Trading and Information Diffusion in Over-the-Counter Markets

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Abstract

We model trading and information diffusion in OTC markets, when dealers can engage in many bilateral transactions at the same time. We show that information diffusion is effective, but not efficient. While each bilateral price partially reveals all dealers' private information after a single round of trading, dealers could learn more even within the constraints imposed by our environment. This is not a result of dealers' market power, but arises from the interaction between decentralization and differences in dealers' valuation of the asset. We apply our framework to confront several explanations for the disruption of OTC markets with stylized facts from the empirical literature. We find more support for narratives emphasizing increased counterparty risk as opposed to increased informational frictions.

JEL Classifications: G14, D82, D85

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1 Introduction

A vast proportion of assets is traded in over-the-counter (OTC) markets. The disruption of several of these markets (e.g. credit derivatives, asset backed securities, and repo agreements) during the financial crisis of 2008, has highlighted the crucial role that OTC markets play in the financial system. The defining characteristic of OTC markets is that trade is decentralized. Dealers engage in bilateral trades with a subset of other dealers, resulting in different prices for each transaction.

In this paper we explore a novel approach to model OTC markets allowing dealers to employ a rich set of trading strategies. Our approach emphasizes that dealers can engage in many bilateral trades at the same time and the terms and information content of all the trades are interconnected. On the theoretical side, our main focus is to study the amount of information that is revealed through trading in OTC markets. We show that information diffuses through the network of trades very effectively. After a single round of trading, each bilateral price partially aggregates the private information of all the dealers in the market, even when they are not a counterparty in the respective transaction. Yet, typically, information diffusion is not efficient. Dealers could learn more, even within the constraints imposed by our environment. We also show that this systematic distortion in information diffusion is not an outcome of dealers' market power. Instead, it arises from the interaction between decentralization and differences in dealers' valuation of the asset. On the applied side, we emphasize that the richness of our model predictions provides new avenues to confront theory with data. As an illustration we re-examine several explanations behind the disruption of OTC markets in financial crisis in light of the stylized facts of the empirical literature. We find more support for narratives emphasizing increased counterparty risk as opposed to increased informational frictions.

In our main specification, there are n risk-neutral dealers organized in a dealer network. Intuitively, a link between i and j indicates that they are potential counterparties in a trade. There is a single risky asset in zero net supply. The final value of the asset is uncertain and interdependent across dealers with an arbitrary correlation coefficient between 0 and 1.[1] Each dealer observes a private signal about her value, and all dealers

[1] In OTC markets, agents may value the same asset differently depending, for instance, on how they

have the same quality of information. Since values are interdependent, inferring each others' signals is valuable. Values and signals are drawn from a known multivariate normal distribution. Dealers simultaneously choose their trading strategy, understanding her price effect given other dealers' strategies. For any private signal, each dealer's trading strategy is a generalized demand function which specifies the quantity of the asset she is willing to trade with each of her counterparties depending on the vector of prices in the transactions she engages in. For example, in a star network the central dealer trades with all the other $n-1$ dealers and her generalized demand function maps $n-1$ prices to $n-1$ quantities. Any of the other dealers trades only with center, and her demand functions maps the respective price to a quantity. Each dealer, in addition to trading with other dealers, also trades with price sensitive costumers. In equilibrium prices and quantities have to be consistent with the set of generalized demand functions and the market clearing conditions for each link. We refer to this structure as the *OTC game*. The OTC game is, essentially, a generalization of the Vives (2011) variant of Kyle (1989) to networks.[2] Our main results in the OTC game apply to *any* network.[3]

We show that equilibrium beliefs, on one hand, and prices and quantities, on the other hand, can be determined in two steps. First, we work-out the equilibrium beliefs in the OTC game. For this, we specify a simpler, auxiliary game in which dealers, connected in the same network and operating in the same informational environment as in the OTC game, do not trade. Instead, they make a best guess of their own value conditional on their signals and the guesses of the other dealers they are connected to. We label this structure the conditional-guessing game. We then establish an equivalence between the equilibrium beliefs in the OTC game and the equilibrium beliefs in the conditional-guessing game. The equilibrium in the conditional guessing game is the vector of guesses which is optimal when agents can learn from the equilibrium guesses of their neighbors. As each dealer's equilibrium guess depends on her neighbors' guesses, and through those, depends on her

use it as collateral, on which techonologies to repackage and resell cash-flows they have, or on what risk-management constraints they face. Moreover, differences in asset valuations vary across markets and states.

[2] A useful property of this variant is that were dealers trade on a centralized market, prices would be privately fully revealing. This provides a clear benchmark for our analysis.

[3] We use specific examples only to illustrate how the structure of the dealer network affects the trading outcome.

neighbors' neighbors' guesses, etc., each equilibrium guess must partially incorporate the private information of all the dealers in a connected network. Moreover, in the common value limit, a belief system where each dealer's guess is proportional to the average of all signals is an equilibrium in any network. This is because the average of all signals is a sufficient statistic for the common value. Thus, if a dealer chooses this guess, it is optimal for all of her neighbors to choose the same guess. When the correlation between values is imperfect, each agent increases the weight on her own signal, since her signal is the most informative for her value, by definition. Hence, her guess is closer to her own value but further from the average signal. Her guess is, thus, less informative for her neighbors who are interested in the average signal. This is a learning externality implying that a planner minimizing the sum of guessing errors would instruct each agent to put less weight on her own signals and more weight on the guesses of her contacts.

Second, once we know the equilibrium expectations, the price and quantities in any bilateral trade are determined as weighted sums of the expectations of the respective counterparties. In particular, the price is close to the weighted average of the two expectations of the counterparties, while the position of each dealer is proportional to the difference between her expectation and the price. Therefore, a dealer with many neighbors sells at a price higher than her belief to those with a better private signal and buys at a price lower than her belief from those with a lower private signal. This gives rise to dispersed prices and profitable intermediation for well connected dealers, as it is characteristic of real-world OTC markets.

Intuitively, the difference between the equilibrium expectations of the two counterparties defines the per-unit gains from trade. The price determines how the gain is shared among the counterparties. As dealers are risk-neutral, if they were not to account for the informational content of the strategies of their counterparties, they would take infinite positions for any positive gains from trade. However, the more they worry about adverse selection, the smaller the position they take. The strength of this effect depends both on their signals and on the relative position of the two counterparties in the network. For example, in a star network, the central dealer is less worried about adverse selection than the others because she has $n-1$ transactions to learn from. This asymmetry does not constrain trade, because the central dealer can make price concessions to compensate for the differences in desired quantities. Similarly, in contrast to standard arguments, in our model larger asymmetry in the quality of information across trading partners (larger adverse selection) tends not to decrease trading volume. As dealers implicitly negotiate both quantities and prices, when one desires to take a larger position, she can offer sufficient concessions in the price to induce her counterparty to trade.

An attractive feature of our model is that it gives a rich set of empirical predictions. Namely, for a given information structure and dealer network, our model generates the full list of demand curves, the joint distribution of bilateral prices and quantities, and measures of price dispersion, intermediation, trading volume etc. As a simple illustration of the range of potential applications, we confront narratives about potential mechanisms behind OTC market distress with the observed stylized facts. The stylized picture[4] is that in a financial crisis price dispersion tends to increase and liquidity (i.e. the inverse of price impact) and volume tend to (weakly) decrease. We show that this picture can be consistent with increased counterparty risk to the extent that it can be captured by removing links from the network. In contrast, in our model any shifts in the informational structure (increased uncertainty, increased adverse selection etc.) tend to move price dispersion, liquidity and volume in the same direction. This is because, as we noted above, volume and liquidity is larger when dealers care less about the information of others. However, this also implies that dealers learn less, resulting in more heterogenous posterior beliefs and larger price dispersion.

The fact that in our model the conceptually complex problem of finding the equilibrium price and quantity vectors is solved in a single shot game, is an abstraction. We prefer to think about the OTC game as a reduced form of the real-world determination of prices and quantities potentially involving complex exchanges of series of quotes across multiple potential partners. We justify this approach by constructing a quasi-rational, but more realistic dynamic protocol which leads to the same outcome. Under this protocol, in each period each dealer sends a message to each of her potential counterparties. In

[4] See Afonso and Lagos (2012), Agarwal, Chang and Yavas (2012), Friewald, Jankowitsch and Subrahmanyam (2012) and Gorton and Metrick (2012).

each subsequent period, each dealer updates her message using a pre-specified rule, given her signal and the set of messages she has received in the previous round. Messages can be interpreted, for instance, as quotes that dealers exchange with their counterparties. A rule, that is common knowledge among dealers, maps messages into prices and quantities, for each pair of connected dealers. Trade takes place when no dealer wants to significantly revise her message based on the information she receives. We show that the dynamic protocol leads to the same traded prices and quantities as in the one-shot OTC game, when dealers use as an updating rule the equilibrium strategy in the conditional-guessing game, and the rule that maps messages into prices is the same as the one that maps expectations into prices in the OTC game. Interestingly, even if the updating rule is not necessarily optimal each round it is used in, we show that when trade takes place, dealers could not have done better.

Related literature

Most models of OTC markets are based on search (e.g. Duffie, Garleanu and Pedersen (2005); Duffie, Gârleanu and Pedersen (2007), Lagos, Rocheteau and Weill (2008), Vayanos and Weill (2008), Lagos and Rocheteau (2009), Afonso and Lagos (2012), and Atkeson, Eisfeldt and Weill (2012)). The majority of these models do not analyze learning through trade. Important exceptions are Duffie, Malamud and Manso (2009) and Golosov, Lorenzoni and Tsyvinski (2009). Their main focus is the time-dimension of information diffusion either between differentially informed agents, or from homogeneously informed to uninformed agents. A key assumption in these models is that there exists a continuum of atomistic agents on the market. This assumption implies that as an agent infers her counterparties' information from the sequence of transaction prices, she does not have to consider the possibility that any of her counterparties traded with each other before. Thus, in these models agents can infer an independent piece of information from each bilateral transactions.[5] In contrast, in our model all the meetings take places between a

[5] An interesting example of a search model where repeated transactions play a role is Zhu (2012) who analyzes the price formation in a bilateral relationship where a seller can ask quotes from a set of buyers repeatedly. In contrast to our model, Zhu (2012) considers a pure private value set-up. Thus, the issue of information aggregation through trade, which is the focus of our analysis, cannot be addressed in his model.

finite set of strategic dealers, but are collapsed in one period. Our results are a direct consequence of the fact that each dealer understands that her counterparties have overlapping information as they themselves have common counterparties, or their counterparties have common counterparties, etc. Our argument is that this insight is potentially crucial for the information diffusion in OTC markets where typically a small number of sophisticated financial institutions are responsible for the bulk of the trading volume. Therefore, we consider that search models and our approach are complementary.

Decentralized trade that takes place in a network has been studied by Gale and Kariv (2007), and Gofman (2011) with complete information and by Condorelli and Galeotti (2012) with incomplete information. These papers are interested in whether the presence of intermediaries affects the efficient allocation of assets, when agents trade sequentially one unit of the asset. Intermediation arises in our model as well. However, we allow a more flexible structure as dealers can trade any quantity of the asset they wish, given the price. Moreover, neither of these papers addresses the issue of information aggregation through trade (Condorelli and Galeotti (2012) consider a pure private value set-up), which is the focus of our analysis.

Finally, we would like to mention contemporaneous work by Malamud and Rostek (2012) who also use a multi-unit double-auction setup to model a decentralized market. Malamud and Rostek (2012) study allocative efficiency and asset pricing with risk-averse dealers with homogeneous information; their framework allows for trading environments intermediate between centralized and decentralized. In contrast, we study how information about an asset diffuses through trading with differentially informed, but risk-neutral dealers.

The paper is organized as follows. The following section introduces the model set-up and the equilibrium concept. In Section 3, we describe the conditional-guessing game, and we show the existence of the equilibrium in the OTC game. We characterize the informational content of prices in Section 4.3. Section 5 provides dynamic foundations for our main specification. In section 4 we illustrate the properties of the OTC game with some simple examples and discusses potential applications.

2 A General Model of Trading in OTC Markets

2.1 The model set-up

We consider an economy with n risk-neutral dealers that trade bilaterally a divisible risky asset in zero net supply. All trades take place at the same time. Dealers, apart from trading with each other, also serve their price sensitive customer-base. Each dealer is uncertain about the value of the asset. This uncertainty is captured by θ_i, referred to as dealer i's value. We assume that θ_i is normally distributed with mean 0 and variance σ_θ^2. Moreover, we consider that values are interdependent across dealers. In particular, $\mathcal{V}(\theta_i, \theta_j) = \rho\sigma_\theta^2$ for any two agents i and j, where $\mathcal{V}(\cdot, \cdot)$ represents the variance-covariance operator, and $\rho \in [0, 1]$. Differences in dealers' values reflect, for instance, differences in usage of the asset as collateral, in technologies to repackage and resell cash-flows, in risk-management constraints.[6]

We assume that each dealer receives a private signal, $s_i = \theta_i + \varepsilon_i$, where $\varepsilon_i \sim IID\, N(0, \sigma_\varepsilon^2)$ and $\mathcal{V}(\theta_j, \varepsilon_i) = 0$.

Dealers are organized into a trading network, g where g_i denote the set of $i's$ links and $m_i \equiv |g_i|$ the number of $i's$ links. A link ij implies that i and j are potential trading partners. Intuitively, agent i and j know and sufficiently trust each other to trade in case they find mutually agreeable terms. Each dealer i seeks to maximize her final wealth.

$$\sum_{j \in g_i} q_i^j \left(\theta_i - p_{ij}\right)$$

where q_i^j is the quantity traded in a transaction with dealer j at a price p_{ij}. A network is characterized by an adjacency matrix, which is a $n \times n$ matrix

$$A = (a_{ij})_{ij \in \{1, \ldots, n\}}$$

where $a_{ij} = 1$ if i and j have a link and $a_{ij} = 0$ otherwise. While our main results hold for any network, throughout the paper, we illustrate the results using two types of networks

[6] As we show in the Appendix, our formalization of the information structure is equivalent with setting $\theta_i = \hat{\theta} + \eta_i$, where $\hat{\theta}$ is the common value element, while η_i is the private value element of $i's$ valuation.

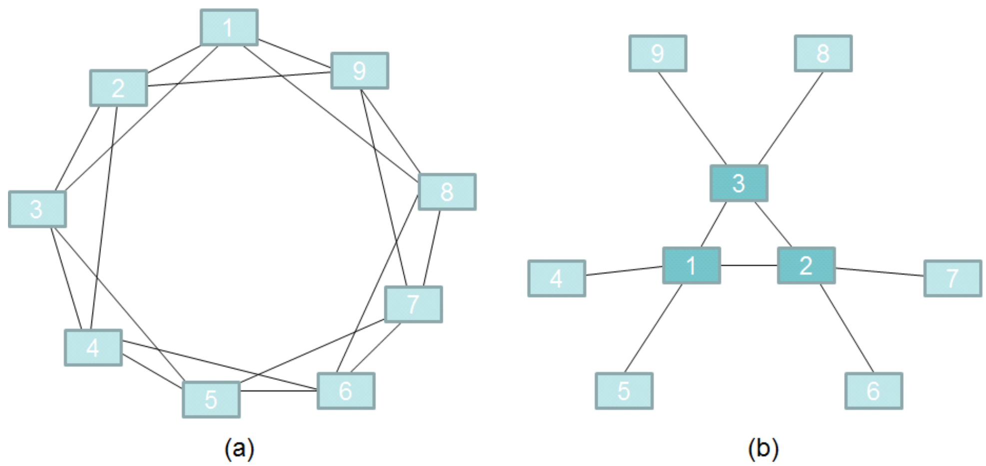

Figure 1: This figures shows two examples of networks. Panel (a) shows a $(9,4)$ circulant network. Panel (b) shows a $(9,3)$ core-periphery network.

as examples.

Example 1 *The first type of networks is the family of* ***circulant networks.*** *In an (n,m) circulant network each dealer is connected with $m/2$ other dealers on her left and $m/2$ on her right. Note that the $(n,2)$ circulant network is the circle and the $(n,n-1)$ circulant network is the complete network. (A $(9,4)$ circulant network is shown panel (a) of Figure 1.)*

Example 2 *The second type of networks is the family of* ***core-periphery networks****. In an (n,r) core-periphery network there are r fully connected agents (the core) each of them with links to $\frac{n-r}{r}$ dealers (the periphery) and no other links exist. Note that the $(n,1)$ core-periphery network is an n-star network where one dealer is connected with $n-1$ other dealers. (A $(9,3)$ core periphery network is shown in panel (b) of Figure 1.)*

These two types of simple networks allow us to isolate the effect of different features of OTC markets in trade and information diffusion. In a circulant network, we isolate the effects of network density and of distance between dealers in a symmetric setting. In contrast, in a star network, we capture information asymmetries that arise among dealers due to their position in the network.

We define a one shot game where each dealer chooses an optimal trading strategy, provided she takes as given others' strategies but she understands that her trade has a price effect. In particular, the strategy of a dealer i is a map from the signal space to the space of generalized demand functions. For each dealer i with signal s_i, a generalized demand function is a continuous function $Q_i : R^{m_i} \rightarrow R^{m_i}$ which maps the vector of prices[7], $p_{g_i} = (p_{ij})_{j \in g_i}$, that prevail in the transactions that dealer i participates in network g into vector of quantities she wishes to trade with each of her counterparties. The j-th element of this correspondence, $Q_i^j(s_i; p_{g_i})$, represents her demand function when her counterparty is dealer j, such that

$$\mathbf{Q}_i(s_i; \mathbf{p}_{g_i}) = \left(Q_i^j(s_i; \mathbf{p}_{g_i}) \right)_{j \in g_i}.$$

Note that our specification of generalized demand functions allows for a rich set of strategies. First, a dealer can buy a given quantity at a given price from one counterparty and sell a different quantity at a different price to another at the same time. Second, the fact that the quantity that dealer i trades with dealer j, $q_i^j = Q_i^j(s_i; p_{g_i})$, depends on all the prices p_{g_i}, captures the potential interdependence across all the bilateral transactions of dealer i. For example, if k is linked to i who is linked to j, a high demand from dealer k might raise the bilateral price p_{ki}. This might make dealer i to revise her estimation of her value upwards and adjust her supplied quantity both to k and to j accordingly. Third, the fact that $Q_i^j(s_i; \mathbf{p}_{g_i})$ depends only on $\mathbf{p}_{g_i}$ but not on the full price vector emphasizes the critical feature of OTC markets that the price and the quantity traded in a bilateral transaction are known only by the two counterparties involved in the trade and are not revealed to all market participants. Fourth, the fact that dealers choose demand schedules implies that dealers effectively bargain both over prices and quantities. This has a crucial role in our results. It is also in contrast with most other models of OTC markets where the traded quantity is fixed and agents bargain only over the price.

Apart from trading with each other, dealers also serve a price-sensitive customer base. In particular, we assume that for each transaction between i and j the customer base generates a downward sloping demand

$$D_{ij}(p_{ij}) = \beta_{ij} p_{ij}, \tag{1}$$

[7] A vector is always considered to be a column vector, unless explicitly stated otherwise.

with an arbitrary constant $\beta_{ij} < 0$. In our analysis costumers play a pure technical role: the exogenous demand (1) ensures the existence of the equilibrium.[8]

The expected payoff for dealer i with signal s_i corresponding to the strategy profile $\{\mathbf{Q}_i(s_i; \mathbf{p}_{g_i})\}_{i\in\{1,\dots,n\}}$ is

$$E\left[\sum_{j\in g_i} Q_i^j(s_i;\mathbf{p}_{g_i})(\theta_i - p_{ij})\,|s_i\right]$$

where p_{ij} are the elements of the bilateral clearing price vector $\mathbf{p}$ defined by the smallest element of the set

$$\widetilde{\mathbf{P}}\left(\{\mathbf{Q}_i(s_i;\mathbf{p}_{g_i})\}_i, \mathbf{s}\right) \equiv \left\{\mathbf{p}\,\middle|\, Q_i^j(s_i;\mathbf{p}_{g_i}) + Q_i^j(s_j;\mathbf{p}_{g_j}) + \beta_{ij}p_{ij} = 0,\, \forall\, ij \in g\right\}$$

by lexicographical ordering[9], if $\widetilde{\mathbf{P}}$ is non-empty. If $\widetilde{\mathbf{P}}$ is empty, we pick $\mathbf{p}$ to be the infinity vector and say that the market brakes down and define all dealers' payoff to be zero. We refer to the collection of these rules defining a unique $\mathbf{p}$ for any given signal and strategy profile as $\mathbf{p} = \mathbf{P}\left(\{\mathbf{Q}_i(s_i;\mathbf{p}_{g_i})\}_i, \mathbf{s}\right)$.

2.2 Equilibrium Concept

The environment described above represents a Bayesian game, henceforth the OTC game. The risk-neutrality of dealers and the normal information structure allows us to search for a linear equilibrium of this game defined as follows.

Definition 1 *A Linear Bayesian Nash equilibrium of the OTC game is a vector of linear generalized demand functions* $\{\mathbf{Q}_1(s_1;\mathbf{p}_{g_1}), \mathbf{Q}_2(s_2;\mathbf{p}_{g_2}), \dots, \mathbf{Q}_n(s_n;\mathbf{p}_{g_n})\}$ *such that* $\mathbf{Q}_i(s_i;\mathbf{p}_{g_i})$ *solves the problem*

$$\max_{(Q_i^j)_{j\in g_i}} E\left\{\left[\sum_{j\in g_i} Q_i^j(s_i;\mathbf{p}_{g_i})(\theta_i - p_{ij})\right] |s_i\right\} \tag{2}$$

where $\mathbf{p} = \mathbf{P}(\cdot, \mathbf{s})$.

[8] It has been known since Kyle (1989) that with only two agents there is no linear equlibrium in a demand submission game. This is no different under our formulation. We follow Vives (2011) and introduce an exogenous demand curve to overcome this problem. This assumption has a minimal effect on our analysis. As we show in Corollary 1, prices and beliefs do not depend on β_{ij} and quantities scale linearly even when $\beta_{ij} \to 0$.

[9] The specific algorithm we choose to sellect a unique price vector is immaterial. We just have to ensure that our game is well defined even for strategies which are allowed, but never part of the equilibrium.

A dealer i chooses a demand function for each transaction ij, in order to maximize her expected profits, given her information, s_i, and given the demand functions chosen by the other dealers. Then, an equilibrium of the OTC game is a fixed point in demand functions.

3 The Equilibrium

In this section, we derive the equilibrium in the OTC game. We proceed in steps. First, we derive the equilibrium strategies as a function of posterior beliefs. This step is standard. Second, we solve for posterior beliefs. For this, we introduce an auxiliary game in which dealers, connected in the same network and operating in the same informational environment as in the OTC game, do not trade. Instead, they make a best guess of their own value conditional on their signals and the guesses of the other dealers they are connected to. We label this structure the *conditional-guessing game.* Third, we establish equivalence between the posterior beliefs in the OTC game and those in the conditional-guessing game and provide sufficient conditions for existence of the equilibrium in the OTC game for any network. We discuss the main properties of the equilibrium in Section 4.

3.1 Derivation of demand functions

Our derivation follows Kyle (1989) and Vives (2011) with the necessary adjustments. We conjecture an equilibrium in demand functions, where the demand function of dealer i in the transaction with dealer j is given by

$$Q_i^j(s_i; \mathbf{p}_{g_i}) = b_i^j s_i + \left(\mathbf{c_i^j}\right)^T \mathbf{p}_{g_i} \tag{3}$$

for any i and j, where $(\mathbf{c_i^j}) = (c_{ik}^j)_{k \in g_i}$.

As it is standard in similar models, we simplify the optimization problem of (2) which is defined over a function space, to finding the functions $Q_i^j(s_i; \mathbf{p}_{g_i})$ point-by-point. That is, for each realization of the signals, $\mathbf{s}$, we solve for the optimal quantity q_i^j that each dealer i demands when trading with a counterparty j. The idea is as follows. Given the conjecture (3) and market clearing

$$Q_i^j(s_i;\mathbf{p}_{g_i}) + Q_j^i(s_j;\mathbf{p}_{g_j}) + \beta_{ij}p_{ij} = 0, \tag{4}$$

the residual inverse demand function of dealer i in a transaction with dealer j is

$$p_{ij} = -\frac{b_j^i s_j + \sum_{k\in g_j, k\neq i} c_{jk}^i p_{jk} + q_i^j}{c_{ji}^i + \beta_{ij}}. \tag{5}$$

Denote

$$I_i^j \equiv -\left(b_j^i s_j + \sum_{k\in g_j, k\neq i} c_{jk}^i p_{jk}\right) / \left(c_{ji}^i + \beta_{ij}\right) \tag{6}$$

and rewrite (5) as

$$p_{ij} = I_i^j - \frac{1}{c_{ji}^i + \beta_{ij}} q_i^j. \tag{7}$$

The uncertainty that dealer i faces about the signals of others is reflected in the random intercept of the residual inverse demand, I_i^j, while her capacity to affect the price is reflected in the slope $-1/\left(c_{ji}^i + \beta_{ij}\right)$. Thus, the price p_{ij} is informationally equivalent to the intercept I_i^j. This implies that finding the vector of quantities $\mathbf{q}_i = \mathbf{Q}_i(s_i;\mathbf{p}_{g_i})$ for one particular realization of the signals, $\mathbf{s}$, is equivalent to solving

$$\max_{(q_i^j)_{j\in g_i}} E\left[\sum_{j\in g_i} q_i^j \left(\theta_i + \frac{1}{c_{ji}^i + \beta_{ij}} q_i^j - I_i^j\right) |s_i, \mathbf{p}_{g_i}\right]$$

or

$$\max_{(q_i^j)_{j\in g_i}} \sum_{j\in g_i} q_i^j \left(E\left(\theta_i | s_i, \mathbf{p}_{g_i}\right) + \frac{1}{c_{ji}^i + \beta_{ij}} q_i^j - I_i^j\right).$$

From the first order conditions we derive the quantities q_i^j for each link of i and for each realization of $\mathbf{s}$ as

$$2\frac{1}{c_{ji}^i + \beta_{ij}} q_i^j = I_i^j - E\left(\theta_i | s_i, \mathbf{p}_{g_i}\right).$$

Then, using the definition of I_i^j from above, we can find the optimal demand function

$$Q_i^j(s_i;\mathbf{p}_{g_i}) = -\left(c_{ji}^i + \beta_{ij}\right)\left(E(\theta_i\, | s_i, \mathbf{p}_{g_i}) - p_{ij}\right) \tag{8}$$

for each dealer i when trading with dealer j.

At this point we depart from the standard derivation. The standard approach is to determine the coefficients in the demand function (3) as a fixed point of (8), given that $E(\theta_i \,| s_i, \mathbf{p}_{g_i})$ can be expressed as a function of coefficients b_i^j and $\mathbf{c_i^j}$. This procedure is virtually intractable for general networks. Instead, our approach is to solve directly for the beliefs in the OTC game. In fact, we can find the equilibrium beliefs in the OTC game without considering the profit motives and the corresponding trading strategies of agents. For this, in the next section we specify an auxiliary game labeled the *conditional-guessing game.*

3.2 The conditional-guessing game

The conditional guessing game is the non-competitive counterpart of the OTC game. The main difference is that instead of choosing quantities and prices to maximize trading profits, each agent aims to guess her value as precisely as she can. Importantly, agents are not constrained to choose a scalar as their guess. In fact, each dealer is allowed to choose a conditional-guess function which maps the guess of each of her neighbors into her guess.

Formally, we define the game as follows. Consider a set of n agents that are connected in the same network g as in the corresponding OTC game. The information structure is also the same as in the OTC game. Before the uncertainty is resolved, each agent i makes a guess, e_i, about her value of the asset, θ_i. Her guess is the outcome of a function that has as arguments the guesses of other dealers she is connected to in the network g. In particular, given her signal, dealer i chooses a guess function, $\mathcal{E}_i$, which maps the vector of guesses of her neighbors, $\mathbf{e}_{g_i}$, into a guess e_i. When the uncertainty is resolved, agent i receives a payoff

$$-\left(\theta_i - e_i\right)^2.$$

Definition 2 *An equilibrium of this game is given by a strategy profile $(\mathcal{E}_1, \mathcal{E}_2, ..., \mathcal{E}_n)$ such that each agent i chooses strategy $\mathcal{E}_i : R \times R^{m_i} \to R$ in order to maximize her expected payoff*

$$\max_{\mathcal{E}_i} \left\{ -E\left(\left(\theta_i - e_i\right)^2 |s_i \right) \right\},$$

where e_i is the guess that prevails when

$$e_i = \mathcal{E}_i\left(\mathbf{e}_{g_i}\right) \tag{9}$$

for all $i \in \{1, 2, ..., n\}$.

We assume that if a fixed point in (9) does not exist, then dealers don't make any guesses and their profits are zero. Essentially, the set of conditions (9) is the counterpart in the conditional-guessing game of the market clearing condition in the OTC game.

As in the OTC game, we simplify this optimization problem and find the guess functions $\mathcal{E}_i\left(s_i; \mathbf{e}_{g_i}\right)$ point-by-point. The procedure is as follows. An agent i chooses a guess function that maximize her expected profits, given her information, s_i, and given the guess functions chosen by the other agents evaluated at the fixed point determined by the set of conditions (9). Moreover, her guess function must be optimal for each realization of the other dealers' signals s_j, as it is reflected in the vector of guesses $\mathbf{e}_{g_i}$. Therefore, her optimal guess is then given by

$$e_i = E\left(\theta_i | s_i, \mathbf{e}_{g_i}\right). \tag{10}$$

In the next proposition, we state that the guessing game has an equilibrium in any network.

Proposition 1 *In the conditional-guessing game, for any network g, there exists an equilibrium in linear guess functions, such that*

$$\mathcal{E}_i\left(s_i, \mathbf{e}_{g_i}\right) = \bar{y}_i s_i + \bar{\mathbf{z}}_{g_i} \mathbf{e}_{g_i}$$

for any i, where $\bar{y}_i$ is a scalar and $\bar{\mathbf{z}}_{g_i} = \left(\bar{z}_{ij}\right)_{j \in g_i}$ is a row vector of length m_i.

It is easy to see that a collection of expectations e_i satisfying (10) for all i, must be a linear combination of the signals

$$e_i = \mathbf{v}_i \mathbf{s},$$

where $\mathbf{v}_i$ is a row vector of length n. Therefore, we can think of the conditional guessing game as as a standard fixed point problem in the space of $n \times n$ matrices. For example, let us pick an arbitrary row vector $\mathbf{v}_i^0$ of size n, for each agent i and conjecture that the guess of agent j is

$$e_j^0 = \mathbf{v}_j^0 \mathbf{s}. \tag{11}$$

Given that this conjecture describes the guesses of each of the neighbors of dealer i, her best conditional guess for θ_i minimizing $E\left((\theta_i - e_i)^2 \,| s_i\right)$ is

$$e_i^1 = E\left(\theta_i | s_i, \mathbf{e}_{g_i}^0\right). \tag{12}$$

Since each element of $e_{g_i}^0$ is a linear function of the signals and the conditional expectation is a linear operator for jointly normally distributed variables, $E\left(\theta_i | s_i, \mathbf{e}_{g_i}^0\right)$ determines a unique vector $\mathbf{v}_i^1$ which can satisfy

$$e_i^1 = \mathbf{v}_i^1 \mathbf{s}. \tag{13}$$

This holds for every agent, and the the $n \times n$ matrix $V^0 = \left[\mathbf{v}_i^0\right]_{i=1,..n}$ is mapped to a new matrix of the same size $V^1 = \left[\mathbf{v}_i^1\right]_{i=1,..n}$. We show that this mapping has a fixed point. An equilibrium of the conditional guessing game is given by the coefficients of s_i and e_{g_i} in $E\left(\theta_i | s_i, \mathbf{e}_{g_i}\right)$ at this fixed point.

In the next section we establish an equivalence between the equilibria of the OTC game and the conditional guessing game. Therefore, in Section 4, we will be able to use the properties of the conditional guessing game to learn about beliefs in the OTC game

3.3 Equivalence and existence

In this part we prove the main results of this section. First, we show that if there exists a linear equilibrium in the OTC game then the posterior expectations form an equilibrium expectation vector in the corresponding conditional guessing game. Second, we provide sufficient conditions under which we can construct an equilibrium of the OTC game building on an equilibrium of the conditional guessing game.

Proposition 2 *In any Linear Bayesian Nash equilibrium of the OTC game the vector with elements e_i defined as*

$$e_i = E(\theta_i \,| s_i, \mathbf{p}_{g_i})$$

is an equilibrium expectation vector in the conditional guessing game.

The idea behind this proposition is as follows. We have already showed that in a linear equilibrium demand functions are given by (8). Substituting this into the bilateral market clearing conditions give

$$p_{ij} = \frac{\left(c_{ji}^{i} + \beta_{ij}\right) E(\theta_i \,|s_i, \mathbf{p}_{g_i}) + \left(c_{ij}^{j} + \beta_{ij}\right) E(\theta_j \,|s_i, \mathbf{p}_{g_j})}{c_{ji}^{i} + c_{ij}^{j} + 3\beta_{ij}},$$

implying that a price p_{ij} is a linear combination of the posteriors of i and j, $E(\theta_i \,|s_i, \mathbf{p}_{g_i})$ and $E(\theta_j \,|s_j, \mathbf{p}_{g_j})$. Therefore, a dealer can infer the belief of her counterparty from the price, given that she knows her own belief. When choosing her generalized demand function, she essentially conditions her expectation about the asset value on the expectations of the other dealers she is trading with. Consequently, the set of posteriors that the OTC game implies works also as an equilibrium in the conditional guessing game.

Proposition 3 *Let $\bar{y}_i$ and $\bar{\mathbf{z}}_{g_i} = (\bar{z}_{ij})$ the coefficients that support an equilibrium in the conditional-guessing game and let $e_i = E(\theta_i \,|s_i, \mathbf{e}_{g_i})$ the corresponding equilibrium expectation of agent i. Then, there exists a Linear Bayesian Nash equilibrium in the OTC game, whenever $\rho < 1$ and the following system*

$$\begin{aligned} \frac{y_i}{\left(1 - \sum\limits_{k \in g_i} z_{ik} \frac{2 - z_{ki}}{4 - z_{ik} z_{ki}}\right)} &= \bar{y}_i \\ z_{ij} \frac{\frac{2 - z_{ij}}{4 - z_{ij} z_{ji}}}{\left(1 - \sum\limits_{k \in g_i} z_{ik} \frac{2 - z_{ki}}{4 - z_{ik} z_{ki}}\right)} &= \bar{z}_{ij}, \forall j \in g_i \end{aligned} \tag{14}$$

has a solution $\{y_i, z_{ij}\}_{i=1,..n, j \in g_i}$ such that $z_{ij} \in (0,2)$. The equilibrium demand functions are given by (3) with

$$\begin{aligned} b_i^j &= -\beta_{ij} \tfrac{2 - z_{ji}}{z_{ij} + z_{ji} - z_{ij} z_{ji}} y_i \\ c_{ij}^j &= -\beta_{ij} \tfrac{2 - z_{ji}}{z_{ij} + z_{ji} - z_{ij} z_{ji}} \left(z_{ij} - 1\right) \\ c_{ik}^j &= -\beta_{ij} \tfrac{2 - z_{ji}}{z_{ij} + z_{ji} - z_{ij} z_{ji}} z_{ik}. \end{aligned} \tag{15}$$

and the equilibrium prices and quantities are

$$p_{ij} = \frac{\left(c^i_{ji} + \beta_{ij}\right) e_i + \left(c^j_{ij} + \beta_{ij}\right) e_j}{c^j_{ij} + c^i_{ji} + 3\beta_{ij}} \tag{16}$$

$$q^j_i = -\left(c^i_{ji} + \beta_{ij}\right)\left(e_i - p_{ij}\right). \tag{17}$$

Note that these two propositions prove the equivalence between our two games in both directions. Proposition 2 shows that one can construct an equilibrium of the conditional guessing game from an equilibrium of the OTC game. Proposition 3 shows that, under some conditions, the reverse also holds. The extra conditions are a consequence of the fact that in the reverse direction we are transforming n expectations, e_i, from the conditional guessing game into $M \geq n$ prices in the OTC game. The conditions make sure that we can do it in a consistent way. While we do not have a general proof that this condition holds for any network, we have no reason to suspect that it does not hold.[10]

The next proposition strengthen the existence result for our specific examples.

Proposition 4 *1. In any network in the circulant family, the equilibrium of the OTC game exists.*

2. Whenever $\bar{z}_{ij} \in [0, 1]$, then for any network in the core-periphery family, the equilibrium of the OTC game exists.

The conceptual advantage of our way of constructing the equilibrium over the standard approach is that it is based on a much simpler and, as we will see in the next section, much more intuitive fixed point problem. Note also that Proposition 3 also describes a simple numerical algorithm to find the equilibrium of the OTC game for any network.[11] In particular, the conditional guessing game gives parameters $\bar{y}_i$ and $\bar{z}_{ij}$, conditions (14) imply parameters y_i and z_{ij}, then (15) give parameters of the demand function implying prices and quantities by (16)-(17).

[10] Our numerical algorithm gives a well behaving solution in all our experiments including a wide range of randomly generated networks. In Appenedix C, we also give analytical expressions for the equilibrium objects in some specific networks.

[11] The Matlab code runs in a fraction of a second for any network we experimented with and it is available from the authors.

Before proceeding to the detailed analysis of the features of the equilibrium in the next section, we make two simple observations.

First, the equivalence of beliefs on the two games implies that any feature of the beliefs in the OTC game must be unrelated in any way to price manipulation, imperfect competition or other profit related motives. It is so, because these considerations are not present in the conditional guessing game.

Second, consumers' demand has a pure technical role in our analysis. While there is no equilibrium for $\beta_{ij} = 0$, for any $\beta_{ij} < 0$ prices and beliefs are the same and $\frac{q_i^j}{\beta_{ij}}, \frac{q_j^i}{\beta_{ij}}$ remain constant. This is evident by simple observation of expressions (16)- (15). We summarize this in the following Corollary.

Corollary 1 *For any collection of non-zero* $\{\beta_{ij}\}_{ij \in g}$ *including the limit where all* $\beta_{ij} \to 0$, *prices,* p_{ij} *and beliefs do not change and quantities scale linearly. That is*

$$\frac{q_i^j}{\beta_{ij}}, \frac{q_j^i}{\beta_{ij}}$$

do not change with β_{ij}.

4 Characterization and discussion

In this section, we analyze the properties of the equilibrium. First, we start with a review of the centralized market benchmark, the Vives (2011) model. Then we proceed to the discussion of how the decentralized trading environment affects prices, volume and intermediation. In the last part, we focus on information transmission through trading.

4.1 A benchmark: the centralized market

When trade takes place in a centralized market, our environment collapses to the risk-neutral case in Vives (2011). The main difference in his model compared to ours is that in a centralized market agents submit simple demand functions to a market maker and the market clears at a single price. For completeness, we summarize the properties of the equilibrium in centralized markets in the following proposition. The derivation follows the standard approach that we described in Section 3.1 and the interested reader can find it in Vives (2011).

Proposition 5 (Vives, 2011) *Let be $\rho < 1$. In a centralized market there is a linear demand function equilibrium if and only if*

$$n - 2 < \frac{n\rho\sigma_\varepsilon^2}{(1-\rho)\left(\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_\theta^2\right)}. \tag{18}$$

The demand functions and the price are given by

$$\begin{aligned} Q_i(s_i;p) &= -\left(c+(n-1)\beta\right)\left(E\left(\theta_i|s_i,p\right)-p\right) = bs_i + cp && (19)\\ p &= \frac{\left(nc+n\left(n-1\right)\beta\right)}{nc+n\left(n-1\right)\beta+\beta}\frac{1}{n}\sum_{i=1}^{n}E\left(\theta_i|s_i,p\right) && (20) \end{aligned}$$

where β is the slope of the exogenous demand curve and

$$\begin{aligned} c &= \beta\frac{(1-\rho)\left(\sigma_\varepsilon^2+(1+(n-1)\rho)\sigma_\theta^2\right)-\rho\sigma_\varepsilon^2}{(n-2)(1-\rho)\left(\sigma_\varepsilon^2+(1+(n-1)\rho)\sigma_\theta^2\right)-n\rho\sigma_\varepsilon^2} && (21)\\ b &= \frac{(1-\rho)\sigma_\theta^2}{(1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2}\left(-\beta-c\left(n-1\right)\right). \end{aligned}$$

The price is fully privately revealing in the sense of

$$E\left(\theta_i|s_i,p\right) = E\left(\theta_i|\mathbf{s}\right).$$

The first thing to note is that in centralized markets the price is always fully privately revealing. As we will discuss in length in part 4.3, this is typically not the case in a decentralized market

However, this benchmark also highlights a few important insights which are partially or fully inherited by our structure. Just as in our case, there are gains from trade whenever $\rho < 1$. As the price is close to the average valuation of agents, the first expression in (19) implies that agents with higher than average signals (optimists) tend to buy and agents with lower than average signals (pessimists) tend to sell the asset. In fact, given that agents are risk-neutral, two critical features prevent optimists and pessimists from taking infinite positions. The first feature is that dealers can learn from prices and the second one is that they take into account that their trade has a price effect. To see this, note that comparing the two expressions for agents demand in (19) gives

$$c = ((n-1)\,c + \beta)\,(1-z) \tag{22}$$

where z is the coefficient of price in the conditional expectation

$$E(\theta_i | s_i, p) = y s_i + z p.$$

That is, the slope, c, of the demand curve of a given dealer is the product of the inverse of her price effect (given that all the other agent have a slope of c) and an inverse measure of the informational content of prices, $(1-z)$. If dealers were price takers or they did not care about the information content of prices, there would be no equilibrium with finite quantities

From (21), it is clear that larger ρ, σ_ε^2 or smaller σ_θ^2 increases c (i.e., typically decreases its absolute value). This is an instance of the well known result since Akerlof (1970) that adverse selection limits trade. All these changes of parameters imply that a given agent finds the signals of others more informative for the estimation of her own valuation.[12] For example, suppose that a dealer with high initial beliefs considers to buy. If many other dealers desire to sell at the same time, pushing prices downward, the first dealer typically increases her demand as a response. However, this adjustment will be weaker, if she is worried that the large supply indicates a low value for the asset. The stronger the information content of other agents' signals, the smaller the quantity response, i.e., the absolute value of c. It turns out that when adverse selection is not strong enough there is no solution in (22). This is the intuitive content of condition (18). As ρ and σ_ε^2 decrease or σ_θ^2 increases, the informational content in others' signals decreases and as condition

[12] In fact, there are two opposite forces at work when σ_ε^2 decreases. It makes the dealer's own signal more precise, decreasing the relevant information content in prices, but at the same time it makes all other dealers' signals more precise, increasing the relevant information content in prices. However, the first effect always dominates, because the effect of the others' signals largely cancels out when they are averaged.

(18) gets binding, adverse selection limits trade less and less and $(-c)$ increases without bounds.

In the next part, we discuss to what extent the mechanism of price formation and trading volume changes in a decentralized market.

4.2 Price dispersion, volume and intermediation

Just as in the centralized market benchmark, it is the interdependent value environment with $\rho < 1$ that ensures that agents trade in our case. Also, the interaction of adverse selection and imperfect competition is still the force which prevents agents from taking infinite positions. However, there are also important differences implied by our structure.

As an illustration, we consider a numerical example with the simplest possible network depicted on Figure 2. There are three dealers organized in a $(2, 1)$ core-periphery network (also known as a line and a 3-star) depicted by the connected squares. For simplicity, we index them by their position: $L(eft)$, $C(entral)$, $R(ight)$. The number in each square is the realization of their signal; $s_L = -2$, $s_C = 0$, $s_R = 1$. We picked $\beta_{L,C} = \beta_{R,C} = \beta = -5$. Prices are in the rhombi located on the links and demand curves are at the bottom of the figure in the form

$$q_i^j = t_i \left(E(\theta_i \left| s_i, \mathbf{p}_{g_i} \right.) - p_{ij} \right) \tag{23}$$

where $t_i \equiv -\left(c_{ji}^i + \beta_{ij} \right)$ is the trading intensity of trader i (i.e., $t_L = t_R = 10.8$ and $t_C = 10.5$). This corresponds to the interpretation that agent i trades t_i units for every unit of perceived gain, $E(\theta_i \left| s_i, \mathbf{p}_{g_i} \right.) - p_{ij}$. Substituting in the prices and signals gives the traded quantities in the first line of the rectangles. Below the quantities in brackets, we calculated the profit or loss realized on that particular trade in the case when the realized value, θ_i, is zero for all dealers. All quantities are rounded to the nearest decimal. For example, the central dealer forms the posterior expectation of -1, buys 3 units from the left and sells 2.4 units to the right at prices -0.4 and 0.1 respectively, earning 2.9 unit of profit in total in the trades. Note, that while counterparties hold a position of opposite sign and same order of magnitude, bought and sold quantities over a given link does not add up to 0. It is so, as the net of the two positions is sold to the customers.

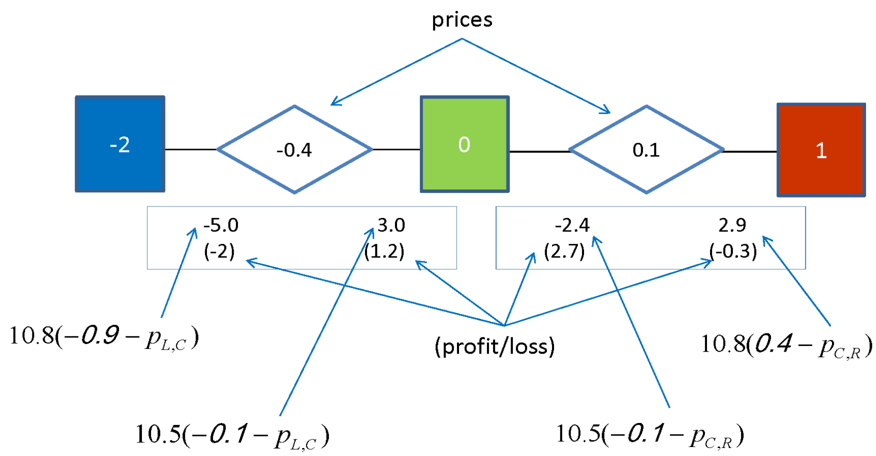

Figure 2: The connected squares depict three dealers organized in a $(2, 1)$-core-periphery network. Their realized signals are in the middle of the square. Prices are in rhombi, demand curves are at the bottom of the figure, with the posterior expectations in italic. The traded quantities are in the first line of the rectangles. Below, the profit or loss realized on that particular trade in the case when $\theta_i = 0$ for all i.Parameters are $\rho = -.5, \sigma_\theta^2 = \sigma_\varepsilon^2 = 1, \beta_{ij} = -5$.

There are a number of observations which generalize to other examples. First, price dispersion arises naturally in this model. The central dealer is trading the same asset at two different prices, because she is facing two different demand curves. Just as a monopolist does in a standard price-discrimination setting, the central agent sets a higher price in the market where demand is higher. In fact, from (16), we can foresee that the price dispersion in our framework must be closely related to the dispersion of posterior beliefs.

Second, profitable intermediation by central agents also arises naturally. That is, the central dealer's net position $(3 - 2.4)$ is significantly lower than her gross position $(3 + 2.4)$ as she trades not only to take a speculative bet but also to intermediate between her counterparties.

Third, the decentralized structure introduces a natural asymmetry in trading. That is, even if all dealers have the same quality of information, trading intensities are different. To see this, consider the decentralized version of equation (22) coming from (8) describing the best response slope $c_{L,C}^{C}$ of dealer L in transaction with C to the same slope of dealer

$C, c^L_{C,L}$

$$c^C_{L,C} = \left(c^L_{C,L} + \beta\right)(1 - z_L).$$

where z_L is the coefficient of the price between C and $L, p_{L,C}$ in the expectation $E(\theta_L|s_L, p_{L,C})$. Using our definition of (23), we rewrite this expression[13] and the corresponding one on $c^L_{C,L}$ as

$$\begin{aligned} -t_C &= t_L (z_L - 1) + \beta \\ -t_L &= t_C (z_C - 1) + \beta \end{aligned}$$

or

$$\begin{aligned} t_C &= -\beta \frac{2 - z_L}{z_C + z_L - z_L z_C} \\ t_L &= -\beta \frac{2 - z_C}{z_C + z_L - z_L z_C}. \end{aligned}$$

Clearly, the reason behind the trading intensity of the central agent being smaller than that of the other agents must be that the central agent relies on each price as a source of learning less than the others, $z_L = z_R > z_C$. Indeed, this is the case, as the central agent can learn from two prices. Therefore, she is less subject to adverse selection. Thus, an extra unit sold by the left agent triggers a smaller price-adjustment by the central agent inducing the left agent to trade more aggressively. This explains $t_L > t_C$.

Fourth, we can understand the joint determination of quantities and prices at each link as the outcome of a fictional bargaining process. To see this, using (16) and (23) we rewrite the price between L and C as a weighted average of the posterior expectations of L, C and 0 (the bliss point of customers)

$$p_{L,C} = \frac{t_L e_L + t_C e_C + (-\beta)\, 0}{t_L + t_C + (-\beta)}.$$

The expression for the price between R and C is analogous. This expression shows that the more aggressive an agent trades, the closer the price is to her expectation, decreasing

[13] In $E(\theta_C|s_C, p_{L,C}, p_{R,C})$ the coefficients of the two prices are equal. Thus, with a slight abuse of notation we denote both z_C in this simple example.

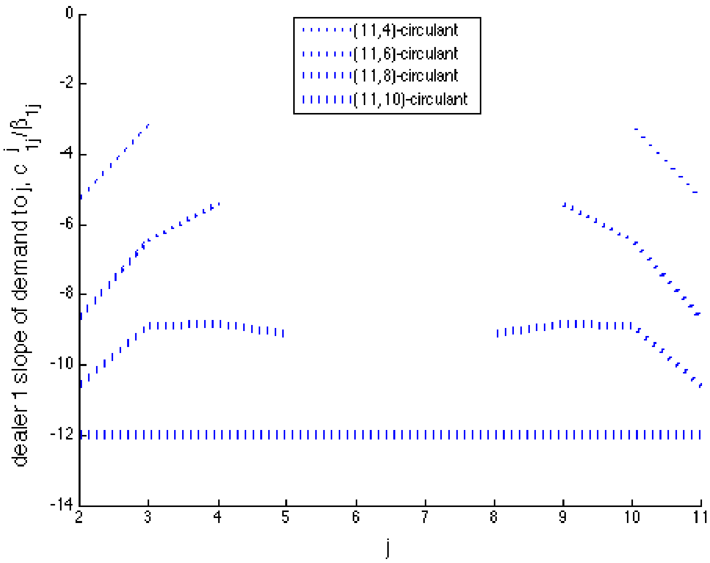

Figure 3: The figure shows the slope, $\frac{c^j_{1j}}{\beta_{1j}}$ of the demand curve, Q^j_1 submitted by agent 1 for the trade where the counterparty is j in the $(11, m)$ circulant networks. Other parameters are $\sigma^2_\varepsilon = \sigma^2_\theta = 1$, $\rho = 0.5$.

her perceived per-unit profit $|e_i - p_{ij}|$. Intuitively, in our example, the pessimistic agent prefers to sell assets to the center agent with larger trading intensity than the trading intensity the central agent is willing to buy with. As markets has to clear, the only way they can agree on the terms, if the pessimistic agent, L, gives a price concession to agent C.

Note also, that in this simple example while the slope of the demand curves differ across dealers, the slope of the demand function of the central dealer is the same when trading with each of her counterparties. This does not has to be the case. Even in symmetric networks, such as the family of circulants, dealer i's demand functions has a different slope depending whom she is trading with. We illustrate this on Figure 3.[14] This suggests that the price effect of an additional unit traded over the counter depends on the particular pair of dealers that are transacting.

Finally, note that in the decentralized case there can be an equilibrium even if ρ or σ^2_ε

[14]In figure 3, for simplification, we normalize the slope with the size of outside demand and plot c^j_{ij}/β_{ij}, as this ratio is independent of β_{ij}.

is very small. That is, in the decentralized case there is no analogous condition to (18). This is a consequence of trade being bilateral.

In Appendix B we provide more examples involving other networks to illustrate the effects of these observations.

4.3 Prices and information transmission

In this part we focus on the characteristics of equilibrium beliefs in OTC games and its implications on the informational efficiency of prices.

We start with two results on the equilibrium of the conditional guessing game.

Lemma 1 *In the conditional guessing game the following properties hold.*

1. *In any connected network g each dealer's equilibrium guess is a linear combination of all signals*
$$e_i = \mathbf{v}_i \mathbf{s},$$
where $\mathbf{v}_i$ is a row vector of length n and $\mathbf{v}_i > 0$.

2. *In any connected network g when $\rho = 1$, there exists an equilibrium where each element of the vectors $\mathbf{v}_i$ is equal to $\frac{\sigma_\theta^2}{n\sigma_\theta^2+\sigma_\varepsilon^2}$. In this equilibrium each expectation efficiently aggregates all the private information in the economy.*

As we will argue, in the OTC game the following corresponding claims also hold.

Proposition 6 *Suppose that there exists an equilibrium in the OTC game. Then,*

1. *in any connected network g each bilateral price is a linear combination of all signals in the economy, with a positive weight on each signal;*

2. *in any connected network g prices are privately fully revealing when $\rho \to 1$, as*
$$\lim_{\rho \to 1} \left(\mathcal{V}\left(\theta_i | s_i, \mathbf{p}_{g_i}\right) - \mathcal{V}\left(\theta_i | \mathbf{s}\right) \right) = 0.$$

These results suggest that a decentralized trading structure can be surprisingly effective in transmitting information. Consider first result 1 of the Proposition 6. This shows that although we consider only a single round of transactions, each price partially incorporates all the private signals in the economy. A simple way to see this is to consider the residual demand curve and its intercept, I_i^j, defined in (6)-(7). This intercept is stochastic and informationally equivalent with the price p_{ij}. The chain structure embedded in the definition of I_i^j is critical. The price p_{ij} gives information on I_i^j which gives some information on the prices agent j trade at in equilibrium. For example, if agent j trades with agent k then p_{jk} affects p_{ij}. By the same logic, p_{jk} in turn is affected by the prices agent k trades at with her counterparties, etc. Therefore, p_{ij} aggregates the private information of signals of every agent, dealer i is indirectly connected to, even if this connection is through several intermediaries.

This property of the equilibrium does not imply that dealers learn all the relevant information in the economy, as it happens in a centralized market. In particular, it follows from Proposition 6 that in a network g, a dealer i can use only m_i linear combinations of the vector of signals, $\mathbf{s}$, to infer (a sufficient statistics of) the other $(n-1)$ signals. Except in two special cases, this is generally not sufficient for the dealer to learn all the relevant information in the economy. One trivial special case is when each agent has $m_i = n-1$ neighbors, that is, when the network is complete. The second special case is highlighted as result 2 in the Proposition 6. It claims that in the common value limit, the decentralized structure does not impose any friction on the information transmission process in any network. To shed more light on the intuition behind these results, we have to understand better the learning process in the conditional guessing game.

Consider the case when $\rho = 1$. The expressions (11)-(13) can be seen as an iterated algorithm to find the equilibrium of the conditional guessing game in an arbitrary network. That is, in round 0 each agent i receives an initial vector of messages $e_{g_i}^0$ from her neighbors. Given that, each of agent i chooses her best, e_i^1, guess as in (12). The vector of messages $e_{g_i}^1$ given by (13) is the starting point for i in the following round. By definition, if this algorithm converges to a fixed point, then this is an equilibrium of the conditional guessing game. According to result 2 in Lemma 1, when $\rho = 1$, the equal-weighted sum of signals,

$\frac{\sigma_\theta^2}{n\sigma_\theta^2+\sigma_\varepsilon^2}1^\top s$, is a fixed point. The reason is simple. With common values, $\frac{\sigma_\theta^2}{n\sigma_\theta^2+\sigma_\varepsilon^2}1^\top s$ is the best possible guess for each agent given the information in the system. In addition, as the sum of signals is a sufficient statistic, the expectation operator (12) keeps this guess unchanged. Since the equilibrium of the conditional guessing game is continuos in ρ, information is aggregated efficiently also in the OTC game in the common value limit. Clearly though, exactly at $\rho = 1$ there is no equilibrium by the Grossman paradox.

Now we depart from the common value limit case. In this case, information transmission is only partial. In particular, if agent k is located further from agent i, her signal is incorporated to a smaller extent into agent i's belief. To see the intuition, we apply the iterated algorithm defined by (11)-(13) for the example of a circle-network of 11 dealers. We illustrate the steps of the iteration in Figure 4.3 from the point of view of dealer 6. We plot the weights with which signals are incorporated in the guess of dealer 5, 6 and 7. In each figure the dashed lines show messages sent by dealer 5 and 7 in a given round, and the solid line shows the guess of agent 6 given the messages she receives. In round 0, we start the algorithm from the common value limit, $\frac{\sigma_\theta^2}{n\sigma_\theta^2+\sigma_\varepsilon^2}1^T s$, illustrated by the straight dashed lines that overlap in panel A. When $\rho < 1$, in contrast to the common value limit, $\frac{\sigma_\theta^2}{n\sigma_\theta^2+\sigma_\varepsilon^2}1^T s$ is not a best guess of θ_6 anymore. The reason is that dealer 6's own signal, s_6, is more correlated to her value, θ_6, than the rest of the signals are. Therefore, the best guess of dealer 6 is a weighted sum of the two equal-weighted messages and her own signal. This is shown by the solid line peaking at s_6 in Panel A. Clearly, this is not a fixed point as all other agents choose their guesses in the same way. Thus, in round 2, agent 6 receives messages that are represented by the dashed lines shown on Panel B; these are the mirror images of the round-1 guess of dealer 6. Note that these new messages are less informative for dealer 6 than the equal-weighted messages $\frac{\sigma_\theta^2}{n\sigma_\theta^2+\sigma_\varepsilon^2}1^T s$. The reason is that even when $\rho < 1$, for dealer 6 the average of the other 10 dealers' signals is a sufficient statistic for her about all the information which is in the system apart from her own signal, which she observes anyway. So from the round-0 messages, she could learn everything she wanted to learn. From the round-1 messages she cannot. The extra weight that dealer 5 and 7 place on their own private signals jams the information content of the messages for dealer 6. Nevertheless, the round-2 messages are informative, and dealer 6 puts some positive weight on those, and a larger weight on her own signal as the solid line on Panel B shows. This guess has a "kink" at s_5 and s_7, because in this round dealer 6 conditions on messages which overweight these two signals. Since all other agents choose their guess in a similar way in round 2, the messages that dealer 6 gets in round 3 are a mirror image of her own guess, as shown by the dashed lines in Panel C. The solid line in Panel C represents dealer's 6 guess in round 4. On Panel D, we depict the guess of dealer 6 in each round until round-5, where we reach the fixed point. Note that it has all the properties we claimed: positive weight on each agents' signals, but decreasing in the distance from dealer 6.

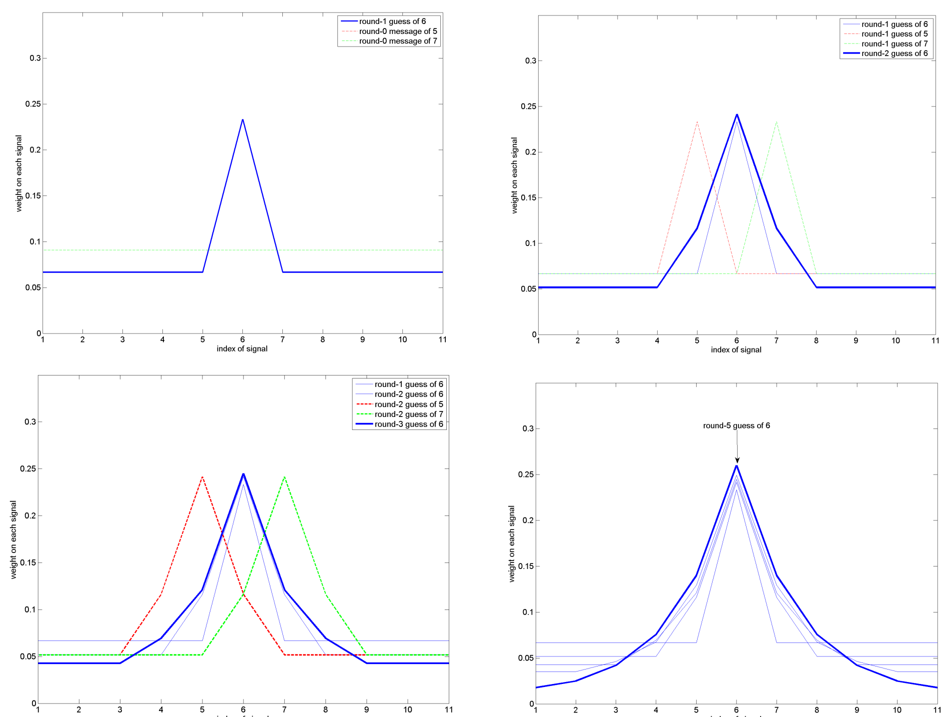

Figure 4: An iterated algorithm to find the equilibrium of the conditional guessing game in a 11-circle. Each line shows weights on a given signal in a given message or guess. Dashed lines denote messages dealer 6 recevies from her contacts of dealer 5 and 7, and the solid line denotes her best response guess of her value. Panel A,B,C illustrates round 1,2 and 3 of iteration, respectively, while panel D illustrates all rounds until convergence. Parameters are $n = 11$, $\rho = -.8$, $\sigma_\theta^2 = \sigma_\epsilon^2 = 1$, $\beta_{ij} = -\frac{10}{11}$.

Away from the common value limit, information transmission is not even "constrained informationally efficient". That is, dealers equilibrium guesses are different than the solu-

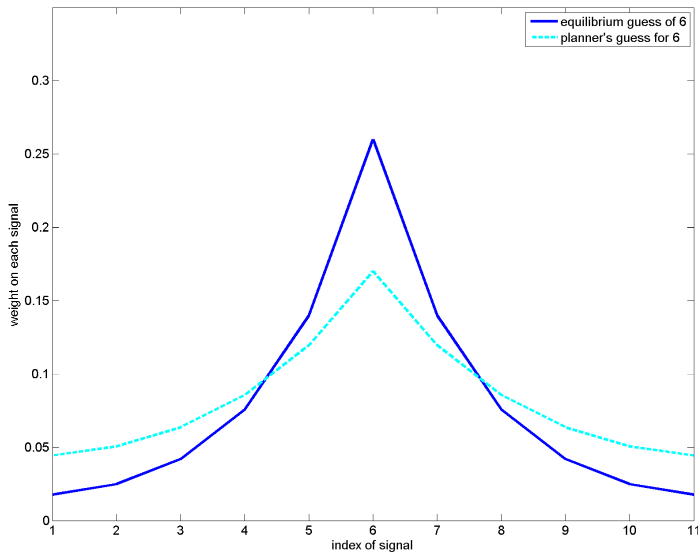

Figure 5: The solid curve depicts the equlibrium weights on each of the signals of dealer 6 in the conditional guessing game, $\mathbf{v}_6$. The dashed curve shows the weights in the solution minimizing the expected total squared errors of guesses for all i. Parameters are $n = 11$, $\rho = -.8$, $\sigma_\theta^2 = \sigma_\varepsilon^2 = 1$, $\beta_{ij} = -\frac{10}{11}$.

tion to the problem of a planner searching for the linear functions $\{\mathcal{E}_i(s_i, \mathbf{e}_{g_i})\}_{i=1\ldots n}$ which minimize the expected sum of squared errors

$$E\left[\sum_i (\theta_i - e_i)^2\right]$$

subject to (9). To see this, we continue the previous example by comparing the equilibrium guess with the solution of the planner's problem on Figure 5. As it is apparent, dealers put too much weight on their own signal from a social learning point of view. The reason is clear from the above explanation. When dealers' distort messages towards their own signals, they do not internalize that they reduce the information content of these guesses for others.

By Proposition 3, the properties of prices in the OTC game are implied by the properties of the equilibrium in the conditional guessing game. That is, the correlation between prices p_{ij} and p_{kl} tend to be lower if the link ij is further from kl. Also, prices could transmit more information even within the constraint imposed by our network structure. From the intuition gathered from the conditional guessing game, it is clear that this distortion is not a result of imperfect competition, strategic trading or anything else connected to the profit motives of agents. Instead, it is a consequence of the learning externality arising from the interaction between the interdependent value environment and the network structure of information sharing.

4.4 An application: what goes wrong in OTC markets in a liquidity crisis?

An attractive feature of our model is that it gives a rich set of empirical predictions. Namely, for any given information structure and dealer network, our model generates the full list of demand curves and the joint distribution of bilateral prices and quantities, and measures of price dispersion, intermediation, trading volume etc. Therefore, there is a wide range of potential applications. For example, one could use our model as the main building block for the analysis of endogenous network formation given that our model gives the payoffs for any given network. Alternatively, with a sufficiently detailed data-set in hand, one could use our framework for a structural estimation of the parameters in different states of the economy. We leave these possibilities for future work. Instead, as a simpler illustration of this richness, we confront narratives on the potential mechanisms behind OTC market distress to the observed stylized facts emerging from existing empirical analyses.

As a starting point, in the following table we summarize the findings of four recent empirical papers that investigate the effect of a liquidity event on market indicators. We focus on three indicators: price dispersion, price impact and volume.[15]

[15] The interested reader can find futher analyses on the effect of changing information structure on intermediation an profits in Appendix B.

	Friewald et al (2012)[16]	Afonso et al. (2012)	Gorton and Metrick (2012)[17]	Agarwal et al. (2012)[18]
Market	Corporate bonds	Fed Funds	Repo	MBS
Event	Subprime & GM/Ford	Lehman	Subprime	Subprime
Price dispersion	$\nearrow$	$\nearrow$	N/A	N/A
Price impact	$\nearrow$	N/A	$\nearrow$	N/A
Volume	$\leftrightarrow$	$\leftrightarrow$	N/A	$\searrow$

The stylized picture that emerges is that in a financial crisis price dispersion and price impact tends to increase and volume tend to decrease or stay constant. Measures for these indicators differ in each of these papers. Instead of picking one of them to match, we pick the measure which best catches the economic content of these indicators in our framework. As in our model prices are jointly normally distributed, for price dispersion we use the standard measure of the determinant of the correlation matrix[19]. For volume, we use the average gross volume per dealer $\sum_{j \in g_i} \left| q_{ij}^j \right|$. For price impact, we use the slope of the inverse residual demand curve a customer face at a given link, $\frac{-1}{c_{ji}^i + c_{ij}^j}$. Given these measures, we confront four potential narratives with the stylized facts:

1. Fundamental uncertainty increases around a crisis, that is, σ_θ^2 increases.

2. The idiosyncratic component of valuations increase, for example, because of larger role of differences in probability and in value of potential bail-outs or of forced liquidation. That is, ρ decreases.

3. Larger adverse selection implied by more asymmetry in information similar to Dang, Gorton and Holmström (2009). That is, some of the dealers have relative disadvan-

[16] The authors show that all the effects are stronger for the subprime crisis than for the GM/Ford. Also, in crisis while volume at the bond level stays the same, the market-wide volume goes down.

[17] The authors proxy price impact through haircut, i.e. larger haircuts imply a larger price impact.

[18] The proxy for volume here is the number of prime and subprime loans that lenders keep in their portfolio. The proportion increases in 2007 relative to previous years, indicating that securitization channels have tightened.

[19] An alternative would be the determinant of the variance-covariance matrix. That choice would leave the qualitative picture and our conclusions unchanged. We opt to use the correlation matrix as it emphasizes the dispersion of prices as opposed to the variance of each individual price.

tage of valuating the securities compared to others. In particular, we increase each of *some* dealers' σ_ϵ^2

4. Counterparty risk increases. That is, certain institutions find some others too risky to trade with. We capture this by dropping some of the links from the network.

The nine panels of Figure 6 depict the result of our experiments. Each column corresponds to a measure and each row corresponds to a narrative (in the last row we depict narrative 3 and 4 together). First note that price impact and volume move in opposite direction in each of the scenarios. This is a direct consequence of the fact that from (23) volume is positively related to trading intensity, while price impact is inversely related to the average trading intensity of the counterparties. The inverse relationship between volume and price impact is consistent with the stylized facts. Now consider first panels in the first and second rows where we plot the results both for the circle and the star networks. It is apparent that neither narrative 1 or 2 is consistent with the stylized facts above. There is the fundamental problem that both these shifts tend to move dispersion and volume into the same direction. Thus, none of this scenarios can generate larger price dispersion and smaller volume at the same time. It is so, because, as we noted above, volume is larger when dealers care less about the information of others. However, this also implies less learning resulting in more heterogenous posterior beliefs and larger price dispersion.

In the third row, we confront narratives 3 and 4 with the stylized facts. For narrative 3, we take a circle and increase the noise in the private signal, σ_ε^2 for each dealer but 6. The solid line on the first panel in row three shows that this decreases price dispersion. The dotted curve on the second panel depicts the case of $\sigma_\varepsilon^2 = 3$ for all but dealer 6 and shows that this decreases trading volume for all except of dealer 6. There are two observations to make about this curve. First, we see that just as with narratives 1 and 2, the shift in the information structure moved the average volume and the price dispersion in the same direction. This makes narrative 3 inconsistent with the stylized facts and the reason is the same as before. Second, comparing trading volume at the links of dealer 6 to trading volume at other links, we see that, in contrast to the standard argument, larger

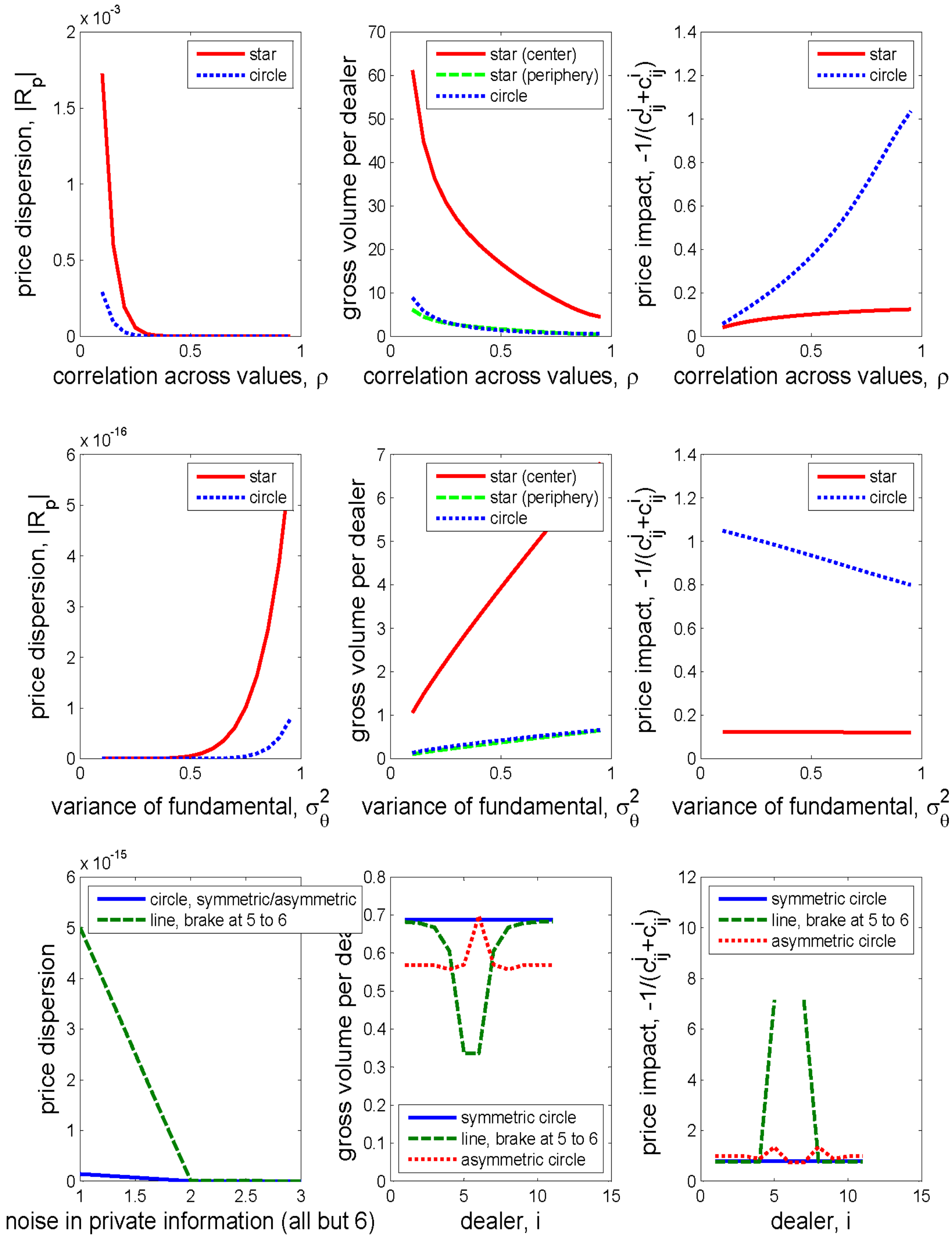

Figure 6: The first and second rows for panels depict comparative statics of price dispersion, trading volume per dealer and price impact per link as the correllation across values, ρ, fundamental uncertainty, σ_θ^2 changes in star (solid) and circle (dashed) networks. The last row shows the change in the same measures when noise in private signals, σ_ε^2 increases for all delaers but 6 (solid) and when the link between delaers 5 and 6 disappears (dashed). In the last two panels trading volume and price impact is shown by dealer and by the link between a given dealer and the one to the left in the circle or line. Parameters are $n = 11$, $\rho = -.8$, $\sigma_\theta^2 = \sigma_\varepsilon^2 = 1$, $\beta_{ij} = -\frac{10}{11}$.

asymmetry in information increases trading volume. The reason is clear given our model. The counterparties of dealer 6 would like to trade more as dealer 6 react to price changes less as she is less worried about adverse selection. An agreement involving larger trading volume is possible, because dealer 5 and 7 can compensate dealer 6 with a better price.

Let us consider now the dashed curves which depict the exercise of removing the link between dealers 5 and 6. The idea is that dealer 5 considers that there is too much counterparty risk in trading with dealer 6. Comparing the solid line and the dashed line in the first figure in row three shows that (this specific interpretation of) counterparty risk indeed increases price dispersion, decreases trading volume and increases price impact. Just as the empirical evidence finds. The difference between the intuitions in narratives 3 and 4 are clear. With counterparty risk, we assumed away the possibility that dealer 6 can provide sufficiently attractive terms at which dealer 5 is ready to trade. Thus, we exogenously reduced the possible trading opportunities which also reduced the possible learning opportunities. The earlier decreased volume, the latter increased the heterogeneity of posterior expectations, and, therefore, increased price dispersion.

Finally, let us emphasize that although we have found more support for counterparty risk than for information based stories, our comparative statics exercises are simplistic interpretations of these narratives. While we believe that and the intuition behind our conclusions are relevant in a more general context, undoubtedly, more research is needed to conclude that information based stories cannot be the reason behind market breakdowns in OTC markets.

5 Dynamic Foundations

The fact that in our model the conceptually complex problem of finding the equilibrium price and quantity vectors is solved in a single shot game, is an abstraction. We prefer to think about the OTC game as a reduced form of the real-world determination of prices and quantities potentially involving complex exchanges of series of quotes across multiple potential partners. In this section we justify our approach by introducing a quasi-rational, but realistic dynamic protocol producing the same outcome than our OTC game.

Suppose that time is discrete, and in each period there are two stages: the morning stage and the evening stage. In the evening, each dealer i sends a message, $h_{i,t}$, to all counterparties she has in the network g. In the morning, each dealer receives these messages. In the following evening, a dealer can update the message she sends, possibly taking into account the information received in the morning. Messages can be interpreted, for instance, as quotes that dealers exchange with their counterparties. Upon receiving a set of quotes, a dealer might decide to contact her counterparties with other offers, before trade actually occurs. The protocol stops if there exists an arbitrarily small scalar $\delta_i > 0$, such that $|h_{i,t} - h_{i,t_{\boldsymbol{\delta}}}| \leq \delta_i$ for each i, in any subsequent period $t \geq t_{\boldsymbol{\delta}}$. That, is the protocol stops when no dealer wants to significantly revise her message in the evening after receiving information in the morning.

Importantly, there exists a rule that maps messages into prices and quantities for each pair of dealers that have a link in the network g, and this rule is common knowledge for all dealers. When the protocol stops, trading takes place at the prices determined by this rules,and quantities are allocated accordingly. No transactions take place before the protocol stops.

Suppose that there exists an equilibrium in the one-shot OTC game. Let dealers use their equilibrium strategy in the conditional guessing game to update each evening the messages they send based on the messages they receive in the morning, such that

$$h_{i,t} = \bar{y}_i s_i + \bar{\mathbf{z}}_{g_i}^T \mathbf{h}_{g_i,t-1}, \ \forall i$$

where $\mathbf{h}_{g_i,t} = (h_{j,t})_{j \in g_i}$, and $\bar{y}_i$ and $\bar{z}_{ij}$ have been characterized, for any i and $j \in g_i$, in Proposition 1. Further, consider a rule based on (16) that determines the price between a pair of agents ij that have a link in the network g as follows

$$p_{ij,t} = \frac{(2 - z_{ji})}{4 - z_{ij} z_{ji}} h_{i,t} + \frac{(2 - z_{ij})}{4 - z_{ij} z_{ji}} h_{j,t},$$

where the relationship between z_{ij} and $\bar{z}_{ij}$ has been characterized in Proposition 3. Given the prices, the quantity that agent i would receive in the transaction with j is $q_{i,t}^j(s_i, \mathbf{p}_{g_i,t})$, where the the function has been characterized in Corollary A.5. As before, agents seek to maximize their expected utility, given their private signal, and the messages they observe.

Proposition 7 *Let $\mathbf{h}_t = (h_{i,t})_{i\in\{1,2,\dots,n\}}$ be the vector of messages sent at time t, and $\boldsymbol{\mu} = (\mu_i)_{i\in\{1,2,\dots,n\}}$ be a vector of IID $N(0,\sigma_\mu^2)$ random normal variables. Suppose that $\rho < 1$. Then*

1. *If $\mathbf{h}_t = \left(I - \bar{Z}\right)^{-1} \bar{Y}\mathbf{s}$, then $\mathbf{h}_{t+1} = \left(I - \bar{Z}\right)^{-1} \bar{Y}\mathbf{s}$, for any t.*

2. *If $\mathbf{h}_{t_0} = \left(I - \bar{Z}\right)^{-1} \bar{Y}\left(\mathbf{s} + \boldsymbol{\mu}\right)$, then there exists a vector of arbitrarily small scalars $\boldsymbol{\delta} = (\delta_i)_{i\in\{1,2,\dots,n\}}$ such that trading takes place in period $t_{\boldsymbol{\delta}}$ and*

$$\left|\mathbf{h}_{t_{\boldsymbol{\delta}}} - \left(I - \bar{Z}\right)^{-1} \bar{Y}\mathbf{s}\right| < \frac{1}{2}\boldsymbol{\delta}.$$

3. *If $\mathbf{h}_{t_0} = \left(I - \bar{Z}\right)^{-1} \bar{Y}\left(\mathbf{s} + \boldsymbol{\mu}\right)$, then there exists a vector of arbitrarily small scalars $\boldsymbol{\delta} = (\delta_i)_{i\in\{1,2,\dots,n\}}$ such that trading takes place in period $t_{\boldsymbol{\delta}}$ and*

$$\left|E\left(\theta_i | s_i, \mathbf{h}_{g_i,t_0}, \mathbf{h}_{g_i,t_0+1}, \dots, \mathbf{h}_{g_i,t_\delta}\right) - E\left(\theta_i | s_i, \mathbf{p}_{g_i}\right)\right| < \frac{1}{2}\boldsymbol{\delta},$$

where $\mathbf{p}_{g_i}$ are the equilibrium prices in the one-short OTC game.

Thus, this dynamic protocol leads to the same traded prices and quantities, independently from which vector of messages we start the protocol at. Interestingly, even if this updating rule is not necessarily optimal in every round when it is used, we show that ex-post, when transactions occur, dealers could not do better.

6 Conclusion

In this paper we present a model of strategic information diffusion in over-the-counter markets. In our set-up a dealer can trade any quantity of the asset she finds desirable, and understands that her trade may affect transaction prices. Moreover, she can decide to buy a certain quantity at a given price from one counterparty and sell a different quantity at a different price to another.

We show that the equilibrium price in each transaction partially aggregates the private information of all agents in the economy. The informational efficiency of prices is the highest in networks where each agent trades with every other agent, or in the common value limit, regardless of the network structure. Otherwise, agents tend to overweight their own signal compared to the outcome which would maximize information transmission.

As an illustration for the possible range of applications, we compare the model generated economic indicators under different scenarios of changing economic environment with the stylized facts in recent empirical papers. We find more support for the arguments that the increased counterparty risk was the main determinant of the distress of OTC markets in the recent financial crisis, as opposed to narratives based on the deterioration of the informational environment.

An attractive feature of our model is that it gives a rich set of empirical predictions. Namely, for any given information structure and dealer network, our model generates the full list of demand curves and the joint distribution of bilateral prices and quantities Therefore, there is a wide range of potential applications. For future research, one could use our model as the main building block for the analysis of endogenous network formation given that our model gives the payoffs for any given network. Alternatively, with a sufficiently detailed data-set in hand, one could use our framework for a structural estimation of the parameters in different states of the economy.

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A Appendix: Proofs

Throughout several of the following proofs we often decompose θ_i into a common value component, $\hat{\theta}$, and a private value component, η_i, such that

$$\theta_i = \hat{\theta} + \eta_i$$

and

$$s_i = \hat{\theta} + \eta_i + \varepsilon_i$$

with $\hat{\theta} \sim N(0, \sigma_{\hat{\theta}}^2)$, $\eta_i \sim IID\ N(0, \sigma_\eta^2)$ and $\mathcal{V}\left(\eta_i, \eta_j\right) = 0$. This implies that

$$\left(1-\rho\right)\sigma_\theta^2 = \sigma_\eta^2$$

Further, we generalize the notation $\mathcal{V}$ to be the variance-covariance operator applied to vectors of random variables. For instance, $\mathcal{V}(\mathbf{x})$ represents that variance-covariance matrix of vector $\mathbf{x}$, and $\mathcal{V}(\mathbf{x}, \mathbf{y})$ represents the covariance matrix between vector $\mathbf{x}$ and $\mathbf{y}$.

A.1 Proof of Proposition 1

We need the following Lemmas for the construction of our proof.

Lemma A.1 *Consider the jointly normally distributed variables $(\theta_i, \mathbf{s})$. Let an arbitrary weighting vector $\boldsymbol{\omega} > \mathbf{0}$. Consider the coefficient of s_i in the projection of $E\left(\theta | s_i\right)$. Adding $\boldsymbol{\omega}^T\mathbf{s}$ as a conditioning variable, additional to s_i, decreases the coefficient of s_i, that is,*

$$\frac{\partial E\left(\theta_i | s_i, \boldsymbol{\omega}^T\mathbf{s}\right)}{\partial s_i} < \frac{\partial E\left(\theta_i | s_i\right)}{\partial s_i}$$

Proof. From the projection theorem

$$E\left(\theta_i | s_i, \boldsymbol{\omega}^T\mathbf{s}\right) = E\left(\theta_i | s_i\right) + \frac{\mathcal{V}\left(\theta_i, \boldsymbol{\omega}^T\mathbf{s} | s_i\right)}{\mathcal{V}\left(\boldsymbol{\omega}^T\mathbf{s} | s_i\right)}\left(\boldsymbol{\omega}^T\mathbf{s} - E\left(\boldsymbol{\omega}^T\mathbf{s} | s_i\right)\right)$$

consequently

$$\frac{\partial E\left(\theta_i | s_i, \boldsymbol{\omega}^T\mathbf{s}\right)}{\partial s_i} = \frac{\partial E\left(\theta_i | s_i\right)}{\partial s_i} - \frac{\mathcal{V}\left(\theta_i, \boldsymbol{\omega}^T\mathbf{s} | s_i\right)}{\mathcal{V}\left(\boldsymbol{\omega}^T\mathbf{s} | s_i\right)} \frac{\mathcal{V}\left(s_i, \boldsymbol{\omega}^T\mathbf{s}\right)}{\mathcal{V}\left(s_i\right)}.$$

Thus, it is sufficient to show that $\mathcal{V}\left(\boldsymbol{\omega}^T\mathbf{s}, s_i\right) > 0$ and $\mathcal{V}\left(\theta_i, \boldsymbol{\omega}^T\mathbf{s} | s_i\right) > 0$. For the former, we know that

$$\mathcal{V}\left(\boldsymbol{\omega}^T\mathbf{s}, s_i\right) = \omega_i\left(\sigma_\varepsilon^2 + \left(1-\rho\right)\sigma_\theta^2\right) + \rho\sigma_\theta^2\boldsymbol{\omega}^T\mathbf{1} > 0$$

Then, we use the projection theorem to show that

$$\begin{aligned}\mathcal{V}\left(\theta_i, \boldsymbol{\omega}^T\mathbf{s} | s_i\right) &= \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2\boldsymbol{\omega}^T\mathbf{1} \\ \sigma_\theta^2\boldsymbol{\omega}^T\mathbf{1} & \mathcal{V}\left(\boldsymbol{\omega}^T s\right)\end{pmatrix} \\ &\quad -\frac{1}{\sigma_\theta^2 + \sigma_\varepsilon^2}\begin{pmatrix} \sigma_\theta^2 \\ \left(\omega_i\left(\sigma_\varepsilon^2 + \left(1-\rho\right)\sigma_\theta^2\right) + \rho\sigma_\theta^2\boldsymbol{\omega}^T\mathbf{1}\right)\end{pmatrix}\begin{pmatrix} \sigma_\theta^2 & \left(\omega_i\left(\sigma_\varepsilon^2 + \left(1-\rho\right)\sigma_\theta^2\right) + \rho\sigma_\theta^2\boldsymbol{\omega}^T\mathbf{1}\right)\end{pmatrix}\end{aligned}$$

implying that

$$\mathcal{V}\left(\theta_i, \boldsymbol{\omega}^T \mathbf{s}|s_i\right) = \sigma_\theta^2 \boldsymbol{\omega}^T \mathbf{1} - \frac{\left(\omega_i \left(\sigma_\varepsilon^2 + (1-\rho)\,\sigma_\theta^2\right) + \rho\sigma_\theta^2 \boldsymbol{\omega}^T \mathbf{1}\right)\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} = \sigma_\theta^2 \frac{\sigma_\varepsilon^2 + (1-\rho)\,\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2}\left(\boldsymbol{\omega}^T \mathbf{1} - \omega_i\right) > 0.$$

■

Lemma A.2 *Take the jointly normally distributed system* $\begin{pmatrix} \hat{\theta} \\ \mathbf{x} \end{pmatrix}$ *where* $\mathbf{x} = \hat{\theta}\mathbf{1} + \boldsymbol{\varepsilon}' + \boldsymbol{\varepsilon}''$, *with the following properties*

- $E\begin{pmatrix} \hat{\theta} \\ \mathbf{x} \end{pmatrix} = 0$, $\mathcal{V}\left(\hat{\theta}, \boldsymbol{\varepsilon}'\right) = 0$ *and* $\mathcal{V}\left(\hat{\theta}, \boldsymbol{\varepsilon}''\right) = 0$;
- $\mathcal{V}(\boldsymbol{\varepsilon}')$ *is diagonal, and* $\mathcal{V}(\mathbf{x}) \geq \mathcal{V}\left(\hat{\theta}\mathbf{1} + \boldsymbol{\varepsilon}'\right)$.

Then the vector $\boldsymbol{\omega}$ *defined by*

$$E\left(\hat{\theta}|\mathbf{x}\right) = \boldsymbol{\omega}^T \mathbf{x}$$

has the properties that $\boldsymbol{\omega}^T \mathbf{1} < 1$ *and* $\boldsymbol{\omega} \in (0,1)^n$.

Proof. By the projection theorem, we have that

$$\boldsymbol{\omega}^T = \mathcal{V}\left(\hat{\theta}, \mathbf{x}\right)\left(\mathcal{V}(\mathbf{x})\right)^{-1}.$$

Then

$$\mathcal{V}\left(\hat{\theta}, \mathbf{x}\right)\left(\mathcal{V}(\mathbf{x})\right)^{-1} \leq \mathcal{V}\left(\hat{\theta}, \hat{\theta}\mathbf{1} + \boldsymbol{\varepsilon}' + \boldsymbol{\varepsilon}''\right)\left(\mathcal{V}\left(\hat{\theta}\mathbf{1} + \boldsymbol{\varepsilon}'\right)\right)^{-1} = \mathcal{V}\left(\hat{\theta}, \hat{\theta}\mathbf{1}\right)\left(\mathcal{V}\left(\hat{\theta}\mathbf{1} + \boldsymbol{\varepsilon}'\right)\right)^{-1}.$$

The inequality comes from the fact that both $\mathcal{V}(\mathbf{x})$ and $\mathcal{V}\left(\hat{\theta}\mathbf{1} + \hat{\boldsymbol{\varepsilon}}\right)$ are positive definite matrixes and that $\mathcal{V}(\mathbf{x}) \geq \mathcal{V}\left(\hat{\theta}\mathbf{1} + \hat{\boldsymbol{\varepsilon}}\right)$. (See Horn and Johnson (1985), Corollary 7.7.4(a)).

Since

$$\mathcal{V}\left(\hat{\theta}, \hat{\theta}\mathbf{1}\right)\left(\mathcal{V}\left(\hat{\theta}\mathbf{1} + \boldsymbol{\varepsilon}'\right)\right)^{-1}\mathbf{1} = 1 - \frac{\frac{1}{\sigma_{\hat{\theta}}^2}}{\frac{1}{\mathcal{V}(\boldsymbol{\varepsilon}')}\mathbf{1} + \frac{1}{\sigma_{\hat{\theta}}^2}} < 1,$$

then

$$\boldsymbol{\omega}^T \mathbf{1} < 1$$

which implies that

$$\boldsymbol{\omega} \in (0,1)^n.$$

■

Lemma A.3 *For any network* g, *define a mapping* $F : R^{n \times n} \to R^{n \times n}$ *as follows. Let* $\mathbf{V}$ *be an* $n \times n$ *matrix with rows* $\mathbf{v}_j$ *and*

$$e_j = \mathbf{v}_j \mathbf{s}$$

for each $j = 1, ...n$. *The mapping* $F(\mathbf{V})$ *is given by*

$$\begin{pmatrix} E(\theta_1|s_1, \mathbf{e}_{g_1}) \\ E(\theta_2|s_2, \mathbf{e}_{g_2}) \\ ... \\ E(\theta_n|s_n, \mathbf{e}_{g_n}) \end{pmatrix} = F(\mathbf{V})\,\mathbf{s}.$$

Then, the mapping F *is a continuos self-map on the space* $[0,1]^{n\times n}$.

Proof. Let

$$\mathbf{v}_j^0 = \begin{pmatrix} v_{j1}^0 & v_{j2}^0 & ... & v_{jn}^0 \end{pmatrix}$$

and consider that

$$\begin{aligned} e_j &= \mathbf{v}_j^0 \mathbf{s} \\ &= \mathbf{v}_j^0 \left(\hat{\theta}\mathbf{1} + \boldsymbol{\varepsilon} + \boldsymbol{\eta}\right) \end{aligned}$$

where $\mathbf{1}$ is a column vector of ones and

$$\boldsymbol{\eta} = \begin{pmatrix} \eta_1 & \eta_2 & ... & \eta_n \end{pmatrix}^T$$

and

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & ... & \varepsilon_n \end{pmatrix}^T$$

Let

$$\hat{e}_j = \frac{e_j}{\mathbf{v}_j^0\mathbf{1}} = \hat{\theta} + \frac{\mathbf{v}_j^0}{\mathbf{v}_j^0\mathbf{1}}(\boldsymbol{\varepsilon} + \boldsymbol{\eta})$$

and

$$\hat{\mathbf{e}}_{g_i} = (\hat{e}_j)_{j\in g_i}$$

To prove the result, we apply Lemma A.2 for each $E(\theta_i|s_i, \mathbf{e}_{g_i})$. In particular, for each i, we construct a vector $\boldsymbol{\varepsilon}'_{g_i}$ with the first element $(\varepsilon_i + \eta_i)$ and the j-th element equal to $\frac{v_{jj}^0}{\mathbf{v}_j^0\mathbf{1}}\left(\varepsilon_j + \eta_j\right)$ with $j \in g_i$, and a vector $\boldsymbol{\varepsilon}''_{g_i}$ with the first element 0 and the j-th element equal to $\frac{\mathbf{v}_j^0}{\mathbf{v}_j^0\mathbf{1}}\left(\boldsymbol{\varepsilon} + \boldsymbol{\eta} - \left(\varepsilon_j + \eta_j\right)\mathbf{1}_j\right)$ with $j \in g_i$ ($\mathbf{1}_j$ is a column vector of 0 and 1 at position j). Then, we have that

$$\begin{pmatrix} s_i \\ \hat{\mathbf{e}}_{g_i} \end{pmatrix} = \hat{\theta}\mathbf{1} + \boldsymbol{\varepsilon}'_{g_i} + \boldsymbol{\varepsilon}''_{g_i}$$

Below, we show that the conditions in Lemma A.2 apply.

First, by construction, $\boldsymbol{\varepsilon}'_{g_i}$ has a diagonal variance-covariance matrix. Next, we also show that $\mathcal{V}\begin{pmatrix} s_i \\ \hat{\mathbf{e}}_{g_i} \end{pmatrix} \geq \mathcal{V}\left(\hat{\theta}\mathbf{1} + \boldsymbol{\varepsilon}'_{g_i}\right)$ element by element. Indeed

$$\begin{aligned}\mathcal{V}(\hat{e}_j) &= \sigma^2_{\hat{\theta}} + \left(\frac{v^0_{jj}}{\mathbf{v}^0_j\mathbf{1}}\right)^2 \left(\sigma^2_\varepsilon + \sigma^2_\eta\right) + \left(\frac{\mathbf{v}^0_j}{\mathbf{v}^0_j\mathbf{1}}\right)\mathcal{V}\left(\boldsymbol{\varepsilon} + \boldsymbol{\eta} - \left(\varepsilon_j + \eta_j\right)\mathbf{1}_j\right)\left(\frac{\mathbf{v}^0_j}{\mathbf{v}^0_j\mathbf{1}}\right)^T \\ &\quad + \frac{v^0_{jj}}{\mathbf{v}^0_j\mathbf{1}}\mathcal{V}\left(\left(\varepsilon_j + \eta_j\right), \left(\boldsymbol{\varepsilon} + \boldsymbol{\eta} - \left(\varepsilon_j + \eta_j\right)\mathbf{1}_j\right)\right)\left(\frac{\mathbf{v}^0_j}{\mathbf{v}^0_j\mathbf{1}}\right)^T\end{aligned}$$

and

$$\begin{aligned}\mathcal{V}(\hat{e}_j, \hat{e}_k) &= \sigma^2_{\hat{\theta}} + \frac{v^0_{kk}}{\mathbf{v}^0_k\mathbf{1}}\mathcal{V}\left(\left(\varepsilon_k + \eta_k\right), \left(\boldsymbol{\varepsilon} + \boldsymbol{\eta} - \left(\varepsilon_j + \eta_j\right)\mathbf{1}_j\right)\right)\left(\frac{\mathbf{v}^0_j}{\mathbf{v}^0_j\mathbf{1}}\right)^T \\ &\quad + \frac{v^0_{jj}}{\mathbf{v}^0_j\mathbf{1}}\mathcal{V}\left(\left(\varepsilon_j + \eta_j\right), \left(\boldsymbol{\varepsilon} + \boldsymbol{\eta} - \left(\varepsilon_k + \eta_k\right)\mathbf{1}_j\right)\right)\left(\frac{\mathbf{v}^0_k}{\mathbf{v}^0_k\mathbf{1}}\right)^T \\ &\quad + \frac{\mathbf{v}^0_j}{\mathbf{v}^0_j\mathbf{1}}\mathcal{V}\left(\left(\boldsymbol{\varepsilon} + \boldsymbol{\eta} - \left(\varepsilon_j + \eta_j\right)\mathbf{1}_j\right), \left(\boldsymbol{\varepsilon} + \boldsymbol{\eta} - \left(\varepsilon_k + \eta_k\right)\mathbf{1}_j\right)\right)\left(\frac{\mathbf{v}^0_k}{\mathbf{v}^0_k\mathbf{1}}\right)^T,\end{aligned}$$

which implies that

$$\mathcal{V}(\hat{e}_j) > \sigma^2_{\hat{\theta}} + \left(\frac{v^0_{jj}}{\mathbf{v}^0_j\mathbf{1}}\right)^2 \left(\sigma^2_\varepsilon + \sigma^2_\eta\right) \tag{A.1}$$

and

$$\mathcal{V}(\hat{e}_j, \hat{e}_k) > \sigma^2_{\hat{\theta}}. \tag{A.2}$$

This is because

$$\mathcal{V}\left(\left(\varepsilon_j + \eta_j\right), \left(\boldsymbol{\varepsilon} + \boldsymbol{\eta} - \left(\varepsilon_j + \eta_j\right)\mathbf{1}_j\right)\right) = 0$$

and

$$\mathcal{V}\left(\eta_i, \eta_j\right) = 0 \text{ and } \mathcal{V}\left(\varepsilon_i, \varepsilon_j\right) = 0 \ \forall i, j.$$

Moreover,

$$\mathcal{V}(s_i, \hat{e}_j) = \sigma^2_{\hat{\theta}} + \frac{v^0_{ji}}{\left(\mathbf{v}^0_j\right)^T\mathbf{1}}\left(\sigma^2_\varepsilon + \sigma^2_\eta\right) > \sigma^2_{\hat{\theta}}. \tag{A.3}$$

From (A.1), (A.2), and (A.3), it follows that $\mathcal{V}\begin{pmatrix} s_i \\ \hat{\mathbf{e}}_{g_i} \end{pmatrix} \geq \mathcal{V}\left(\hat{\theta}\mathbf{1}^T + \boldsymbol{\varepsilon}'_{g_i}\right)$. Then, for each i there exists a column vector $\boldsymbol{\omega}_{g_i} = \left(\omega_{ij}\right)_{j\in\{i\cup g_i\}}$ with the properties that $\boldsymbol{\omega}^T_{g_i}\mathbf{1} < 1$ and $\boldsymbol{\omega}_{g_i} \in (0,1)^{m_i+1}$, such that

$$E\left(\hat{\theta}|s_i, \hat{\mathbf{e}}_{g_i}\right) = \boldsymbol{\omega}^T_{g_i}\begin{pmatrix} s_i \\ \hat{\mathbf{e}}_{g_i} \end{pmatrix}.$$

It is immediate that

$$E\left(\theta_i|s_i,\mathbf{e}_{g_i}\right)=E\left(\theta_i|s_i,\hat{\mathbf{e}}_{g_i}\right)=E\left(\hat{\theta}|s_i,\hat{\mathbf{e}}_{g_i}\right)+E\left(\eta_i|s_i\right)$$

where

$$E\left(\eta_i|s_i\right)=\frac{\sigma_\eta^2}{\sigma_{\hat{\theta}}^2+\sigma_\eta^2+\sigma_\varepsilon^2}s_i.$$

Then, from Lemma A.1,

$$v_{ii}=\omega_{ii}+\sum_{k\in g_i}\omega_{ik}\frac{v_{ki}^0}{\mathbf{v}_k^0\mathbf{1}}+\frac{\sigma_\eta^2}{\sigma_{\hat{\theta}}^2+\sigma_\eta^2+\sigma_\varepsilon^2}=\frac{\partial E\left(\theta_i|s_i,\mathbf{e}_{g_i}\right)}{\partial s_i}<\frac{\partial E\left(\theta_i|s_i\right)}{\partial s_i}<1 \tag{A.4}$$

and

$$v_{ij}=\sum_{k\in g_i}\omega_{ik}\frac{v_{kj}^0}{\mathbf{v}_k^0\mathbf{1}}<\sum_{k\in g_i}\omega_{ik}<1,\forall j\in g_i. \tag{A.5}$$

Thus far, we have used only that $v_{ij}^0\geq 0$ (and not that $v_{ij}^0\in[0,1]$). This implies that if $v_{ij}^0\geq 0$, then $v_{ij}\in[0,1]$. Then it must be true also that $\forall v_{ij}^0\in[0,1]$, then $v_{ij}\in[0,1]$. This concludes the proof. ■

Now we are ready to prove the statement.

An equilibrium exists if there exists a matrix V and matrices $\bar{Y}$ and $\bar{Z}$ such that

$$V\mathbf{s}=\left(\bar{Y}+\bar{Z}V\right)\mathbf{s},$$

and

$$F\left(V\right)\mathbf{s}=\left(\bar{Y}+\bar{Z}V\right)\mathbf{s},$$

where $F\left(\cdot\right)$ is the mapping introduced in Lemma A.3. The first condition insures that

$$\mathbf{e}=V\mathbf{s}$$

is a fixed point in (9), and the second condition insures that first order conditions (10) are satisfied.

We construct an equilibrium for $\rho<1$ and for $\rho=1$ as follows.

Case 1: $\rho<1$

By Brower's fixed point theorem, the mapping $F\left(\cdot\right)$ admits a fixed point on $[0,1]^{n\times n}$. Let $V^*\in[0,1]^{n\times n}$ be a matrix such that

$$F(V^*)=V^*.$$

Let $\bar{Y}$ be a diagonal matrix with elements

$$\bar{y}_i=\omega_{ii}+\frac{\sigma_\eta^2}{\sigma_{\hat{\theta}}^2+\sigma_\eta^2+\sigma_\varepsilon^2}$$

and let $\bar{Z}$ have elements

$$\bar{z}_{ij}=\begin{cases} \frac{\omega_{ij}}{\left(\mathbf{v}_j^*\right)^T \mathbf{1}}, \text{ if } ij \in g \\ 0, \text{ otherwise} \end{cases}$$

where ω_{ii} and ω_{ij} have been introduced the proof above. Both matrices $\bar{Y} \geq 0$ and $\bar{Z} \geq 0$.

Substituting V^* in (A.4) and (A.5), it follows that

$$V^* = \bar{Y} + \bar{Z}V^*,$$

and since $F(V^*) = V^*$, then

$$F(V^*) = \bar{Y} + \bar{Z}V^*.$$

Case 2: $\rho = 1$

Let $\bar{Y} = 0$ and $\bar{Z}$ have elements $\bar{z}_{ij}$ if $ij \in g$, and 0 otherwise, with $\sum_{j \in g_i} \bar{z}_{ij} = 1$. Let V^* be a matrix with $v_{ij}^* = \frac{\sigma_{\hat{\theta}}^2}{n\sigma_{\hat{\theta}}^2 + \sigma_{\varepsilon}^2}$ for any i and j.

It is straightforward to see that

$$V^* = \bar{Z}V^*.$$

Next we show that

$$F(V^*) = V^*.$$

Next, we show that the matrix V^* with $v_{ij}^* = \frac{\sigma_{\hat{\theta}}^2}{n\sigma_{\hat{\theta}}^2 + \sigma_{\varepsilon}^2}$ for any i and j satisfies

$$F(V^*) = V^*.$$

By definition, matrix V^* with $v_{ij}^* = \frac{\sigma_{\hat{\theta}}^2}{n\sigma_{\hat{\theta}}^2 + \sigma_{\varepsilon}^2}$ is a fixed point of the mapping $F(\cdot)$, if

$$e_i = v^* \sum_{i=1}^{n} s_j, \ \forall i \in \{1, 2, ..., n\}$$

then

$$E\left(\theta_i | s_i, \mathbf{e}_{g_i}\right) = v^* \sum_{j=1}^{n} s_j, \forall i \in \{1, 2, ..., n\}.$$

As $\rho = 1$, then $\theta_i = \hat{\theta}$ for any i and

$$\begin{aligned} E\left(\theta_i | s_i, \mathbf{e}_{g_i}\right) &= E\left(\hat{\theta} | v^* \sum_{i=1}^{n} s_i\right) \\ &= \frac{1}{v^*} \frac{\sigma_{\hat{\theta}}^2}{n\sigma_{\hat{\theta}}^2 + \sigma_{\varepsilon}^2} v^* \sum_{i=1}^{n} s_i \\ &= v^* \sum_{i=1}^{n} s_i. \end{aligned}$$

It follows that

$$F(V^*) = \bar{Z}V^*.$$

This concludes the proof.

In the remaining proofs, we use the results in the following Lemma

Lemma A.4 *In the conditional guessing game, the following properties hold whenever $\rho < 1$.*

1. $\bar{Y} \in (0,1]^{n\times n}$,
2. $\lim_{n\to\infty} \bar{Z}^n = 0$ *and* $(I - \bar{Z})$ *is invertible, and*
3. $\mathbf{e} = (I - \bar{Z})^{-1} \bar{Y}\mathbf{s}$, *where* $\mathbf{e} = (e_i)_{i\in\{1,2,\ldots,n\}}$, $\bar{Y}$ *is a matrix with elements* $\bar{y}_i$ *on the diagonal and* 0 *otherwise, and* $\bar{Z}$ *is a matrix with elements* $\bar{z}_{ij}$, *when* i *and* $j (\neq i)$ *have a link and* 0 *otherwise.*

Proof.

1. From (A.4) it also follows that

$$\bar{y}_i < \frac{\partial E(\theta_i|s_i, \mathbf{e}_{g_i})}{\partial s_i} < 1.$$

   Moreover, as $\rho < 1$, then $\sigma_\eta^2 > 0$, which implies that $\bar{y}_i > 0$. It follows that $\bar{Y}$ is invertible.

2. We first show that matrix V^* is nonsingular. For this we construct a matrix $W^* = \bar{Y}^{-1}(I - \bar{Z})$ and show that $W^*V^* = I$. Indeed, the element on the position (i,i) on the diagonal of W^*V^* is equal to

$$\begin{aligned}
\frac{1}{\bar{y}_i}\left(v_{ii}^* - \sum_{k\in g_i} \bar{z}_{ik} v_{ki}^*\right) &= \frac{1}{\bar{y}_i}\left(\omega_{ii} + \sum_{k\in g_i} \omega_{ik} \frac{v_{ki}^*}{(\mathbf{v}_k^*)^T \mathbf{1}} + \frac{\sigma_\eta^2}{\sigma_{\hat{\theta}}^2 + \sigma_\eta^2 + \sigma_\varepsilon^2} - \sum_{k\in g_i} \frac{\omega_{ik}}{(\mathbf{v}_k^*)^T \mathbf{1}} v_{ki}^*\right) \\
&= \frac{1}{\bar{y}_i}\left(\omega_{ii} + \frac{\sigma_\eta^2}{\sigma_{\hat{\theta}}^2 + \sigma_\eta^2 + \sigma_\varepsilon^2}\right) \\
&= 1
\end{aligned}$$

   while the element on the position (i,j) off the diagonal of W^*V^* is equal to

$$\frac{1}{\bar{y}_i}\left(v_{ij}^* - \sum_{k\in g_i} \bar{z}_{ik} v_{kj}^*\right) = \frac{1}{\bar{y}_i}\left(\sum_{k\in g_i} \omega_{ik} \frac{v_{kj}^*}{(\mathbf{v}_k^*)^T \mathbf{1}} - \sum_{k\in g_i} \frac{\omega_{ik}}{(\mathbf{v}_k^*)^T \mathbf{1}} v_{kj}^*\right) = 0$$

   where we used again the fact that V^* is a fixed point in (A.4) and (A.5).

3. Since V^* is nonsingular, then $(I - \bar{Z})$ is also nonsingular as

$$(I - \bar{Z}) = \bar{Y}(V^*)^{-1}.$$

Given that $\bar{Z} \geq 0$ this implies, as shown in Meyer (2000), that the largest eigenvalue of $\bar{Z}$ is strictly smaller than 1. This is a useful result, as it is sufficient to show that

$$\lim_{n\to\infty} \bar{Z}^n = 0_{n\times n}$$

and that

$$\left(I - \bar{Z}\right)^{-1} = \sum_{n=1}^{\infty} \bar{Z}^n.$$

(For both claims see Meyer (2000) pp. 620 & 618.)

4. The equilibrium outcome guess vector is, by construction

$$\mathbf{e} = V^*\mathbf{s}$$

which implies that

$$\mathbf{e} = \left(I - \bar{Z}\right)^{-1} \bar{Y}.$$

■

A.2 Proof of Proposition 2

In an equilibrium of the OTC game, prices and quantities satisfy the first order conditions (8) and must be such that all bilateral trades clear.

Since market clearing conditions (4) are linear in prices and signals, we know that each price (if an equilibrium price vector exists) must be a certain linear combination of signals. Thus, each price is normally distributed.

From the first order conditions we have that

$$q_i^j(s_i, \mathbf{p}_{g_i}) = -\left(c_{ji}^i + \beta_{ij}\right)\left(E(\theta_i \,|s_i, \mathbf{p}_{g_i}) - p_{ij}\right).$$

The bilateral clearing condition between a trader i and trader j that have a link in network g implies that

$$-\left(c_{ji}^i + \beta_{ij}\right)\left(E(\theta_i \,|s_i, \mathbf{p}_{g_i}) - p_{ij}\right) - \left(c_{ij}^j + \beta_{ij}\right)\left(E(\theta_j \,|s_j, \mathbf{p}_{g_j}) - p_{ij}\right) + \beta_{ij} p_{ij} = 0$$

and solving for the price p_{ij} we have that

$$p_{ij} = \frac{\left(c_{ji}^i + \beta_{ij}\right) E(\theta_i \,|s_i, \mathbf{p}_{g_i}) + \left(c_{ij}^j + \beta_{ij}\right) E(\theta_j \,|s_j, \mathbf{p}_{g_j})}{c_{ji}^i + c_{ij}^j + 3\beta_{ij}}$$

Since agent i knows $E(\theta_i \,|s_i, \mathbf{p}_{g_i})$, by definition, the vector of prices $\mathbf{p}_{g_i}$ is informationally equivalent for her with the vector of posteriors of her neighbors $\mathbf{E}_{g_i} = \left\{E(\theta_j \,|s_j, \mathbf{p}_{g_j})\right\}_{j\in g_i}$. This implies that

$$E(\theta_i \,|s_i, \mathbf{p}_{g_i}) = E(\theta_i \,|s_i, \mathbf{E}_{g_i}).$$

Note also that as each price is a linear combination of signals and $E(\theta_j|\cdot)$ is a linear operator on jointly normal variables, there must be a vector $\mathbf{w}_i$ that $E(\theta_i|s_i,\mathbf{p}_{g_i}) = E(\theta_i|s_i,\mathbf{E}_{g_i}) = \mathbf{w}_i\mathbf{s}$. That is, the collection of $\{\mathbf{w}_i\}_{i=1,\dots n}$ has to satisfy the system of n equations given by

$$\mathbf{w}_i\mathbf{s} = E(\theta_i \left| s_i, \{\mathbf{w}_j\mathbf{s}\}_{j\in g_i}\right.)$$

for every i. However, the collection $\{\mathbf{w}_i\}_{i=1,\dots n}$ that is a solution of this system, is also an equilibrium of the conditional guessing game by construction.

Lemma A.5 *Suppose that there exists a linear equilibrium in the OTC game in which the belief of each dealer i is given by*

$$E(\theta_i|s_i,\mathbf{p}_{g_i}) = y_i s_i + \sum_{j\in g_i} z_{ij}p_{ij}.$$

Then the equilibrium demand functions

$$Q_i^j(s_i;\mathbf{p}_{g_i}) = b_i^j s_i + \left(\mathbf{c_i^j}\right)^T \mathbf{p}_{g_i}$$

must be such that

$$\begin{aligned} b_i^j &= -\beta_{ij}\tfrac{2-z_{ji}}{z_{ij}+z_{ji}-z_{ij}z_{ji}}y_i \\ c_{ij}^j &= -\beta_{ij}\tfrac{2-z_{ji}}{z_{ij}+z_{ji}-z_{ij}z_{ji}}(z_{ij}-1) \\ c_{ik}^j &= -\beta_{ij}\tfrac{2-z_{ji}}{z_{ij}+z_{ji}-z_{ij}z_{ji}}z_{ik}. \end{aligned}$$

Proof. Taking into account that agents' beliefs have an affine structure and identifying coefficients in (8) we obtain that

$$\begin{aligned} b_i^j &= -\left(c_{ji}^i + \beta_{ij}\right) y_i \\ c_{ij}^j &= -\left(c_{ji}^i + \beta_{ij}\right)(z_{ij}-1) \\ c_{ik}^j &= -\left(c_{ji}^i + \beta_{ij}\right) z_{ik} \end{aligned}$$

for any i and $j \in g_i$. Therefore, for any pair ij that has a link in the network g, the following two equations must hold at the same time

$$\begin{aligned} c_{ij}^j &= -\left(c_{ji}^i + \beta_{ij}\right)(z_{ij}-1) \\ c_{ji}^i &= -\left(c_{ij}^j + \beta_{ij}\right)(z_{ji}-1). \end{aligned}$$

which implies that

$$\begin{aligned} c_{ij}^j &= \frac{(z_{ij}-1)(z_{ji}-2)}{z_{ij}+z_{ji}-z_{ij}z_{ji}}\beta_{ij} \\ c_{ji}^i &= \frac{(z_{ji}-1)(z_{ij}-2)}{z_{ij}+z_{ji}-z_{ij}z_{ji}}\beta_{ij}. \end{aligned}$$

A simple manipulations shows that

$$c_{ji}^{i}+\beta_{ij}=\frac{2-z_{ji}}{z_{ij}+z_{ji}-z_{ij}z_{ji}}\beta_{ij}$$

and

$$\frac{\left(c_{ji}^{i}+\beta_{ij}\right)}{c_{ji}^{i}+c_{ij}^{j}+3\beta_{ij}}=\frac{2-z_{ji}}{4-z_{ij}z_{ji}}.$$

■

A.3 Proof of Proposition 3 and Corollary 1

We show that given an equilibrium of the conditional-guessing game and the conditions of the proposition, we can always construct an equilibrium for the OTC game with beliefs given by

$$E(\theta_i\,|s_i,\mathbf{p}_{g_i})=E(\theta_i\,|s_i,\mathbf{e}_{g_i}).$$

To see this, consider an equilibrium of the conditional-guessing game in which

$$E(\theta_i\,|s_i,\mathbf{e}_{g_i})=\bar{y}_i s_i+\sum_{k\in g_i}\bar{z}_{ik}E(\theta_k\,|s_k,\mathbf{e}_{g_k})$$

for every i. If the system (14) has a solution, then

$$E(\theta_i\,|s_i,\mathbf{e}_{g_i})=\frac{y_i}{\left(1-\sum_{k\in g_i}z_{ik}\frac{2-z_{ki}}{4-z_{ik}z_{ki}}\right)}s_i+\sum_{k\in g_i}z_{ij}\frac{\frac{2-z_{ij}}{4-z_{ij}z_{ji}}}{\left(1-\sum_{k\in g_i}z_{ik}\frac{2-z_{ki}}{4-z_{ik}z_{ki}}\right)}E(\theta_k\,|s_k,\mathbf{e}_{g_k}) \tag{A.6}$$

holds for every realization of the signals, and for each i. Using that from Lemma A.5

$$\frac{2-z_{ki}}{4-z_{ik}z_{ki}}=\frac{\left(c_{ki}^{i}+\beta_{ik}\right)}{c_{ki}^{i}+c_{ik}^{k}+3\beta_{ik}},$$

we can rewrite (A.6) as

$$E(\theta_i\,|s_i,\mathbf{e}_{g_i})=y_i s_i+\sum_{k\in g_i}z_{ik}\frac{\left(c_{ki}^{i}+\beta_{ik}\right)E(\theta_i\,|s_i,\mathbf{e}_{g_i})+\left(c_{ik}^{k}+\beta_{ik}\right)E(\theta_k\,|s_k,\mathbf{e}_{g_k})}{c_{ki}^{i}+c_{ik}^{k}+3\beta_{ik}}.$$

Now we show that picking the prices and demand functions

$$p_{ij} = \frac{\left(c_{ki}^{i}+\beta_{ik}\right)E(\theta_i\,|s_i,\mathbf{e}_{g_i})+\left(c_{ik}^{k}+\beta_{ik}\right)E(\theta_k\,|s_k,\mathbf{e}_{g_k})}{c_{ki}^{i}+c_{ik}^{k}+3\beta_{ik}} \tag{A.7}$$

$$Q_i^j(s_i;\mathbf{p}_{g_i}) = -\left(c_{ji}^{i}+\beta_{ij}\right)\left(E(\theta_i\,|s_i,\mathbf{e}_{g_i})-p_{ij}\right) \tag{A.8}$$

is an equilibrium of the OTC game.

First note that this choice implies

$$E(\theta_i \left| s_i, \mathbf{e}_{g_i} \right.) = y_i s_i + \sum_{k \in g_i} z_{ik} p_{ij} = E(\theta_i \left| s_i, \mathbf{p}_{g_i} \right.). \tag{A.9}$$

The second equality comes from the fact that the first equality holds for any realization of signals and the projection theorem determines a unique linear combination with this property for a given set of jointly normally distributed variables. Thus, (A.8) for each ij link is equivalent with the corresponding first order condition (8). Finally, (A.9) also implies that the bilateral clearing condition between a dealer i and dealer j that have a link in network g

$$-\left(c_{ji}^i + \beta_{ij}\right)\left(E(\theta_i \left| s_i, \mathbf{p}_{g_i} \right.) - p_{ij}\right) - \left(c_{ij}^j + \beta_{ij}\right)\left(E(\theta_j \left| s_j, \mathbf{p}_{g_j} \right.) - p_{ij}\right) + \beta_{ij} p_{ij} = 0$$

is equivalent to (A.7). That concludes the statement.

Proof of Corollary 1.

This follows straightforwardly from Lemma A.5.Substituting b_i^j and c_{ij}^j in (16) and (17) we obtain that

$$\begin{aligned} p_{ij} &= \frac{(2 - z_{ji}) e_i + (2 - z_{ij}) e_j}{4 - z_{ij} z_{ji}} \\ Q_i^j(s_i; \mathbf{p}_{g_i}) &= -\frac{2 - z_{ji}}{z_{ij} + z_{ji} - z_{ij} z_{ji}} \beta_{ij} \left(e_i - p_{ij}\right). \end{aligned}$$

This implies that prices do not depend on β_{ij} and that equilibrium quantities are linear in β_{ij}. ■

A.4 Proof of Proposition 4

Case 1: Circulant networks

In circulant networks, it is easy to see that beliefs must be symmetric in the equilibrium of the conditional guessing game in the sense of

$$\bar{z}_{ij} = \bar{z}_{ji}.$$

First, we show that this implies that each $\bar{z}_{ij} \in [0, 1]$. To see this, consider any symmetric matrix X with each of its elements non-negative. Suppose that there is a $x_{ij} = x_{ji} \geq 1$ (including the case when i.and j are equal). It is easy to see that in X^2 the $i-th$ diagonal is

$$\sum_{k=1}^{n} x_{ik}^2 \geq 1.$$

As $\bar{Z}$ is also symmetric and non-negative element by element, a simple induction argument shows that there cannot be a $\bar{z}_{ij} \geq 1$ element, because in this case in all matrices $\bar{Z}^2, \bar{Z}^4, \bar{Z}^8 ... \bar{Z}^{2^k}$ there is at least one diagonal not smaller than 1. This would be in contradiction with the property that $Z^k \to 0$.

Now, we search for equilibria such that beliefs are symmetric, that is

$$z_{ij} = z_{ji}$$

for any pair ij that has a link in network g.

The system (14) becomes

$$\begin{aligned}
\frac{y_i}{\left(1 - \sum\limits_{k \in g_i} z_{ik} \frac{2 - z_{ik}}{4 - z_{ik}^2}\right)} &= \bar{y}_i \\
z_{ij} \frac{\frac{2 - z_{ij}}{4 - z_{ij}^2}}{\left(1 - \sum\limits_{k \in g_i} z_{ik} \frac{2 - z_{ik}}{4 - z_{ik}^2}\right)} &= \bar{z}_{ij}
\end{aligned}$$

for any $i \in \{1, 2, ..., n\}$. Working out the equation for z_{ij}, we obtain

$$\frac{z_{ij}}{2 + z_{ij}} = \bar{z}_{ij} \left(1 - \sum_{k \in g_i} \frac{z_{ik}}{2 + z_{ik}}\right)$$

and summing up for all $j \in g_i$

$$\sum_{j \in g_i} \frac{z_{ij}}{2 + z_{ij}} = \sum_{j \in g_i} \bar{z}_{ij} \left(1 - \sum_{k \in g_i} \frac{z_{ik}}{2 + z_{ik}}\right).$$

Denote

$$S_i \equiv \sum_{k \in g_i} \frac{z_{ik}}{2 + z_{ik}}.$$

Substituting above and summing again for $j \in g_i$

$$S_i \left(1 + \sum_{j \in g_i} \bar{z}_{ij}\right) = \sum_{j \in g_i} \bar{z}_{ij}$$

or

$$S_i = \frac{\sum\limits_{j \in g_i} \bar{z}_{ij}}{\left(1 + \sum\limits_{j \in g_i} \bar{z}_{ij}\right)}.$$

We can now obtain

$$z_{ij} = \frac{2\bar{z}_{ij}\left(1 - S_i\right)}{1 - \bar{z}_{ij}\left(1 - S_i\right)} \tag{A.10}$$

and

$$y_i = \bar{y}_i \left(1 - S_i\right).$$

Finally, the following logic show that $z_{ij} \leq 2$. As $\bar{z}_{ij} < 1$, $2\bar{z}_{ij} < \left(1 + \sum_{j \in g_i} \bar{z}_{ij}\right)$ implying that $2\bar{z}_{ij}\left(1 - S_i\right) < 1$ or $2\bar{z}_{ij}\left(1 - S_i\right) < 2\left(1 - \bar{z}_{ij}\left(1 - S_i\right)\right)$, which gives the result by A.10.

Case 2: Core-periphery networks

There exist at least one equilibrium of the conditional guessing game such that for all i, j in the core

$$\bar{z}_{ij} = \bar{z}_{ji} = \bar{z}_c \tag{A.11}$$

and for all player i in the periphery

$$\begin{aligned} \bar{z}_{ij} &= \bar{z}_p \\ \bar{z}_{ji} &= \bar{z}_{cp} \end{aligned} \tag{A.12}$$

Then the system (14) becomes

$$\frac{y_c}{1 - (r-1)z_c \frac{2-z_c}{4-z_c^2} - \frac{n-r}{r} z_{cp} \frac{2-z_p}{4-z_{cp}z_p}} = \bar{y}_c \tag{A.13}$$

$$z_c \frac{\frac{2-z_c}{4-z_c^2}}{1 - (r-1)z_c \frac{2-z_c}{4-z_c^2} - \frac{n-r}{r} z_{cp} \frac{2-z_p}{4-z_{cp}z_p}} = \bar{z}_c \tag{A.14}$$

$$z_{cp} \frac{\frac{2-z_{cp}}{4-z_c z_{cp}}}{1 - (r-1)z_c \frac{2-z_c}{4-z_c^2} - \frac{n-r}{r} z_{cp} \frac{2-z_p}{4-z_{cp}z_p}} = \bar{z}_{cp} \tag{A.15}$$

for agents in the core and

$$\frac{y_p}{1 - z_p \frac{2-z_{cp}}{4-z_p z_{cp}}} = \bar{y}_p \tag{A.16}$$

$$z_p \frac{\frac{2-z_p}{4-z_p z_{cp}}}{1 - z_p \frac{2-z_{cp}}{4-z_p z_{cp}}} = \bar{z}_p \tag{A.17}$$

for agents in the periphery.

From equation (A.17) it is easy to see that

$$z_p = 2\bar{z}_p \tag{A.18}$$

Substituting this back to the equations (A.14) and (A.15) we get

$$z_c \frac{2 - z_c}{4 - z_c^2} = \bar{z}_c \left(1 - (r-1)z_c \frac{2 - z_c}{4 - z_c^2} - \frac{n-r}{r} z_{cp} \frac{2 - 2\bar{z}_p}{4 - 2z_{cp}\bar{z}_p}\right) \tag{A.19}$$

$$z_{cp} \frac{2 - z_{cp}}{4 - 2z_{cp}\bar{z}_p} = \bar{z}_{cp} \left(1 - (r-1)z_c \frac{2 - z_c}{4 - z_c^2} - \frac{n-r}{r} z_{cp} \frac{2 - 2\bar{z}_p}{4 - 2z_{cp}\bar{z}_p}\right) \tag{A.20}$$

From (A.19)

$$z_c \frac{2 - z_c}{4 - z_c^2} = \frac{\bar{z}_c(4 - 2z_{cp}\bar{z}_p) - \frac{n-r}{r} z_{cp}\bar{z}_c(2 - 2\bar{z}_p)}{(1 + (r-1)\bar{z}_c)(4 - 2z_{cp}\bar{z}_p)} \tag{A.21}$$

Substituting this back to (A.20) one gets

$$(1+(r-1)\bar{z}_c)z_{cp}^2+\left(\bar{z}_{cp}(2\bar{z}_p-2)\frac{n-r}{r}-2-2\bar{z}_p\bar{z}_{cp}-2(r-1)\bar{z}_c\right)z_{cp}+4\bar{z}_{cp}=0 \quad \text{(A.22)}$$

what has got real solution if the discriminant is non-negative for any $\bar{z}_p, \bar{z}_c, \bar{z}_{cp} \in [0,1]$, so when

$$\left(\bar{z}_{cp}(2\bar{z}_p-2)\frac{n-r}{r}-2-2\bar{z}_p\bar{z}_{cp}-2(r-1)\bar{z}_c\right)^2-16\bar{z}_{cp}-16(r-1)\bar{z}_p\bar{z}_{cp}\geq 0 \quad \text{(A.23)}$$

This discriminant is decreasing in $\bar{z}_p$ if $\bar{z}_p \in [0,1]$ and non-negative for $\bar{z}_p = 1$ therefore the equation has real solution.

Note also, that at $z_{cp} = 2$, (A.22) equals $4\bar{z}_{cp}\left(\bar{z}_p-1\right)\frac{n-2r}{r}$, which is negative for $\bar{z}_p \in [0,1]$. As the intercept of this quadratic equation is positive, this implies that when it has a real solution, it also has a real solution for which $z_{cp} < 2$.

We have to check whether $z_c < 2$ only to the case of $r \geq 2$, as when $r = 1$ this coefficient does not exist. To see that for $r \geq 2$, $z_c < 2$ indeed holds, substituting the solution z_{cp} back to (A.21) is gives

$$z_c = 2\frac{\frac{\bar{z}_c(4-2z_{cp}\bar{z}_p)-\frac{n-r}{r}z_{cp}\bar{z}_c(2-2\bar{z}_p)}{(1+(r-1)\bar{z}_c)(4-2z_{cp}\bar{z}_p)}}{1-\frac{\bar{z}_c(4-2z_{cp}\bar{z}_p)-\frac{n-r}{r}z_{cp}\bar{z}_c(2-2\bar{z}_p)}{(1+(r-1)\bar{z}_c)(4-2z_{cp}\bar{z}_p)}}.$$

It is easy to see that $z_c < 2$ is equivalent to

$$-(4-2z_{cp}\bar{z}_p)\bar{z}_c\left(\frac{1}{2}(r-2)\right)-\frac{n-r}{r}z_{cp}\bar{z}_c(2-2\bar{z}_p)<0$$

which always holds for $r \geq 2$.

Finally, given z_c, the solutions for (A.13) and (A.16) follow immediately.

A.5 Proof of Lemma 1

1. Suppose network g is connected. This implies that between any two agents i and j, there exists a sequence of dealers $\{i_1, i_2, ..., i_r\}$ such that $ii_1 \in g$, $i_k i_{k+1} \in g$, and $i_r j \in g$ for any $k \in \{1, 2, ..., r\}$. The sequence $\{i_1, i_2, ..., i_r\}$ forms a *path* between i and j. The length of this path, r, represents the distance between i and j.

   Let V^* be the fixed point, as introduced in the proof of Proposition 1. Then, the equilibrium guess vector is given by

   $$\mathbf{e} = V^*\mathbf{s}.$$

   Suppose that there exists an equilibrium

   $$v_{ij}^* = 0$$

for some i and j at distance r from each other. Then from (A.5) if follows that

$$\sum_{k \in g_i} \omega_{ik} \frac{v_{kj}^*}{\left(\mathbf{v}_k^*\right)^T \mathbf{1}} = 0,$$

and, since $\omega_{ik} > 0$ for $\forall i, k \in \{1, 2, ..., n\}$, then it must be that

$$v_{kj}^* = 0, \ \forall k \in g_i.$$

This means that all the neighbors of agent i place 0 weight on j's information. Further, this implies

$$\sum_{l \in g_k} \omega_{il} \frac{v_{lj}^*}{\left(\mathbf{v}_l^*\right)^T \mathbf{1}} = 0,$$

and

$$v_{lj}^* = 0, \ \forall l \in g_k.$$

Hence, all the neighbors and the neighbors of the neighbors of agent i place 0 weight on j's information. We can iterate the argument for r steps, and show that it must be that any agent at distance at most r from i places 0 weight on j's information. Since the distance between i and j is r, then

$$v_{jj}^* = 0,$$

which is a contradiction, since (A.4) must hold and $\rho < 1$ ($\sigma_\eta^2 > 0$). This concludes the first part of the proof.

2. See Case 2 in the proof of Proposition 1.

A.6 Proof of Proposition 6

1. From Proposition 2 we know that in any equilibrium of the OTC game

$$E(\theta_i | s_i, \mathbf{p}_{g_i}) = E(\theta_i | s_i, \mathbf{e}_{g_i}).$$

Lemma 1 shows that each equilibrium expectation in the conditional guessing game is a linear combination of all signals in the economy

$$E(\theta_i | s_i, \mathbf{e}_{g_i}) = \mathbf{v}_i \mathbf{s}$$

where $\mathbf{v}_i > 0$ for all i. Since the equilibrium price in any trade between two dealers i and j is a weighted sum of their respective beliefs, as in (16), and the weights are positive, then the result follows immediately.

2. As $\rho \to 1$, we show that there exists an equilibrium such that

$$\lim_{\rho \to 1} E\left(\theta_i | s_i, \mathbf{p}_{g_i}\right) = v^* \sum_{i=1}^{n} s_i, \ \forall i \in \{1, 2, ..., n\}$$

where $v^* = \frac{\sigma_\theta^2}{n\sigma_\theta^2 + \sigma_\varepsilon^2}$.

If there exists an equilibrium in the OTC game, then it follows from the proof of Proposition 1 that

$$E(\theta_i \,| s_i, \mathbf{p}_{g_i}) = \bar{y}_i s_i + \sum_{k \in g_i} \bar{z}_{ik} E(\theta_k \,| s_k, \mathbf{p}_{g_k}).$$

or

$$E(\theta_i \,| s_i, \mathbf{e}_{g_i}) = \bar{y}_i s_i + \sum_{k \in g_i} \bar{z}_{ik} E(\theta_k \,| s_k, \mathbf{e}_{g_k}).$$

Taking the limit as $\rho \to 1$, and using Case 2 in the proof of Proposition 1, we have that

$$\lim_{\rho \to 1} E\left(\theta_i | s_i, \mathbf{p}_{g_i}\right) = \frac{\sigma_\theta^2}{n\sigma_\theta^2 + \sigma_\varepsilon^2} \sum_{i=1}^{n} s_i.$$

Given that

$$\lim_{\rho \to 1} E\left(\theta_i | s_i, \mathbf{p}_{g_i}\right)$$

The conditional variance is

$$\mathcal{V}\left(\theta_i | s_i, \mathbf{p}_{g_i}\right) = \sigma_\theta^2 - \mathcal{V}\left(E\left(\theta_i | s_i, \mathbf{p}_{g_i}\right)\right)$$

and taking the limit $\rho \to 1$, we obtain

$$\lim_{\rho \to 1} \mathcal{V}\left(\theta_i | s_i, \mathbf{p}_{g_i}\right) = \sigma_\theta^2 - \left(\frac{\sigma_\theta^2}{n\sigma_\theta^2 + \sigma_\varepsilon^2}\right)^2 n\left(\sigma_\varepsilon^2 + n\sigma_\theta^2\right).$$

and

$$\begin{aligned}
\lim_{\rho \to 1} \mathcal{V}\left(\theta_i | \mathbf{s}\right) &= \sigma_\theta^2 - \mathcal{V}\left(E\left(\hat{\theta} | \mathbf{s}\right)\right) \\
&= \sigma_\theta^2 - \left(\frac{\sigma_\theta^2}{n\sigma_\theta^2 + \sigma_\varepsilon^2}\right)^2 n\left(\sigma_\varepsilon^2 + n\sigma_\theta^2\right) \\
&= \sigma_\theta^2 \frac{\sigma_\varepsilon^2}{n\sigma_\theta^2 + \sigma_\varepsilon^2}
\end{aligned}$$

A.7 Proof of Proposition 7

Dealers revise their messages according to the rule that

$$h_{i,t} = \bar{y}_i s_i + \bar{\mathbf{z}}_{g_i}^T \mathbf{h}_{g_i, t-1}, \ \forall i.$$

or, in matrix form

$$\mathbf{h}_{t+1} = \bar{Y}\mathbf{s} + \bar{Z}\mathbf{h}_t.$$

1. Since $\mathbf{h}_{t_0} = \left(I - \bar{Z}\right)^{-1}\bar{Y}\mathbf{s}$, then

$$\begin{aligned}
\mathbf{h}_{t_0+1} &= \bar{Y}\mathbf{s} + \bar{Z}\mathbf{h}_{t_0} \\
&= \bar{Y}\mathbf{s} + \bar{Z}\left(I - \bar{Z}\right)^{-1}\bar{Y}\mathbf{s} \\
&= \bar{Y}\mathbf{s} + \left(I - \left(I - \bar{Z}\right)\right)\left(I - \bar{Z}\right)^{-1}\bar{Y}\mathbf{s} \\
&= \bar{Y}\mathbf{s} + \left(I - \bar{Z}\right)^{-1}\bar{Y}\mathbf{s} - \bar{Y}\mathbf{s} \\
&= \left(I - \bar{Z}\right)^{-1}\bar{Y}\mathbf{s}
\end{aligned}$$

   It follows straightforwardly, from an inductively argument that

$$\mathbf{h}_t = \left(I - \bar{Z}\right)^{-1}\bar{Y}\mathbf{s}.$$

2. From

$$\mathbf{h}_{t_0+1} = \bar{Y}\mathbf{s} + \bar{Z}\mathbf{h}_{t_0}$$

   it follows that

$$\mathbf{h}_{t_0+n} = \left(I + \bar{Z} + ... + \bar{Z}^{n-1}\right)\bar{Y}\mathbf{s} + \bar{Z}^n\mathbf{h}_{t_0}$$

   In the limit as $n \to \infty$, from Proposition 1 we know that

$$\lim_{n\to\infty} \mathbf{h}_{t_0+n} = \left(I - \bar{Z}\right)^{-1}\bar{Y}\mathbf{s}.$$

   This implies that for any vector $\boldsymbol{\gamma} \in R_+^n$, there exists an $n_{\boldsymbol{\gamma}}$ such that

$$\left|\mathbf{h}_{t_0+n} - \left(I - \bar{Z}\right)^{-1}\bar{Y}\mathbf{s}\right| < \boldsymbol{\gamma}, \forall n \geq n_{\boldsymbol{\gamma}}.$$

   Fix an arbitrarily small vector $\boldsymbol{\gamma}$. Then

$$-\boldsymbol{\gamma} < \left(I - \bar{Z}\right)^{-1}\bar{Y}\mathbf{s} - \mathbf{h}_{t_0+n_{\boldsymbol{\gamma}}} < \boldsymbol{\gamma}$$

   and

$$-\boldsymbol{\gamma} < \mathbf{h}_{t_0+n} - \left(I - \bar{Z}\right)^{-1}\bar{Y}\mathbf{s} < \boldsymbol{\gamma}, \forall n \geq n_{\boldsymbol{\gamma}}.$$

   Adding up these two inequalities we have that

$$-2\boldsymbol{\gamma} < \mathbf{h}_{t_0+n} - \mathbf{h}_{t_0+n_{\boldsymbol{\gamma}}} < 2\boldsymbol{\gamma}, \forall n \geq n_{\boldsymbol{\gamma}}.$$

   This shows that there exists $\boldsymbol{\delta} = 2\boldsymbol{\gamma}$ and $t_{\boldsymbol{\delta}} = t_0 + n_{\boldsymbol{\gamma}}$ such that

$$|\mathbf{h}_t - \mathbf{h}_{t_{\boldsymbol{\delta}}}| < \boldsymbol{\delta}, \forall t \geq t_{\boldsymbol{\delta}}.$$

   which implies that the protocol stops at $t_{\boldsymbol{\delta}}$.

3. We start by observing that

$$E\left(\theta_i | s_i, \mathbf{h}_{g_i,t_0}, \mathbf{h}_{g_i,t_0+1}, ..., \mathbf{h}_{g_i,t_0+n}\right) = E\left(\theta_i | s_i, \mathbf{h}_{g_i,t_0+n}\right), \forall n \geq 0.$$

Further, in the limit $n \to \infty$, we have that

$$\lim_{n\to\infty} \mathbf{h}_{t_0+n} = \left(I - \bar{Z}\right)^{-1} \bar{Y}\mathbf{s} = \mathbf{e},$$

and subsequently

$$\lim_{n\to\infty} \mathbf{h}_{g_i,t_0+n} = \mathbf{e}_{g_i}, \forall i.$$

Then

$$\lim_{n\to\infty} E\left(\theta_i | s_i, \mathbf{h}_{g_i,t_0+n}\right) = E\left(\theta_i | s_i, \mathbf{e}_{g_i}\right) = E\left(\theta_i | s_i, \mathbf{p}_{g_i}\right).$$

As above, we can construct $t_{\boldsymbol{\delta}}$ such that protocol stops and show that

$$\left|E\left(\theta_i | s_i, \mathbf{h}_{g_i,t_0+n}\right) - E\left(\theta_i | s_i, \mathbf{p}_{g_i}\right)\right| < \frac{1}{2}\boldsymbol{\delta}.$$

B Appendix: Numerical examples and more comparative statics

B.1 Examples

In this part, we compare trading and information diffusion in the two simple examples of networks. In particular, we compare the equilibrium when seven dealers with the same realizations of signals trade in a (7,2)-circulant network (i.e. a circle) versus in a 7-star. We use Figure 7 and Table 1 to illustrate the analysis.

The two graphs illustrate the equilibrium of the OTC game in a $(7,2)$ circulant network (panel (a)) and in a $7-$star network (panel (b)). In each case, $\theta_i = 0$ for each dealer and $s_i = i - 4$, thus blue (red) nodes denote pessimist (optimist) dealers.

In this example, the realization of the value of each of the 7 dealers is $\theta_i = 0$, but they receive the signals

$$s_i = i - 4.$$

That is, their signals are ordered as their index is, dealer seven is the most optimistic, dealer 1 is the most pessimistic and dealer 4 is just right. To visualize this, in Figure 7 nodes corresponding to pessimist (optimist) dealers are blue (red). For illustrative purposes, we placed pessimists and optimists on nodes in a way that sometimes dealers with very large informational differences trade. The price, p_{ij} of each transaction is in the rhombi on the link. The quantities traded by each of the counterparties, q_j^j, q_j^i and the corresponding profits a given trader makes on the given transaction are in the rectangles near the links. Profits are in brackets. Each number is rounded to the nearest decimal.

For example, when dealers 6 and 3 trade in a circle network, dealer 6 takes the long position of $q_6^3 = 2.3$ at price $p_{36} = 0.1$ leading to the loss of

$$q_6^3\left(\theta_6 - p_{36}\right) = (0 - 0.1)(2.3) \approx -0.2$$

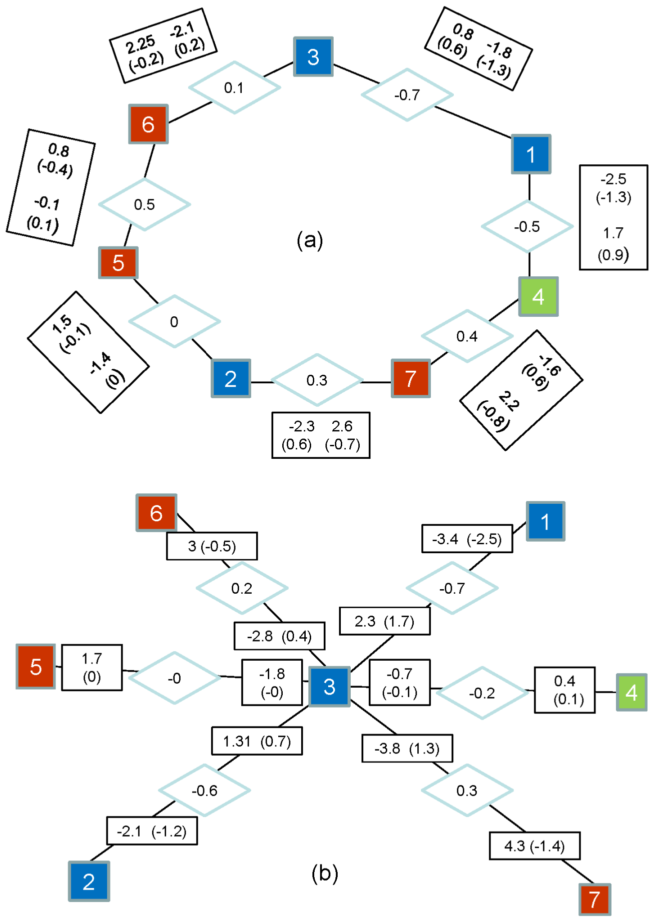

Figure 7: Values in the rhombi on each edge are equilbrium prices. The quanty q_{ij} and the corresponding profit earned (in brackets) by a given dealer in a given trade are in the rectangles. Other parameter values are $\rho = 0.5$, $\sigma_\theta^2 = \sigma_\varepsilon^2 = 1$, $\beta_{ij} = -10$.

Dealer	s_i	$E(\theta_i \vert s_i, \mathbf{p}_{g_i})$		$\mathbf{1}^T \mathbf{q}_i$		$\mathbf{1}^T \vert\mathbf{q}_i\vert$		$\mathbf{1}^T \mathbf{q}_i (\theta_i - \mathbf{p}_{g_i})$	
		circle	star	circle	star	circle	star	circle	star
3	-1	-0.5	-0.3	-1.3	-5.4	2.9	12.7	0.8	4
1	-3	-1.2	-1.3	-4.3	-3.4	4.3	3.4	-2.7	-2.5
4	0	-0.1	-0.1	0.1	0.4	3.3	0.4	1.6	0.1
7	3	1	1.1	4.8	4.3	4.8	4.3	-1.5	-1.4
2	-2	-0.4	-0.9	-3.7	-2.1	3.7	2.1	0.6	-1.2
5	1	0.5	0.3	1.4	1.7	1.6	1.7	0	0
6	2	0.7	0.7	3.1	3	3.1	3	-0.6	-0.5

Table 1: The table summarizes the signal, the expectation, the net position, the gross position and the total profit if each dealer in the two cases when they arranged into a (7,2)-circle network and a 7-star network.

While dealer 3 takes the short position of $q_3^1 = 0.8$ at the same price leading to a profit of 0.6.

In Table 1, we summarize the signal, the expectation, the net position, the gross position and the total profit if each dealer. Looking at the Figure 7 and Table 1, we can make several intuitive observations.

First, while in most transactions one of the counterparties gain and the other loose, the profits and quantities do not add up to zero. It is because the excess demand or supply is the quantity $\beta_{ij} p_{ij}$ bought by the consumers.

Second, when a dealer has counterparties both whom are more and less optimistic than her, this dealer will intermediate trades between them. That is, the size of her gross positions will be larger than the size of her net positions. This is the case of dealer 3, 4 and 5 in the circle network. In the star network, dealer 3 is in an ideal position to intermediate trades. This is explains her enormous gross position compared to others in that network.

Third, the profits dealer make, apart from the accuracy of their information, depends heavily on their position in the network. For example, dealer 4 is very lucky in the circle network not only, because her information is accurate, but also because she turns out to be connected with two dealers of extreme opinion. So she can take both a large long and a large short position of almost equal sizes and make a large profit on both. That explains why she makes the largest profit in the circle network. However, in the star network her profit is much smaller than the profit of the central dealer 3, even if the guess of dealer 3 is less accurate.

Forth, the after-trade expectation of each dealer depends both on the shape of the network and her position in it. That is apparent in Table 1 by comparing $E(\theta_i|s_i, \mathbf{p}_{g_i})$ across networks. In general, the extent of adjustment of their post trade expectations compared to their pre-trade expectations depends on the degree of pessimism and optimism of their counterparties and also on how much their counterparties know. For example, dealer 6 arrives to the same expectation in the two networks because the effect of learning from more agents in the circle network is offset by the effect of learning from the better informed dealer 3 in the star network.

B.2 Profits and intermediation in circulants

In this part we illustrate how changes in the information structures affect profit and intermediation in we check how the total expected profit and expected intermediation changes with the level of correlation across values in these networks. The top panels in Figure 8 show expected total profit in the $(11, m)$-circulant networks and the $11-$star network as ρ changes (right panel) and as σ_θ^2 increases (left panel).[20]

Expected profits tend to decrease with the correlation across values, and when the fundamental uncertainty is lower. The reason is that as correlation decreases (the fundamental uncertainty increases), agents are less worried about adverse selection and trade more aggressively. This enforces the intuition of section B that intermediating trades is extremely profitable. To see this point better, the top panels show the expected intermediation, that is, the ratio of absolute net positions to total gross positions,

$$E\left(\frac{\left|\sum_{j\in g_i} q_i^j\right|}{\sum_{j\in g_i}\left|q_i^j\right|}\right),$$

for each of these networks. The smaller is this measure, the larger is the intermediation level. As we see, intermediation increases in m in the (n, m) circulants. The reason is that even in the complete network prices for each transaction are different, and as agents are better informed, they take larger positions. Intermediation levels are relatively high for intermediate level of ρ. However, intermediation decreases as σ_ε^2 increases, which is consistent with the drop in trading activity during the financial crisis.

C Appendix: Analytical solutions for the star, circle and complete networks

In this section we provide a step by step derivation of the equilibrium in closed form for the star and the complete network. The procedure has been outlined in Section 3.3. For both the star and complete network we first derive the parameters $\bar{y}_i$ and $\bar{z}_{ij}$. Then conditions (14) imply parameters y_i and z_{ij}, then (15) give parameters of the demand function implying prices and quantities by (16)-(17).

Star networks

Without loss of generality, we characterize a star network with dealer 1 at the centre. Dealer 1 chooses her demand function conditional on the beliefs of the other $(n-1)$ dealers. Given that she knows s_1, she can invert the signals of all the other dealers. Hence, her belief is given by

$$E(\theta_1|s_1, \mathbf{e}_{g_1}) = E(\theta_1|\mathbf{s}) = \sigma_\theta^2 \frac{1-\rho}{\sigma_\theta^2+\sigma_\varepsilon^2-\rho\sigma_\theta^2}\left(s_1 + \frac{\rho\sigma_\varepsilon^2}{(1-\rho)\left(\sigma_\theta^2+\sigma_\varepsilon^2-\rho\sigma_\theta^2+n\rho\sigma_\theta^2\right)}\sum_{i=1}^n s_i\right).$$

[20] We keep the sum $\Sigma_i\Sigma_{g_i}\beta_{ij}$ constant. Intuitively, this is equivalent of dividing a constant mass of consumers across the various links in different networks. We do this to make sure that it is not the increasing number of consumers drive the results.

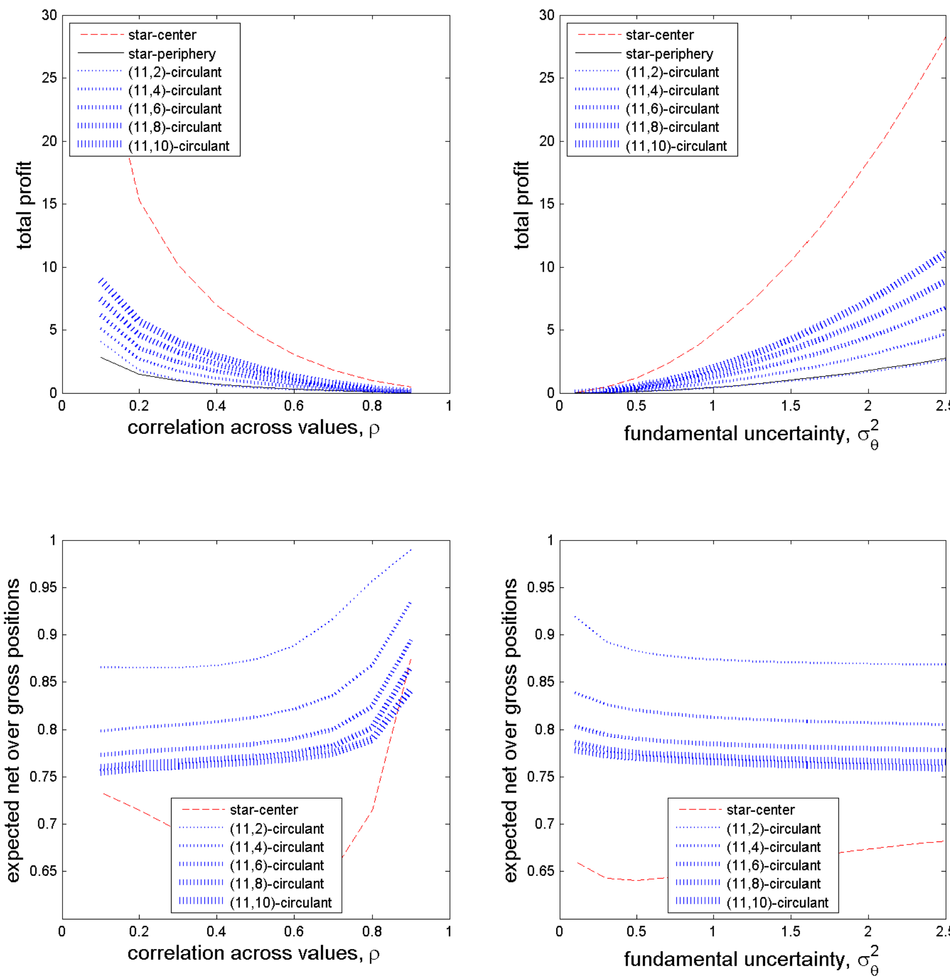

Figure 8: The plots compare the total profits and expected intermediation when dealers trade in (11,m)-circulant networks and in star-networks as ρ increases (left panels) and as σ_θ^2 increases (right panels). Whenever they are kept fixed, other parameters are $n = 11$, $\sigma_\epsilon^2 = \sigma_\theta^2 = 1$ and $\rho = 0.5$ The sum of the coefficients β_{ij} over all links are kept constant across each network at -10.

Then, the belief of a periphery dealer i is given by

$$E(\theta_i|s_i,e_1)=\begin{pmatrix}\sigma_\theta^2\\ \mathcal{V}(\theta_i,e_1)\end{pmatrix}^T\begin{pmatrix}\sigma_\theta^2+\sigma_\varepsilon^2 & \mathcal{V}(s_i,e_1)\\ \mathcal{V}(s_i,e_1) & \mathcal{V}(e_1)\end{pmatrix}^{-1}\begin{pmatrix}s_i\\ e_1\end{pmatrix},$$

where

$$\mathcal{V}(e_1)=\sigma_\theta^4\frac{\sigma_\theta^2(1-\rho)(1+(n-1)\rho)+\sigma_\varepsilon^2\left(1+(n-1)\rho^2\right)}{\left(\sigma_\theta^2+\sigma_\varepsilon^2-\rho\sigma_\theta^2\right)\left(\sigma_\theta^2+\sigma_\varepsilon^2-\rho\sigma_\theta^2+n\rho\sigma_\theta^2\right)}$$

$$\mathcal{V}(s_i,e_1)=\rho\sigma_\theta^2$$

$$\begin{aligned}\mathcal{V}(\theta_i,e_1)&=\rho\sigma_\theta^4\frac{\sigma_\theta^2(1-\rho)(1+(n-1)\rho)+\sigma_\varepsilon^2(2+(n-2)\rho)}{\left(\sigma_\theta^2+\sigma_\varepsilon^2-\rho\sigma_\theta^2\right)\left(\sigma_\theta^2+\sigma_\varepsilon^2-\rho\sigma_\theta^2+n\rho\sigma_\theta^2\right)}\\&=\mathcal{V}(e_1)-\sigma_\theta^4\frac{(1-\rho)^2}{\sigma_\theta^2+\sigma_\varepsilon^2-\rho\sigma_\theta^2}.\end{aligned}$$

This implies that

$$E(\theta_i|s_i,e_1)=\sigma_\theta^2\frac{\mathcal{V}(e_1)-\mathcal{V}(\theta_i,e_1)\rho}{\mathcal{V}(e_1)\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)-\rho^2\sigma_\theta^4}s_i+\frac{\mathcal{V}(\theta_i,e_1)\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)-\rho\sigma_\theta^4}{\mathcal{V}(e_1)\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)-\rho^2\sigma_\theta^4}e_1$$

Keeping the same notation from the proof of Proposition (4), then we have[21]

$$\begin{aligned}\bar{y}_p&=\sigma_\theta^2\frac{\mathcal{V}(e_1)-\mathcal{V}(\theta_i,e_1)\rho}{\mathcal{V}(e_1)\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)-\rho^2\sigma_\theta^4}\\ \bar{z}_p&=\frac{\mathcal{V}(\theta_i,e_1)\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)-\rho\sigma_\theta^4}{\mathcal{V}(e_1)\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)-\rho^2\sigma_\theta^4}\end{aligned}$$

Moreover, since

$$e_1=E(\theta_1|s_1,\mathbf{e}_{g_1})=\bar{y}_c s_1+\sum_{j=2}^{n}\bar{z}_c e_j=\bar{y}_c s_1+\sum_{j=2}^{n}\bar{z}_c\left(\bar{y}_p s_i+\bar{z}_p e_1\right)$$

then

$$E(\theta_1|s_1,\mathbf{e}_{g_1})=\frac{\bar{y}_c}{1-(n-1)\bar{z}_c\bar{z}_p}s_1+\sum_{j=2}^{n}\frac{\bar{z}_c\bar{y}_p}{1-(n-1)\bar{z}_c\bar{z}_p}s_i$$

This implies that

$$\bar{z}_c=\frac{v_{cp}}{\bar{y}_p+(n-1)\bar{z}_p v_{cp}}$$

where

$$v_{cp}=\sigma_\theta^2\frac{1-\rho}{\sigma_\theta^2+\sigma_\varepsilon^2-\rho\sigma_\theta^2}\frac{\rho\sigma_\varepsilon^2}{(1-\rho)\left(\sigma_\theta^2+\sigma_\varepsilon^2-\rho\sigma_\theta^2+n\rho\sigma_\theta^2\right)}$$

[21] In addition, since $-\sigma_\theta^4\frac{(\rho-1)^2}{\sigma_\theta^2+\sigma_\varepsilon^2-\rho\sigma_\theta^2}<0\Leftrightarrow\left(\mathcal{V}(\theta_i,e_1)-\mathcal{V}(e_1)\right)\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)<0$, then $\frac{\mathcal{V}(\theta_i,e_1)\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)-\rho\sigma_\theta^4}{\mathcal{V}(e_1)\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)-\rho^2\sigma_\theta^4}<1$, or $\bar{z}_p<1$.

and

$$\bar{y}_c = \frac{v_c \bar{y}_p}{\bar{y}_p + (n-1)\,\bar{z}_p v_{cp}}$$

where

$$v_c = \sigma_\theta^2 \frac{1-\rho}{\sigma_\theta^2 + \sigma_\varepsilon^2 - \rho\sigma_\theta^2}\left(1 + \frac{\rho\sigma_\varepsilon^2}{(1-\rho)\left(\sigma_\theta^2 + \sigma_\varepsilon^2 - \rho\sigma_\theta^2 + n\rho\sigma_\theta^2\right)}\right).$$

Further, the parameters y_i, and z_{ij} are given by the equations (A.13) and (A.14) for $r = 1$ for the central dealer, and by equations (A.16) and (A.17) for the periphery dealers.

Complete network

In the complete network, each dealer i chooses her demand function conditional on the beliefs of the other $(n-1)$ dealers. Given that she knows s_i, she can invert the signals of all the other dealers. Hence, her belief is given by

$$E(\theta_i|s_i, \mathbf{e}_{g_i}) = E(\theta_i|\mathbf{s}) = \sigma_\theta^2 \frac{1-\rho}{\sigma_\theta^2 + \sigma_\varepsilon^2 - \rho\sigma_\theta^2}\left(s_i + \frac{\rho\sigma_\varepsilon^2}{(1-\rho)\left(\sigma_\theta^2 + \sigma_\varepsilon^2 - \rho\sigma_\theta^2 + n\rho\sigma_\theta^2\right)}\sum_{i=1}^{n} s_i\right).$$

Then, following the same procedure as above (for a star), and taking into account that in a star trading strategies are symmetric, we obtain that

$$E(\theta_i|s_i, \mathbf{e}_{g_i}) = \bar{y}s_i + \bar{z}\sum_{\substack{j=1\\ j\neq i}}^{n} e_j$$

where

$$e_j = E(\theta_j|s_j, \mathbf{e}_{g_j})$$

and

$$\begin{aligned}
\bar{y} &= \frac{\sigma_\theta^2(1-\rho)(1+(n-1)\rho)}{\sigma_\theta^2 + \sigma_\varepsilon^2 + \rho^2\sigma_\theta^2 - 2\rho\sigma_\theta^2 - 2\rho\sigma_\varepsilon^2 - n\rho^2\sigma_\theta^2 + n\rho\sigma_\theta^2 + n\rho\sigma_\varepsilon^2}\\
\bar{z} &= \frac{\rho\sigma_\varepsilon^2}{\sigma_\theta^2 + \sigma_\varepsilon^2 + \rho^2\sigma_\theta^2 - 2\rho\sigma_\theta^2 - 2\rho\sigma_\varepsilon^2 - n\rho^2\sigma_\theta^2 + n\rho\sigma_\theta^2 + n\rho\sigma_\varepsilon^2}.
\end{aligned}$$

Solving the system (14), we obtain

$$\begin{aligned}
y_i &= \frac{\sigma_\theta^2(1-\rho)(1+(n-1)\rho)}{\left(\sigma_\theta^2(1-\rho)(1+(n-1)\rho) + \sigma_\varepsilon^2(1+2(n-3)\rho)\right) + 3\rho\sigma_\varepsilon^2}, \forall i\\
z_{ij} &= \frac{2\rho\sigma_\varepsilon^2}{\left(\sigma_\theta^2(1-\rho)(1+(n-1)\rho) + \sigma_\varepsilon^2(1+2(n-3)\rho)\right) + 2\rho\sigma_\varepsilon^2}, \forall ij.
\end{aligned}$$

Substituting in the expressions for b_i^j, c_{ij}^j, and c_{ij}^k, respectively, in Proposition (3) we obtain

$$
\begin{aligned}
b_i^j &= -\frac{\beta_{ij}}{2}\frac{\sigma_\theta^2(1-\rho)}{\rho\sigma_\varepsilon^2}\frac{(1+(n-1)\rho)\left(\sigma_\theta^2(1-\rho)(1+(n-1)\rho)+(1+2n\rho-4\rho)\sigma_\varepsilon^2\right)}{\sigma_\theta^2(1-\rho)(1+(n-1)\rho)+(1+2n\rho-3\rho)\sigma_\varepsilon^2} \\
c_{ij}^i &= \beta_{ij}\frac{1}{2\rho\sigma_\varepsilon^2}\left(\sigma_\theta^2(1-\rho)(1+(n-1)\rho)+\sigma_\varepsilon^2(1+2(n-3)\rho)\right) \\
c_{ij}^k &= -\beta_{ij}.
\end{aligned}
$$

Circle network

In a circle network the equilibrium is symmetric. That is beliefs are symmetric with

$$
\begin{aligned}
y_i &= y, \forall i \\
z_{ij} &= z. \forall ij
\end{aligned}
$$

and demand functions are symmetric with

$$
\begin{aligned}
b_i^j &= b, \forall i, j \\
c_{ij}^j &= c, \forall i, j \\
c_{ik}^j &= d, \forall i, k.
\end{aligned}
$$

Taking $\beta_{ij} = \beta$ for any ij, from the FOC we have that

$$
\begin{aligned}
b &= -(c+\beta)y \\
c &= -(c+\beta)(z-1) \\
d &= -\beta.
\end{aligned}
$$

where

$$
z = -\frac{\rho\sigma_\theta^2\sigma_\varepsilon^2\left(-\frac{4}{2+C}+\frac{1}{n}\sum_{j=0}^{n-1}\frac{2+\lambda_j}{C+\lambda_j}\right)}{\left(\frac{b}{m}\right)\left(\begin{array}{c}\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\rho\sigma_\theta^2\frac{(4)^2}{(2+C)^2}+\left((1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2\right)\frac{1}{n}\sum_{j=0}^{n-1}\left(\frac{\lambda_j+2}{\lambda_j+C}\right)^2\right) \\ -\left(2\rho\sigma_\theta^2\frac{2}{2+C}+\left((1-\rho)\sigma_\theta^2+\sigma_\varepsilon^2\right)\frac{1}{n}\sum_{j=0}^{n-1}\frac{2+\lambda_j}{C+\lambda_j}\right)^2\end{array}\right)}
$$

$$y=-\frac{\begin{pmatrix}\left(\rho\sigma_\theta^2\frac{4}{2+C}+(1-\rho)\,\sigma_\theta^2\frac{1}{n}\sum_{j=0}^{n-1}\frac{2+\lambda_j}{C+\lambda_j}\right)\left(2\rho\sigma_\theta^2\frac{2}{2+C}+\left((1-\rho)\,\sigma_\theta^2+\sigma_\varepsilon^2\right)\frac{1}{n}\sum_{j=0}^{n-1}\frac{2+\lambda_j}{C+\lambda_j}\right)\\ -\sigma_\theta^2\left(\rho\sigma_\theta^2\frac{(4)^2}{(2+C)^2}+\left((1-\rho)\,\sigma_\theta^2+\sigma_\varepsilon^2\right)\frac{1}{n}\sum_{j=0}^{n-1}\left(\frac{\lambda_j+2}{\lambda_j+C}\right)^2\right)\end{pmatrix}}{\begin{pmatrix}\left(\sigma_\theta^2+\sigma_\varepsilon^2\right)\left(\rho\sigma_\theta^2\frac{(4)^2}{(2+C)^2}+\left((1-\rho)\,\sigma_\theta^2+\sigma_\varepsilon^2\right)\frac{1}{n}\sum_{j=0}^{n-1}\left(\frac{\lambda_j+2}{\lambda_j+C}\right)^2\right)\\ -\left(2\rho\sigma_\theta^2\frac{2}{2+C}+\left((1-\rho)\,\sigma_\theta^2+\sigma_\varepsilon^2\right)\frac{1}{n}\sum_{j=0}^{n-1}\frac{2+\lambda_j}{C+\lambda_j}\right)^2\end{pmatrix}}$$

where λ_j as the the jth eigenvalue of the adjacency matrix A, and $C=\frac{-\beta-2c}{\beta}$.

Proposition C.1 *A vector of demand functions $\{\mathbf{Q}_1(s_1;\mathbf{p}_{g_1}),\mathbf{Q}_2(s_2;\mathbf{p}_{g_2}),...,\mathbf{Q}_n(s_n;\mathbf{p}_{g_n})\}$ is an equilibrium of the OTC game for a circle network if and only if*

$$C<1$$

where C is a solution of

$$\left(\rho\frac{4}{2+C}+(1-\rho)\,\frac{1}{n}\sum_{j=0}^{n-1}\frac{2+\lambda_j}{C+\lambda_j}\right)\left(2\rho\sigma_\theta^2\frac{2}{2+C}+\left((1-\rho)\,\sigma_\theta^2+\sigma_\varepsilon^2\right)\frac{1}{n}\sum_{j=0}^{n-1}\frac{2+\lambda_j}{C+\lambda_j}\right)$$
$$-\left(\rho\sigma_\theta^2\frac{(4)^2}{(2+C)^2}+\left((1-\rho)\,\sigma_\theta^2+\sigma_\varepsilon^2\right)\frac{1}{n}\sum_{j=0}^{n-1}\left(\frac{\lambda_j+2}{\lambda_j+C}\right)^2\right)+\rho\sigma_\varepsilon^2\left(-\frac{4}{2+C}+\frac{1}{n}\sum_{j=0}^{n-1}\frac{2+\lambda_j}{C+\lambda_j}\right)=0.$$

Proof. A proof can be provided upon request. ■