The Natural Rate and its Usefulness for Monetary Policy Making

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Wicksell characterizes the natural rate as the real interest rate that yields price stability and would equate real saving and investment in an (otherwise equivalent) nonmonetary Walrasian economy.

> *There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them. This is necessarily the same as the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods* (Knut Wicksell- Interest and Prices, 1898 p .102)

In identifying the price stabilizing rate with the real general equilibrium rate Wicksell gets close to describing what has been labeled the divine coincidence of the benchmark New Keynesian Model (Woodford, 2003; Blanchard and Gali, 2007). In this context, the notion of a natural rate of interest is unambiguous and most clearly meaningful. Therefore, for motivation and as a point of comparison with the richer model used later in the quantitative analysis, we begin by describing the natural rate in the canonical New Keynesian (NK, hereafter) model without wage stickiness and capital accumulation as the real interest rate prevailing in an economy with flexible prices.

Following the derivations of the canonical NK model in Gali (2008), we characterize the expressions for the marginal cost, the Euler equation for consumption and the Phillips Curve in terms of the gaps between output and the real interest rate from their natural counterparts. Substituting the consumer's static labor supply condition $w_t - p_t = (s^{-1}y_t + \varphi n_t)$ into the usual Cobb Douglas specification of log real marginal cost, mc_t and rearranging, mc_t can be expressed in terms of (log) output y_t and technology a_t as $\left(s^{-1} + \frac{\varphi+\alpha}{1-\alpha}\right) y_t - \frac{1+\varphi}{1-\alpha} a_t - \log(1-\alpha)$.[1] Setting this expression equal to the inverse desired markup $-\mu$ (presumed constant over time) and solving for the associated flexible price level of output y_t^n, we characterize natural output as $y_t^n = \psi_{ya}^n a_t + \vartheta_y^n$, where $\psi_{ya}^n \equiv \frac{1+\varphi}{s^{-1}(1-\alpha)+\varphi+\alpha}$ and ϑ_y^n is (for our purposes) an inessential constant.

The consumer Euler equation with $c_t = y_t$, including a time-varying second-order precautionary saving term associated with the conditional variance, $Var_t\left[y_{t+1}\right]$, is $y_t = E_t\left[y_{t+1}\right] - s\left(i_t - E_t\left[\pi_{t+1}\right] - \rho_t\right) - \frac{1}{2}s^{-1}Var_t\left[y_{t+1}\right]$. The *natural rate of interest* r_t^n must satisfy $E_t\left[\Delta y_{t+1}^n\right] = s\left(r_t^n - \rho_t\right) + \frac{1}{2}s^{-1}Var_t\left[y_{t+1}^n\right]$; where ρ_t is the (possibly time-varying) subjective rate of time preference. Simple algebraic manipulations lead to $r_t^n = \rho_t + s^{-1}E_t\left(\Delta y_{t+1}^n\right) - \frac{1}{2}s^{-2}Var_t\left[y_{t+1}^n\right]$. Substituting in for the growth of natural output one can write the following definition for the natural rate r_t^n:

$$\begin{aligned} r_t^n &= \rho_t + s^{-1}\psi_{ya}^n E_t\left[\Delta a_{t+1}\right] \\ &\quad -\frac{1}{2}s^{-2}\left(\psi_{ya}^n\right)^2 Var_t\left[a_{t+1}\right]. \end{aligned} \tag{1}$$

If we write the Euler equation in terms of the output gap, $\tilde{y}_t \equiv y_t - y_t^n$, i.e. the difference between actual and natural output, and impose $\lim_{T\to\infty} E_t\left[\tilde{y}_{t+T}\right] = 0$ so that the gap-creating

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[1] We follow Gali's (2008) notation, with n and φ denoting labor and the inverse Frisch elasticity, respectively, but depart from him by using s for the intertemporal elasticity of substitution. The loglinear aggregate production function reads $y_t = a_t + (1-\alpha)\, n_t$.

consequences of sticky prices do not last forever, we can also see that

$$\tilde{y}_t = -s \sum_{k=0}^{\infty} E_t \left(r_{t+k} - r_{t+k}^n \right) . \quad (2)$$

The last expression makes evident that the output gap is the sum of all future real interest rate gaps, defined as the deviations of the ex-ante real rate, $i_t - E_t \pi_{t+1}$, from the natural rate, r_t^n. Finally, from the NK Phillips curve, $\pi_t = \beta E_t \left[\pi_{t+1} \right] + k \tilde{y}_t$, closing the output gap $\tilde{y}_t$ also stabilizes inflation. Thus we have shown:

- The natural rate is increasing in the subjective rate of time preference, ρ_t, and the expected growth rate of technology, $E_t \left[\Delta a_{t+1} \right]$, and decreasing in the conditional variance of future technology, $Var_t \left[a_{t+1} \right]$. Increases in patience, i.e. declines in ρ_t, (often labelled discount factor or "beta" shocks) lower r^n, as does a reduction in expected productivity growth and higher uncertainty about future productivity (due to an increase in precautionary savings, or equivalently the increased attractiveness of a safe asset).

- An interest rate path in which the actual real rate is always equal to the natural rate achieves both an output gap of zero (in the sense that output is at natural, i.e. flexible price equilibrium level) and zero inflation.

Equation (1) shows that an uncertainty shock is isomorphic to the discount shock and indeed may provide one attractive structural interpretation of that rather reduced-form construct. Although movements in r_t^n due to realistic *aggregate* technological uncertainty is rather small for reasonable calibrations, it can be far larger if heterogeneous agents face *idiosyncratic* shocks (Aiyagari and Gertler 1991 and Huggett 1993). Thus in a richer economy uncertainty shocks can have a significant depressing effect on the natural rate (Guerrieri and Lorenzoni 2011).[2]

I. A State-of-the-Art DSGE Model

Though the canonical New Keynesian model of the last section provides motivation and intuition for the natural rate and its determinants, it is far too stylized to be taken directly to the data. For that purpose, we build on the well-known framework by Smets and Wouters (2007), which has been shown to fit the data well. Compared with the stylized model of the last section, the Smets and Wouters' model includes price and wage stickiness, backward-looking components in wage and price setting, habit formation, non-separable utility in consumption and leisure as well as investment subject to adjustment costs. In addition to stationary variations in the level of technology, it is buffeted by shocks to the marginal efficiency of investment and stochastic variations in wage and price markups.

There is an additional disturbance that Smets and Wouters call a "risk shock". As described below, in our model risk shocks play a prominent role in explaining business cycle fluctuations and a major role in triggering the Great Recession. Although this shock is of course not identical to the uncertainty shock in the stylized model (indeed, we log-linearize, thereby removing the role of risk) – it is analogous on the consumer side, in that it lowers the required return to saving and reduces consumption. The "risk shock", however, is not simply isomorphic to a "beta shock" because it acts as a wedge and thus does not imply that the reduction in consumption in recessions gets channelled into investment.

We introduce some important departures in specification from Smets and Wouters (2007), which are now described, together with the empirical rationale for their inclusion and the data brought to inform them. The reader is referred to the Online Appendix for additional details. Beginning with the interest rate rule, policymakers are assumed to respond to four quarter averages of current, expected and two lags of inflation, and the deviation of GDP from the model's linear trend.[3] Second, based on the evidence pre-

[2] At least two other disturbances that could provide an underlying foundation for discount shocks come to mind Eggertsson and Krugman (2012) sketch a two agent economy in which a loss of net worth forces deleveraging and hence reduced consumption on the part of the borrower group, requiring a fall in the real interest rate to induce a compensating increase in the consumption of the lenders. But even when agents purchase durables with their own funds, efforts to reduce the durables stock in response to downward revisions of permanent income also lower the natural rate much in the fashion of a beta shock (Hall 2011).

[3] Smets and Wouters assume instead responses to quarter over quarter inflation and deviations of output –unadjusted for utilization– from the model-based natural level of output, as well

sented in Gurkaynak, Sack and Swanson (2005), we incorporate Forward Guidance (henceforth, FG) regarding the future conduct of monetary policy. Following the methodology in Campbell et al. (2012) and Campbell, Fisher and Justiniano (2012), agents receive news regarding the future paths of the federal funds rate, governed by two latent variables referred to as the *Target* and *Path* factors.[4] FG is informed with market-based expectations of the fed funds rate obtained from Fed Funds, Eurodollar and OIS futures contracts. Hence, the model accounts for agents' evolving expectations regarding the duration of the zero lower bound (ZLB) since the Great Recession.

Third, we introduce a slow moving inflation drift in the policy rule. This primarily accounts for the stability of long-run expected inflation since 1997, but will also capture, in reduced form, the effects –if any–of unmodeled uncoventional policy actions on agents' inflation expectations. The drift is informed by matching model-based average expected inflation over the next 40 quarters with median 10 year expected inflation from the Survey of Professional Forecasters.

Fourth and finally, instead of matching the model's concepts of price and wage inflation with a single series for each, we rely on multiple indicators. This approach diminishes the importance of markup shocks for cyclical fluctuations, as shown by Boivin and Giannoni (2006) for the case of goods prices, and by Justiniano, Primiceri and Tambalotti (2013) for wages. For this reason, our estimate of the natural rate is quite robust to the interpretation of these disturbances as efficient or inefficient (Section IV).

The estimation sample is 1990q1-2013q2, allowing for a break in all parameters in 2008q3, and centering the prior for the admittedly short second subsample at the first subsample estimates.[5] To the seven observables used in Smets and Wouters we add two price and one wage series, long-run expected inflation, as well as 4 and 10 quarters of market-based federal funds rate expectations in the first and second subsamples, respectively.

Defining the Natural Rate in the Richer Economy.

Unlike the canonical model in the first part of the paper, a richer economy, which is subject to *inefficient supply shocks* (e.g., shocks to markup or other cost-push shocks), does not appear to have a unique, unambiguous definition of the natural interest rate (or output). One might define the natural rate as the rate that would prevail if both wages and prices were perfectly flexible (and, as Woodford 2003 stresses, expected to be so henceforth). However, if "cost-push" –also known as markup– shocks create inefficiencies, the associated flexible wage and price equilibrium would not be "welfare relevant" (Wooodford, 2003). Thus, in our empirical model we choose to define the natural real interest rate and level of output, as those that would have prevailed in an economy with neither nominal rigidities nor shocks to price and wage markups. As such, we could have referred to these natural levels as (2nd best) "efficient".[6]

II. The Natural Rate Is Volatile and Procyclical

Figure 1 presents the filtered (one-sided) and smoothed (two-sided) estimates of the natural rate (on a quarterly basis), which follows a highly procyclical pattern characterized by fairly pronounced swings. Perhaps surprisingly, we do not observe a substantially larger drop during the Great Recession than in the previous two downturns. However, in stark contrast with earlier recessions, it has remained persistently negative since 2008. This last finding is mainly explained by the highly persistent negative risk shock that according to our model triggered the Great Recession and is responsible for the ensuing slow

as the change in this gap.

[4]The target factor is the only common component correlated with changes in the current federal funds rate. Therefore, the path factor is more intepretable as the forward guidance component of monetary policy.

[5]More precisely, we first estimate through 2008q3 using a prior almost identical to Smets and Wouters. This mode becomes the center of the prior for the second subsample, with the initialization of the filter based on the mean and variance of the state in 2008q3. The beginning of the sample is determined by the availability of fed funds futures data.

[6]As noted by Blanchard and Gali (2007), this is not the fully efficient "first best" equilibrium because of the steady state markup associated with monopolistic competition, but that distortion is not under the control of the monetary authority, and would require a fiscal solution.

recovery.

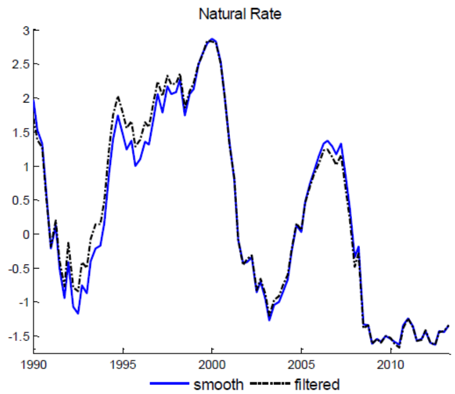

Figure 1: One-sided (filtered) and two-sided (smoothed) estimate of the natural rate.

III. Stabilization Properties of the Wicksellian Policy

As discussed in the introductory part of the paper, tracking the natural rate would accomplish full macroeconomic stabilization in models where the divine coincidence holds. Unfortunately, even in the absence of inefficient markup shocks, this policy does not necessarily deliver such a desirable outcome in the presence of both price and wage rigidities (Woodford, 2003 p. 443, Gali 2008, Chapter 6). Additional trade-offs arise from the presence of real inefficiencies –as we have interpreted markups–which do not affect the (second best) efficient economy and hence the natural rate of interest. In section IV we gauge the robustness of our results to the alternative interpretation of markups.

Even if setting the nominal interest rate to target the natural rate is not guaranteed to achieve full stabilization of inflation and the output gap, according to our model pursuing this policy would have considerably diminished the volatility of these variables in the last twenty-five years, including the Great Recession. Therefore, despite departures from the divine coincidence - and abstracting from the implementation issues that we address in section IV - the natural rate of interest still seems to be a useful summary statistic for poliycmakers.

In what follows we compare macroeconomic outcomes depending on the assumptions regarding the conduct of monetary policy. The solid line in the top panel of Figure 2 shows the inferred output gap –defined as the difference between actual and the natural level of output— under the estimated interest rate rule. The dashed line captures instead the counterfactual output gap that would have arisen under a *quasi Wicksellian rule* of the form

$$i_t = r_t^n + \phi_\pi E_t \pi_{t+1} \tag{3}$$

that is, had the Federal Reserve tracked the natural rate period by period.[7] The counterfactual economy is characterized by a considerable reduction in the output gap, even during the Great Recession, as well as by remarkably reduced variability thereof. The bottom panel shows the *inefficiency gap* (also known as the labor wedge), which following Gali, Gertler and Lopez-Salido (2007), is defined as the wedge between the marginal rate of substitution between consumption and leisure and the marginal productivity of labor. This turns out to be essentially the reciprocal of the output gap and once again it is drastically reduced under rule (3).

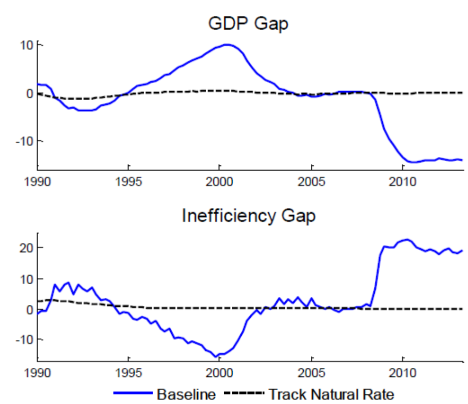

Figure 2: Output and Inefficienct Gaps under estimated interest rule and when track the natural rate.

As previously mentioned, in principle closing the output or inefficiency gap could bring about significant costs in terms of the stabilization of price and wage inflation. However, in our model

[7] We use the term *quasi Wicksellian* since in our model such a rule need not stabilize the price level, and, relatedly, to differentiate it with Giannoni's (2012) Wicksellian rules that explicitly respond to deviations of the price level from trend. We feed the smoothed sequence of all shocks (including markups) and replace the estimated interest rule with (3), setting $\phi_\pi = 1.0001$.

this is not the case. Indeed, the ratios of observed (nominal) wage inflation variance to the counterfactual variance had policy tracked the natural rate are 3.7 and 23.3 in the first and second subsamples respectively, suggesting that a considerable degree of wage inflation stabilization can also be achieved by following the rule (3). The fall in the corresponding ratios for price inflation is more moderate at 1.9 and 1.7.

Overall, our results echo the findings in Justiniano, Primiceri and Tambalotti (2013), and as in their case, rest on the predominance of demand fluctuations, such as the risk shock, which move price and quantities in same direction, and on the smaller contribution of price and wage markup shocks to fluctuations in economic activity. These authors further characterize optimal policy–a fairly involved task in our model–and show that it essentially involves closing the output gap, as we have defined it here. Our findings suggest in turn that tracking the natural rate would stabilize this output gap as well as inflation in prices and wages. However, characterizing the distance of this prescription from optimal monetary policy –and particularly the extent to which it implies a larger degree of inflation stabilization– is beyond the scope of this short paper.

We conclude by addressing the desirability of *history dependence* by augmenting the quasi Wicksellian rule with a term corresponding to the previous period's output gap,

$$i_t = r_t^n + E_t \pi_{t+1} + \phi_x x_{t-1} \tag{4}$$

History dependence is a common theme in the optimal design of monetary policy rules in the presence of inefficient disturbances (Woodford, 2003, Chapter 7). We obtain that the inertial rule above would allow policymakers to attain even further reductions in the variability of both output and inefficiency gaps, at the expense of no discernible increase in the variability of wage and price inflation, for values of ϕ_x ranging between 0.1 and 0.3.

IV. Implementability

In Section III, we showed that a Generalized Wicksellian policy rule would accomplish substantial macroeconomic stabilization compared to our empirical Taylor-type reaction function. But, can such a policy prescription be implemented in practice?

A natural question is the extent to which the ZLB on nominal interest rates represents a constraint. Figure 3 sheds light on this issue by presenting the actual path of the federal funds rate (on a quarterly basis) together with the counterfactual rate had monetary policy tracked the natural rate. The ZLB would have bound our quasi Wicksellian policy rule in the three recessions of the sample, questioning the feasibility and the stabilizing properties of this rule in practice.

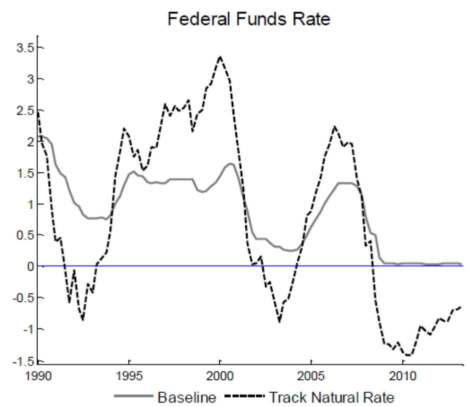

Figure 3: Actual and counterfactual Federal Funds Rate (quarterly basis).

The pervasiveness of the ZLB in the last twenty-five years requires the central bank to be able to raise short-term inflation expectations in recessions so as to attain a negative *real* rate as required by the quasi Wicksellian rule. Following Eggertsson and Woodford (2003), this can be achieved, for instance, through a commitment by the central bank to raising the nominal interest rate slowly at the time the natural rate will become positive in order to affect expectations already when the ZLB is binding.[8] A way to implement the policy suggested by Eggertsson and Woodford is for the central bank to make explicit statements, often called *forward guidance*,

[8] Krugman (1998) and Werning (2012) have also advocated the use of this type of policy to provide more accommodation when the policy rate is stuck at its zero bound. Bianchi and Melosi (2013) show that committing to systematically inflating away the portion of public debt accumulated during severe economic downturns would also be a powerful device to raise short-term inflation expectations.

about the future path of short-term interest rates. Indeed, our model suggests that Forward Guidance contributed considerably to real activity in 2003-04 as well as since the more explicit language adopted by the FOMC in August 2011.[9]

Another concern for implementability is the availability of real time estimates of this latent variable as well as the seemingly implausible requirement of discerning efficient from inefficient fluctuations.[10] On the first issue, note that one-sided and two-sided estimates of the natural rate are reasonably close (Figure 1). On the second, it is interesting that overall the contours of the natural rate as well as the stabilization properties of (3) for the output gap are quite robust to assuming all markups are not efficient (baseline) or the polar opposite case in which they are treated as efficient fluctuations, and therefore included in the definition of the natural rate of interest and output.[11] As mentioned earlier, this robustness property stems from the diminished importance of these shocks –compared to models without multiple indicators and idiosyncractic disturbances– in explaining real fluctuations.

Finally, in this short article we do not directly address other important issues such as the appropriate characterization of the uncertainty surrounding our estimates of the natural rate, including its sensitivity to alternative specifications of the model.

V. Concluding Remarks

A fairly rich DSGE model indicates that since 1990 the natural real rate of interest, defined as the real rate of an efficient economy with neither nominal stickiness nor cost-push distortions has been quite variable and highly procyclical. The natural rate turned negative in the last three recessions and has remained persistently depressed since 2008. We find that the natural rate could be a useful summary statistic for the Federal Reserve in so far as policy designed to track it would significant stabilize the output and inefficient gaps while also decreasing the variability of price and wage inflation. Nevertheless, the recurrently binding zero lower bound, and the difficulty of computing the natural rate in real time pose non-trivial challenges for adopting the natural interest rate as an implementable target of monetary policy**.**

[9] According to our estimates forward guidance (given by the *path factor*) contributed almost a full percentage point to quarterly GDP growth (on a quartertly basis) in 2003 and since 2011. Conversely, it dragged GDP by roughly 1 percent at the trough of the Great Recession.

[10] As noted by Woodford (2003, p.454), it is often difficult quantitatively to tell whether a particular real disturbance distorts the economy towards inefficiency or simply affects the efficient level of output and interest rates.

[11] More precisely, when treating price markup shocks as efficient, the natural rate exhibits the same cyclical pattern, but is slightly more volatile, particularly during times of large increases in oil prices, e.g. 2006. Our quasi Wicksellian rule continues to almost fully stabilize the output gap, somewhat less so the inefficiency gap, and achieves a bigger reduction in the variability of price and wage inflation. Wage markup shocks leads to negligible differences with our baseline.

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