Bequest motives in a life-cycle model with intergenerational interactions *

Loretti I. Dobrescu†, Fedor Iskhakov‡

PRELIMINARY AND INCOMPLETE
December 31st, 2013

Abstract

This paper revisits the debate on altruism vs exchange motive for bequest and studies to what extent the saving behavior of elderly Europeans is different in the presence or absence of adult children. To this purpose, we develop a structural model with two overlapping generations, namely elderly parents and their adult children. For each generation, we formulate a separate lifecycle model in which individuals consume and save. Children care about their elderly parents’ health and may choose to support them via money transfers and time assistance. Parents, on the other hand, must cover the health costs resulting from heterogeneous health and medical spending shocks, and they can do so via formal insurance (purchased beforehand), informal insurance (provided through time and money transfers by their children) and out-of-pocket. We join the two lifecycle models in a dynamic game between parents and children, which we show has a unique Markov perfect equilibrium. We estimate the model on SHARE data using the simulated method of moments. Preliminary results show a significant bequest motive for savings in Europe. In an altruistic world, children have a considerable incentive to provide both time and financial help, and this greatly impacts the parents’ savings behavior. Moreover, health, medical spending and health insurance also appear to be crucial in determining the old age saving patterns. Finally, counterfactual experiments show that considering only the strategic motive for bequest cannot explain the slow wealth decumulation in old age or the children’s transfers patterns.

Key Words: bequest, health, health insurance, intergenerational transfers, family interactions, dynamic game, method of moments.

JEL Classification: D1, D31, E27, H31, H51, I1.

*The authors are especially grateful to Orazio Attanasio, Chris Carroll, Maria Cristina De Nardi, Eric French, Michalis Halissos, John Jones, Alberto Motta, Makoto Nakajima, Jose-Victor Rios-Rull, John Rust, Xiaodong Fan, and participants at the 2013 Australasian Meeting of the Econometric Society, 2013 Annual NBER Summer Institute Workshop on Aggregate Implications of Microeconomic Behavior for helpful comments and suggestions. Research support from the Australian Research Council Centre of Excellence in Population Ageing Research (CEPAR), under project CE110001029 is gratefully acknowledged.

†CEPAR and School of Economics, UNSW, Sydney 2052, Australia. dobrescu@unsw.edu.au
‡CEPAR, UNSW, Sydney 2052, Australia. f.iskhakov@unsw.edu.au
1 Introduction

As the world grows old, the question of why do the elderly dissave so slowly is becoming increasingly important. Do they keep wealth to support themselves in retirement or to leave bequests to their children? The economic literature has investigated both these precautionary savings and bequest explanations extensively (Kotlikoff and Summers, 1981; Bernheim et al., 1985; Dynan et al., 2002). Among the sources of risk that induce the elderly to engage in precautionary savings, health and medical spending have long been recognized as two of the most significant (Hubbard et al., 1994, 1995; Palumbo, 1999; Dynan et al., 2004; De Nardi et al., 2010). Recently, the availability of health insurance, both formal and informal, also proved to have a sizeable effect on wealth decumulation in old age (Guariglia and Rossi, 2004; Dobrescu, 2012).

In terms of bequests, the key issue of whether they result from a deliberate motive or from unpredictable death has not yet garnered an univoqual answer (see Hurd, 1987, 1989; Hendricks, 2002; Kopczuk and Lupton, 2007; Laitner and Sonnega, 2012; Lokwood, 2012). And if people do save to leave bequests, the question is why. They could be motivated by pure altruism, e.g., parents care about the well-being of their grown up children (Becker, 1974; Barro, 1974; Laitner, 1992; 2001). Or there could be a strategic (or exchange) motive where parent-to-child emotional and social ties favor non-market exchanges that may generate bequests, e.g., bequests may emerge as payments to heirs for personal services rendered (Bernheim et al., 1985; Kotlikoff and Spivak, 1981). Finally, another alternative is the “warm glow” motive, where parents derive pleasure from making transfers to their adult children, but that pleasure is not specifically dependent upon the children’s utility gain (Blinder, 1974).

The current paper revisits the ‘altruism vs. exchange motive for bequest’ and studies to what extent the saving behavior of elderly Europeans (and relatedly their bequest motive) is different in the presence or absence of adult children. To this purpose, we develop a structural model with two overlapping generations, namely elderly parents and

---

1 For instance, Hurd (1987) finds that people with children decumulate their wealth faster than people without children, while Hurd (1989) finds the bequest motive to be economically trivial. On the contrary, Kopczuk and Lupton (2007) find that 79 percent of households with children have a bequest motive compared with 63 percent of childless households.

2 We note that individuals with adult children, unlike those with underage children, do not have generally higher expenses than the childless, but they do have a potential bequest motive in their saving behavior.
their adult children. In our life cycle model, children care about the health of their parents (Johnson and Lo Sasso, 2000; Bonsang, 2007). As a result, they consume and save, but may also choose to support their parents via money transfers or time spent providing care to them. Parents, on the other hand, consume, save and also have to cover the health costs resulting from heterogeneous health and medical spending shocks. There are three options to fund health costs: i) formal insurance that has to be purchased beforehand, ii) informal insurance provided through time and money transfers by their children, and iii) out-of-pocket. We also include a “warm-glow”-type of parental altruism to study the extent to which such a bequest motive can explain the slow dissaving observed in old age. Finally, we join the two models in a dynamic game between parents and children.

This is obviously not the first attempt to construct an overlapping generations model with bequests and intergenerational transfers (Laitner, 1992, 2001; De Nardi, 2004; Fuster et al., 2002; Nishiyama, 2002). Up to our best knowledge, however, this is the first study to develop a dynamic game between parents and children, in which the interaction involves intergenerational transfers of both time and money and the size of bequeathable wealth. Thus, this paper also contributes to the extensive literature on inter vivos transfers and on their impact on savings (Kotlikoff, 1988; Gale and Scholz, 1994; Rendall and Bahchieva, 1998) and bequests (Brown, 2006; 2007).³

We estimate the model on data from the Survey of Health, Ageing and Retirement in Europe (SHARE), using the simulated method of moments (SMM) for three European regions: Scandinavia - Denmark and Sweden, Central Europe - Austria, Belgium, France, Germany, Netherlands, Switzerland, and the Mediterranean - Italy, Spain and Greece.⁴ This country classification corresponds to several interesting patterns observed in the SHARE data. Specifically, there seems to be a very clear North-South gradient in formal and informal care provision: formal insurance appears more prevalent in Denmark and Sweden, whereas the Mediterraneans rely more on informal arrangements. When looking more closely at informal care, time transfers appear to be much more common than financial transfers, with higher levels of time assistance in Italy, Spain and Greece.⁵

³Rendall and Bahchieva (1998) for instance argue that without transfers from children (and relatives or friends), the U.S. elderly poverty rate would be double. In return, parents are more likely to transfer their bequest to children who provide them with regular care (Brown, 2006).
⁴See Gullestad and Segalen (1997) for institutional differences between the three regions.
⁵In SHARE, 32.8 percent of the single individuals aged 65 and above receive help in the form of time,
Preliminary results from our structural model show a significant bequest motive for savings in Europe. In an altruistic world, children have a considerable incentive to provide both time and financial help, and this greatly impacts the parents’ savings behavior. Moreover, health, medical spending and health insurance also appear to be crucial in determining the old age saving patterns. Finally, counterfactual experiments show that considering only the strategic motive for bequest cannot explain the slow wealth decumulation in old age or the children’s transfers patterns.

*Final results (and more findings) to be provided once the simulations have converged.*

From a policy perspective, the reason why the elderly dissave so slowly and how they use their wealth will determine not only their own well-being, but will also affect the living standards of their children, the resources available and the level of investment capital in the economy. In order to predict the impact of policy changes on future wealth accumulation, it is important to understand not only the disposition of wealth, but also why and how people save (or dissave) in the first place. In this sense, a model that is capable of explaining the choices of European elderly can significantly improve our understanding and design of reforming policies.

The remainder of the paper is organized as follows: Section 2 gives an overview of the health care systems and their funding in Europe. Section 3 develops the dynamic model, and Section 4 describes the data. Section 5 presents the estimation method, using the SMM methods. Results are illustrated in Section 6, and experiments are conducted in Section 7. Section 8 concludes.

## 2 Some European facts

Health care arrangements vary enormously across Europe, providing an ideal setting to study the effect of institutional and cultural differences across countries. Table 1 provides selected statistics related to the prevalence of formal versus informal, and public versus private health care funding across the 11 European countries in SHARE.

A quick glance reveals that the public policy in these countries is based on the principle of health care funded by the state or by social insurance, made available to all individuals which is almost seven times higher than the prevalence of financial help.
and covering most of the major health shock. There are three types of public health-care systems, following a north-south gradient: i) national health services in Scandinavia (Denmark and Sweden), ii) social-insurance systems in Central Europe (Austria, Belgium, France, Germany, the Netherlands and Switzerland), and iii) a mixed type of systems that can be seen as ‘third way’ (Freeman, 2000), established in the early 1980s in the Mediterranean (Italy, Spain and Greece).

The common feature of these systems is that they provide almost universal health care, across two dimensions. On the one hand, being financed through taxation or contributions from employers and employees, participation in the public system is usually mandatory. On the other hand, these systems cover all severe (life-threatening) medical conditions, offering comprehensive benefits that account for more than 70 percent of the total health care expenditure.

The existence of near universal public coverage in Europe reduces the basic need for additional insurance. However, the exclusion of certain health services from the statutory coverage, like specialist or diagnostic outpatient services, drugs, dental care, medical appliances, glasses, alternative medicine, occasional choice of better or faster inpatient care for important interventions (Paccagnella et al., 2013) has led to the development of a private health insurance market. As a result, 32.7% of the roughly 3,800 elderly single individuals represented in SHARE Wave 1 hold private health insurance (as shown in Table 2). This contract can be offered as a short-term or as a long-term arrangement, with premiums almost always set as a flat rate per month (or year) and used to finance health care costs. The norm for private health insurance in the European Union is short-term contracts (typically annual), with roughly €700 premiums on average in our sample. The lowest prevalence of private health insurance is registered in the Mediterranean countries and Sweden where less than 6.5% of respondents hold health insurance. In Sweden however the small size of the private insurance market is traditionally attributed to the generosity of the public system: health care is predominantly financed through national

---

6 Around 99 percent of the European population is covered by these schemes. The exceptions are Germany and the Netherlands, where people with income above a certain threshold have to be privately insured.

7 The exceptions are Switzerland that has a private health care system, compulsory for everyone, and Greece that has a mix of national health, social and private insurance system.
and local general taxation\textsuperscript{8} that cover roughly 82% of the total health expenditure.

Despite the relatively high prevalence of formal health insurance, the benefits paid accounted for less than 5% of the total expenditure on health in most countries (Dobrescu, 2012). Thus, private expenditure is largely generated by out-of-pocket payments (as seen in Table 2), with above 70% of the elderly in our sample having annual median out-of-pocket expenses of roughly €600.

In recent years, several empirical studies have documented the strong correlation between the amount of care provided via the formal schemes and the care supplied informally by close relatives or neighbors. For instance, Bolin et al. (2008) finds that informal and formal home care are substitutes, while informal care is a complement to doctor and hospital visits. These relationships also differ according to a European north-south gradient: compared to those residing in Italy, Spain or Greece, the negative effect of informal care on formal home care is significantly lower for Central Europeans and absent for Danish and Swedish. On the other hand, Bonsang (2009) shows that informal care is an effective substitute for long-term care only as long as the needs of the elderly are low and require unskilled type of care.

Informal care can be provided in the form of time and financial transfers given to parents. Elderly people who receive financial help from their children represent a small minority of less than 4% in all countries (see Table 3), except in Greece (15.1%), Austria (5.1%) and Spain (4.9%). Variation across countries is relatively large in terms of financial transfers prevalence, but regime patterns are difficult to discern. The story is quite different however in terms of the amounts of financial help received. For instance, Danish and Swedish are clearly the ones that offer the highest amount of financial support (both in median or average terms). On the other hand, in the Southern group – with the exception of Italy – the amount of financial transfers from children to parents is not significantly different from the Continental average level of transfers: €950 in Greece and €1,000 in Spain vs. roughly €950 in Continental Europe.

Regarding time assistance, on average, the elderly singles receive above 50% of their informal care from relatives and friends (as shown in Table 4). Children provide the remaining 50% of care, except in Greece, where they provide almost 73%. The country

\textsuperscript{8}In Sweden local taxes in 2003-2004 were approximately 72% (Glennård et al. 2005).
patterns of social support follow only partially the proposed distinction between the three types of welfare regimes. For the prevalence of support, these regimes seem not to matter: on average, 24.1% of the elderly singles in our sample receive help from children, and this proportion is significantly higher in Denmark, Sweden, Belgium, Germany and Greece than in Switzerland, Italy and Spain. On the other hand, the intensity of social support matches well the three regime clusters. The lowest group in terms of the average number of help hours is that of Denmark, Sweden and the Netherlands, while the highest one is Italy, Spain and Greece, with all the continental countries in between.

All these country-specific factors have a strong impact on the elderly wealth decumulation patterns. Thus, the model will be estimated separately for the Mediterranean, Central Europe and Scandinavia.

3 The Model

We model the two generations of parents (p) and children (k) as unitary decision making agents who live separately. When making their every period choices of consumption, purchase of formal health insurance or covering health expenses out-of-pocket, parents eventually spend down their wealth, thus decreasing the size of potential bequest for the children. In realization of this fact, as well as for the altruistic reasons, children may decide to provide financial support for their parents or to provide care for them by sacrificing some of their working hours.

3.1 Lifecycle model for the parents

When modeling the parent’s generation we focus on consumption, health insurance and savings decisions, and disregard choices related to labour supply, timing of retirement and household dynamics. Thus, we focus on single retired individuals, and fix the retirement age at 65, with certain death occurring at the age of 100, i.e. \( t \in [65, 100] \).

Preferences: In each time period the parent seeks to maximize the expected utility over the remainder of her life, by choosing the levels of consumption \( C_t \) and formal health insurance premium \( f_t \), while children are providing a time transfer \( T_t \) and a money transfer \( M_t \) to help with illness episodes and other expenses. The future is discounted
with a constant discount factor $\beta^p \in [0, 1]$, and within period utility is given by

$$u^p_t(m_t, C^p_t, T_t) = \delta_t \left[ \left( \frac{(C^p_t)^{\theta_t} + \alpha_t \cdot (T_t)^{\theta_t}}{1 - \gamma^p} \right)^{\frac{1}{\gamma^p}} - 1 \right], \quad (1)$$

where $1/\gamma^p$ is the intertemporal elasticity of substitution, $1/(1 - \theta_t)$ is the elasticity of substitution between consumption $C^p_t$ and the value of time assistance provided by children $T_t$, and $\alpha_t$ is scale parameter. We note that the time dependent coefficients $\theta_t$ and $\alpha_t$ vary as health deteriorates over time. This implies that in worse health states children’s assistance may become relatively more important than physical consumption.

When the parent dies, all bequeathable wealth (net of inheritance taxes) $B_t$ is transferred to her children. The utility from such accidental bequest $u_b(B_t)$ is given by

$$u_b(B_t) = b \left( \frac{(B_t + k)^{1-\gamma^p} - 1}{1 - \gamma^p} \right), \quad (2)$$

where $b$ is the intensity of the bequest motive and $k$ determines the extent to which bequests are luxury goods (De Nardi et al., 2010).

In each period, the parent can be in one of four possible health states (Dobrescu, 2012): i) good health, $m_t = \mathbb{G}$; ii) fair health (some medical problems, but no need for long-term care), $m_t = \mathbb{F}$; iii) poor health (invalidity or some form of long-term care required), $m_t = \mathbb{I}$; iv) death, $m_t = \mathbb{D}$. We treat symbols $\mathbb{G}$, $\mathbb{F}$, $\mathbb{I}$ and $\mathbb{D}$ as scalar parameters and impose the following normalization and ordering constraint:

$$0 = \mathbb{D} < \mathbb{I} < \mathbb{F} < \mathbb{G} = 1. \quad (3)$$

We assume that risk aversion $\gamma^p$ is time invariant and that the coefficient $\delta_t$ is given by (Palumbo, 1999)

$$\delta_t = \begin{cases} 
1 + m_t & \text{if } 0 < m_t \leq 1, \\
0 & \text{if } m_t = 0,
\end{cases} \quad (4)$$

i.e., higher utility is associated with better health, and utility drops to zero at death.

---

9We impose a strict restriction $0 < \theta_t < 1$ to ensure that first, even with no assistance time $T_t = 0$, utility of consumption is well defined, and second, with any amount of time assistance, utility of zero consumption is infinitely negative (i.e., marginal utility of consumption approaches plus infinity when $T_t = 0$ and $C^p_t \to 0^+$).
The parameter $\alpha_t$ denotes the value of time assistance relative to consumption, and takes the form

$$\alpha_t = \alpha \cdot (2 - m_t), \quad \alpha \geq 0,$$

allowing for consumption to become relatively less important than children’s assistance in bad health states.

Finally, we assume that elasticity of substitution between consumption and the value of children’s time assistance falls with health, namely

$$\theta_t = \theta_0 + \theta \cdot m_t, \quad \theta_0, \theta \geq 0.$$  

In other words, more consumption has to be given up to retain the same level of utility in worse health if children choose not to increase the time support for parents because of, for example, less costly options (like hiring a caretaker) have to be chosen.

**Health transitions:** There are two sources of uncertainty in the parent’s problem. First, health status $m_t$ evolves as a Markov chain with transition probability

$$p_{jk}^m(t, s_t^p) = \Pr(m_{t+1} = k | m_t = j, t, s_t^p),$$

that allows the next period health to depend on age $t$, current health $m_t$ and parent’s end-of-period wealth $s_t^p$. We do not model investments in health directly, but instead we use end-of-period wealth to proxy for them. As it will be clear below, this also makes the parent’s health transitions dependent on monetary transfers from children, and thus children may affect parent’s health dynamics. To keep the model computationally tractable, we disregard other factors that may influence health transitions.\(^\text{10}\)

We parametrize the transition probability matrix $[p_{jk}^m(t, s_t^p)]_{j,k \in \{G,F,I,D\}}$ by assuming that age and wealth effects on health can be expressed with two separate adjustment

\(^\text{10}\)One other important factor that might affect health transitions is education. In Europe, however, Avendano et al. (2009) find however that education is not significantly associated with transitions to or out of (long-term) illness. As for disability, switching from primary school to postgraduate levels matters only in Central Europe and the Mediterranean, and only for transitions into this state (not out of it). The relative association between education and health also diminishes with age (Huisman et al., 2005a) and even disappears after retirement (due to stable incomes and universal health insurance coverage). Since we focus on retirees, not accounting for education at this stage seemed reasonable.
matrices, $A_1(t)$ and $A_2(s^p_t)$, as follows

$$[p_{jk}^m(t, s^p_t)]_{j,k \in \{G,F,I,D\}} = A_0 + A_1(t) + A_2(s^p_t), \quad (8)$$

where $A_0$ is the initial health transition probability matrix at age 64. We impose the following parametrization to the initial transition probability matrix $A_0$:

$$A_0 = \begin{bmatrix}
  c_1(1 - \delta_0) & \frac{1-c_1}{2}(1 - \delta_0) & \frac{1-c_1}{2}(1 - \delta_0) & \delta_0 \\
  \frac{1-c_2}{4}(1 - \delta_0) & c_2(1 - \delta_0) & \frac{3(1-c_2)}{4}(1 - \delta_0) & \delta_0 \\
  \frac{1-c_3}{4}(1 - \delta_0) & \frac{3(1-c_3)}{4}(1 - \delta_0) & c_3(1 - \delta_0) & \delta_0 \\
  0 & 0 & 0 & 1
\end{bmatrix}, \quad (9)$$

where $\delta_0$ is calibrated to the survival probability between age 64 and 65.\footnote{When choosing the specification of the initial health transition probability matrix $A_0$ we were constrained first by the limitations of our data that precluded the estimation of health transitions between 64 and 65 directly, and second by considerations of computational tractability of our model and identification of the large number of parameters in the health transition process.}

Following Ameriks et al. (2005), we model the health dependence on age in (8) with an age adjustment matrix

$$A_1(t) = \begin{bmatrix}
  -c_4 \tau^{\alpha_m} & \frac{c_4 c_5 c_6}{1+c_5+c_5c_6} & \frac{c_4 c_5 c_6}{1+c_5+c_5c_6} & \frac{c_4 c_5 c_6}{1+c_5+c_5c_6} \\
  0 & -c_4 \tau^{\alpha_m} & \frac{c_4 c_5 c_6}{1+c_5} & \frac{c_4 c_5 c_6}{1+c_5} \\
  0 & 0 & -c_4 \tau^{\alpha_m} & c_4 \tau^{\alpha_m} \\
  0 & 0 & 0 & 0
\end{bmatrix}, \quad (10)$$

which shifts the transition probability mass towards worse health, relative to the transitions at age 65. The variable $\tau = t - 65$ denotes the number of years since retirement, while $\alpha_m$ is a fixed parameter that allows for faster than linear shifting in health as one becomes older. The other three parameters in (10) have the following roles: $c_4$ controls the transition from invalidity to death as age increases, $c_5$ determines how much more likely is death relative to invalidity when in fair or good health, and $c_6$ determines an individual’s chance to persist in good health.
Finally, the wealth adjustment matrix $A_2(s^p_t)$ is given by

$$
A_2(s^p_t) = \begin{bmatrix}
    c_7 \ln(s^p_t + 1) & -c_7 \ln(s^p_t + 1) & 0 & 0 \\
    c_8 \ln(s^p_t + 1) & -c_8 \ln(s^p_t + 1) & 0 & 0 \\
    0 & c_9 \ln(s^p_t + 1) & -c_9 \ln(s^p_t + 1) & 0 \\
    0 & 0 & 0 & 0
\end{bmatrix},
$$

where the level of parents’ end-of-period wealth shifts the probability mass in health transitions towards transitions to better health.\footnote{Despite being theoretically possible, the adjusted values for health transition} probabilities never reach 0 or 1 during the simulations or estimation of the model.

We assume that higher wealth increases the chances for healthy to stay in good health (first row) and decreases the chance of the next worse health stata ($F$). For the individuals in fair ($F$) or poor ($I$) health, higher wealth provides better opportunity for recovery (i.e., higher chances of transitioning into $G$ or $F$, respectively). The last row in (11) contains no adjustments because $D$ is an absorbing state.

**Health spending:** The second source of uncertainty in the parents problem besides health transitions is out-of-pocket medical spending $oop_t(m_t)$. We define $oop_t$ as the part of total health costs $h(m_t)$ not covered by formal insurance $F_t(f_{t-1}, m_t)$. Out-of-pocket costs may only appear in bad health states ($m_t < 1 = G$), except that in the event of death ($m_t = 0 = D$) only a one time deterministic funeral cost $h_0$ is incurred, i.e.,

$$
oop_t(m_t) = \begin{cases}
    h_0 & \text{if } m_t = 0 = D, \\
    \max\{0, h(m_t) - F_t(f_{t-1}, m_t) + \psi_t\} & \text{if } m_t \in (0, 1), \\
    0 & \text{if } m_t = 1 = G.
\end{cases}
$$

Health cost $hc(m_t)$ is an exogenous deterministic process and health spending shock $\psi_t$ follows a Gaussian autoregressive process

$$
\ln \psi_t = (1 - \rho_\psi) \ln \overline{\psi} + \rho_\psi \ln \psi_{t-1} + \varepsilon^p_t, \varepsilon^p_t \sim N(0, \sigma^2_\varepsilon),
$$

where $\rho_\psi$, $\overline{\psi}$ and $\sigma_\varepsilon$ are fixed parameters. Without loss of generality, we assume that $\psi_t = 0$ when $m_t \in \{0, 1\}$.\footnote{Despite being theoretically possible, the adjusted values for health transition probabilities never reach 0 or 1 during the simulations or estimation of the model.}
**Budget constraint and health insurance:** The model allows for two links between each pair of consequent years. The first link is due to the standard consumption-savings process: parents’ end-of-period wealth $s^p_t$ at time $t$ grows with constant risk-free interest rate $r$, so that at period $t+1$, they start with wealth level $s^p_t (1 + r)$.

The second link is due to formal medical insurance. Following Dobrescu (2012), we represent a typical contract in the following simplified way. In exchange for paying an annual premium $f_t$, individuals have a share of their next period medical costs covered by formal health insurance $F_{t+1}(f_t, m_{t+1})$. Besides the private component, this contract includes a minimum health care consumption floor $F(m_{t+1})$ provided by the public health system, as follows:

$$ F_{t+1}(f_t, m_{t+1}) = \begin{cases} F(m_{t+1}) + \frac{\omega}{m_{t+1}} f_t, & \text{if } m_{t+1} \in (0, 1), \\ 0, & \text{if } m_{t+1} \in \{0, 1\}, \end{cases} \quad (14) $$

where $\frac{\omega}{m_{t+1}}$ is the inverse of the loading factor.\footnote{The loading factor captures the health-specific ratio of formal premium to coverage, net of any administrative costs.}

Any medical expenditures in excess of the formal health coverage must be paid out-of-pocket. To help with expenses children can provide to their elderly parents not only time assistance $T_t$, but also financial help $M_t$. Let $w^p_t$ denote the wealth of parents net of the transfer from children. Assuming that resources can only be spent on consumption and formal insurance premium, the end-of-period wealth is

$$ s^p_t = w^p_t + M_t - C^p_t - f_t, \quad (15) $$

and the intertemporal budget constraint is given by

$$ w^p_{t+1} = \max \left\{ c^p_{\text{floor}}, \ (w^p_t + M_t - C^p_t - f_t)(1 + r) + y_{t+1} - \text{oop}_{t+1}(m_{t+1}) \right\} \quad (16) $$

$$ = \max \left\{ c^p_{\text{floor}}, \ s^p_t (1 + r) + y_{t+1} - \text{oop}_{t+1}(m_{t+1}) \right\}, $$

where $y_t$ is pension benefit in period $t$ and $\text{oop}(m_t)$ is out-of-pocket medical spending defined in (12). Wealth must satisfy the borrowing constraint $s^p_{t+1} \geq 0$, which eliminates the possibility of parents dying in debt. Because out-of-pocket spending is not bounded...
from above, we include a possibility for personal bankruptcy by introducing a consumption floor denoted $c_{\text{floor}}^p$ at the level that corresponds to $\frac{1}{100}$ of the average pension level. Because according to (16) the pension payment is spent for covering out-of-pocket health spending, personal bankruptcy is effectively punished by at least one period of consumption at level $c_{\text{floor}}^p$.

**Recursive form of the parent’s problem:** The Bellman equation of the parent’s problem takes the form (for $t < 100$):

$$V_t^p(w_t^p, m_t, \psi_t; M_t, T_t) = \max_{C_t^p, f_t} \left\{ u_t^p(m_t, C_t^p, T_t) + \beta^p (1 - p_{m_tD}) E_{m, \varepsilon^p} \left[ V_{t+1}^p(w_{t+1}^p, m_{t+1}, \psi_{t+1}; M_{t+1}, T_{t+1}) \mid m_{t+1} \neq D \right] + \beta^p p_{m_tD}^m u_b(B_{t+1}) \right\}$$

subject to $C_t^p \geq 0$, $f_t \geq 0$ and the credit constraint $w_t^p + M_t - C_t^p - f_t \geq 0$. The expectation is taken over health transitions and health spending shock $\varepsilon_t^p$ in (13). Note that the last two arguments of value function $V_t^p(w_t^p, m_t, \psi_t; M_t, T_t)$ are marked out to emphasize the fact that they are determined by the actions of the children. When $t = 100$ the value function is given by

$$V_{100}^p(w_{100}^p, m_{100}, \psi_{100}; M_{100}, T_{100}) = \max_{C_{100}^p} \left\{ u_{100}^p(m_{100}, C_{100}^p, T_{100}) + \beta u_b(B_{100}) \right\},$$

subject to $C_{100}^p \geq 0$ and the credit constraint $w_{100}^p - C_{100}^p \geq 0$. Because the decision maker dies with certainty after the termination period, it is never optimal to “waste” resources by purchasing formal insurance, i.e., $f_{100} = 0$.

### 3.2 Lifecycle model for the children

We model children as one representative unit with unitary preferences, i.e. as if each parent had either none or a single child. We focus on modelling children’s choices of hours of labour supply and transfers to parents after their parents retire at age 65. When parents die, children inherit the remained of parent’s wealth, and do not have to make any transfers thereafter. Children’s interactions with parent in the model are driven by both strategic and altruistic motives. By providing time and money transfers children
can affect the rate of parent’s wealth decumulation in the anticipation of larger bequest. The altruistic motive is related to the fact that children care about their parents’ health, and this plays a crucial role in the decision to help with money and/or time (Johnson and Lo Sasso, 2000; Bonsang, 2007).

Preferences: In each period $t$, children’s within period utility is given by

$$u^k_t(C^k_t, m_t) = \frac{\left(C^k_t\right)^{1-\gamma^k} - 1}{1 - \gamma^k} + \eta m_t,$$  

where $1/\gamma^k$ is the intertemporal elasticity of substitution of children’s consumption $C^k_t$, while $\eta$ quantifies the altruism motive in children’s preferences.

Transfers to parents: Besides consumption, in every period $t$, children choose how much time to spend helping their parents and how much money to transfer to them (i.e., they choose $M_t$ and $T_t$). These transfers will be primarily directed to cover parents’ health spending, but we do not condition them on parent’s health status $m_t$.

Monetary transfers $M_t$ are made out of children’s beginning-of-period wealth $w^k_t$, thus

$$w^k_t = C^k_t + M_t + s^k_t,$$  

where, analogous to parents problem, $s^k_t$ denotes end-of-period wealth.

When it comes to time transfers $T_t$, we focus on the trade-off between wage earnings and potential bequest, and therefore assume that the time children choose to spend assisting their parents is taken out of their working time (Bolin et al., 2008; Johnson and Lo Sasso, 2000; Carmichael and Charles, 1998, 2003a,b; Heitmueller, 2007; Heitmueller and Inglis, 2007). For simplicity, we disregard children’s leisure.\textsuperscript{14} Let $L$ denote per period working time endowment, and $H_t$ the time actually spent at work. Then the time budget is

$$H_t + T_t = L.$$  

\textsuperscript{14}SHARE does not collect data on time use, and so we do not have any information on children’s time allocation between work, leisure and help provided to their elderly parents. The literature on informal care however has extensively documented the negative impact of informal care on carers work hours. Thus, assuming that children’s time assistance would reduce their work time is both in line with previous empirical evidence (see Bolin et al. 2008 for evidence on Europe) and it ultimately avoids the identification issues raised by the lack of data.
**Wage equation:** We use a traditional Mincer (1958) type specification for the children’s wage equation. Specifically, the wage $W_t$ follows an age profile with idiosyncratic log-normal shocks, namely

$$
\ln W_t = \lambda_0 + \lambda_1 (t - \Delta) + \lambda_2 (t - \Delta)^2 + \lambda_3 \text{edu} + \varepsilon_t^k, \quad (22)
$$

where $\Delta$ is the offset of the child’s age from the parent’s age, $\text{edu}$ denotes the time invariant education level of children, and $\varepsilon_t^k \sim N(0, \sigma_{\varepsilon_t}^2)$.

**Budget constraint and bequest:** Similarly to parents, children earn the risk-free interest rate $r$ on their savings, while their incomes consist of wage earnings and anticipated bequest. We make a common assumption that wage is paid after a waiting time of one period.\(^{15}\) Therefore, next period wealth for children is given by

$$
w_{t+1}^k = (w_t^k - C_t^k - M_t)(1 + r) + W_{t+1}H_t + B_{t+1}(m_{t+1})
= s_t^k \cdot (1 + r) + W_{t+1}H_t + B_{t+1}(m_{t+1}), \quad (23)
$$

where $B_{t+1}(m_{t+1})$ is the bequest the children expect to receive if the parent dies. The size of the bequest is

$$
B_{t+1} = \begin{cases} 
0 & \text{if } m_{t+1} > 0, \\
(1 - \zeta) \max \{0, s_t^p (1 + r) + y_{t+1} - h_0\} & \text{if } m_{t+1} = 0 = \mathbb{D},
\end{cases} \quad (24)
$$

where $m_{t+1} > 0$ is realized with probability $1 - p_{m_{t-1}}^m$ and $m_t = 0$ is realized with probability $p_{m_{t-1}}^m$. The parameter $\zeta$ denotes the effective inheritance tax rate on wealth transferred as bequest.

**Recursive form of the children’s problem:** The Bellman equation of the children’s problem takes the form (for $t < 100$)

$$
V_t^k(w_t^k, \text{edu}; m_t, s_t^p) = \max_{C_t^k; M_t, T_t} \left\{ w_t^k(C_t^k, m_t) + \beta^k E_{\varepsilon_t^k} [V_{t+1}^k(w_{t+1}^k, \text{edu}; m_{t+1}, s_{t+1}^p)] \right\}, \quad (25)
$$

\(^{15}\)The essential part of this assumption is that labour supply choices are made before the wage shock is realized. This allows for treatment of wage shock as idiosyncratic and simplifies the numerical solution because shocks do not have to be included in the state space. An alternative interpretation is that wages are paid in the end of the period with a multiplicator $\frac{1}{1+r}$ and idiosyncratic shocks are relabelled with one period offset.
subject to $C^k_t \geq 0$, credit constraint $s^k_t = w^k_t - C^k_t - M_t \geq 0$ and time budget $0 \leq T_t \leq L$.

Note that similarly to parent’s problem, the last two arguments of the value function $V^k_t(w^k_t, edu; m_t, s^k_t)$ are marked out to emphasize that they are determined by the actions of the parents.

The future in children’s problem is discounted with discount factor $\beta^k$ that may not equal parent’s discount factor $\beta^p$. For simplicity, we disregard the possibility of parents outliving their children and so, survival probability is not present in children’s Bellman equation.

Denote by $t_d \leq 100 - \Delta$ the age of the children when parents die, (i.e. the first time when $m_t = D$ is realized). In this period, bequests are realized according to (24), and from this period on the action space of the children is reduced to a consumption-savings choice only: for $t \geq t_d$, children solve a 'cake eating' problem with the size of the cake increasing every period by a random amount corresponding to full wage (22). We use Carroll’s (2006) endogeneous grid method to compute the numerical solution for this reduced problem for children, and use the computed value functions as the children’s terminal values when the parents die. Effectively, we can terminate the children’s problem at time $t_d$ and start backwards induction for the children’s problem from the terminal value at that time.

### 3.3 Dynamic intergenerational game

While making their individual choices, parents and children account for the impact of these choices on the other party. Thus, in our model parents and children find themselves in a dynamic intergenerational finite horizon game.

**Common state space and common knowledge:** As it follows from the Bellman equations of both parents and children’s problems in (17) and (25), the actions of the opponent enter the state space of each problem. When we consider the two problems together within the intergenerational game, these actions are endogenized, and thus, the state space of the game is given by

$$\Omega_t = \{w_t^p, m_t, \psi_t, w_t^k, edu, \Delta\}.$$ (26)
We note that the age difference $\Delta$ between parents and children enters necessarily into the state space of the game to facilitate the alignment of their life cycles in the unified model, as in (22). Further, we assume that idiosyncratic shocks $\varepsilon^p_t$ and $\varepsilon^k_t$ to health expenditures and to wages are independent and are common knowledge within the family once they are realized. In other words, the information sets of parents and children are identical at each moment in time, forming a complete information game.

**Timing of the events and the order of moves in the game:** The sequence of events in the intergenerational game is the following: Period $t$ starts with the realization of (i) the health shock that determines $m_t$, and (ii) the shock $\varepsilon^k_t$ in the wage equation (22). Current incomes (i.e., the pension benefit $y_t$ for parents and the wage earnings $W_t H_{t-1}$ for children) are then immediately realized.

If the parent dies ($m_t = \mathcal{D}$) bequest is realized according to (24), and children’s help is no longer needed ($M_t = 0$ and $T_t = 0$). Children’s wealth $w^k_t$ is then fully determined by (23), and they are assigned with a termination value. The game ends.

In case parents survive ($m_t > 0$), medical spending shock $\psi_t$ is realized, and out-of-pocket expenses $oopt_t(m_t)$ are determined according to (12), taking into account the formal health insurance $f_{t-1}$ purchased in the previous period. Also, by this time all components in (23), in particular $B_t = 0$, are known and children simultaneously choose time assistance $T_t$ (which determines work hours $H_t$), consumption $C^k_t$ and money transfers $M_t$ to parents.

We assumw that children’s choices become known to the parents before they have to move, and thus parents know not only their wealth $w^p_t$ given by (16), but also the amount $M_t$ transferred from children, which together define their consumable resources. Parents then choose consumption $C^p_t$ and purchase formal insurance at premium $f_t$. Period $t$ ends after consumption has taken place, and wealth decreases to its end-of-period values $s^p_t$ and $s^k_t$. The game continues to the next period.

**Equilibrium concept:** The fact that in each period parents observe children’s decisions ($C^k_t$, $M_t$ and $T_t$) before making their own choices ($C^p_t$ and $f_t$), together with the assumption of complete information, ensures that the information sets of the players are singletons. Thus, the intergenerational game belongs to the class of finite dynamic games.
of perfect information. Consequently we only consider the pure strategy equilibria of the game, and following the Markov nature of the dynamic problems at hand, we adopt the concept of Markov Perfect Equilibrium (MPE).

**Definition 1** The pure strategies MPE equilibrium in the dynamic intergenerational game is given by a pair of strategies \((\varphi^p_t, \varphi^k_t)\)

\[
\begin{align*}
\varphi^p_t : \Omega_t &\rightarrow (C^p_t, f_t), \\
\varphi^k_t : \Omega_t &\rightarrow (C^k_t, M_t, T_t), \\
\end{align*}
\]

which map the state space into the action space of parents and children for each \(t\) and constitute the solutions of the Bellman equations of these two problems given in (18-17) and (25), conditional on the endogenously determined actions of the opponent.

### 3.4 Solution method

According to Definition 1, the solution of our model is a pair of policy functions that jointly solve the system of Bellman equations given in (17-18) and (25). To find the solution, we develop a backwards induction algorithm that sequentially searches for Markov perfect equilibria in subgames spanning over parents’ ages \(t\) to 100, where we allow \(t\) to iteratively decrease from 100 to 65. Consequently, we first solve for an equilibrium in a “terminal game” when \(t = 100\), which forms the base of the induction. Then given the value functions \(V^p_{t+1}(\Omega_{t+1})\), \(V^k_{t+1}(\Omega_{t+1})\) and the policy functions \(\varphi^p_{t+1}\), \(\varphi^k_{t+1}\) we solve for all equilibria in period \(t\) subgame by computing the mutual best responses in period \(t\) conditional on strategies \(\varphi^p_{t+1}\), \(\varphi^k_{t+1}\) to be used at period \(t + 1\) and beyond.16 Proposition 1 below ensures the uniqueness of the fixed point in the mutual best responses of children and parents arising from the maximization problems in (18-17) and (25), thus establishing the uniqueness of the equilibrium in every subgame \(t \in \{100, 99, \ldots, 65\}\), and eventually the uniqueness of the MPE in the whole game.

**Characterization of stage equilibrium:** The equilibrium in a given period \(t\) is comprised of a pair of policy functions (27) that defines the correspondence between the points

---

16Theorem 5 in Iskhakov, Rust and Schjerning (2013) provides justification of this algorithm in computing subgame equilibria.
in the state space (26) and the decision vectors \((C_t^p, f_t, C_t^k, M_t, T_t)\). We construct this correspondence by solving for the optimal decisions (fixed point in mutual best responses) in every point of the discretized state space.

We discretize the continuous dimensions in the state space \(\Omega_t\) using both fixed and endogenous grids, following the endogenous grid point method developed by Carroll (2006). Specifically, the grid over \(\psi_t\) is fixed, and the grids over wealth variables \(w_t^p\) and \(w_t^k\) are determined in the solution algorithm as explained below. The state variables \(m_t\), \(edu\) and \(\Delta\) are discrete.

Given a point in the discretized state space, the optimal decisions of parents and children are characterized by a complex system of non-linear equations (28). This system consists of the Euler equations for parents' and children's problems (18-17) and (25) with respect to each of the five continuous decision variables.\(^{18}\) In the numerous cases

\(^{17}\)The full system of first order conditions derived in sections A.1 and A.2 in the Appendix is

\[
\begin{align*}
\frac{\partial u_t^p(m_t, C_t^p, T_t)}{\partial c_t^p} &= \beta^p(1+r)E_{m_t} \left[ \frac{\partial u_{t+1}^p(m_{t+1}, C_{t+1}^p, T_{t+1})}{\partial c_{t+1}^p} \right]_{m_{t+1} \not= 0} \\
&+ \beta^p(1+r)p_{t+1}^m(1-c)u_{t+1}^p(B_{t+1}) \\
&+ \beta^p \sum_{m' \neq 0} \frac{\partial u_t^p}{\partial m'} E_{t+1}(w_{t+1}, m', \psi_{t+1}; M_{t+1}, T_{t+1}), \quad (a) \\
0 &= -(1+r)E_{m_t} \left[ \frac{\partial u_{t+1}^p(m_{t+1}, C_{t+1}^p, T_{t+1})}{\partial c_{t+1}^p} \right]_{m_{t+1} \not= 0} - (1+r)p_{t+1}^m(1-c)u_{t+1}^p(B_{t+1}) \\
&- \beta^p \sum_{m' \neq 0} \frac{\partial u_t^p}{\partial m'} E_{t+1}(w_{t+1}, m', \psi_{t+1}; M_{t+1}, T_{t+1}) \\
&+ \tilde{\tau}E_t \left[ \frac{\partial u_{t+1}^p(b_t, C_{t+1}^p, T_{t+1})}{\partial b_{t+1}} \right]_{b_{t+1} = \tilde{F}} \exp(\varepsilon_t^p) > \Lambda(f_t) \\
&+ \tilde{\tau}E_t \left[ \frac{\partial u_{t+1}^p(c_t^k, C_{t+1}^p, T_{t+1})}{\partial c_t^k} \right]_{c_t^k = \tilde{F}} \exp(\varepsilon_t^p) > \Lambda(F_t) \quad (b) \\
\frac{\partial u_t^k(c_t^k, m_t)}{\partial c_t^k} &= \beta^k(1+r)E_{m_t} \left[ \frac{\partial u_{t+1}^k(c_{t+1}^k, m_{t+1})}{\partial c_{t+1}^k} \right]_{m_{t+1} = \tilde{D}} \\
0 &= (1+r)E_{m_t} \left[ \frac{\partial u_{t+1}^k(c_{t+1}^k, m_{t+1})}{\partial c_{t+1}^k} \right]_{m_{t+1} = \tilde{D}} - (1+r)\frac{\partial P}{\partial m_t} E_t \left[ \frac{\partial u_{t+1}^k(c_{t+1}^k, m_{t+1})}{\partial c_{t+1}^k} \right]_{m_{t+1} = \tilde{D}} \\
&- \frac{\partial P}{\partial m_t} \sum_{m'} \frac{\partial u_t^k}{\partial m'} E_t V_{t+1}(w_{t+1}, m', \psi_{t+1}; s_{t+1}) \\
0 &= E_{m_t} \left[ W_{t+1} \frac{\partial u_{t+1}^k(c_{t+1}^k, m_{t+1})}{\partial c_{t+1}^k} \right]_{m_{t+1} = \tilde{D}} - (1+r)\frac{\partial P}{\partial m_t} E_t \left[ \frac{\partial u_{t+1}^k(c_{t+1}^k, m_{t+1})}{\partial c_{t+1}^k} \right]_{m_{t+1} = \tilde{D}} \\
&- \frac{\partial P}{\partial m_t} \sum_{m'} \frac{\partial u_t^k}{\partial m'} E_t V_{t+1}(w_{t+1}, m', \psi_{t+1}; s_{t+1}). \quad (d)
\end{align*}
\]

where the expectations are taken over idiosyncratic shocks in wage equation \(\varepsilon_t^k\), shocks \(\varepsilon_t^p\) in the health spending equation where indicated by \(\varepsilon\) and health transitions where indicated by \(m\), \(\Lambda(m_{t+1}, f_t)\) is given in (34), and we use the shortcut notation \(\bar{E}(\cdot|s) = \bar{E}(\cdot|s)P(s)\). In addition, 47 corner solutions are characterized by the same system after replacing equations (a)-(c) in (28) according to the rules given by all combinations of the cases in the following three groups

\[
\begin{align*}
(a, b) &\rightarrow (36), w_{t+1}^p = C_{t+1}^p + f_t, \quad (e) \rightarrow T_t = 0, \quad (c, d) \rightarrow (41), w_{t+1}^k = C_{t+1}^k + M_t, \\
(a, b) &\rightarrow (a), f_t = 0, \quad (e) \rightarrow T_t = L, \quad (c, d) \rightarrow (c), M_t = 0, \\
(a, b) &\rightarrow f_t = 0, C_{t+1}^p = w_{t+1}^p, \quad (e) \text{ as is}, \quad (c, d) \rightarrow M_t = 0, C_{t+1}^k = w_{t+1}^k, \\
(a, b) \text{ as is}, \quad (e) \text{ as is}, \quad (c, d) \text{ as is}. \quad (29)
\end{align*}
\]

\(^{18}\)We do not discretize any of the decision variables in our model except time transfers \(T_t\).
of corner solutions, arising from non-negativity and credit constraints in both problems, as well as time budget constraint in children’s problem, the system of first order conditions also includes the corresponding binding constraints.

**Proposition 1**  The MPE equilibrium in the intergenerational bequest game is unique.

**Proof.** We provide an intuition instead of a rigorous proof here. First we show that the system (28) has a unique solution in every t given the continuation strategies \( \varphi^p_{t+1}, \varphi^k_{t+1} \). Then, because the equilibrium path necessarily passes through the fixed points in mutual best responses in every period, it follows that the whole dynamic MPE equilibrium is unique as well.

To show the former, note that system (28) can be rearranged to yield optimal values of each of the decision variables one by one similar to a triangular system (see Appendix A for the details). Therefore it suffices to show the uniqueness of solution of each equation in (28).

The complete description of the solution algorithm is given in Appendix A.

4 Data

The data we use to estimate the model are drawn from the first two waves of the Survey of Health, Ageing and Retirement in Europe (SHARE).\(^ {19} \) SHARE is a longitudinal, multidisciplinary and cross-national survey representing the population of individuals aged 50 and over in Europe. The first wave of SHARE took place in 2004, with the second one following in 2006. There are 11 countries participating in both waves, namely Austria, Belgium, Denmark, France, Germany, Greece, Italy, the Netherlands, Spain, Sweden and Switzerland. The total number of individuals interviewed in 2004 (Release 3) is 28,517.

Our analysis focuses on the sample of single, non-institutionalized individuals,\(^ {20} \) with ages 65 and above. The raw data on this sample cover about 4,500 individuals. When the elderly receive help from co-residing children or other co-residing adults such as siblings, we do not observe the amount of hours they provide. Therefore, we exclude

\(^ {19} \)http://www.share-project.org/

\(^ {20} \)van Houtven and Norton (2004) point out that the types of care needed by institutionalized individuals differ substantially from those of the elderly living at home. Nursing-home residents for instance may have less access to care from children, and so, we exclude them from the sample. We note however that the institutionalized subsample represents only 2.6% of the full sample we consider.
about 13.7% of the individuals that have adult children living in the same household. As a last restriction, we impose that each parent must have non-negative net worth (Brown, 2005), which leads to a final sample of 3,796 individuals, of whom 921 are men and 2,875 are female. Of these individuals, 3,628 are still alive in 2006.

On the parents side, the four variables of interest for our model are: i) total net worth in PPP adjusted € (wealth henceforth), ii) total expenditures on non-durables (consumption henceforth), iii) annual voluntary (supplementary) private insurance premium (premium henceforth), and iv) total amount spent out-of-pocket on medical goods and services. Wealth represents the value of all financial and real assets, net of any debts and liabilities (i.e., loan repayment, mortgage, taxes). Pension benefits, on the other hand, include any public and private pensions or invalidity benefits, such as old age, early or pre-retirement pension, disability benefits, survivor and war pension, private annuities, etc. Out-of-pocket medical spending was computed as the total amount spent on drugs, inpatient and outpatient care, as well as on nursing home care, day-care and home care. We note that they include medical expenses incurred during respondent’s last year of life.

To insure cross-country comparability, we follow Browning et al. (2003) and calculate total consumption as the sum of the amount spent on food (at home and outside home) and phone bills, adjusted by country-specific weights. These weights represent OLS coefficients of a regression of total non-durable consumption on a subset of household expenses (i.e., groceries, eating out, phone bills and other utilities, transportation, clothing, entertainment, etc.) using national expenditure survey data. We use the IS-TAT2004 survey for Italy, Spain and Greece, and the Dutch Budget Survey (DBS) for the remaining countries.

Finally, the question on the amount spent on premium was only asked in the first SHARE wave. Moreover, the corresponding data clearly reflects the limited ability of older parents to report this cost. Although continuous information on premiums is solicited, the responses contain many missing values. Given the stability of insurance profiles in old age (Paccagnella et al., 2013), we impute the missing values in Wave 1 and the corresponding values for Waves 2 using the coefficients of age, education, health, wealth and consumption from an OLS regression on existing premium data.\textsuperscript{21}

\textsuperscript{21}The imputation affected 64.7% of the whole sample of individuals reporting wealth and consumption
Given its focus on ageing, SHARE provides only limited information on the respondents’ adult children. For instance, there is no indication of children’s earnings or consumption value, but there is detailed demographics data on age, gender, marital status, number of children, education and occupation for up to four children. To retrieve the wage and consumption profiles, we use two sets of data, as follows: i) the ISTAT2004 and DBS data, to compute consumption, and ii) the European Union - Statistics on Income and Living Conditions (EU-SILC) data, to compute wages. The procedure employed to obtain the consumption profiles of adult children (of age 25-64) is simple. For each possible combination of the demographic characteristics that are reported in SHARE, we compute the median value of consumption using the ISTAT2004 (for Italy, Spain and Greece) and the DBS data (for the other countries). We then assign to each individual in SHARE the correspondent imputed consumption value obtained from these alternative datasets, based on his/her specific demographic characteristics.\footnote{Using this procedure, we manage to match 2,282 individuals that represent 78.7\% of the sample of children. We note that this is a conservative matching procedure (i.e., we match across six demographic dimensions), but given the importance of the data profiles for children, we opt for matching accuracy over sample size.}

To impute the wages of children, we follow a similar procedure to the one used to retrieve consumption for their elderly parents. Specifically, we model the wage profiles as a function of age, gender, marital status, number of children, education, occupation and country of residence. Next, we estimate the wage equation on country-specific data from the EU-SILC and use the coefficients to retrieve the wage profiles of SHARE individuals, based on the common demographic variables available for children.

Importantly, we model children as one representative unit, i.e., as if each parent had either none or a single child. As a result, for parents with more children, we consider as representative the child who provides the most time and/or money transfers, or the oldest if none of the children provide help. If the most time and money transfers come however from different children, we take their median.

One of the advantages of using SHARE is that it contains detailed information on financial transfers and social support from the perspective of the ‘parents’ aged 65 and above. In this paper we use the information on: (i) the frequency and amount of financial transfers in Wave 1. Alternative specifications were explored, but the one implemented fitted best the existing data. More details on the imputation procedure are available from the authors.
or material gifts or support (other than for shared housing and food) of at least €250 (or the equivalent in local currency) from children living outside the household; (ii) the frequency and amount (in hours) of social support received in any of the three forms – personal care, practical household help and help with paperwork – from children living outside the household. Both these questions refer to the 12 months prior to the interview.

We note that when asked about the financial transfers’ amount, some respondents refuse to answer or encounter difficulties in remembering exact amounts. In these cases, a series of brackets unfolds, with the respondent being asked whether the amount is similar to, lower or higher than or in-between these bracket values. This information can be used to reduce the number of missing values (Christelis, 2011). Thus, we use the SHARE generated data module that contains the imputed values for the three financial transfers reported by each respondent. Finally, among the transfers received from children we select the one provided by the representative child.

In terms of time transfers, for each person from whom the respondent received social support information is collected on: i) the identity of the caregiver and his/her relationship with the respondent, and ii) the frequency of this help and its average amount in hours. Respondents can receive help “almost daily”, “almost every week”, “almost every month” and “less often”. Depending on the answer to this question, respondents are asked about the number of hours on a “typical day/week/month” or “in the last twelve months”. In order to obtain the number of hours of support in the last 12 months, we therefore multiply the average number of hours provided daily, weekly and monthly by 365 days, 52 weeks and 12 months, respectively. Receiving time assistance from more than one child is not common: among the households which receive informal home care, only 1.5% report having two or three children providing help. Similar to financial transfers, we construct the variable denoting time assistance from children as equal to the annual hours provided by the representative child.

In addition to constructing moment conditions, we also use the SHARE data to construct the region-specific initial distributions of wealth, age difference (between parents and children), pensions and out-of-pocket medical expenses, by (children’s) education and (parents) health to start off our simulations. As a result, each simulated individual will be endowed with a state vector drawn from the joint distribution of the state variables
observed in 2004.

5 Calibrations and estimation methodology

The life cycle literature based on European data is quite limited and the institutional differences are potentially significant. Therefore, we estimate most of the model parameters for each region separately, and calibrate only those parameters that appear as instruments for the dynamic programming model (Gourinchas and Parker, 2002; Cagetti, 2003; French and Jones, 2011). We set the real risk-free asset return at $(1 + r) = 1.04^{23}$, $\alpha_m$ at 1.5 (Ameriks et al., 2005) and $\zeta$ at the levels reported by AGN International (2013). For each European region, we follow Dobrescu (2012) and set the public insurance coverage $\bar{F}$ to the adjusted mean 2004 level of: i) public curative and rehabilitation (CR henceforth) expenditure per capita, if in fair health; ii) public long-term care (LTC henceforth) expenditure per capita, if in poor health. Similarly, the exogenous total medical spending $h$ matches the adjusted country-specific 2004 level of: i) total CR costs per capita, if in fair health; ii) total LTC per capita, if in poor health; iii) funeral costs, if dead (Bjørnerud et al., 2005).\footnote{The average long-term interest rate in 2004 for the Mediterranean countries was 4.2 percent, for Central Europe 3.9 percent and for Scandinavia 4.4 percent (OECD Statistics 2010, Key Economic Indicators).} Since $\bar{F}$ and $h$ represent absolute per capita amounts (OECD, 2006), the adjustment involved re-weighting their value with the Eurostat (2012) share of population reporting good or fair health (for CR costs) and bad or very bad health (for LTC costs). The parameter $\bar{\psi}$ is set to the OECD 2004 PPP adjusted total expenditure on health per capita, while $\rho_\psi$, $\sigma_{\psi}^2$, $\gamma^p$ and $\beta^p$ are taken from Dobrescu (2012). Finally, $\delta_0$ is calibrated to the average survival probability between age 64 and 65 in Europe using Eurostat and Human Mortality Database ($\delta_0 = 0.00914$ for Mediterranean countries, $\delta_0 = 0.01036$ in Central Europe, and $\delta_0 = 0.01098$ in Scandinavia), whereas the age difference $\Delta$ between parents and children is taken from the SHARE data.

The aim of the analysis is to explain the wealth decumulation profiles, as well as the formal and informal insurance decisions in old age. Hence, we match total wealth, consumption, formal premium, out-of-pocket medical spending, amounts and prevalence of time and financial transfers, as well as children’s consumption, conditional on age and

\footnote{Further details on health and funeral costs are provided in Appendix B.}
education. To calculate the empirical moments, we first break the data into five cohorts and three levels of education. The 1st cohort consists of individuals with ages 65-69 in 2004, the 2nd cohort contains ages 70-74, the 3rd cohort contains ages 75-79, the 4th cohort contains ages 80-84, and the 5th cohort, for sample size reasons, contains ages 85+. We use the data for two different years corresponding to the two SHARE waves, namely 2004 and 2006. To calculate the moments of our variables of interest, we take cell medians by cohort and education, for surviving individuals in each calendar year.

To calculate the simulated moments, for each region, we proceed as follows: To compute the optimal choices, we use the Gauss-Hermite quadrature method to discretize the state space of the shocks. We solve the model numerically by backward induction and simulate wealth, consumption, formal premium, health and medical spending histories for \( N = 10,000 \) artificial individuals, using Monte Carlo draws for the two stochastic variables. Additionally, we also simulate consumption, wages and time and money transfers profiles of their children. For each artificial profile, we compute cell medians by cohort and education level conditional on the initial values of the state variables \( \boldsymbol{\Omega}_0 \) and on model parameters \( \phi' = (\theta_0, \theta, \alpha, \omega, b, k, c_{i=1:3}, \gamma^k, \beta^k, \eta) \in \mathbb{R}^{18} \).

Finally, parameters are chosen to minimize the difference between these artificial moments and their empirical counterparts. The goodness of fit between the two series is assessed by a \( \chi^2 \)-test statistic or corresponding \( p \)-value. This statistic assesses whether or not the true data moments \( (m_T) \) are equal to the realized data moments \( (m_n(\Omega_0, \phi'), n = 1, N) \), given the stochastic processes for which the true time series is only one realization. Analytically, as \( NT \to \infty \), keeping the number of random sequences fixed, if the weighting matrix \( W \) is chosen optimally, then

\[
T \cdot \arg \min_{\phi'} \left[ \left( m_T - \frac{1}{N} \sum_{n=1}^{N} m_n(\Omega_0, \phi') \right)^\prime \tilde{W} \left( m_T - \frac{1}{N} \sum_{n=1}^{N} m_n(\Omega_0, \phi') \right) \right] \to \chi^2(j - k),
\]

where \( j \) is the number of moments, \( k \) is the number of estimated parameters and \( \phi' \in \mathbb{R}^k \) is the unknown parameter vector.\(^{27}\)

\(^{25}\)We define education via a variable denoting whether respondents have secondary education, finished high school or whether they pursued further education after high school.

\(^{26}\)Cells with less than 10 observations are excluded from the moment conditions. Due to the low prevalence of financial transfers, the medians for this variable were calculated only by education.

\(^{27}\)The standard errors of the parameters are obtained using the Newey and West (1994) weighting
6 Estimation results

Our estimation procedure is still currently running, but preliminary results are available from the authors upon request.

7 Counterfactual simulations

To be completed.

8 Conclusions

To be completed.

References


matrix $W$ and the first order derivatives of the moments conditions with respect to each parameter:

$$se = \frac{1}{\sqrt{T}}(DW^{-1}D')^{-1} \text{ with } D' = \begin{pmatrix} D'_{mT} & \frac{1}{2}m_{N}(\Theta_{0}, \Theta') \end{pmatrix} \bigg|_{\Theta' = \bar{\Theta}}$$
[61] Human Mortality Database. University of California, Berkeley (USA), and Max Planck
A Derivations and proofs

A.1 Euler equations in the parent’s problem

The parents problem is given in the Bellman equations (17-18). Consider first the case of interior solution where the constraint \( w^p_t - C^p_t - f_t \geq 0 \) does not bind.\(^{28}\) The standard combination of the FOCs for the optimal level of consumption with the envelope condition leads to a standard identity between marginal utility evaluated at the optimal consumption level and the partial derivative of the value function with respect to consumable wealth in the same period. Euler equation that characterizes the optimal levels of consumption in two adjacent time periods, however, is complicated by the utility of bequest and the fact that health transition probabilities are dependent on the consumption level through end-of-period wealth. For \( t < 100 \) we have

\[
\frac{\partial u^p_t}{\partial C^p_t}(m_t, C^p_t, T_t) = \beta^p (1 + r) \tilde{E}_{m,\epsilon} \left[ \frac{\partial u^p_{t+1}}{\partial C^p_{t+1}}(m_{t+1}, C^p_{t+1}, T_{t+1}) \right | m_{t+1} \neq \emptyset ] + \beta^p (1 + r) P_{m}\epsilon \tilde{e}_t (1 - \varsigma) u''_b (B_{t+1}) + \beta^p \sum_{m' \neq \emptyset} P_{m'} \tilde{e}_{m'} E_{v} V^p_{t+1}(w^p_{t+1}, m', \psi_{t+1}; M_{t+1}, T_{t+1}),
\]

where \( \tilde{E}(\bullet | \text{condition}) = E(\bullet | \text{condition}) \cdot P(\text{condition}) \) is a shorthand notation used henceforth to denote conditional expectation multiplied with the probability of the condition. The expectations in (30) are taken over the health transitions governed by transition probabilities (7), idiosyncratic component \( \varepsilon^p_t \) in the out-of-pocket medical shocks (13), and independent idiosyncratic component \( \xi^p_t \) in wage equation (22). The latter enters the

\(^{28}\)We assume that when parameters of the utility function (1) are in the range of interest, the non-negativity constraint for consumption never binds, i.e. \( C^p_t > 0 \) unless \( w^p_t = 0 \).
expression because next period assistance time $T_{t+1}$ depends on the children’s circumstances.

The same argument, i.e., the combination of FOCs for $f_t$ and envelope condition, leads to the second Euler equation that characterizes the optimal level of formal insurance $f_t$, namely

$$0 = \beta^p E_{m,z} \left[ \frac{\partial u_{t+1}^p(m_{t+1}, C_{t+1}^p, T_{t+1})}{\partial C_{t+1}^p} \left( -(1 + r) - \frac{\partial \text{oop}_{t+1}(m_{t+1})}{\partial f_t} \right) \right]_{m_{t+1} \neq \mathbb{D}}$$

$$- \beta^p (1 + r) p^m_{m', D}(1 - \zeta) u'_b(B_{t+1}) - \beta^p \sum_{m' \neq D} \frac{\partial p^m_{m', m'}_{p, D}}{\partial s^p_t} E V_{t+1}^p(w_{t+1}^{p, m'}, \psi_{t+1}; M_{t+1}, T_{t+1}),$$

(31)

where the expectation is taken over the same random variables. Rearranging and expanding the first expectation using the transition probabilities (7) and plugging in expression (12) for $\text{oop}_{t+1}(m_{t+1})$ we get

$$(1 + r) E_{m,z} \left[ \frac{\partial u_{t+1}^p(m_{t+1}, C_{t+1}^p, T_{t+1})}{\partial C_{t+1}^p} \right]_{m_{t+1} \neq \mathbb{D}} + (1 + r) p^m_{m', D}(1 - \zeta) u'_b(B_{t+1}) =$$

$$- p^m_{j\in\mathbb{D}}(t) E_{\tilde{e}} \left[ \frac{\partial u_{t+1}(m_{t+1}, C_{t+1}^p, T_{t+1})}{\partial C_{t+1}^p} \cdot 0 \right]$$

$$- p^m_{j\in\mathbb{D}}(t) E_{\tilde{e}} \left[ \frac{\partial u_{t+1}(\mathbb{I}, C_{t+1}^p, T_{t+1})}{\partial C_{t+1}^p} \cdot \frac{\partial \text{oop}(\mathbb{I}) - F_{t+1}(f_t, \mathbb{I}) + \psi_{t+1}}{\partial f_t} \right]$$

$$- p^m_{j\in\mathbb{D}}(t) E_{\tilde{e}} \left[ \frac{\partial u_{t+1}(\mathbb{F}, C_{t+1}^p, T_{t+1})}{\partial C_{t+1}^p} \cdot \frac{\partial \text{oop}(\mathbb{F}) - F_{t+1}(f_t, \mathbb{F}) + \psi_{t+1}}{\partial f_t} \right]$$

$$- p^m_{j\in\mathbb{D}}(t) E_{\tilde{e}} \left[ \frac{\partial u_{t+1}(\text{G}, C_{t+1}^p, T_{t+1})}{\partial C_{t+1}^p} \cdot 0 \right]$$

$$- \beta^p \sum_{m' \neq D} \frac{\partial p^m_{m', m'}_{p, D}}{\partial s^p_t} E V_{t+1}^p(w_{t+1}^{p, m'}, \psi_{t+1}; M_{t+1}, T_{t+1}),$$

(32)

The second and fifth lines in (32) disappear due to the fact that $\text{oop}_{t+1}(1) = \text{oop}_{t+1}(0) = 0$. Further, it follows from (14) that the partial derivatives in the middle two lines equal either 0 or $-\frac{w}{m_{t+1}}$ depending on the sign of the out-of-pocket expenditures. Therefore the
expectations in the middle lines can be expressed using conditional expectations as

\[
\begin{align*}
(1 + r)\tilde{E}_{m,e} \left[ \frac{\partial u_{t+1}^p(m_{t+1}, C_{t+1}^p, T_{t+1})}{\partial C_{t+1}^p} \right]_{m_{t+1} \neq \mathbb{D}} &+ (1 + r)\bar{p}_{m_{t+1}}^m (1 - \zeta) u'_b(B_{t+1}) = \\
+ \frac{\omega}{\mathbb{I}} \tilde{E}_{e} \left[ \frac{\partial u_{t+1}^p(\mathbb{I}, C_{t+1}^p, T_{t+1})}{\partial C_{t+1}^p} \right]_{m_{t+1} = \mathbb{I} \text{ and } h_c(\mathbb{I}) - F_{t+1}(f_t, \mathbb{I}) + \psi_{t+1} > 0} &+ (1 + r)\bar{p}_{m_{t+1}}^m (1 - \zeta) u'_b(B_{t+1}) \\
+ \frac{\omega}{\mathbb{F}} \tilde{E}_{e} \left[ \frac{\partial u_{t+1}^p(\mathbb{F}, C_{t+1}^p, T_{t+1})}{\partial C_{t+1}^p} \right]_{m_{t+1} = \mathbb{F} \text{ and } h_c(\mathbb{F}) - F_{t+1}(f_t, \mathbb{F}) + \psi_{t+1} > 0} &+ (1 + r)\bar{p}_{m_{t+1}}^m (1 - \zeta) u'_b(B_{t+1})
\end{align*}
\]

\[ (33) \]

and further substituting in the expression for \( \psi_{t+1} \) (13) and for \( F_{t+1}(f_t, m_{t+1}) \) (14) to

\[
(1 + r)\tilde{E}_{m,e} \left[ \frac{\partial u_{t+1}^p(m_{t+1}, C_{t+1}^p, T_{t+1})}{\partial C_{t+1}^p} \right]_{m_{t+1} \neq \mathbb{D}} &+ (1 + r)\bar{p}_{m_{t+1}}^m (1 - \zeta) u'_b(B_{t+1}) \\
+ \beta^p \sum_{m' \neq \mathbb{D}} \frac{\partial p_{mm'}^m}{\partial s_t^p} E_v V_{t+1}^p(w_{t+1}, m', \psi_{t+1}; M_{t+1}, T_{t+1}),
\]

\[ (34) \]

\[ \Lambda(m_{t+1}, f_t) = \frac{\mathbb{F}(m_{t+1}) + \frac{\omega f_t}{m_{t+1} - h_c(m_{t+1})}}{(\psi)^{1-p_{c^p} \psi_{t+1}^p}} \]

The resulting expression \( (34) \) is an equation with one unknown choice variable in period \( t \), namely \( f_t \), and therefore can be used to compute the optimal level of formal insurance purchase in period \( t \) conditional on period \( t \) state \( (j = m_t, \psi_t) \), optimal levels of decision variables in period \( t + 1 \) \((C_{t+1}^p, T_{t+1})\), and state transitions from period \( t \) to period \( t + 1 \).

Consider next the first corner solution when the credit constraint is the binding constraints, i.e. \( w_t^p = C_t^p + f_t \). In this case the problem is much simplified due to the fact that \( s_t^p = 0 \) and neither the amount of bequest nor health transition probabilities depend on the choices of consumption \( C_t^p \) or formal health insurance \( f_t \), and the next period value function depends on these choices only through the out-of-pocket health spending. Bearing in mind that \( \frac{\partial h}{\partial C_t^p} = -1 \), we have the Euler equation of the form

\[
\frac{\partial u_t^p(m_t, C_t^p, T_t)}{\partial C_t^p} = \beta^p \tilde{E}_{m,e} \left[ \frac{\partial u_{t+1}^p(m_{t+1}, C_{t+1}^p, T_{t+1})}{\partial C_{t+1}^p} \right]_{m_{t+1} \neq \mathbb{D}} \cdot \frac{\partial o_{t+1}(m_{t+1})}{\partial f_t}
\]

\[ (35) \]
Performing similar expansion of the expectation with respect to transition probabilities of health leads to

\[
\frac{\partial u^p_t(m_t, C^p_t, T_t)}{\partial C^p_t} = \\
= \beta p^\omega \left[ \frac{\partial u^p_{t+1}(I, C^p_{t+1}, T_{t+1})}{\partial C^p_{t+1}} \right] \bigg| m_{t+1} = I \text{ and } \varepsilon_{t+1}^p > \Lambda(I, f_t) \bigg] + \\
+ \beta p^\omega \left[ \frac{\partial u^p_{t+1}(F, C^p_{t+1}, T_{t+1})}{\partial C^p_{t+1}} \right] \bigg| m_{t+1} = F \text{ and } \varepsilon_{t+1}^p > \Lambda(F, f_t) \bigg].
\]

(36)

Next consider the second corner solution, where the binding constraint is \( f_t = 0 \). In this case, the Euler equation (30) is derived in the similar way as in the interior solution, and the level of formal insurance is determined by the binding constraint itself.

Finally, in the third case, when both the above constraints bind, i.e. \( w^p_t = C^p_t + f_t \) and \( f_t = 0 \), the level of consumption is trivially determined from the constraints. In this case the resources are spent solely for consumption.

To summarize, the interior solution is characterized by the system of equations (30) and (34) which equate marginal utility of current consumption and appropriately scaled expected marginal utility of consumption or bequest in the next period with appropriate adjustments for the effects on health transition probabilities, and further to discounted expected of marginal utility of consumption in the next period conditional on positive out-of-pocket health spending. In the two first corner solutions, either former or latter equality is relaxed, and the solution is characterized by a system of binding constraint and a single equation, namely (30) or (36). In the third case, both of the first order condition equations do not hold, and the solution is uniquely determined by the binding constraints.

### A.2 Euler equations in the children’s problem

The children’s problem is given in the Bellman equation (25). Similarly to the parents problem, first consider interior solution case when neither of the constraints is binding.\(^{29}\)

\(^{29}\)We disregard the non-negativity constraint on consumption for the same reasons as in the parents problem, see footnote 28.
Again, standard argument leads to the standard Euler equation

\[
\frac{\partial u^k(C^k_t, m_t)}{\partial C^k_t} = \beta^k (1 + r) E_{m,e} \left[ \frac{\partial u^k_{t+1}(C^k_{t+1}, m_{t+1})}{\partial C^k_{t+1}} \right],
\]

(37)

where the expectation is taken over the shocks in the wage equation (22), out-of-pocket health spending (7) and parent’s health process including survival.

Following a similar argument, namely the combination of first order conditions and the envelope condition, leads to the Euler equation characterizing the optimal level of transfers. For \(M_t\) we have

\[
\beta^k E_{m,e} \left[ \frac{\partial u^k_{t+1}(C^k_{t+1}, m_{t+1})}{\partial C^k_{t+1}} \right] \left( - (1 + r) + \frac{\partial B_{t+1}(m_{t+1})}{\partial M_t} \right) + \beta^k \frac{\partial s^p}{\partial M_t} \sum_{m'} \frac{\partial p_{mm'}^m}{\partial s_t^p} E_x V^k_{t+1}(w^k_{t+1}, edu; m', s^p_t) = 0.
\]

(38)

The derivative of the bequest (24) can be expressed through the derivative of end-of-period assets of the parents \(s^p_t\). Rearranging and expanding the expectation over the health state \(m_{t+1}\) we get

\[
(1 + r) E_{m,e} \left[ \frac{\partial u^k_{t+1}(C^k_{t+1}, m_{t+1})}{\partial C^k_{t+1}} \right] = (1 + r) \frac{\partial s^p}{\partial M_t} E_x \left[ \frac{\partial u^k_{t+1}(C^k_{t+1}, m_{t+1})}{\partial C^k_{t+1}} \right] \bigg|_{m_{t+1} = \mathbb{D}} + \frac{\partial s^p}{\partial M_t} \sum_{m'} \frac{\partial p_{mm'}^m}{\partial s_t^p} E_x V^k_{t+1}(w^k_{t+1}, edu; m', s^p_t).
\]

(39)

Similarly, for \(T_t\) we have

\[
E_{m,e} \left[ W_{t+1} \frac{\partial u^k_{t+1}(C^k_{t+1}, m_{t+1})}{\partial C^k_{t+1}} \right] = (1 + r) \frac{\partial s^p}{\partial T_t} \tilde{E}_x \left[ \frac{\partial u^k_{t+1}(C^k_{t+1}, m_{t+1})}{\partial C^k_{t+1}} \right] \bigg|_{m_{t+1} = \mathbb{D}} + \frac{\partial s^p}{\partial T_t} \sum_{m'} \frac{\partial p_{mm'}^m}{\partial s_t^p} E_x V^k_{t+1}(w^k_{t+1}, edu; m', s^p_t).
\]

(40)

The resulting expressions (39) and (40) are both equations with one unknown choice variable in period \(t\), namely \(M_t\) and \(T_t\), and therefore can be used to compute the optimal level of transfers in period \(t\) conditional on period \(t\) state, optimal levels of decision variables in period \(t + 1\) \((C^p_{t+1}, T_{t+1})\), and state transitions from period \(t\) to period \(t + 1\). Moreover, the unknowns \(M_t\) and \(T_t\) only enter in the derivatives of the parent’s end of period assets \(s^p_t\), which implies that solving (39) and (40) entails simple search for a
point in which the function of parent’s end-of-period assets has a particular value of the corresponding partial derivative.

The number of corner solutions in the children’s problem is higher than in the parent’s problem due to the fact that the corner solutions in monetary transfers may coincide with the corner solutions in time transfers. We consider the two sets of corner solutions separately.

The two corner solutions in time transfer are obtained when constraints \( T_t = 0 \) or \( T_t = L \) bind. The level of time transfer is then trivially determined by the binding constraint that replace equation (40). Equations (37) and (39) remain in the system of first order conditions in this case.

The first corner solution in \( M_t \) follows from the binding credit constraint \( w_t^k = C_t^k + M_t \). Bearing in mind that \( \frac{\partial M_t}{\partial C_t^k} = -1 \), and again combining first order conditions for \( C_t^k \) with envelope condition, we have the following Euler equation for this case

\[
\frac{\partial u_t^k(C_t^k, m_t)}{\partial C_t^k} = \beta^k (1 + r) \frac{\partial s_t^p}{\partial M_t} \hat{E}_t \left[ \frac{\partial u_{t+1}^k(C_{t+1}^k, m_{t+1})}{\partial C_{t+1}^k} \right] m_{t+1} = \mathbb{D}
\]

The second corner solution in monetary transfer is given by the binding constraint \( M_t = 0 \) that replaces equation (39). The optimal level of other controls are determined by equations (37) and (40). Finally, in the third corner solutions in \( M_t \), both of the above constraints bind leading to \( C_t^k = w_t^k \). In this case both equations (37) and (40) are replaced by the binding constraints, so that the level of consumption is equal to the total level of recourses \( w_t^k \), and the optimal level of time transfer is determined by (40) (unless \( T_t \) is at the boundary).

To summarize, the optimal decision \( (C_t^k, M_t, T_t) \) by children is characterized by the system of three equations, namely (37), (39) and (40) if the solution is in the interior. In addition, there are 6 corner solutions, each characterized by the same system in which one, two or all three equations are replaced by the corresponding binding constraints or equation (41). The structure of the system is similar to that of the parents, however the role of formal health insurance \( f_i \) is taken by the monetary transfer \( M_t \), and there is an additional equation to determine the optimal level of assistance time \( T_t \).
A.3 Solution algorithm

Despite that the system of 5 first order conditions (28) is rather complex, and is further complicated by the numerous potential corner solutions, we are able to solve it relatively quickly by implementing the endogenous grid point approach in the following computational algorithm:

1. Compute the policy function in the terminal game when \( t = 100 \).
   This is the base for backward induction.

2. Begin/continue the outer loop over period \( t = t + \Delta \) discrete and discretized state points \( (m_t, \psi_t, edu, \Delta) \). For each point:

3. Compute solution for the parents problem conditional on a grid of possible values of \( T_t \) and \( s^k_t \) (parent’s best response). Namely:
   (a) Fix an auxiliary grids over potential current period assistance time \( T_t \) and over children’s end-of-period wealth \( s^k_t \). For each point:
   (b) Fix a grid over parent’s end-of-period wealth \( s^p_t \). For each point:
   (c) Numerically solve the system of two Euler equations for the parent’s problem (28.a, 28.b) taking into account 3 potential corner solutions as defined by the first column in (29).

The system is solved in the following way. First we solve equation (28.b) which has one unknown current decision \( f_t \) (because current consumption does not enter the next period budget other than through \( s^p_t \) which is fixed at previous step). Given the end-of-period wealth \( s^p_t \) and \( s^k_t \), which enter into the intertemporal budget constraints (16) and (23), the next period state \( \Omega_{t+1} \) is known up to the random transitions of \( m_t \) and idiosyncratic shocks \( \varepsilon^p_t \) and \( \varepsilon^k_t \). We discretize the distribution of the shocks using Gaussian quadrature\(^{30}\) and use transition probabilities (7) of \( m_t \) to compute the expectations that enter the equation. For each plausible point in \( \Omega_{t+1} \) the choice variables, namely \( C^p_{t+1}, T_{t+1} \) and \( M_{t+1} \) are given by the policy functions computed on the previous

\(^{30}\)We make three sets of quadrature points for \( \varepsilon_t^p \) in order to preserve the accuracy of integration in the conditional expectations.
iteration of the backward induction loop started in step 2. Thus, equation (28.b) has only one unknown and can be solved independently.

We then solve equation (28.a) using exact same procedure, given the found optimal level of formal insurance $f_t$ that enters the intertemporal budget (16). One unfortunate deviation from the Carroll (2004) is that the marginal utility function is not invertible in $C^p_t$, so the equation also has to be solved numerically.

(d) One corner solution case requires special treatment. When the credit constraint binds ($s^p_t = 0$) but the level of formal insurance is strictly positive, i.e. first case in first column in (29), in step 3b the grid is fixed over $f_t$ instead of $s^p_t$, and the next step is performed the same way.

(e) Once the optimal levels of current purchase of formal insurance $f_t$ and current consumption $C^p_t$ are found, the amount of consumable resources to which they are attributed is computed using intra-temporal budget (15) completely analogously to the original EGM method. Thus, $w^p_t + M_t = s^p_t + C^p_t + f_t$, and the triplet $(w^p_t + M_t, s^p_t, C^p_t)$ is recorded in the memory as one of potential optimal points conditional on the grid values of step 3a and the amount of monetary transfers $M_t$.

(f) For the low values of parents wealth $w^p_t$ and children’s transfers $M_t$ it is not clear in which order the three corner solutions follow. Presumably, for very low wealth it holds $C^p_t = w^p_t$, $s^p_t = f_t = 0$; for large values of wealth, the interior solution is most likely. In any case, to distinguish between the relevant corner solutions, we invoke the upper envelope calculation procedure which compares value functions implied by the different solutions, and computes the the thresholds separating them.

(g) Proceed with the next point in steps 3b and 3a until the points are exhausted.

4. With the parent’s solution at hand, and in particular the optimal reaction function $s^p_t (w^p_t + M_t, T_t, s^k_t)$, which is calculated on the fixed grid over $(T_t, s^k_t)$ and the endogenous grid over $(w^p_t + M_t)$, proceed to the solution of children’s problem. Namely,
(a) For every point \((w^p_t + M_t)\) of the endogenous grid produced in step 3:

(b) Fix a grid over children’s end-of-period wealth \(s^k_t\), preferably the same grid as in step 3a. For each point:

(c) Numerically solve the system of three Euler equations for the children’s problem (28.c, 28.d, 28.e) taking into account 11 potential corner solutions as defined by the second and third columns in (29).

The system is solved in a similar way to the one in parent’s problem. First we solve equation (28.e) which has one unknown current decision \(T_t\) (because current consumption and monetary transfers do not enter the next period budget other than through \(s^k_t\) which is fixed at previous step). A given trial value \(T_t\) together with \((w^p_t + M_t)\) and \(s^k_t\) determine parent’s purchase of formal insurance \(f_t\) and end-of-period wealth \(s^p_t\). On the other hand, the trial value \(T_t\) affects the next period wealth of the children. Thus, the next period state \(\Omega_{t+1}\) is known up to the random transitions of \(m_t\) and shocks \((\varepsilon^p_t, \varepsilon^k_t)\). We again use two dimensional Gaussian quadrature for the shocks and transition probabilities (7) of \(m_t\) to compute the expectations that enter the equation. For each plausible point in \(\Omega_{t+1}\) the choice variables, namely \(C^{k}_{t+1}\) which enters marginal utility, is given by the policy functions computed on the previous iteration of the backward induction loop started in step 2. Thus, equation (28.e) has only one unknown and is solved independently.

Equations (28.c) and (28.d) can then be solved in any order. Given the optimal choice of assistance time \(T_t\), the right hand side in (28.c) can be computed directly, and because the marginal utility of children’s consumption is invertible, the optimal \(C^k_t\) is computed directly.

Finally, equations (28.d) entails finding the point \(M_t\) at which the derivative \(\frac{\partial s^p_t}{\partial M_t} = \frac{\partial s^p_t}{\partial (w^p_t + M_t)}\) takes a particular value, which can be calculated directly. The derivative is approximated from the solution for the parent’s problem, given already known \(T_t\) and fixed \(s^k_t\). Because the computed policy functions for the parent’s problem are not conditional on \(M_t\), the solution is determined up to a constant, namely only \((w^p_t + M_t)\) is determined.
(d) Similar to the parent’s problem, one corner solution case also requires special treatment here. When credit constraint binds \( s^k_t = 0 \) but the level of monetary transfer is strictly positive, i.e. first case in third column in (29), in step 4b the grid is fixed over \( M_t \) instead of \( s^k_t \), and the next step is performed the same way.

(e) Once the optimal levels of the “aim” for consumable resources for parents \( (w^p_t + M_t) \) are computed, corresponding level of children’s resources is computed using intra-temporal budget \( w^k_t = s^k_t + C^k_t + M_t \). Even though \( M_t \) is not determined exactly, the vector \( (w^p_t + M_t, w^k_t + w^p_t, C^k_t, M^k_t, T^k_t) \) is recorded in the memory as one of potential optimal points conditional on the grid values of step 4a and the amount of monetary transfer \( M_t \).

(f) Again, to distinguish between the relevant corner solutions, we invoke the upper envelope calculation procedure which compares value functions implied by the different solutions, and computes the thresholds separating them.

(g) Proceed with the next point in steps 4b and 4a until the points are exhausted.

5. Combine the solutions of parent’s and children’s problems through and additional grid on \( M_t > 0 \). For each point \( M' \) on this grid, compute the corresponding values of parent’s and children’s wealth from the endogenous grid points \( (w^p_t + M_t, w^k_t + w^p_t) \) found in steps 3 and 4, i.e. \( w^p_t = w^p_t + M_t - M' \), \( w^k_t = (w^k_t + w^p_t) - (w^p_t + M_t - M') \), and associate them with the corresponding values of the choice variables from the solutions found on these grids. This concludes the computations for given point in the discretized state space.

6. Proceeds with the next iteration of the backward induction started in step 2 until \( t = 65 \) is reached.

**B Health and Funeral Costs**

When no country data on long term care or curative and rehabilitation care was available, we used the benchmark countries below. The primary data for funeral costs in the OECD countries analyzed are drawn from the AGIR dataset (Westerhout and Pellikaan 2005, based on EPC 2001) for EU-15 countries and from OECD calculations for 2005. To obtain
the 2004 funeral costs, we applied the health expenditure real growth rate to the 2005 series (OECD Health Data 2008). The cost of death for the oldest group (95+) is assumed to be the lowest and was proxy by their observed health expenditure per person, when available. For France, Germany, Italy, Spain and Netherlands, where expenditure data for the oldest group were not available, the cost of people aged 75-79 was taken as a proxy. In fact, when available, expenditure at age 95+ is roughly equal to the level of expenditure at age 75-79. For the countries with no data available, the cost of death for the oldest group was estimated by taking three times the average health expenditure per capita, adjusted by the country-specific residual (Björnerud et al., 2005; OECD, 2006).

<table>
<thead>
<tr>
<th>Country estimated</th>
<th>Benchmark countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>Netherlands</td>
</tr>
<tr>
<td>Denmark</td>
<td>average (Norway, Sweden)</td>
</tr>
<tr>
<td>France</td>
<td>Germany</td>
</tr>
<tr>
<td>Greece</td>
<td>Spain</td>
</tr>
<tr>
<td>Italy</td>
<td>average (Germany, Spain)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>Germany</td>
</tr>
</tbody>
</table>
### Table 1. Health Care in Europe, Sources of Funding

<table>
<thead>
<tr>
<th>Country</th>
<th>Average Health Care Costs per Capita (€)</th>
<th>Public Expenditure%</th>
<th>% of Population with Private Medical Insurance</th>
<th>Private Premium per Insured (€)</th>
<th>Average Benefits paid+</th>
<th>Claims Ratio$</th>
<th>% of Population with Private Insurance</th>
<th>Out-of-Pocket Costs +^</th>
<th>% Receiving Transfers from Family</th>
<th>Median Amount from Family (€)</th>
<th>% Receiving Transfers from Family</th>
<th>Median Amount from Family (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>2,658</td>
<td>73.8%</td>
<td>98.0%</td>
<td>3.2%</td>
<td>529</td>
<td>4.5%</td>
<td>86%</td>
<td>31.1%</td>
<td>16.9%</td>
<td>6.0%</td>
<td>800.0</td>
<td>32.2%</td>
</tr>
<tr>
<td>Belgium</td>
<td>2,639</td>
<td>75.6%</td>
<td>99.0%</td>
<td>4.9%</td>
<td>119</td>
<td>1.7%</td>
<td>94%</td>
<td>44.1%</td>
<td>17.6%</td>
<td>1.5%</td>
<td>1250.0</td>
<td>41.1%</td>
</tr>
<tr>
<td>Switzerland</td>
<td>4,527</td>
<td>58.5%</td>
<td>100.0%</td>
<td>8.7%</td>
<td>2,370</td>
<td>8.7%</td>
<td>76%</td>
<td>22.9%</td>
<td>31.9%</td>
<td>5.9%</td>
<td>847.4</td>
<td>23.7%</td>
</tr>
<tr>
<td>Germany</td>
<td>2,723</td>
<td>73.4%</td>
<td>89.8%</td>
<td>9.3%</td>
<td>1,087</td>
<td>9.3%</td>
<td>124%</td>
<td>29.6%</td>
<td>19.2%</td>
<td>3.8%</td>
<td>600.0</td>
<td>36.6%</td>
</tr>
<tr>
<td>Denmark</td>
<td>3,166</td>
<td>84.1%</td>
<td>100.0%</td>
<td>1.5%</td>
<td>n.a.</td>
<td>6.6%</td>
<td>n.a.</td>
<td>n.a.</td>
<td>14.0%</td>
<td>2.2%</td>
<td>1344.3</td>
<td>31.4%</td>
</tr>
<tr>
<td>Spain</td>
<td>1,565</td>
<td>70.1%</td>
<td>99.5%</td>
<td>5.9%</td>
<td>373</td>
<td>4.5%</td>
<td>76%</td>
<td>25.6%</td>
<td>23.0%</td>
<td>5.6%</td>
<td>1532.0</td>
<td>22.1%</td>
</tr>
<tr>
<td>France</td>
<td>2,317</td>
<td>78.5%</td>
<td>99.9%</td>
<td>13.2%</td>
<td>451</td>
<td>3.1%</td>
<td>79%</td>
<td>21.9%</td>
<td>21.2%</td>
<td>2.1%</td>
<td>1500.0</td>
<td>34.9%</td>
</tr>
<tr>
<td>Italy</td>
<td>1,921</td>
<td>75.0%</td>
<td>100.0%</td>
<td>*(2006) 0.9%</td>
<td>n.a.</td>
<td>1.0%</td>
<td>80%</td>
<td>*(2006) 6.1%</td>
<td>21.3%</td>
<td>0.9%</td>
<td>578.4</td>
<td>23.4%</td>
</tr>
<tr>
<td>Netherlands</td>
<td>2,321</td>
<td>70.1%</td>
<td>71.2%</td>
<td>13.2%</td>
<td>590</td>
<td>18.1%</td>
<td>83.3%</td>
<td>87.3%</td>
<td>18.8%</td>
<td>2.4%</td>
<td>2250.0</td>
<td>32.6%</td>
</tr>
<tr>
<td>Greece</td>
<td>1,608</td>
<td>61.8%</td>
<td>100.0%</td>
<td>*(2002) 1.6%</td>
<td>n.a.</td>
<td>n.a.</td>
<td>n.a.</td>
<td>*(2002) 12%</td>
<td>5.9%</td>
<td>16.8%</td>
<td>821.3</td>
<td>44.0%</td>
</tr>
<tr>
<td>Sweden</td>
<td>2,837</td>
<td>82.3%</td>
<td>100.0%</td>
<td>*(2006) 0.3%</td>
<td>417</td>
<td>0.1%</td>
<td>n.a.</td>
<td>1.9%</td>
<td>17.0%</td>
<td>3.3%</td>
<td>762.5</td>
<td>38.0%</td>
</tr>
</tbody>
</table>

+ As percent of total health care expenditure. § As percent of premium.

Sources: CEA Statistics No. 41: The European Health Insurance Market in 2008, ^ OECD Health Data: Health expenditure and financing,
* Thomson and Mossialos (2009), # Share 2004 data (singles, age 65+, transfers received annually)
## Table 2. Formal Premium and Out-of-Pocket Medical Spending - SHARE

<table>
<thead>
<tr>
<th>Country</th>
<th>Prevalence (%) of Formal premium*</th>
<th>OOP costs^</th>
<th>Premium per insured (€) Median</th>
<th>Mean</th>
<th>Individual OOP costs (€) Median</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweden</td>
<td>1.8</td>
<td>91.7</td>
<td>960.00</td>
<td>1961.74</td>
<td>2600.00</td>
<td>4436.66</td>
</tr>
<tr>
<td>Denmark</td>
<td>26.6</td>
<td>86.1</td>
<td>2000.00</td>
<td>1995.74</td>
<td>2000.00</td>
<td>3497.34</td>
</tr>
<tr>
<td>Germany</td>
<td>17.5</td>
<td>88.0</td>
<td>800.00</td>
<td>1869.34</td>
<td>140.00</td>
<td>357.01</td>
</tr>
<tr>
<td>Netherlands</td>
<td>74.3</td>
<td>46.8</td>
<td>440.00</td>
<td>741.82</td>
<td>226.10</td>
<td>545.00</td>
</tr>
<tr>
<td>Belgium</td>
<td>70.9</td>
<td>96.1</td>
<td>181.70</td>
<td>260.58</td>
<td>400.00</td>
<td>893.29</td>
</tr>
<tr>
<td>France</td>
<td>85.2</td>
<td>37.3</td>
<td>715.80</td>
<td>804.05</td>
<td>120.00</td>
<td>409.53</td>
</tr>
<tr>
<td>Switzerland</td>
<td>49.7</td>
<td>74.5</td>
<td>450.00</td>
<td>1296.52</td>
<td>460.00</td>
<td>1328.40</td>
</tr>
<tr>
<td>Austria</td>
<td>21.3</td>
<td>73.0</td>
<td>600.00</td>
<td>912.50</td>
<td>175.00</td>
<td>459.29</td>
</tr>
<tr>
<td>Italy</td>
<td>3.3</td>
<td>77.1</td>
<td>400.00</td>
<td>406.13</td>
<td>200.00</td>
<td>429.82</td>
</tr>
<tr>
<td>Spain</td>
<td>6.4</td>
<td>28.1</td>
<td>700.00</td>
<td>831.70</td>
<td>138.70</td>
<td>485.07</td>
</tr>
<tr>
<td>Greece</td>
<td>3.2</td>
<td>90.7</td>
<td>309.40</td>
<td>533.27</td>
<td>256.50</td>
<td>624.14</td>
</tr>
<tr>
<td>Mean across Europe</td>
<td>32.7</td>
<td>71.8</td>
<td>686.99</td>
<td>1055.76</td>
<td>610.57</td>
<td>1224.14</td>
</tr>
</tbody>
</table>

* Share of people insured privately in the full sample

^ Share of people with positive out-of-pocket medical spending in the full sample
Table 3. Financial Transfers (FT) - SHARE

<table>
<thead>
<tr>
<th>Country</th>
<th>Prevalence (%) of FT from kids*</th>
<th>Amount of € from kids Median</th>
<th>Amount of € received Mean</th>
<th>Amount of € from kids Median</th>
<th>Amount of € from kids Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>from kids in total FT^</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td>2.3</td>
<td>5,000.00</td>
<td>26,562.13</td>
<td>7,000.00</td>
<td>9,548.53</td>
</tr>
<tr>
<td>Denmark</td>
<td>1.5</td>
<td>9,000.00</td>
<td>13,494.64</td>
<td>9,000.00</td>
<td>10,670.70</td>
</tr>
<tr>
<td>Germany</td>
<td>1.9</td>
<td>600.00</td>
<td>987.19</td>
<td>500.00</td>
<td>1,239.74</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1.8</td>
<td>894.70</td>
<td>1,473.98</td>
<td>894.70</td>
<td>973.79</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.6</td>
<td>1,671.60</td>
<td>7,168.27</td>
<td>500.00</td>
<td>1,072.10</td>
</tr>
<tr>
<td>France</td>
<td>1.0</td>
<td>2,016.00</td>
<td>2,589.76</td>
<td>2,016.00</td>
<td>2,683.31</td>
</tr>
<tr>
<td>Switzerland</td>
<td>2.7</td>
<td>1,126.00</td>
<td>2,078.76</td>
<td>1,300.00</td>
<td>2,829.27</td>
</tr>
<tr>
<td>Austria</td>
<td>5.1</td>
<td>500.00</td>
<td>2,128.61</td>
<td>480.00</td>
<td>911.60</td>
</tr>
<tr>
<td>Italy</td>
<td>1.6</td>
<td>350.00</td>
<td>437.09</td>
<td>532.90</td>
<td>584.90</td>
</tr>
<tr>
<td>Spain</td>
<td>4.9</td>
<td>1,087.80</td>
<td>1,808.47</td>
<td>1,000.00</td>
<td>1,737.12</td>
</tr>
<tr>
<td>Greece</td>
<td>15.1</td>
<td>950.00</td>
<td>1,264.12</td>
<td>950.00</td>
<td>1,302.43</td>
</tr>
<tr>
<td>Mean across Europe</td>
<td>3.5</td>
<td>2,108.74</td>
<td>5,453.91</td>
<td>2,197.60</td>
<td>3,050.32</td>
</tr>
</tbody>
</table>

* Share of people receiving positive financial transfers from kids in the full sample
^ Share of people receiving positive financial transfers from kids in the sample of people receiving financial transfers
<table>
<thead>
<tr>
<th>Country</th>
<th>Prevalence (%) of TT</th>
<th>Amount of hours received</th>
<th>Amount of hours from kids</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>from kids</strong></td>
<td><strong>from kids in total TT</strong></td>
<td><strong>Median</strong></td>
</tr>
<tr>
<td>Sweden</td>
<td>27.0</td>
<td>61.5</td>
<td>53</td>
</tr>
<tr>
<td>Denmark</td>
<td>25.5</td>
<td>57.4</td>
<td>60</td>
</tr>
<tr>
<td>Germany</td>
<td>30.8</td>
<td>62.5</td>
<td>220</td>
</tr>
<tr>
<td>Netherlands</td>
<td>20.0</td>
<td>46.3</td>
<td>52</td>
</tr>
<tr>
<td>Belgium</td>
<td>28.1</td>
<td>55.1</td>
<td>116</td>
</tr>
<tr>
<td>France</td>
<td>23.8</td>
<td>59.5</td>
<td>156</td>
</tr>
<tr>
<td>Switzerland</td>
<td>17.4</td>
<td>52.0</td>
<td>52</td>
</tr>
<tr>
<td>Austria</td>
<td>21.9</td>
<td>57.0</td>
<td>260</td>
</tr>
<tr>
<td>Italy</td>
<td>13.9</td>
<td>47.9</td>
<td>312</td>
</tr>
<tr>
<td>Spain</td>
<td>17.2</td>
<td>56.8</td>
<td>260</td>
</tr>
<tr>
<td>Greece</td>
<td>39.3</td>
<td>72.8</td>
<td>275</td>
</tr>
<tr>
<td>Mean across Europe</td>
<td>24.1</td>
<td>57.2</td>
<td>165</td>
</tr>
</tbody>
</table>

* Share of people receiving positive time transfers from kids in the full sample

^ Share of people receiving positive time transfers from kids in the sample of people receiving time transfers