Together at Last: Trade Costs, Demand Structure, and Welfare

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International trade, like many other branches of applied theory, has made enormous progress in recent decades by building on a central insight of Dixit and Stiglitz (1977): a taste for variety, the essential foundation for a theory of monopolistic competition, can be modeled parsimoniously using a conventional utility function with convex indifference curves defined over the quantities of all potential commodities. To explore the implications of this, they considered two alternative specifications of the industry utility function: the CES case, and what they called “a more general additive form.” The latter approach was explored in Krugman (1979), one of the first applications of monopolistic competition to trade. However, he assumed that trade was unrestricted, and modeled trade liberalization only as an expansion of the global economy. When he and others turned to examine restrictions to trade, it became the norm to consider only the CES case: in the words of Krugman (1980), “it seems worth sacrificing some realism to gain tractability.” The result is paradoxical. We now have a clear understanding of many issues in trade under monopolistic competition, and more recently, thanks to Arkolakis, Costinot and Rodríguez-Clare (2012) and others, a clear basis for quantifying the gains from trade, but only under CES assumptions, with their unsatisfactory implication that firms’ price-cost mark-ups are invariant to shocks.

A number of authors have considered particular alternative specifications to the CES. By contrast, the case of general demands, especially when combined with trade costs, has received relatively little attention. In this paper we show that trade costs and additively separable preferences can be combined in a simple model, one which is tractable without sacrificing too much realism. Section I sketches the model and introduces the key concepts of super-convex demand and superconcave utility. Section II compares the implications of integrated and segmented markets for pricing and mark-ups. Section III shows how the pattern of sales across markets responds to globalization and trade cost shocks. Section IV derives the implications for the gains from trade, while Section V discusses the problems that arise in calibrating the gains.

I. Preliminaries

Except for allowing trade costs, the setting is the same as in Krugman (1979). In each of $\kappa + 1$ identical countries, there is a

2The case of general additive preferences has been reexamined by Neary (2009), Zhelobodko et al. (2012), Dhingra and Morrow (2011), and Mrázová and Neary (2013), but without trade costs. Bertoletti and Epifani (2012) take a similar approach to ours but do not consider welfare. Arkolakis et al. (2012) adopt a general specification of demand, but neither their approach nor ours nests the other: they assume that demand functions exhibit a “choke price” and are less convex than the CES case. Most of these papers are more general than ours in allowing for heterogeneous firms, but, as we hope to show, many interesting issues arise even when we abstract from this. Related results have been independently presented in Russian by Evgeny Zhelobodko and Sergey Kokovin with Maxim Goryunov and Alexey Gorn.

3Technical details are relegated to footnotes or to an online appendix, which also contains detailed references which for reasons of space have had to be omitted from the text.
single monopolistically competitive industry, with a measure $n$ of identical firms, each producing a single symmetrically differentiated variety. International trade incurs symmetric iceberg trade costs $\tau$, but no fixed costs. It follows that trade is all-or-nothing: except when trade costs are prohibitive, every consumer in the world consumes each of the $N \equiv (\kappa + 1)n$ varieties produced in the world, where $N \in \mathcal{N}$, the set of all potential varieties. Why do they bother? Because they have a taste for varieties.

The supply of identical worker-consumers in each country is fixed at $L$. Technology follows the Dixit-Stiglitz specification, perhaps the simplest possible way of allowing for increasing returns. Each firm requires $f$ workers to operate, and $c$ workers to produce a unit of output. Labor-market clearing in every country therefore implies:

$$L = n (f + cy)$$

We follow the Marx-Keynes-Krugman-Melitz convention of measuring nominal variables in labor units, so the wage is set equal to one by choice of numéraire.

The elasticity of substitution is a sufficient statistic for the comparative statics implications of a CES utility function. With general additive preferences, we need to know at least two statistics to understand the positive effects of exogenous shocks: the elasticity $\varepsilon(x) \equiv -\frac{p(x)}{xp(x)}$ and the convexity $\rho(x) \equiv -\frac{xp''(x)}{p'(x)}$ of demand. In addition, to understand normative implications, we need to know the elasticity of the subutility function $\xi(x) \equiv \frac{xu'(x)}{u(x)}$, this must lie between zero and one, and is an inverse measure of the consumer’s taste for diversity. All three parameters are defined at a point only, so we need to know their values evaluated at the consumption of both home and imported varieties: we will write $\xi^* = \xi(x^*)$ and so on for the parameters pertaining to imports.

As shown in Mrázová and Neary (2013), many implications of these preference and demand parameters can be summarized in terms of two key properties. The first we call “superconvexity” of demand: we define a demand function as superconvex at a point if it is more convex than a CES demand function with the same elasticity, otherwise it is subconvex. Formally, the con-

\[ L = n (f + cy) \]

With so much symmetry assumed, the general-equilibrium structure of the model is straightforward. Goods-market clearing requires that the output of each firm, denoted by $y$, meet global demand for its product: $x$ from each consumer in its home market, and $x^*$ from each consumer in its $\kappa$ export markets, with the proviso that $\tau x^*$ units must be shipped abroad to ensure that $x^*$ arrive:

$$ y = L(x + \kappa \tau x^*) $$

Labor is the only factor of production, and
tion for strict superconvexity is \( \rho > \frac{\varepsilon + 1}{\varepsilon} \).  
Superconvexity implies that the elasticity of demand rises as per capita consumption increases, and so it is crucial for the difference between a firm’s mark-ups on its home and foreign sales, as we discuss in Section II. The second key property we call “superconcavity” of utility: we define a subutility function as superconcave at a point if it is more concave than a CES subutility function with the same elasticity, otherwise it is subconcave. Formally, the condition for strict superconcavity is \( \xi > \frac{\varepsilon + 1}{\varepsilon} \). Superconcavity implies that the elasticity of utility falls as per capita consumption, as we shall see in Section IV.

II. Integrated or Segmented Markets?

The first issue that arises when we combine trade costs and general demands is whether firms view their home and foreign markets as integrated or segmented. Integrated markets imply that prices are equalized allowing for transport costs: \( p^* = \tau p \). Segmented markets imply that marginal revenues are equalized allowing for transport costs: \( r^* = \tau r \). (Sales revenue at home is denoted by \( r(x) \equiv xp(x) \), and similarly for \( r(x^*) \) abroad.) These coincide if and only if preferences are CES, since otherwise the ratio of price to marginal revenue differs across markets:

\[
(4) \quad p = \frac{\varepsilon}{\varepsilon - 1} r, \quad p^* = \frac{\varepsilon^*}{\varepsilon^* - 1} r^*
\]

What does this imply for the pattern of price-cost mark-ups across markets? When markets are integrated, mark-ups are the same at home and abroad: \( \xi = \frac{\varepsilon}{\varepsilon - 1} \). However, when markets are segmented, mark-ups differ in a way that depends on the concavity of demand. With subconvexity, the elasticity is lower in export markets and so from (4) the mark-up is higher there. Moreover, the price charged abroad is lower than the transport-cost-inclusive home price: \( p^* < \tau p \). Both these outcomes represent a form of reciprocal dumping which does not assume oligopolistic behavior as in the classic treatment of Brander and Krugman (1983). All these statements are reversed if demands are superconvex. Now margins are higher abroad, and prices there exceed the transport-cost-inclusive home price if markets are segmented.

III. Globalization or Colder Icebergs?

For reasons of space, we focus on the case of segmented markets, probably a more realistic description of world markets that exhibit non-competitive behavior. We consider two alternative kinds of trade liberalization: an increase in the number of countries \( \kappa \), which we call “globalization,” and a reduction in trade costs \( \tau \). As we will see, these have very different effects on the conditions for firm and industry equilibrium.

At the firm level, profit maximization equalizes \( \tau \)-inclusive marginal revenues across markets, as we have seen. Totally differentiating, using “hats” to denote proportional changes (\( \hat{x} \equiv d \log x, x \neq 0 \)):

\[
\hat{x} + \tau \hat{x} = \hat{\tau} \quad \Rightarrow \quad \eta \hat{x} = \eta^* \hat{x}^* + \hat{\tau}
\]

where \( \eta \equiv -\frac{\theta \tau}{1-\rho} = \frac{2-\rho}{1-\rho} \) is the elasticity of marginal revenue at home.  
Equation (5) shows that profit maximization implies a positive relationship between home and foreign sales, as illustrated by the curves labeled “MR=MC” in Figure 1. These are shifted upwards by reductions in trade costs, but are unaffected by changes in the number of countries.

At the industry level, free entry requires that operating profits in all markets combined, denoted \( \pi + \kappa \pi^* \), equal fixed costs \( f \). Using the first-order condition, we can write operating profits in a typical foreign

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5 Recall footnote 3.
6 The concavity of an arbitrary sub-utility function is \( -\frac{\varepsilon^*}{\varepsilon^* - 1} = \frac{1}{2} \); while, from footnote 3, the concavity of a CES sub-utility function with elasticity \( \xi \) is \( 1 - \xi \). The former is greater than the latter when the condition in the text holds.
7 The firms’ first- and second-order conditions require \( \varepsilon > 1 \) and \( \rho < 2 \) respectively, so \( \eta \) must be positive.
market as: \( \pi^* = (p^* - \tau c)Lx^* = \frac{x_l x^*}{x + L} \), and analogously at home. Totally differentiating the free-entry condition:

\[
\omega_{\pi} \varepsilon \eta \hat{x} + (1 - \omega_{\pi}) \varepsilon^* \eta^* \hat{x}^* = -(1 - \omega_{\pi})(\hat{k} + \hat{\tau})
\]

where \( \omega_{\pi} \equiv \frac{\pi}{\pi + \kappa \pi^*} \) is the contribution of home-market sales to total operating profits. This implies a negative relationship between home and foreign sales, as illustrated by the curves labeled “\( \Pi = 0 \)” in Figure 1. These are affected in the same way by increases in \( \tau \) and \( \kappa \): both shocks cause the curve to pivot anti-clockwise around the x-axis (the zero-profit point conditional on not exporting).

Combining these results we can deduce the effects on home and export sales:

\[
\varepsilon_x \eta \hat{k} = (1 - \omega_{\pi})[-\hat{k} + (\varepsilon^* - 1) \hat{\tau}]
\]

\[
\varepsilon_x \eta \hat{x}^* = -(1 - \omega_{\pi}) \hat{k}^* - [1 + \omega_{\pi}(\varepsilon - 1)] \hat{\tau}^*
\]

Here \( \varepsilon_x \) is an aggregate elasticity weighted by profit shares: \( \varepsilon_x \equiv \omega_{\pi} \varepsilon + (1 - \omega_{\pi}) \varepsilon^* \). Equations (7) and (8) can be interpreted using terms borrowed from the literature on the effects of a currency devaluation. Globalization leads to “expenditure-reduction”: the increase in total varieties in the world reduces spending on each individual variety, so both \( x \) and \( x^* \) fall. As for a fall in trade costs, the case illustrated in Figure 1, this leads to “expenditure-switching”: consumption of imported varieties rises at the expense of home-produced ones.

The two kinds of shocks clearly have very different qualitative effects on home and foreign sales. Moreover, inspection of the equations shows that it is not possible to aggregate them in general, because the elasticities of marginal revenue differ between the two markets. There are two exceptions to this rule. One is the case of free trade, for any demand function. The elasticities of marginal revenue are now the same on both home and export sales, and the two loci in Figure 1 are straight lines with slopes of 45 degrees. The other exception is the CES case. Both loci are now straight lines for any value of trade costs. The \( MR=MC \) locus reduces to \( x^* = \tau^{-1} x \), while the zero-profit locus simplifies to \( x + \kappa \tau x^* = (\sigma - 1) \frac{L}{\kappa L} = \frac{y}{L} \). Eliminating \( \tau \), we can solve for the locus from \( A \) to \( F \) along which \( x \) and \( x^* \) adjust as \( \tau \) falls: \( x^* = \left( \frac{y/L - x}{\kappa} \right) \frac{\pi^*}{\varepsilon^*} x^{-1/\varepsilon^*} \). This smooth adjustment of sales to trade costs mirrors their smooth adjustment along a straight-line \( MR=MC \) line to changes in the number of countries. It explains why, as noted by Arkolakis, Costinot and Rodríguez-Clare (2012), the effects of both shocks on aggregate welfare in the CES case are isomorphic in that they can be summarized in terms of their effects on the share of domestic goods in total output. However, this isomorphism between changes in trade costs and in the size of the world economy breaks down when demands are not CES.

With changes in per capita sales pinned down, it is straightforward to deduce the implications for prices, output, and firm numbers. Prices are directly linked to sales by the firms’ first-order conditions; differentiating (4) gives:

\[
\hat{p} = \frac{\varepsilon + 1 - \varepsilon \rho}{\varepsilon (\varepsilon - 1)} \hat{x}; \quad \hat{p}^* = \frac{\varepsilon^* + 1 - \varepsilon^* \rho^*}{\varepsilon^* (\varepsilon^* - 1)} x^* + \hat{\tau}^*
\]

Both are increasing with sales if and only if demands are subconvex: as consumption rises, the elasticity of demand falls, and so price-cost mark-ups increase. Is this the more likely case? There is no clear consensus in the literature, though the balance of empirical and other evidence suggests that subconvex demands are more relevant than superconvex. Provided subconvexity holds, we can conclude that both globalization and lower trade costs reduce all prices.

\[
\text{in the CES case, so the coefficients of both } \hat{x} \text{ and } \hat{x}^* \text{ in (7) and (8) reduce to unity.}
\]

\[\text{With homogeneous firms, this share is simply } \frac{1}{\pi}. \]

\[\text{in the CES case, since } y \text{ is fixed, the change in this is the CES specialization of (7), } \hat{x} = (1 - \omega_{\pi})[-\hat{k} + (\sigma - 1) \hat{\tau}]. \]

\[\text{See Mrázová and Neary (2013) for more details.} \]

\[\text{The effect of lower trade costs on import prices is always negative: though the mark-up on foreign sales may rise or fall, the direct effect of the change in trade costs always dominates. From (8) and (9), } \hat{p}^*/x = 1 - \]
As for adjustment at the intensive and extensive margins, these follow directly from the market-clearing conditions (2) and (3). From (2), the change in firm output is a weighted average of the changes in sales to home and foreign consumers:

\[ \hat{y} = \omega_x \hat{x} + (1 - \omega_x) (\hat{\kappa} + \hat{\tau} + \hat{x}^*) \]

where the weight \( \omega_x \equiv \frac{x}{x + \tau + \kappa} \) is the share of home sales in total output. This in turn directly determines the number of active firms and so of produced varieties in each country from the full-employment condition (3):

\[ \hat{n} = -\psi \hat{y}, \quad \psi = \frac{\bar{\varepsilon} - 1}{\bar{\varepsilon}} \]

Here, \( \psi \equiv \frac{\varepsilon w}{\varepsilon r + \varepsilon y} \) is the share of variable costs in total costs, which is an inverse measure of returns to scale. It is increasing in the aggregate elasticity \( \varepsilon_h \), which is a sales-weighted harmonic mean of the demand elasticities in the home and foreign markets: \( \varepsilon_h \equiv [\omega_x \varepsilon^{-1} + (1 - \omega_x) (\varepsilon^*)^{-1}]^{-1} \).

Just as the changes in mark-ups given by (9) hinge on sub- or superconvexity of demand, so too do those of output and firm numbers given by (10) and (11). In the CES case, output is fixed, and both types of shock merely reallocate sales: globalization encourages firms to sell to more markets, but less in each; while a rise in trade costs induces a reduction in production for exports which exactly offsets the increase in home sales. If instead demand is subconvex, mark-ups rise as per capita sales fall. Hence, for a globalization shock, the negative effect on profits of a reduction in sales is less than the positive effect of an expansion in the number of markets; so, to keep overall profits equal to zero, total output must increase. As for a rise in trade costs, to keep profits constant requires sales to fall by less in declining markets than they rise in expanding markets, so once again total output rises.

These results take a particularly simple form in the neighborhood of free trade: \(^{12}\)

\[ \left. \frac{\hat{y}}{\tau} \right|_{\tau = 1} = (1 - \omega) \left( 1 - \frac{1}{\varepsilon^*} \right) (\hat{\kappa} + \hat{\tau}) \]

The key expression on the right-hand side is positive if and only if the elasticity of marginal revenue \( \eta \) exceeds the elasticity of inverse demand \( \frac{1}{\varepsilon^*} \), which is equivalent to demand being subconvex. \(^{13}\) Note the implication that “trade liberalization” has opposite effects on total output, and so, from (11), on the number of firms per country, depending on whether it involves an increase in \( \kappa \) or a reduction in \( \tau \). Paradoxically, a reduction in trade costs in the neighborhood of free trade reduces firm output and increases the number of domestic firms if and only if demand is subconvex. When trade costs are initially positive, equation (12) has to be qualified to take account of the differences in the importance of the home market in output and profits (i.e., the difference between \( \omega_x \) and \( \omega_y \)). The full expressions are given in the appendix.

IV. Gains from Trade

When we come to evaluate the gains from trade, we need for the first time to take account of the elasticity of utility. We measure the gains by the change in equivalent income, \( \hat{Y} \), that would keep the typical consumer at their initial level of utility: \(^{14}\)

\[ \hat{Y} = \left( \frac{\bar{\varepsilon}_z}{\bar{\varepsilon}_u} \frac{1}{\xi_u} - 1 \right) \hat{N}_Y - \omega_Y \hat{p} - (1 - \omega_Y) \hat{p}^* \]

\(^{12}\) At free trade, both sets of weights reduce to: \( \omega_x = \omega_y = \omega = \frac{1}{\varepsilon^* + 1} \).

\(^{13}\) Recalling the definition of \( \eta \) following equation (5): \( \eta - \frac{1}{\varepsilon} = \frac{x + 1 - \varepsilon x^*}{\varepsilon x - 1} \).

\(^{14}\) For small changes the equivalent and compensating variations are the same. Formally, \( \hat{Y} \) is calculated by first substituting the Frisch demand functions into the utility function (1) to derive what we call the “Frisch indirect utility function” \( V^N(N, p, \hat{p}, \lambda) \); then using the budget constraint to solve for \( \lambda \) and substituting to get the indirect utility function \( V(N, p, \hat{p}, z) \), where \( z \) is the typical consumer’s income or expenditure; and finally solving for \( Y \) as the solution to: \( V(N, p, \hat{p}, z/Y) = U^o \) for given \( z \) and \( U^o \). See Mrázová and Neary (2013) for details.
where $\hat{N}_Y$ is a composite term in the change in number of varieties.\textsuperscript{15} Qualitatively, the change in real income is identical to that in the free-trade case in Mrázová and Neary (2013). Consumers gain at the extensive margin when the number of varieties increases, and by more the lower is the elasticity of utility $\xi$, that is, the more they care about diversity. They also gain at the intensive margin from any falls in prices. Quantitatively, however, matters are more complicated. The elasticity of utility in (13), $\xi_u$, is a weighted average of those for home and imported varieties, where the weights are the shares of each group in utility, themselves weighted by the elasticities of demand.\textsuperscript{16} This elasticity is also adjusted in (13) to take account of any difference between the expenditure- and utility-weighted average demand elasticities $\bar{\varepsilon}_x$ and $\bar{\varepsilon}_u$.\textsuperscript{17} Finally, the welfare effects of price changes depend on the shares of each good in expenditure, also adjusted to take account of differences between expenditure- and utility-weighted average demand elasticities.\textsuperscript{18}

Equation (13) is the last building block needed to calculate explicitly the gains from trade. The change in firm numbers is given by (11) as a function of firm output, which in turn is related to sales by (10). Prices are also related to sales by (9), and the structure is completed by equations (7) and (8), which relate changes in sales to changes in exogenous variables only. Clearly, the differences between weights generate income effects which make even qualitative statements problematic. However, we can provide intuition for the effects by considering changes in the neighborhood of free trade:

\begin{equation}
\hat{Y} \bigg|_{\tau=1} = \psi - \xi \hat{n} + (1 - \omega) \left( 1 - \frac{\xi}{\xi_u} \hat{k} - \hat{\tau} \right)
\end{equation}

Here the change in real income is expressed as a function of changes in the exogenous variables and of the number of varieties produced at home. Two sufficient conditions for gains from trade follow immediately. First is when the term in $\hat{n}$ is zero, so the initial equilibrium is efficient: for given values of the exogenous variables there is no change that can raise welfare. Recalling the discussion of the elasticity of utility in Section I, the coefficient of $\hat{n}$ is zero if and only if preferences are CES: a familiar result from Dixit and Stiglitz (1977). Alternatively, an activist anti-trust policy could set firm numbers in each country at just the right level to ensure efficiency. A second sufficient condition for gains from trade implied by (14) is that the expressions $\psi - \xi$ and $\hat{n}$ have the same sign. For example, both are positive when utility is subconcave (so $\psi > \xi$, implying that consumers desire more variety), and demand is subconvex (so, from Section III, trade liberalization increases the number of varieties).

\section{V. Calibrating the Gains from Trade}

Qualitative results such as those in the previous section are valuable for giving intuition, but the complexity of the general expressions when trade costs are initially positive means that we have to resort to calibration. Space constraints preclude our presenting detailed results, so instead we note some general considerations relating to calibrating the gains from trade.

A key feature of our results is that all the local comparative statics properties of the model can be expressed in terms of a relatively small number of parameters: a range of different home-market share parameters, as well as the elasticity and convexity of both utility and demand for both home and imported varieties. While this is not bad news for calibrationists, the key phrase is...
“relatively small”. Arkolakis, Costinot and Rodríguez-Clare (2012) showed that only two parameters are needed to calibrate the gains from trade in CES-based models: the share of home goods in output, and the elasticity of substitution itself. Here, the same parameters arise, but many variants of each are required.

Consider first the various weights we have encountered. These simplify greatly in two central cases. With free trade, country symmetry implies that all the weights collapse to the share of each country in world GNP, $\frac{1}{\sum_i \omega_i}$, irrespective of the form of demand. With CES preferences, all the weights are also identical, this time to $\frac{1}{\sum_i \omega_i}$, irrespective of the form of demand. With CES preferences, all the weights are also identical, this time to $\frac{1}{\sum_i \omega_i}$, irrespective of the form of demand.

VI. Conclusion

In this paper we have used the tools and techniques introduced in Mrázová and Neary (2013) to explore the implications of combining two real-world features typically studied in isolation in general-equilibrium trade models: variable elasticities of demand on the one hand, and barriers to international trade on the other. Even in our simple setting, we have shown that relaxing the assumption of CES preferences in monopolistic competition has surprising implications when trade is restricted. Integrated and segmented markets behave very differently, the latter typically exhibiting a stronger form of reciprocal dumping. Globalization and lower trade costs have very different effects: the former reduces spending on all existing varieties, the latter switches spending from home to imported varieties; in the plausible case where demands are subconvex, globalization raises firm output whereas lower trade costs reduce it. Finally, calibrating gains from trade is harder than in the CES case. Many more parameters need to be calibrated, while import demand elasticities are likely to overestimate the true elasticities, and so underestimate the gains from trade. Hopefully, our approach points the way towards much-needed robustness checks of calibration exercises to take account of simultaneous departures from CES demands and zero trade costs.
REFERENCES


### Table 1—Ranking of Weights.

<table>
<thead>
<tr>
<th>Demand Convexity</th>
<th>Utility</th>
<th>Subconcave</th>
<th>Superconcave</th>
</tr>
</thead>
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<td>Sub-ωπ</td>
<td>ωπ &gt; ωz &gt; ωx, ωu &gt; ωx, x*, x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Super-ωπ</td>
<td>ωπ &gt; ωz &gt; ωx, ωu &gt; ωx, x*</td>
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**Figure 1. Globalization or Cooler Icebergs**