, the fixed component, L, becomes less important. In the argument below, we drop this term, and prove the existence of BGP given Λ = λy. In the presence of L, we have only an approximate BGP which converges to actual BGP as y goes to infinity.

Recall that the developer’s problem is laid out in equations (3) - (7). To simplify the notation, we omit the subscript t from the notation. In addition, lease age τ is dropped because it is no longer relevant.

For a given structure in location x, the developer’s decision depends on size of the structure (h), age of the structure (a), the sequences of income \( \{y_j\}_{j=t}^{\infty} \) and the sequence of rental rates \( \{Q_j\}_{j=t}^{\infty} \). For simplicity of notations, we use y and Q to denote the sequences. These state variables are summarized in \( (h, x, a, y, Q) \).

For undeveloped land in location x, \( h = 0 \) and \( a = 0 \)

We summarize the developer’s decision by \( \{I_{\text{action}}, k^*\} \). \( I_{\text{action}} \) is the indicator function, with \( I_{\text{action}} = 0 \) if developer takes no action and \( I_{\text{action}} = 1 \) if the developer takes an action, either to redevelop the existing residential area or to convert new land. If the decision is to take an action, then \( k^* \) represents the capital
input that produces a new structure of size $h' = Ak^\alpha$. In case of no action, $k^* = 0$.

**Claim (1)** If $\{I_{\text{action}}, k^*\}$ solves the developer’s problem in state $(h, x, a, y, Q)$, then $\{I_{\text{action}}, k^*G_K\}$ solves the developer’s problem in state $(hG_H, x, a, yG_Y, QG_Q)$, with $G_H = G_Y^\gamma$ and $G_Q = G_Y^{1-\alpha}$. In addition, $v(hG_H, x, a, yG_Y, QG_Q) = G_Y v(h, x, a, y, Q)$ and $v^D(x, yG_Y, QG_Q) = G_Y v^D(x, y, Q)$.

Our proof uses a backward induction argument. Based on standard contraction mapping theory, we can start from a terminal condition with zero continuation value, then we iterate backwards until discounting makes any assumption on the terminal condition irrelevant. When the continuation value is zero, if no redevelopment is planned, then the developer’s value is

$$v_R(h, x, a, y, Q) = \max_k -mhy^{1-\alpha} - k + QA k^\alpha$$

where $k^*$ is the optimal capital input.

In state $(hG_H, x, a, yG_Y, QG_Q)$, the value of re-development becomes

$$v^R(hG_H, x, a, yG_Y, QG_Q) = \max_k -mG_Hh(G_Y^\gamma y)^{1-\alpha} - k + (QG_Q)Ak^\alpha$$

$$= \max_k -mG_Y^\gamma h(G_Y^\gamma y)^{1-\alpha} - k + G_Y^{1-\alpha}QA k^\alpha$$

$$= G_Y \left( \max_k -mhy^{1-\alpha} - k + QA \left( \frac{k}{G_Y^\gamma} \right)^\alpha \right).$$

(20)

$k = k^*G_Y$ solves problem (20), and

$$v^R(hG_H, x, a, yG_Y, QG_Q) = G_Y v^R(h, x, a, y, Q)$$

It is straightforward to show that

$$v^{NR}(hG_H, x, a, yG_Y, QG_Q) = G_Y v^{NR}(h, x, a, y, Q)$$

Since the overall value is $v(h, x, a, y, Q) = \max\{v^R(h, x, a, y, Q), v^{NR}(h, x, a, y, Q)\}$, we have

$$v(hG_H, x, a, yG_Y, QG_Q) = G_Y v(h, x, a, y, Q).$$

(21)

For any un-developed location, the state vector is simplified to $(x, y, Q)$. The value of converting land is

$$v^D(x, y, Q) = \max_k -\lambda y - k + QA k^\alpha + \frac{1}{1+r} v(QA k^\alpha, x, 1)$$

$$= -\lambda y - k + QA k^\alpha + \frac{1}{1+r} v(QA k^\alpha, x, 1)$$

where $k^*$ is the optimal level of capital input.

In state $(x, yG_Y, QG_Q)$, the value of converting land is

$$v^D(x, yG_Y, QG_Q) = \max_k -\lambda(G_Y y) - k + (QG_Q)Ak^\alpha + \frac{1}{1+r} v(G_QQA k^\alpha, x, 1)$$

$$= G_Y \left( \max_k -\lambda y - k + QA \left( \frac{k}{G_Y} \right)^\alpha + \frac{1}{1+r} v(QA \left( \frac{k}{G_Y} \right)^\alpha, x, 1) \right)$$

$$= G_Y v^D(x, y, Q)$$

(22)
The first equality in (22) is true because
\[ v(G_Q Q^a_k, x, 1) = v(G_Y^{\frac{1}{1-\alpha}} Q A G_Y^\alpha \left(\frac{k}{G_Y}\right)^\alpha, x, 1) \]
\[ = v(G_Y Q A \left(\frac{k}{G_Y}\right)^\alpha, x, 1) \]
\[ = G_Y v(Q A \left(\frac{k}{G_Y}\right)^\alpha, x, 1) \]

From (22), it’s clear that \( k = G_Y k^* \) solves the problem in state \( (x, G_Y y, G_Q Q) \). In addition, if it is optimal to convert new construction land in state \((x, y, Q)\), i.e., if \( v^D(x, y, Q) > 0 \), then it is also optimal to convert new construction land in state \((x, G_Y y, G_Q Q)\).

Combining the above argument and equations (21) and (22), we conclude that the developer’s actions in state \( \{h, x, a, y, Q\} \) are the same as their actions in state \( \{G_H h, x, y Y, Q G_Q\} \).

Q.E.D.

Claim (2) If income grows at constant factor \( G_Y \) and price grows at constant factor \( G_Q = G_Y^{1-\alpha} \), then no land conversion takes place.

This can be seen from (22). Since \( v^D(x, y, y, Q) = G_Y v^D(x, y, Q) \), if \( v^D(x, y, Q) < 0 \), i.e., a parcel of land is not developed in state \((x, y, Q)\), then it won’t be developed in the next period in state \((x, G_Y y, G_Q Q)\) either, nor will it be developed in the future as income grows at \( G_Y \) and price grows at \( G_Q \).

Claim (3) If income grows at constant factor \( G_Y \) and price grows at constant factor \( G_Q = G_Y^{1-\alpha} \), then aggregate housing supply grows by the factor of \( G_H = G_Y^\alpha \).

In claim (2) we already know that conversion of new land does not happen if income and price grow at constant rates \( G_Y \) and \( G_Q \). Therefore we need only to show the supply from existing residential area grows at the factor of \( G_H \).

Let us revisit the value of redevelopment, \( v^R(h, x, a, y, Q) \).

\[ v^R(h, x, a, y, Q) = \max_k \left[ -m h y^{1-\alpha} - k + Q A k^\alpha + \frac{1}{1+r} v(A k^\alpha, x, 1, y', Q') \right] \]
\[ = -m h y^{1-\alpha} + \max_k \left[ -k + Q A k^\alpha + \frac{1}{1+r} v(A k^\alpha, x, 1, y', Q') \right] \] (23)

Denote the optimal capital input \( k^* \). Notice that \( k^* \) does not depend on \( h \), the size of existing structure.
Now we turn to the same problem given state \((h, x, a, yG_Y, QG_Q)\).

\[
v^R(h, x, a, yG_Y, QG_Q) = \max_k \left[ -mhy^{1-\alpha} G_Y^{1-\alpha} - k + QG_Q Ak^\alpha + \frac{1}{1 + r} v(Ak^\alpha, x, 1, y' G_Y, Q' G_Q) \right]
\]

\[
= -mhy^{1-\alpha} G_Y^{1-\alpha} + \max_k \left[ -k + QG_Y^{1-\alpha} Ak^\alpha + \frac{1}{1 + r} v \left( \frac{Ak^\alpha}{G_H}, x, 1, y' G_Y, Q' G_Q \right) \right]
\]

\[
= -mhy^{1-\alpha} G_Y^{1-\alpha} + G_Y \max_k \left[ -\frac{k}{G_Y} + QA \left( \frac{k}{G_Y} \right)^\alpha + \frac{1}{1 + r} v \left( A \left( \frac{k}{G_Y} \right)^\alpha, x, 1, y', Q' \right) \right]
\]

(24)

where the second-to-last equation holds because of Claim (1).

It clear that if \(k^*\) is the optimal capital input in problem (23), then \(k^*G_Y\) is the optimal capital input in problem (24). Let \(h^* = Ak^*\), then the optimal housing size in problem (24) is

\[
h^* = A(k^*G_Y)^\alpha = Ak'^*G_Y^\alpha
\]

(25)

In other words, the optimal level of housing size grows at the factor of \(G_Y^\alpha\).

Next, we show that aggregate housing supply grows at the factor of \(G_H\). To understand the evolution of aggregate housing supply, we assume without loss of generality that there exist \(n\) units of houses in total at time \(t\), labeled \(h_1, h_2, ..., h_n\) ranked according to the value of redevelopment relative to non-redevelopment \((v^R - v^{NR})\), from the least to the most. That is, in the first row, \(h_n\) is the unit that should be redeveloped earliest, due to the unit being older, smaller or having a relatively longer remaining lease or all of these factors. In the next row, for period \(t + 1\), with unit \(h_n\) already redeveloped optimally, the table shows that a different unit should be redeveloped earliest. For simplicity, in the analysis below we assume in each period, exactly one unit is redeveloped. For the case of multiple redevelopment or zero redevelopment, the same logic would apply without change.

Table 4 illustrates how the aggregate supply converges to BGP in which housing supply grows at a constant rate. The first row shows the size of housing units at period \(t\). In period \(t + 1\), \(h_n\) is redeveloped and the size becomes \(h^*\). Generally the newly developed unit is the least one to redevelop among the new pools due to younger age and larger size\(^{20}\). Thus \(h^*\) is placed in the first column of second row where the order of redevelopment in period \(t + 1\) is shown. Notice that that table shows how the other units depreciates over time, so the real supply of old houses from last period are multiplied by \(\psi\). Now the unit with most redevelopment value is \(\psi h_{n-1}\) in the end of the row. When that unit is redeveloped on the BGP, its new unit quality would be \(h^*G_h\). There would be a different unit most in need of redevelopment and so on through time.

In period \(t + 2\), \(h_{n-1}\) is redeveloped, with the size of new house being \(h^*G_h\). This pattern of redevelopment goes on. The last two rows of the table show the supply in period \(t + n\) and \(t + n + 1\). It is easy to see

\(^{20}\)Recall that the size of new housing increase with income and house price, both are higher in period \(t + 1\) relative to period \(t\).
that aggregate supply grows at the factor of $G_h$ from period $t + n$ on. Therefore, no matter what the initial housing units are, given that price grows at the factor of $G_Q = G_Y^{1-\alpha}$ and income grows at the factor of $G_Y$, the aggregate supply will grow at the factor of $G_H = G_Y^\alpha$.

### Table 4: Illustration of Housing Supply

<table>
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<tr>
<th>$t$</th>
<th>$h_1$</th>
<th>$h_2$</th>
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<th>$h_n$</th>
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</thead>
<tbody>
<tr>
<td>$t+1$</td>
<td>$h^*$</td>
<td>$\psi h_1$</td>
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<td>$\psi h_{n-2}$</td>
<td>$\psi h_{n-1}$</td>
</tr>
<tr>
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<td>$h^*G_h$</td>
<td>$\psi h^*$</td>
<td>$\psi^2 h_1$</td>
<td>...</td>
<td>$\psi^2 h_{n-3}$</td>
<td>$\psi^2 h_{n-2}$</td>
</tr>
<tr>
<td>$t+3$</td>
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<td>$\psi h^*G_h$</td>
<td>$\psi^2 h^*$</td>
<td>...</td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t+n$</td>
<td>$h^*G_h^{n-1}$</td>
<td>$\psi h^*G_h^{n-2}$</td>
<td>$\psi^2 h^*G_h^{n-3}$</td>
<td>...</td>
<td>$\psi^{n-2} h^*G_h$</td>
<td>$\psi^{n-1} h^*$</td>
</tr>
<tr>
<td>$t+n+1$</td>
<td>$h^*G_h^n$</td>
<td>$\psi h^*G_h^{n-1}$</td>
<td>$\psi^2 h^*G_h^{n-2}$</td>
<td>...</td>
<td>$\psi^{n-2} h^*G_h^n$</td>
<td>$\psi^{n-1} h^*G_h$</td>
</tr>
</tbody>
</table>

Q.E.D.

**Claim (4)** If income grows at the factor of $G_Y$ and rental rate grows at the factor of $G_Q = G_Y^{1-\alpha}$, then aggregate housing demand grows at the factor of $G_{Hd} = G_Y^\alpha$.

At any point of time, the consumer’s problem is characterized in equation (10): if income grows by the factor of $G_Y$ and price grows by the factor of $G_Q = G_Y^{1-\alpha}$, then satisfying the budget constraint requires that housing demanded grows by the factor of $G_H = G_Y^\alpha$.

Q.E.D.

**Claim (5)** If income grows by the constant factor of $G_Y$, then the economy converges to a balanced growth path in which the rental rate grows by the factor of $G_Q = G_Y^{1-\alpha}$, and aggregate housing supply and demand grow at the factor of $G_H = G_Y^\alpha$.

Let $Q$ be the market clearing price in period $t$, with housing supply and demand being $H^s$ and $H^d$ respectively. Based on Claim (3)-(4), in period $t + 1$, $QG_Q$ clears the market with supply and demand being $H^sG_H$ and $H^dG_H$. Therefore the balanced growth path exists.

To see convergence to BGP, recall that no matter what the current housing stock is, if the developer or a consumer decides to take an action, the future housing stock will be proportional to income up to the scale of $G_Y^\alpha$ which is on the BGP. In addition, the arguments related to table reveals that no matter what the distribution of houses is in the dimensions are age, size and location, the supply will converge to BGP.

Q.E.D.
A.2. Properties

On the BGP, Property (1) is clear from equation (22). If $v^D(x, y, Q) < 0$, then $v^D(x, G_y y, G_Q Q) < 0$ also holds, and vice versa. Property (2), (3), (4) and (5) are already clear in Claims (1) and (5).

Q.E.D.

B. Characteristics of the Value and Policy Functions on BGP

We summarize some characteristics of BGP by plotting value functions and policy functions, shown in Figure 6. The figure is based on the baseline parameterizations and different functions consider either different locations ($x$) or different house ages ($a$).

The horizontal axis in all panels is the “height” of a house in the beginning of a period. The left panels are value functions, with vertical axis being the discounted sum of future flows of rent. The right side panels are policy functions which show the “height” in the end of the period. Notice that whenever the policy function coincides with the 45 degree line, the height at the beginning is the same as the height at the end because the developer chooses not to redevelop the structure.

A couple of features are evident. First, everything else equal, a structure located near the urban core ($x = 1$) has higher value than one that is farther away ($x = 30$). This is shown in the upper-left panel. Next, as shown in the upper-right panel, a structure near the urban core is more likely to be redeveloped. In addition, if redeveloped, the new structure is taller compared to those farther away from urban core.

The third and fourth features are evident from the lower panels of Figure 6. In the same location ($x = 1$), a newer structures has higher value, unless the structure has very small height $h$. In the lower-left panel, when $h < 0.5$, a structure of age $a = 1$ has the same value as that of age $a = 30$. This is because when $h$ is small, it is optimal to redevelop it. In this case, the existing structure is to be torn down and its age becomes irrelevant. Correspondingly, the policy functions in the lower-right panel indicate that redevelopment is optimal when $h < 0.5$. The fourth feature is that older structures are more likely to be redeveloped, as less of the policy line labeled $a = 30$ coincides with the 45 degree line.

A final feature to notice is the non-monotonicity of value functions. When the optimal decision is to redevelop an existing structure, the value function decreases with $h$, the height of existing structure. This is because demolition cost increases with height. On the other hand, when the optimal decision is not to redevelop, value function increases with $h$, because from larger $h$ comes more rental income.

C. Other Considerations and Some Model Extensions

Although our main results are qualitatively robust to various parameter values, we recognize some weakness of our exercise. A few are technical while others focus on the specification of the model and the relevance of its application to China.
The figure plots the value and policy functions when the economy operates in the balanced growth path.

To some, there may be a concern about model developers who claim to look forward as much as 70 years when many economic models find it difficult to make accurate predictions one or two years into the future. In response, we note that our discussion focuses on the comparative static and comparative dynamic properties of different institutional arrangements. We focus on the difference between a government which allows owning versus one which allows leasing land when investment decisions involve multi-year projections along with an attempt to account for possible outcomes many years down the road. While we may not be able to accurately predict specific events in a specific location, we believe that 50 or 70 years is long enough for market forces to produce an equilibrium outcome for whichever institution is in force.

Below we discuss a number of issues that are not directly considered in our model, but are interesting enough for future explorations.
C.1. Cooperation among Residents

Most Chinese live in apartments, and redeveloping a residential structure usually requires all the residents in a building block to cooperate. In our model, the demolition cost parameter \((m)\) partially accounts for the cost of coordination. We pin down demolition cost by assuming redevelopment occurs 5 year from now, and the implied rate of redevelopment is around 0.01 on average under leasehold. Because land lease has been a common practice only in the past 10 years, little data is available regarding the historic redevelopment rate. With a higher coordination cost (hence higher \((m)\), redevelopment rate might be even lower. The 70-year land lease may facilitate redevelopment by offering a natural time to redevelop, at the termination of land lease when all rights return to the government and cooperation is not an issue.\(^{21}\) It would be interesting to explore the conditions under which this channel has the strongest effect on rate of redevelopment and on growth patterns of Chinese cities.

C.2. Acquisition of Rural Land

Acquisition of rural land is typically done in two steps. In the first step, a local governments acquire land from rural residents, paying due compensation and tearing down the existing structure. Next, the land use right is sold to developers, typically through a auction process.\(^{22}\)

Due to the complexity of acquisition process and the unpredictability of policies, cost of land acquisition is uncertain and changes substantially over time. In the model, we use zero profit condition (the value of new development in location \(x = 1\) in period \(t = 1\) equals zero) to determine land acquisition cost. This is obviously a simplified short cut. Cai, Henderson, and Zhang (2013) discuss whether the auction mechanism used to allocate land to developers in China is being manipulated so that the highest value user is not necessarily the winning bidder. Future work could incorporate more detailed modeling of land acquisition process.

C.3. Population and Consumer Characteristics

We do not consider the effects of population growth or of changes in household size which have been evident in China. The shift from the rural areas to urban areas in China has been massive, especially in the Tier 1 and Tier 2 cities. This increase in the number of people who need a place to live increases the current demand, independent of the land use rights setting. Perhaps in this environment, the land use rights setting and the political and practical challenge of implementing change makes the issue of property rights a problem of second order importance.

\(^{21}\)We thank Andres Almazan and Sheridan Titman for pointing this out.

\(^{22}\)Since 2010, the sale of use right of “gross land” is strictly prohibited. Gross land refers to land with existing structure and/or with existing residents who are not compensated properly. See http://www.mlr.gov.cn/zwgk/zytz/201009/t20100927_772366.htm
The migrants to Tier 1 cities come from Tier 3 cities and rural areas. The challenge in such cities is that, since housing is a durable and immobile good, the only ways to change the stock of housing are to let it depreciate or to redevelop at a cost. The results of our model have the same implications for Tier 3 cities while it does not apply to rural land markets.

Government policy in China has reduced the size of families and the extent of the effects is unclear. Our model assumes that the relevant “consumer” is a family and family size is not in any way endogenous. In practice, having smaller families allows more living space per person and the housing services derived per person can increase if the size of a housing unit has not changed. This difference may reduce the pressure to develop or redevelop a location sooner but this effect has a natural limit. We note that, even in the U.S., changes in household size have had unexpected effects (Goodman (2005)).

The change in family composition may have a secondary effect. The increase in competition amongst young men for marriage partners has caused some families to transfer resources across the generations. As has been historically true in many countries, competition in the marriage market has favored men with more financial resources or more status. This motive should exaggerate any demand for newer and relatively larger homes. This suggests that the land use setting that produces more new buildings would allow those new units to better suit evolving preferences. These new buildings could be on the periphery but, at market clearing prices, a redeveloped location with a pre-existing favorable reputation may be even more attractive to this segment of consumers. We leave the specific effects of this issue as a question for future research.

C.4. Income and Wealth

Although the number of people who are migrating within China is enormous, we suggest that the effects of income growth will become more dramatic. Our model uses a very conservative assumption that the annual growth rate of income will decelerate from 8 percent to 2 percent within 30 years. This is consistent with the prediction of Hans Rosling who predicted that per capital income in China would catch up to that in the U.S. in 2044 the same period. Higher growth rate of income will be reflected by higher housing demand in the model, and hence higher price growth. This will affect the timing and frequency of redevelopment. And, unlike migration, income growth can be expected to continue for the foreseeable future.

Many people are also concerned about the effects of land policy on consumers with different levels of income and wealth. Given our assumption of a homothetic utility function, our model of consumer utility allows for a particularly simple aggregation condition on behavior. If income elasticity of housing demand were not 1.0 then the growth in income could interact with the difference in land use setting to increase or decrease the aggregate quantity demanded of housing. Differences in income are often associated with differences in wealth, access to capital markets or mobility. If, for any of these reasons, higher income consumers find it easier to move then any policy which caused more redevelopment would impose an extra inconvenience on poorer consumers who, disproportionately, would have been living in the older and less

expensive locations for a long time.

Our discussion above noted that we assume consumers are renters in order to avoid the complexity of modeling the inter-temporal effects on decision making of consumers that arise from capital investment decisions tied to the consumption of housing services. Housing is a significant source of savings and investment for many people but, in part, this behavior is affected by imperfections in local capital markets. We suggest that redevelopment opportunities would be more common in a fee simple setting, otherwise, the impact on wealth of the difference between leasing and owning would be slight since any difference should be capitalized.

C.5. Uncertainty

Our model ignores uncertainty. With uncertainty, the issues discussed in our paper would become more important. An increase in uncertainty tends to increase the value of a redevelopment option and the value of developing a new property (with its implied option to redevelop in the future). When the terms of a lease create a finite horizon, it is reasonable to expect a developer to either redevelop sooner than is efficient or to not redevelop a location. In either case, the terms of a lease makes the value of developing a new location relatively more valuable than investing the same resources into redeveloping a previously developed location. In other words, if there is more uncertainty then a leasehold setting is likely to result in a less dense city with an older urban core in equilibrium than would be the case in a fee simple setting.

C.6. Location and Transportation Cost

Our representation of the transportation cost function, \( f(.) \), is very simple and represents a potentially important difference among cities. \( f(0) \) is fixed at 1, as a reference point. If the level of the transportation cost in one city were higher for all \( x > 0 \) then previous research has shown that that city would be smaller as people accept less housing in return for living closer to the center. If \( f(.) \) were steeper, the penalty for living away from the center would be higher and the equilibrium price differential in any period would widen to compensate.

These effects are independent of the land use setting. Since \( f(.) \) is fixed in our model, these increases would increase the density on land which is occupied but the magnitude of the effect would depend on the land use setting. In a leasehold setting, the city is naturally larger and the characteristics of \( f(.) \) for \( x \) on the periphery have effects which would not exist in a fee simple setting.

Different specifications of \( f(.) \) would imply that the effects of excess demand may be expressed mostly in terms of a price change or mostly in terms of a change in the total quantity or types of housing. For example, in an extreme case where \( f(x) \) became infinite for a finite value of \( x \), the maximum size of the city would be predefined. Since an important difference between leasing and owning is the impact on the redevelopment decision and the redevelopment decision depends on the growth rate of prices, the dynamic effects of different institutions differ with \( f(.) \). For example, continuing with the extreme case, the price would rise quickly but that incentive would not be enough as the lease nears its end. In a fee simple setting, redevelopment
would occur. Therefore, in a leasehold setting, cities with these extreme $f(.)$ would reach the maximum size sooner and, once at the maximum, the same growth in income would produce less redevelopment and a faster increase in equilibrium prices.

C.7. Land Price, Developers and Local Government

Our model posits a simple conceptualization of developers who, as a group, build and own the property rented by consumers. An alternate model might allow developers to compete to own the best locations and to build the most suitable housing in those locations. Introducing that dimension into our model would enable us to consider a related, but more complicated, issue jointly. As Dale-Johnson and Brzeski (2002) note, long term land leases are also a revenue source for a government both when the rights are sold and on an ongoing basis. In this context, it might be valuable to explore the effectiveness of land leasehold as a means of generating government revenue. Note that in a fee simple setting the property tax is the usual mechanism for generating local revenue.

Two issues should be considered when recognizing the role of local government more fully. First, developers in our model make decisions to maximize their value function taking prices as given. Thus, our model already accounts for some of the most critical aspects of a perfectly competitive model when deciding what to offer and when to offer it. Like many simple pricing models, we essentially assume any excess profits of developers would be transferred to the government through the equilibrium price of land. In a more complex model, it is not guaranteed that developers would earn zero economic profit or that the full value of the surplus would be transferred efficiently to the local government. An alternative specification might assume that, rather than developers paying for access to land whenever they choose, the government might make a fixed quantity of land available and let the land price adjust for any excess demand. Others have solved this kind of dynamic monopoly problem which represents an intra- and inter-temporal trade off of price versus quantity (Lichtenberg and Ding (2009), Turnbull (2004) or, even, Hotelling (1931)).

It may be more accurate to claim that developers bargain with the government. The relative bargaining power of a government versus developers depends on the excess profits, the government’s spending plans and the surplus created by any action (or inaction). Bargaining power also depends on the relative patience of different actors and that aspect of the problem could be addressed with a more detailed understanding of inter-temporal decision making. Any resolution to this aspect of the model would have clear implications for a question we do not discuss: whether or under what conditions a lease can be renewed after its 70 year term expires.

Second, the issue of whether local governments are solvent is an important issue in China currently (e.g., Zhu (2011)). Many governments use the funds raised from land auctions to pay for current services and to build infrastructure which will be used productively in the future. There are also concerns of “off-balance sheet” accounting based on optimistic projections. The careers of some government officials depend in part on their ability to manage the revenue from land. Lichtenberg and Ding (2009) study this issue in more
depth. Studying local government revenue mechanisms is likely to be an avenue for fruitful research.

We do not consider the activities of the local government for a final reason which is related to our model of dynamics with a long horizon. Much prior research has considered the interaction between developers and a local government when developing a new location to determine the appropriate level of public services and infrastructure for the proposed development. To compute a solution and to by-pass the “hold-up issue” which has been well-studied in a static model, our model assumes that the level of public services is independent of the developer’s decision. By implication, the local governments choice between adding infrastructure at the periphery or upgrading of old town infrastructure is also independent. In China, the sale of land use rights and the ongoing fees are the primary source of funding for local governments. We presume that local governments develop budgets for capital and operating expenditure that, in turn, determine the level of fees charged. This is another fertile area for research. Once built, the location and quality of infrastructure impacts the consumer’s location decision which makes our assumption of a mono-centric city somewhat simplistic. Nonetheless, all else equal, we present strong evidence that a leasehold setting will yield a larger, flatter city with relatively older properties in the urban core.

References


