1 Introduction

The provision of international public goods, such as financial stability, international security or environmental protection, has been salient in international negotiations over the past decades. International public goods have the property that all countries benefit from aggregate provisions, while the costs are only carried by those actually providing. For instance, a country’s provision of military forces to a peace keeping mission also benefits countries that do not participate in the mission. Or, if some countries invest in clean technology and enforce strict regulations with respect to greenhouse gas emissions, all countries benefit from reduced pollution on a global scale while only those countries investing in emission reduction pay the costs.

The potential divergence between those who carry the costs for provision and those who benefit is prevalent in public goods problems. It leads to a strong incentive to free-ride on others’ contributions, and typically induces underprovision and significant welfare losses. While on a national scale governments might be able to coerce cooperation on certain issues through specific policies, the sovereignty of nations makes this impossible on an international scale. The only way to limit inefficiencies resulting from free riding incentives in international public goods provision is therefore the formation of self-enforcing agreements.

A prominent example of such a self-enforcing agreement can be seen in the Montreal Protocol on Substances that Deplete the Ozone Layer. It was first signed
in 1987 and has since experienced a steady increase in membership as well as good compliance rates. On the contrary, negotiations on climate change abatement have proven difficult. With the United Nations Framework Convention on Climate Change, negotiations on greenhouse gas abatement received a baseline structure in 1992. Five years later, the Kyoto Protocol was adopted in an effort to restrict greenhouse gas emissions in developed countries. However, compliance rates with the Kyoto Protocol have been low, and finding a follow up agreement which includes a common standard of greenhouse gas abatement for all countries has proven difficult.

In this paper, we analyze the impact heterogeneity in underlying country characteristics has on the formation of an agreement and we discuss the consequences this, in turn, has on the provision of public goods and welfare. We start by analyzing the role of economic heterogeneity (costs and endowments) and preferences (valuations of the public good). We then take the analysis of heterogeneity a step further in focusing on the role the political process plays in shaping the way negotiation outcomes are influenced by heterogeneity.

To describe our analysis in more details, we first consider the role of heterogeneity in either the provision costs or the valuation of the public good. We observe that a country with higher costs or a lower valuation is less likely to join an agreement. Using this observation, we can show that an increase in cost or valuation heterogeneity makes agreement formation more difficult. We furthermore derive results on the effects of level changes that impact all countries. Among other things, we show that shift towards lower costs, due to the introduction of new technologies, leads to a decreased incentive for a high cost country to participate; that is, it makes agreement formation more difficult. A shift towards higher valuation due to, for instance, new information about the relevance of the public good has an equivalent impact on agreement formation.

Second, we look at the impact of endowment heterogeneity on negotiation outcomes. We show that countries with large endowments are more willing to participate in negotiations. Based on this, we argue that agreement formation should become easier as countries grow richer. Subsequently, we look at the way endowment heterogeneity interacts with cost and valuation heterogeneity. A straightforward analysis of this combined heterogeneity delivers a (to us) counterintuitive result: introducing heterogeneity over endowments can make countries more willing to participate in agreements, when other forms of heterogeneity are already present. This is the case if endowments and costs are positively correlated.

Third, we consider the role the political process plays in shaping the way heterogeneity influences negotiation outcomes. As pointed out by Stavins (2011), difficulties in the formation of international agreements seem to be less induced
by economical and technological feasibility questions, but rather appear to be due to political coordination problems. We therefore explicitly model the voting procedure. In our baseline model, we assume that decisions are made by majority rule, but we also consider voting rules with different distributions of political power; specifically, we compare outcomes under majority rule with weighted or unweighted voting to dictator rule results, and analyze the way the distribution of political power interacts with changes in cost heterogeneity and valuation heterogeneity. Among other things, we can show that the negative impact on the possibility of agreement formation that follows from an increase in heterogeneity is reduced if political power is more concentrated to certain countries.

Our work builds on, and draws together, three different strands of literature. The first of these strands is the literature on public provision of public goods, which refers to the formation of agreements to provide public goods. The most closely related paper here is Kosfeld et al. (2009), where the authors consider heterogeneity with respect to preferences. They do not, however, analyze endowment or cost heterogeneity and they do not explicitly model the political process.

In addition to the public goods literature, our work also builds on the literature on union formation in the context of federalism and the development of new political confederations. The most closely related paper here is Alesina et al. (2005). While Alesina et al. consider a setting with heterogeneity and explicitly analyze the political process, our model differs from theirs in several ways, perhaps most importantly in that we assume spillovers between agreement members and non-members while they do not. We consider such spillovers a crucial part of a model of international public goods. Other related papers in this strand of literature are Cremer and Palfrey (1999, 2006) and Hafer and Landa (2007); these papers differ for our work in similar ways.

A third closely related literature is the set of papers on self-enforcing international environmental agreements, which originated in the 1990s with seminal contributions by Carraro and Siniscalco (1993) and Barrett (1994). Almost all of the models with this focus have assumed homogeneous agents. Among the few papers that do consider heterogeneity, Barrett (1997) analyzes how benefits from cooperation can be divided between countries in an agreement using simulation-based results. Botteon and Carraro (2001) and McGinty (2006) extend Barrett’s framework to a larger number of countries, but also use simulations to generate their results. Kolstad (2010) is, to our knowledge, the first to present a model of agreement formation under heterogeneity with closed-form results. However, Kolstad limits heterogeneity to size and marginal damage and considers only two different types of countries, and he does not develop an explicit model of the political process. We add to these papers by incorporating a more general approach to heterogeneity and a more detailed model of the political process, while
allowing for spillovers between agreement members and non-members in a way that distinguishes us from the political confederation literature.

2 The Model

There exist $N \geq 2$ countries which we, in slight abuse of notation, label $i = 1, ..., N$. A country $i$’s preferences with respect to the provision of a public good are given by the following Cobb-Douglas function:

$$U_i = \ln \left( \frac{w_i - (1 + c_i)g_i}{n_i} \right) + a_i \ln \left( \sum_{j=1}^{N} g_j \right) - K 1_{\{g_i > 0\}} + B 1_{\{agreementmember\}}$$

where $w_i$ is endowment, $c_i$ are costs, $a_i$ is valuation, $n_i$ is the number of citizens in country $i$, and $g_i$ is a country’s contribution. The first term of the expression denotes the utility received from private consumption, that is, from the part of the endowment that is not invested in the public good. The second part of the expression is the utility received from the aggregate amount provided, weighted by the respective country’s valuation for the good. We furthermore assume that there may be additional fixed costs and benefits associated with provision and agreement membership. $K$ is a fixed cost that arises if a country provides a positive amount to the public good, either on its own or in an agreement. This fixed cost may, for instance, capture the need for technological investments or other start-up costs associated with provision. $B$ is an exogenous benefit from being in an agreement. This benefit may, for instance, pick up non-modelled favorable treatment for an agreement member by other members in different policy areas, such as additional trade. It may also capture a desire by countries, or more specifically country leaders, to be held in positive regard by other countries, to do what is perceived to be the morally correct thing, or simply to not stand out in a negative way on the international stage. We make no attempt to distinguish between these possible intrinsic benefits of agreement membership, but simply represent their joint effect with one parameter, $B$.

In order to analyze how an agreement for the provision of such a public good might form, we introduce a three stage setup. In the first stage, countries decide simultaneously and independently whether they wish to enter a process of agreement formation. If at least three countries enter, they become participants of a political process which takes place in the second stage. Participants vote on a common fraction $t \in [0, 1]$ of endowment they wish to contribute$^1$. In our baseline

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$^1$In reality, countries participating in agreement negotiations of course bargain in more complex and multidimensional ways. In particular, the level of emissions is typically an important additional aspect in the determination of each country’s specified contributions (or emissions
model decisions are made by majority rule. Countries vote in favor if they are indifferent between forming an agreement or not. Countries participating in the second-stage political process cannot leave during this stage and non-participants cannot enter. If an agreement forms the participants turn into members of the agreement. In the third stage the countries make their contributions, with these contributions conditional on whether an agreement formed and, if so, whether they are members of that agreement. Countries adhere to the contributions they agreed to in the second stage.

Before we proceed, a note on the political process, in particular the majority rule assumption, is in place. In reality, agreements frequently require unanimity for ratification, and one could imagine using some form of unanimity rule to model the second stage here. However, unanimity is not a model of a political process. It simply says that everyone has to prefer the chosen policy to a breakdown of the agreement; it does not offer us any guidance regarding which policy will be selected. And since we are trying to analyze the political outcome, we need an explicit assumption about the political process in the second stage. We use a majority rule assumption for simplicity and transparency; as will be clear, such a setup makes it easy to illustrate the effects of politics on agreement formation. We would, however, like to emphasize that one could substitute the majority rule setup for another model of the political process, such as a model of political bargaining, without changing the key structure and the main insights derived from the setup we use here, although the results might differ quantitatively.

We assume that $K$ is such that countries have no incentive to provide if they are not members of an agreement. To be more precise, we first define $U_{i}^{\text{no}}$ as the utility achieved by country $i$ when no one is providing: $U_{i}^{\text{no}} = \ln\left(\frac{w_i}{n_i}\right)$. We then define $U_{i}^{a}$ as the utility achieved by country $i$ when it alone provides and does so in an optimal way without taking $K$ into consideration; that is, $U_{i}^{a} = \ln\left(\frac{w_i - (1+c_i)g_{i}^{a}}{n_i}\right) + a_i \ln (g_{i}^{a})$, where $g_{i}^{a} = \arg \max_{g_i} \left\{ \ln\left(\frac{w_i - (1+c_i)g_i}{n_i}\right) + a_i \ln (g_i) \right\}$. Using these definitions, we make the following assumption about $K$:

$$K > \max \{ U_{1}^{a} - U_{1}^{\text{no}}, ..., U_{N}^{a} - U_{N}^{\text{no}} \}$$

Now, note that the utility increase the country with the greatest incentives to
provide can achieve by providing is largest when no one else is providing. Hence, if that country has no incentive to provide in this setting it also has no incentive to provide in any other setting. This assumption thus implies that any country outside of an agreement will not choose to provide a positive amount of the public good.

The three stage setup we employ matches the agreement formation process outlined in most of the literature on international environmental agreements (Wagner 2001) and international public goods provision (Kosfeld et al. 2009). However, we introduce an explicit voting procedure and thereby add more detail to the political process described in the second stage. This helps us to analyze the impact of heterogeneity in costs, endowments and valuation on agreement formation and the way this underlying heterogeneity interacts with the political structure.

3 Homogeneous Countries

We start by considering the case of homogeneous countries, that is, we assume all countries to have equal cost, endowment and valuation. If an agreement forms, it provides according to all countries’ optimal provision amount. This follows because all voting participants have the same optimal $t$. There is no disagreement and the agreed upon $t$ trivially equals all countries’ optimal $t$, given by

$$t = \frac{a}{(1 + a)(1 + c)}.$$

As endowments are assumed to be the same for all countries, also the actual optimal provision amount is the same across all countries. We now look at how many countries can form an agreement, such that this agreement is stable.

In the appendix we show that there exists an upper bound $\bar{S}$ on internal stability; that is, there is an upper bound on the area for which no country in an agreement would have an incentive to leave the agreement. At the same time, there exists a lower bound $\underline{S}$ on external stability; that is, there is a lower bound on the number of countries in an agreement for which no non-member would have an incentive to join. These bounds coincide in the sense that the smallest integer fulfilling $\bar{S} \geq S^*$ is at the same time the largest integer fulfilling $S \leq \bar{S}$. Thus, $\bar{S} \geq S^* \geq \underline{S}$ defines the unique stable agreement size.

**Proposition 1.** If all countries are homogeneous and $B - K$ is not too small, there exists a unique number of countries that can form a stable agreement, defined by $\bar{S} \geq S^* \geq \underline{S}$.

The number of countries in the agreement is independent of costs and endowments, but depends on the valuation of the respective public good and on the
additional benefit $B$ received if being in an agreement. If $B - K$ is larger, the stable agreement size is larger. The number of countries is also increasing in $a$; that is, the higher the good is valued the more countries can form a stable agreement in equilibrium.

A special case of this proposition is the formation of an agreement including all countries, which we refer to as the “grand coalition”. We can show that there exists parameter values such that the grand coalition forming is the unique equilibrium outcome. Such an agreement forms if the number of countries is lower than the upper bound for internal stability, that is $N \leq \bar{S}$. In that case all countries prefer participating in an agreement over not doing so.

**Corollary 1.** If $N \leq \bar{S}$ and $B - K$ not too small, an agreement with $S = N$ forms.

If an agreement with $S = N$ forms, its members agree on the socially optimal fraction of endowment to contribute to the public good. The socially optimal fraction $t$ is determined through maximizing $\sum_{j \in C} U_{\text{agreement members}} + \sum_{j \not\in C} U_{\text{non-members}}$, which yields

$$t_{\text{opt}} = \frac{N}{S} a \frac{(1 + c)(1 + N)}{1} a.$$  

In case an agreement with $S = N$ forms this equals exactly the individually optimal $t$. In case an agreement with $S < N$ forms, however, $t_{\text{opt}} > t^*$ and thus there is underprovision in equilibrium.

## 4 Heterogeneous Countries

### 4.1 Cost and Valuation Heterogeneity

We now introduce heterogeneity over costs and valuation, respectively. Countries might differ with respect to their costs of provision. That is, while one country might be able to reach some provision amount fairly easily given the production technology it uses, the same provision might cause much higher costs for another country. These differences are important to take into consideration as they largely influence countries’ willingness to engage in provision.

Equivalently, countries might have different valuations of a public good. For instance with respect to climate change, some countries are likely to be more impacted than others and, at the same time, some countries seem to have a higher intrinsic value they attach to sustainable behavior. Similar considerations hold
true for other international public goods. Differences along dimensions such as geographic position, economic development or general social norms prevalent in a country’s culture are likely to lead to different valuations for a public good.

We assume costs and valuations to be uniformly and equidistantly distributed. This allows us to order countries from highest costs to lowest costs and lowest valuations to highest valuations, respectively. As preferences are single-peaked, countries can also be ordered according to their optimal \( t \). Larger costs induce the fraction of endowment a country wishes to contribute to decrease, while a higher valuation leads this fraction to increase. As the policy space is linear and countries’ preferences are single-peaked, the median voter theorem holds. If an agreement forms, it thus provides \( G_C = t_M Sw \) where \( t_M \) is the fraction of endowment the median country would like to provide, and \( Sw \) is the aggregate endowment of agreement members. In the case of an even number of countries joining, \( t_M \) is an average of the two countries around the median position.

When evaluating whether a country would like to enter into an agreement, we can distinguish a private consumption effect, a political effect and an agreement size effect. These effects describe the difference in utility between participating and not participating in the political process. The private consumption effect symbolizes the decrease in private consumption due to participation. This effect is always negative, meaning that it pushes countries towards not wanting to participate. The political effect characterizes the difference in the median \( t \) between participation and non-participation of a country; it can have a positive or negative effect on a country’s willingness to participate. The agreement size effect comes from the change in the membership number when an additional country decides to participate; this agreement size effect is always positive.

Based on the distinction of these effects, we can see that, in the case of cost heterogeneity, the country with the highest costs in a conjectured agreement is the least likely to be willing to be a member of that agreement. If a country with a \( t > t_M \) were to not participate the new median \( t \), denoted by \( t'_M \), would be lower than the old median \( t \), denoted by \( t_M \), so the fraction contributed by the agreement is lower if the country does not participate. If, however, a country with \( t < t_M \) does not participate, then \( t'_M > t_M \). That is, the fraction contributed by the agreement is larger if the country does not participate. Thus, participation incentives resulting from the political effect are smaller for countries with \( t < t_M \). To this we need to add the private consumption effect, which captures the decrease in private consumption that follows from being inside the agreement. The larger a country’s costs in comparison to median costs, the larger its decrease in private consumption due to participating. Hence, the country with the highest costs in a conjectured agreement is the least likely to participate.
The case of *valuation* heterogeneity is similar, though there are differences. In particular, the private consumption effect is solely determined by the median valuation and does not depend on a country’s own valuation. This (negative) effect becomes greater if the median valuation is larger. Another difference between the two types of heterogeneity can be found in the political effect. In opposition to the dynamics of cost heterogeneity, the sign of the political effect switches depending on whether a country’s valuation is larger or smaller than the median valuation. If \( a_i < a_M \), this effect is negative, which means that country \( i \) likes the effect its own decision to *stay out* has on the political process. This makes country \( i \) less willing to participate. If \( a_i > a_M \), the effect on the political process is the opposite.

In addition to the restrictions imposed by the setup, we now make the following assumption:

\[
A1 : \quad \frac{S}{S-1} \geq \frac{a_m}{a_m + 1} \cdot \frac{a_m + 1}{a_m + 1}
\]

The consequence of this assumptions is that the combined political and agreement size effects yield a positive incentive to join the agreement. We assume that the \( a_i \)'s are closely packed and \( a_i \in (0, \infty) \) in order to ensure that A1 holds for the broadest range of agreement sizes possible. Under A1, the country with the lowest valuation in any given agreement is the one who receives the lowest utility from participating. The lower a country’s valuation the lower the positive impact of the combined political and agreement size effects, while the negative impact of the private consumption effect does not vary with the respective country’s own valuation. Thus, the lower a country’s own valuation the higher is the weight of the negative private consumption effect relative to the positive combined political and agreement size effects, creating stronger incentives not to participate. That is, the country with the lowest valuation in a conjectured agreement is the least likely to participate.

Under this assumption, whenever countries have an incentive to form an agreement, one possible agreement forming is one comprising the countries with the lowest costs or, respectively, the highest valuations. A country with larger costs or a lower valuation has a lower incentive to be in an agreement, while a country with lower costs or a higher valuation has a larger incentive to be in an agreement. An agreement can thus include all countries from the one with lowest costs up to some threshold above which all countries are non-members, or respectively it can include all countries from the highest valuation down to some threshold below which all countries are non-members. We call such an agreement of countries with adjacent cost or valuation levels a *contiguous* agreement. In the following we focus our analysis on this type of agreement.
We now turn to the impact of heterogeneity on welfare. For the case of valuation heterogeneity, we can rewrite the expression for the socially optimal $t$, derived above for the case of homogeneity, to

$$t^{opt} = \frac{N\bar{a}}{(1+c)(1+N\bar{a})},$$

where $\bar{a}$ denotes the average valuation of all countries. Given that $a$ is equidistantly distributed, the agreed upon $t$ is socially optimal in the case of an agreement with $S = N$. If an agreement with $S < N$ forms, however, there is underprovision with $a_M < \frac{N}{S}\bar{a}$. In principle, there could also be cases where an agreement with $S < N$ provides optimally or even overprovides, i.e. where $a_M = \frac{N}{S}\bar{a}$ or $a_M > \frac{N}{S}\bar{a}$. This does not occur in equilibrium under the assumption of equidistantly distributed valuation parameters, but could occur under different parameter assumptions. For the case of cost heterogeneity a similar reasoning holds; that is, agreements that form in equilibrium will, in specific situations, provide optimally but otherwise underprovide under our parameter assumptions.

Subsequently, we analyze the impact of a median-preserving increase of heterogeneity on agreement formation. Specifically, we analyze what happens to an existing (equilibrium) agreement if we increase heterogeneity while keeping the median of the agreement fixed. (Although we keep the median of the agreement fixed, the increase in heterogeneity extends to the whole population; that is, it is not restricted to members of the agreement.) The effects of a median-preserving increase of heterogeneity highlight relevant differences between cost and valuation heterogeneity. To make these differences clear, we consider the effect of an increase in heterogeneity on each side of the median separately.

For cost heterogeneity, an increase of heterogeneity on the low-cost side induces the political effect to be stronger while the private consumption effect stays unchanged. A stronger political effect here means that the positive political effect for high-cost countries of choosing not to enter becomes greater. An increase on the high-cost side leads to a larger decrease in private consumption for high-cost countries if they choose to enter, while the political effect stays unchanged. The political and consumption effects thus go in the same direction: they decrease the incentives for high-cost countries, i.e. the countries least interested in participating in agreements, to stay outside. Hence, an increase in heterogeneity on both sides of the median leads to lower participation incentives for the countries that are least likely to want to participate.

For valuation heterogeneity, the private consumption effect, i.e. the negative effect on a country’s incentive to participate that follows from the private consumption it would have to give up, does not change with a median-preserving
increase in heterogeneity, as it depends only on the median valuation. There are, however, political and agreement size effects. Increasing heterogeneity on the side of the countries with valuations above the median leads to a greater political effect; specifically, it increases the impact on the political outcome from a low-valuation country’s decision. Hence, the country with the lowest valuation in an agreement (that existed before the change in heterogeneity) now has a lower incentive to participate. Increasing heterogeneity for countries with valuations below the median does not change the relevant political effect; specifically, it does alter the impact on the political outcome from a low-valuation country’s decision. However, it still leads to a decrease in the incentive for a low-valuation country to participate, as it mutes the (positive) combined political and agreement size effects. Hence, the effects from increasing heterogeneity on both sides of the median both lead to a lower incentive to participate for the country with the lowest valuation, which is the country with the greatest incentive to participate among all the agreement members. The total effect of an increase in valuation heterogeneity on agreement formation is thus negative. We show this formally in the appendix and summarize in the following proposition:

**Proposition 2.** A median preserving increase in the degree of cost or valuation heterogeneity makes it harder to reach an agreement.

This proposition says that the existence of an equilibrium in which an agreement forms depends on the degree of heterogeneity between countries. If heterogeneity is sufficiently large, agreement formation becomes impossible. The impact of a change in heterogeneity on welfare depends, however, on the size of the change. To think about changes in welfare, it is useful to distinguish small changes that are not large enough to change agreement size, and larger changes that alter the number of countries willing to join. For valuation heterogeneity, a small change does not alter welfare in the case of an agreement with $S = N$, but it leads to a decrease in welfare in the case of an agreement with $S < N$. For cost heterogeneity, a small change decreases welfare both in the case of an agreement with $S = N$ and $S < N$. This divergence follows from the difference in the way the cost and valuation parameters impact individual utility: while valuation enters the utility function through the political and agreement size effects, costs impact private consumption and therefore utility directly.

To analyze the effect of a larger change in heterogeneity, we consider an increase in heterogeneity that leads to a decrease in the agreement size. It is useful to start by isolating the effect of the decrease in the size, by ignoring the direct effects on utility from the parameter changes. In the case of valuation heterogeneity, A1 then implies that the utility of countries that do not change their membership status decreases if the agreement size decreases. For countries that do change their membership status, the utility of a country changing its membership status
is larger in the new status. This follows directly from a revealed preference argument; a country would only change its decision to be on the inside if it was better off doing so. In the case of cost heterogeneity, we impose a parallel assumption to A1:

$$A2 : \frac{1 + c_M}{1 + c_M} < \frac{S}{S - 1}$$

This assumption ensures that larger agreement leads to greater contributions.

We now leave the welfare analysis and turn to the analysis of variation-preserving level changes. We start by considering changes in costs. A variation-preserving level change in costs could, for instance, be a new technology that makes overall production of the public good cheaper. For the case of climate change, new advances in clean energy technologies might pose an example of such a change in cost levels.

If the cost level of public good production decreases, the private consumption effect gets larger; that is, just considering the decrease in private consumption that follows from participating in an agreement, each country has a greater incentive not to participate. At the same time, the political effect of not being member of an agreement is larger; that is, each country’s decision to participate has a greater impact on the political process. As explained above, creates an incentive not to participate for high-cost countries. Hence, while one, perhaps, might expect a cost decrease to have a positive effect on agreement formation, our model predicts that it makes agreement formation more difficult.

In the case of valuation heterogeneity, a variation-preserving change could, for instance, be the result of new information becoming available, or it could come from
a general increase in awareness of the relevance of the public good. A variation-preserving shift in the valuation level leads the members of a given agreement to provide more because the median $t$ is larger than before. However, this increase in the median $t$ might at the same time decrease participation incentives for the country with the lowest valuation. The reasoning for this is as follows: A larger median $t$ leads the private consumption effect to be more negative and the political effect to be less negative. At the same time, the own valuation of the lowest-valuation country in a conjectured agreement is higher due to the shift. This higher own valuation is relevant as a factor enhancing the political effect. Even though the political effect as such is now less negative, after the higher own valuation of the lowest-valuation country is taken into consideration the factored political effect is more negative than before the shift. Thus, the combined own consumption and factored political effect of a shift towards higher valuations is negative, leaving the lowest-valuation country in an agreement with a stronger incentive not to participate.

**Proposition 3.** A variation-preserving decrease in the level of production costs, or a variation-preserving increase in valuation of the public good, makes it harder to reach an agreement.

The welfare effect of such a level shift is not clear. If we consider only a change in agreement size, $A_1$ and $A_2$ ensure that both agreement members and non-members are worse off if less countries participate. However, if we additionally take the actual shift in parameters into account, we can no longer make a clear statement on the impact on countries’ utilities.

### 4.2 Endowment Heterogeneity

Differences in endowment between countries constitute the most visible form of heterogeneity on the international stage. We assume endowment to be equidistantly distributed across countries. Differences in endowment do not have any direct impact on the fraction of endowment a country is willing to contribute. However, they do influence the overall amount a country contributes and through that lead to different externalities caused by countries participating or not. For a country with a large endowment the utility difference between being member of an agreement and not being member is larger, that is, being in an agreement is relatively more profitable for a rich country than for a poor country. Thus, a country with a larger endowment is more likely to be in an agreement than one with a lower endowment.

Whether the members of an agreement provide what would be socially optimal depends on the relation between the endowment of agreement members and the aggregate endowment of all countries. That is, an agreement with $S = N$
provides optimally, while an agreement with $S < N$ underprovides because its aggregate endowment is always smaller than the overall endowment of the whole population.

We then consider the effect of a median-preserving increase in endowment heterogeneity. Again, we study the effect of a change around the median of an existing agreement and as the question: is this agreement/coalition still an equilibrium outcome after the change in heterogeneity? Countries above the median have a higher endowment after such an increase, leading them to be more willing to participate. Countries below the median, however, have a lower endowment, leading them to be less likely to participate. The country with the lowest endowment in a given agreement is thus less likely to be a member of the agreement after a median-preserving increase of endowment heterogeneity, leading agreement formation to be more difficult.

We begin the analysis of this subsection by looking at the way a shift in average income influences agreement formation. A shift towards higher income means that every country possesses a higher endowment. That leads every individual country to be more willing to participate. Thus, a shift towards higher income leads agreement formation to be easier.

**Proposition 4.** An increase in income for all countries leads agreement formation to be easier.

This has a positive effect on contributions and welfare. Consider a small change in endowment that does not lead to a change in agreement size. The larger endowment of agreement members as such increases contributions and leaves everybody better off. If more countries join the agreement formation process due to their higher endowments, contributions and welfare further increase.

We now analyze the effects of combining endowment and cost heterogeneity on the one hand, and endowment and valuation heterogeneity on the other. A relevant determinant is the correlation between the respective variables. Depending on the specific scenario, the correlation can be positive or negative. When considering the interaction of endowment and cost heterogeneity, a positive correlation means that countries with higher average costs have a higher endowment, while a negative correlation means that countries with higher average costs have a lower endowment. A country with a high endowment has higher average cost than a country with a low endowment if marginal cost is increasing, while it has lower average cost if marginal cost is decreasing. A positive correlation between endowment and costs is thus equivalent to increasing marginal costs, and a negative correlation is equivalent to decreasing marginal costs.
If endowments are equal across countries, we have seen that countries with higher costs is less likely to participate. If endowments are positively correlated with costs, a country with high costs is more willing to participate than if endowments are equal across countries. Hence, a positive correlation between costs and endowment has a positive effect on agreement formation (and a negative correlation has the opposite effect). Endowment and valuation heterogeneity interact in a similar manner. While a lower valuation in itself leads a country to be less willing to participate, a higher endowment can counter this effect. We show this formally in the appendix and summarize in the following proposition:

**Proposition 5.** An increase in the degree of endowment heterogeneity makes agreement formation less difficult if marginal costs are increasing in endowment or if valuation is decreasing in endowment. It makes agreement formation more difficult if marginal costs are decreasing in endowment or if valuation is increasing in endowment.

In order to analyze the welfare effects of an interaction between endowment and costs or endowment and valuation, we compare the case of additional endowment heterogeneity to the case where only costs or valuation are distributed heterogeneously, while endowment is homogeneous. Endowment heterogeneity has two effects; first, it influences private consumption utility; secondly, it impacts how much the members of an agreement provide. Assume, for instance, there exists an equilibrium agreement with \( S < N \) in a scenario with only cost heterogeneity. Adding endowment heterogeneity then has a negative effect on aggregate private consumption utility if the correlation between costs and endowments is negative. If the correlation is positive, it has a negative effect up to some number of countries being outside of the agreement. Above that number of outside countries, the effect becomes positive. In cases where aggregate contributions increase in agreement size while countries with high costs have a smaller endowment, welfare decreases. In cases where aggregate contributions increase in agreement size while countries with high costs have a larger endowment, the effect on welfare is unclear. The analysis for valuation heterogeneity proceeds along the same lines.

### 5 The Political Process

We have so far analyzed the way cost, valuation and endowment heterogeneity influence agreement formation, and we have looked at the way endowment heterogeneity interacts with cost or valuation heterogeneity. Now we turn to the question how heterogeneity over costs, valuation and endowment interact with the political process. We argue that endowment as a measure of economic power constitutes a valuable proxy for political power. That is, we see endowment as
a measure for a country’s relevance on the international stage: a large endowment country is likely to have a larger weight in international negotiations than a country with a small endowment. While potentially neglecting some other aspects that might cause differences in political weights, such as for instance historical relevance, we believe this approach reflects much of the political heterogeneity observed on the international stage.

Heterogeneity in political power might make agreement formation more or less difficult, depending on the correlation between weights and costs or valuation. We therefore consider in detail the way heterogeneity over political power interacts with heterogeneity over these underlying parameters. In order to consider the combined effect of cost or valuation heterogeneity and political power heterogeneity, we compare the utility of a high cost or low valuation country in the case of equally distributed voting weights to the utility it receives under weighted voting.

We first consider the case of cost heterogeneity. If weight is larger for countries with high cost, $t_{M_{\text{weighted}}} < t_M$. That is, $t_{M_{\text{weighted}}}$ is closer to a high cost country’s optimal $t$ than $t_M$. This induces a high cost country which participates in an agreement to be better off in the case with weighted voting than in the case without. If a country with high costs opts against participating, on the other hand, it causes the median to change more, leaving it worse off not participating than what it is in the unweighted voting case. Thus, a high cost country is more likely to be in an agreement under weighted voting than under unweighted voting. If weight is larger for countries with low costs, we have the opposite result; that is, the countries (with high costs) that have the lowest incentive to enter an agreement become less interested in participating.

We then turn to the case of valuation heterogeneity. If valuation and voting weights are positively correlated, that is, if countries with a higher valuation have larger political power, a low valuation country is worse off if it participates under weighted voting, but better off if it does not participate under weighted voting. If valuation and voting weights are negatively correlated, that is if countries with a higher valuation have lower political power, the country with the largest incentive not to participate is better off if it participates under weighted voting, but worse off if it does not participate under weighted voting compared to the unweighted case.

**Proposition 6.** Weighted voting makes agreement formation easier if countries with high costs or low valuation have greater political power, while it makes agreement formation more difficult if countries with low costs or high valuation have greater political weight.

If high cost countries have a larger political weight the fraction $t_M$ weakly
decreases, while agreement size weakly increases. Whether aggregate contributions increase or decrease in either of these cases depends on which of the effects is stronger: if the agreement size effect is stronger, aggregate contributions decrease when low cost countries have a larger weight, while they increase if high cost countries have a larger weight. In comparison to the case of evenly distributed political weights, aggregate utility received from agreement provisions then decreases when low cost countries have a larger weight, while it increases if high cost countries have a larger weight.

The private provision utility is larger than in the homogeneity case if high cost countries have a larger political weight, while it is lower if low cost countries have a larger political weight. We can thus say that, given the agreement size effect is stronger than the political effect, utility of countries that do not change membership status is decreasing if more weight is given to low cost countries, while it increases if more weight is given to high cost countries. For countries changing membership status, we can say that they are better off in the new status given that weights have changed. However, we can not directly compare to the homogeneity case. A clear statement on welfare is therefore dependent on the number of countries keeping and changing membership status as well as the size of their loss or gain in utility. The reasoning for the case of valuation heterogeneity proceeds along the same lines.

We consider now how the effect of an increase in heterogeneity depends on the distribution of voting weights. If voting weights are more concentrated on high cost countries, this has a mitigating effect on the impact of an increase in heterogeneity. On the other hand, voting weights being more concentrated on low cost countries has an aggravating effect on the impact of an increase in heterogeneity.

**Corollary 2.** If political weights are more concentrated on high cost countries, the negative effect of an increase in cost heterogeneity on agreement formation is less harsh than under equally distributed political weights. The negative impact of an increase in valuation heterogeneity is stronger if political weights are more concentrated on high valuation countries.

In order to illustrate this statement, we can consider the most extreme case of power concentration, that is, the existence of a dictator. If there exists a dictator in an agreement with high costs or low valuation, agreement formation becomes easier, while it becomes more difficult if a dictator has low costs or high valuation. Similarly, if there exists a high cost dictator in a given agreement, this has a mitigating effect on the negative impact from an increase in heterogeneity, while the existence of a high valuation dictator has an aggravating effect. Note, however, that these are the effects on agreement size. The effects on aggregate provision and welfare are ambiguous.
6 Conclusion

We show that if countries are homogeneous, there exists a unique number of countries that can form a stable agreement in equilibrium. If we allow for heterogeneity over costs or valuation, agreement formation becomes more difficult. We see that the country with the highest costs or the lowest valuation is the least likely to be willing to be a member of an agreement. The existence of an equilibrium in which an agreement forms thus depends on the degree of heterogeneity between countries. If heterogeneity is sufficiently large, agreement formation may become impossible.

Further, we look at the impact of endowment heterogeneity on agreement formation. We first consider pure endowment heterogeneity and then analyzed the way it interacts with cost or valuation heterogeneity. Endowment heterogeneity leads countries with a large endowment to be more willing to participate. The effect of an interaction between endowment heterogeneity and cost or valuation heterogeneity depends on whether high cost countries are associated with a high or a low endowment and, respectively, whether high valuation countries are associated with a high or a low endowment. We show that introducing endowment heterogeneity in addition to cost or valuation heterogeneity might make a large cost country more willing to participate if it leads to an increase in the large cost or low valuation country’s endowment.

Cost and valuation heterogeneity have a direct negative effect on agreement formation. Heterogeneity over costs or valuation causes countries to differ with respect to the fraction of endowment they are willing to contribute to a public good. Such differences in willingness to contribute lead agreement formation to be fairly difficult, especially if differences are substantial. Endowment heterogeneity, on the other hand, has a more indirect effect. It does not change countries direct willingness to contribute, but has an indirect effect which is based on the externalities caused by a country. A larger country causes larger positive externalities for other countries if it provides. If it does not provide, however, the aggregate amount provided decreases more than if a smaller country decided against participating. In that sense, a larger country can be seen as more decisive for the outcome of negotiations. That idea can be reflected through weighted voting.

We conduct a detailed analysis of heterogeneity over political power. We model political coordination through introducing an explicit voting procedure as a way to predict the outcome of negotiations, where political heterogeneity is introduced through different voting weights. We are able to show that the political process plays a crucial role in agreement formation. If a country with a large endowment has a larger political weight on the international stage, agreement outcomes are shifted in favor of high endowment countries. Introducing different
voting weights might lead a country with larger costs or lower valuation to be more likely to be in an agreement than under unweighted voting, given that it also has a larger endowment. Thus, if political power is positively correlated with costs or negatively correlated with valuation, agreement formation is less difficult under weighted voting than under unweighted voting.

The impact of an increase in heterogeneity depends largely on the specific political situation. Political weights that are more concentrated on high cost countries lead the impact of an increase in cost heterogeneity to be less harsh than under equally distributed political weights. Equivalently, political weights that are more concentrated on low cost countries lead the impact of an increase in cost heterogeneity to be harsher than under equally distributed political weights. In the case of valuation heterogeneity, the negative impact of an increase in heterogeneity is stronger if political weights are more concentrated on high or median valuation countries.

APPENDIX

Proposition 1: Homogeneity. The proposition follows from considering internal and external stability of an agreement. The members of an agreement provide $G_C = S t^* w$ where $S$ is the number of members and

$$t^* = \frac{a}{(1 + a)(1 + c)}$$

is the agreed upon fraction of endowment that is contributed. A country’s utility from being in an agreement is then

$$U_C = \ln \left( \frac{w}{n} - \frac{a}{1 + a} \frac{w}{n} \right) + a \ln \left( \frac{Sw}{(1 + a)(1 + c)} \right) + (B - K),$$

while being outside of an agreement yields utility

$$U_O = \ln \left( \frac{w}{n} \right) + a \ln \left( \frac{Sw}{(1 + a)(1 + c)} \right).$$

Comparing the utility a country in an agreement with $S$ members receives to the utility a country being outside of an $S - 1$ agreement receives, yields

$$\ln \left( 1 - \frac{a}{1 + a} \right) + a \ln(Sw) + (B - K) \geq a \ln((S - 1)w).$$

Comparing the utility a country outside of an agreement with $S$ members receives to that which a country participating in an agreement of $S + 1$ receives, yields

$$\ln \left( 1 - \frac{a}{1 + a} \right) + a \ln((S + 1)w) + (B - K) \leq a \ln(Sw).$$
We call the value of $S$ for which the first expression holds with equality $\bar{S}$, and the value of $S$ for which the second holds with equality $\underline{S}$. An agreement is internally stable for $S \leq \bar{S}$ and externally stable for $S \geq \underline{S}$. As the largest integer fulfilling $S \leq \bar{S}$ is at the same time the smallest integer fulfilling $S \geq \underline{S}$, the unique stable agreement size is defined by $\bar{S} \geq S^* \geq \underline{S}$.

For the case where $B - K = 0$ this can be expressed as

$$\frac{1}{1 - \left(\frac{1}{1+a}\right)^{\frac{1}{a}}} \geq S^* \geq \frac{1}{(1+a)^{\frac{1}{a}} - 1} \quad \text{for} \quad a \geq 4.$$ 

\[\square\]

**Proposition 2: Cost and valuation heterogeneity.** We first consider cost heterogeneity and then valuation heterogeneity.

1. **Cost heterogeneity**

Consider internal and external stability:

$$\ln(1 - \frac{1 + c_i}{1 + c_M} \frac{a}{1 + a}) + a \ln(\frac{a}{(1+c_M)(1+a)}) + a \ln((S - 1)w) + (B - K) \geq \ln(\frac{a}{(1+c_M)(1+a)}) + a \ln(Sw) + (B - K)$$

and

$$\ln(1 - \frac{1 + c_i}{1 + c_M} \frac{a}{1 + a}) + a \ln(\frac{a}{(1+c_M)(1+a)}) + a \ln((S + 1)w) + (B - K).$$

If heterogeneity on the low cost side increases, the distance between countries with respect to costs increases. That induces the political effect to be stronger: the difference between $t_M$ when the country with the highest costs in a conjectured agreement participates and $\hat{t}_M$ when it does not participate increases. That makes it more profitable for the country to not participate.

If heterogeneity on the high cost side increases, the difference between the median country and the country with the largest costs increases. Thus, $\frac{1 + c_i}{1 + c_M}$ increases, which leads the utility received from private consumption to decrease more due to
contributing. This makes a country more likely not to participate. Furthermore, the political effect remains unchanged, as the country most inclined to drop out of an agreement is found on the high-cost side, which means that if it was to change its mind the change in the median would be determined by the distance on the low-cost side.

In the case of a two-sided heterogeneity increase both effects work together.

2. Valuation heterogeneity

Consider internal and external stability:

\[
\ln \left(1 - \frac{a_M}{1 + a_M}\right) + a_i \ln \left(\frac{a_M}{(1+c)(1+a_M)}\right) + a_i \ln(Sw) + (B - K) \geq a_i \ln \left(\frac{a_M}{(1+c)(1+a_M)}\right) + a_i \ln((S - 1)w) \tag{3}
\]

and

\[
\ln \left(1 - \frac{a_M}{1 + a_M}\right) + a_i \ln \left(\frac{a_M}{(1+c)(1+a_M)}\right) + a_i \ln(Sw) \geq a_i \ln \left(\frac{a^*}{(1+c)(1+a_M)}\right) + a_i \ln((S + 1)w) + (B - K) \tag{4}
\]

An increase of heterogeneity on the right side of the median induces the difference between \(a^*_M\) and \(a_M\) to be larger. The political effect on agreement formation is therefore negative; the (low-valuation) member of the agreement who is most likely to change her mind will, after the increase in heterogeneity, induce a greater increase in the median voter decision of \(t\) by deviating (i.e. by not participating). As the relevant \(a_i\) stays unchanged, the overall effect is negative.

An increase of heterogeneity on the left side of the median leaves the difference between \(a^*_M\) and \(a_M\) unchanged and, therefore, has a neutral political effect. The relevant \(a_i\) is lower, however, and therefore the overall effect is negative.

A heterogeneity increase on both sides of the median induces a negative political effect and a lower relevant \(a_i\). The overall effect of an increase in heterogeneity is thus negative.
Proposition 3: Endowment heterogeneity. Consider internal and external stability:

\[
\ln(1 - \frac{a}{1 + a}) + a \ln(\sum_{j \in S} w_j) + (B - K) \geq a \ln(\sum_{j \in S-1} w_j)
\]

and

\[
a \ln(\sum_{j \in S} w_j) \geq \ln(1 - \frac{a}{1 + a}) + a \ln(\sum_{j \in S+1} w_j) + (B - K).
\]

The difference between the utility resulting from being in an agreement and not being in an agreement is larger for a country with a larger endowment. That is, a country with a larger endowment has a larger incentive to be part of the agreement.

Proposition 4: Combining cost or valuation heterogeneity and endowment heterogeneity.

1. Combining cost and endowment heterogeneity

Consider internal and external stability:

\[
\ln(1 - \frac{1 + c_i}{1 + c_M} \frac{a}{1 + a}) + a \ln(\frac{a}{(1+c_M)(1+a)}) + a \ln(\sum_{j \in S} w_j) + (B - K) \geq a \ln(\sum_{j \in S} w_j)
\]

and

\[
a \ln(\sum_{j \in S} w_j) \geq \ln(1 - \frac{1 + c_i}{1 + c_M} \frac{a}{1 + a}) + a \ln(\sum_{j \in S-1} w_j)
\]

Increasing marginal costs lead to higher average costs for a large endowment country. Decreasing marginal costs lead to lower average costs. If a country with high average costs has a high endowment it is more likely to participate than under equal distribution of endowments, while if it has a low endowment it is even less likely to participate than it would be under income homogeneity.
2. Combining valuation and endowment heterogeneity

Consider internal and external stability:

\[
\ln \left(1 - \frac{a_M}{1 + a_M} \right) + a_i \ln \left(\frac{a_M}{(1+c)(1+a_M)}\right) + a_i \ln \left(\sum_{j \in S} w_j\right) + (B - K) \geq \nln \left(\frac{\tilde{a}_M}{(1+c)(1+\tilde{a}_M)}\right) + a_i \ln \left(\sum_{j \in S-1} w_j\right)
\]

and

\[
\ln \left(1 - \frac{\tilde{a}_M}{1 + \tilde{a}_M} \right) + a_i \ln \left(\frac{\tilde{a}_M}{(1+c)(1+\tilde{a}_M)}\right) + a_i \ln \left(\sum_{j \in S+1} w_j\right) + (B - K)
\]

In any given agreement, the country with the lowest valuation is the least likely to participate. A larger endowment leads that country to be more willing to participate, while a smaller endowment leads that country to be less likely to participate. That is, an increase in endowment heterogeneity makes agreement formation easier or harder, depending on the direction of correlation between endowment and valuation.

\[\Box\]

**Proposition 5: Cost or valuation heterogeneity and weighted voting.** We first consider cost heterogeneity and then valuation heterogeneity.

1. Cost heterogeneity and weighted voting

We first consider the case where weight is larger for countries with large costs and then the case where weight is larger for countries with low costs.

a. Weight is larger for countries with large costs: Assume there exists some agreement $S$. It holds that $t_{M_{\text{weighted}}} < t_M$. For countries with $t^* \geq t_{M_{\text{weighted}}}$ not participating would yield a lower utility than participating. Thus, these countries are more likely to be members of an agreement than to be non-members. We can therefore focus on countries with $t^* < t_{M_{\text{weighted}}}$. As $t_{M_{\text{weighted}}} < t_M$, $t_{M_{\text{weighted}}}$ is closer to these countries’ optimal $t$ than $t_M$. Thus, a high cost country which participates in an agreement is better off in the case with weighted voting than
in the case without. At the same time, the political effect is stronger if a high weight country does not participate, that is, it causes the median to change more if it decides against participating. A country with a larger weight is thus worse off not participating than in the unweighted voting case, while it is better off participating than in the unweighted voting case. Therefore, a large cost country is more likely to be part of an agreement than under unweighted voting.

b. Weight is larger for countries with small costs: Assume there exists some agreement $S$. It holds that $t_{M_{weighted}} > t_M$. As above, for countries with a $t^* \geq t_{M_{weighted}}$ not participating would yield a lower utility than participating. Thus, these countries are more likely to be members of an agreement than to be non-members. We can therefore again focus on countries with $t^* < t_{M_{weighted}}$. As $t_{M_{weighted}} > t_M$, $t_{M_{weighted}}$ is further away from these countries’ optimal $t$. That makes a large cost country worse off in case it participates under weighted voting than what it would be under unweighted voting. If it does not participate, on the other hand, it would be better off than under unweighted voting because it induces a lower increase in the median $t$ through not participating.

2. Valuation heterogeneity and weighted voting

We first consider the case where weight is larger for countries with low valuation and then the case where weight is larger for countries with high valuation.

a. Weight is larger for countries with low valuation: Assume there exists some agreement $S$. It holds that $t_{M_{weighted}} < t_M$. For countries with a $t^* \geq t_{M_{weighted}}$ not participating would yield a lower utility than participating. Thus, these countries are more likely to be members of an agreement than to be non-members. We can therefore focus on countries with $t^* < t_{M_{weighted}}$. As $t_{M_{weighted}} < t_M$, $t_{M_{weighted}}$ is closer to these countries’ optimal $t$ than $t_M$. Thus, a low valuation country which is part of an agreement is better off in the case with weighted voting than in the case without. At the same time, the political effect is stronger if a high weight country does not participate, that is, it causes the median to change more if it decides against participating. A country with a larger weight is thus worse off not participating than in the unweighted voting case, while it is better off participating than in the unweighted voting case. Therefore, a low valuation country is more likely to be in an agreement than under unweighted voting.
b. Weight is larger for countries with high valuation: Assume there exists some agreement $S$. It holds that $t_{M_{\text{weighted}}} > t_M$. As above, for countries with a $t^* \geq t_{M_{\text{weighted}}}$ not participating would yield a lower utility than participating. Thus, these countries are more likely to be members of an agreement than to be non-members. We can therefore again focus on countries with $t^* < t_{M_{\text{weighted}}}$. As $t_{M_{\text{weighted}}} > t_M$, $t_{M_{\text{weighted}}}$ is further away from these countries’ optimal $t$. That makes a low valuation country worse off in case it participates under weighted voting than what it would be under unweighted voting. If it does not participate, on the other hand, it would be better off than under unweighted voting because it induces a lower increase in the median $t$ through not participating. 

References


