Credit Crunches, Asset Prices and Technological Change

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Abstract

We investigate the effects of a credit crunch in an economy where firms can retain a mature technology or adopt a new technology. We show that firms' collateral and credit relationships ease firms' access to credit and investment but can also inhibit firms' innovation. When this occurs, negative collateral and productivity shocks and the resulting drop in the price of collateral assets squeeze collateral-poor firms out of the credit market but foster the innovation of collateral-rich firms. We characterize conditions under which such an increase in firms' innovation activity occurs within existing credit relationships or through their breakdown. The analysis reveals that the credit and asset market policies adopted during the recent credit market crisis can promote investment but slow down firms' innovation.

Keywords: Technological Change, Collateral, Credit Relationships, Credit Crunch.
JEL Codes: E44.

1 Introduction

Since the financial crisis struck in 2008, a major drop in the value of collateral assets, especially real estate, has triggered a severe breakdown of credit relationships and decline in total credit, forcing firms to curtail their investments. The literature offers well-established theoretical arguments for interpreting these effects of a credit crunch. When entrepreneurs cannot commit to repay lenders,

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the availability of collateral assets eases their access to credit (Kiyotaki and Moore, 1997).[1] In turn, credit relationships can enhance this role of collateral. For example, lenders who establish informationally intensive relationships with entrepreneurs can better monitor their assets and recover more value from collateral repossession (Diamond and Rajan, 2001). An implication of these arguments is that aggregate shocks that erode the value of collateral assets and break credit relationships depress total investment by hindering firms' access to external finance (Kiyotaki and Moore, 1997; Lorenzoni, 2008; Holmstrom and Tirole, 1997; Den Haan, Ramey and Watson, 2003).

While useful to explain key mechanisms of transmission of a credit market crisis, these arguments only yield partial insights into the effects of a credit crunch on technological change. Financial crises appear to have contrasting effects on technological change. Field (2011) and Bernstein (1987) document that the Great Depression was actually a period of major innovations for the U.S. economy. These innovations ranged from Teflon in petrochemicals industries to household appliances, such as the radio and refrigerator, and formed the basis for the post–World War II economic expansion. In South Korea and in Finland, the number of innovative firms boomed during and in the immediate aftermath of the 1990s financial crises (OECD, 2009). And, during the current crisis, the Silicon Valley and the San Francisco Bay Area have been experiencing a new wave of innovations in the information technology sector (The Economist, 2012, 2011). The OECD (2009) summarizes this body of evidence stressing that, while on the one hand credit market crises can certainly damage innovative firms, on the other hand they can also be "times of industrial renewal". These observations naturally elicit fundamental questions: Can we build a model economy in which credit markets matter and that can capture the contrasting forces that affect innovation during a credit crunch? In such an economy, under what conditions would credit market crises depress or stimulate innovation?

This paper takes a step towards this objective. We posit an economy where entrepreneurs operate a mature technology or innovate and adopt a new technology. Lenders, in turn, acquire information that is essential for repossessing and liquidating productive assets pledged as collateral when entrepreneurs default (as in Diamond and Rajan, 2001, for example). Lenders' information on collateral assets eases entrepreneurs' access to credit but makes lenders reluctant to finance entrepreneurs' innovation. In fact, the new technology has less assets pledgeable as collateral. Furthermore, the information on the collateral assets of the mature technology is (partially) specific and non-transferable to the assets of the new technology. Therefore, expecting that the information they have accumulated on mature collateral assets will go wasted if entrepreneurs upgrade to the new technology, lenders may hinder entrepreneurs' innovation efforts. In this economy, entrepreneurs can form informationally intensive credit relationships with lenders to transfer them more information on collateral assets and obtain cheaper financing. Yet, the information accumulated within the relationships exacerbates lenders' incentive to inhibit firms' innovation.

The distribution of firms across collateral values replicates salient features of that obtained

[1] Ex post, after entrepreneurs default, lenders can repossess collateral and this compensates for the limited pledgeability of entrepreneurs' output; ex ante, lenders' threat to repossess collateral deters entrepreneurs' misbehavior.

in previous general equilibrium models of the credit market (e.g., Holmstrom and Tirole, 1997). Collateral-poor firms lack access to credit because they cannot pledge enough expected returns to lenders, even when these obtain high quality information on collateral assets. Firms with medium and rich collateral, instead, obtain credit. The novelty consists of firms' technology adoption. While firms with medium collateral value potentially innovate, collateral-rich firms with credit relationships preserve the mature technology. In fact, their lenders expect a large depreciation in the value of their information if the mature technology is abandoned in favor of the new technology.

We study the effects of negative shocks to the liquidity of collateral assets and to productivity. Consider collateral shocks (the reasoning for productivity shocks is similar). Following the drop in the price of collateral assets, the credit relationships of collateral-poor firms break down because these firms can no longer pledge enough expected returns to lenders. This tends to reduce total investment and innovation. Consider next collateral-rich firms. The reduction in the asset price erodes the value of the information acquired by their lenders on mature collateral assets. This mitigates lenders' technological inertia within credit relationships. This also increases the incentive of collateral-rich firms to deliberately break their credit relationships, borrow from new lenders and innovate. Both these effects foster the innovation of collateral-rich firms. If the increase in the innovation of collateral-rich firms outweighs the drop in the innovation of collateral-poor firms, the shock will cause a decline in total investment but a net increase in innovation.

What are the consequences for the credit market? We show that there is a credit regime in which lenders' technological inertia is weak and after the shock collateral-rich firms innovate within their relationships. There is instead a credit regime in which lenders' technological inertia is strong and/or firms derive small benefits from relationships: in this regime, collateral-rich firms innovate by breaking their relationships and borrowing from new lenders. Thus, depending on the credit regime, the increase in innovation activity induced by the shock can entail a moderate or a major breakdown of credit relationships. This is also important for output, because the breakdown of relationships can depress output by raising asset liquidation costs.

In the last part of the paper, we investigate the effects of two unconventional policies carried out by the Federal Reserve and the Treasury during the financial crisis begun in 2008: an intervention in the collateral asset market aimed at sustaining the asset price after the shock and a policy of direct lending to collateral-poor firms. We find that both policies foster total investment but may dampen the increase in the innovation of collateral-rich firms during a credit crunch. The case of the direct lending policy is especially insightful. In our economy, the credit rationing of collateral-poor firms following the shock fosters the innovation of collateral-rich firms by bringing down collateral asset demand and prices. A policy of direct lending dampens this effect.

This paper especially relates to two strands of literature. The first investigates the impact of a disruption in the financial structure on aggregate investment (e.g., Gertler and Karadi, 2010, Gertler and Kiyotaki, 2010). In this literature, we borrow some properties of our modelling strategy, such as the focus on a finite horizon economy, from Holmstrom and Tirole (1997). Den Haan, Ramey and Watson (2003) and dell'Ariccia and Garibaldi (2001) are other related papers in this literature.

These studies analyze the breakdown of credit relationships that can be caused by a recession in economies with search frictions. While in these studies a breakdown of credit relationships depresses investment, in our economy it depresses investment but may also foster technological change.

The second strand of literature analyzes the impact of recessions on firms' innovation. Most of this literature neglects the role of the credit market for technological change. Caballero and Hammour (2004), Ramey (2004) and Barlevy (2003) are exceptions. These studies show that credit frictions can worsen during recessions, hindering aggregate restructuring. Caballero and Hammour (2004) show that, because of credit frictions, production units can be destroyed at an excessive rate during a recession. Ramey (2004) endogenizes financial managers' project selection and shows that, if managers have empire-building incentives, during downturns they can discard efficient projects to preserve the size of their portfolios. Barlevy (2003) finds that during recessions credit frictions can lead to the disruption of high-surplus production units rather than low-surplus ones. This paper endorses a view opposite to these studies: while it negatively affects investment, the breakdown of credit relationships also mitigates lenders' technological inertia.

The remainder of the paper is organized as follows. In Section 2, we outline and discuss the setup. Section 3 solves for the equilibrium. In Section 4, we investigate the effects of shocks. Section 5 analyzes the robustness of the analysis. Section 6 considers the effect of policies in the asset and credit markets. Section 7 concludes. The Appendix contains the main proofs while more technical proofs are relegated to a Supplement.

2 The Model

This section describes the setup of the model. Figure 1 illustrates the timing of events while Table 1 summarizes the notation.

2.1 Agents, goods, and technology

Consider a four-date economy ($t = 0, 1, 2, 3$) populated by a unit continuum of entrepreneurial firms and a continuum of investors of measure larger than one. There is a final consumption good, which can be produced and stored, and productive assets of two vintages, mature and new. Entrepreneurs have no endowment while each investor is initially endowed with an amount ω of final good. All agents are risk neutral and consume on date 3.

Each entrepreneur can carry out one indivisible project. On date 2, an entrepreneur can face an innovation opportunity. If the innovation opportunity arises, the entrepreneur chooses whether to adopt a new technology or retain a mature, less productive technology. If the innovation opportunity does not arise, the entrepreneur has to retain the mature technology. Under the mature (new) technology, on date 3 the entrepreneur transforms an amount $i < \omega$ of final good into one unit of mature (new) assets. With probability $\pi > 1/2$ the project succeeds and the mature (new) assets yield an output y ($y(1+n)$) of final good; otherwise the project fails and the entrepreneur goes out of business. In this case, a fraction a (ϕa) of mature (new) assets can be recovered and

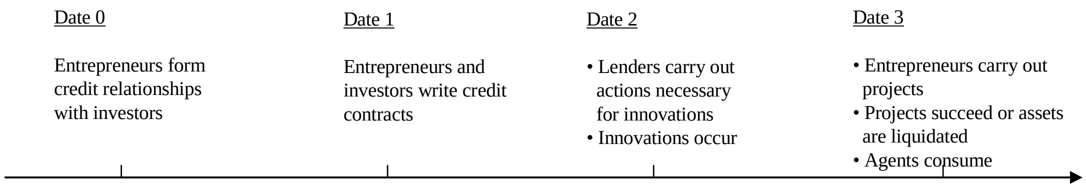

Figure 1: Timing of Events.

redeployed outside the firm. a captures the amount of collateralizable assets of an entrepreneur and is uniformly distributed across entrepreneurs over the domain $[0, 1]$. $\phi \leq 1$ is a parameter that reflects the redeployability of new assets relative to mature assets.

On date 3, each entrepreneur still in business can reuse one unit of liquidated assets, obtaining an amount $\eta\theta$ of final good. θ is uniformly distributed across entrepreneurs over the domain $[0, \overline{\theta}]$; η represents the aggregate productivity of liquidated assets.[2]

2.2 Credit sector

Each entrepreneur can stipulate a credit contract with one investor on date 1. A lender interacts with an entrepreneur along two dimensions besides credit provision: she exerts control and she acquires information. Following an established literature, we allow the lender to exert control over production opportunities (see, e.g., Aghion and Bolton, 1992; Rajan, 1992). Precisely, on date 2 the lender can carry out a costless action that affects the probability of the innovation opportunity: if she carries out this action, the innovation opportunity will arise with probability $1-\sigma$ $(0 < \sigma < 1)$; otherwise, the innovation opportunity cannot arise.

The lender also acquires information as a by-product of her financing activity. As in Diamond and Rajan (2001) and Habib and Jonsen (1999), this information enables her to obtain more than other agents from the liquidation of the entrepreneur's assets, that is, the lender "monitors" collateralizable assets. Precisely, the share of liquidation value that the lender recovers in the event of project failure equals the amount of her information on the assets; the rest of the liquidation value is lost in the form of transaction costs.[3] By contrast, we normalize to zero the net amount of final good that any other agent obtains from asset liquidation.

We allow the entrepreneur to influence the amount of information of the lender by choosing the type of funding, relationship or transactional. Precisely, on date 0 each entrepreneur chooses whether to establish an informationally intensive credit relationship with his financier on date 1 or seek a transactional loan. Consider first mature assets: $\sigma\mu$ (respectively, μ) is the amount of information of a lender if she does (not) carry out the action for the innovation; furthermore, for a relationship lender $\mu = M$ while for a transactional lender $\mu = m$. This specification has

[2] Building on the analysis of collateral asset markets in Shleifer and Vishny (1992), we generate a downward sloping demand for collateral assets by allowing for heterogeneity in entrepreneurs' ability to reuse assets.

[3] For most of the analysis, we do not take a stance on whether transaction costs are transfers or a real resource loss.

TABLE I.
Notation of the Model.

Probability of project success	π
Output of mature technology	y
Productivity edge of new technology	n
Collateral assets of firm	a
Investment outlay of project	i
Share of verifiable output	l
Probability of innovation opportunity	$1-\sigma$
Aggregate productivity liquidated assets	η
Idiosyncratic productivity liquidated assets	θ
Relative redeployability new assets	ϕ
Information of (share of asset rescued by) relationship lender	M
Information of (share of asset rescued by) transactional lender	m

Note. The table describes the symbols used in the model.

two features. First, in an informationally intensive credit relationship a lender acquires more information about the entrepreneur's assets ($M > m$). Second, when a lender allows the innovation, she acquires less information on mature assets ($\sigma < 1$).[4] This reflects the idea that the lender has less opportunities − and with endogenous information acquisition, less incentives − to acquire information on a technology if the entrepreneur is working to abandon it. Consider next new assets: denoting the amount of information by μ_n, we let $\mu_n \phi = 0$. Thus, a lender recovers less value from liquidating new assets than from liquidating mature assets − the normalization to zero is for simplicity.

2.3 Contractual structure

As in Aghion and Bolton (1992) and Diamond and Rajan (2001), a lender cannot contractually commit to carry out her action necessary for the innovation because this action is non-verifiable; moreover, imperfect enforceability limits agents' commitment to pecuniary transfers and, hence, the design of pecuniary incentives for the lender. Specifically, in the event of project success, only a fraction l of the output is verifiable while the rest accrues privately to the entrepreneur. In the event of project failure and asset liquidation, the lender cannot commit the specific liquidation skills tied to her information about the assets. Thus, as in Diamond and Rajan (2001), she can threaten to withhold her skills during the liquidation, forcing a renegotiation of the allocation of the asset liquidation proceeds to appropriate them in full.[5]

[4] Letting the amount of information aquired by the lender on the mature technology be equal to the probability that the mature technology is adopted simplifies the algebra but is not relevant for the results.

[5] As in Diamond and Rajan (2001), for simplicity the lender has all the bargaining power in the renegotiation.

2.4 Discussion

In the real sector, the difference between the two technologies is that the new technology produces more output ($y(1+n) > y$) but its assets have lower liquidation value ($\mu_n \phi pa < \mu pa$, where p denotes the market price of assets). We put forward two interpretations. First, new technologies typically have less assets pledgeable as collateral, i.e., $\phi < 1$ (Hall, 2001; Berlin and Butler, 2002; Rajan and Zingales, 2001).[6] Carpenter and Petersen (2002) argue that "R&D investment, which is predominantly salary payments, has little salvage value in the event of failure. Furthermore, physical investments designed to embody R&D results are likely to be firm specific, and therefore may have little collateral value". Second, for a given liquidation value, lenders typically have less experience in liquidating new vintages of assets than mature ones, i.e., $\mu_n < \mu$.

In the credit sector, there are three features worth discussion: the control exerted by a lender; the characterization of information as asset liquidation skills; the amount of information of a lender. We share with Aghion and Bolton (1992), Rajan (1992) and Bhattacharya and Chiesa (1995), for example, the assumption that a lender carries out an interim action that affects production opportunities. This action has several real world counterparts. It can consist of providing the entrepreneur with advice or information for expanding the firm's technological frontier; in an R&D race, it can consist of concealing the findings of the entrepreneur's internal research from her competitors (Bhattacharya and Chiesa, 1995); if the lender has representatives on the board of the firm, as in the case of German and Japanese banks, it can consist of voting for an innovative strategy. In other circumstances, this action can consist of a refinancing (Rajan, 1992): the need for refinancing is likely for a new technology which generally yields little interim cash flow, especially at the R&D stage (Goodakre and Tonks, 1995). Aghion and Bolton (1992) discuss examples of other actions of lenders which can affect innovation opportunities, such as supporting firms' mergers and spin-offs.

We borrow the characterization of information as asset liquidation skills from Diamond and Rajan (2001) and Habib and Jonsen (1999). The critical feature is that the lender acquires more information than the entrepreneur and the other investors. As for latter, following Diamond and Rajan (2001), our assumption reflects the idea that the lender obtains more information on collateral assets through her financing activity. As for the former, "Because he [the entrepreneur] is a specialist at maximizing the value of the asset in its primary use [...] it is reasonable to assume that he lacks the skill even to identify the asset's next best use or to recognize clearly the occurrence of the bad states, in which case he risks maintaining it in a suboptimal use" (Habib and Johnsen, 1999, p. 145).

The third feature worth discussion is the amount of information of a lender. In several models, the distinct characteristic of a relationship lender is her informational advantage over a transactional lender (Berger and Udell, 1998). Indeed, Berger and Udell argue that "banks may acquire private

[6] In the model, we normalize to zero the value that a lender obtains from the liquidation of new assets. We could allow new assets to have positive liquidation value, though lower than that of mature assets ($0 < \mu_n \alpha_n < 1$). The results would carry through but the analysis would be more cumbersome.

information over the course of a relationship" (1995, p. 352) and "under relationship lending, banks acquire information over time through contact with the firm" (2002, p. 32).[7]

3 Model Solution

Throughout this section, we posit that $\phi = 1$, that is, we let mature and new assets have the same redeployability in the asset market. This means that the difference between the return that a lender obtains from the liquidation of mature assets and the return she obtains from the liquidation of new assets is fully attributable to her poorer information about the new assets. We consider the case in which $\phi < 1$ in the robustness section.

We solve the model in steps. We first study lenders' decisions, namely the choice $\alpha \in \{A, \overline{A}\}$ of a lender on date 2, where A denotes action and $\overline{A}$ inaction, and the choice of a lender whether to finance an entrepreneur. We then study the determination of the market price of assets p. Finally, we consider entrepreneurs' decisions. In particular, we solve for an entrepreneur's choice (r_m, r_n) of contract on date 1, where r_m is the repayment to the lender if the mature technology is successfully operated and r_n is the repayment if the new technology is successfully operated.[8] We also solve for an entrepreneur's choice $\mu \in \{m, M\}$ of funding on date 0. We say that an outcome (μ, α) is feasible if there exists a contract that satisfies the lender's participation constraint (i.e., it is funded) and that induces the lender to choose α.

3.1 Lenders' decisions

Consider the date 2 decision of a lender whether to carry out the action necessary for the innovation. The lender compares her expected return if the innovation can occur with her expected return if the innovation cannot occur. Thus, assuming that she breaks a tie in favour of inaction, the lender will carry out the action if and only if

$$(1-\sigma)\pi r_n + \sigma\left[\pi r_m + (1-\pi)\sigma\mu pa\right] > \pi r_m + (1-\pi)\mu pa, \tag{1}$$

which can be rewritten as

$$r_n - r_m > \frac{1-\pi}{\pi}(1+\sigma)\mu pa. \tag{2}$$

Inequality (2) is the lender's incentive compatibility constraint. The left hand side of (2) is the spread between the repayment in the event of successful adoption of the new technology and the repayment in the event of successful adoption of the mature technology. The right hand side of (2) is (a monotonic transformation of) the reduction in liquidation proceeds that the lender suffers if the entrepreneur innovates and the project fails. This reduction, which is due to the lender's worse ability to liquidate new assets, is positively related to the lender's information μ and to the

[7] See also Guiso and Minetti (2010) for empirical evidence on this.

[8] The contractual structure implies that a contract only sets the loan granted by the lender and the repayment to the lender in the event of project success, contingent on the technology adopted. Since projects are indivisible and each project requires an amount i of final good, without loss of generality we can restrict attention to contracts that set a loan equal to i.

liquidation value pa of mature assets. The lender will allow the innovation if and only if, as in (2), the contract guarantees her a sufficiently higher repayment if the new technology is successfully adopted, compensating her for the reduction in her expected liquidation proceeds.

Lemma 1 characterizes the conditions under which there exists a contract that satisfies the lender's incentive and participation constraints. The proof is in the Appendix.

Lemma 1 *There exists a feasible contract that induces a lender to carry out the action necessary for the innovation if and only if the entrepreneurs's collateral assets satisfy* $a \in [\underline{a}(p, \mu, A), \overline{a}(p, \mu))$*, where*

$$\underline{a}(p, \mu, A) \equiv \frac{i - \pi l y (1 + n - n\sigma)}{\sigma^2 p (1 - \pi) \mu} \tag{3}$$

and

$$\overline{a}(p, \mu) \equiv \frac{\pi l y (1 + n) - i}{p \sigma (1 - \pi) \mu}. \tag{4}$$

The intuition behind Lemma 1 is as follows. The spread $r_n - r_m$ that can be specified in a contract is bounded. On the one hand, the repayment r_n for the new technology is constrained above by the entrepreneur's limited liability constraint. On the other hand, for a given r_n, the repayment r_m for the mature technology is constrained below by the lender's participation constraint.[9] Lemma 1 shows that if $a > \overline{a}(p, \mu)$, in (2) the left hand side falls short of the right hand side for any feasible pair (r_n, r_m). In this region, the lender impedes the entrepreneur's innovation. Inspection of (4) yields two insights. First, for a given market price p of assets, a lender is more likely to induce technological inertia within a credit relationship, that is, $\overline{a}(p, M) < \overline{a}(p, m)$. Second, the lender is more likely to induce technological inertia when an entrepreneur is rich in collateral (has a high a) and when the market price of assets is higher. Intuitively, a lender loses more from the depreciation of her asset liquidation skills when she has better information on the entrepreneur's collateral assets ($M > m$) and the value pa of the assets is higher. Turning to the lower bound $\underline{a}(p, \mu, A)$ in the lemma, this stems from the fact that collateral-poor firms (firms with $a < \underline{a}(p, \mu, A)$) cannot satisfy the participation constraint of a lender and are thus excluded from the credit market. Inspection of (3) reveals that a lender is more likely to provide credit within a credit relationship ($\underline{a}(p, M, A) < \underline{a}(p, m, A)$), when an entrepreneur is rich in collateral (has a high a) and when the market price of assets p is high. Lemma 1 thus illustrates the dual role of collateral and credit relationships in our economy. On the one hand, they ease entrepreneurs' access to credit. On the other hand, an excess of collateral and credit relationships may inhibit entrepreneurs' innovation.

In what follows, to guarantee that there exists a region of parameters in which innovation is feasible, we assume that $\underline{a}(p, \mu, A) < \overline{a}(p, \mu)$, which can be rewritten as

$$\frac{i}{\pi l y} < 1 + \frac{n}{1 + \sigma}. \tag{5}$$

[9] We do not impose a limited liability constraint for the lender. Implicitly, we are assuming that the lender has more than enough funds to make transfers to the entrepreneur on date 3, if needed. Adding a limited liability constraint would not alter the results.

Further, we are interested in a scenario in which not all firms can obtain credit and not all firms can innovate. We thus restrict attention to a region of parameters in which entrepreneurs with small enough collateral do not obtain credit ($\underline{a}(p, M, A) > 0$) and entrepreneurs with high enough collateral cannot innovate ($\overline{a}(p, m) < 1$). The first condition implies

$$\frac{i}{\pi l y} > 1 + n - n\sigma, \tag{6}$$

while the second implies

$$\frac{i}{\pi l y} > 1 + n - \frac{\sigma(1-\pi)mp}{\pi l y}. \tag{7}$$

3.2 Asset price

Thus far, we have treated the market price of assets p as exogenous. We now solve for the asset price and characterize the region of parameters where condition (7) holds.[10] As noted, the assumption $\phi = 1$ implies that the asset redeployability is independent of the technology embodied in the assets. Therefore, while the demand and the supply of assets depend on the measure of entrepreneurs who have access to credit, they do not depend on the distribution of firms between the new and the mature technology.

The demand $D(p)$ of liquidated assets satisfies

$$D(p) = [1 - \underline{a}(p, M, A)]\, \pi \left(1 - \frac{p}{\eta\overline{\theta}}\right). \tag{8}$$

In fact, a measure $1 - \underline{a}(p, M, A)$ of entrepreneurs obtain credit and become active. Moreover, a share π of active entrepreneurs is successful and remains in business. Finally, a share $\left(\eta\overline{\theta} - p\right)/\eta\overline{\theta}$ of the entrepreneurs who remain in business recover an output no lower than p from reusing assets, that is, have a reservation price no lower than p. In turn, the supply $S(p)$ of assets satisfies

$$S(p) = (1-\pi) \int_{\underline{a}(p,M,A)}^{1} a\,da. \tag{9}$$

The supply is given by the probability $1 - \pi$ that an entrepreneur fails times the amount of assets a that are liquidated by a failed entrepreneur, integrated across active entrepreneurs. In the Appendix, we prove that there exists a unique equilibrium with positive asset demand and supply. In this equilibrium, the asset price is

$$p = \frac{\eta\overline{\theta}}{2\pi} \left\{ \frac{3\pi - 1}{2} + \left[\left(\frac{3\pi - 1}{2}\right)^2 - \frac{2\pi}{\eta\overline{\theta}} \frac{i - \pi l y(1 + n - n\sigma)}{\sigma^2 M} \right]^{\frac{1}{2}} \right\}. \tag{10}$$

[10] So far, we have only considered the case where credit is granted under the possibility of innovation. We also need to consider the conditions under which credit is granted under no innovation. In this case, the entrepreneur obtains credit if $\pi r_m^{\text{no}} + (1-\pi)\,\mu p a = i$, where r_m^{no} is the repayment when the entrepreneur anticipates that the lender will not choose the action that allows innovation. Note that since $r_m^{\text{no}} \leq ly$, a lower bound on a that is consistent with access to credit is given by $a \geq \frac{i - \pi l y}{(1-\pi)\mu p} \equiv \underline{a}(p, \mu, \overline{A})$. Now, observe that (5) implies $\underline{a}(p, \mu, \overline{A}) > \underline{a}(p, \mu, A)$: conditional on the funding choice, preventing innovation never increases the possibility of funding a project. Moreover, (5) implies $\underline{a}(p, \mu, \overline{A}) < \overline{a}(p, \mu)$: conditional on the funding choice, credit is always available when innovation is not incentive-compatible. Together, these results imply that the lower bound $\underline{a}(p, \mu, A)$ on the set of firms which have access to credit under innovation includes all firms that can be funded.

In the Appendix, we also replace p in condition (7) and pin down the region of parameters where the condition holds. Intuitively, we only need $\overline{\theta}$ to be sufficiently large. This ensures that the value of collateral assets is high enough that lenders want to prevent collateral-rich entrepreneurs from adopting the new technology.[11]

3.3 Entrepreneurs' decisions

Having studied lenders' decisions, we now determine which contract and which type of funding are chosen by entrepreneurs. The choice of an entrepreneur critically depends on the value of a. Given Lemma 1, the analysis for firms with extreme collateral values is trivial. Collateral-poor firms with $a < \underline{a}(p, M, A)$ do not have access to the credit market. Collateral-rich firms with $a > \overline{a}(p, m)$ cannot innovate, regardless of their funding choice. Thus, since relationship funding is cheaper than transactional funding (because a relationship lender can recover more collateral value), these firms choose $\mu = M$.[12] The non-trivial case occurs for firms with intermediate collateral values ($a \in [\underline{a}(p, M, A), \overline{a}(p, m)]$). We start with establishing the following result.

Lemma 2 *In the region of parameters in which a relationship lender can be induced to carry out the action necessary for the innovation, an entrepreneur prefers innovating if and only if* $l\,(1+\sigma) < 1$.

Henceforth, we let $l\,(1+\sigma) < 1$. Lemma 2 implies that firms with $a \in [\underline{a}(p, M, A), \overline{a}(p, M))$ choose relationship funding and potentially innovate. In fact, relationship lenders allow these firms to adopt the new technology (because $a < \overline{a}(p, M)$) and these firms indeed prefer the new technology. Moreover, relationship funding is cheaper than transactional funding.

Next, we have to consider the firms with collateral $a \in [\overline{a}(p, M), \overline{a}(p, m)]$. These firms face a non-trivial funding choice. They must choose transactional funding if they want to innovate. However, transactional funding is more expensive than relationship funding so these firms may be unable or unwilling to finance themselves this way. Specifically, there are two possible cases. In the first, $\underline{a}(p, m, A) \leq \overline{a}(p, M)$ so all firms with collateral $a \in [\overline{a}(p, M), \overline{a}(p, m)]$ can obtain credit if they choose to innovate under transactional funding. In the second, $\underline{a}(p, m, A) > \overline{a}(p, M)$ so some firms with collateral $a \in [\overline{a}(p, M), \overline{a}(p, m)]$ cannot obtain credit if they want to innovate under transactional funding. We deal with the latter possibility in a discussion in Section 5. In what follows, we instead restrict attention to the case in which, if the innovation of a firm can only be induced under transactional funding, it is not prevented by the firm's lack of access to credit. This means that, if transactional funding is not chosen, it is because it is expensive relative to relationship funding and not due to lack of credit. Formally, the region of parameters in which under transactional funding credit is always available to firms with $a \in [\overline{a}(p, M), \overline{a}(p, m)]$ is given

[11] The choice of $\overline{\theta}$ has no effect on the region of parameters set by (5) and (6). In particular, we choose $\overline{\theta}$ large enough that (7) holds whenever (6) holds.

[12] In this case, the repayment r_m^{no} in case of success is set at a value such that $\pi r_m^{\text{no}} + (1-\pi)\,\mu p a = i$. Since innovation does not occur, the choice of r_n^{no} is immaterial.

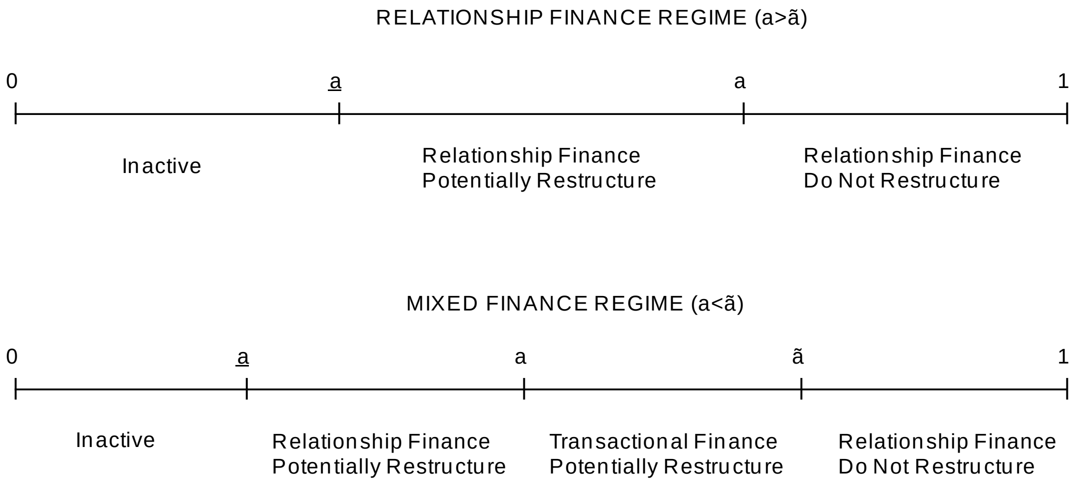

Figure 2: Firm Distribution Across Collateral Values.

by

$$\frac{i}{\pi ly} < 1 + \frac{\left[M + \sigma m - (M - m)\sigma^2\right] n}{(M + \sigma m)\,(1 + \sigma)}. \tag{11}$$

There exists a non-empty region of parameters which satisfy (5), (6), (7), and (11). Further, we can restrict attention to (6) and (11) because (11) implies (5) and, assuming $\overline{\theta}$ large enough, (6) implies (7). Lemma 3 solves for an entrepreneur's choice.

Lemma 3 *Assume that conditions (6) and (11) hold. An entrepreneur will choose transactional funding if and only if* $a \in [\overline{a}(p, M), \overline{a}(p, m)]$ *and* $a < \frac{\pi yn(1-\sigma)}{(1-\pi)(M-\sigma^2 m)p} \equiv \widehat{a}(p)$.

Lemma 3 allows to identify two credit regimes (see also Figure 2). In the first regime, which arises when $\widehat{a}(p) \leq \overline{a}(p, M)$, no entrepreneur chooses transactional funding. We thus label it "relationship finance" regime. This regime arises when lenders are not very inclined to induce technological inertia within credit relationships, that is, relationship lenders prevent only firms with large collateral from innovating. Alternatively, this credit regime arises when entrepreneurs derive a large benefit from credit relationships, that is, they obtain much cheaper financing from relationship lenders. In this credit regime, all firms form credit relationships and firms only differ in their technology choice: those with intermediate collateral potentially innovate whereas those with large collateral adopt the mature technology. In the second credit regime, which arises when $\widehat{a}(p) > \overline{a}(p, M)$, some entrepreneurs avoid the technological inertia induced by relationship lenders by choosing transactional funding. We label it "mixed finance" regime. This regime arises when lenders are very inclined to induce technological inertia within credit relationships or entrepreneurs derive a small benefit from relationships. In this regime, firms with medium collateral value avoid the technological inertia induced by relationship lenders by choosing transactional funding. As in the relationship finance regime, firms with medium collateral value potentially innovate while firms with large collateral adopt the mature technology. In sum, the critical difference between the two credit regimes is that in the relationship finance regime transactional finance is inactive whereas in the mixed finance regime it is active and specializes in the financing of firms that potentially innovate. In both regimes, collateral-rich firms form credit relationships with lenders in which they retain the mature technology.

Lemma 4 characterizes the region of parameters in which the mixed finance and the relationship finance regimes arise.

Lemma 4 *Assume that conditions (6) and (11) hold. Then*

(i) if $l < \frac{(1-\sigma)M}{M-\sigma^2 m}$*, mixed finance is the only credit regime;*

(ii) If $l \in \left[\frac{(1-\sigma)M}{M-\sigma^2 m}, \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}\right]$*, the mixed finance regime occurs if and only if*

$$\frac{i}{\pi l y} \in \left(1 + \frac{\left[1-\sigma\left(1-\sigma\right)M\right]n}{l\left(M-\sigma^2 m\right)}, 1 + \frac{\left[M+\sigma m-(M-m)\sigma^2\right]n}{\left(M+\sigma m\right)\left(1+\sigma\right)}\right); \tag{12}$$

the relationship finance regime occurs otherwise;

(iii) if $l > \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}$*, relationship finance is the only credit regime.*

3.4 Summary

Proposition 1 combines the results of Lemmas 1-4. The proposition characterizes the distribution of firms across all collateral values (a) according to whether a firm obtains credit or not, and, if it obtains credit, according to the type of funding it chooses (relationship or transactional) and the technology it adopts (mature or new). The proof is immediate given the lemmas. Figure 2 illustrates the distribution of firms in the two credit regimes.

Proposition 1 *Assume that (6) and (11) hold. Consider the region of parameters consistent with the relationship finance regime. In this case, the distribution of firms across collateral values is as follows: (i) firms have no access to credit iff* $a < \underline{a}(p, M, A)$*, (ii) firms choose relationship funding and potentially innovate iff* $a \in [\underline{a}(p, M, A), \overline{a}(p, M)]$*, (iii) firms choose relationship funding and do not innovate iff* $a > \overline{a}(p, M)$*. Consider now the region of parameters consistent with the mixed finance regime. In this case, the distribution of firms across collateral values is as follows: (i) firms have no access to credit iff* $a < \underline{a}(p, M, A)$*, (ii) firms choose relationship funding and potentially innovate iff* $a \in [\underline{a}(p, M, A), \overline{a}(p, M)]$*, (iii) firms choose transactional funding and potentially innovate iff* $a \in (\overline{a}(p, M), \min\{\widehat{a}(p), \overline{a}(p, m)\})$*, (iv) firms choose relationship funding and do not innovate iff* $a > \min\{\widehat{a}(p), \overline{a}(p, m)\}$*.*

It is useful to compare the equilibrium in Proposition 1 with the allocation that would be chosen by a social planner. This will also ease the interpretation of the policy analysis of Section 6.

Lemma 5 *Assume that conditions (6) and (11) hold. A social planner would choose an allocation in which all entrepreneurs obtain credit from relationship lenders and potentially innovate.*

The proof of the lemma is in the Appendix. The lemma shows that the decentralized equilibrium in Proposition 1 is characterized by a suboptimally low measure of active firms and a suboptimally low measure of innovative firms.

3.5 A numerical example

While Proposition 1 derives a closed form solution, to fix ideas it can be useful to consider two numerical examples. Consider the parameters in Table II, first column and Panel A (second column). These parameters imply that the probability $1-\pi$ of failure of a project is 8%; when the innovation occurs, the return from a project $y(1+n)/i-1$ in the event of success net of the investment cost equals 114%. If we let a period correspond to four years so that the project lifetime from financing (date 1) to outcome (date 3) equals eight years, this implies a10% annual return. In the event of success, the return of the new technology exceeds that of the mature by 17.5% ($=n$). If the lender carries out the action, the probability $1-\sigma$ of innovation is 5%; 8% $(1-l)$ of the output is non-verifiable; finally, the share of value lost in liquidation by a relationship (transactional) lender amounts to $1-M=20\%$ $(1-m=27\%)$. With this parameter selection the economy is in the relationship finance regime. Consider next the parameters in Table II, first column and Panel B (second column). These parameters are the same as before, except that the share of value lost in liquidation by a relationship (transactional) lender amounts to 23%. With this parameter selection the economy is in the mixed finance regime.

In the two numerical examples, we have kept the values of the technological parameters fixed and we have allowed only the parameters of the credit market M and m to vary. The purpose of this exercise is to disentangle the role of lenders' information. Using the parameterization in Table II, it is possible to show that, for a given level of M, a wider gap between M and m pushes the economy into the relationship finance regime. In fact, for a given degree of technological inertia, a wider gap $M-m$ implies a larger benefit from credit relationships (i.e., a larger gap between the cost of transactional finance and the cost of relationship finance). In turn, for a given gap $M-m$, a higher M pushes the economy into the mixed finance regime. In fact, for a given benefit of credit relationships, a higher value of M exacerbates the technological inertia of relationship lenders. All in all, this implies that in the $(M, M-m)$ space the frontier between the two credit regimes is upward sloping.

4 Impact of Collateral Shocks

We now study the effects of shocks. We are primarily interested in the effects of a shock to collateral that is akin to the collateral squeeze considered in Holmstrom and Tirole (1997) (see also the capital quality shock in Gertler and Karadi, 2010). We let this shock take the form of a drop in the aggregate productivity η of liquidated assets and assume that the drop in η is small so we can evaluate its effects with the help of differential calculus. In the next section, we also investigate the effects of a shock to productivity that takes the form of a drop in y. In practice, as in Holmstrom

TABLE II.

Effects of 1% Collateral Shock

Parameters (*Technology*)	*Parameters* (*Credit Mkt*)	*Thresholds*	*Effects* (*Real Sector*)		*Effects* (*Credit Market*)	
	(Panel A: Relationship Finance Regime)					
$\pi = 0.92$	$M = 0.80$	$\underline{a}(p, M, A) = 0.5396$	$\frac{\Delta I}{I}$	-1.2147%	$\frac{\Delta R_r}{CR}$	-1.2147%
$y = 0.96$	$m = 0.73$	$\overline{a}(p, M) = 0.8555$	$\frac{\Delta N_r}{N}$	$+1.0364\%$	$\frac{\Delta C}{C}$	-1.2147%
$n = 0.175$		$\overline{a}(p, m) = 0.9375$	$\frac{\Delta p}{p}$	-1.0258%		
$i = 0.525$		$\hat{a}(p) = 0.6984$	$\frac{\Delta Y}{Y}$			
$l = 0.92$		$p = 0.9797$				
$\sigma = 0.95$	(Panel B: Mixed Finance Regime)					
$\eta = 1.00$	$M = 0.80$	$\underline{a}(p, M, A) = 0.5396$	$\frac{\Delta I}{I}$	-1.2147%	$\frac{\Delta R_m}{CR}$	-1.3904%
$\overline{\theta} = 1.05$	$m = 0.77$	$\overline{a}(p, M) = 0.8555$	$\frac{\Delta N_m}{N}$	$+1.0364\%$	$\frac{\Delta C}{C}$	-1.2147%
$\phi = 1$		$\overline{a}(p, m) = 0.8888$	$\frac{\Delta p}{p}$	-1.0258%		
		$\hat{a}(p) = 0.9384$	$\frac{\Delta Y}{Y}$			
		$p = 0.9797$				

Note. The table reports a parameter selection (first and second column), implied collateral thresholds (third column), and the effects of a 1% drop in collateral productivity on investment (I), credit (C), measures of innovative firms (N) and credit relationships (R_i), and asset price (p).

and Tirole (1997), for example, we perform comparative statics exercises, comparing the equilibrium that obtains in our economy for two different values of η (in the case of a collateral shock) and for two different values of y (in the case of a productivity shock). For instance, when we say that a credit relationship breaks down after the collateral shock, we mean that for the higher value of η the firm would have borrowed from a relationship lender, while for the lower value of η it borrows from a transactional lender or it is inactive. In the real sector, we focus on the effects on total investment, on the measure of innovative firms, on output and on the asset price. In the credit sector, we focus on the effects on the measure of firms that have access to the credit market and on the measure of firms that engage in credit relationships.

Proposition 2 summarizes the effects of the collateral shock in the two credit regimes. The proof is in the Appendix.

Proposition 2 *In both the relationship finance regime and the mixed finance regime, a negative collateral shock (reduction of η) reduces the asset price, total investment, the measure of firms that have access to credit, and the measure of firms that engage in credit relationships. By contrast, a negative collateral shock increases the measure of innovative firms. The effect on output is ambiguous: the drop in investment tends to reduce output while the increase in the measure of innovative firms tends to increase output.*

The drop in the productivity η of assets induces a fall of the asset price because, for a given price, the demand for liquidated assets shrinks. The measure of firms that have access to the credit market falls with the asset price. In particular, the firms that were "marginal" in the credit market (that is, with initial collateral in the neighborhood of $\underline{a}(p, M, A)$) are denied credit because they can no longer pledge enough expected returns to a lender. These firms drop out of the credit market, their investment is lost and their credit relationships break down. Clearly, the exclusion of these firms from the credit market further reduces the demand and the supply of assets and feeds back on the asset price. This effect of the collateral shock on collateral-poor firms is similar to that obtained by Holmstrom and Tirole (1997).

The new prediction of the model is the impact of the shock on collateral-rich firms. Consider first the relationship finance regime. Since the threshold collateral $\overline{a}(p, M)$ above which collateral-rich firms face technological inertia within credit relationships is negatively related to the asset price, the shock allows firms in the neighborhood of $\overline{a}(p, M)$ to potentially innovate within their credit relationships. Next, consider the mixed finance regime. Since $\min\{\overline{a}(p, m), \widehat{a}(p)\}$ is negatively related to the asset price, firms with initial collateral in the neighborhood of $\min\{\overline{a}(p, m), \widehat{a}(p)\}$ prefer now borrowing from transactional lenders and innovating. These firms deliberately break their credit relationships, borrow from new investors, and innovate. In both credit regimes, the assumption of a uniform distribution of firms' collateral values implies that the measure of additional collateral-rich firms that potentially innovate outweighs the measure of firms squeezed out by the shock from the credit market. Hence, the measure of innovative firms increases.[13] It is also worth observing that in general equilibrium the exclusion of collateral-poor firms from the credit market is beneficial for the innovation of collateral-rich firms. In fact, when collateral-poor firms become inactive, the net demand for collateral assets drops. This further depresses the collateral asset price, and, given the negative relationship between $\overline{a}(p, M)$ and p (in the relationship finance regime) and between $\min\{\overline{a}(p, m), \widehat{a}(p)\}$ and p (in the mixed finance regime), it fosters the innovation of collateral-rich firms. This will be relevant for evaluating the impact of policies that sustain the access of collateral-poor firms to the credit market.

Turning to the impact of the shock on output, in the relationship finance regime this reflects the interaction between two opposite forces. On the one hand, the loss of investment due to the exclusion of collateral-poor firms from the credit market tends to decrease output. On the other hand, the increase of innovation of collateral-rich firms tends to increase output. In the mixed finance regime, if the asset liquidation costs are real, there is also a third additional force that affects output. To understand this third force, however, we need to investigate the effects of the shock in the credit market.

The way the innovation of collateral-rich firms occurs in the credit market depends on the credit regime. In the relationship finance regime, after the shock the innovation of collateral-rich firms occurs within their credit relationships and the breakdown of relationships caused by the shock

[13] With a generic distribution, whether the measure of innovative firms increases or decreases depends on the measure of this group of firms relative to those squeezed out by the shock from the credit market. Of course, there are sufficiently right-skewed distributions such that the measure of innovative firms drops. However, even in this case, the ratio "(innovative firms)/(total firms)" may increase. Finally, if this ratio declines too, the predictions of the model will resemble those of the studies on the negative effects of credit imperfections on innovation activity during recessions (e.g., Caballero and Hammour, 2004; Barlevy, 2003).

is entirely attributable to the exclusion of collateral-poor firms from the credit market. In the mixed finance regime, instead, the innovation of collateral-rich firms entails the breakdown of their credit relationships, so the surge in firms' innovation leads to an additional breakdown of credit relationships besides that induced by the exclusion of collateral-poor firms from the credit market. This additional breakdown is only partially compensated by the fact that in the mixed finance regime firms with initial collateral in the neighborhood of $\overline{a}(p, M)$ are now allowed to innovate by relationship lenders and prefer relationship funding. These arguments are formalized in Proposition 3. The proof is in the Appendix.

Proposition 3 *In the relationship finance regime, a negative collateral shock (reduction of η) induces the same percentage drop in the measure of active firms and in the measure of credit relationships. In the mixed finance regime, the percentage drop in the measure of credit relationships is larger than the percentage drop in the measure of active firms. Precisely, the change in the ratio of credit relationships (R_m) over active firms (C) is given by*

$$\frac{\partial \left(\frac{R_m}{C}\right)}{\partial \eta} = \frac{C - R_m}{C^2} \frac{1}{p} \frac{\partial p}{\partial \eta} > 0. \tag{13}$$

Thus, in the mixed finance regime there can be a third additional force which affects output after the collateral shock. When the liquidation costs are at least partially a real resource loss, the additional breakdown of credit relationships that is induced by firms' innovation activity implies a surge in liquidation costs. To summarize, in the mixed finance regime the output change is the result of three competing forces. Besides the output drop due the decline of investment and the output increase due to the increase in innovation, there is also the increase in transaction costs due to the voluntary breakdown of credit relationships by collateral-rich firms. The latter effect does not occur in the relationship finance regime.

The effects of the shock can be further grasped numerically. Consider again the parameters in Table II, first column and Panel A (second column), that is, the relationship finance regime. A reduction of η by 1% triggers a drop in investment, credit and measure of credit relationships equal to -1.21%. The asset price declines by 1.03%. The measure of innovative firms rises by 1.036%. Consider next the parameters in Table II, first column and Panel B (second column), that is, the mixed finance regime. The fall in investment, credit, asset price and the rise in the measure of innovative firms are the same as in the relationship finance regime. By contrast, the drop in the measure of credit relationships is larger than the drop in the measure of active firms and equals -1.39%.

5 Robustness

This section explores the robustness of the results. We first relax the assumption that mature and new assets have the same redeployability in the asset market. We then illustrate what would happen in credit regimes alternative to the two regimes considered in the previous section. Finally, we study the effects of productivity shocks.

5.1 Asset heterogeneity

In the baseline analysis, we assumed that mature and new assets have the same market redeployability ($\phi = 1$). This implies that the supply of assets does not depend on the distribution of firms between the new and the mature technology, but only on the measure of firms that have access to credit. Hence, when a negative collateral shock hits, there is a contraction in the demand and in the supply of assets due to the dropout of some firms from the credit market. However, the change in the measure of innovative firms has no feedback effect on the asset price. By contrast, in a context in which the redeployability of new assets is lower than that of mature assets ($\phi < 1$), the increase in the measure of innovative firms shrinks the supply of assets because less assets will be resold in the market. For example, in the relationship finance regime the asset supply equals

$$S(p) = (1-\pi)\left[\int_{\underline{a}(p,M,A)}^{\overline{a}(p,M)} \left[(1-\sigma)\,\phi + \sigma\right] ada + \int_{\overline{a}(p,M)}^{1} ada\right]. \tag{14}$$

In turn, the reduction in the supply of assets due to the increase in the innovation of collateral-rich firms tends to sustain the asset price dampening both the increase of innovation and the decline of investment. In Proposition 4, we show that in this alternative case with $\phi < 1$ the qualitative results of Proposition 1 remain unchanged, although the effects of the shock are dampened. The proof is in the Appendix.

Proposition 4 *Suppose $\phi < 1$. Both in the relationship finance regime and in the mixed finance regime, the negative collateral shock has the same effects as those illustrated in Proposition 2. However, all the effects of the shock, including the increase in firms' innovation activity, are smaller.*

An interpretation of this result is that if the innovation is "radical", so that the new assets significantly differ from the mature assets and, hence, are less easily redeployable in the market, the increase in innovation activity induced by a collateral shock can be less important. At the same time, however, Proposition 4 suggests that the drops in investment, asset price and output can also be smaller than in the case of an "incremental" innovation with assets similar to those of the existing technology.

5.2 Other credit regimes

In the baseline analysis, we introduced restrictions on parameters that allowed us to focus on two credit regimes of interest. We now briefly discuss what happens in the complementary regions of the parameter space. A complete characterization of the equilibrium for all the parameter values is provided in the Supplement.

First, we only considered cases in which some innovation activity is feasible. Clearly, if we impose that innovation is never feasible,[14] the only equilibrium outcome in the presence of credit involves $\left(M, \overline{A}\right)$, i.e., relationship funding and no innovation. In this scenario, a negative collateral shock has only the standard effect of a contraction in aggregate investment due to tightening collateral constraints. We also restricted attention to cases in which, whenever transactional funding is required for innovation (i.e., $a \in [\overline{a}(p, M), \overline{a}(p, m))$), it is not prevented by lack of credit. If we drop this restriction, two scenarios can arise. First, it may be that, in a subset of $[\overline{a}(p, M), \overline{a}(p, m))$, the outcome is $\left(M, \overline{A}\right)$ because credit is only available under relationship funding. Second, it may be that the outcome is $\left(M, \overline{A}\right)$ in a subset of $[\overline{a}(p, M), \overline{a}(p, m))$ because, even though transactional funding is feasible, it is not desirable for firms. In both scenarios, the key implication is that now there is a threshold value of collateral $a^{\circ}(p) \in [\overline{a}(p, M), \overline{a}(p, m))$ such that the equilibrium outcome is (m, A) for all $a \in [\overline{a}(p, M), a^{\circ}(p))$, and the equilibrium outcome is $\left(M, \overline{A}\right)$ for all $a \in [a^{\circ}(p), \overline{a}(p, m))$. Broadly speaking, such a distribution of firms across collateral values is not very different from the distribution in the mixed finance regime.

In the baseline analysis, we also assumed that there exists an upper tail in the distribution of collateral values in which the technological inertia of lenders arises (i.e., innovation is not feasible for all firms). In fact, we considered a region of parameters (basically, a value of $\overline{\theta}$ sufficiently large) such that this indeed happens. Whether this assumption is realistic or not $-$ and how fat this upper tail of the distribution of collateral assets is $-$ depends on the structural characteristics of the economy. In an economy in which very few firms have large collateral, or where new technologies have a significant productivity advantage over mature ones, this condition does not hold and the equilibrium outcome in the presence of credit is always (M, A). In this case, the only mechanism at work during a credit crunch is the traditional mechanism of contraction of investment and of innovation activity due to tightening collateral constraints.

Finally, in the baseline analysis we assumed that, if innovation under relationship funding is feasible, it achieves the highest surplus. If we relax this assumption, there will exist a subset of $a \in [\underline{a}(p, M, A), \overline{a}(p, M))$ where, even though the outcome (M, A) is feasible, it is not desirable for firms and the entrepreneurs will prefer relationship funding and no innovation. Broadly speaking, this scenario is not very different from the distribution in the relationship finance regime.

5.3 Productivity shocks

As anticipated, while our primary focus is on collateral shocks, we are also interested in studying the effects of a productivity shock, that is a reduction in y. As above, we assume that the shock is sufficiently small so we can evaluate its effects with the help of differential calculus. Proposition 5 summarizes the results. The proof is in the Appendix.

Proposition 5 *In both credit regimes, a negative productivity shock reduces the asset price, total investment, and the measure of firms that have access to credit. In the relationship finance*

[14] This occurs, for instance, if the output verifiability is very low so that for collateral-poor firms it is actually the mature technology that can offer more pledgeable expected returns to a lender.

regime, a negative productivity shock decreases the measure of credit relationships, while in the mixed finance regime this decrease occurs only if the elasticity of the asset price with respect to y is relatively small. In both regimes, the effect on output and on the measure of innovative firms is ambiguous, and depends on the elasticity of the asset price with respect to y. Finally, regardless of the credit regime, the drop in the measure of innovative firms is smaller than the drop in the measure of active firms as long as the elasticity of the asset price with respect to y is not too small.

Given what said about collateral shocks, the intuition behind Proposition 5 is straightforward. The direct negative effect of a drop in productivity especially hits the return $y(1+n)$ of the new technology and hence tends to depress innovation. However, just like for a collateral shock, the decrease in the asset price tends to promote the innovation of collateral-rich firms. If the latter effect is sufficiently large, that is, the elasticity of the asset price with respect to y is not too small, the drop in innovation will be smaller than the drop in investment.

6 Credit and Asset Market Policy

In 2008 and 2009, the Federal Reserve and the Treasury engaged in two kinds of unconventional policy. First, they intervened directly in asset markets to sustain declining asset prices (for instance, by purchasing mortgage-backed securities). Second, they directly granted loans to firms and non-bank financial institutions to finance asset holdings at margin requirements lower than those applied by financial institutions. The model can help understand the consequences of these two policies for firms' innovation, besides their consequences for total investment. We are going to see that a consequence of these policies could be that while, as intended, they support investment, they also tend to freeze the increase in firms' innovation triggered by a credit crunch.

Throughout this section, we work with the baseline scenario in which $\phi = 1$ and, for conciseness, we also restrict attention to the relationship finance regime (the qualitative insights carry through to the mixed finance regime). Following Krishnamurty (2010), we can think of the first policy (intervention in the asset market) as consisting of the government subsidizing asset purchases in the liquidation market on date 3. Precisely, we posit that the government makes a transfer τ of final good to each entrepreneur who purchases one unit of liquidated assets. We have also to specify how the government finances these subsidies for asset purchases. In our economy, the government can levy non-distortionary (lump-sum) taxes. For example, on date 3 it may tax the revenues of investors, regardless of whether these revenues originate from storage, from the repayment of loans or from asset liquidation; alternatively, it may tax the output of collateral-rich entrepreneurs without incurring the risk that such taxation distorts agents' decisions. The second policy consists of the government directly lending to firms in the credit market at margin requirements lower than private lenders. We model this policy assuming that the government grants credit to firms with $a \in [\underline{a}_G, \underline{a}(p, M, A)]$, where the policy tool is now the $\underline{a}_G$ threshold. Similar to the first policy, we posit that the government finances any loss due to this policy by levying lump-sum taxes. Proposition 6 summarizes the effects of the two policies; the proof is in the Appendix.

Proposition 6 *Policies of subsidies for asset purchases or of direct lending increase the measure of firms with access to credit but reduce the measure of innovative firms. Thus, the total effect of these policies on output is ambiguous. In particular, an optimal policy does not necessarily involve making credit accessible to all firms.*

By sustaining the asset demand, a subsidy raises the asset price. This increases the access of collateral-poor firms to credit but reduces the collateral threshold $\overline{a}(p^*(\tau), M)$ above which entrepreneurs are prevented from adopting the new technology by their relationship lenders. Similarly, direct lending increases the access of collateral-poor firms to credit. However, since in general equilibrium this increases the demand for assets and thus the asset price, this policy reduces the collateral threshold above which relationship lenders prevent entrepreneurs from adopting the new technology. To summarize, because the technological inertia induced by relationship lenders is stronger when the asset price is higher, the two policies stimulate total investment but tend to freeze the increase in the innovation of collateral-rich firms after the collateral shock.

7 Conclusion

We have investigated the role of the credit market in an economy where firms can innovate and switch to new technologies or retain existing less productive technologies. In our economy, credit relationships ease information flows between entrepreneurs and lenders and, hence, entrepreneurs' access to the credit market. However, relationship lenders inhibit the innovation of collateral-rich firms to preserve the value of their information on mature technologies. By depressing the price of collateral assets, a negative collateral shock squeezes collateral-poor firms out of the credit market but fosters the innovation of collateral-rich firms. Depending on the credit regime that prevails in equilibrium, the innovation of collateral-rich firms can occur with or without the breakdown of their credit relationships. In the last part of the analysis, we have also investigated the consequences of policies of direct government intervention in the credit and asset markets. We have found that the credit and asset market policies adopted during the recent credit crunch can promote investment but might also slow down innovation.

The analysis leaves important questions open for future research. While the model can help disentangle the mechanisms through which a credit crunch affects technological change, it cannot offer predictions on the magnitude of the effects. A priority for future research is thus to cast the analysis into a dynamic general equilibrium environment and study the quantitative relevance of the mechanisms investigated in this paper.

8 Appendix

Proof of Lemma 1 Given the repayment r_n, the minimum value of r_m that ensures participation of the lender under the expectation that the lender will implement the action that allows innovation satisfies

$$(1-\sigma)\pi r_n + \sigma[\pi r_m + (1-\pi)p\sigma\mu a] = i,$$

which can be rewritten as

$$r_m = r_m = \frac{i}{\sigma\pi} - \frac{1-\sigma}{\sigma} r_n - \frac{1-\pi}{\pi} p\sigma\mu a. \tag{15}$$

By plugging r_m into the left hand side of (2), we obtain

$$r_n > \frac{i}{\pi} + \frac{1-\pi}{\pi}\sigma\mu p a,$$

which can be rewritten as

$$a < \frac{\pi r_n - i}{(1-\pi)\sigma p\mu}. \tag{16}$$

Since the highest value of r_n consistent with the limited liability of the entrepreneur is $ly(1+n)$, the upper bound on the values of a that satisfy (16) is given by

$$\overline{a}(p,\mu) \equiv \frac{\pi ly(1+n) - i}{(1-\pi)\sigma p\mu}.$$

Finally, since the highest value of r_m consistent with the limited liability of the entrepreneur is ly, we need to make sure that $r_m \leq ly$. Imposing this condition on (15) (and replacing r_n with $ly(1+n)$), we obtain

$$\frac{i}{\sigma\pi} - \frac{1-\sigma}{\sigma} ly(1+n) - \frac{1-\pi}{\pi} p\sigma\mu a \leq ly,$$

which can be rewritten as

$$a \geq \frac{i - \pi ly\left(1+n-n\sigma\right)}{\left(1-\pi\right)\mu\sigma^2 p} \equiv \underline{a}(p,\mu,A).$$

Clearly, among the contracts that allow innovation, the contract above, i.e.,

$$(r_n^{\text{action}}, r_m^{\text{action}}) = \left(ly(1+n), \frac{i}{\sigma\pi} - \frac{1-\sigma}{\sigma} ly(1+n) - \frac{1-\pi}{\pi} p\sigma\mu a \right)$$

is optimal as it induces participation of the lender at the lowest possible cost.

Asset Price Firms' collateral a is uniformly distributed in the interval $[0,1]$ and a firm's ability to reuse assets is given by $\eta\theta$, where θ is uniformly distributed in the interval $[0,\overline{\theta}]$. This implies that the demand for assets is

$$D(p) = \max\left\{ \left[1 - \underline{a}(p,M,A)\right]\pi\left(1 - \frac{p}{\eta\overline{\theta}}\right), 0 \right\},$$

while the supply of assets is

$$S(p) = \max\left\{ (1-\pi) \int_{\underline{a}(p,M,A)}^{1} a da, 0 \right\}.$$

Assumptions (5) and (7) imply that $\underline{a}(p,M,A) < \overline{a}(p,M) < \overline{a}(p,m) < 1$. Thus,

$$D(p) = [1 - \underline{a}(p,M,A)]\,\pi\left(1 - \frac{p}{\eta\overline{\theta}}\right)$$

and

$$S(p) = (1-\pi)\int_{\underline{a}(p,M,A)}^{1} a da,$$

which can be rewritten as

$$S(p) = \frac{1}{2}(1-\pi)\left[1 - (\underline{a}(p,M,A))^2\right].$$

Equating demand and supply, we obtain

$$\frac{\pi}{\eta\overline{\theta}}p^2 + \frac{1-3\pi}{2}p + \frac{1}{2}(1-\pi)\frac{i - \pi ly(1+n-n\sigma)}{\sigma^2(1-\pi)M} = 0$$

from which

$$p = \frac{\eta\overline{\theta}}{2\pi}\left\{\frac{3\pi-1}{2} \pm \left[\frac{(3\pi-1)^2}{4} - \frac{2\pi}{\eta\overline{\theta}}\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2 M}\right]^{\frac{1}{2}}\right\}.$$

We impose the following conditions

$$p_- < \frac{i-\pi ly(1+n-n\sigma)}{\sigma^2(1-\pi)M} \Leftrightarrow \frac{i}{\pi ly} < 1 + \frac{n}{1+\sigma} - n\sigma^2\left[\frac{1}{1+\sigma} - \eta\overline{\theta}\frac{(2\pi-1)(1-\pi)M}{\pi^2 lyn}\right]$$

and

$$p_+ > \frac{i-\pi ly(1+n-n\sigma)}{\sigma^2(1-\pi)M} \Leftrightarrow \frac{i}{\pi ly} < 1 + \frac{n}{1+\sigma} - n\sigma^2\left[\frac{1}{1+\sigma} - \eta\overline{\theta}\frac{(2\pi-1)(1-\pi)M}{\pi^2 lyn}\right].$$

For these conditions to hold, we need $\pi > \frac{1}{2}$ and $\overline{\theta}$ large enough. In particular, (11) implies that it is sufficient to have

$$\frac{1}{1+\sigma} - \eta\overline{\theta}\frac{(2\pi-1)(1-\pi)M}{\pi^2 lyn} < \frac{(M-m)}{(M+\sigma m)(1+\sigma)},$$

which can be rewritten as

$$\eta\overline{\theta} > \frac{m}{M(M+\sigma m)}\frac{\pi}{2\pi-1}\frac{\pi lyn}{1-\pi}.$$

Note also that $p_+ < \eta\overline{\theta}$ always holds. Thus, there exists a unique non-trivial equilibrium price, namely

$$p = \frac{\eta\overline{\theta}}{2\pi}\left\{\frac{3\pi-1}{2} + \left[\frac{(3\pi-1)^2}{4} - \frac{2\pi}{\eta\overline{\theta}}\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2 M}\right]^{\frac{1}{2}}\right\}.$$

Finally, in order to make sure that (7) is satisfied, we simply need $\overline{\theta}$ to be sufficiently large. Precisely, if (6) holds, a sufficient condition for (7) to hold is

$$p > \frac{\pi lyn}{(1-\pi)m},$$

which can be rewritten as

$$\eta\overline{\theta} > \frac{2\pi}{2\pi - (1-\pi)\left[1 + \frac{i-\pi ly(1+n-n\sigma)}{\sigma^2 M}\frac{m}{\pi lyn}\right]}\frac{\pi lyn}{(1-\pi)m}.$$

Proof of Lemma 2 Consider firms with $a \in [\underline{a}(p, M, A), \overline{a}(p, M))$. These firms can choose between innovating and not innovating under relationship funding. Since relationship funding is cheaper, it is the optimal choice for these firms. Note though that (5) implies $\underline{a}(p, M, \overline{A}) < \overline{a}(p, M)$. As a result, since $\underline{a}(p, M, A) < \underline{a}(p, M, \overline{A})$, firms with $a \in \left[\underline{a}(p, M, A), \underline{a}(p, M, \overline{A})\right)$ have no option but to innovate in order to obtain credit. The choice is non-trivial only for firms with $a \in \left[\underline{a}(p, M, \overline{A}), \overline{a}(p, M)\right)$, since in this case credit is available irrespective of the choice of technology. An entrepreneur chooses a contract that allows innovation if and only if

$$(1-\sigma)\pi\left[(1+n)y - r_n^{\text{action}}\right] + \sigma\pi\left(y - r_m^{\text{action}}\right) > \pi\left(y - r_m^{\text{no}}\right),$$

which can be rewritten as

$$a < \frac{\pi n y}{(1-\pi)(1+\sigma) M p} \equiv a^*(p, M).$$

All entrepreneurs with collateral $a \in \left[\underline{a}(p, M, \overline{A}), \overline{a}(p, M)\right)$ choose to innovate iff $a^*(p, M) > \overline{a}(p, M)$. It is straightforward to show that a necessary and sufficient condition for $a^*(p, M) > \overline{a}(p, M)$ to hold under (5), (6) and (7) is that the output pledgeability l is not too large, precisely, $l(1+\sigma) < 1$. In fact, $a^*(p, M) > \overline{a}(p, M)$ holds if and only if $\frac{i}{\pi l y} > 1 + n - \frac{n\sigma}{l(1+\sigma)}$. Condition (6) implies that the lower bound on $\frac{i}{\pi l y}$ is given by $1 + n - n\sigma$. Thus, $a^*(p, M) > \overline{a}(p, M)$ is always true under (6) if and only if $l(1+\sigma) < 1$.

Proof of Lemma 3 An entrepreneur cannot innovate under relationship funding and can innovate under transactional funding if and only if $a \in [\overline{a}(p, M), \overline{a}(p, m))$. In this case, it is easy to see that the condition under which an entrepreneur chooses transactional funding is

$$\pi\left\{(1-\sigma)[y(1+n) - r_n^{\text{action}}] + \sigma(y - r_m^{\text{action}})\right\} > \pi(y - r_m^{\text{no action}}). \tag{17}$$

The repayments r_m and r_n that guarantee zero profits to a transactional lender when the innovation can occur satisfy

$$\pi\left[(1-\sigma)r_n^{\text{action}} + \sigma r_m^{\text{action}}\right] = i - (1-\pi)\sigma^2 p m a, \tag{18}$$

whereas the repayment $r_m^{\text{no action}}$ that guarantees zero profits to a relationship lender when the innovation cannot occur satisfies

$$\pi r_m^{\text{no action}} = i - (1-\pi) M p a. \tag{19}$$

Using (19) and (18) to substitute into (17), we obtain

$$a < \frac{\pi y n(1-\sigma)}{(1-\pi)(M - \sigma^2 m)p} \equiv \widehat{a}(p).$$

Proof of Lemma 4 A mixed finance regime occurs when $\widehat{a}(p) > \overline{a}(p, M)$, which can be rewritten as

$$\frac{i}{\pi l y} > 1 + \frac{n}{1+\sigma} - n\sigma(1-\sigma)\left[\frac{1}{l}\frac{M}{M - \sigma^2 m} - \frac{1}{1-\sigma^2}\right].$$

Note that

$$1 + \frac{n}{1+\sigma} - n\sigma(1-\sigma)\left[\frac{1}{l}\frac{M}{M - \sigma^2 m} - \frac{1}{1-\sigma^2}\right] < 1 + \frac{n}{1+\sigma} - n\sigma\frac{\sigma}{1+\sigma}$$

if and only if

$$l < \frac{M(1-\sigma)}{M-\sigma^2 m}.$$

Thus, whenever $l < \frac{M(1-\sigma)}{M-\sigma^2 m}$, for all parameters consistent with (6) and (11), entrepreneurs with $a \in (\overline{a}(p,M), \min\{\widehat{a}(p), \overline{a}(p,m)\})$ choose transactional funding and set a contract that induces the lender to choose the action that allows innovation. Assume, instead, that $l \geq \frac{M(1-\sigma)}{M-\sigma^2 m}$. One possibility then is that

$$l < \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}.$$

This implies that

$$1+\frac{n}{1+\sigma} - n\sigma(1-\sigma)\left[\frac{1}{l}\frac{M}{M-\sigma^2 m} - \frac{1}{1-\sigma^2}\right] < 1+\frac{n}{1+\sigma} - n\sigma\frac{(M-m)\sigma}{(M+\sigma m)(1+\sigma)},$$

in which case there are two regions of parameters consistent with (6) and (11). In the region

$$\frac{i}{\pi l y} \in \left(1+\frac{n}{1+\sigma} - n\sigma\frac{\sigma}{1+\sigma}, 1+\frac{n}{1+\sigma} - n\sigma(1-\sigma)\left[\frac{1}{l}\frac{M}{M-\sigma^2 m} - \frac{1}{1-\sigma^2}\right]\right],$$

there is no mixed finance regime. In turn, in the region

$$\frac{i}{\pi l y} \in \left(1+\frac{n}{1+\sigma} - n\sigma(1-\sigma)\left[\frac{1}{l}\frac{M}{M-\sigma^2 m} - \frac{1}{1-\sigma^2}\right], 1+\frac{n}{1+\sigma} - n\sigma\frac{(M-m)\sigma}{(M+\sigma m)(1+\sigma)}\right),$$

all entrepreneurs with $a \in (\overline{a}(p,M), \min\{\widehat{a}(p), \overline{a}(p,m)\})$ choose transactional funding and set a contract that induces the lender to choose the action. A last possibility is $l \geq \frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}$. Since we have assumed $l < \frac{1}{1+\sigma}$, there are values of l which satisfy both requirements if and only if

$$\frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m} < \frac{1}{1+\sigma},$$

which can be rewritten as

$$\frac{M-m}{m} > \frac{1-\sigma^2}{\sigma}.$$

In this case,

$$1+\frac{n}{1+\sigma} - n\sigma(1-\sigma)\left[\frac{1}{l}\frac{M}{M-\sigma^2 m} - \frac{1}{1-\sigma^2}\right] \geq 1+\frac{n}{1+\sigma} - n\sigma\frac{(M-m)\sigma}{(M+\sigma m)(1+\sigma)},$$

and, as long as $l \in \left(\frac{(1-\sigma)(M+\sigma m)}{M-\sigma^2 m}, \frac{1}{1+\sigma}\right)$, the mixed finance regime never occurs in the region of parameters consistent with (6) and (11).

Proof of Lemma 5 The proof is immediate. Assumption (5) implies $i < \pi y(1+n)$, which guarantees that even a firm with $a = 0$ should invest and potentially innovate. Further, observe that since the liquidation costs are a transfer, in the event of project failure the asset liquidation return of the new technology is the same as that of the mature technology. In addition, in the event of project success the output of the new technology exceeds that of the mature technology since $y(1+n) > y$.

Proof of Proposition 2 Regardless of the credit regime, the asset price is given by (10). Thus,

$$\frac{\partial p}{\partial \eta}=\frac{1}{\eta}\left\{p+\frac{1}{2}\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2 M}\left[\left(\frac{3\pi-1}{2}\right)^2-\frac{2\pi}{\eta\overline{\theta}}\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2 M}\right]^{-\frac{1}{2}}\right\}>0.$$

The change in investment is computed by multiplying i by the change in the measure of active firms C. Regardless of the credit regime, the measure of firms that obtain credit is

$$C=1-\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2\left(1-\pi\right)Mp}.$$

We obtain

$$\frac{\partial I}{\partial \eta}=i\frac{\partial C}{\partial \eta}=i\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2\left(1-\pi\right)Mp^2}\frac{\partial p}{\partial \eta}>0.$$

In the relationship finance regime, the measure of firms that participate in credit relationships (R_r) is equal to the measure of active firms, that is, $R_r=C$. Thus

$$\frac{\partial R_r}{\partial \eta}=\frac{\partial C}{\partial \eta}=\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2\left(1-\pi\right)Mp^2}\frac{\partial p}{\partial \eta}>0.$$

In the mixed finance regime, this measure is given by

$$R_m=C-\left[\min\left\{\widehat{a}(p),\overline{a}(p,m)\right\}-\overline{a}(p,M)\right].$$

We have

$$R_m=\left\{\begin{array}{c} C-\frac{\pi ly(1+n)-i}{\sigma(1-\pi)m}\frac{M-m}{Mm}\frac{1}{p}\text{ if }\overline{a}(p,m)\leq\widehat{a}(p)\\ C-\frac{1}{1-\pi}\left[\frac{(1-\sigma)\pi ny}{M-\sigma^2 m}-\frac{\pi ly(1+n)-i}{\sigma M}\right]\frac{1}{p}\text{ if }\overline{a}(p,m)>\widehat{a}(p)\end{array}\right..$$

Thus,

$$\frac{\partial R_m}{\partial \eta}=\left\{\begin{array}{c} \frac{\partial C}{\partial \eta}+\frac{\pi ly(1+n)-i}{\sigma(1-\pi)m}\frac{M-m}{Mm}\frac{1}{p^2}\frac{\partial p}{\partial \eta}\text{ if }\overline{a}(p,m)\leq\widehat{a}(p)\\ \frac{\partial C}{\partial \eta}+\frac{1}{1-\pi}\left[\frac{(1-\sigma)\pi ny}{M-\sigma^2 m}-\frac{\pi ly(1+n)-i}{\sigma M}\right]\frac{1}{p^2}\frac{\partial p}{\partial \eta}\text{ if }\overline{a}(p,m)>\widehat{a}(p)\end{array}\right..$$

Condition (5) and the fact that $\frac{(1-\sigma)\pi ny}{M-\sigma^2 m}>\frac{\pi ly(1+n)-i}{\sigma M}$ in the mixed finance regime imply that $\frac{\partial R_m}{\partial \eta}>0$.

In the relationship finance regime, the measure of firms that innovate is

$$N_r=\frac{\pi ly(1+n)-i}{\sigma(1-\pi)Mp}-\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2(1-\pi)Mp}=\frac{\pi ly\left(1+n+\sigma\right)-i\left(1+\sigma\right)}{\sigma^2(1-\pi)Mp}.$$

We obtain

$$\frac{\partial N_r}{\partial \eta}=-\frac{\pi ly\left(1+n+\sigma\right)-i\left(1+\sigma\right)}{\sigma^2(1-\pi)Mp^2}\frac{\partial p}{\partial \eta}.$$

Condition (5) implies that this derivative is negative. In turn, in the mixed finance regime, the measure of firms that innovate is

$$N_m=\min\left\{\widehat{a}(p),\overline{a}(p,m)\right\}-\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2(1-\pi)Mp}.$$

We have

$$N_m = \begin{cases} \left[\frac{\pi ly(1+n)-i}{\sigma(1-\pi)m} - \frac{i-\pi ly(1+n-n\sigma)}{\sigma^2(1-\pi)M}\right]\frac{1}{p} \text{ if } \overline{a}(p,m) \leq \widehat{a}(p) \\ \left[\frac{(1-\sigma)\pi ny}{(1-\pi)(M-\sigma^2 m)} - \frac{i-\pi ly(1+n-n\sigma)}{\sigma^2(1-\pi)M}\right]\frac{1}{p} \text{ if } \overline{a}(p,m) > \widehat{a}(p) \end{cases}$$

and, hence,

$$\frac{\partial N_m}{\partial \eta} = \begin{cases} -\left[\frac{\pi ly(1+n)-i}{\sigma(1-\pi)m} - \frac{i-\pi ly(1+n-n\sigma)}{\sigma^2(1-\pi)M}\right]\frac{1}{p^2}\frac{\partial p}{\partial \eta} \text{ if } \overline{a}(p,m) \leq \widehat{a}(p) \\ -\left[\frac{(1-\sigma)\pi ny}{(1-\pi)(M-\sigma^2 m)} - \frac{i-\pi ly(1+n-n\sigma)}{\sigma^2(1-\pi)M}\right]\frac{1}{p^2}\frac{\partial p}{\partial \eta} \text{ if if } \overline{a}(p,m) > \widehat{a}(p) \end{cases}.$$

Condition (5) implies that $\frac{\pi ly(1+n)-i}{\sigma(1-\pi)m} - \frac{i-\pi ly(1+n-n\sigma)}{\sigma^2(1-\pi)M} > 0$ and $\frac{(1-\sigma)\pi ny}{(1-\pi)(M-\sigma^2 m)} - \frac{i-\pi ly(1+n-n\sigma)}{\sigma^2(1-\pi)M} > 0$. Thus, it is always the case that $\frac{\partial N_m}{\partial \eta} < 0$.

Finally, output is given by

$$Y_i = \omega + \left(\pi y + \frac{1-\pi}{2}\frac{\left(\overline{\theta}\eta\right)^2 - (p)^2}{\overline{\theta}\eta} - i\right) C + N_i (1-\sigma)\pi ny,$$

where $i \in \{r, m\}$ identifies the relationship or mixed credit regime. We obtain,

$$\frac{\partial Y_i}{\partial \eta} = \frac{\partial C}{\partial \eta}(\pi y - i) + \frac{\partial N_i}{\partial \eta}(1-\sigma)\pi ny + \frac{1-\pi}{2}\frac{1}{\overline{\theta}\eta}\left\{\frac{\partial C}{\partial \eta}\left[\left(\overline{\theta}\eta\right)^2 - p^2\right] + C\left[\frac{p^2 + \left(\overline{\theta}\eta\right)^2}{\eta} - 2p\frac{\partial p}{\partial \eta}\right]\right\}.$$

After some algebraic manipulation, we obtain that $\frac{\partial Y_i}{\partial \eta} > 0$ iff

$$\left\{\pi y + (1-\pi) E\left[\eta\theta \middle| \theta \geq \frac{p}{\eta}\right] - i\right\}\frac{\partial C}{\partial \eta} + (1-\pi)\frac{C}{\eta}\left\{E\left[\eta\theta \middle| \theta \geq \frac{p}{\eta}\right] + \frac{p^2}{\overline{\theta}\eta}\left[1 - \frac{\partial p}{\partial \eta}\frac{\eta}{p}\right]\right\} > -\frac{\partial N_i}{\partial \eta}(1-\sigma)\pi ny.$$

Proof of Proposition 3 If $\overline{a}(p,m) \leq \widehat{a}(p)$, we have

$$R_m = \begin{cases} C - \frac{\pi ly(1+n)-i}{\sigma(1-\pi)m}\frac{M-m}{Mm}\frac{1}{p} \text{ if } \overline{a}(p,m) \leq \widehat{a}(p) \\ C - \frac{1}{1-\pi}\left[\frac{(1-\sigma)\pi ny}{M-\sigma^2 m} - \frac{\pi ly(1+n)-i}{\sigma M}\right]\frac{1}{p} \text{ if } \overline{a}(p,m) > \widehat{a}(p) \end{cases},$$

from which

$$\frac{\partial\left(\frac{R_m}{C}\right)}{\partial \eta} = \frac{C\frac{\partial R_m}{\partial \eta} - R_m\frac{\partial C}{\partial \eta}}{C^2} = \frac{C\left[\frac{\pi ly(1+n)-i}{\sigma(1-\pi)m} - \frac{\pi ly(1+n)-i}{\sigma(1-\pi)M}\right]\frac{1}{p^2}\frac{\partial p}{\partial \eta} + \left[\frac{\pi ly(1+n)-i}{\sigma(1-\pi)m} - \frac{\pi ly(1+n)-i}{\sigma(1-\pi)M}\right]\frac{1}{p}\frac{\partial C}{\partial \eta}}{C^2}.$$

This can be rewritten as

$$\frac{\partial\left(\frac{R_m}{C}\right)}{\partial \eta} = \frac{\left[\frac{\pi ly(1+n)-i}{\sigma(1-\pi)m} - \frac{\pi ly(1+n)-i}{\sigma(1-\pi)M}\right]\frac{1}{p^2}\frac{\partial p}{\partial \eta}}{C^2} = \frac{C - R_m}{C^2}\frac{1}{p}\frac{\partial p}{\partial \eta} > 0.$$

If $\overline{a}(p,m) > \widehat{a}(p)$, we have

$$\frac{\partial\left(\frac{R_m}{C}\right)}{\partial \eta} = \frac{C\frac{\partial R_m}{\partial \eta} - R_m\frac{\partial C}{\partial \eta}}{C^2} = \frac{C\left[\frac{(1-\sigma)\pi ny}{(1-\pi)(M-\sigma^2 m)} - \frac{\pi ly(1+n)-i}{\sigma(1-\pi)M}\right]\frac{1}{p^2}\frac{\partial p}{\partial \eta} + \left[\frac{(1-\sigma)\pi ny}{(1-\pi)(M-\sigma^2 m)} - \frac{\pi ly(1+n)-i}{\sigma(1-\pi)M}\right]\frac{1}{p}\frac{\partial C}{\partial \eta}}{C^2}.$$

This can be rewritten as

$$\frac{\partial\left(\frac{R_c^{\overline{a}>\widehat{a}}}{C}\right)}{\partial \eta} = \frac{\left[\frac{(1-\sigma)\pi ny}{(1-\pi)(M-\sigma^2 m)} - \frac{\pi ly(1+n)-i}{\sigma(1-\pi)M}\right]\frac{1}{p^2}\frac{\partial p}{\partial \eta}}{C^2} = \frac{C - R_m}{C^2}\frac{1}{p}\frac{\partial p}{\partial \eta} > 0.$$

Proof of Proposition 4 The demand for assets is the same as in the case with $\phi = 1$, i.e., (8). The supply of assets now depends on the degree of innovation, and is given by

$$S(p) = (1-\pi)\left\{ \int_{\underline{a}(p,M,A)}^{\overline{a}} \left[(1-\sigma)\phi + \sigma\right] ada + \int_{\overline{a}}^{1} ada \right\},$$

where $\overline{a} = \overline{a}(p, M)$ in the relationship finance regime and $\overline{a} = \min\{\widehat{a}(p), \overline{a}(p, m)\}$ in the mixed finance regime. The supply of assets can be rewritten as

$$S(p) = \max\left\{ \frac{1-\pi}{2} \left\{ \left[(1-\sigma)\phi + \sigma\right] \left[\overline{a}^2 - (\underline{a}(p, M, A))^2\right] + 1 - \overline{a}^2 \right\}, 0 \right\}$$

which implies

$$S(p) = \frac{1-\pi}{2}\sigma \left[1 - (\underline{a}(p, M, A))^2\right] + \frac{1-\pi}{2}(1-\sigma)\left\{\phi\left[\overline{a}^2 - (\underline{a}(p, M, A))^2\right] + 1 - \overline{a}^2\right\}.$$

Equating demand and supply, we obtain

$$[1 - \underline{a}(p, M, A)]\,\pi\left(1 - \frac{p}{\eta\overline{\theta}}\right) = \frac{1-\pi}{2}\left\{ \begin{array}{c} \sigma\left[1 - (\underline{a}(p, M, A))^2\right] + \\ \frac{1-\pi}{2}(1-\sigma)\left\{\phi\left[\overline{a}^2 - (\underline{a}(p, M, A))^2\right] + 1 - \overline{a}^2\right\} \end{array} \right\}.$$

First, note that, at any given price, $\sigma = 1$ implies a supply of assets equal to that when $\phi = 1$, while $\sigma = 0$ implies a supply of assets that is below the supply when $\phi = 1$. Thus, the equilibrium price when $\phi < 1$ must be larger than the equilibrium price when assets have the same degree of redeployability. Note also that $\phi < 1$ only affects the distribution of firms through a different price. In fact, $(\underline{a}(p, M, A))^2$ and $\overline{a}$ do not depend directly on ϕ. This implies that, in order to compare the distribution when $\phi < 1$ with the distribution when $\phi = 1$, we only need to examine how the asset price affects the distribution of firms. Since the asset price is higher under asset heterogeneity, there will be more active firms, more investment, more credit relationships and less innovation.

It remains to examine the impact of a negative collateral shock, i.e., a decrease in η. First, a decrease in η leads to a reduction of the demand for assets at any price level, but it does not affect the position of the supply curve. In other words, the movement on the supply of assets is a movement along the supply curve, driven by the reduction of the asset price. If assets are homogeneous, the only impact of a reduction of the asset price on the supply of assets is caused by a reduction in the number of firms that have access to credit. If, instead, assets are heterogeneous, the reduction of the asset price not only reduces the number of firms that have access to credit (and by the same amount that it does so under homogeneous assets, since the expression for $\underline{a}(p, M, A)$ only depends on η through the asset price), but it also reduces the supply of assets due to the increase in innovation (this is so because $\overline{a}$ is decreasing in p). As a result, a negative collateral shock has a smaller impact on the asset price when assets are heterogeneous. Finally, since the change in η only affects the distribution of firms through the change in the asset price, it must be that all the effects in Proposition 2 are dampened when assets are heterogeneous.

Proof of Proposition 5 Regardless of the credit regime, the asset price is given by (10), hence

$$\frac{\partial p}{\partial y}=\frac{1}{2y}\left[\left(\frac{3\pi-1}{2}\right)^2-\frac{2\pi}{\eta\overline{\theta}}\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2 M}\right]^{-\frac{1}{2}}\frac{\pi ly(1+n-n\sigma)}{\sigma^2 M}>0.$$

The change in investment is computed by multiplying i by the change in the measure of active firms C. Regardless of the credit regime, the measure of firms that obtain credit is

$$C=1-\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2\,(1-\pi)\,Mp}.$$

We obtain

$$\frac{\partial I}{\partial y}=i\frac{\partial C}{\partial y}=i\frac{\pi ly(1+n-n\sigma)+[i-\pi ly(1+n-n\sigma)]\,\frac{\partial p}{\partial y}\frac{y}{p}}{\sigma^2\,(1-\pi)\,Mpy}>0.$$

In the relationship finance regime, the measure of firms that participate in credit relationships (R_r) is equal to the measure of active firms, that is, $R_r=C$. Thus

$$\frac{\partial R_r}{\partial y}=\frac{\partial C}{\partial y}=\frac{\pi ly(1+n-n\sigma)+[i-\pi ly(1+n-n\sigma)]\,\frac{\partial p}{\partial y}\frac{y}{p}}{\sigma^2\,(1-\pi)\,Mpy}>0.$$

We now look at the effects of the shock on the measure of firms that innovate in the relationship finance regime. The measure of firms that innovate is

$$N_r=\overline{a}(p,M)-\underline{a}(p,M,A)=\frac{\pi ly(1+n)-i}{p\sigma(1-\pi)M}-\frac{i-\pi ly(1+n-n\sigma)}{p\sigma^2(1-\pi)M}=\frac{\pi ly\,(1+n+\sigma)-i\,(1+\sigma)}{\sigma^2(1-\pi)Mp}.$$

Note that, since p increases in y, an increase in y has a potentially ambiguous effect on N_r.We obtain that the measure of firms who innovate increases in y iff

$$\frac{y}{p}\frac{\partial p}{\partial y}<\frac{1+\frac{n}{1+\sigma}}{1+\frac{n}{1+\sigma}-\frac{i}{\pi ly}}. \tag{20}$$

Since

$$\frac{y}{p}\frac{\partial p}{\partial y}=\frac{\frac{\pi}{\eta\overline{\theta}}\frac{\pi ly(1+n-n\sigma)}{\sigma^2 M}}{\frac{3\pi-1}{2}\left[\left(\frac{3\pi-1}{2}\right)^2-\frac{2\pi}{\eta\overline{\theta}}\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2 M}\right]^{\frac{1}{2}}+\left[\left(\frac{3\pi-1}{2}\right)^2-\frac{2\pi}{\eta\overline{\theta}}\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2 M}\right]},$$

by making $\overline{\theta}$ large enough, we can ensure that (20) holds.

We now turn to the mixed finance regime, considering first the measure of firms that participate in credit relationships. This is given by

$$R_m=C-\left[\min\left\{\overline{a}(p,m),\widehat{a}(p)\right\}-\overline{a}(p,M)\right].$$

If $\min\left\{\overline{a}(p,m),\widehat{a}(p)\right\}=\overline{a}(p,m)$, we have

$$R_m=1-\left\{i-\pi ly(1+n-n\sigma)+\frac{(M-m)\,\sigma}{m}\left[\pi ly(1+n)-i\right]\right\}\frac{1}{\sigma^2(1-\pi)Mp}.$$

We obtain $\frac{\partial R_m}{\partial y} < 0$ iff

$$\frac{y}{p}\frac{\partial p}{\partial y} < \frac{\frac{(M-m)\sigma}{m}\pi l y(1+n) - \pi l y(1+n-n\sigma)}{i - \pi l y(1+n-n\sigma) + \frac{(M-m)\sigma}{m}\left[\pi l y(1+n) - i\right]}. \tag{21}$$

As in (20), by making $\overline{\theta}$ large enough we can ensure that (21) holds. If, instead, $\min\{\overline{a}(p,m), \widehat{a}(p)\} = \widehat{a}(p)$, we have

$$R_m = 1 - \left\{i - \pi l y(1+n-n\sigma) + \sigma\left\{\pi l y\left[n\frac{\sigma(1-\sigma)M}{l(M-\sigma^2 m)} - (1+n)\right] - i\right\}\right\}\frac{1}{\sigma^2(1-\pi)Mp}.$$

We obtain $\frac{\partial R_m}{\partial y} < 0$ iff

$$\frac{y}{p}\frac{\partial p}{\partial y} < \frac{\sigma\pi l y\left[n\frac{\sigma(1-\sigma)M}{l(M-\sigma^2 m)} - (1+n)\right] - \pi l y(1+n-n\sigma)}{i - \pi l y(1+n-n\sigma) + \sigma\left\{\pi l y\left[n\frac{\sigma(1-\sigma)M}{l(M-\sigma^2 m)} - (1+n)\right] - i\right\}}. \tag{22}$$

Once more, by making $\overline{\theta}$ large enough we can ensure that (22) holds. Consider now the measure of firms that innovate in the mixed finance regime. It is given by (if $\min\{\overline{a}(p,m), \widehat{a}(p)\} = \widehat{a}(p)$)

$$N_m = \widehat{a}(p) - \underline{a}(p,M,A) = \frac{(1-\sigma)\pi n y}{(1-\pi)(M-\sigma^2 m)p} - \frac{i - \pi l y(1+n-n\sigma)}{\sigma^2(1-\pi)Mp},$$

which can be rewritten as

$$N_m = \frac{1}{(1-\pi)p}\left[\frac{(1-\sigma)\pi n y}{M-\sigma^2 m} - \frac{i - \pi l y(1+n-n\sigma)}{\sigma^2 M}\right].$$

We obtain that $\frac{\partial N_m}{\partial y} > 0$ iff

$$\frac{y}{p}\frac{\partial p}{\partial y} < \frac{\frac{(1-\sigma)\pi n y}{M-\sigma^2 m} + \frac{\pi l y(1+n-n\sigma)}{\sigma^2 M}}{\frac{(1-\sigma)\pi y n}{M-\sigma^2 m} - \frac{i-\pi l y(1+n-n\sigma)}{\sigma^2 M}}. \tag{23}$$

Again, by making $\overline{\theta}$ large enough we can ensure that (23) holds. A similar reasoning applies in the case where $\min\{\overline{a}(p,m), \widehat{a}(p)\} = \overline{a}(p,m)$. Finally, we consider output. It is given by

$$Y_i = \omega + \left[\pi y - i + (1-\pi)\Pr(\theta\eta > p^*)E(\theta\eta \mid \theta\eta > p^*)\right]C + N_i(1-\sigma)\pi n y,$$

where $i \in \{r, m\}$ identifies the relationship or mixed credit regime. We obtain.

$$\frac{\partial Y_i}{\partial y} = \frac{\partial C}{\partial y}(\pi y - i) + \frac{\partial N_i}{\partial y}(1-\sigma)\pi n y + \pi\left[C + N_i(1-\sigma)n\right] + (1-\pi)\frac{\partial \Pr(\theta\eta > p^*)E(\theta\eta \mid \theta\eta > p^*)C}{\partial y}.$$

The overall effect of an increase in y is potentially ambiguous, due to the direct impact through asset prices. However, as shown above, as long as $\overline{\theta}$ is not too small, an increase in y leads to an increase in output. It does so because it increases the measure of firms with access to credit and the measure of firms that innovate.

It remains to prove that, irrespective of the credit regime, the ratio N/C decreases in y. Consider, first, the relationship finance regime. We have

$$\frac{\partial\left(\frac{N_r}{C}\right)}{\partial y} = \frac{\partial\left(\frac{\overline{a}(p,M) - \underline{a}(p,M,A)}{1 - \underline{a}(p,M,A)}\right)}{\partial y} < 0$$

iff

$$[1-\underline{a}(p,M,A)]\,\frac{\partial \overline{a}(p,M)}{y}<[1-\overline{a}(p,M)]\,\frac{\partial \underline{a}(p,M,A)}{y},$$

which, after some algebraic manipulation, can be rewritten as

$$\frac{\partial p}{\partial y}\frac{y}{p}>\frac{1+\frac{n}{1+\sigma}-\frac{n}{1+\sigma}\frac{i}{(1-\pi)Mp}}{1+\frac{n}{1+\sigma}-\frac{i}{\pi ly}}.$$

Consider now the mixed finance regime. We have

$$\frac{\partial\left(\frac{N_m}{C}\right)}{\partial y}=\frac{\partial\left(\frac{\min\{\overline{a}(p,m),\widehat{a}(p)\}-\underline{a}(p,M,A)}{1-\underline{a}(p,M,A)}\right)}{\partial y}<0$$

iff

$$[1-\underline{a}(p,M,A)]\,\frac{\partial \min\left\{\overline{a}(p,m),\widehat{a}(p)\right\}}{y}<[1-\min\left\{\overline{a}(p,m),\widehat{a}(p)\right\}]\,\frac{\partial \underline{a}(p,M,A)}{y}.$$

After some tedious algebraic manipulation, which parallels the one made in the case of the relationship finance regime, we obtain that this condition holds as long as $\frac{\partial p}{\partial y}\frac{y}{p}$ is not too small.

Proof of Proposition 6 We start with the first policy (subsidies for asset purchases). The demand for assets is

$$D(p)=[1-\underline{a}(p,M,A)]\,\pi\left(1-\frac{p-\tau}{\eta\overline{\theta}}\right),$$

while the supply of assets is

$$S(p)=\frac{1}{2}\,(1-\pi)\left[1-(\underline{a}(p,M,A))^2\right].$$

Equating demand and supply, we obtain

$$\frac{\pi}{\eta\overline{\theta}}p^2+\left(\frac{1-3\pi}{2}-\frac{\pi}{\eta\overline{\theta}}\tau\right)p+\frac{1}{2}\,(1-\pi)\,\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2\,(1-\pi)\,M}=0,$$

from which

$$p=\frac{-\left(\frac{1-3\pi}{2}-\frac{\pi}{\eta\overline{\theta}}\tau\right)\pm\left[\left(\frac{1-3\pi}{2}-\frac{\pi}{\eta\overline{\theta}}\tau\right)^2-\frac{1}{\eta\overline{\theta}}2\pi\,(1-\pi)\,\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2(1-\pi)M}\right]^{\frac{1}{2}}}{2\frac{\pi}{\eta\overline{\theta}}}.$$

We impose the following conditions

$$p_-<\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2(1-\pi)M}\Leftrightarrow\frac{i}{\pi ly}<1+n-n\sigma+\eta\overline{\theta}\frac{\left(2\pi-1+\frac{\pi}{\eta\overline{\theta}}\tau\right)(1-\pi)\sigma^2M}{\pi^2ly}$$

and

$$p_+>\frac{i-\pi ly(1+n-n\sigma)}{\sigma^2(1-\pi)M}\Leftrightarrow\frac{i}{\pi ly}<1+n-n\sigma+\eta\overline{\theta}\frac{\left(2\pi-1+\frac{\pi}{\eta\overline{\theta}}\tau\right)(1-\pi)\sigma^2M}{\pi^2ly}.$$

For these conditions to hold, it suffices that $\pi > \frac{1}{2}$ and $\overline{\theta}$ large enough. Note also that $p_+ < \eta\overline{\theta}$ always holds. Thus, there exists a unique non-trivial equilibrium price, namely

$$p^* = \frac{1}{\frac{2\pi}{\eta\overline{\theta}}} \left\{ \pi - \frac{1-\pi}{2} + \frac{\pi}{\eta\overline{\theta}}\tau + \left[\left(\pi - \frac{1-\pi}{2} + \frac{\pi}{\eta\overline{\theta}}\tau \right)^2 - \frac{2\pi}{\eta\overline{\theta}} \frac{i - \pi l y(1+n-n\sigma)}{\sigma^2 M} \right]^{\frac{1}{2}} \right\}.$$

Again, we assume that $\overline{a}(p^*, m) < 1$, which is always satisfied as long as $\pi > \frac{1}{2}$ and $\overline{\theta}$ is large enough. Clearly, an increase in τ leads to an increase in the asset price. It is thus immediate that

$$\frac{\partial C(\tau)}{\partial \tau} = \frac{i - \pi l y(1+n-n\sigma)}{\sigma^2 (1-\pi) M [p^*(\tau)]^2} \frac{\partial p^*(\tau)}{\partial \tau} > 0,$$

so an increase in subsidies lead to an increase in the measure of firms with access to credit; and

$$\frac{\partial N(\tau)}{\partial \tau} = -\frac{\pi l y (1+n+\sigma) - i(1+\sigma)}{\sigma^2 (1-\pi) M [p^*(\tau)]^2} \frac{\partial p^*(\tau)}{\partial \tau} < 0,$$

and an increase in subsidies lead to a decrease in the measure of firms that innovate. Finally, if a subsidy τ is granted, output is given by

$$Y(\tau) = \omega + C(\tau) \left[\pi y + \frac{1-\pi}{2} \frac{\left(\overline{\theta}\eta\right)^2 - (p^*(\tau) - \tau)^2}{\overline{\theta}\eta} - i \right] + N(\tau)(1-\sigma)\pi n y,$$

where

$$C(\tau) = 1 - \underline{a}(p^*(\tau), M, A)$$

is the measure of firms with access to credit and

$$N(\tau) = \overline{a}(p^*(\tau), M) - \underline{a}(p^*(\tau), M, A)$$

is the measure of firms that innovate. The change in output due to the subsidy is given by

$$\frac{\partial Y(\tau)}{\partial \tau} = \left\{ \begin{array}{c} \frac{\partial C(\tau)}{\partial \tau} \left[\pi y + \frac{1-\pi}{2} \frac{\left(\overline{\theta}\eta\right)^2 - (p^*(\tau)-\tau)^2}{\overline{\theta}\eta} - i \right] \\ + \frac{\partial N(\tau)}{\partial \tau} (1-\sigma) \pi n y \end{array} \right\} - (1-\pi) C(\tau) \left[\frac{p^*(\tau) - \tau}{\overline{\theta}\eta} \right] \left[\frac{\partial p^*(\tau)}{\partial \tau} - 1 \right],$$

where

$$\frac{\partial C(\tau)}{\partial \tau} = \frac{i - \pi l y(1+n-n\sigma)}{\sigma^2 (1-\pi) M [p^*(\tau)]^2} \frac{\partial p^*(\tau)}{\partial \tau} > 0,$$

and

$$\frac{\partial N(\tau)}{\partial \tau} = -\frac{\pi l y (1+n+\sigma) - i(1+\sigma)}{\sigma^2 (1-\pi) M [p^*(\tau)]^2} \frac{\partial p^*(\tau)}{\partial \tau} < 0.$$

An increase in τ may lead to a decrease in output as long as $-\frac{\partial N(\tau)}{\partial \tau}$ is sufficiently larger than $\frac{\partial C(\tau)}{\partial \tau}$. This occurs, for instance, if i is close enough to $\pi l y(1+n-n\sigma)$. In this case, $\frac{\partial p^*(\tau)}{\partial \tau} \approx 1$, $\frac{\partial C(\tau)}{\partial \tau} \approx 0$ and $\frac{\partial N(\tau)}{\partial \tau} \approx -\left[\frac{\eta\overline{\theta}}{\pi} \left(\frac{3\pi - 1}{2} + \frac{\pi}{\eta\overline{\theta}}\tau \right) \right]^{-2} \frac{\pi l y n}{(1-\pi)M} < 0$. This concludes the proof for the first policy.

Let us turn to the second policy (direct lending). The demand for assets is

$$D(p) = (1 - \underline{a}_G)\pi \left(1 - \frac{p}{\eta\overline{\theta}} \right),$$

while the supply of assets is

$$S(p) = \frac{1}{2}(1-\pi)\left(1-\underline{a}_G^2\right).$$

Equating demand and supply, we obtain

$$(1-\underline{a}_G)\,\pi\left(1-\frac{p}{\eta\overline{\theta}}\right) = \frac{1}{2}(1-\pi)\left(1-\underline{a}_G^2\right),$$

which can be rewritten as

$$p^*(\underline{a}_G) = \frac{\eta\overline{\theta}}{2\pi}\left[3\pi - 1 - (1-\pi)\,\underline{a}_G\right].$$

We assume that $\overline{a}(p^*(\underline{a}_G), m) < 1$, which is always satisfied as long as $\pi > \frac{1}{2}$ and $\overline{\theta}$ is large enough. Clearly, a reduction in $\underline{a}_G$ leads to an increase in the asset price. It is thus immediate that

$$\frac{\partial C(\tau)}{\partial \tau} = \frac{\partial\left[1-\underline{a}_G\right]}{\partial \tau} = \frac{\eta\overline{\theta}}{2\pi}(1-\pi) > 0,$$

and

$$\frac{\partial N(\tau)}{\partial \tau} = -\frac{\pi l y\,(1+n+\sigma) - i\,(1+\sigma)}{\sigma^2(1-\pi)M\left[p^*(\underline{a}_G)\right]^2}\frac{\partial p^*(\underline{a}_G)}{\partial \tau} < 0,$$

and an increase in direct lending leads to a decrease in the measure of firms that innovate. Finally, under direct lending, output is given by

$$Y(\underline{a}_G) = \omega + C(\underline{a}_G)\left[\pi y + \frac{1-\pi}{2}\frac{\left(\overline{\theta}\eta\right)^2 - \left(p^*(\underline{a}_G)\right)^2}{\overline{\theta}\eta} - i\right] + N(\underline{a}_G)\,(1-\sigma)\,\pi n y,$$

where $C(\underline{a}_G) = 1-\underline{a}_G$ is the measure of firms with access to credit and $N(\underline{a}_G) = \overline{a}(p^*(\underline{a}_G), M) - \underline{a}_G$ is the measure of firms that innovate. The change in output due to the direct lending policy is

$$\frac{\partial Y(\underline{a}_G)}{\partial \underline{a}_G} = \left\{\begin{array}{c} \frac{\partial C(\underline{a}_G)}{\partial \underline{a}_G}\left[\pi y + \frac{1-\pi}{2}\frac{\left(\overline{\theta}\eta\right)^2 - \left(p^*(\underline{a}_G)\right)^2}{\overline{\theta}\eta} - i\right] \\ +\frac{\partial N(\underline{a}_G)}{\partial \underline{a}_G}(1-\sigma)\,\pi n y \end{array}\right\} - (1-\pi)\,C(\underline{a}_G)\frac{p^*(\underline{a}_G)}{\overline{\theta}\eta}\frac{\partial p^*(\underline{a}_G)}{\partial \underline{a}_G}.$$

Note that

$$\frac{\partial Y(0)}{\partial \underline{a}_G} = -\left\{\begin{array}{c} \pi y + (1-\sigma)\,\pi n y \\ +\overline{\theta}\eta\frac{1-\pi}{2}\left[1-\left(\frac{3\pi-1}{2\pi}\right)^2\right] - i \end{array}\right\} + \frac{1-\pi}{3\pi-1}\left[\overline{a}(p^*(0), M)\,(1-\sigma)\,\pi n y + (1-\pi)\,\overline{\theta}\eta\left(\frac{3\pi-1}{2\pi}\right)^2\right]$$

is positive if n is large enough, precisely, if

$$n > \left\{(1-\sigma)\,\pi y\left[\frac{2\pi}{(3\pi-1)^2}\frac{\pi l y(1+n) - i}{\sigma M\overline{\theta}\eta} - 1\right]\right\}^{-1}\left\{\pi\left[y + \overline{\theta}\eta\left(\frac{1-\pi}{2\pi}\right)^3\right] - i\right\}.$$

In this case, the optimal policy does not involve offering credit to all firms. This concludes the proof for the second policy.

References

Aghion, P. and P. Bolton (1992) "An incomplete contracts approach to financial contracting," Review of Economic Studies 59, 3, 473-494.

Barlevy, G. (2003) "Credit market frictions and the allocation of resources over the business cycle," Journal of Monetary Economics 50, 8, 1795-1818.

Berger, A. N. and G. F. Udell (2002) "Small business credit availability and relationship lending: The importance of bank organisational structure," Economic Journal 102, 2, 32-53.

Berger, A. N. and G. Udell (1998) "The economics of small business finance: The roles of private equity and debt markets in the financial growth cycle," Journal of Banking and Finance 22, 6-8, 613-673.

Berger, A. and G. Udell (1995) "Relationship lending and lines of credit in small firm finance," Journal of Business 68, 3, 351-381.

Berlin, M. and A. Butler (2002) "Collateral and competition," Working paper 02-22, Federal Reserve Bank of Philadelphia.

Bernstein, M. A. (1987) The Great Depression. Delayed Recovery and Economic Change in America, 1929–1939. Princeton University, New Jersey.

Bhattacharya, S. and G. Chiesa (1995) "Proprietary information, financial intermediation, and research incentives," Journal of Financial Intermediation 4, 4, 328-357.

Caballero, R. and M. Hammour (2005) "The costs of recessions revisited: a reverse-liquidationist view," Review of Economic Studies 72, 2, 313-341.

Carpenter, R. and B. Petersen (2002) "Capital market imperfections, high-tech investment, and new equity financing," Economic Journal 102, 2, 54-72.

Dell' Ariccia, G. and P. Garibaldi (2004) "Gross credit flows," Review of Economic Studies 72, 3, 665-685.

Dell' Ariccia, G. and P. Garibaldi (2001) "Bank lending and interest rate changes in a dynamic matching model," International Monetary Fund, Working Paper 98/93.

Den Haan, W. J., G. Ramey and J. Watson (2003) "Liquidity flows and fragility of business enterprises," Journal of Monetary Economics 50, 6, 1215-1241.

Diamond, D. and R. Rajan (2001) "Liquidity risk, liquidity creation and financial fragility: a theory of banking," Journal of Political Economy 109, 2, 287-327.

Field, A. (2011) A Great Leap Forward: 1930s Depression and U.S. Economic Growth. Yale University Press.

Gertler, M. and P. Karadi (2011) "A model of unconventional monetary policy," Journal of Monetary Economics 58, 1, 17-34.

Gertler, M. and N. Kiyotaki (2011) "Financial intermediation and credit policy in business cycle analysis," Working paper, New York University.

Goodacre, A. and I. Tonks (1995) "Finance and technological change," in P. Stoneman, ed.: Handbook of the economics of innovation and technological change. Blackwell, Oxford.

Guiso, L. and R. Minetti (2010) "The structure of multiple credit relationships: Evidence from U.S. firms," Journal of Money, Credit and Banking 42, 6, 1037-1071.

Habib, M. A. and D. B. Johnsen (1999) "The financing and redeployment of specific assets," Journal of Finance 54, 2, 693-720.

Holmstrom, B., and J. Tirole (1997) "Financial intermediation, loanable funds and the real sector," Quarterly Journal of Economics 112, 3, 663-691.

Kiyotaki, N. and J. Moore (1997) "Credit cycles," Journal of Political Economy 105, 2, 211-48.

Krishnamurthy, A. (2010) "Amplification mechanisms in liquidity crises," American Economic Journal: Macroeconomics, 2, 3, 1-30.

Lorenzoni, G. (2008) "Inefficient credit booms," Review of Economic Studies, 75, 3, 809-833.

Manove, M. and A. J. Padilla (1999) "Banking (conservatively) with optimists," Rand Journal of Economics 30, 2, 324-350.

Manove, M., A. J. Padilla and M. Pagano (2001) "Collateral versus project screening: a model of lazy banks," Rand Journal of Economics, 32, 4, 726-744.

OECD (2009) Policy Responses to the Economic Crisis: Investing in Innovation for Long-Term Growth, Paris, France.

Rajan, R. (1992) "Insiders and outsiders: the choice between informed and arm's length debt," Journal of Finance 47, 4, 1367-1400.

Rajan, R. and L. Zingales (2001) "Financial systems, industrial structure, and growth," Oxford Review of Economic Policy 17, 4, 467-482.

Ramey, G. (2004) "Financial intermediation in a restructuring economy," Unpublished working paper, University of California San Diego.

The Economist (2011) "Another digital gold rush," May 12th.

The Economist (2012) "Hot times in Silicon Valley," June 25th.