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Macroprudential Policy: Its Effects and Relationship to Monetary Policy*

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Abstract

This paper examines the interactions of macroprudential policy and monetary policy in a New Keynesian DSGE model with financial frictions. Macroprudential policy can stabilize credit cycles. However, a macroprudential instrument that aims to stabilize a specific segment of the credit market can cause regulatory arbitrage, that is, a reallocation of credit to a less regulated part of the market. Within this model, welfare-maximizing monetary policy aims to stabilize only inflation and macroprudential policy only stabilizes credit. Two aspects of the model account for this dichotomy. First, credit stabilization is welfare improving because lower volatility is compensated by higher mean equilibrium credit and capital. Second, monetary policy is sub-optimal for credit stabilization. The reason is that it operates on the decisions of borrowers and savers, while macroprudential policy operates only on the decisions of borrowers.

JEL Classification: E44, E50, E61

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1. Introduction

The recent financial crisis shows that problems in the financial sector can damage the real economy. In response, policymakers and economists are reconsidering the role of policies aimed at financial stability. Among these are macroprudential policies. Macroprudential policies are regulatory instruments mainly imposed on the credit intermediation process to ex-ante prevent the buildup of risks that can produce financial crises. In this paper, I focus on a specific question regarding macroprudential policy, that is, how macroprudential policy and monetary policy should interact to jointly achieve financial stability and existing mandates of monetary policy, such as inflation and output gap stability.

Woodford (2012) and Svensson (2012) debate the roles of monetary and macroprudential policies in achieving financial stability. Their debate concerns whether financial stability is better achieved through monetary policy adjusting short-term interest rates or using macroprudential policy to control credit intermediation. Woodford argues that using interest rate policy to maintain financial stability can be justified even with macroprudential instruments, as long as the latter cannot provide a complete solution for financial stability. On the other hand, Svensson argues that it is more efficient to assign monetary policy to focus on inflation stability alone and use macroprudential policy for financial stability because the latter policy directly affects leverage.

In this paper, I address the above question using a New Keynesian dynamic stochastic general equilibrium (NKDSGE) model with a Bernanke, Gertler, and Gilchrist (1999) financial accelerator mechanism. This type of model has been popular in studying the implication of financial frictions in macroeconomics, for example, Christiano, Motto, and Rostagno (2008) and Fernandez-Villaverde (2010). I investigate the effects of macroprudential policy on the dynamics of the economy using this model. Next, welfare-maximizing optimal monetary and macroprudential policy rules are constructed conditional on this model. My focus is the interactions between interest rate policy and credit intermediation policy, within the Bernanke-Gertler-Gilchrist (BGG) NKDSGE model, under the criterion of optimal policy. Within this framework, I address the issue of whether it is optimal for monetary policy to react countercyclically to movements in credit demand.

The BGG-NKDSGE model has financial accelerator mechanisms in business and household lending. These mechanisms generate an interaction between default possibility and borrower's net worth (in business lending) or collateral value (in mortgage lending). Financial intermediaries are exposed to aggregate uncertainty. Bank capital functions as a buffer stock to absorb the risks in this environment. The macroprudential policy instruments available to the policymakers are countercyclical capital requirement and loan-to-value (LTV) ratio regulations. I assume, for comparison purposes, that the former is universal regulation affecting the household and business credit markets, while the latter is a market-specific regulation applied only to household credit.

The results in this paper indicate that rule-based countercyclical macroprudential policy plays a stabilization role for business and credit cycles. However, LTV ratio regulation, a household credit market specific instrument, increases the volatility of the business sector by generating regulatory arbitrage. In this case, regulatory arbitrage involves reallocating credit from the household sector to the business sector.

My analysis shows that macroprudential policy is welfare improving. Welfare gains mostly come from the countercyclical capital requirement regulation, but the gains from the LTV ratio regulation are small. The optimal policy combination features monetary policy stabilizing only inflation and the capital requirement regulation stabilizing only credit. There is a separation of objectives between the two policies in this case. Two components of the model account for this dichotomy. First, stabilizing credit is welfare improving. This is a property of BGG financial contract in which default costs limit the financial intermediary's ability to supply credit, and reduced uncertainty in credit activity is compensated by higher credit in equilibrium. Second, macroprudential policy is a better tool for credit stabilization than monetary policy. Although monetary policy has spillover effects in that it alters the decisions of borrowers and savers, macroprudential policy only operates on the decision of borrowers. Thus, using monetary policy to stabilize credit is 'too blunt' an instrument to achieve financial stability.

2. Macroprudential Policy: Concepts, Instruments and Issues

As mentioned, macroprudential policy refers to a set of regulatory policies imposed mainly on the credit intermediation process for purposes of macroeconomic stabilization. According to the BIS (2010), there are two, not mutually exclusive, objectives of macroprudential policy: to strengthen the financial system's resilience against adverse shocks in the economy and to actively limit the buildup of financial systemic risk.[1] Being preventive in nature, it aims for ex-ante stabilization and should be distinguished from an ex-post crisis management policy. Macroprudential policy instruments include a wide range of financial regulation measures. Table 1 summarizes the examples of macroprudential policy instruments that are already in practice or proposed. They can be classified by the channels in

[1] BIS (2010) defines systemic risk as "a risk of disruption to financial services that is caused by an impairment of all or parts of the financial system and has the potential to have serious negative consequences for the real economy."

which they operate, such as financial intermediaries' balance sheets, the terms and conditions of credit contracts and transactions, and market structure.

Table 1: Examples of macroprudential policy instruments

Category	Instruments	Description	Adopted by
Balance sheet instruments	Countercyclical capital buffer	Increase capital buffer in expansion, release it in downturn	Basel III Spain(2000, provisioning)
	Sectoral capital requirements	Require additional capital on lending to specific sectors	Australia (2004), India (2005)
	Maximum leverage ratio	Cap on the ratio of total (non-risk-adjusted) assets to bank equity	Basel III, Canada (1980s), Swiss (2011)
	Time-varying liquidity buffer	Increase liquidity ratio in expansion, decrease it in downturn	Croatia (2003), New Zealand (2008)
Terms and conditions of transactions	Loan to value ratio	Cap on the ratio of loan value to collateral value	Many countries
	Debt service to income ratio	Cap on the ratio of debt service to borrower's income	Many countries
Market structure	Use of central counterparty	Financial trade using centralized clearing center rather than OTC	Many countries

Source: Bank of England (2010)

Although some of these macroprudential policy measures are already being implemented in practice, there are still several questions about macroprudential policy in need of further study. In particular, the following issues are especially of interest in this paper. First is the question regarding the joint implementation of monetary and macroprudential policy mentioned in the introduction. Second, policymakers should take into account the general equilibrium effect from regulatory arbitrage when implementing sector- or market-specific policy instruments. Some macroprudential instruments are designed as sector- or market-specific, aiming to adjust the imbalance within a particular sector or market. Examples of such instruments are sectoral capital requirements, loan-to-value (LTV) or debt-service-to-income (DTI) caps for household lending. However, these instruments can create a regulatory arbitrage, a movement of funds from heavily regulated markets to less regulated markets.

In the next section, I suggest a model that can be used to assess the above issues about macroprudential policy.[2]

3. Model

The model in this paper is based on the BGG financial accelerator mechanism in a New Keynesian framework. The financial intermediation in this paper mainly follows that of Zhang (2009), who supplements the traditional BGG mechanism by introducing risk sharing into the banking sector. Bank capital then functions as a buffer stock to absorb the profit or loss caused by the difference between the expected and realized value in aggregate return. In addition, I distinguish between saving households and borrowing households, so that borrowing households and entrepreneurs are dual borrowers in the economy (household and business credit). Housing goods are not only in the utility function but also used as collateral for the borrowing households. Monetary policy, by choosing the nominal interest rate, influences saving households' real deposit rate, which also affects lending rates. Macroprudential policy affects bank real lending rates by imposing regulations on the bank. A summary of variables used in the model is provided in appendix A.1.

3.1. Households

There exist two types of households, savers (s) and borrowers (b), who are distinguished by time preference parameters. Borrowing households are less patient about future consumption than saving households ($\beta_b < \beta$). This distinction by time preference parameters is often used in credit friction models, as in Iacoviello (2005). Because of different time preferences, saving households will always save and borrowing households will always borrow in the steady state and its neighborhood. I assume that both types have the same population, as if there is one saving member and one borrowing member in a single household. Agents do not move between the two groups.

[2]There are also several other issues about macroprudential policy that are not explicitly addressed in this paper. One example is the rule vs. discretion in policy action. In most countries, the financial regulatory instruments mentioned above are conducted largely at the policymaker's discretion, except for Spain's dynamic provisioning rule (IMF (2010)), which requires banks to set aside additional provisions according to a formula during phases of rapid credit expansion. Despite these practices, a rule-based approach can still be an appealing option. It can better anchor agents' expectations about future regulatory policies, and overcome the bias for the regulator's inaction facing strong political and market resistance. This paper assumes rule-based macroprudential policy.

3.1.1. *Saving Household*

Saving households (denoted by s) with future discount rate β solve

$$\max_{C_s,H_s,N_s,I_s^H,B,D,e} E_o \left\{ \sum_{t=0}^{\infty} \beta^t [\gamma \log C_{t,s} + (1-\gamma\epsilon_t^\gamma) \log H_{t,s} + \varphi \log(1-N_{t,s})] \right\} \tag{1}$$

subject to budget constraint

$$C_{t,s} + P_t^H I_{t,s}^H + \frac{B_t}{P_t} + D_t + e_t + T_{t,s} \leq R_{t-1}^N \frac{B_{t-1}}{P_t} + R_{t-1}^D D_{t-1} + R_{t-1}^e e_{t-1} + w_t N_{t,s} + Div_t \tag{2}$$

and the law of motion for the housing stock

$$H_{t,s} = (1-\delta_H) H_{t-1,s} + I_{t,s}^H. \tag{3}$$

C, H, N denotes consumption goods, housing goods and labor supply, respectively. ϵ^γ is a preference shock on housing goods, such that a positive ϵ^γ shock can cause housing demand to drop. In the budget constraint, P^H is the relative price of housing goods in terms of final consumption goods and I^H is the investment in housing goods. Saving households can invest in an asset portfolio that consists of nominal assets (B), real bank deposits (D) and real bank equity capital (e). Each asset yields return R^N, R^D and R^e. For the rest of the paper I assume all credits in the economy are real credits, by assuming that the equilibrium quantity of nominal assets is zero. w is the real wage, Div is dividends from entrepreneurs, and T_s is a lump-sum tax equally imposed on both savers and borrowers. Appendix A.2 provides the details of the saving households' optimization problem.

3.1.2. *Borrowing Household*

Borrowing households (denoted by b) with future discount rate β_b maximize

$$\max_{C_b,H_b,N_b,I_b^H,L^H} E_o \left\{ \sum_{t=0}^{\infty} \beta_b^t [\gamma \log C_{t,b} + (1-\gamma\epsilon_t^\gamma) \log H_{t,b} + \varphi \log(1-N_{t,b})] \right\} \tag{4}$$

subject to

$$C_{t,b}+P_t^H I_{t,b}^H+\int_0^{\bar{\omega}_t^{H,b}} \omega^H P_t^H H_{t,b} f(\omega^H) d\omega^H+[1-F(\bar{\omega}_t^{H,b})] R_{t-1}^{LH} L_{t-1}^H+T_{t,b} \leq w_t N_{t,b}+L_t^H. \quad (5)$$

and

$$H_{t,b} = (1-\delta_H) H_{t-1,b} + I_{t,b}^H. \quad (6)$$

L^H, R^{LH} denote real household borrowing and its interest rate. Other terms are defined similarly to the saving households' problem. ω^H is an idiosyncratic shock in the housing price, with $E(\omega^H) = 1$ and hits each household after aggregate variables are determined. Borrowing households use housing goods as collateral, and default occurs when ω^H falls below the default threshold $\bar{\omega}^{H,b}$, which is set according to the debt repayment value. When default occurs, the lender claims the remaining value of housing stock ($=\int_0^{\bar{\omega}_t^{H,b}} \omega^H P_t^H H_{t,b} f(\omega^H) d\omega^H$). For computational convenience, I assume that defaulting households pay the cash value of their housing goods to the lender rather than losing actual housing stock. This way they keep their housing stock and resume economic activity. A more detailed description of the household borrowing contract is given in section 3.3.2. Finally, $[1 - F(\bar{\omega}_t^{H,b})]$ is the fraction of households that avoid default and they redeem $[R_{t-1}^{LH} L_{t-1}^H]$ amount of the debt obligations. In appendix A.2, the borrowing households' optimization problem is provided in detail. In the steady state, borrowing households have to provide more labor and consume less than saving households to pay interest on their debt.

3.2. Entrepreneurs

Entrepreneurs produce intermediate goods using capital (K), labor (N), entrepreneurs' labor (N_e) and bankers' labor (N_f). The production technology includes entrepreneurs' and bankers' labor, and those labor incomes are added to entrepreneurs' net worth and bank capital. However, their contribution to aggregate output is assumed to be very small ($\alpha_{ne} = \alpha_{nf} = 0.01$).

$$Y_t = A_t (K_{t-1}^{\alpha_k})(N_t^{\alpha_n})(N_{t,e}^{\alpha_{ne}})(N_{t,f}^{\alpha_{nf}}), \quad \alpha_k + \alpha_n + \alpha_{ne} + \alpha_{nf} = 1. \quad (7)$$

Gross return from one unit of capital is defined by:

$$R_t^K = \frac{z_t + (1-\delta)q_t}{q_{t-1}}, \tag{8}$$

where q is the price of capital in terms of consumption goods, and z_t is the value of the marginal product of capital ($z_t = mc_t \cdot \alpha_k Y_t/K_{t-1}$). R_t^K is the return on capital chosen in the last period (K_{t-1}), determined only when this period's aggregate shocks are realized. It reflects the price changes (q_t/q_{t-1}) of capital as well as z_t. Given real marginal cost mc_t, labor demand for each type of labor is given by

$$w_t = mc_t \cdot \alpha_n \frac{Y_t}{N_t}, \quad w_{t,e} = mc_t \cdot \alpha_{ne} \frac{Y_t}{N_{t,e}}, \quad w_{t,f} = mc_t \cdot \alpha_{nf} \frac{Y_t}{N_{t,f}}. \tag{9}$$

Here labor supply of the entrepreneurs and the bankers is fixed at 1.

Entrepreneurs' net worth, denoted by W, is determined by their labor income and retained earnings in the investment project.

$$W_t = \upsilon V_t + w_{t,e} \tag{10}$$

where $(1-\upsilon)$ is the fraction that is paid to saving households as dividends ($Div_t = (1-\upsilon)V_t$), and V is the return from each period's project net of the borrowing cost.

$$V_t = \int_{\bar{\omega}_t^b}^{\infty} \omega R_t^K q_{t-1} K_{t-1} f(\omega) d\omega - (1 - F(\bar{\omega}_t^b)) R_{t-1}^{LB} L_{t-1}^B. \tag{11}$$

The first term on the right-hand side is the gross payoff for entrepreneurs, and the second term is the debt repayment obligation to the lender. L^B is the amount of borrowing and R^{LB} is its interest rate. ω is an idiosyncratic shock that hits each entrepreneur and $\bar{\omega}^b$ is the default threshold determined by the debt repayment obligation ($R_{t-1}^{LB} L_{t-1}^B = \bar{\omega}^b R_t^K q_{t-1} K_{t-1}$). If an entrepreneur defaults, ($\omega < \bar{\omega}^b$), he will end up with nothing and the remaining value of the project will be accrued to the bank. The lending contract between entrepreneurs and the financial intermediary is further discussed below.

3.3. Financial Contract and Banking Sector: BGG Mechanism

Financial contracts in this paper are based on Zhang (2009), who refines the BGG model by introducing a risk-sharing banking sector and bank capital. In the original BGG model, there is no need for the financial intermediary ('bank' hereafter) to set aside buffer capital to perform the intermediation, since it can diversify its idiosyncratic risk and is fully insured against aggregate risk. The bank is insured against aggregate risk since risk-neutral entrepreneurs offer an aggregate-state-contingent default threshold and debt repayment value to guarantee the bank zero profit in all states. However, in the real world, it would be natural to believe that the bank is also exposed to aggregate risk, and its profit and capital are affected. Zhang's model has a financial contract in which the debt repayment value is not state-contingent and is set according to the next period's expected aggregate return. Thus, there are ex-ante default threshold that determines the debt repayment value and the ex-post default threshold that determines the actual default. The forecast error in the next period's return on capital or housing value creates a discrepancy between the expected default rate and the actual default rate, causing profit or loss to the bank, which is reflected in the changes in bank capital.

3.3.1. *Financial Contract: Business Loan*

The size of a business loan is defined by the difference between the size of the investment project and the entrepreneurs' net worth. That is, $L_t^B = q_t K_t - W_t$. The debt repayment value is not state-contingent, and entrepreneurs offer the bank an ex-ante default threshold, chosen from the distribution of the idiosyncratic shock (ω) given the next period's expected aggregate return on capital ($E_t R_{t+1}^K$). Denoting this ex-ante default threshold by $\bar{\omega}_t^a$, the relationship between the gross loan repayment value and the expected project return is

$$R_t^{LB} L_t^B = \bar{\omega}_t^a E_t R_{t+1}^K q_t K_t. \tag{12}$$

Entrepreneurs' optimization problem becomes

$$\max_{K_t, \bar{\omega}_t^a} \int_{\bar{\omega}_t^a}^{\infty} \omega E_t R_{t+1}^K q_t K_t f(\omega) d\omega - (1 - F(\bar{\omega}_t^a)) R_t^{LB} L_t^B. \tag{13}$$

subject to the bank's ex-ante participation incentive constraint

$$R_t^f(q_tK_t - W_t) = (1 - F(\bar{\omega}_t^a))R_t^{LB}L_t^B + (1-\mu)\int_0^{\bar{\omega}_t^a} \omega E_t R_{t+1}^K q_t K_t f(\omega)d\omega. \tag{14}$$

R^f is the funding rate of the bank. μ represents the monitoring cost that the bank has to pay when it claims the post-default investment project. This 'costly state verification' problem by Townsend (1979) is the core friction in the BGG model. Banks maximize profit but competition among banks will lead them to accept this zero profit participation constraint. However, the zero profit condition holds only ex-ante, and profit or loss can occur once aggregate return R_{t+1}^K is realized and different from $E_tR_{t+1}^K$. In period t+1, after R_{t+1}^K is realized, the ex-post actual default threshold $\bar{w}_{t+1}^b$ is defined as

$$\bar{\omega}_{t+1}^b = \frac{R_t^{LB}L_t^B}{R_{t+1}^K q_t K_t} = \bar{\omega}_t^a \frac{E_t R_{t+1}^K}{R_{t+1}^K}. \tag{15}$$

Details of the entrepreneurs' optimization problem, given a lognormal assumption for ω, are provided in appendix A.3.

3.3.2. *Financial Contract: Household Loan*

Similar to the BGG mechanism applied to the business sector, we can model the household loan contract where household lending is subject to default. Suppose an idiosyncratic housing price shock hits each borrowing household after aggregate variables are determined. The ith borrowing household will face foreclosure if the value of the idiosyncratic price shock ($\omega^{H,i}$) is less than some threshold level ($\bar{\omega}^{H,i}$). It is assumed that there is a redistribution of wealth among borrowing households every period so that every borrowing household is homogeneously endowed before the idiosyncratic shock. Then we can drop the i superscript and the household lending contract can be written as

$$R_t^{LH}L_t^H = \bar{\omega}_t^{H,a} E_t P_{t+1}^H H_{t+1,b}. \tag{16}$$

Here $E_tP_{t+1}^H H_{t+1,b}$ is the expected value of the collateral owned by borrowing households, and $\bar{\omega}_t^{H,a}$ is the ex-ante default threshold. The zero profit condition for the bank becomes

$$(R_t^f + \nu_c)L_t^H = (1 - F(\bar{\omega}_t^{H,a}))R_t^{LH} L_t^H + (1 - \mu^H) \int_0^{\bar{\omega}_t^{H,a}} \omega^H E_t P_{t+1}^H H_{t+1,b} f(\omega^H) d\omega^H. \quad (17)$$

ν_c is a markup in household lending so that the steady-state risk premium in the household lending rate matches the historical U.S. observation. It is assumed that the profit from this markup is redistributed to saving households as dividends. After aggregate shocks are realized, the ex-post default threshold is defined by $\bar{\omega}_{t+1}^{H,b} = (R_t^{LH} L_t^H)/(P_{t+1}^H H_{t+1,b})$.

3.3.3. *Financial Intermediary*

Similar to the original BGG model, the bank has the ability to diversify idiosyncratic risk and can insure savers against it. In contrast to the BGG model, as mentioned, the bank shares aggregate risk. The difference between the expected value and the realized value in aggregate return is reflected in the changes in bank capital. The bank has two means of financing, deposits (D_t) and equity capital (e_t), and its asset side consists of business (L_t^B) and household lending (L_t^H). The bank capital ratio is defined by $\kappa_t \equiv e_t/L_t$, where L_t is total lending ($L_t = L_t^B + L_t^H$). Also, the asset side of the bank must be balanced with its liability and equity side, that is, $D_t + e_t = L_t$. It is assumed that the bank's funding cost is affected by the bank's capital structure in a reduced form. The bank's funding rate (R^f) is determined by adding a markup to the actual funding rate (the weighted average of the deposit rate and the return on bank capital).

$$R_t^f = \kappa_t R_t^e + (1 - \kappa_t) R_t^D + s(\kappa_t, \overline{\kappa}_t). \quad (18)$$

s is a markup function that captures the effect of the bank's capital structure on the bank's funding rate. It is decreasing in κ, implying that if the bank is badly capitalized, it will face a higher cost of funding. It reflects the positive probability given to the event where bank capital reaches zero and the bank fails to function. s is also a function of $\bar{\kappa}$, the capital requirement ratio, to capture the effect of bank capital regulation imposed by the regulatory authority. The violation of the capital requirement regulation will lead the bank to face a corrective measure, adversely affecting its reputation or constraining its managerial decisions. This, in turn, worsens the bank's funding cost as the bank is charged a higher markup. Thereby I call s 'regulatory markup function.' Details about this regulatory markup are further discussed in section 3.8.

Bank capital is required by the regulator, owned by households, and functions as a buffer stock that absorbs the forecast error in aggregate return. The law of motion for bank capital is given as below.

$$
\begin{aligned}
e_t &= (1-\phi)e_{t-1} + w_{t,f} - R^f_{t-1}(L^B_{t-1} + L^H_{t-1}) \\
&\quad + R^{LB}_{t-1}L^B_{t-1}(1 - F(\bar{\omega}^b_t)) + (1-\mu)\int_0^{\bar{\omega}^b_t} \omega R^K_t q_{t-1}K_{t-1}f(\omega)d\omega \\
&\quad + R^{LH}_{t-1}L^H_{t-1}(1 - F(\bar{\omega}^{H,b}_t)) + (1-\mu^H)\int_0^{\bar{\omega}^{H,b}_t} \omega^H P^H_t H_{t,b} f(\omega^H)d\omega^H + \epsilon^e_t.
\end{aligned} \tag{19}
$$

Net profit from lending activity - inflows from loans minus outflows to depositors and shareholders - affects bank capital. This net profit is nonzero when there are forecast errors in the return on capital ($R^K_t - E_{t-1}R^K_t$) or housing value ($P^H_t H_{t,b} - E_{t-1}P^H_t H_{t,b}$). Bankers' labor income is also added to the capital. Note that since the profit from lending activity is zero in the steady state, the steady-state bank capital level is entirely dependent on bankers' labor income. To prevent capital overaccumulation, it is assumed that bankers spend ϕ fraction of bank capital every period. It should be noted that the adjustment process of bank capital is slow when ϕ and $w_{t,f}$ are small. Therefore, the bank complies with the capital requirement ratio obligation mainly by adjusting assets rather than bank capital. ϵ^e_t is an exogenous shock to bank capital, which captures the type of shocks originating from the financial sector. As an example, one can imagine changes in bank capital due to changes in asset quality or due to the differences between the ex-ante and ex-post variance of idiosyncratic shocks.

3.4. Capital Producer

At the beginning of each period, the capital producer purchases I_t amounts of consumption goods at a price of one and turns them into the same amount of new capital. Transformation costs arise during the process, and at the end of the period, she resells new capital to entrepreneurs at price q_t. The law of motion for capital stock is given by

$$K_t = I_t + (1-\delta)K_{t-1}. \tag{20}$$

Appendix A.4 discusses further details about the capital producer's optimization problem.

3.5. Retail Goods Producers

Each retail goods producer (i) purchases intermediate goods and turns them into retail goods ($Y_t(i)$) in a monopolistically competitive market. Total final usable goods Y_t are the following composite of retail goods.

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \tag{21}$$

Only (1-θ) fraction of retail goods producers are allowed to change price, à la Calvo (1983). Appendix A.5 discusses the details of the retail goods producers' optimization problem, using the method used by Schmitt-Grohe and Uribe (2004a).

3.6. Exogenous Processes and Market Clearing

I assume shocks to productivity, housing preference and government spending follow stationary AR(1) processes in a log-linearized form:

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \epsilon_t^A, \; \hat{\epsilon}_t^\gamma = \rho_\gamma \hat{\epsilon}_t^\gamma + \xi_t^\gamma, \; \hat{G}_t = \rho_G \hat{G}_{t-1} + \epsilon_t^G. \tag{22}$$

where $\hat{X}$ stands for the log-linear deviation of any variable X. Aggregate housing investment demand is the sum of the housing investments of both types of households ($I_t^H(D) = I_{t,s}^H + I_{t,b}^H$). It is possible to invent a production technology for housing goods. However, for simplicity, I assume the housing investment supply is given exogenously ($I_t^H(S) = \varrho Y$). In equilibrium, $I_t^H(D) = I_t^H(S)$. The government budget is balanced every period as government expenditure is financed by lump-sum taxes from households.

$$G_t = T_{t,s} + T_{t,b}. \tag{23}$$

Finally, we have market clearing conditions for goods and labor markets. Note that the aggregate resource constraint contains terms for monitoring costs and regulatory markups.

$$\begin{aligned} Y_t/s_t &= C_{t,s} + C_{t,b} + q_t I_t + I_t^H + G_t + \phi e_{t-1} \\ &\quad + \mu \int_0^{\bar{\omega}_t^b} \omega R_t^K q_{t-1} K_{t-1} dF(\omega) + \mu^H \int_0^{\bar{\omega}_t^{H,b}} \omega^H P_t^H H_{t,b} dF(\omega^H) + \varsigma_t. \end{aligned} \tag{24}$$

(ς_t denotes the terms representing the resource usage by regulatory markups.)

For the labor market, $N_{t,s} + N_{t,b} = N_t$.

3.7. Monetary Policy

Monetary policy is set to follow an extended Taylor rule in log-linearized form. The central bank sets the nominal interest rate according to the log deviation of inflation, output, and credit from their steady-state values.

$$\hat{R}_t^N = \rho_r \hat{R}_{t-1}^N + (1-\rho_r)[\phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t + \phi_L \hat{L}_t] + \epsilon_t^R. \tag{25}$$

3.8. Macroprudential Policy

The regulatory authority sets the capital requirement ratio ($\bar{\kappa}_t$) and target loan-to-value ratio ($\overline{ltv}_t$) dynamically according to simple rules that systematically react to observable macro variables such as output, credit, or housing prices. The set of variables that the macroprudential policy reacts to is chosen with practical considerations, from among those that are used or are likely to be used in practice. As mentioned in 3.3.3, the bank has to pay higher funding costs when regulatory markup increases. I assume the functional form of this capital requirement regulatory markup in equation (18) to be

$$s(\overline{\kappa}_t, \kappa_t) = \nu_a^I \exp[\nu_b^I(\overline{\kappa}_t - \kappa_t)/\kappa]. \tag{26}$$

Here ν_a^I determines the level of intervention[3] and ν_b^I determines the responsiveness of regulation. The required capital ratio is set as a simple function of output and gross credit ($\bar{\kappa}_t = \zeta_\kappa(Y_t, L_t)$). This idea of a dynamic capital requirement ratio is close to the countercyclical capital buffer suggested in Basel III.[4] Specifically, the requirement ratio is a function of output and gross credit.

$$\hat{\bar{\kappa}}_t = \rho_\kappa \hat{\bar{\kappa}}_{t-1} + \phi_Y^\kappa \hat{Y}_t + \phi_L^\kappa \hat{L}_t. \tag{27}$$

The LTV ratio regulation is a market-specific regulation pertaining only to the house-

[3] In the steady state, it is assumed that $\bar{\kappa} = \kappa$; therefore, the steady-state markup level is $s(\overline{\kappa}, \kappa) = \nu_a^I$.
[4] See BIS (2011)

hold lending market. LTV is defined as a ratio of household lending to the value of the borrower's housing goods ($ltv_t = L_t^H / P_t^H H_{t,b}$). I assume that this LTV regulation operates as a markup over the bank funding rate that applies to household lending. With this regulatory markup, the bank's ex-ante zero-profit condition in the household loan contract (17) should be modified as below.

$$[R_t^f + \nu_c + Q(ltv_t, \overline{ltv}_t)] L_t^H = (1 - F(\bar{\omega}_t^{H,a})) R_t^{LH} L_t^H + (1 - \mu^H) \int_0^{\bar{\omega}_t^{H,a}} \omega^H E_t P_{t+1}^H H_{t,b} f(\omega^H) d\omega^H. \tag{28}$$

Here Q is the regulatory markup function, a function of the actual LTV ratio and the target LTV ratio.[5] Borrowing households have to pay a regulatory penalty for taking a higher LTV ratio than the target LTV ratio set by the financial regulator. The functional form of this regulatory markup is assumed to be

$$Q(ltv_t, \overline{ltv}_t) = \nu_a^H \exp[\nu_b^H (ltv_t - \overline{ltv}_t)/ltv]. \tag{29}$$

Again, the target LTV ratio is set using a simple rule where the policymaker cares only about housing prices ($\overline{ltv}_t = \zeta_{ltv}(P_t^H)$). This specification of the LTV rule corresponds with policy practices in Asian countries such as Hong Kong and Korea, which use LTV regulation in household credit as a tool to stabilize housing prices.[6] Specifically, in log-linear form,

$$\hat{\overline{ltv}}_t = \rho_{ltv} \hat{\overline{ltv}}_{t-1} - \phi_{PH}^{ltv} \hat{P^H}_t. \tag{30}$$

It should be emphasized that each government policy affects different agents and interest rates differently. Monetary policy, by determining the nominal interest rate, influences the real deposit rate through the Fisher equation. The deposit rate affects the funding rate of the bank and is thereby passed through to lending rates. The capital requirement ratio regulation directly affects the bank's funding rate and thus has a direct effect on both business and household borrowing conditions. The LTV regulation is imposed on household lending and has a direct effect only on household borrowing conditions.

[5]Again, in the steady state, $\overline{ltv} = ltv$.

[6]see Gerlach and Peng (2005).

4. Calibration

The parameters in the utility function, housing and production sector are chosen in a range that accords with the standard values found in the literature. They are adjusted so that the variables are presented as their quarterly values. The future discount factors are chosen to be 0.99 for saving households and 0.9885 for borrowing households. γ, which determines the weight of consumption goods and housing goods in the utility function, is calibrated as 0.9. The weight of labor in the utility function (φ) is assumed to be 2. The rate of depreciation for capital goods (δ) is chosen to be 0.025, implying that it takes 10 years to completely depreciate. The rate of depreciation for housing goods (δ^H) is 0.0125. In the production sector, the share of capital, labor, entrepreneur's labor, and banker's labor in a Cobb-Douglas production function is chosen to be 0.31, 0.67, 0.01, and 0.01, respectively. The dividend ratio of the entrepreneurial sector ($1 - \upsilon$) is 0.027 and the capital adjustment cost parameter (χ_K) is 4. The fraction of retail goods producers who can reset the sale price each period (θ) is 0.25, and retailers' degree of monopolistic power (ϵ) is chosen so that the steady-state real markup is 1.1. For the exogenous processes, autoregressive coefficients are $\rho_A = 0.85$, and $\rho_G = 0.8$, $\rho_\gamma = 0.95$. The degree of inertia in monetary policy (ρ_r) is 0.75. The standard deviations of the technology shock, the government spending shock, the housing demand shock and the bank capital shock are chosen so that they can match the historical volatility of output, government expenditure, housing price and nonfinancial credit liabilities[7] given that the monetary and macroprudential policies follow the baseline specification that is shown in section 5. The standard deviation of the monetary shock is chosen to be 25bps when annualized. In addition, the parameters for the financial contract and the banking sector are calibrated so that they imply certain spread levels in the steady state that match the historical data[8]. Table 5 in appendix A.1 provides a summary of these parameters and their calibrated values.

In the steady state, the borrowing households/saving households ratio of consumption (C^B/C^S) is 0.78, and that of labor supply is (1.78). Regarding credit variables, the business credit/household credit ratio (L^B/L^H) is 3.27, entrepreneurs' capital-net worth ratio (K/W) is 1.66, and household debt/annualized GDP ratio (L^H/Y) is 0.19. The probability of default

[7]During 1982-2011, the standard deviation of quarterly output, government expenditure, housing price, and nonfinancial credit liabilities is measured to be 1.1%, 1.2%, 2.2% and 1.3%, respectively. (Source: National Accounts, U.S. Bureau of Economic Analysis, House Price Index, New Single-family Houses Sold, U.S. Census Bureau, Flow of Funds, Federal Reserve)

[8]As proxies for the interbank spread, business lending spread and household lending spread, I choose the federal funds rate - financial CP spread, 6-month Treasury bill - prime loan spread, and 6-month Treasury bill - 30 year fixed mortgage spread. During 1982-2011, the average of each spread is 0.1pp, 2.9pp and 3.6pp, respectively. (Source : Federal Reserve Board)

for entrepreneurs and households is 3.0% and 0.7%, respectively. For prudential variables, the steady-state bank capital ratio (κ) is 0.080 and the LTV ratio is 0.708. Steady-state interest rates and variable ratios are provided in table 6 in appendix A.1.

5. Effects of Macroprudential Policy

In this section, I provide simulation results to evaluate the effects of macroprudential policy on the economy. The second-order perturbation method is used to solve the model. Three policy specifications are considered. They differ by the different parameterization of the regulatory markup functions. The first is the baseline regime with static capital regulation and no LTV regulation ('BL'), which mimics the current regulatory stance in many countries. It has $\nu_a^I = \nu_a^H = 0.0025$ for the degree of intervention, and $\nu_b^I = 25$ and $\nu_b^H = 0$ for the sensitivity of regulation to the movement of the target variable. The second specification is a dynamic capital requirement ratio rule reacting to gross credit ('DCRR'). Again, it has $\nu_a^I = \nu_a^H = 0.0025$, $\nu_b^I = 25$ and $\nu_b^H = 0$, but now the capital requirement ratio moves over time in response to gross credit. The third is a dynamic LTV ratio rule reacting to housing price ('DLTV'). It has $\nu_a^I = \nu_a^H = 0.0025$, $\nu_b^I = \nu_b^H = 25$ and target LTV ratio moves over time in response to housing price. It is assumed that monetary policy responds to inflation and output ($\phi_\pi = 1.5, \phi_Y = 0.1$) in all three cases.

5.1. Effects of The Capital Requirement Ratio Rule

In figures 1 and 2, I compare the impulse response functions from the baseline model (BL) and the dynamic capital requirement ratio model (DCRR). Impulse responses are measured in percentage changes from steady-state values and the magnitude of the shocks is scaled as one standard deviation for each shock. Here parameters in the capital requirement ratio rule (27) are chosen so that the capital requirement ratio reacts countercyclically to gross credit ($\phi_L^\kappa = 1.5$) but does not react to output ($\phi_Y^\kappa = 0$). The capital requirement ratio has a high degree of inertia ($\rho_\kappa = 0.9$).

Figure 1 shows the impulse response functions from a positive productivity shock. In the baseline model, consumption, investment and output increase and inflation falls. Housing price rises because housing supply is given exogenously in this model and housing price is completely determined by its demand. With respect to the household optimization problem, it is straightforward that housing demand must increase when consumption increases and the marginal utility of consumption decreases. In credit markets, both business and household lending increase. With DCRR macroprudential policy, the capital requirement ratio rises as it responds to the increase in gross credit. Because of the regulation, credit expansion is dampened in the DCRR model compared to the baseline model. Consequently, we have less persistent movements in consumption, investment, output and housing price. Since the regulation requires a higher capital ratio, bank capital is higher with the DCRR policy.

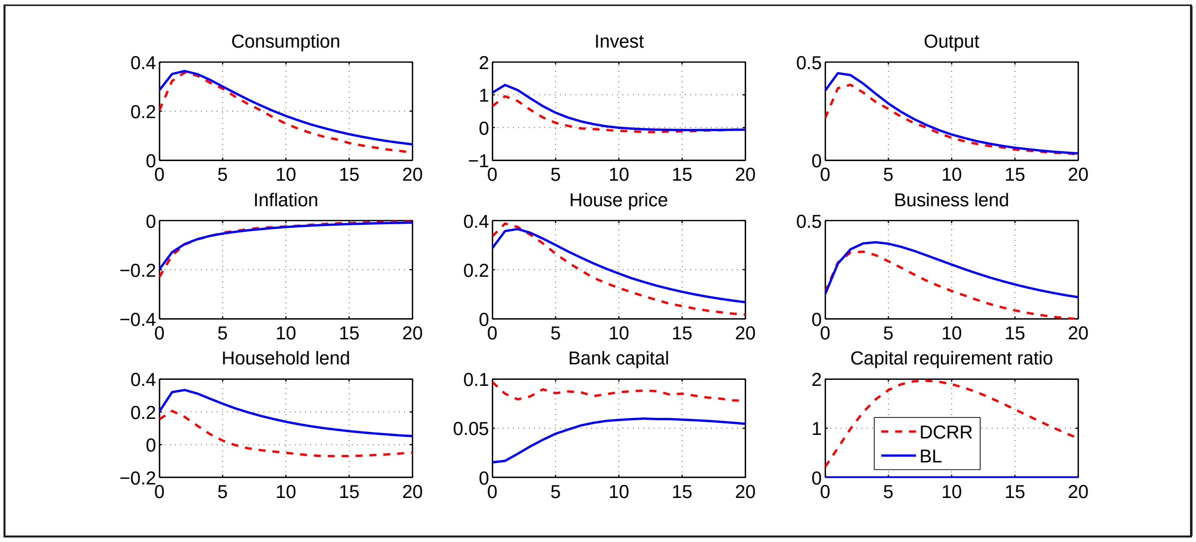


Figure 1: Impulse response functions given productivity shock, from baseline ('BL') model and dynamic capital requirement ('DCRR') model

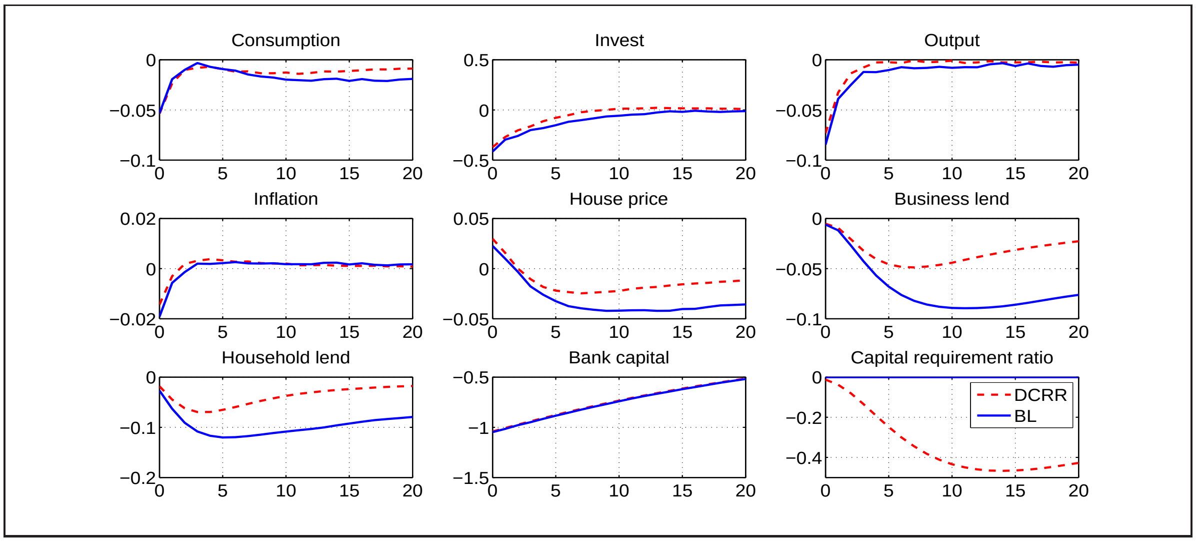


Figure 2: Impulse response functions given bank capital shock, from baseline ('BL') model and dynamic capital requirement ('DCRR') model

Figure 2 shows the response of the economy to a negative shock to bank capital, which represents a shock originating from the financial intermediary sector. As bank capital is reduced, both business lending and household lending shrink. It generates recessionary pressure as consumption, investment, output decrease and inflation falls. Here DCRR policy helps faster recovery by lowering the capital requirement ratio. With DCRR policy, the decline in lending is not as great as in the baseline model and the recessionary effect on investment, output and inflation is less severe.

5.2. Effects of LTV Ratio Rule

Figures 3 and 4 present impulse responses from the model with the DLTV macroprudential policy. Here the DLTV rule is assumed to react to housing price ($\phi_{PH}^{LTV} = 1.5$) with a high degree of inertia ($\rho_{LTV} = 0.9$). Figure 3 shows that this DLTV rule is successful in dampening housing price upon a productivity shock. A positive productivity shock raises housing price and the target LTV ratio falls in response. Stronger regulation dampens the expansion in household lending and the rise in housing price. It is noticeable that regulatory arbitrage - a flow of credit from the heavily regulated (household) market to the less heavily regulated (business) market - occurs here. As a result, business lending and investment are more volatile than in the baseline model. Lower growth in household lending due to the LTV regulation induces a relatively higher bank capital ratio, giving the bank more room to increase business lending, which is weakly regulated.

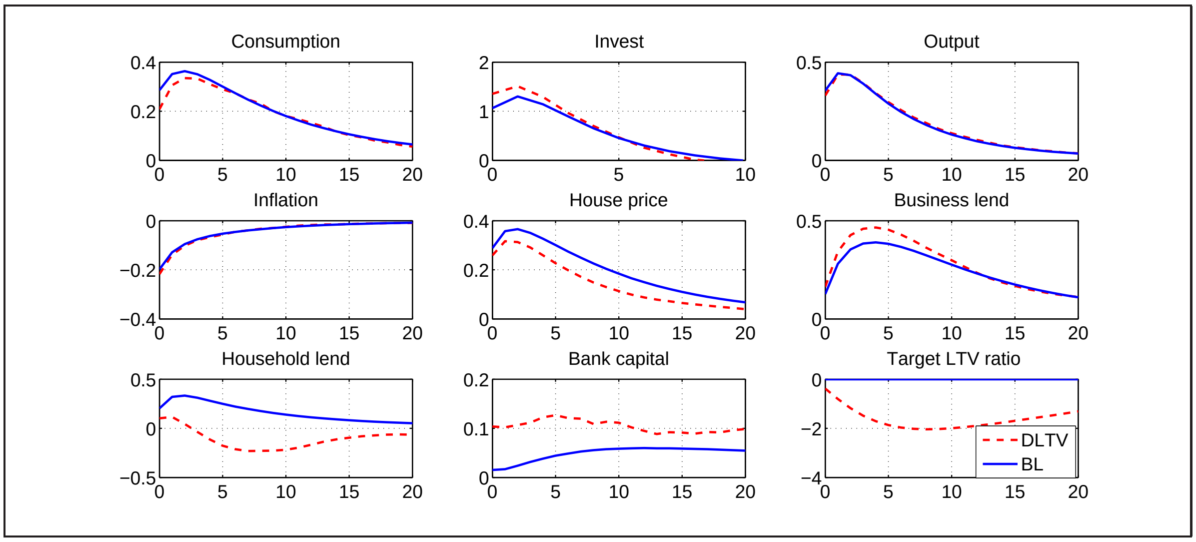


Figure 3: Impulse response functions given productivity shock, from baseline ('BL') model and dynamic LTV ('DLTV') model

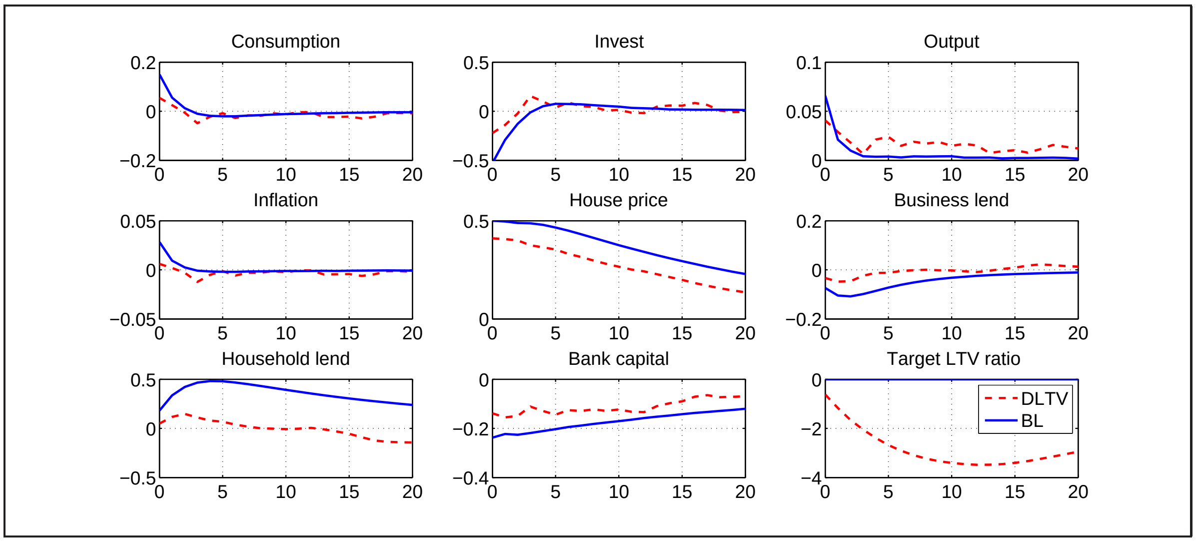


Figure 4: Impulse response functions given housing demand shock, from baseline ('BL') model and dynamic LTV ('DLTV') model

In figure 4, positive housing demand shock, a housing market-specific shock, is imposed. The effect of this housing demand shock is to raise housing prices and increase credit to borrowing households. With the DLTV policy, the target LTV ratio goes down with a higher housing price. This in turn curbs lending to borrowing households and eventually lowers housing price. Again, regulatory arbitrage occurs from household credit to business credit, in this case offsetting the original credit shift from the business to the household credit market. Both figures 3 and 4 indicate that while the DLTV policy is effective in stabilizing housing price, which it targets, policymakers should be aware that it can trigger regulatory arbitrage because of its market-specific nature.

5.3. Effects on Economic Volatility

As expected from the above impulse response functions, macroprudential poliy can function as a built-in stabilizer. Table 2 identifies the effect of macroprudential policy on the volatility of major variables. The numbers in the table are unconditional standard deviations of consumption (σ_C), output (σ_Y), investment (σ_I), inflation (σ_π), housing price (σ_{P^H}), business lending (σ_{L^B}), and household lending (σ_{L^H}). It is observed that the DCRR policy reduces the volatility of consumption, output, investment, housing price, and credit and raises the volatility of inflation. In particular, the standard deviation of business and household lending significantly decreases. However, the fact that the volatility of inflation increases with the DCRR policy suggests a possible conflict with the monetary policy objective. A detailed discussion about this conflict will be presented in later sections. On the other hand, the effect of the DLTV macroprudential policy is rather mixed. Clearly, this type of policy can stabilize household credit and housing price, as those variables show a significant reduction in volatility. However, it can amplify the volatility of the real sector, as the volatility of output, investment and business lending increases. This result strongly suggests that when implementing a regulatory measure that only influences a partial segment of the economy, policymakers need to be careful about the possibility of regulatory arbitrage.

Table 2: Effects of macroprudential policy on economic volatility

	σ_C	σ_Y	σ_I	σ_π	σ_{P^H}	σ_{L^B}	σ_{L^H}
Baseline (A)	0.010	0.015	0.006	0.003	0.026	0.048	0.022
DCRR Macroprudential (B)	0.009	0.014	0.005	0.003	0.024	0.034	0.018
Change ((B-A)/A))	-8.59%	-9.17%	-18.63%	8.01%	-4.82%	-29.29%	-17.36%
DLTV Macroprudential (C)	0.009	0.015	0.006	0.003	0.019	0.052	0.010
Change ((C-A)/A))	-4.69%	1.16%	4.83%	4.34%	-24.86%	6.53%	-56.21%

* Figures are unconditional standard deviations of consumption (σ_C), output (σ_Y), investment (σ_I), inflation (σ_π), housing price (σ_{P^H}), business lending (σ_{L^B}), and household lending (σ_{L^H}).

6. Welfare Analysis and Optimal Policy

In this section, I analyze the welfare-maximizing optimal combination of monetary and macroprudential policies using the second-order approximation of the equilibrium. Policy rules are designed to be simple and implementable, as in Schmitt-Grohe and Uribe (2004a). They define a policy as being simple when rules are set as a function of a small number of easily observable macroeconomic indicators and being implementable when it delivers the uniqueness of the rational expectations equilibrium. Here macroprudential policy is simple because it is a function of observable macro variables (output, credit and housing price) and implementable because policy coefficients are restricted to a range that guarantees a unique bounded second-order approximated solution. Details of the advantage that the second-order approximation method offers over the linear-quadratic method is shown in Schmitt-Grohe and Uribe (2004b).

6.1. Optimal Monetary and Macroprudential Policy Combination

As a welfare measure, the unconditional expectation of average household utility in period zero is calculated.[9] That is,

$$E_0 V = E_0 \left\{ \sum_{t=0}^{\infty} \tilde{\beta}^t [\gamma \log \tilde{C}_t + (1 - \gamma \epsilon_t^{\gamma}) \log \tilde{H}_t + \varphi \log(1 - \tilde{N}_t)] \right\} \tag{31}$$

where $\tilde{X}$ denotes the average household variable $((X_s + X_b)/2)$. This average household concept is consistent with the model assumption that there are the same number of saving and borrowing households. The policy gain is measured by the fraction of consumption goods that households have to give up with an inferior policy, that is, the value of λ in the equation below.

$$E_0 V(\tilde{C}(A)) = E_0 V((1 - \lambda)\tilde{C}(B)). \tag{32}$$

Here A and B represent two different government policies such that B is superior in terms of a welfare measure, and $\tilde{C}(\cdot)$ represents the consumption stream associated with each policy. For macroprudential policy parameters, the range of ϕ_Y^{κ}, ϕ_L^{κ}, ϕ_{PH}^{ltv}, the reactions of the capital requirement ratio and target LTV ratio to output, gross credit and housing price, are restricted to the interval between 0 and 2.[10] This range is then partitioned with grids of size 0.05. For monetary policy parameters, a range between 1 and 3 of grid size 0.05 for ϕ_π and a range between 0 and 1 with grid size 0.05 for ϕ_Y and ϕ_L are examined. For each combination of monetary and macroprudential policy, I calculate $E_0 V$ and define the optimal policy as the policy combination that maximizes $E_0 V$. Macroprudential policies are assumed to have inertia ($\rho_\kappa = \rho_{ltv} = 0.9$).

Table 3 suggests optimal policy combinations and welfare gains. There are five different panels according to the regulation regime. The 'Baseline' panel features the baseline policy calibration in the previous section. Monetary policy parameters are given as $\phi_\pi = 1.5, \phi_Y = 0.1$, and the regulatory regime is given by the static capital requirement regulation and no

[9] An alternative is to find the conditional expectation given initial values at the Ramsey steady state. However, in this paper, results from conditional expectation do not substantially differ from results from unconditional expectation.

[10] For example, $\phi_L^{\kappa} = 2$ means that the capital requirement ratio increases by 2 percent of its steady-state value in response to a 1 percent increase in credit.

LTV regulation. ($\nu_a^I = \nu_a^H = 0.0025, \nu_b^I = 25, \nu_b^H = 0$, $\phi_Y^\kappa = \phi_L^\kappa = 0$.) In the 'Monetary policy only' panel, optimal monetary policy based on the same regulatory regime is found. Thus policies in this panel are the optimal monetary policy without a macroprudential policy. In the three lower panels, the macroprudential policy is in action. To see how much each instrument contributes to welfare, I separately calculate the optimal policy and welfare gains when only either the DLTV or the DCRR policy is in action, and when both policies operate together.[11] Welfare gains in each panel are the increase in welfare compared to welfare in the 'Baseline' panel. The measure of welfare gain (λ) is the fraction of the consumption stream defined in (32). For each panel, three different scenarios regarding the volatility of the housing and financial markets are suggested. In the 'Stable' environment, housing demand and bank capital shocks are assumed to be nonexistent, providing stable conditions for both markets. The 'Normal' environment follows the basic calibration for the standard deviation of those shocks ($\sigma^\gamma = 0.0021$ and $\sigma^e = 0.004$). In the 'Volatile' scenario, both σ^γ and σ^e are doubled to 0.0042 and 0.008, respectively.

Table 3: Optimal monetary (MOP) and macroprudential policy (MPP)

Regulation Regime	Volatility	Policy Parameters: MOP ϕ_π	ϕ_Y	ϕ_L	MPP ϕ_{PH}^{ltv}	ϕ_Y^κ	ϕ_L^κ	Welfare	Welfare Gains : $\lambda(\%)^*$
Baseline	Stable	1.5	0.1	0.0	-	-	-	-130.29	-
	Normal	1.5	0.1	0.0	-	-	-	-130.41	-
	Volatile	1.5	0.1	0.0	-	-	-	-130.75	-
Monetary policy only	Stable	3.0	0.15	0.0	-	-	-	-130.22	0.09
	Normal	3.0	0.15	0.0	-	-	-	-130.33	0.10
	Volatile	3.0	0.15	0.0	-	-	-	-130.63	0.14
Monetary Policy + DLTV	Stable	3.0	0.1	0.0	0.1	-	-	-130.21	0.10
	Normal	3.0	0.1	0.0	0.1	-	-	-130.31	0.13
	Volatile	3.0	0.1	0.0	0.1	-	-	-130.58	0.21
Monetary Policy + DCRR	Stable	3.0	0.1	0.0	-	0.0	0.05	-130.22	0.09
	Normal	3.0	0.0	0.0	-	0.0	2.0	-130.25	0.19
	Volatile	3.0	0.0	0.0	-	0.0	2.0	-130.28	0.58
Monetary Policy + DLTV, DCRR	Stable	3.0	0.1	0.0	0.1	0.0	0.05	-130.21	0.10
	Normal	3.0	0.0	0.0	1.05	0.0	2.0	-130.25	0.20
	Volatile	3.0	0.0	0.0	0.35	0.0	2.0	-130.27	0.59

* Welfare gains compared with the baseline policy regime

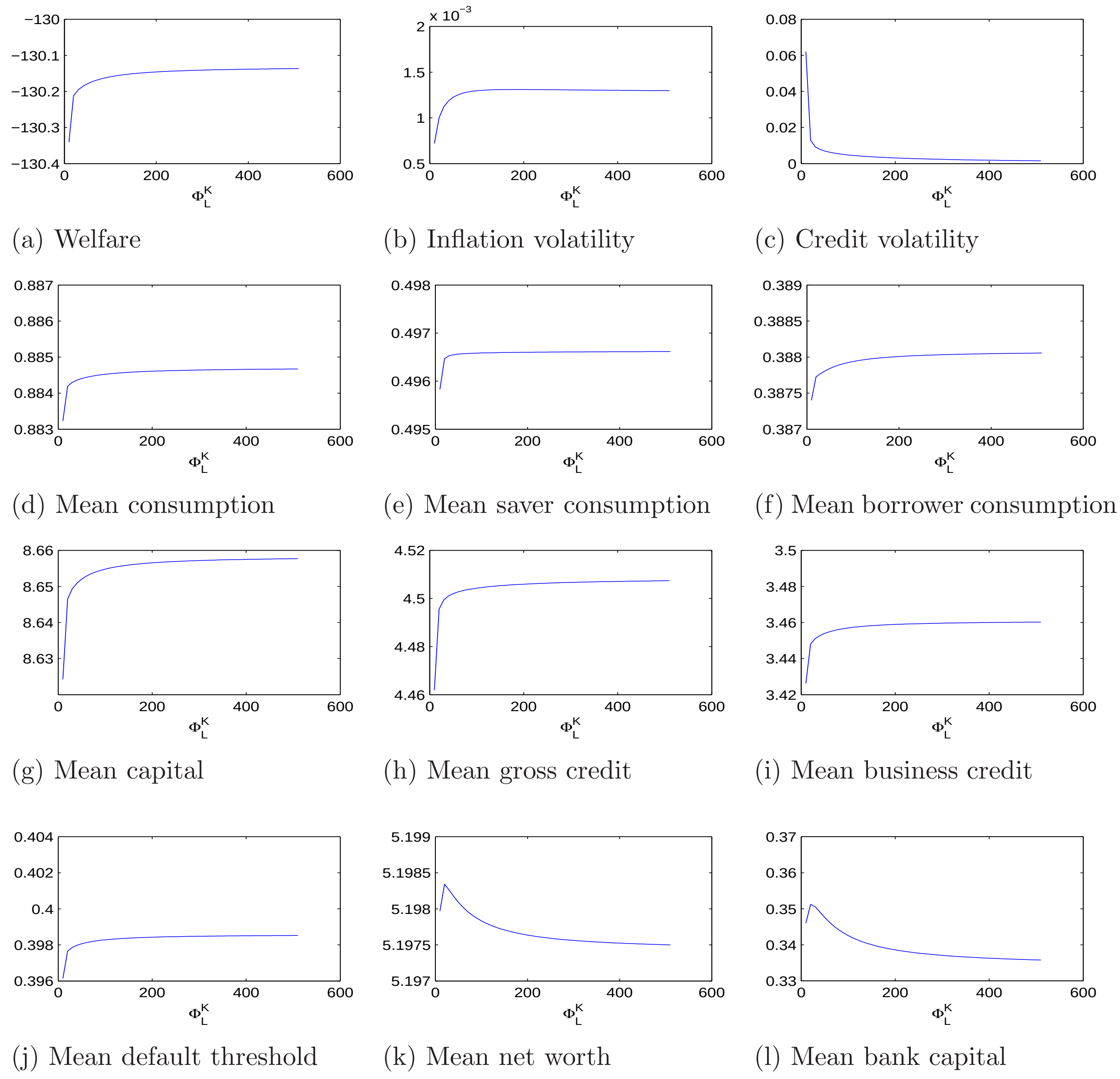
In the 'Monetary policy only' panel when macroprudential policy is nonexistent, the

[11] Note that the nonstochastic steady state associated with each panel is the same.

optimal monetary reacts aggressively to inflation ($\phi_\pi = 3$) and positively to output ($\phi_Y = 0.15$). Note the optimal response of monetary policy to output is nonzero but the reaction to gross credit (ϕ_L) is zero in the optimal rule. The welfare gain from optimal monetary policy in terms of λ, compared to the 'Baseline' policy specification, is around 0.1%. In the 'Monetary policy+DLTV' panel, DLTV is the only macroprudential instrument. The additional welfare gain of the DLTV policy over optimal monetary policy is less than 0.1pp. Optimal monetary policies are similar to 'Monetary policy only' case and the optimal DLTV policy response to housing price (ϕ^{ltv}_{PH}) is quite weak at 0.1. In the 'Monetary policy+DCRR' panel, the DCRR policy is the only macroprudential instrument. The additional welfare gain of the DCRR policy over optimal monetary policy increases as the volatility in lending activity and the housing market increases; it reaches 0.44pp in the 'Volatile' scenario. In the 'Normal' and 'Volatile' scenarios, optimal monetary policy does not react to output ($\phi_Y = 0.0$) and the reaction of the capital requirement ratio to credit reaches its ceiling ($\phi^{\kappa}_L = 2.0$). When both DCRR and DLTV are allowed to operate ('Monetary policy+DLTV+DCRR'), the welfare gains are not substantially larger than those in 'Monetary policy+DCRR,' indicating that most gains are from the DCRR policy. The optimal monetary and DCRR policies remain the same with 'Monetary policy+DCRR.' The optimal DLTV coefficient varies and is positive. In section 6.2, I explain how a credit-stabilizing DCRR policy has welfare gains in this model. After all, the result in table 3 can support the argument for using a macroprudential policy against asset price and financial market disturbances, as it shows that the welfare gain from the macroprudential policy is greater when we have high volatility in lending activity and the housing market.

6.2. Interpreting Gains from DCRR Policy

The optimal policy result shows that when there are disturbances in housing price and bank capital, monetary policy focusing on inflation stabilization and a DCRR policy focusing on credit stabilization are optimal in terms of welfare. Inflation stabilization reduces welfare loss from relative price dispersion when price rigidity is present. But how does credit stabilization using DCRR policy improve welfare? This question is especially intriguing because a DCRR policy to stabilize credit may increase inflation volatility, a tradeoff shown in the previous section. To further analyze this question, I examine how expected values and standard deviations of major macro variables change with the aggressiveness of the DCRR policy against credit. Those moments are calculated assuming that the volatility of housing demand and bank capital follows the 'Normal' calibration.



(a) Welfare (b) Inflation volatility (c) Credit volatility

(d) Mean consumption (e) Mean saver consumption (f) Mean borrower consumption

(g) Mean capital (h) Mean gross credit (i) Mean business credit

(j) Mean default threshold (k) Mean net worth (l) Mean bank capital

Figure 5: Reaction of capital requirement ratio to credit (ϕ_L^κ), economy and welfare

In figure 5, I display the mean and standard deviation of key variables, varying the response of the capital requirement ratio to credit (ϕ_L^κ) from 0 to 500. As the DCRR response increases, the welfare measure and the expected values of consumption, capital, credit, and the default threshold monotonically increase and credit volatility monotonically decreases. One possible interpretation of this result is that the credit stability due to the DCRR policy yields a less conservative financial contract, represented by a higher expected value of the default threshold and credit. In other words, there is a compensation mechanism that a policy-induced reduction in risk leads to a larger expected credit. Larger credit in the economy results in a welfare gain through high investment, capital and consumption. Note that the welfare gain comes from the capital accumulation of the business sector, thus saving households and borrowing households are both better off with the macroprudential policy. Inflation volatility shows a monotonic increase as ϕ_L^κ increases, implying that this is a policy region where the welfare gain from credit stability outweighs the loss from larger inflation volatility. More analysis about this tradeoff follows in Section 7, where the interaction between monetary and macroprudential policies is discussed. Finally, since the expected values of the entrepreneur's net worth or bank credit are not monotonic, it is not clear whether and how they affect welfare.

Here I suggest a partial equilibrium analysis to show that the above linkage between a policy-induced volatility reduction and larger expected credit is a property of the BGG financial contract.

$$R_t^f(q_tK_t - W_t) = \underbrace{\left[(1 - F(\bar{\omega}_t^a))\bar{\omega}_t^a + (1-\mu)\int_0^{\bar{\omega}_t^a} \omega f(\omega)d\omega\right]}_{(a)} E_tR_{t+1}^K q_tK_t. \tag{33}$$

Equation (33) is the bank zero profit condition (14), after substituting out the debt repayment value $R_t^{LB}L_t^B$ using its definition in (12). Suppose $E_tR_{t+1}^K, R_t^f, q_t$ and W_t are given. Then equation (33) defines the relationship between the level of the investment project (K_t) and the default threshold ($\bar{\omega}_t^a$). A higher default threshold raises the expected return of the bank by increasing the non-default payoff ($\bar{\omega}_t^a E_tR_{t+1}^K q_tK_t$), but it also raises the probability of default ($F(\bar{\omega}_t^a)$). Given parameter values and with some general regularity conditions for the distribution of ω, it is possible to show that (a) - the expected return to the bank for one unit of gross return - is increasing and concave in $\bar{\omega}_t^a$ (Bernanke, Gertler, and Gilchrist (1999)).

By rearranging equation (33), we have

$$K_t = \frac{-R_t^f W_t}{\left[(1 - F(\bar{\omega}_t^a))\bar{\omega}_t^a + (1-\mu)\int_0^{\bar{\omega}_t^a} \omega f(\omega)d\omega\right] E_tR_{t+1}^K q_t - R_t^f q_t} = h(\bar{\omega}_t^a) \tag{34}$$

where h is concave and a strictly increasing function of $\bar{\omega}_t^a$. This concavity implies that the ability of the bank to intermediate credit to finance a larger investment project (K_t) is restricted by the corresponding higher default risk. As the default threshold increases, the more that will be reflected in the increase in the lending rate R_t^{LB} rather than in the size of lending $(L_t^B = q_t K_t - W_t)$.

Suppose that K_t is a random variable; thus, so is $\bar{\omega}_t^a$. Macroprudential policy can reduce the volatility of lending activities, as the previous results suggest. Suppose that there is a macroprudential-policy-induced volatility reduction with regard to K_t and $\bar{\omega}_t^a$. More specifically, this volatility reduction preserves the expected value of $\bar{\omega}_t^a$. Since h is concave, this will raise the expected value $E_t h(\bar{\omega}_t^a)$; hence, $E_t(K_t)$ must increase. Therefore, this policy-induced volatility reduction is compensated by less conservative financial intermediation represented by higher expected capital and lending. Macroprudential policy can be welfare improving, as it helps to facilitate more credit by lowering the uncertainty related to lending activities. It should be noted that this source of the welfare gain is different from the externality in credit boom-bust cycles emphasized in Lorenzoni (2008) or Jean and Korinek (2011).

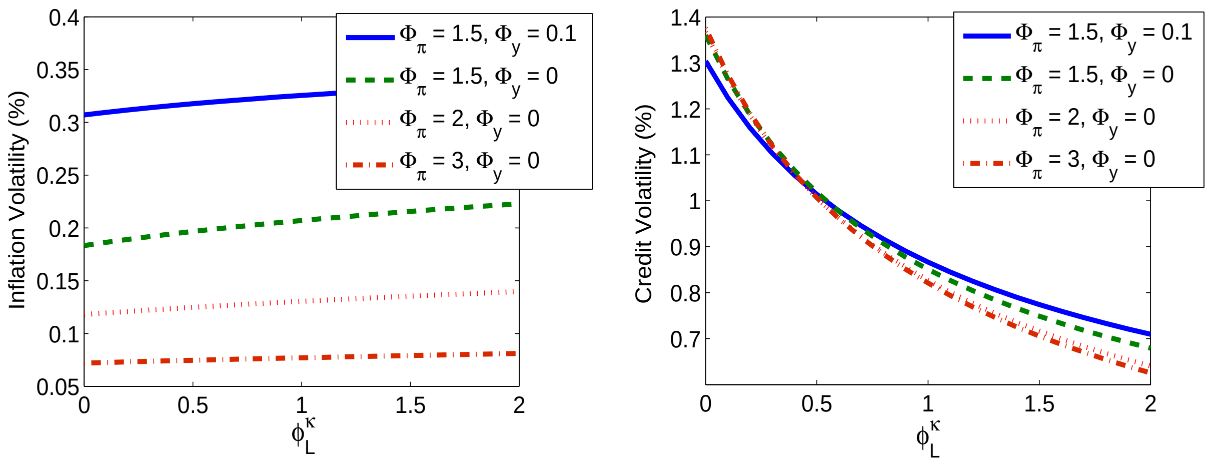
7. The Interaction between Macroprudential Policy and Monetary Policy

This section examines the relationship between monetary and macroprudential policies. To make the analysis more tractable, I rule out the DLTV policy, which has little welfare implication, and I assume that the DCRR is the only macroprudential policy tool. The analysis in the previous section shows that it is optimal to assign monetary policy to exclusively target inflation stabilization and macroprudential policy to exclusively target credit stabilization. Based on this separation optimality result, I discuss how each policy affects the performance of the other policy. Also, I analyze whether a monetary policy rule reacting to credit can better function in terms of inflation and credit stabilization. Results show why it is optimal to assign a single target variable for each policy. It allows each instrument to stabilize the target variable with only a limited effect on the other variable. The monetary policy rule reacting to credit is not only non-optimal but also ineffective as a credit stabilization instrument.

In order to see how the DCRR policy affects inflation stability, I calculate the unconditional standard deviation of inflation (σ_π) while varying the response of the capital requirement ratio to gross credit (ϕ_L^κ). Panel (a) in figure 6 shows the relationship for four different monetary policy specifications; a strict and aggressive inflation targeting ($\phi_\pi = 3$, $\phi_Y = 0$), weaker responses to inflation ($\phi_\pi = 2$, $\phi_Y = 0$), ($\phi_\pi = 1.5$, $\phi_Y = 0$), and a weaker response to inflation and a positive response to output ($\phi_\pi = 1.5$, $\phi_Y = 0.1$).

As expected, it is observed that inflation volatility is lower when the monetary policy response to inflation is stronger and the response to output is weaker. The stronger DCRR comes with higher inflation volatility, as the graph shows that inflation volatility is monotonically increasing in ϕ_L^κ. However, it also shows that when monetary policy has a large enough reaction to inflation, it can still effectively stabilize inflation even with the DCRR policy. For example, inflation volatility for ($\phi_\pi = 3$, $\phi_Y = 0$, $\phi_L^\kappa = 2$) is still smaller than ($\phi_\pi = 2$, $\phi_Y = 0$, $\phi_L^\kappa = 0$). This result indicates that the macroprudential policy does not necessarily impede inflation stability as long as monetary policy has a strong enough commitment to inflation stability.

In addition, I examine how different monetary policies affect the credit stabilization of the macroprudential policy. Panel (b) in figure 6 shows the unconditional standard deviation of credit (σ_L) while varying the response of the capital requirement ratio to gross credit (ϕ_L^κ). Credit volatility monotonically decreases as ϕ_L^κ increases. Interestingly, credit volatility reaches its lowest value when monetary policy has the most aggressive response to inflation, which is the optimal monetary policy with DCRR macroprudential policy.

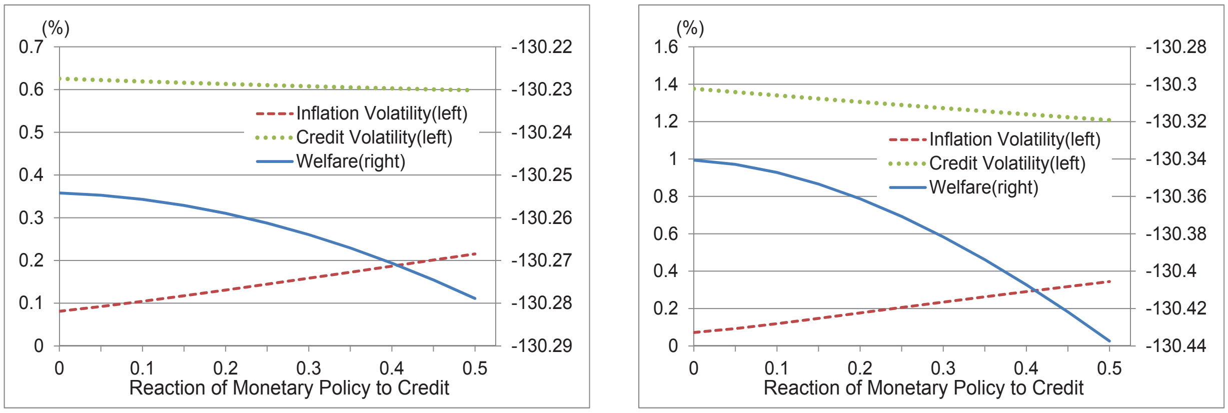


(a) Standard deviation: inflation

(b) Standard deviation: gross credit

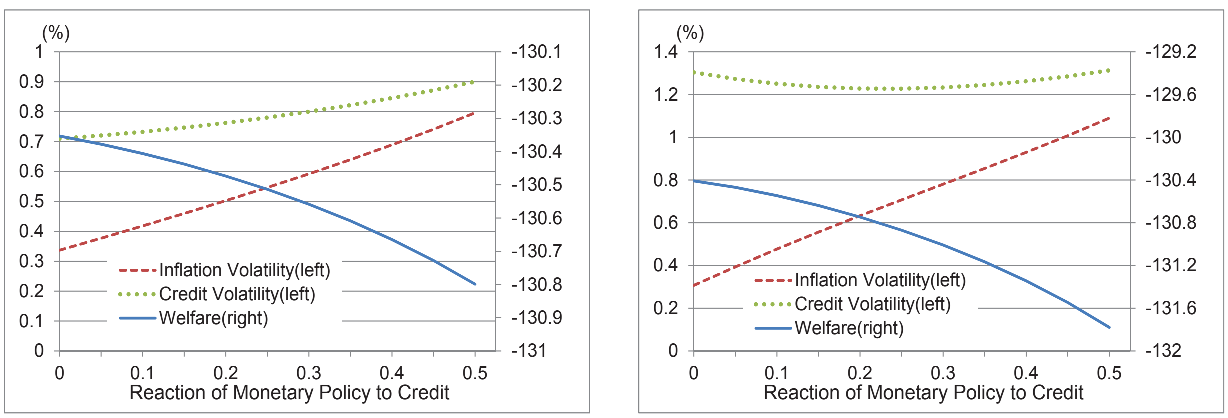
Figure 6: DCRR response to credit (ϕ_L^κ), welfare, inflation volatility, credit volatility, given different monetary policy parameters (ϕ_π, ϕ_Y)

Finally, figures 7 and 8 suggest how monetary policy rule reacting to credit affects welfare, inflation and credit volatility. In figure 7, monetary policy's reaction to inflation and output is set at $\phi_\pi = 3$, $\phi_Y = 0$ and its reaction to credit (ϕ_L) varies between 0 and 0.5 on the horizontal axis. The macroprudential policy exists in panel (a) and is absent in panel (b). As ϕ_L increases, the welfare measure decreases, credit volatility decreases and inflation volatility increases in both panels. Although monetary policy can reduce credit volatility, the effect is smaller than what the macroprudential policy can bring. Regardless of ϕ_L, credit volatility in panel (a) with the macroprudential policy is overall about half the size of that in panel (b) without the macroprudential policy. The figure shows that monetary policy reacting to credit is a less efficient tool in credit stabilization than macroprudential policy, and it comes with the cost of greater inflation instability and welfare loss.



(a) MOP ($\phi_\pi = 3, \phi_Y = 0$), MPP ($\phi_L^\kappa = 2$) (b) MOP ($\phi_\pi = 3, \phi_Y = 0$), MPP ($\phi_L^\kappa = 0$)

Figure 7: Reaction of monetary policy to credit (ϕ_L), inflation volatility, credit volatility and welfare



(a) MOP ($\phi_\pi = 1.5, \phi_Y = 0.1$), MPP ($\phi_L^\kappa = 2$) (b) MOP ($\phi_\pi = 1.5, \phi_Y = 0.1$) , MPP ($\phi_L^\kappa = 0$)

Figure 8: Reaction of monetary policy to credit (ϕ_L), inflation volatility, credit volatility and welfare

Figure 8 shows the same relationship with a different monetary policy reaction to inflation and output ($\phi_\pi = 1.5$, $\phi_Y = 0.1$). In both panel (a) (with the macroprudential policy) and panel (b) (without the macroprudential policy), the welfare measure decreases and inflation volatility increases as ϕ_L increases. Credit volatility increases in panel (a) and is non-monotonic in panel (b). A panel-by-panel comparison between figures 7 and 8 shows that the welfare measure is lower and inflation volatility is higher in figure 8.

Results in figures 7 and 8 support the idea that macroprudential policy is a more efficient tool to control credit. This result stems from the design of macroprudential policy in the model. While monetary policy influences both the supply (savers) and the demand (borrowers) for credit, the macroprudential policy operates only on credit demand. This gives an advantage to the macroprudential policy to control credit while affecting inflation less. Therefore, there is an intrinsic dichotomy between monetary policy and macroprudential policy regarding inflation and credit. Suh (2012) discusses this issue analytically using a simple New Keynesian model.

8. Conclusion

Results in this paper indicate that countercyclical macroprudential policy can stabilize the credit cycle. However, the LTV ratio reacting to housing price, which is a market-specific instrument targeting a market-specific asset price, produces regulatory arbitrage. This result poses a challenge for regulators tasked with the responsibility of maintaining financial stability. A welfare analysis shows that credit stabilization using macroprudential policy is welfare improving. Macroprudential policy helps facilitate more lending and capital accumulation by reducing the uncertainty related with lending activity. Optimal policy separates macroprudential from monetary policy objectives. The reason is that these instruments are used to stabilize different target variables.

These results are specific to the BGG-NKDSGE models. Nonetheless, this paper reveals the strengths and limits of using this class of models to study macroprudential policy, to explore its implications for macroprudential risk and its interactions with monetary policy. This matters because the BGG-NKDSGE model is a model of choice for macroeconomists wanting to examine the impact of financial frictions on the aggregate economy. Moreover, the BGG-NKDSGE model is a workhorse model for studying issues surrounding macroprudential risk. However, it remains the case that there may very well be better DSGE models with fiancial frictions waiting to be built.

Finally, macroprudential risk and regulatory arbitrage are topics needing further study. Along with the example in this paper, regulatory arbitrage can occur in an economy with non-negligible secondary credit markets that are, in general, subject to weaker regulation. Likewise, it can be found in international financial markets where countries adopt different regulatory standards. These examples suggest that there is large payoff to research that explores the issues raised in this paper. The hope is that this paper provides a basis on which to conduct future reseach on macroprudential risk.

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Appendix

A.1. Notations and Parameter Calibrations

Table 4: Notations

Variable	Description	Variable	Description
Y	Output	z	Marginal return of capital
I	Investment	q	Capital price
K	Capital	V	Net return to entrepreneurs
N	Aggregate Labor	W	Net worth of entrepreneurs
N_s	Labor, savers	B	Nominal bond
N_b	Labor, borrowers	D	Bank deposit
N_e	Labor, entrepreneurs	L^H	Household lending
N_f	Labor, bankers	L^B	Business lending
C_s	Consumption, savers	L	Aggregate lending (credit)
C_b	Consumption, borrowers	e	Bank capital
P^H	Housing price	κ	Capital ratio
H	Aggregate housing	$\bar{\omega}^{H,a}$	Ex-ante default threshold
H_s	Housing, savers		– household
H_b	Housing, borrowers	$\bar{\omega}^{H,b}$	Ex-post default threshold
I_s^H	Housing investment, savers		– household
I_b^H	Housing investment, borrowers	$\bar{\omega}^a$	Ex-ante default threshold
I^H	Aggregate housing investment		– business
T_s	Tax, savers	$\bar{\omega}^b$	Ex-post default threshold
T_b	Tax, borrowers		– business
w	Wage, households	A	Productivity shock
w_e	Wage, entrepreneurs	G	Government spending shock
w_f	Wage, bankers	ϵ^γ	Housing demand shock
mc	Marginal cost	ϵ^e	Bank capital shock
R^N	Nominal interest rate	ϵ^R	Monetary shock
R^D	Deposit rate		
R^e	Return on bank capital		
R^{LH}	Household lending rate		
R^{LB}	Business lending rate		
R^f	Interbank rate		
R^K	Return on capital		

Table 5: Parameter calibrations in the baseline model: static CRR, no LTV

β	0.99	Discount factor, savers	ρ_A	0.85	Autocorrelation, productivity
β_b	0.9885	Discount factor, borrowers	ρ_γ	0.95	Autocorrelation, housing demand
γ	0.9	Weight of housing in the utility	ρ_G	0.8	Autocorrelation, G
φ	2	Weight of labor in the utility	ρ_r	0.75	Autocorrelation, monetary policy
α_k	0.31	Weight, capital in production	ρ_κ	0.9	Autocorrelation, CRR
α_n	0.67	Weight, households' labor in production	ρ_{ltv}	0.9	Autocorrelation, target LTV ratio
α_e	0.01	Weight, entrepreneurs' labor in production	ϕ_π	1.5	Monetary policy response to inflation
α_f	0.01	Weight, bankers' labor in production	ϕ_Y	0.1	Monetary policy response to output
δ	0.025	Depreciation, capital	ϕ_L	0	Monetary policy response to credit
ν	0.973	Entrepreneur retention rate	SE_G	0.007	SE, government spending shock
σ	0.44	SD, idiosyncratic shock, business	SE_r	0.0006	SE, monetary shock
σ_H	0.13	SD, idiosyncratic shock, housing price	SE_e	0.004	SE, bank capital shock
ϕ	0.035	Bank dividend rate	SE_A	0.0067	SE, productivity shock
ψ_k	4	Capital adjustment cost	SE_γ	0.0021	SE, housing demand shock
$\frac{IH}{Y}$	0.03	Investment / output	ν_a^I	0.0025	Degree of regulatory intervention, CRR
$\frac{G}{Y}$	0.17	Government Spending / output	ν_b^I	25	Degree of regulatory sensitivity, CRR
μ	0.09	Default cost, business	ν_a^H	0.0025	Degree of regulatory intervention, LTV
μ^H	0.15	Default cost, household	ν_b^H	0	Degree of regulatory intervention, LTV
ν_c	0.0071	Markup in household lending			

Table 6: Steady-state interest rates and variable ratios

Variable	Description	Value
R	Deposit rate	1.04
R^f	Bank funding rate	R+0.001
R^{LH}	Household borrowing rate	R+0.036
R^{LB}	Entrepreneur borrowing rate	R+0.029
C/Y	Consumption-output ratio	0.64
I/Y	Investment-output ratio	0.16
I^H/Y	Housing investment-output ratio	0.03
G/Y	Investment-output ratio	0.17
K/W	Entrepreneur's capital-net worth ratio	1.66
L^B/L^H	Business lending-household lending ratio	3.27
L^H/Y	Household debt-output ratio	0.19

Note: all values are in real, annualized terms

A.2. Households' Optimization Problem

Saving households (denoted by s) with future discount rate β maximize

$$\max_{C_s,H_s,N_s,I_s^H,B,D,e} E_o\left\{\sum_{t=0}^{\infty}\beta^t[\gamma\log C_{t,s}+(1-\gamma\epsilon_t^{\gamma})\log H_{t,s}+\varphi\log(1-N_{t,s})]\right\} \tag{35}$$

subject to

$$C_{t,s}+P_t^H I_{t,s}^H+\frac{B_t}{P_t}+D_t+e_t+T_{t,s}\leq R_{t-1}^N\frac{B_t}{P_t}+R_{t-1}^D D_{t-1}+R_{t-1}^e e_{t-1}+w_t N_{t,s}+Div_t \tag{36}$$

We can set up the Lagrangian as

$$\begin{aligned}\mathcal{L}= \quad : \quad & E_o\sum_{t=0}^{\infty}\beta^t[\gamma\log C_{t,s}+(1-\gamma\epsilon_t^{\gamma})\log H_{t,s}+\varphi\log(1-N_{t,s}) \qquad (37)\\ & +\lambda_t(R_{t-1}^N\frac{B_{t-1}}{P_t}+R_{t-1}^D D_{t-1}+R_{t-1}^e e_{t-1}+w_t N_{t,s}+Div_t\\ & -C_{t,s}-P_t^H I_{t,s}^H-\frac{B_t}{P_t}-D_t-e_t-T_{t,s})+\mu_t((1-\delta_H)H_{t-1,s}+I_{t,s}^H-H_{t,s})]\end{aligned}$$

where the Lagrange multiplier λ is on the household budget constraint and μ is on the law of motion for housing stock.

The FOCs are shown below:

$$C_{t,s} \quad : \quad \frac{\gamma}{C_{t,s}}=\lambda_t \tag{38}$$

$$N_{t,s} \quad : \quad \frac{\varphi}{1-N_{t,s}}=\lambda_t w_t \Rightarrow \frac{\varphi}{1-N_{t,s}}=\frac{\gamma}{C_{t,s}}w_t \tag{39}$$

$$D_t \quad : \quad \lambda_t=\beta E_t\lambda_{t+1}R_t^D \Rightarrow \frac{1}{C_{t,s}}=\beta E_t\frac{1}{C_{t+1,s}}R_t^D \tag{40}$$

$$B_t \quad : \quad \frac{\lambda_t}{P_t}=\beta E_t\lambda_{t+1}\frac{R_t^N}{P_{t+1}} \Rightarrow \frac{1}{C_{t,s}}=\beta E_t\frac{1}{C_{t+1,s}}R_t^N\frac{P_t}{P_{t+1}} \tag{41}$$

$$e_t \quad : \quad \lambda_t=\beta E_t\lambda_{t+1}R_t^e \Rightarrow \frac{1}{C_{t,s}}=\beta E_t\frac{1}{C_{t+1,s}}R_t^e \tag{42}$$

$$I_{t,s}^H \quad : \quad \lambda_t P_t^H=\mu_t \tag{43}$$

$$H_{t,s} \quad : \quad \mu_t=\frac{1-\gamma\epsilon_t^{\gamma}}{H_{t,s}}+\beta E_t\mu_{t+1}(1-\delta_H) \tag{44}$$

Combining (38) and (39) gives us the labor supply decision. (40) and (41) are Euler equations for real deposits and nominal assets. Combining them gives us the Fisher equation. (42) is the Euler equation with respect to bank capital, which implies that the equilibrium return from bank capital should be the same as the deposit rate. (43) and (44) show that the shadow price of housing goods in the current period is given by the sum of the current period's marginal utility from housing goods and the discounted value of the next period's expected shadow price.

Borrowing households' Lagrangian is given as below.

$$\begin{aligned}\mathcal{L}^{\lfloor} = \ : \ & E_o \sum_{t=0}^{\infty} \beta_b^t [\gamma \log C_{t,b} + (1-\gamma\epsilon_t^{\gamma}) \log H_{t,b} + \varphi \log(1-N_{t,b}) \\ & + \lambda_t^b (w_t N_{t,b} + L_t^H - C_{t,b} \\ & - P_t^H [I_{t,b}^H + \int_0^{\bar{\omega}_t^{H,b}} \omega H_{t,b} f(\omega) d\omega] - [1 - F(\bar{\omega}_t^{H,b})] R_{t-1}^{LH} L_{t-1}^H - T_{t,b})]\end{aligned} \tag{45}$$

The optimal intertemporal borrowing decision of the borrowing household is derived considering the possibility of default.

$$\frac{1}{C_{t,b}} = \beta_b E_t [\frac{1}{C_{t+1,b}} (1 - F(\bar{\omega}_{t+1}^{H,b})) R_t^{LH}]. \tag{46}$$

The first-order condition for housing goods also becomes different from that of the saving household because of possible default.

$$\frac{\gamma}{C_{t,b}} P_t^H (1 + \int_0^{\bar{\omega}_t^{H,b}} \omega f(\omega) d\omega) = \frac{1-\gamma\epsilon_t^{\gamma}}{H_{t,b}} + \beta E_t \frac{\gamma}{C_{t+1,b}} P_{t+1}^H (1-\delta_H). \tag{47}$$

The optimal labor decision of borrowing households is similar to that of saving households.

A.3. Financial Accelerator Mechanism in the Business Sector

For simplification, define the bank's claim when non-default (ND^b) and default (D^b) as $ND^b \equiv (1 - F(\bar{\omega}))$, $D^b \equiv \int_0^{\bar{\omega}} \omega f(\omega) d\omega$, and the entrepreneur's claim when non-default (ND^e) as $ND^e \equiv \int_{\bar{\omega}}^{\infty} \omega f(\omega) d\omega - ND^b(\bar{\omega})\bar{\omega}$. If ω is assumed to follow a lognormal distribution $\big(\ln(\omega) \sim N(-\frac{1}{2}\sigma^2, \sigma^2), E(\omega) = 1\big)$, and define an auxiliary variable $x = (\ln(\bar{\omega}) + 0.5\sigma^2)/\sigma$, then it can be shown that

$$ND^b(\bar{\omega}) = 1 - \Phi(x), \quad D^b(\bar{\omega}) = \Phi(x - \sigma), \quad ND^e(\bar{\omega}) = 1 - D^b(\bar{\omega}) - ND^b(\bar{\omega})\bar{\omega}. \tag{48}$$

Entrepreneur's optimization problem is

$$\max ND_t^e(\bar{\omega}_t^a) E_t [R_{t+1}^K q_t K_t] \tag{49}$$

subject to

$$R_t^f(q_tK_t - W_t) = \{ND_t^b(\bar{\omega}_t^a)\cdot\bar{\omega}_t^a + (1-\mu)D_t^b(\bar{\omega}_t^a)\}E_t[R_{t+1}^K q_t K_t]. \tag{50}$$

The first-order conditions are given by

$$\partial\bar{\omega}_t^a \quad : \quad ND_t^{e\prime}(\bar{\omega}_t^a) = -\lambda[ND_t^b(\bar{\omega}_t^a) + ND_t^{b\prime}(\bar{\omega}_t^a)\bar{\omega}_t^a + (1-\mu)D_t^{b\prime}(\bar{\omega}_t^a)], \tag{51}$$

$$\partial K_t \quad : \quad ND_t^e(\bar{\omega}_t^a)\frac{E_tR_{t+1}^K}{R_t^f} = \lambda[1 - \{ND_t^b(\bar{\omega}_t^a)\cdot\bar{\omega}_t^a + (1-\mu)D_t^b(\bar{\omega}_t^a)\}\frac{E_tR_{t+1}^K}{R_t^f}], \tag{52}$$

$$\partial\lambda \quad : \quad [ND_t^b(\bar{\omega}_t^a)\cdot\bar{\omega}_t^a + (1-\mu)D_t^b(\bar{\omega}_t^a)]\frac{E_tR_{t+1}^K}{R_t^f}\frac{q_tK_t}{W_t} = \frac{q_tK_t}{W_t} - 1, \tag{53}$$

where

$$ND_t^{e\prime}(\bar{\omega}_t^a) = -D^{b\prime}(\bar{\omega}_t^a) - ND_t^b(\bar{\omega}_t^a) - ND_t^{b\prime}(\bar{\omega}_t^a)\bar{\omega}_t^a, \tag{54}$$

$$D^{b\prime}(\bar{\omega}_t^a) = (\bar{\omega}_t^a)f((\bar{\omega}_t^a)) = \frac{f(x_t-\sigma)}{\sigma\bar{\omega}} = \frac{f(x_t)}{\sigma}, \tag{55}$$

$$ND_t^{b\prime}(\bar{\omega}_t^a) = -\frac{f(x)}{\bar{\omega}_t^a\sigma}. \tag{56}$$

Substituting λ out, we have two equilibrium conditions

$$\frac{ND_t^b}{ND_t^b - \mu f(x_t)/\sigma} = \frac{ND_t^e(E_tR_{t+1}^K/R_t^f)}{1 - \{ND_t^b\cdot\bar{\omega}_t^a + (1-\mu)D_t^b\}(E_tR_{t+1}^K/R_t^f)} \tag{57}$$

and

$$[ND_t^b\cdot\bar{\omega}_t^a + (1-\mu)D_t^b]\frac{E_tR_{t+1}^K}{R_t^f}\frac{q_tK_t}{W_t} = \frac{q_tK_t}{W_t} - 1. \tag{58}$$

From (57) and (58) it is possible to derive the BGG financial accelerator equation.

$$E_t R_{t+1}^K = S(\frac{q_t K_t}{W_t}) R_t^f. \tag{59}$$

S is an increasing function in $(q_t K_t / W_t)$, implying that the external finance premium $(E_t R_{t+1}^K / R_t^f)$ is increasing in the asset-net worth (leverage) ratio. This is the reason why the mechanism is called a financial accelerator. If a positive shock that improves the net worth of the entrepreneur is realized, with better balance-sheet conditions she can further increase investment with a lower external finance premium.

A.4. Capital Goods Producer's Optimization Problem

Capital good producer's optimization problem is given by

$$\max_I (q_t - 1) I_t - f(\frac{I_t}{K_{t-1}}) K_{t-1}. \tag{60}$$

The first-order condition is written as

$$q_t = 1 + f'(\frac{I_t}{K_{t-1}}). \tag{61}$$

The function f is assumed to have a simple quadratic form,

$$f(\frac{I_t}{K_{t-1}}) = \frac{\chi_k}{2}(\frac{I_t}{K_{t-1}} - \delta)^2 K_{t-1}. \tag{62}$$

A.5. Retailers' Optimization Problem

Each retail goods producer (i) purchases intermediate goods and turns them into retail goods $(Y_t(i))$ in a monopolistically competitive market. Total final usable goods Y_t are the following composite of retail goods:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \tag{63}$$

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{\epsilon-1}} \tag{64}$$

Only (1-θ) fraction of retail goods producers are allowed to change price, à la Calvo (1983).

Define a measure of the resource cost induced by price dispersion (s_t) as

$$s_t = \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\epsilon} = (1-\theta)(p_t^*)^{-\epsilon} s_{t-1} \tag{65}$$

and introduce state variables x_t^1 and x_t^2 such that

$$x_t^1 = (p^*{}_t)^{-1-\epsilon} \frac{Y_t mc_t}{s_t} + \theta\beta E_t \frac{\lambda_{t+1,b}}{\lambda_{t,b}} \pi_{t+1}^{\epsilon} \left(\frac{p_t^*}{p_{t+1}^*}\right)^{-1-\epsilon} x_{t+1}^1 \tag{66}$$

$$x_t^2 = (p^*{}_t)^{-\epsilon} \frac{Y_t}{s_t} + \theta\beta E_t \frac{\lambda_{t+1,b}}{\lambda_{t,b}} \pi_{t+1}^{\epsilon-1} \left(\frac{p_t^*}{p_{t+1}^*}\right)^{-\epsilon} x_{t+1}^2 \tag{67}$$

and

$$\frac{\epsilon}{\epsilon-1} x_t^1 = x_t^2. \tag{68}$$

The aggregate price will be determined as

$$1 = \theta\pi_t^{-1+\epsilon} + (1-\theta)(p_{t-1}^*)^{1-\epsilon}. \tag{69}$$

A.6. Steady-state Conditions

$$A = \pi = p^* = s = q = N_e = N_f = 1, \quad mc = (\epsilon-1)/\epsilon. \tag{70}$$

$$x_1 = mc \cdot Y/(1-\theta\beta), \quad x_2 = \epsilon/(\epsilon-1)x_1. \tag{71}$$

$$N = N_b + N_s, \quad C = C_b + C_s, \quad H = H_b + H_s. \tag{72}$$

$$C_b = \frac{\gamma w(1-N_b)}{\varphi}, \quad C_s = \frac{\gamma w(1-N_s)}{\varphi}. \tag{73}$$

$$w = mc \cdot \alpha_n \frac{Y}{N}, \quad w_f = mc \cdot \alpha_{nf} \frac{Y}{N_f}, \quad w_e = mc \cdot \alpha_{ne} \frac{Y}{N_e}. \tag{74}$$

$$R^D = R^N = \frac{1}{\beta}, \quad R^{LH} = \frac{1}{\beta_b \cdot ND^{B,H}}. \tag{75}$$

Ex-ante and ex-post default thresholds are the same in the steady state.

$$\bar{\omega} \equiv \bar{\omega}^a = \bar{\omega}^b, \quad \bar{\omega}^H \equiv \bar{\omega}^{H,a} = \bar{\omega}^{H,b}. \tag{76}$$

$$I_s^H = I^H / \left[1 + \frac{C_b}{C_s} \cdot \frac{1-\beta(1-\delta_H)}{1+D^{B,H}-\beta_b(1-\delta_H)} \right]. \tag{77}$$

$$I^H = \frac{I^H}{Y} \cdot Y, \quad I^H = I_s^H + I_b^H, \quad H_s = I_s^H/\delta_H, \quad H_b = I_b^H/\delta_H. \tag{78}$$

$$P^H = \frac{1-\gamma}{\gamma} \frac{C_s}{H_s}(1-\beta(1-\delta_H)). \tag{79}$$

$$N_b = \left[\frac{\gamma w}{\varphi} + (R^{LH} ND^{B,H} - 1)L^H + P^H(I_b^H + D^{B,H} H_b) + \frac{1}{2} G \right] / (w + \frac{\gamma w}{\varphi}). \tag{80}$$

$$R^{LH} L^H = \bar{\omega}^H P^H H_b. \tag{81}$$

$$[R^f + \nu_c + \nu_a^H] L^H = ND^{B,H} \cdot R^{LH} L^H + (1-\mu) D^{B,H} \cdot P^H H_b. \tag{82}$$

$$R_t^f = \kappa_t R_t^e + (1-\kappa_t) R_t^D + s(\overline{\kappa}_t - \kappa_t). \tag{83}$$

$$e = w_f/\phi, \quad L = L^B + L^H, \quad D + e = L, \quad \kappa = \frac{e}{L}. \tag{84}$$

$$z = mc \cdot \alpha \frac{Y}{K}, \quad R^K = z + (1 - \delta). \tag{85}$$

$$V = (1 - D^b) R^K K_{t-1} - (ND^b) R^{LB} L^B, \quad W = \upsilon V + w_e, \quad L^B = K - W. \tag{86}$$

$$R^f L^B = (ND^b \bar{\omega} + (1 - \mu) D^b) R^K K. \tag{87}$$

$$\frac{ND^b}{ND^b - \mu f(x)/\sigma} = \frac{ND^e (R^K/R^f)}{1 - (\bar{\omega} \cdot ND^b + (1 - \mu) D^b)(R^K/R^f)}. \tag{88}$$

Here $f(x)$ is the pdf of the standard normal distribution. See the appendix of Bernanke, Gertler, and Gilchrist (1999).

$$Y = K^{\alpha_k} N^{\alpha_n}. \tag{89}$$

$$K = I/\delta, \quad g = \frac{g}{Y} Y, \quad I^H = \frac{I^H}{Y} Y. \tag{90}$$

$$Y = C_s + C_b + I + G + \mu D^B R^K K + \mu^H D^{B,H} P^H H + I^H + \nu_a^I L + \nu_a^H L^H. \tag{91}$$