Imperfect Competition in Selection Markets

Neale Mahoney† E. Glen Weyl‡

December 27, 2013

Abstract

Many standard intuitions about the distortions created by market power and selection are reversed when these forces co-exist. Adverse selection may be socially beneficial under monopoly, for example, and market power may be beneficial in the presence of advantageous selection. We develop a simple, but quite general, model of symmetric imperfect competition in selection markets that parameterizes the degree of both market power and selection. We derive basic comparative statics verbally and illustrate them graphically to build intuition. We emphasize the relevance of the most counter-intuitive effects with a calibrated model of the insurance market and empirical results from the credit card industry. Among other policy insights, we show that in selection markets four core principles of the United States Horizontal Merger Guidelines are reversed.

Keywords: selection market, imperfect competition, mergers, risk-adjustment, risk-based pricing

JEL classifications: D42, D43, D82, I13, L10, L41

†Weyl acknowledges the financial support of the Ewing Marion Kauffman foundation which funded the research assistance of Kevin Qian. We are grateful to André Veiga for helpful comments. All errors are our own.
‡Chicago Booth and NBER. Email: neale.mahoney@gmail.com
‡University of Chicago. Email: weyl@uchicago.edu
1 Introduction

Adverse selection, or the tendency of the costliest consumers to also be most eager to enter the market, can limit the power of a monopolist to raise price, as higher prices entail facing costlier consumers. As a result, standard policies that aim to reduce the degree of adverse selection, such as risk-adjustment or risk-based pricing in insurance markets, may allow firms to charge higher prices and thereby reduce consumer, or even social, surplus. Conversely, market power itself may help mitigate the tendency of advantageous selection, where the cheapest consumers are the most eager to enter the market, to create excessive supply as firms chase the most profitable, infra-marginal consumers. Thus, traditional competition policy that aims to reduce market power can lower social surplus in the presence of advantageous selection.

This paper studies imperfect competition in selection markets. Market power is perhaps the central topic in industrial organization, with a history tracing back to Cournot (1838). More recently, many leading scholars in the field have turned their attention to quantifying the welfare effects of selection (Einav, Finkelstein and Levin, 2010). Yet despite the striking contradictions of conventional wisdom about both selection markets and market power that arise when the two forces interact, we are not aware of a systematic investigation in the literature of the normative consequences of imperfect competition in selection markets. This paper provides such a treatment, derives from it several basic comparative statics and draws out from these several implications for competition and selection policy that contrast with the conventional wisdom in these areas.

We begin in the next section by presenting a general model of symmetric imperfect competition in selection markets. Building on Weyl and Fabinger (2013) and Bresnahan (1989), we propose a model that nests standard micro-foundations of market structure including monopoly, perfect competition and versions of symmetric Cournot competition (with or without conjectural variations) and differentiated products Bertrand competition. Allowing for selection requires strengthening the notion of symmetry in a way first proposed by Rochet and Stole (2002) and generalized by White and Weyl (2012) in the context of preferences for non-price product characteristics. In particular we assume that, at symmetric prices, all firms receive a representative sample of all consumers purchasing the product in terms of their cost, and that a firm cutting its price steals consumers with similarly representative distribution of costs from its competitors. This allows a simple parameterization of
the “extent” of both market power and selection, each with a single parameter, $\theta$ and $\sigma$ respectively.

We use this model in Section 3 to derive comparative statics that sometimes match, and sometimes contradict, standard intuitions.

1. Under adverse selection, social surplus is (weakly) decreasing in market power. Adverse selection leads to undersupply and market power only worsens this problem.

2. Under advantageous selection, social surplus is inverted-U-shaped in market power. Advantageous selection leads to oversupply, thus market power is socially beneficial up to a point as it offsets the natural tendency towards excessive supply.

3. Despite its direct costs, increasing the extent of adverse selection may benefit consumers, and even society, if market power and equilibrium quantity are both sufficiently high, as increased selection makes the marginal consumer less costly to serve, thereby lowering price and offsetting market power.

4. Conversely advantageous selection is beneficial if the market is sufficiently competitive or quantity is sufficiently low, because increased selection lowers the cost of the marginal consumer and directly lowers firm costs by creating a better selection of consumers in the market.

To illustrate the implications of these comparative statics, in Subsection 4.1 we apply them to a canonical problem in competition policy: the evaluation of a merger of two firms. We show that several standard intuitions embodied in the latest revision of the United States Horizontal Merger Guidelines (United States Department of Justice and Federal Trade Commission, 2010) are partially or fully reversed in selection markets. For example, we show that large “Upward Pricing Pressure” (UPP), which is a standard indicator of a prospective merger’s harm, can instead be generated by advantageous selection. The means that UPP can be large exactly in settings where there can too much competition and additional market power can be socially beneficial. And we show that mergers between firms selling highly substitutatable products, which is typically viewed as most harmful, should be interpreted positively in settings with advantageous selection. This is because advantageously selected markets with highly substitutable products are most likely to be those where industry supply exceeds the socially optimal level.

Beyond these theoretical points, we verify the practical relevance of our theoretical arguments
in a calibrated model of a monopolized health insurance industry (Subsection 4.2) and using empirical data on credit cards (COMING SOON). The calibrated model of health insurance generates the counterintuitive result that eliminating adverse selection by implementing back-end risk-adjustment raises prices and reduces quantity in the market, thereby harming consumers. Normatively, the results suggest that while risk-adjustment may modestly increase or decrease overall surplus depending on the normative interpretation, the most striking effect is a transfer of surplus from consumers to producers. Indeed, we find that risk-adjustment reduces consumer surplus by nearly 10%. Allowing firms to segment the market and implement risk-based pricing has qualitatively identical, but quantitatively larger, effects because it also allows price discrimination.

Our paper is mostly closely related to Einav, Finkelstein and Cullen (2010) and Einav and Finkelstein (2011), who conduct a general analysis of a perfectly competitive selection markets that builds on the classical theory of a natural monopoly regulated to charge average cost prices (Dupuit, 1849; Hotelling, 1938).\textsuperscript{1} A constraint in applying this framework is that the assumption of perfect competition is problematic in classic selection markets such as insurance and consumer credit.\textsuperscript{2} Perhaps because of this, existing work on imperfect competition in selection markets has typically taken an approach that relies more heavily on structural assumptions about firm and consumer behavior (e.g., Lustig, 2010; Starc, Forthcoming).

Our main contribution is to provide a general understanding of the interaction between selection and imperfect competition. To do so we extend the price theoretic approach of Einav and Finkelstein—all of our main results can be understood as applications of classical price theory and are thus conveyed with simple graphs and verbal descriptions. As a result, the text contains a minimum of formalism, with formal mathematical statements and proofs presented in the appendix. We hope that in addition to providing general but sharp results about the welfare effects of imperfect competition in selection markets, our approach also helps build broader intuition and thereby provides a foundation for empirical work in a range of socially important markets.

\textsuperscript{1}This is an application of Marshall (1890)'s famous observation that competitive industries with economies or diseconomies of scale that are external to an individual firm's production would operate identically to a monopolist regulated to charge a price at average cost.

\textsuperscript{2}In their survey on empirical models of insurance markets, Einav, Finkelstein and Levin (2010) write that “there has been much less progress on empirical models of insurance market competition, or on empirical models of insurance contracting that incorporate realistic market frictions. One challenge is to develop an appropriate conceptual framework. Even in stylized models of insurance markets with asymmetric information, characterizing competitive equilibrium can be challenging, and the challenge is compounded if one wants to allow for realistic consumer heterogeneity and market imperfections.”
2 Model

In this section we describe a general model of symmetric imperfect competition that nests monopoly, perfect competition and common models of imperfect competition including Cournot and differentiated products Bertrand competition. By placing these models in a common framework, we are able to develop results in the next section that are robust to the details of the industrial organization. Our model follows closely that in Weyl and Fabinger (2013), with modifications necessary to allow for the selection effects that are the focus of our work here.

Consider an industry with symmetric firms that produce symmetric, though not necessarily identical, products. When firms produce symmetric quantities, prices are given by \( P(q) \), where \( q \in [0, 1] \) denotes the fraction of consumers served by the market. We do not specify the cardinality of the firms in the market to minimize the notational burden.

As in Einav and Finkelstein (2011), and as described more formally by Veiga and Weyl (2013a), every individual served in the market potentially has a different cost but the total cost of the industry is summarized by an aggregate cost function \( C(q) \), given by the aggregation of the cost of all individuals served, and associated marginal cost function \( MC(q) \equiv C'(q) \) and an average cost \( AC(q) \equiv \frac{C(q)}{q} \). These may be increasing or decreasing in aggregate quantity depending on whether selection is respectively “advantageous” or “adverse”. We assume that firms have no internal economies or diseconomies of scale, and thus no fixed costs. At a symmetric equilibrium, firms supply segments of the market that are equivalent in terms of their distribution of costs and thus have average costs equal to \( AC(q) \).

Industry profits are \( qP(q) - C(q) = q[P(q) - AC(q)] \). A competitive equilibrium requires that firms earn zero profits and is characterized by \( P(q) = AC(q) \). A monopolist or collusive cartel chooses \( q \) to maximize profit by equating marginal revenue to marginal cost

\[
P(q) + qP'(q) \equiv MR(q) = MC(q).
\]

---

\(^3\)Some consumers may favor one product over another, but there must be an equal number of consumers who have the symmetric opposite preference.

\(^4\)It is possible that these slopes have different signs over different ranges or over a particular range different signs of slopes between the two over a particular range. All of these cases do not fall cleanly into one category or the other and are not our focus in what follows. It would be interesting to extend our analysis to such cases.
We refer to \(-qP'(q)\) as the *marginal consumer surplus* \(MS(q)\).\(^5\) Panel A of Figure 1 shows the perfectly competitive and monopoly equilibria in the case of “advantageous selection” where \(AC'(q) > 0\) and the consumers with the highest willingness-to-pay are least costly. Panel B shows the perfectly competitive and monopoly equilibria in the case of “adverse selection” where \(AC'(q) < 0\) and the consumers with the highest willingness-to-pay are least costly.\(^6\)

2.1 Imperfect Competition (\(\theta\))

We can nest the monopoly optimization and competitive equilibrium conditions into a common framework by introducing a parameter \(\theta \in [0, 1]\). The parameter indexes the degree of competition in the market with \(\theta = 0\) under perfect competition and \(\theta = 1\) under monopoly. Equilibrium prices are given by

\[
P(q) = \theta [MS(q) + MC(q)] + (1 - \theta)AC(q).
\]

(1)

Below we discuss how Equation 1 is a reduced-form representation of two canonical models of imperfect competition. Formal derivations of these representations appear in Appendix Section A.

1. *Cournot:* There are \(n\) symmetric firms that each choose a quantity \(q_i > 0\), taking the quantity chosen by other firms as given. Price is set by Walrasian auction to clear the market so that the price is \(P(q)\) where \(q = \sum_i q_i\). If we assume that each firm gets a random sample of all consumers who purchase the product, then the equilibrium is characterized by Equation 1 with \(\theta \equiv \frac{1}{n}\). Intuitively, just as in the standard Cournot model, firms internalize their impacts on aggregate market conditions proportional to their market share (\(\frac{1}{n}\) at equilibrium) and otherwise act as price- and average cost-takers. This model can easily be extended to incorporate conjectural variations; see Weyl and Fabinger (2013) for details.

2. *Differentiated Product Bertrand:* There are \(n\) single-product firms selling symmetrically differentiated products. Each firm chooses a price \(p_i\) taking as given the prices of all other firms.

Consumers have a type that determines their utility for each product and their cost to firms.

\(^5\)Note that the marginal social value, gross of costs, created by increasing \(q\) is \(P(q)\). The part of this captured by firms is the marginal revenue \(P(q) + qP'(q)\). The rest, \(-qP'(q)\) is captured by consumers. Another way to see this is that consumer surplus is \(\int_0^q P(x) - P(q)dx\) so differentiating with respect to \(q\) yields \(-\int_0^q P'(q)dx = -qP'(q)\).

\(^6\)We follow Einav and Finkelstein in defining the sign of selection in terms of the slope of the average cost curve as this determines the sign of the marginal distortion under perfect competition as \(AC'(q) = \frac{MC(q) - AC(q)}{q}\).
The distribution of consumer types is symmetric in the interchange of any two products. In addition to these traditional assumptions of the symmetrically differentiated Bertrand model we add two additional assumptions proposed by White and Weyl (2012) that imply our representation is valid. First, the distribution of costs is orthogonal to the distribution of preferences across products given the highest utility a consumer can earn from any product. Second, the distribution of utility among the switching consumers that definitely will buy one product but are just indifferent between any two products is identical to that among the set of all consumers who are currently purchasing. These two assumptions imply that the average cost of consumers that switch between firms in response to a small price change is the same as the average cost among all participating consumers.

In Appendix A we provide two micro-foundations for these assumptions. The first is a re-normalized version of the Chen and Riordan (2007) “spokes” model that generalizes Hotelling (1929)’s linear city model in which the dimensions of consumer’s type other than her spatial position are orthogonal to her spatial position as in Rochet and Stole (2002). The second is a discrete choice, random utility model in the spirit of Anderson, de Palma and Thisse (1992) in which, rather than utility draws being independent across products as in Perloff and Salop (1985), the relative utility of different products is independent of the draw of the first-order statistic of utilities and the distribution of consumer costs is mean-independent of relative utilities conditional on the first-order statistic.

In this case, again, our representation is valid if $\theta \equiv 1 - D$ where the aggregate diversion ratio $D \equiv -\sum_{j \neq i}\frac{Q_i}{\partial Q_j/\partial p_j}$, which is independent of the $i$ chosen at symmetric prices by symmetry. Note that, unlike in the previous case, $\theta$ will not be constant in this case; it will typically increase in price and thus decline in quantity (Weyl and Fabinger, 2013).

### 2.2 Selection ($\sigma$)

Selection arises from the fact that consumers have different costs that may be correlated with their willingness-to-pay. If all consumers had identical cost, $AC$ and $MC$ would be constant. A natural way to parameterize selection $\sigma$ is for $\sigma = 0$ to represent a situation in which costs are homogeneous across individuals and $\sigma = 1$ to be normalized to represent a situation in which costs are “fully” heter-
erogeneous (i.e., either “fully” adversely or “fully” advantageously selected). Below, for example, we normalize \( \sigma = 1 \) to represent the baseline scenario in which there is no risk adjustment to compensate firms for the differential costs of consumers that they receive or there is perfect correlation between risk and willingness-to-pay.

For many counterfactuals involving changes in the extent of selection, it is natural to hold fixed the average cost of all individuals in the population, \( AC(1) \). Doing so implies that if \( \sigma = 0 \), then average and marginal costs at any \( q \) are equal to average costs in the population \( AC(1) \). Thus to parameterize the degree of selection we can replace average costs with \( \sigma AC(q) + (1 - \sigma) AC(1) \) and marginal costs with \( \sigma MC(q) + (1 - \sigma) AC(1) \). Substituting these terms for into Equation 1 yields

\[
P(q) = \theta MS(q) + \sigma \left[ \theta MC(q) + (1 - \theta) AC(q) \right] + (1 - \sigma) AC(1).
\]

so that we have representation for price where \( \theta \) indexes the degree of market power and \( \sigma \) indexes the degree of selection in the market. As in the case of our parameterization of competition, this formulation of selection can be motivated in two ways:

1. **Risk-adjustment**: Suppose that firms selling insurance receive a risk-adjusted payment for each customer in their plan that partially accounts for the difference between the customer’s marginal costs and average costs in the population. For example, the Medicare Advantage system makes risk-adjustment payments based on demographics and ex ante health conditions and the national health systems in the Netherlands and Germany implement similar systems (Ellis and Van de Ven, 2000).

These payments often do not fully account for selection, both because the payments are not always given full weight in payment formulas and because consumers have private information that makes it difficult to predict expected costs. Let \( (1 - \sigma) \) indicate the fraction of the difference between expected average and population average costs that is compensated for by risk adjustment. The average risk adjustment payments are then \( ARA(q) \equiv (1 - \sigma) [AC(q) - AC(1)] \) with \( \sigma = 1 \) normalized to a setting where firms receive no risk adjusted payments and \( \sigma = 0 \) indicating a setting where firms are fully compensated for any differential selection their receive. Firm average costs are the difference between their individual average cost and the average risk
adjustment:
\[
\hat{AC}(q) = AC(q) - ARA(q) = \sigma AC(q) + (1 - \sigma) AC(1).
\]

Similarly industry marginal cost is the same weighted average of marginal cost and \(AC(1)\), as in Equation 2.

Note that the risk-adjustment is a cost that must be paid, or a benefit collected, by some external to the system, such as the government. We do not include such costs or benefits on our welfare analysis, though we flag in our application how including it would impact our results: namely it makes them even more striking. We plan to deal with this issue in greater detail in the next draft.

2. Degree of correlation: Building upon work by Chiappori and Salanié (2000), a rapidly growing literature estimates the correlation between demand and marginal costs in a wide variety of selection markets (e.g., Finkelstein and Poterba, 2004; Bundorf, Levin and Mahoney, 2012).

Consider a standard econometric model of product choice.
\[
v = \tilde{\beta}_0 + \tilde{\beta}_1 (c - \mu_c) + \epsilon.
\]

Here willingness-to-pay \(v\) depends linearly on expect costs \(c\), which are distributed normally in the population \(c \sim \mathcal{N}(\mu_c, V_c)\), and a mean-zero idiosyncratic taste parameter \(\epsilon\), which is independent of costs and normally distributed \(\epsilon \sim \mathcal{N}(0, V_\epsilon - \tilde{\beta}_1^2 V_c)\). In this formulation, we parameterize the variance of \(v\) with \(V_v\), rather than parameterizing the variance of \(\epsilon\), so that the correlation between \(c\) and \(v\) may be adjusted holding fixed the marginal distribution of \(v\). Similarly, we normalize \(\tilde{\beta}_0\) and \(\tilde{\beta}_1\) so that changing \(\tilde{\beta}_1\) does not impact the mean of the marginal distribution of \(v\).

Consumers purchase the product if and only if their willingness to pay is greater than the price:
\[
q = 1 \iff v > p \iff \tilde{\beta}_0 + \tilde{\beta}_1 c + \epsilon > p.
\]

If we divide through by the standard deviation of the taste parameter \(\sqrt{V_\epsilon} = \sqrt{V_\epsilon - \tilde{\beta}_1^2 V_c}\) and define \(\beta_2 = 1/\sqrt{V_\epsilon - \tilde{\beta}_1^2 V_c}\) and the coefficients \(\beta_i = \beta_2 \tilde{\beta}_i\) for \(i = 0, 1\), the model can be estimated
by a Probit regression of product choice on expected costs and premiums, assuming we have a
source of exogenous variation in premiums:

$$\Pr(q = 1|c, p) = \Phi(\beta_0 + \beta_1 c - \beta_2 p),$$

and the parameters $\mu_c$ and $V_c$ can be estimated directly from the data: $\tilde{\beta}_1 = \beta_1 / \beta_2$ and $V_v = 1/\beta_2^2 + \tilde{\beta}_1^2 V_c$.

Using standard properties of the normal distribution, we show in the Appendix that these esti-
mates imply that the industry marginal cost is

$$\hat{MC}(q) = \mathbb{E}[c|v = P(q)] = \tilde{\beta}_1 \sqrt{\frac{V_c}{V_v}} \left[ \sqrt{V_c} \Phi^{-1} (1 - q) \pm \mu_c \right] + \left( 1 - |\tilde{\beta}_1| \sqrt{\frac{V_c}{V_v}} \right) \mu_c$$

and average cost is

$$\hat{AC}(q) = \mathbb{E}[c|v \geq P(q)] = \tilde{\beta}_1 \sqrt{\frac{V_c}{V_v}} \left[ \sqrt{V_c} e^{-\frac{[\Phi^{-1} (1-q)]^2}{2}} \pm \mu_c \right] + \left( 1 - |\tilde{\beta}_1| \sqrt{\frac{V_c}{V_v}} \right) \mu_c$$

where the $\pm$ has the sign of $\tilde{\beta}_1$. Thus defining $\sigma$ as $|\tilde{\beta}_1| \sqrt{\frac{V_c}{V_v}}$, which is always between 0 and 1 because it is the absolute value of the correlation between $v$ and $c$, $MC(q) \equiv \sqrt{V_c} \Phi^{-1} (1 - q) \pm \mu_c$ and

$$AC(q) \equiv \sqrt{V_c} e^{-\frac{[\Phi^{-1} (1-q)]^2}{2}} \pm \mu_c$$

fits our model, with positive sign if selection is adverse and negative if selection is advanta-
geous. Note that here we have normalized perfect correlation between willingness-to-pay and
cost, the standard uni-dimensional model of heterogeneity in Akerlof (1970) for adverse and
de Meza and Webb (1987) for advantageous selection, as $\sigma = 1$.

Because the degree of correlation is a property of a market, and not the result of a policy inter-
vention, this interpretation of $\sigma$ is most useful for studying comparative statics across markets
rather than the impacts of policy interventions. For example, Hendren (2013) compares out-
comes in markets with different degrees of correlation under the assumption of perfect com-
petition; our comparative statics with respect to $\sigma$ would allow such analysis to be extended to
imperfect competition.\footnote{Hendren’s set-up is less parametric than the one we describe here and thus our parametrization would not fit exactly. However our calibrated results in Subsection 4.2 do suggest that the presence of a realistic degree of market power could substantially alter his results.}

Of course these are only two possible environmental changes that could impact selection. Others commonly-discussed are changes in the permitted extent of risk-based pricing (Finkelstein and Poterba, 2006) and changes in consumers’ knowledge of their own costs (Handel and Kolstad, 2013). Unlike the micro-foundations above, these interventions will not only result in a change in the cost curves but will also shift the demand curves. In the first case this is because the same characteristics that are used to price risk can also be used to price discriminate and in the second case because greater knowledge by consumers of their health risks will shift the distribution of willingness-to-pay for insurance, not only the correlation of this distribution with cost.

In some cases, these discriminatory motives will offset or even reverse the results we derive about the effects of selection under market power; see Appendix B for an example. However, as we show in Subsection 4.2, allowing for a price-discrimination effect will often strengthen our main results, especially our most counterintuitive result that eliminating adverse selection may harm consumers.\footnote{However, as discussed in Subsection 4.2, price discrimination will typically increase social welfare (?)} However in what follows we focus attention on cost-side effects, both because price discriminatory effects are already well understood in the literature and because the policy interventions we are most interested in primarily impact firm costs.\footnote{For a detailed analysis of the interaction between cost-based pricing and price discrimination in the monopoly setting see Chen and Schwartz (2013).}

In the next section we thus study equilibrium as characterized by Equation 2. To ensure a unique such equilibrium exists, we impose global stability conditions that, while not necessary for our results, simplify the analysis. In particular we assume that $P' < \min\{AC', MC', 0\}$ and $MR' < \min\{MC', 0\}$. Under these conditions there is a unique equilibrium for a constant value of $\theta$, the case we focus on below. While $\theta$ is not constant in the Bertrand case, all of our results below can be extended to the case of non-constant $\theta$ with appropriately generalized stability conditions at the cost of some notational complexity.
3 Results

In this section, we present results on the welfare effects of (i) market power in industries with selection and conversely (ii) selection in industries with market power. To do so, we build on the notation, equilibrium and stability conditions of the previous section. To ease the exposition, all propositions are stated verbally. When possible, the results are illustrated graphically assuming linear demand and costs, and often focusing on the extreme cases of monopoly and perfect competition. Formal statements and proofs of all results appear in Appendix Section C.

3.1 Imperfect Competition

Proposition 1. Market power increases producer surplus and decreases consumer surplus

As firms gain market power, they increasingly internalize the impact of their output decisions on equilibrium price and quantity. This leads them to raise their price so long as price slopes downward more quickly than does average cost ($AC' > P'$), as implied by our stability assumptions. This internalization directly leads to higher producer surplus. The higher price that results reduces consumer surplus by the logic of the envelope condition.

Proposition 2. Under adverse selection, social surplus falls with market power. Any time a market would collapse as a result of adverse selection no monopolist would choose to operate.

With perfect competition, adverse selection leads to too little equilibrium quantity, as shown in Panel (b) of Figure 1. Since market power reduces quantity, market power only further reduces social surplus. An implication is if the market collapses under perfect competition (Akerlof, 1970), and therefore the market generates no social surplus, then no amount of market power will restore the market and enable it to contribute to aggregate welfare (Dupuit, 1844).

Thus, at least under adverse selection, standard intuitions about the undesirability of market power are confirmed. However, while these results are in this sense unsurprising, they contrast with intuitions in the contract theory literature that market power may be beneficial under adverse selection. For example, Rothschild and Stiglitz (1976) argue that imperfect competition may be necessary to sustain the existence of markets under adverse selection when non-price product characteristics are endogenous, and Veiga and Weyl (2013b) show that imperfect competition can indeed restore
the first-best, albeit in a stylized model. However, Veiga and Weyl assume a covered market and thus assume away the deleterious effects our results show that market power has on the number of individuals covered in the market.

Under advantageous selection our analysis more directly contradicts conventional intuitions on the impact of market power.

**Proposition 3.** Under advantageous selection, social surplus in inverse-U shaped in market power. There is a socially optimal degree of market power strictly between monopoly and perfect competition. Additional market power is beneficial socially below this level and socially harmful if it is above this level. The optimal degree of market power is increasing in the degree of advantageous selection.

Under advantageous selection, perfect competition leads to excessive output because in an attempt to cream skim from their rivals, competitive firms attract higher marginal cost consumers into the market (de Meza and Webb, 1987). On the other hand, a monopolist, who internalizes the industry cost and revenue curves, will produce too little. As a result, there is an intermediate degree of market power that leads to the optimal quantity being produced.

Figure 2 shows this result graphically. The monopoly equilibrium, determined by $MR = MC$, results in too little quantity. The perfectly competitive equilibrium, determined by $P = AC$, results in too much. An intermediate level of market power $\theta = \theta^*$, which leads to the equilibrium determined by $\theta^*MR + (1 - \theta^*)P = \theta^*MC + (1 - \theta^*)AC$, results in the same equilibrium level of quantity as the equilibria achieved by setting $P = MC$ and is therefore socially optimal. Because advantageous selection always pushes firms towards excessive production, the degree of market power required to offset this selection and restore optimality increases with the extent of advantageous selection.

### 3.2 Selection

Our results about the impact of changing the extent of selection are easiest to state verbally for the cases of monopoly and perfect competition. We thus confine attention here to these extreme cases. Results for intermediate cases are a natural interpolation between these and are stated and proved in the formalization of these propositions in Appendix C.

**Proposition 4.** Under monopoly, increasing the extent of adverse selection reduces profits but can raise or lower consumer surplus. Increasing the degree of adverse selection in more likely to benefit consumers when
the monopolist’s optimal quantity is high. When quantity is sufficiently high, increasing the degree of adverse selection can raise social surplus.

Figure 3 shows the effect of increasing the degree of adverse selection. Panels in the left column show the scenario in which there is no selection and the average and marginal cost curves are horizontal. Panels in the right column show the effect of increasing the degree of selection of adverse selection, depicted by a clockwise rotation of the average cost curve around the point \( AC(1) \) and a corresponding shift in the marginal cost curve. The resulting average cost curve is downward sloping (indicating adverse selection) and has unchanged population average costs (\( AC(1) \) is the same). Panels in the top row show the effect of this shift when the equilibrium quantity is low and panels in the bottom row show the effect when the optimal quantity is high.

When the equilibrium quantity is low, the increase in selection raises the cost of the average marginal consumer because adverse selection implies that the first consumers into the market are the costliest. This increases equilibrium quantity and lowering price. When the equilibrium quantity is high, the increase in selection lowers the cost of the average marginal consumer, increasing equilibrium quantity. In this setting with linear costs, increasing the degree of selection increases quantity whenever the optimal monopolist quantity is greater than \( \frac{1}{2} \). More generally, increasing the degree of adverse selection increases quantity and reduces prices whenever the population average consumer has costs that are higher than the average marginal consumer at the optimal level of quantity.

By the envelope theorem, we can determine the effect of increased adverse selection on a monopolist’s profits holding fixed the quantity the monopolist optimally chooses. Because an increase in selection raises average costs, as those participating in the market are selected adversely, producer surplus is necessarily reduced. An increase in the degree of adverse selection raises welfare if the increase in consumer surplus is large enough to offset the decline in firm profits, which only happens when optimal quantity is sufficiently high because in that case both the fall in marginal cost is large and the change in average cost is small as the firm’s average consumers are nearly representative of the whole population.

When there is advantageous selection, the conditions under which an increase in the degree of selection raises consumer surplus are reversed.

**Proposition 5.** Under monopoly, increasing the extent of advantageous selection raises a monopolist’s profits but can raise or lower consumer surplus. Increasing the degree of advantageous selection is more likely to
benefit consumers when the monopolist’s optimal quantity is lower. When quantity high, raising the degree of advantageous selection can lower social surplus.

The graphs for this scenario are analogous to those for adverse selection and shown in Appendix Figure A1. Increasing in the degree of advantageous selection rotates the average cost curve around AC(1) in a counter-clockwise direction. When the optimal quantity is low, this rotation reduces the cost of the average marginal consumer, reducing prices and raising equilibrium quantity. When the optimal quantity is high, the increase in the degree of selection raises the cost of the average marginal consumer, raising prices, and lowering quantity. Increased advantageous selection raises industry profits by the same envelope logic discussed above. Increased advantageous selection raises welfare except when quantity is sufficiently high, in which case the increase in producers surplus outweighs the decline in consumer surplus.

The results under adverse and advantageous selection can be thought about by noticing that an increase in the degree of selection increases the cost heterogeneity in the population, moving away from all individuals having the population average cost. Because the monopolist internalizes the costs of the average marginal consumer, increasing selection will increase this marginal cost exactly when the average marginal consumer is more costly than the population average consumer. Under adverse selection the average marginal consumer has lower cost at high quantity and under advantageous selection the average marginal consumer has higher cost at higher quantity. Therefore benefits from selection occur at high equilibrium quantities under adverse and low equilibrium quantities under advantageous selection.

**Proposition 6.** Under perfect competition, increasing adverse selection lowers consumers surplus and is socially harmful. Increasing advantageous selection raises consumer surplus and is socially beneficial. Producer surplus is always zero under perfect competition.

Under perfect competition, firms make no profits and thus the effect of selection on welfare is driven entirely by the impact of selection on average cost and therefore on the price faced by consumers. If consumers are adversely selected, for any quantity less than 1, consumers are always more costly than the population average. Therefore increasing the degree of adverse selection always raises price under competition, making consumers and society worse off. The reverse is true of advantageous selection.
4 Applications

4.1 Merger Analysis

In this subsection we discuss how the results we developed above should change approaches to a classic area of competition policy: the welfare evaluation of mergers. In particular, we examine a number of central principles articulated in the most recent revision of the United States Horizontal Merger Guidelines (United States Department of Justice and Federal Trade Commission, 2010) and show that many qualitative findings are altered or reversed in an industry with selection.

To facilitate the analysis, we focus on symmetrically differentiated Bertrand industry in which a potential merger changes the industry from a duopoly to a monopoly. This is not intended to be a realistic applied merger model, but simply to illustrate our argument in the cleanest and simplest case that has also been emphasized in previous theoretical merger analysis (Farrell and Shapiro, 1990; Werden, 1996; Farrell and Shapiro, 2010a).

1. Price-raising incentives are harmful: A basic principle of merger analysis is that the stronger are firms’ incentives to raise prices as a result of a merger, the more suspect antitrust authorities should be of the merger. However, to the extent that the incentive to raise prices is stronger because of selection, rather than because of demand-side substitution patterns, mergers are likely to be more beneficial the stronger the incentive to raise prices.

To see this, consider the “first-order” incentive of a firm to raise prices after a merger (Farrell and Shapiro, 2010a; Jaffe and Weyl, 2013), or “Upward Pricing Pressure” (UPP), measured by the externality a firm imposes on its rivals when it increases its sales by one (infinitesimal) unit. When a firm increases its sales by one (infinitesimal) unit, it diverts $D$ units from its rivals, where $D$ is the aggregate diversion ratio. In a market without selection, the markup associated with this unit is $M = P - MC$ so that the sale exerts a negative externality on its rivals of $DM = D(P - MC)$. In a market with selection, the marginal cost perceived by an individual firm is

$$\sigma (D(q)AC(q) + [1 - D(q)]MC(q)) + (1 - \sigma)AC(1),$$

so we if plug this measure of marginal costs into the standard UPP formula and drop arguments
we get
\[ DM = D \left( P - \sigma [DAC + (1 - D) MC] \right). \]

However, in selection markets, our assumption that switching consumers are representative of all consumers and have costs given by \( AC \) means that the incremental profit from this unit is \( P - \sigma AC - (1 - \sigma) AC \) and the sale creates a negative externality on rivals of \( D [P - \sigma AC - (1 - \sigma) AC] \). As a result, the relevant UPP in selection markets is

\[
\text{UPP in Selection Markets} = D [P - \sigma AC - (1 - \sigma) AC] = \\
= D \left( P - \sigma [DAC + (1 - D) MC] \right) + \sigma D (1 - D) (MC - AC) \\
= \text{Standard UPP} + \sigma D (1 - D) (MC - AC),
\]

which is the standard measure plus an additional term \( \sigma D (1 - D) (MC - AC) \).

It is this additional term which reverses the standard logic that a greater incentive to increase prices makes a merger more suspect. To see this, note that increasing advantageous selection (raising \( \sigma \) when \( MC > AC \)) creates more upward pricing pressure, yet is precisely the setting where market power can be desirable because firms exert real externalities on other firms by skimming their inframarginal consumers. Conversely, greater adverse selection (raising \( \sigma \) when \( MC < AC \)) reduces upward pricing pressure but at the same time is the setting where market power is most harmful because it further distorts the incentive to price above marginal cost which occurs even in a perfectly competitive market. Thus, to the extent that it is selection rather than changes in \( D \) or \( M \) that generate upward pricing pressure, a merger is actually most desirable when pricing pressure is large rather than small. For the rest of this subsection, we assume \( \sigma = 1 \) and drop the \( q \) arguments to reduce notation.

2. **Competition-reduction is harmful**: A second principle of merger analysis is that when the services supplied by the merging firms are closer substitutes, antitrust authorities should be more suspect of the merger. However, in settings with advantageous selection, mergers between firms producing highly substitutable products are exactly the settings in which there may be too much competition and increases in market power may be more beneficial.

This point can be seen using the UPP framework discussed above. Standard analysis suggests
that the larger is $D$ the more problematic a merger because it leads to a larger value of $UPP = D(P - MC)$. However, recall that $D = 1 - \theta$ and that under advantageous selection social surplus is inverse-U shaped in market power with an optimal level of $\theta = \theta^*$ strictly between 0 and 1. Thus if $D$ is sufficiently small, and as a result $\theta = 1 - D$ is larger than $\theta^*$, then the resulting merger will alway be harmful because it will further increase $\theta$ above its optimal level. And if $D$ is very large, and as a result $\theta = 1 - D$ is smaller than $\theta^*$, then the resulting merger may be preferable because it will reduce the externalities firms impose on each other in their efforts to cream skim the lowest cost customers. Thus, while under adverse selection the standard intuition is still valid, under advantageous selection it may be reversed: mergers may be socially beneficial (absent other efficiencies) if and only if $D$ is large enough.

3. **Marginal costs should be used to calculate mark-ups:** A third principle of merger analysis is that firm’s marginal rather than average cost should be used to assess their mark-ups in determining the incentive they will have to raise prices upon merging. In selection markets, recall that the valid upward pricing pressure we computed above is $D(P - AC)$ not $D(P - (DAC + (1 - D)MC))$. Thus, if we want to use the simple formula suggested by Farrell and Shapiro (2010a) to calculate upward pricing pressure, we should use average cost not marginal cost to calculate firms’ mark-ups.

Of course we have throughout ruled out non-linearities in cost at the firm level (non-additive-across-consumer cost structures); firm-level non-linearities from forces other than selection would still require attention to an adjusted notion of marginal cost. Nonetheless even in this case firm-level marginal costs would be inappropriate and some notion of average cost is likely to be more accurate in many cases.

4. **Demand data is preferable to administrative data:** As a result of the focus on marginal costs, demand side data is often preferred to administrative data to evaluate the impact of a potential merger (Nevo, 2001). The reason is that marginal costs are hard to measure from firm administrative data (Laffont and Tirole, 1986). Therefore a standard approach to measuring marginal costs suggested by Rosse (1970) is to use demand-side data to estimate the firm’s mark-up and recover marginal costs from first-order conditions. For example, Nevo (2001) backs out mark-ups from a structural model of pricing of cereals and uses these to conduct a merger analysis.
However, in markets with selection, this approach identifies the mark-up in

\[ D(P - [DAC + (1 - D)MC]) \]

and not the relevant mark-up over average cost need need to calculate \( D(P - AC) \). Thus administrative data that reveals \( P \) and \( AC \) is not only sufficient to calculate valid upward pricing pressure in this context; it is necessary to do so and demand data will not suffice. Thus the administrative data obtained in many studies of selection markets recently (Einav, Finkelstein and Levin, 2010) are likely to prove particularly valuable for antitrust purposes, as well as the measurement of selection on which the literature has typically focused.

Our discussion above focuses on the lowest-hanging fruit that can most easily be derived from the simplest extension to the most canonical models. Many other standard antitrust intuitions, both within and beyond merger policy, should be reexamined in markets where selection is an important concern.

### 4.2 Health Insurance

In this subsection we examine impact of market power on the desirability of standard selection policies with a calibrated model of health insurance coverage. We find that for standard parameters, we obtain the counterintuitive results that reducing adverse selection (through risk-rating or risk-based pricing) harms consumers, though it raises profits and aggregate social surplus.

#### 4.2.1 Model

There is market of potential consumers who decide whether to purchase an annual health insurance contract. We assume that consumers are expected utility maximizers with constant absolute risk aversion (CARA) . Consumers are heterogeneous in their absolute risk aversion, denoted \( \alpha \), and their health-type, denoted \( \lambda \). In particular, we assume that these parameters are jointly log normally
Consumers purchase the policy if and only if their willingness to pay is greater than the premium ($q = 1 \iff v \geq p$). The distribution of willingness-to-pay provides us with demand and marginal revenue curves for the industry according to the standard identities.$^{10}$

To emphasize the effects of market power maximally and to simplify the analysis, we assume that the industry is monopolized. Industry average costs are $AC(q) = \mathbb{E}[c - \kappa(c)|v \geq p]$ and marginal costs are $MC(q) = AC'(q)q + AC(q)$. As shown in Section 2, equilibrium price is determined by

---

$^{10}$Viz. $Q(p) = \mathbb{P}(v \geq p)$, $P(q) = Q^{-1}(p)$ and $MR(q) = P(q) + P'(q)q$. 

Equation 2:

\[ P(q) = MS(q) + \sigma MC(q) + (1 - \sigma) AC(1). \]

where we normalize \( \sigma = 1 \) to the baseline degree of selection in the market coming directly from our calibration, which we now discuss.

### 4.2.2 Calibration

We calibrate the distributions of risk aversion using values from the literature and the distribution of health types and medical spending using values from the 2009 Medical Expenditure Panel Survey. Table 1 summarizes the exact calibrated variables. Below we discuss the calibrated values in more detail.

- **Risk aversion (\( \alpha \)).** We calibrate the distribution of absolute risk aversion to the values estimated by Handel, Hendel and Whinston (2013), which are estimated using over-time variation in the choice set of health insurance plans offered to employees at a large firm. These values are similar to those estimated by Cohen and Einav (2007). The mean value of \( \alpha = 0.000439 \) implies indifference between a 50-50 gamble for \{\$100, −\$77\} and \$0 with certainty.

- **Realized costs (\( c \)).** We calibrate the distribution of realized medical costs \( c \) to match the population mean and standard deviation of medical spending for non-elderly individuals without public insurance in the 2009 MEPS. The mean level of spending for this sample is $3,139 and the standard deviation in $10,126.

- **Health-type (\( \lambda \)).** To calibrate the degree of private information, we assume that consumers knowledge of their future health costs is similar to that which can be predicted by standard risk adjustment software.\(^{11}\) The 2009 MEPS provides information on individual’s Relative Risk Scores, which is calculated using the Hierarchical Clinical Classification (HCC) model that is also used to risk-adjust Medicare Advantage payments.

- **Correlation between risk aversion and health-type (\( \rho_{\alpha,\lambda} \)).** We assume that risk aversion and health

---

\(^{11}\)This assumption follows standard practice in the literature (Handel, 2013; Handel, Hendel and Whinston, 2013) and is supported by the finding from Bundorf, Levin and Mahoney (2012) of little private information conditional on an industry standard measure of predicted health risk.
risk are uncorrelated in the population. This is probably a reasonable assumption given the diverging estimates of the sign of this correlation in the literature.

- **Correlation between realized costs and health-type** \((\rho_{\lambda,c})\). Following our model, we estimate the correlation \(\rho_{\lambda,c}\) with a regression of log realized health costs on the log Relative Risk Score, where both variables are normalized by subtracting the mean and dividing by the standard deviation. We estimate a coefficient of \(\rho_{\lambda,c} = 0.498\). This estimate, combined with information on the mean and standard deviation of the Relative Risk Scores and realized costs, allow us to simulate the joint distributions of \(c\) and \(\lambda\).

- **Cost sharing** \((\kappa(c))\). We calibrate the structure of the insurance plan to cover approximately 60% of the costs of medical care for the population on average, assuming no moral hazard from the insurance contract.\(^\text{12}\) We use a plan with co-insurance of 40% up to a out-of-pocket max of $6,000 per year.

### 4.2.3 Results

Figure 4 shows the results of calibrations of the model. In the top panel, we show the equilibrium with a monopolist health insurance provider and the baseline level of adverse selection \((\sigma = 1)\). In the bottom panel, we implement perfect risk adjustment \((\sigma = 0)\) so that the monopolist faces a market of consumers with constant marginal costs equal to the population average. In the baseline scenario with no risk adjustment, premiums are $8,804 and 73.8% of the population has coverage. Because marginal costs are below average population costs at this equilibrium, implementing full risk adjustment increases the cost of the marginal consumer, raising premiums by 2.8% to $9,047 and reducing quantity by 2.4 percentage points to 71.4%.

Another standard policy that is used to address selection is risk-based pricing of insurance. Segmenting the market not only allows prices to reflect cost differences across consumers but also allows the monopolist to price-discriminate by charging different markups to different market segments based on demand elasticity. It thus does not correspond cleanly to our pure cost-side parameter \(\sigma\). To implement segmentation we segmented the distribution of \(\lambda\) into quartiles and allowed the firm to charge a profit-maximizing price each market thus defined. We find that each segmented market has

\(^\text{12}\)This is the level of generosity required by the “Bronze” plans available under the ACA.
essentially no selection (a more-or-less flat cost curve), so that the results under segmentation reflect
the elimination of selection as well as any price discriminatory effects.

Table 2 examines the normative implications these two interventions. The first column shows
surplus under the baseline scenario with no risk adjustment, the second column shows surplus with
full risk adjustment, the third column shows values under segmentation. All values are presented as
a percent of total surplus under the first best scenario with no risk adjustment, where the premium is
determined by the intersection of willingness-to-pay and marginal costs. Under the baseline scenario,
market power reduces total surplus to 82% of the first best level. Producers capture more than two-thirds of this surplus, while consumers capture less than one-third of the surplus created by the
market. Because risk-adjustment raises costs and premia, it reduces consumer surplus by 9% when
compared to the baseline. Segmentation further reduces consumer surplus by 19%.

Despite these harms to consumers, risk-adjustment and especially segmentation significantly
increase profits. Risk-adjustment increases producer surplus by 8% compared to the baseline and
segmentation increase it a further 14%. These effects outweigh the losses to consumers and thus
total social surplus rises 3% from risk-adjustment and an additional 5% from segmentation. How-
ever, if we counted the cost to the government of providing risk-adjustment, this would wipe out
more than all of the gain in producer surplus of risk-adjustment, leading more than all of the reduc-
tion in consumer surplus as a fall in social surplus. Thus, depending on the welfare interpretation,
risk-adjustment may even reduce consumer surplus by 2.5%, though risk-based pricing clearly does
increase social surplus, at the cost of further-reduced consumer surplus.

These effects are far from universal. As discussed in Section 3 eliminating selection may raise or
lower consumers and social surplus, and the same is famously true of the price discriminatory effects
of market segmentation (Aguirre, Cowan and Vickers, 2010). However, in our calibrated model, (i)
eliminating selection with risk adjustment and (ii) allowing price discrimination have essentially
the same qualitative effects. While firms profits and social surplus are increased (unless one counts
the government’s transfers), addressing selection with these polices reduces consumers surplus. In a
future draft we plan to investigate other selection-reducing interventions, such as changing consumer
information and changing the correlation between preferences and risk-type.
5 Conclusion

This paper makes three contributions. First, we propose a simple but general model nesting a variety of forms of imperfect competition in selection markets. Second, we derive from this model several basic, and often counter-intuitive, comparative statics. Third, we show the empirical and policy relevance of these comparative statics by applying them to merger policy, a calibrated model of health insurance and an empirical analysis of the credit card industry (COMING SOON).

Our work here suggests several other directions for future research. We have shown calibrated and empirical examples where the counter-intuitive comparative statics we derived are relevant. However, it is not clear how prevalent such examples are or the breadth with which the issues we raise are first-order in determining optimal competition policy or selection policy. Further empirical research is important to investigate this.

We have also focused on a small number of policy instruments: merger policy, risk-rating and cost-based pricing. While these may be the most canonical policies for addressing selection and market power, many others, such as subsidies for group care and restraints on exclusive dealing, play an important role. Investigating the affect of market power on the first policy and selection on the second would be informative.

Finally our paper contributes to a growing literature that connects issues of contemporary interest, such as selection and imperfect competition, to classical price theory. While we primarily used this connection to draw out the implications of contemporary interest, our results also have implications for the classical theory of regulation of natural monopolies, as our monopoly and competition models correspond, respectively, to an unregulated monopoly and one bound to average cost pricing. In particular, to the best of our knowledge, the welfare implications of average cost pricing, when compared to unregulated monopoly, in a region of a monopoly’s cost curve where cost is increasing (corresponding to advantageous selection) have not been explored in previous literature. Exploring the relationship between these literatures further would be an interesting topic for future research.
References


Figure 1: Selection

Note: Panel (a) shows the perfectly competitive, monopoly, and socially optimal equilibria in the case of advantageous selection where \( AC'(q) > 0 \) and the consumers with the highest willingness-to-pay have the lowest marginal costs. Panel (b) shows the perfectly competitive, monopoly, and socially optimal equilibria in the case of adverse selection where \( AC'(q) < 0 \) and the consumers with the highest willingness-to-pay have the highest marginal costs.
Figure 2: Optimal Market Power with Advantageous Selection

Note: This figure shows that under advantageous selection, there is a socially optimal degree of market power strictly between monopoly and perfect competition. The monopoly equilibrium, determined by $\text{MR} = \text{MC}$, results in too little quantity. The perfectly competitive equilibrium, determined by $P = AC$, results in too much. There is intermediate level of market power $\theta = \theta^*$, which leads to the equilibrium determined by $\theta^* \text{MR} + (1 - \theta^*) P(Q) = \theta^* \text{MC} + (1 - \theta^*) \text{AC}$, and results in the same equilibrium level of quantity as the socially optimal, which is determined by $P = MC$. 
Figure 3: Increasing Adverse Selection under Monopoly

Note: Figure shows the effect of introducing adverse selection into a market served a monopoly provider. Panels (a) and (b) consider a setting where the equilibrium quantity is low and introducing adverse selection increases price and reduces quantity. Panels (c) and (d) consider a setting where the equilibrium quantity is high and introducing adverse selection reduces price and increases quantity.
Figure 4: Risk Adjustment in Health Insurance

Note: Figure shows results from calibrated health insurance model. Panel (a) shows the baseline scenario with adverse selection. Panel (b) shows a scenario with perfect risk adjustment in which the marginal costs to the firm equal average costs in the population. See Subsection 4.2 for more details.
**Table 1: Calibrated Variables**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Absolute risk aversion</td>
<td>4.39E-04</td>
<td>6.63E-05</td>
<td>Estimates of absolute risk aversion from Table 3 of Handel, Hendel and Whinston (2013).</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Privately known health type</td>
<td>0.979</td>
<td>1.378</td>
<td>Values for Relative Risk Score (HCC, Private) in the 2009 MEPS.</td>
</tr>
<tr>
<td>$c$</td>
<td>Realized medical spending</td>
<td>$3,138$</td>
<td>$10,125$</td>
<td>Realized medical spending for non-elderly population without public insurance in 2009 MEPS.</td>
</tr>
<tr>
<td>$\rho_{\lambda,c}$</td>
<td>Correlation of log($\lambda$) and log($c$)</td>
<td>0.498</td>
<td>-</td>
<td>Estimated from a regression of normalized log realized medical spending on normalized log Relative Risk Scores in the 2009 MEPS.</td>
</tr>
</tbody>
</table>

**Note:** Table show calibration values for the health insurance model. See Subsection 4.2 for more details.
### Table 2: Welfare

<table>
<thead>
<tr>
<th></th>
<th>Percent of First Best Total Surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline</td>
</tr>
<tr>
<td>Consumer Surplus</td>
<td>24.8%</td>
</tr>
<tr>
<td>Producer Surplus</td>
<td>56.7%</td>
</tr>
<tr>
<td>Total Surplus</td>
<td>81.5%</td>
</tr>
</tbody>
</table>

**Note:** Table show consumer, producer, and total surplus under three different pricing scenarios. The first columns shows surplus under the baseline scenario with no risk adjustment, the second column shows surplus with full risk adjustment, the third column shows values under the scenario where the market is segmented into four equally-sized risk quartiles, which eliminates most of the selection from the market. All values are presented as a percent of total surplus under the first best scenario with no risk adjustment, where the premium is determined by the intersection of willingness-to-pay and marginal costs. See Subsection 4.2 for more details.
APPENDIX

A Model

This appendix formally micro-founds the representations in the text.

A.1 Cournot model

Potential consumers of a homogeneous service are described by a multi-dimensional type \( t = (t_1, \ldots, t_T) \) drawn from a smooth and non-atomic distribution function \( f(t) \) with full support on a hyper-box \((t_1, T_1) \times \cdots \times (t_T, T_T) \subseteq \mathbb{R}^T\). Consumers receive a quasi-linear utility \( u(t) - p \) if they purchase the service for price \( p \). When the prevailing price is \( p \), therefore, the set of consumers purchasing the service is \( T(p) = \{t : u(t) \geq p\} \) and the number of purchasers \( Q(p) = \int_{T(p)} f(t) dt \). \( T(p) \) is clearly decreasing in \( p \) in the strong set order so that by our assumption of full support \( Q(p) \) is strictly decreasing. Thus we can define the inverse demand function \( P(q) \) as the inverse of \( Q(p) \).

Each consumer also carries with her a cost of service, \( c(t) > 0 \) that must be incurred to supply the service to her by any supplier. Thus the average cost of all individuals served when the aggregate quantity is \( q \) is

\[
AC(q) = \frac{\int_{T(P(q))} c(t) f(t) dt}{Q(P(q))}.
\]

There are \( n \) firms that can each choose a quantity \( q_i \) of the service to supply non-cooperatively. If \( q = \sum_i q_i < 1 \) then the prevailing market price is set by market clearing as \( P(q) \). If \( q > 1 \) then price is 0. Clearly no equilibrium can involve \( q > 1 \) as all firms would make losses. Firms receive a uniform random sample of all customers who are in the market at the prevailing prices and thus earn profits \( q_i [P(q) - AC(q)] \). Thus, to maximize profits non-cooperatively they must satisfy

\[
P(q) - AC(q) + P'(q) q_i - \frac{MC(q) - AC(q)}{q} q_i = 0.
\]

At a symmetric equilibrium where \( q_i = \frac{q}{n} \) for all \( i \) this becomes

\[
P(q) - \left(1 - \frac{1}{n}\right) AC(q) - \frac{MS(q)}{n} - \frac{MC(q)}{n} = 0
\]

as claimed in the text.

A.2 Differentiated Bertrand model

There are \( n \) firms \( i = 1, \ldots, n \) each selling a single service. Consumers are described by two types, each possibly multidimensional, \((t, e)\). \( t \) is drawn as in the Cournot case. \( e \) consists of two components: \( e = (l, e) \) where \( l \) is an integer between 1 and \( L \), with each value of \( l \) having equal probability, and \( e \) is drawn from a real hyper rectangle in \( E \) dimensions. The distribution of \( e \) is atomless, symmetric in all
coordinates, independent of the value of \( l \) and given by the distribution function \( g \). The distributions of \( t \) and \( e \) are independent.

Consumers may consume at most a single service and receive a quasi-linear utility from consuming the service of firm \( i \), \( u_i(t,e) - p_i \), where \( p_i \) is the price charged for service \( i \). Let the first order statistic of utility \( u^*(t,e) \equiv \max_i u_i(t,e) \). We assume (without loss of generality yet) that \( u^*(t,e) = u^*(t) \); that is that the value of the first-order statistic depends only on \( t \) and not on \( e \). Second, and this does entail a loss of generality, we make the following assumption.

**Assumption 1.** \( u_i = u^*(t) + \hat{u}_i(e) \) so that all valuations shift up uniformly with a shift in \( u^* \) induced by changes in \( t \).

This implies that the relative utility of services other than the one the individual most prefers, compared to that which she most prefers, are determined purely by \( e \) and not \( t \). Third we assume, with only a modest loss of generality, that \( u^*(t) \) is smooth in \( t \) and that \( \partial u^*/\partial t \geq k > 0 \) for some constant \( k \). This implies that raising \( t_T \) sufficiently causes \( u^* > u \) for any fixed \( u \) and lowering it sufficiently causes the reverse to be true.

Services are symmetrically differentiated in the sense that distribution of \( u(t,e) = (u_1(t,e), \ldots, u_n(t,e)) \) induced by the distribution of \( (t,e) \) is symmetric in permutations of coordinates. The set of individuals purchasing service \( i \) is

\[
T_i(p) = \{(t,e) : u_i(t,e) \geq p_i \land i \in \arg\max_j u_j(t,e) - p_j \}
\]

and the demand for good \( i \) is thus \( Q_i(p) = \int_{T_i(p)} f(t,e) d(t,e) \).

As in the Cournot example, the cost of serving a consumer depends on her type. However, we make the substantive assumption now that cost depends only on \( t \) and not on \( e \).

**Assumption 2.** The cost of serving a consumer of type \( (t,e) \) is \( c(t) \) and thus the total cost faced by firm \( i \) is \( C_i(p) = \int_{T_i(p)} c(t) f(t,e) d(t,e) \).

This assumption states that only the determinants of the highest possible utility a consumer can achieve, and not of her relative preferences across services, may directly determine her cost to firms. Given the independence of \( t \) and \( e \), this assumption implies a clean separation between determinants of relative “horizontal” preferences across services and “vertical” utility for the most preferred service that also determines the cost of service. Absent this assumption it is possible that the consumers that firms attract from their rivals when lowering their price are very different in terms of cost from the average consumers of the service more broadly.

Let \( 1 \equiv (1, \ldots, 1) \). Then by symmetry \( Q_i(p1) = Q_j(p1) \forall i,j \) and similarly for \( C_i \) and \( C_j \). Let the aggregate demand \( Q(p) \equiv nQ_i(p1) \) for any \( i \) and similarly for aggregate cost. Then we define the inverse demand function \( P(q) \) as the inverse of the aggregate demand. Average cost is then \( AC(q) \equiv C'(P(q))/q \) and marginal cost \( MC(q) \equiv C'(P(q)) P'(q) \).

We now describe two particular models satisfying these assumptions and show how they yield the reduced-form representation we use in the text. Any other micro-foundation of these assumptions should also yield our representation, but the notation required to encompass different cases is
sufficiently abstract and not relevant enough to any results we derive. We thus omit it here and focus on specific micro-foundations.

First consider a random utility model in the spirit of Anderson, de Palma and Thisse (1992) proposed by White and Weyl (2012) in the context of heterogeneity of preferences for non-price product characteristics. \( L = n \) and the value of \( l \) represents which product is the individual’s favorite. \( e = (e_1, \ldots, e_E) \) and \( E \geq n - 1 \). We assume that

\[
u_i (u^*(t), l, e)\]

is increasing in \( e_i \), where \( i^* \) is \( i \) if \( i < l \) and is \( i - 1 \) if \( i > l \) and that it is constant in all other \( e_i \) where \( i \leq n - 1 \) and not \( i^* \). We also assume that \( u_i \) is smooth in its arguments other than \( l \), bounded and that and that \( \lim_{e_i \to -\infty} u_i (u^*(t), l, e) = u^*(t) \) and \( \lim_{e_i \to \infty} u_i (u^*(t), l, e) = 0 \) for any value of the other entries \( u^*, l \) and \( e \). This implies that raising \( e_i \) sufficiently for any \( i \) while holding fixed the other components of \( e \) makes (in the limit) service \( i \) equally desirable to the most desirable service for the individual and lowering it makes it always uncompetitive with the best service regardless of the price differential.

An individual firm \( i \)'s profits are \( p_i Q_i (p_1, \ldots, p_i, \ldots, p_n) - C_i (p_1, \ldots, p_i, \ldots, p_n) \). Thus the first-order condition for the optimization of any firm \( i \) is

\[
 p_i \frac{\partial Q_i}{\partial p_i} + Q_i = \frac{\partial C_i}{\partial p_i} \tag{3}
\]

Because price does not appear in the interior of the integrals defining \( Q_i \) and \( C_i \), the derivatives of these with respect to \( p_i \) is, by the Leibniz Rule applied to multidimensional integrals (Veiga and Weyl, 2013a), given by the sum of the effects of the extensive margin effects from the change in the boundaries of integration. There are many such boundaries, so we use a shorthand notation for them. \( \partial T^X_i (p) \equiv \{(t, e) \in T_i (p) : u^* (t) = p_i \} \) denotes the set of exiting consumers from product \( i \) who are just indifferent between buying service \( i \) and no service. \( \partial T^S_i (p) \equiv \{(t, e) \in T_i (p) \cap T_j (p) \} \) denotes the set of switching consumers between services \( i \) and \( j \) who are just indifferent between the two services, but prefer purchasing one over purchasing nothing. To formally define the density of consumers on such boundaries it is useful to express the multidimensional integrals representing \( Q_i \) and \( C_i \) more explicitly.

At symmetric prices \( p \), every individual \( i \) with \( t_T \) above this threshold buys from her most preferred services \( l \) and any individual below this threshold buys no service. If a single price \( p_i \) is elevated to \( p_i + \delta \) then all individuals with \( l \neq i \) continue to buy their preferred product as at symmetry. However, individuals with \( l = i \) and \( u^* (t) \in (p, p + \delta) \) will stop consuming any service and those with \( l = i \) and \( e_i \) sufficiently close to \( \varepsilon_i \) will switch to purchasing service \( j \). Let \( t^*_T (p; t_{-T}) \) be defined implicitly by \( u^* (t, t^*_T (p; t_{-T})) = p \) and let \( e^*_j (\Delta; u^* (t), E - \{1, \ldots, n - 1\}) \) be implicitly defined for positive \( \Delta \) by

\[
u^* (t) - u_j \left( u^* (t), e^*_j (\Delta; u^* (t), E - \{1, \ldots, n - 1\}) e_{-\{1, \ldots, n - 1\}} \right) = \Delta
\]
where \( e_{-\{1, \ldots, n-1\}} \) is all components of \( e \) other than the first \( n-1 \) and the dependence of \( u_j \) on the other components of \( e \) is dropped as these do not impact \( u_j \).

Then when prices are symmetric except for price \( p_i \) being above the other prices, we can write

\[
Q_i(p, \ldots, p_i, \ldots, p) = \frac{1}{n} \int_{T^{-}} \int_{e_{-\{1, \ldots, n-1\}}} t_j^{1}(p_i, t) \int_{t_j} e_j(p_i, t) \int_{t_j} \cdots \int_{t_j} f(t) g(e) d(t, e) \]

and similarly

\[
C_i(p, \ldots, p_i, \ldots, p) = \frac{1}{n} \int_{T^{-}} \int_{e_{-\{1, \ldots, n-1\}}} t_j^{1}(p_i, t) \int_{t_j} e_j(p_i, t) \int_{t_j} \cdots \int_{t_j} c(t) f(t) g(e) d(t, e). \]

To fill in the first-order condition (Equation 3), we need to differentiate these using the Leibniz rule.

\[
\frac{\partial Q_i}{\partial p_i} \begin{pmatrix} p \end{pmatrix} = -\frac{1}{n} \left[ \int_{\partial t_j^{1}(p_i)} \frac{f(t, t_j^{1}(p; t^{-})) g(e)}{\partial u_j/\partial t_j(t, t_j^{1}(p; t^{-}))} \frac{c(t) f(t; e)}{\partial e} d(t, e) \right] + (n-1) \int_{\partial t_j^{1}(p_i)} \frac{f(t; e)}{\partial e} d(t, e) \]

for any \( j \neq i \) by symmetry and similarly

\[
\frac{\partial C_i}{\partial p_i} \begin{pmatrix} p \end{pmatrix} = \frac{1}{n} \left[ \int_{\partial t_j^{1}(p_i)} \frac{c(t, t_j^{1}(p; t^{-})) (t, t_j^{1}(p; t^{-}))}{\partial u_j/\partial t_j(t, t_j^{1}(p; t^{-}))} \frac{c(t) f(t; e)}{\partial e} d(t, e) \right]. \]

By contrast and following the same logic

\[
\frac{\partial Q_i}{\partial p} \begin{pmatrix} p \end{pmatrix} = -\frac{1}{n} \int_{\partial t_j^{1}(p_i)} \frac{f(t, t_j^{1}(p; t^{-})) g(e)}{\partial u_j/\partial t_j(t, t_j^{1}(p; t^{-}))} \frac{c(t, t_j^{1}(p; t^{-})) f(t; e)}{\partial e} d(t, e) \]

and

\[
\frac{\partial C_i}{\partial p} \begin{pmatrix} p \end{pmatrix} = -\frac{1}{n} \int_{\partial t_j^{1}(p_i)} \frac{c(t, t_j^{1}(p; t^{-})) f(t; e)}{\partial e} d(t, e). \]

Thus by symmetry

\[
Q'(p) = -\int_{\partial t_j^{1}(p_i)} \frac{f(t, t_j^{1}(p; t^{-})) g(e)}{\partial u_j/\partial t_j(t, t_j^{1}(p; t^{-}))} d(t, e) \]

and

\[
C'(p) = -\int_{\partial t_j^{1}(p_i)} \frac{c(t, t_j^{1}(p; t^{-})) f(t; e)}{\partial e} d(t, e). \]
so that
\[
MC(Q(p)) = \frac{\int_{\partial T_i^T(p_1)} c(t-T_i^T(p_1))f(t-T_i^T(p_1))g(e) \, d(t-T,e)}{\int_{\partial T_i^T(p_1)} f(t-T_i^T(p_1))g(e) \, d(t-T,e)}.
\]
Furthermore
\[
\int_{\partial T_i^T(p_1)} \frac{c(t) f(t) g(e_{-i}, \bar{e}_j)}{\partial u_i/\partial c_j (t, e_{-i}, \bar{e}_j)} \, d(t, e_{-i}) = n \int_{T_i(p)} c(t) f(t) \int_{e_{-i}} \frac{g(e_{-i}, \bar{e}_j)}{\partial u_i/\partial c_j (t, e_{-i}, \bar{e}_j)} \, d(t, e_{-i}) = AC(Q(p)) Q(p) \int_{e_{-i}} \frac{g(e_{-i}, \bar{e}_j)}{\partial u_i/\partial c_j (t, e_{-i}, \bar{e}_j)} \, d(t, e_{-i}) = -AC(Q(p)) s(p)
\]
where \(s(p)\) is the density of consumers diverted to a rival from a small increase in one firms price starting from symmetric prices \(p\). Thus we can rewrite Expression 4 as
\[
Q'(p) - (n - 1)s(p) = \frac{Q'(p) - 1}{n [1 - D(Q(p))]},
\]
where \(D(q) = \frac{(n - 1)s(P(q))}{Q'(P(q)) - (n - 1)s(P(q))}\) is the aggregate diversion ratio (Farrell and Shapiro, 2010b), the fraction of consumers lost to a small increase in prices by a single first that go to rivals rather than the outside good. We can also rewrite expression 5 as
\[
Q'(p) \frac{MC(Q(p)) + \frac{D(Q(p))}{1 - D(Q(p))} AC(Q(p))}{n}.
\]
Then Equation 3 becomes, at symmetric prices
\[
Q'(p) \frac{p}{n [1 - D(Q(p))]} + \frac{Q(p)}{n} = Q'(p) \frac{MC(Q(p)) + \frac{D(Q(p))}{1 - D(Q(p))} AC(Q(p))}{n} \Rightarrow
\]
that at any symmetric equilibrium
\[
\frac{P(q)}{1 - D(q)} - MS(q) = MC(q) + \frac{D(q)}{1 - D(q)} AC(q)
\]
because \(MS(q) = \frac{Q'(P(q))}{Q'(P(q))}\). Letting \(\theta(q) = 1 - D(q)\) this becomes
\[
P(q) - \theta(q) MS(q) = \theta(q) MC(q) + [1 - \theta(q)] AC(q)
\]
as reported in the text.

A second model that delivers our form builds on the Chen and Riordan (2007) “spokes” extension of the Hotelling linear city model, combining it with modifications from Rochet and Stole (2002). There are \(n\) firms \(i = 1, \ldots, n\). For every pair of firms, \((i, j)\) with \(i < j\) there is a line segment of unit length of potential consumers who will only consider purchasing either service \(i\) or service \(j\). Thus
there are $n(n-1)$ such segments and we denote the segment $(i, j)$ by the integer $\frac{i(i-1)}{2} + (j \mod i)$. $e = (l, e)$ where $l$ is the integer representing the line segment on which the consumer lives and $e \in (0, 1)$ is the distance of the consumer from $i$ or $1-$ her distance from $j$. In particular let $i(l) \equiv \max_{i \in \mathbb{Z}: \frac{i(i-1)}{2} < l} \frac{i(i-1)}{2}$ and let $j(l) \equiv l \mod i(l)$; then $e$ is the distance of the consumer from $i(l)$. There are an equal number of consumers on each segment so $\frac{2n(n-1)}{n(n-1)}$ of the consumers are on each segment.

In addition to maintaining our assumptions about $t$ and $e$, we make two modifications to the set-up of Chen and Riordan:

1. We modify the exact form of consumer utility. In particular, $u^*(t)$ is the utility a consumer earns from service $i(l)$ if $e \leq \frac{1}{2}$ and from good $j(l)$ if $e \geq \frac{1}{2}$ regardless of the other details of her position. This contrasts with the standard Chen and Riordan, and Hotelling (1929), model because it implies no transport cost to an individual’s most preferred service.

2. Consumers’ highest possible utility is not constant across consumers but instead follows a distribution $u^*(t)$.

3. The gross utility a consumer derives from purchasing from $j(l)$ if $e < \frac{1}{2}$ is $u^*(t) - (1 - 2e)t$, where $t$ is a transportation cost parameter absent in the Chen and Riordan model. If $e > \frac{1}{2}$ the consumer derives gross utility of $u^*(t) - (2e - 1)t$ from purchasing from $i(l)$.

4. We allow arbitrary smooth and symmetric-about-$\frac{1}{2}$ distributions of $e$ on the unit interval, as long as this distribution is the same for all $l$.

Calculations to derive the representation in the text are tedious and extremely similar to those in our modified Anderson, de Palma and Thisse model above. We therefore omit these calculations and simply explain why there results are the same. At symmetric prices, every consumer purchases from her most preferred firm, $i(l)$ if $e \leq \frac{1}{2}$ and $j(l)$ if $e \geq \frac{1}{2}$. All consumers with the same $t$ make the same purchase decision at this price because only $u^*$ impacts their total utility. Consumers with $e = \frac{1}{2}$ are “switchers” between a pair of firms (if $u^*(t) \geq p$) and have the same distribution of $t$ as all purchasers by the independence of $e$ and $t$. Thus switchers will be representative of the full population of consumers and exiters everywhere will be on average identical. This is precisely what gave rise to our structure above.

### B Example with Large Demand-Driven Effects of Selection

One intervention commonly applied in selection markets is cost-based pricing. In Subsection ?? we showed that these discriminatory effects may reinforce the cost-based effects of selection. In this appendix we discuss how price discriminatory effects of cost-based pricing may instead reverse the results we established about the impact of changing the degree of selection in Section 3.

Proposition 5 states that increasing advantageous selection increases monopoly profits. However, clearly allowing cost-based pricing may never hurt a monopolist as she may maintain uniform pricing. It will generically aid the monopolist. Thus her gains from price discrimination swamp the effects we highlight.
To see that impacts on consumers may also be reversed by price discriminatory effects, consider our result (Proposition 4 and 5) that decreasing adverse and advantageous selection may both benefit consumers, depending on the equilibrium quantity. This may be true of cost-based pricing, but not in one simple, extreme case. Suppose that there is only a single dimension of heterogeneity determining both cost and valuation and that we move from uniform pricing to full cost-based pricing. This operates as perfect, first-degree price discrimination, extracting all surplus from consumers regardless of the equilibrium quantity and thus contradicting the natural extrapolation of our result.

Thus cost-based pricing cannot cleanly be interpreted as an example of increasing selection in our framework; price discrimination may be more important in some cases than are cost-based effects. However, in the leading counter-intuitive case we emphasize, the two effect reinforce one another.

C Proofs

Throughout we assume that $\theta, \sigma \in [0, 1]$, that selection is either globally adverse or advantageous (either $AC', MC' > 0$ or $AC', MC' < 0$ for all $q$) and impose a global equilibrium stability condition: $P' < \min\{AC', MC', 0\}$ and $MR' < \min\{MC, 0\}$. Most of the results may be obtained absent these global monotonicity assumptions, but the additional expositional complexities add little insight. We also assume that $\theta$ and $\sigma$ are constant parameters, independent of $q$; all results can be extended to the case when this fails, but, again, the additional notation is cumbersome.

**Lemma 1.** Let $F(q) \equiv P(q) - \sigma (\theta MC(q) + (1 - \theta) AC(q)) + (1 - \sigma) AC(1) - \theta MS(q)$. $F' < 0$.

**Proof.** The derivative of the expression is

$$P' - \sigma \theta MC' - \sigma (1 - \theta)AC' - \theta MS' =$$

$$\sigma \left[ \theta (MR' - MC') + (1 - \theta)(P' - AC') \right] + [1 - \sigma] \left[ \theta MR' + (1 - \theta)P' \right] < 0,$$

by our monotonicity assumptions. 

**Proposition (Formal) 1.** For $\theta \in (0, 1)$, $\frac{\partial PS}{\partial \theta} \geq 0 \geq \frac{\partial CS}{\partial \theta}$, with strict inequality if $q^* > 0$.

**Proof.** By the implicit function theorem

$$Fq^* \frac{\partial q^*}{\partial \theta} - [MC(q^*) - AC(q^*)] - MS(q^*) = 0 \implies \frac{\partial q^*}{\partial \theta} = \frac{MS(q^*) + MC(q^*) - AC(q^*)}{F'}.$$

Focusing on the denominator

$$MS + MC - AC = -P'q + AC'q > 0$$

by our monotonicity assumptions. Thus $\frac{\partial q^*}{\partial \theta} < 0$ if $q \neq 0$ and weakly if $q = 0$. This immediately implies that price rises in $\theta$ by monotonicity and thus that CS falls. Producer surplus is

$$PS(q) = q [P(q) - \sigma AC(q) - (1 - \sigma)AC(1)]$$
Proof. \( SS'(q) = P(q) - \sigma AC(q) - (1 - \sigma) AC(1) + q [P'(q) - \sigma AC'(q)] = \\
\quad \quad \quad P(q) - MS(q) - \sigma MC(q) - (1 - \sigma) AC(1) < F(q) \)
as \( MS + MC - AC > 0 \) by the argument above. Thus at \( q^* \) for any \( \theta < 1 \), \( PS' < 0 \). \( \square \)

**Proposition (Formal) 2.** If \( AC' < 0 \) and \( \theta \in (0, 1), \frac{\partial SS}{\partial \sigma} \leq 0 \), strictly if \( q^* > 0 \).

Proof. \( SS(q) = \int_0^q P(q) - [\sigma MC(q) + (1 - \sigma) AC(1)] dq \) so \( SS'(q) = P(q) - \sigma MC(q) - (1 - \sigma) AC(1) \). Thus
\[
SS'(q^*) = \sigma (1 - \theta) [AC(q^*) - MC(q^*)] + \theta MS(q^*) > 0.
\]
Thus the result follows from the fact that \( \frac{\partial q^*}{\partial \sigma} < 0 \) as shown in the proof of the previous proposition. \( \square \)

**Proposition (Formal) 3.** If \( AC' > 0 \) and \( q^* > 0 \) for every \( (\theta, \sigma) \in (0, 1)^2 \), \( \exists \theta^* \in (0, 1) \) such that \( \frac{\partial SS}{\partial \sigma} > (\leq / =)0 \) if \( \theta < (\geq / =)\theta^* \). \( \frac{\partial q^*}{\partial \sigma} > 0 \) if \( \sigma \in (0, 1) \).

Proof. By the logic of the previous proof, \( SS'(q) = P(q) - \sigma MC(q) - (1 - \sigma) AC(1) \) so
\[
SS''(q) = P'(q) - \sigma MC'(q) = (1 - \sigma) P'(q) + \sigma [P'(q) - MC'(q)] < 0.
\]
Thus social surplus is concave in quantity. Quantity is below its optimal level at \( \theta = 1 \) by the standard monopoly argument and quantity is above its optimal level at \( \theta = 0 \) by the argument in the proof of the previous proposition. Thus the result follows from the fact, shown in the proof of Proposition 1 that \( \frac{\partial q^*}{\partial \sigma} < 0 \). \( \square \)

**Proposition (Formal) 4.** \( \frac{\partial q^*}{\partial \sigma} \) has the same sign as \( AC(1) - \theta MC(q^*) - (1 - \theta) AC(q^*) \). If \( AC' < 0 \) then providing a specific subsidy to the industry can only cause the sign of \( \frac{\partial q^*}{\partial \sigma} \) to move from being negative to being positive; a sufficiently large such subsidy guarantees this sign is positive. If \( AC' > 0 \) then levying a specific tax on the industry can only cause the sign of \( \frac{\partial q^*}{\partial \sigma} \) to move from being positive to being negative; a sufficiently large such tax guarantees this sign is negative.

Proof. With a specific tax (negative specific taxes are specific subsidies), the equilibrium condition is
\[
P(q) - \sigma (\theta MC(q) + (1 - \theta) AC(q)) + (1 - \sigma) AC(1) - \theta MS(q) - t = 0.
\]
Thus by the Implicit Function Theorem
\[
F'(q^*) \frac{\partial q^*}{\partial \sigma} - [\theta MC(q^*) + (1 - \theta) AC(q^*) - AC(1)] = 0 \quad \Rightarrow \quad \frac{\partial q^*}{\partial \sigma} = \frac{\theta MC(q^*) + (1 - \theta) AC(q^*) - AC(1)}{F'(q^*)}.
\]
Because \( F' < 0 \) by Lemma 1, this has the same sign as \( AC(1) - \theta MC(q^*) - (1 - \theta) AC(q^*) \) regardless of the degree of tax or subsidy. By the same arguments as above, \( \frac{\partial q^*}{\partial \sigma} < 0 \). Thus if \( AC', MC' < (\geq)0 \) the sign of this expression can only move, with an increase in tax, from being positive to being negative (from being negative to being positive). \( \square \)
**Proposition (Formal) 5.** If \( \theta = 1 \) then \( \frac{\partial PS}{\partial \sigma} \) has the same sign as \( AC' \).

**Proof.** As before \( PS(q) = q [P(q) - \sigma AC(q) - (1 - \sigma) AC(1)] \). When \( \theta = 1 \) profits are maximized over \( q \) so by the envelope theorem we can calculate \( \frac{\partial PS}{\partial \sigma} \) while holding fixed \( q^* \) yielding \( AC(1) - AC(q^*) \). For \( q^* < 1 \) this clearly has the same sign as \( AC' \). \( \square \)

**Proposition (Formal) 6.** If \( \theta = 0 \) then \( \frac{\partial SS}{\partial \sigma} \) has the same signs as \( AC' \). If \( \theta = 1 \) then a large enough upward vertical shift in \( P \) makes \( \frac{\partial SS}{\partial \sigma} > 0 \).

**Proof.** At \( \theta = 0 \) there is no producer surplus so only the impact on consumer surplus is relevant, which is signed the same as \( \frac{\partial q^*}{\partial \sigma} \) as in Proposition 4. When \( \theta = 1 \)

\[
\frac{\partial SS}{\partial \sigma} = \frac{\partial CS}{\partial \sigma} + \frac{\partial PS}{\partial \sigma} = q^* P'(q^*) \frac{\partial q^*}{\partial \sigma} + AC(1) - AC(q^*) = \frac{MC(q^*) - AC(1)}{F'(q^*)} q^* P'(q^*) + AC(1) - AC(q^*) .
\]

Assume that \( \frac{q^* P'(q^*)}{F'(q^*)} \) is bounded away from 0 as \( q \rightarrow 1 \); clearly this occurs in most standard cases; for example it is trivially satisfied in the linear case near 1. Then as a vertical upward shift (which impacts first-order conditions equivalently to a subsidy; see Weyl and Fabinger (2013)) increases and \( q^* \rightarrow 1 \) the second term approaches 0 and the first term is dominant. When \( AC' < 1 \) this must be positive near \( q^* = 1 \) because

\[
\int_0^1 MC(q) = C(1) = 1AC(1)
\]

so that if \( MC' < 0 \), \( MC \) must be below \( AC(1) \) near 1. \( \square \)
Figure A1: Increasing Advantageous Selection under Monopoly

Note: Figure shows the effect of introducing advantageous selection into a market served a monopoly provider. Panels (a) and (b) consider a setting where the equilibrium quantity is low and introducing advantageous selection lowers price and raises quantity. Panels (c) and (d) consider a setting where the equilibrium quantity is high and introducing advantageous selection increases price and reduces quantity.