Optimal Financial Knowledge and Wealth Inequality*

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Abstract

While financial knowledge is strongly positively related to household wealth, there is also considerable cross-sectional variation in both financial knowledge and net asset levels. To explore these patterns, we develop a calibrated stochastic life cycle model featuring endogenous financial knowledge accumulation. The model generates substantial wealth inequality, over and above that of standard life cycle models; this is because higher earners typically have more hump-shaped labor income profiles and lower retirement benefits which, when interacted with precautionary saving motives, boost their need for private wealth accumulation and thus financial knowledge. Our simulations show that endogenous financial knowledge accumulation has the potential to account for a large proportion of wealth inequality. The fraction of the population which is rationally financially “ignorant” depends on the generosity of the retirement system and the level of means-tested benefits. Educational efforts to enhance financial savvy early in the life cycle so as to produce one percentage point excess return per year would be valued highly by people in all educational groups.

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1 Introduction

Americans are increasingly being asked to manage their own financial wellbeing, both during their working years and in retirement. This process has been hastened by the movement away from defined benefit (DB) pensions toward defined contribution (DC) plans: in 1980, about 40% of private-sector pension contributions went to DC plans, but two decades later, almost 90% of these contributions flowed to DC plans (which were mainly 401(k) plans; cf., Poterba, Venti, and Wise 2007). At the same time, financial markets have become more complex, expanding the set of instruments that households can use to save and invest. The trend toward more individual responsibility means that people’s financial decisions made early in life can have long-term consequences. For example, if young workers direct their pension contributions to equities instead of money market accounts, this will likely produce very different wealth levels by retirement age. Moreover, in this new financial environment, investments in financial knowledge can have important consequences for retirement wellbeing, by influencing people’s ability to save and invest. When the decision to invest in financial literacy alters life cycle wealth profiles, individuals with similar initial circumstances will end up with very different wealth holdings at retirement. To the extent that this mechanism is at work, understanding it will help explain wealth inequality.

This paper proposes that financial knowledge itself should be modeled as an endogenous choice variable akin to human capital investment. The mechanism we posit is that financial knowledge can enable individuals to better allocate resources over their lifetimes in a world of imperfect insurance and uncertainty. Our approach uses an explicit multiperiod theoretical model which allows us to explore two important questions: (1) What forces shape financial knowledge accumulation over the life cycle?, and (2) How much wealth inequality can be attributable to resulting differences in financial knowledge? We also evaluate which types of consumers would benefit most from investment in financial knowledge and the use of

1 An earlier version of this paper was circulated under the title “Optimal Financial Literacy and Saving for Retirement” (Lusardi, Michaud, and Mitchell, 2011).
2 Wealth levels do vary considerably across both workers and retirees; see Moore and Mitchell (2000) and Venti and Wise (2001).
sophisticated investment products. These issues have not been explored previously in a rich theoretical context with uncertainty, and our answers shed light on the important issue of wealth disparities over the life cycle.

We build and calibrate a stochastic life cycle model featuring uncertainty in income, capital market returns, and medical expenditures; we also incorporate an endogenous knowledge accumulation process and a sophisticated saving technology. In the model, financial knowledge permits consumers to use more sophisticated financial products which can help them raise the return earned on financial assets. Individuals who wish to transfer resources over time by saving will benefit most from financial knowledge. Moreover, because of how the U.S. social insurance system works, better-educated individuals have the most to gain from investing in financial knowledge. As a result, making financial knowledge accumulation endogenous allows for an amplification of differences in accumulated retirement wealth over the life cycle.

Our contributions to the literature are several. First, we explain why many consumers lack knowledge about key aspects of financial markets. Several papers have reported that a large proportion of the population is not financially literate and cannot grasp the concepts of inflation and risk diversification (Lusardi and Mitchell 2007, 2011a; Lusardi, Mitchell, and Curto 2010). Second, we show that some level of financial ignorance may, in fact, be optimal. That is, we explain why consumers may rationally fail to invest in knowledge, since it is expensive to acquire financial knowledge and not everyone will benefit from greater financial sophistication. Third, our model can account for a large share of wealth inequality without appealing to exogenous preference differences or heterogeneity in fixed costs of investing (cf., Cagetti 2003; Vissing-Jorgensen 2003). In support of this approach, Venti and Wise (2001) show that permanent income differences and chance alone can explain only 30-40% of observed differences in retirement wealth, implying that other factors should be taken into account. Fourth, our model generates wealth inequality above and beyond what traditional models of saving normally deliver. Thus it helps account for some of the large differences in wealth reported in the literature, by recognizing that individuals do not start their economic
lives with full financial knowledge and knowledge can be acquired endogenously over the life cycle.

Finally, we show that financial knowledge can be an important public policy lever. For example, reducing the cost of financial knowledge by providing financial education in high school could have potentially large effects on wealth accumulation and welfare. For instance, we report that a 25-year-old college graduate would be willing to pay more than half of his initial wealth to boost financial knowledge, if it offered an expected permanent increase of 1% in his annual rate of return. Our estimates also suggest that a large portion of wealth inequality, over half, can be attributed to financial knowledge that helps people access a sophisticated technology generating higher returns. Policies such as personal accounts under Social Security and increased reliance on individually managed retirement accounts, for example, would be anticipated to lead to higher financial knowledge.

In the remainder of the paper we first briefly summarize prior studies; then we offer evidence on the life cycle path of assets, consumers’ use of financial products, and financial knowledge accumulation by education group. Next we present our model, outline the model calibration, and report simulation results. The paper then offers conclusions and implications.

2 Prior Literature

Our research builds on several related literatures including research on household life cycle saving patterns. We depart from conventional intertemporal models in that we allow for the endogenous choice of a saving technology with returns and costs that depend on a consumer’s level of financial knowledge. In this way, we extend the portfolio choice model (e.g. Cocco, Gomes, and Maenhout 2005) in which returns are assumed to be exogenous and consumers decide only how much they will invest in risky assets. Our work is also informed by prior studies that examined patterns of financial knowledge in the general population. For

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3See, for instance, Cagetti (2003); DeNardi, French, and Jones (2011); Gourinchas and Parker (2002); Hubbard, Skinner, and Zeldes (hereafter HSK; 1995); and Scholz, Seshadri, and Khitatrakun (hereafter SSK; 2006).
instance, Bernheim (1995, 1998) was among the first to document that many U.S. consumers display low levels of financial literacy. Hilgert, Hogarth, and Beverly (2003) showed that most Americans do not understand basic financial concepts including key aspects of bonds, stocks, and mutual funds. The National Council for Economic Education’s report (NCEE, 2005) detailed widespread knowledge gaps regarding fundamental economic concepts among high school students, as did Mandell (2008). Lusardi and Mitchell’s (2008, 2011a) Health and Retirement Study (HRS) modules on planning and financial literacy confirm that many older individuals (age 50+) cannot do simple interest-rate computations such as calculating how money grows at an interest rate of 2% per year, nor do they have a grasp of inflation and risk diversification concepts. These findings have also been confirmed for younger adults (Lusardi, Mitchell, and Curto 2010; Lusardi and Mitchell 2011a).

We are not the first to suggest that financial knowledge is an endogenously determined choice variable. For example, Delavande, Rohwedder, and Willis (2008) posited that investment in financial knowledge is akin to human capital investment, but their static model cannot trace life cycle wealth patterns. Jappelli and Padula (2011) discussed investments in financial knowledge, but they used a life cycle model with certainty and no borrowing constraint and did not evaluate whether differences in knowledge levels produce wealth inequality. Both papers built on the seminal work of Ben-Porath (1967) and Becker (1975) who modeled the economic decision to invest in human capital by linking education to wages. By contrast, we dynamically model investments in financial knowledge in a rich intertemporal setting with decisionmaking under many sources of uncertainty, an approach that allows us to evaluate the quantitative importance of financial knowledge and to perform several important policy experiments. Our work also helps explain recent empirical findings regarding financial knowledge and economically consequential outcomes. For example, our model is consistent with evidence of a positive empirical link between financial knowledge and wealth.
holdings. Additionally, our model explains why highly knowledgeable consumers may be more likely to participate in the stock market, which in our model is represented by the use of a sophisticated investment technology.

Finally, our analysis speaks to the difficulty that standard life cycle models have when attempting to account for observed wealth inequality using heterogeneity in education and permanent income. In view of the conventional model’s shortcomings, some researchers have invoked different factors including impatience in the form of hyperbolic discounting (Angeletos, Laibson, Repetto, Tobacman, and Weinberg 2001) or means-tested programs (HSZ, 1995). Still others assume that consumers use rules of thumb when making saving decisions (Campbell and Mankiw 1989). By contrast, our approach draws on the fact that risk-adjusted expected returns from financial products can differ across income groups. For example, Yitzhaki (1987) established that higher earners enjoyed higher returns on stock market transitions. In field experiments, Choi, Laibson, and Madrian (2010) and Hastings, Mitchell, and Chyn (2011) showed that more financially knowledgeable individuals paid lower fees for mutual funds. Since such fees can substantially reduce net returns on such investments, this implies that financial knowledge boosts investors’ net returns. Moreover, financial knowledge may also have an effect on diversification, which can produce higher risk-adjusted returns. For example, Calvet, Campbell, and Sodini (2009) showed that better-educated Swedish households held more stock than others. Using Dutch data, van Gaudecker (2011) looked at the relationship between investment diversification (return loss), financial knowledge, and financial advice, and he reported that the least financially informed were unlikely to do well on diversification. Such differences in returns can produce a considerable amount of wealth inequality: for example, a dollar invested at a 6% versus a 2% return over 50 years grows to be nearly seven times as large. To simply assume that

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5See for instance Behrman, Mitchell, Soo, and Bravo (2012), Lusardi and Mitchell (2011a and b), and van Rooij, Lusardi, and Alessie (2012).


7One could argue that financial knowledge would not be needed if individuals could rely on financial advisers. Our model, explained in detail below, therefore incorporates the cost of obtaining financial advice. Yet it is worth noting that there are impediments to obtaining good financial advice if consumers lack financial knowledge (Collins 2012; U.S. GAO 2011). For this reason, financial literacy is plausibly a complement to, rather than a substitute for, financial advice.
there is substantial heterogeneity in returns does not help much in explaining wealth heterogeneity, since that merely replaces one source of unexplained heterogeneity with another. Instead, our approach generates such heterogeneity arising from endogenous accumulation of financial knowledge.

3 Life Cycle Wealth and Financial Knowledge

3.1 The Evolution of Income and Assets by Education

The basic life cycle economic model posits that individuals will save in order to transfer resources to life stages where the marginal utility of consumption is highest. Given concavity of the utility function, consumers seek to transfer resources from periods of their lives when they earn substantial incomes to periods when they earn less. We illustrate typical household income profiles over the life cycle in Figure 1, which plots median net household income by education group constructed from the Panel Study of Income Dynamics (PSID).\footnote{Educational attainment refers to three sets of household heads: those who had not completed their high school education, high school graduates, and those with at least some college. We focus on white males throughout this paper to keep our sample as homogeneous as possible. We also drop individuals with business assets and censor all variables at the 99th percentile.}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Median household income by education group.}
\end{figure}

As is evident, life cycle household income for this cohort is hump-shaped. It also rises at a faster rate for the college-educated than for the less-educated, and from around age 50 onward, incomes slowly decrease for all groups. Post-retirement, income falls due to the fact that Social Security and pension benefit amounts are generally less than labor earnings. In the U.S., old-age benefit replacement rates are higher for the least-educated due to the progressivity of public safety net programs, so better-educated consumers see their incomes

\footnote{These calculations use the PSID CNEF files from 1980 to 1997 (in $2004). To generate the figure, we first run median regressions with age and cohort effects, and then we predict incomes for the 1935-1945 cohort. Age dummies are smoothed with a lowess filter.}

\footnote{Including those with business assets skews the interpretation of saving motives compared to the general population, because of the large amount of wealth held in these ventures as well as the volatility of this income (see Hurst, Kennickell, Lusardi, and Torralba 2010).}
fall relatively more after retirement. Additionally, net household income declines somewhat for all groups in retirement, probably because of changes in household composition (e.g. loss of a spouse).

Figure 2 traces life cycle paths of median net wealth (defined as the sum of bank accounts, stocks, IRAs, mutual funds, bonds, and net real estate, minus debt) for these same individuals. For the typical household, wealth grows steadily up to the mid-60s and then flattens or declines. Again, there are striking differences by educational attainment, with the median college-educated household having more than $375,000 in wealth at age 65 (in $2004). By contrast, high school dropouts at the same age had accumulated less than $125,000, with most of that in the form of housing wealth.

In the simplest version of a life cycle economic model, individuals will optimally consume only a portion of their lifetime incomes each period, borrowing in some periods and saving in others. A key prediction of this framework is that the life cycle path of assets normalized by lifetime income should be the same across groups. Therefore, as noted by HSZ (1995), higher earners would be predicted to have wealth-to-income profiles that simply scale-up lower earners’ paths. Yet our data indicate non-proportional wealth-to-income profiles, implying that the simple model cannot explain observed wealth heterogeneity. This confirms evidence from Dynan, Skinner, and Zeldes (2004) who report large differences in saving rates across education groups, using different datasets (both CEX and PSID).

More sophisticated models allow for a precautionary saving motive, which comes into play when income is uncertain and borrowing is difficult. In this circumstance, some individuals will want to save more anticipating that they will have a very high marginal utility of consumption when future income is low. Given a concave utility function exhibiting prudence, such a consumer will save more in anticipation of this possibility. While precautionary saving can explain some of the heterogeneity observed in the data, it still falls short of explaining wealth differences among those facing similar uncertain income profiles.

In what follows, we refer interchangeably to net wealth, net assets, net worth, and household wealth.

Yet another explanation for why the less-educated fail to save was offered by HSZ (1995), who noted that the U.S. social insurance system protects families with limited resources against bad states of the world. That is, means-tested and redistributive transfer programs such as Social Security, Medicaid, and Supplemental Security Income provide an explicit consumption floor in the event that households fall into poverty. In turn, the existence of this consumption floor dampens consumers’ precautionary saving motives, particularly when people are rather likely to become eligible for such benefits. Though this does help explain why the less-educated save little, it cannot explain wealth inequality in the upper half of the income distribution, where the consumption floor is less likely to be reached.

Other authors resort to differences in preferences to explain observed wealth inequality patterns. For example, Cagetti (2003) posits that consumers have different high rates of time preference and low rates of risk aversion, and he suggests that this combination could lead to small precautionary saving for the less-educated and younger consumers. Differences in household composition over the life cycle can also affect consumption by directly changing discount factors or the marginal utility of consumption: inasmuch as household size is negatively correlated with education, this could also account for some portion of wealth inequality (Attanasio, Banks, Meghir, and Weber 1999; SSK 2006). Another potential channel generating wealth inequality might be differences in anticipated mortality patterns. It is well-known that the more educated live longer (De Nardi, French, and Jones 2011), which might also account for a portion of the observed divergence in wealth accumulation (and decumulation) across groups. Our analysis below incorporates many of these differences, to assess how their impacts compare to those of endogenous financial knowledge as a separate channel explaining wealth inequality.

3.2 Differentials in Sophisticated Financial Products by Education

In view of the income paths illustrated above, it should be apparent that college-educated consumers would optimally do relatively more saving (and borrowing), compared to the less-educated. In turn, this could make the better-educated group more interested than their
less-educated peers in a technology that enhanced returns on resources transferred across periods. Table 1 shows the fraction of PSID respondents holding stocks, mutual funds, bonds, and/or individual retirement accounts (IRAs), arrayed by age and education. We denote these products as relatively “sophisticated,” compared to having only a bank account (or no saving at all).

[Table 1 here]

From these data, it is evident that college-educated households are much more likely to use a sophisticated technology for saving, compared to high school dropouts. In particular, more than three-quarters of the older college-educated respondents (age 55-65) use sophisticated products, compared to fewer than one-third of the high school dropouts of the same age. And even small differences in returns over the working life can generate substantial differences in wealth levels at retirement, holding saving rates constant.

The ability of the highly educated and better paid to enjoy higher risk-adjusted returns may result from greater knowledge about financial products. Some authors surmise that limited numeracy and lack of financial sophistication can explain people’s generally low levels of investment and low participation in the stock market (van Rooij, Lusardi, and Alessie 2011). Since equity investments provide higher expected returns, one would therefore anticipate that highly educated households would likely earn higher returns on their saving. But such differences in financial holdings are insufficient to generate observed wealth inequality patterns. For example, Venti and Wise (2001) report that including controls for stock ownership contributes little to explaining the dispersion of wealth across households: that is, adding “investment choices” as controls (in addition to lifetime earnings) reduces the unconditional standard deviation of wealth at retirement by only 8%, at the margin. Accordingly, if the differential take-up of sophisticated products is to account for more of wealth inequality, modelers must allow for the possibility that returns are persistently heterogeneous across households in a predictable way. In what follows, we endogenize the motivation to take up sophisticated financial products as a way to motivate the emergence and persistence

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12See also Curcuru, Heaton, Lucas, and Moore (2005) and Campbell (2006).
of wealth differences over the life cycle.

### 3.3 Financial Knowledge and Wealth Accumulation

To understand how financial knowledge can alter the invariance of wealth to income in a standard life cycle model, we first build an illustrative two-period example, and below we extend this to a richer framework. Accordingly, assume that the individual receives labor income $y$ only in the first period. Denoting wealth in period 2 as $w$, we seek to understand wealth accumulation in period 2 as a function of lifetime income. The consumer can choose how much to consume, $c$, in the first period, and how much to invest in raising $R$, the return factor on saving, $s$. Thus, $w = Rs$ and $c = y - \pi R - w/R$ where $\pi$ is the monetary cost of raising $R$. Assuming the consumer has a discount factor $\beta$, he maximizes:

$$
\max_{w,R} u(y - \pi R - w/R) + \beta u(w)
$$

From the first order conditions to this problem and assuming power utility, $u(c) = c^{1-\sigma}/1-\sigma$, we obtain the following condition for optimal wealth:

$$
w^{1-\frac{1}{2\sigma}} (y - 2\sqrt{\pi w}) = \left(\frac{\sqrt{\pi}}{\beta}\right)^{1/\sigma}
$$

While the right-hand side of the equation is constant with wealth, the left-hand side is not: the left-hand side is decreasing in $w$ for reasonable values of $\sigma$ and $\pi$. A rise in income increases the left-hand side for a given wealth level. If the wealth ratio is to increase to equal the right-hand side, wealth must rise by more than income. We use simulations to show that this is indeed always the case for reasonable parameter values. Figure 3 illustrates how wealth-to-income ratios vary with income levels, given $\sigma = 1.6$, $\beta = 0.96$, and $\pi = 2.5$; these are all reasonable parameter values (as discussed below).

[Figure 3 here]

The slope of this line is positive, and the intuition behind the result is clear. There is a complementarity between an agent’s need to save and his willingness to invest in raising $R$. 

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For high values of $y$, the reward to investing in $R$ rises because saving needs are relatively important. In this two-period model, where lifetime income and the income trajectory are the same, it is not higher income *per se* that raises the incentive to invest in financial knowledge but rather the need to smooth marginal utility across periods. The need is greater when there is a larger gap between first- and second-period consumption. Accordingly, heterogeneity in retiree benefit replacement rates can affect the incentives to invest in financial knowledge; in turn, this can lead to additional differences in wealth accumulation. The same can be said of differences in demographic factors that shift the marginal utility of consumption over the life cycle, as well as differences in expected mortality.

A richer setting with uncertainty and borrowing constraints offers additional motivations to save. If consumers are liquidity constrained, they may be unwilling to invest in financial knowledge. Faced with uncertainty, a consumer might also wish to save more and invest more in financial knowledge for precautionary reasons. Furthermore, the sensitivity of saving to the interest rate can be smaller than in the certainty case (Cagetti 2003), which may also affect incentives to invest in knowledge. Accordingly, we next turn to a richer model of saving to investigate the effects of financial knowledge on wealth inequality.

4 The Model

To allow cross-sectional variation in both financial knowledge and wealth levels, we extend the two-period example above in several directions. First, we introduce uncertainty regarding asset returns, household income, and out-of-pocket medical expenditures. The consumer is assumed to choose his consumption stream by maximizing expected discounted utility, where utility flows are discounted by $\beta$. Second, the individual also faces stochastic mortality risk, and decisions are made from time $t = 0$ (age 25) to age $T$ (or as long as the consumer is still alive; $T = 100$). Third, and adding to the heterogeneity created by the stochastic components, we also examine three different education groups (less than high school, high school, and college). Across these, we allow for heterogeneity in income,
mortality, demographics, out-of-pocket medical expenditure levels, and risk. Importantly, to highlight how investment in financial knowledge affects outcomes, we do not allow for differences in preferences, and we assume consumers start their life cycles with no financial knowledge.

The utility function is assumed to be strictly concave in consumption and defined as $n_t u(c_t/n_t)$, where $n_t$ is an equivalence scale capturing (known) changes in demographics (SSK 2006). The marginal utility of consumption is $u'(c_t/n_t)$ and thus rises with $n_t$. Since the path of $n_t$ is hump-shaped over the life cycle, this contributes to generate a hump-shaped consumption profile with age (Attanasio, Banks, Meghir, and Weber 1999).

The consumer may elect to invest his resources in two different investment technologies. The first is a basic technology (for example, a checking account) which yields a certain (low) return $\bar{r}$ ($\bar{R} = 1 + \bar{r}$). The second is a more sophisticated technology which enables the consumer to receive a higher expected return, increasing in financial knowledge, $f$, but it comes at a price. Specifically, the consumer must pay a direct cost (fee) to use the technology, $c_d$, and he must also invest time and money in acquiring the knowledge.

Obtaining knowledge, in the form of investment, $i_t$, has a cost of $\pi_i(i_t)$; we assume that this cost function is convex, reflecting decreasing returns in the production of knowledge. Relatively little is known about how this cost might vary across individuals; for instance, it could either rise or fall depending on the level of education. Clearly the opportunity cost of time is higher for higher earners, but education might be a complement in the production of knowledge, making it easier for the better-educated to learn. We remain agnostic about whether the average cost of investing in additional knowledge is higher or lower for more educated households; rather, we assume initially that all households face the same cost function, and subsequently we explore alternative formulations in sensitivity analyses.

The rate of return of the sophisticated technology is stochastic, and the expectation of

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13 A convex cost has the advantage of avoiding bang-bang solutions where consumers invest massively in one period; hence it encourages the smoothing of investment over time (cf., Delavande, Rohwedder, and Willis 2008).

14 Alternatively one might allow for a direct disutility of investing in financial knowledge and try to estimate it from the data. Here, because we are concerned mainly with the model’s properties rather than its precise fit to the data, we abstract from the direct disutility channel.
the return depends on the agent’s level of financial knowledge at the end of \( t \), \( \tilde{R}(f_{t+1}) \). Thus the stochastic return function is given by

\[
\tilde{R}(f_{t+1}) = \tilde{R} + r(f_{t+1}) + \sigma_\varepsilon \varepsilon_{t+1}
\]

where \( \varepsilon_{t+1} \) is a \( N(0,1) \) iid shock and \( \sigma_\varepsilon \) is the standard deviation of returns on the sophisticated technology. The function \( r(f_{t+1}) \) is increasing in \( f_{t+1} \) and it can be interpreted as an excess return function. Since the variance is assumed fixed, this also implies that agents with higher financial literacy obtain a higher Sharpe ratio (higher risk-adjusted returns) on their investments. We denote by \( \kappa_t = 1 \) an indicator that the consumer invests in the sophisticated technology in period \( t \), and \( \kappa_t = 0 \) if not.\(^{15}\)

Financial knowledge evolves according to:

\[
f_{t+1} = \delta f_t + i_t
\]

where \( \delta \) is a depreciation factor and \( i_t \) is gross investment. Depreciation exists both because consumer financial knowledge may decay, and also because some knowledge may become obsolete as new financial products are developed.

The consumer receives a government transfer \( tr_t \) which guarantees a minimum consumption floor of \( c_{min} \) (as in HSZ 1995). This consumption floor may lower the expected variance of future consumption, which diminishes the precautionary motive for saving. Transfers are defined as \( tr_t = \max(c_{min} - x_t, 0) \) where cash on hand is:

\[
x_t = a_t + y_t - oop_t
\]

where \( y_t \) is net household income and \( oop_t \) represents out-of-pocket medical expenditures. Both variables are stochastic over and above a deterministic trend. The sophisticated tech-

\(^{15}\)In this paper we model only the extensive margin of investment in the sophisticated technology, since exploratory work with the PSID showed that differences across education groups are larger at the extensive rather than the intensive margin of “how much” to invest. Prohibitive computational requirements put the latter model beyond current reach.
nology cannot be purchased if \( x_t - c_d < c_{\text{min}} \) (that is, the government will not pay for costs of obtaining the technology). End-of-period assets are given by

\[
a_{t+1} = \tilde{R}_\kappa(f_{t+1})(x_t + tr_t - c_t - \pi(i_t) - c_d\kappa_t)
\]

where \( \tilde{R}_\kappa(f_{t+1}) = (1 - \kappa_t)\overline{R} + \kappa_t\overline{R}(f_t) \). We impose a borrowing constraint on the model such that assets \( a_{t+1} \) must be non-negative.

As in many papers in this literature, we posit that during the work life, the individual’s net income equation (in logs) is given by a deterministic component which depends on education, age, and an AR(1) stochastic process:

\[
\log y_{e,t} = g_{y,e}(t) + \mu_{y,t} + \nu_{y,t}
\]

\[
\mu_{y,t} = \rho_{y,e}\mu_{y,t-1} + \varepsilon_{y,t}
\]

\[
\varepsilon_{y,t} \sim N(0, \sigma_{y,\varepsilon}^2), \nu_{y,t} \sim N(0, \sigma_{y,v}^2)
\]

Here \( e \) represents the education group, and \( g_{y,e}(t) \) is an age polynomial (quadratic). The error term \( \eta_{y,t} \) is the sum of a persistent component \( \mu_{y,t} \) and an idiosyncratic component \( \nu_{y,t} \). Retirement is exogenous at age 65. After retirement, the individual receives retirement income which is a function of pre-retirement income.

A similar stochastic AR(1) process is assumed for out-of-pocket medical expenditures. Log out-of-pocket expenditures follow the process:

\[
\log oop_{e,t} = g_{o,e}(t) + \mu_{o,t} + \nu_{o,t}
\]

\[
\mu_{o,t} = \rho_{o,e}\mu_{o,t-1} + \varepsilon_{o,t}
\]

\[
\varepsilon_{o,t} \sim N(0, \sigma_{o,\varepsilon}^2), \nu_{o,t} \sim N(0, \sigma_{o,v}^2).
\]

Because these expenditures are generally low prior to retirement (and to save on computation time), we allow only for medical expenditure risk after retirement (as in HSZ, 1995). Again,
the error term \( \eta_{o,t} \) is the sum of a persistent component \( \mu_{o,t} \) and an idiosyncratic component \( \nu_{o,t} \). Finally, we allow for mortality risk at all ages, denoting \( p_{e,t} \) as the one-year survival probability. Mortality risk is allowed to differ across education groups.

The state-space in period \( t \) is defined as \( s_t = (\eta_y,t, \eta_o,t, e, f_t, a_t) \). The consumer’s decisions are given by \( (c_t, i_t, \kappa_t) \). Hence there are two continuous control variables (consumption and investment) and a discrete one (participation). There are five state variables. We represent the problem as a series of Bellman equations such that, at each age, the value function has the following form:[16]

\[
V_d(s_t) = \max_{c_t, i_t, \kappa_t} n_{e,t}u(c_t/n_{e,t}) + \beta p_{e,t} \int_{\varepsilon} \int_{\eta_y} \int_{\eta_o} V(s_{t+1})dF_e(\eta_o)dF_e(\eta_y)dF(\varepsilon)
\]

\[
a_{t+1} = \bar{R}_K(f_{t+1})(a_t + y_{e,t} + tr_t - c_t - \pi(i_t) - c_d I(\kappa_t > 0)), \quad a_{t+1} \geq 0
\]

\[
f_{t+1} = \delta f_t + i_t
\]

\[
\bar{R}_K(f_{t+1}) = (1 - \kappa_t)\bar{R} + \kappa_t \bar{R}(f_t).
\]

We index variables by \( e \) where education differences are assumed to be present.[17]

The model is solved by backward recursion after discretizing the continuous state variables. At each point in the state-space, we use a grid-search method to search for the optimal solution of consumption, financial knowledge investment, and investment in the sophisticated technology. We solve for optimal decisions for a grid of 40 net asset points and 25 financial knowledge points. Bi-linear interpolation is used to find the value function when net assets or the financial knowledge stock at \( t + 1 \) fall off the grid; the value function behaves smoothly and is concave except at low levels of net assets where liquidity constraints and the consumption floor bind. Accordingly, the grid for assets in the state-space is defined as equally spaced points on \( a^{0.5} \), which leads to more points at lower levels of net assets.

[16] This formulation abstracts from bequest motives. While an extension to include bequests could be interesting, the evidence suggests that this would have a minimal effect on wealth decumulation among the elderly (DeNardi, French, and Jones 2011). Moreover, incorporating bequests would only increase wealth inequality, without changing the qualitative nature of our results.

[17] There are four sources of risk over which the value function is integrated: mortality, rate of return, out-of-pocket medical expenditures, and income. These risks are assumed to be independent.
We use the method proposed by Tauchen (1986) to discretize the processes for income and out-of-pocket median expenditures (with nine points each). Finally, we use three points for rate of return shocks.\textsuperscript{18}

\section{Calibration}

Our goals are to show how endogenous financial knowledge affects wealth holding, and to understand the determinants of financial knowledge accumulation patterns. Since we lack information on individual returns over the life cycle by education group, we do not estimate all relevant parameters of the model. Rather, we proceed with a calibration using plausible values from the literature for preferences and constraints for our base case. Additionally, we provide an extensive sensitivity analysis in Section 6.

To implement the model in the base case, we assume that $u(c_t/n_t)$ has a CRRA form with relative risk aversion $\sigma$. The value of 3 for this parameter used by HSZ (1995) is reasonable in their context, since their main mechanism for creating dispersion in saving patterns is the differential impact of the precautionary saving motive due to a consumption floor. Accordingly, in their setting, the precautionary saving motive governed by the coefficient of relative prudence, $1 + \sigma$, needed to be large. By contrast, our model has an additional channel for creating wealth dispersion, so there is no need for such a strong precautionary saving motive. We use a value of $\sigma = 1.6$ in the base case, which is close to that estimated by Attanasio, Banks, Meghir, and Weber (1999) using consumption data. It is worth noting that the portfolio choice literature typically assumes risk aversion parameters in excess of four (e.g. Campbell and Viceira 2002; Cocco, Gomes, and Maenhout 2005), but we do not require such a high degree of risk aversion in our model. One reason is that agents with low financial knowledge already face low returns if they used the sophisticated technology; hence they will not adopt it. Additionally, the cost of participating in the sophisticated technology

\textsuperscript{18}The fact that even low literacy individuals act as though they can solve the complex model above may seem incompatible with their lack of sophistication. But an approximation to these people’s optimal decision rules can be quite simple in our setup, as noted by Deaton (1992) in his discussion of complex precautionary saving models.
reduces the incentives to use it (Vissing-Jorgensen 2003). Both factors mean that we can fit market participation patterns relatively well using the sophisticated technology proposed here, without resorting to high values of risk aversion.

Following SSK (2006), we define an equivalence scale that accounts for consumption differences in household size by education group and changes in demographics over the life cycle. Let \( z(j, k) = (j + 0.7k)^{0.75} \) where \( j \) is the number of adults in the household and \( k \) is the number of children (under 18 years old). We then define \( n_{e,t} = z(j_{e,t}, k_{e,t})/z(2, 1) \) where \( j_{e,t} \) and \( k_{e,t} \) are the average number of adults and children in the household by age and education group. We use PSID data to estimate the time series of average equivalence scales by education group. The age profile of those scales is hump-shaped and more amplified for less-educated households. For the base case, we use a discount factor of 0.96, as in SSK (2006) and Campbell and Viceira (2002). The annual minimum consumption floor is set at $10,000 per couple with one child.

Computing post-retirement income as a function of pre-retirement income is notoriously difficult because retirement is endogenous. Here we abstract from this and estimate fixed-effect regressions of net household income on age and a retirement dummy, analyzed separately by education level. This produces replacement rates of 0.81 for dropouts, 0.72 for high school graduates, and 0.68 for college graduates. These are higher than rates based only on Social Security benefits, since older households have additional sources of retirement income (e.g. spousal earnings, employer pension benefits, annuities, etc.). The replacement rates are close to those based on total retirement income in the literature (cf. Aon Consulting 2003). In the PSID, we compute the average number per household of adults and children (under 18 years old) by the head’s education and age. We then implement the equivalence scale according to the formula in the text. This value is derived from the Office of the Assistant Secretary for Planning and Evaluation (ASPE, 2008), where the maximum monthly benefit payable to a couple with one child under the Temporary Assistance for Needy Families (TANF) program was $495 (in $2006). The average monthly benefit of recipients on food stamps (for a 3-person household) was $283. Hence, prior to age 65, the sum of TANF and food stamp benefits totaled $778/month for a 3-person household or $9,336/year (omitting the lifetime TANF receipt limit). The Social Security Administration (http://www.ssa.gov/pressoffice/factsheets/colafacts2004.htm) reports that the 2004 maximum monthly federal payment for SSI was $552 for a single household and $829 for couples; including food stamps yields an annual total of $7,620 for singles and $12,180 for couples. Accordingly we use a value of $10,000/year, comparable to the $12,000 used by HSZ (1995) in $2004.
Following retirement, we let income decline at the rate estimated in PSID data controlling for educational groups and cohort effects; that pattern is mostly due to changes in household composition (e.g. widowhood).

The return on the safe asset is set to $r = 2\%$ (as in Campbell and Viceira 2002). The form of the excess return function is not available from previous studies. The range of risk-adjusted portfolio returns reported, for example, by van Gaudecker (2011) ranges from -0.017 to 0.054. Therefore we set the range of possible returns using the technology between 0\% and 4\%; we choose the latter figure because it roughly matches the equity premium used in other studies on portfolio choice. Consequently we do not assume a range of returns that would make financial knowledge artificially important (say, by assuming households could persistently earn excess returns over 10\%).

Another issue has to do with the functional form for the return function that depends on financial knowledge, $r(f_{t+1})$. As data are not available to calibrate this function, we employ a linear function by setting $r_{\text{max}} = r(f_{\text{max}}) = 0.04$ and $r_{\text{min}} = r(f_{\text{min}}) = 0$. Below, we choose a convex cost function for investing in financial knowledge, which therefore embodies decreasing returns to producing knowledge. Accordingly, even if the production function is linear, agents will seek to smooth their investments in financial knowledge over the life cycle. We adopt this simpler form of the production function in order to show the basic mechanisms of the model. In the robustness analysis of Section 6, we show how allowing for an elasticity below one affects results regarding the role of financial knowledge in explaining wealth inequality.

To compute the deterministic part of net household income, we draw on data from Cross-National Equivalent files of the PSID, pooling all available waves (1980-2005). The NBER’s Taxsim program is used to compute net household income. We account for cohort effects when computing income profiles, setting the cohort effect for our calibration baseline.

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22 We thank Hans-Martin van Gaudecker for sharing the Dutch portfolio data with us. Using expected portfolio returns and standard deviation (computed using CAPM and historical returns), we then evaluated the Sharpe ratio for each individual portfolio (the ratio of expected returns to the standard deviation). We then used a standard deviation of 0.16, as in Campbell and Viceira (2002), to compute risk-adjusted returns.


to the 1935-1945 birth group. For comparability with prior studies, we use the AR(1) error structure estimated by HSZ (1995) for net household income prior to age 65. We use HRS data to compute the profile of household out-of-pocket medical expenditures allowing for cohort effects; we predict the profiles for those born between 1935 and 1945, again using the error structure estimated by HSZ (1995). Both income and out-of-pocket expenditures prove to be highly persistent, and differences in persistence and variance across education groups are relatively small. Following the literature (HSZ 1995; SSK 2006), we set the variance of the transitory error component to zero in the simulations since most of it likely reflects measurement error.

Estimating the price of acquiring financial knowledge from available data is difficult because little information is available on inputs to the production process – time and expenditures on financial services – let alone data on investments in, as opposed to the stock of, financial knowledge. According to Turner and Muir (2012), the cost of a one-hour financial advice consultation averages about $250. Veritat.com offers financial planning at $25 a month for singles and $40 for families ($35 for retirees), after an initial meeting fee of $250. Accordingly, the cost ranges from $550 for singles and $730 for families. Less-expensive alternatives include financial advice software such as ESPlanner, where a one-year license costs $40 (the upgraded ESPlanner costs $149; see esplanner.com/product_catalog). In our analysis, we seek to match an average annual expenditure of $500 on financial literacy. We use the function \( \pi(i_t) = 100i_t^{1.75} \) which matches this average expenditure in the simulations and yields a smooth financial literacy investment age profile. For the participation cost of the sophisticated technology \((c_d)\), we use the median estimate of $750 (in $2004) from Vissing-Jorgensen (2003).

We also require an estimate of the depreciation factor for financial knowledge, \( \delta \), but there is little information on the size of this parameter. One study reported that undergraduates’ economic learning depreciated at 4-10% annually (Kipps, Kohen, and Paden 1984). Wage and labor supply information have also been used to measure human capital depreciation; for instance, Heckman (1976) estimated annual depreciation rates of 3-7%. For the
sensitivity analysis, we start with a value of 6% and study whether results vary from that baseline. We could permit the depreciation rate to rise with age to reflect the possibility of cognitive decline, but this is not needed to produce a hump-shaped financial knowledge profile. Furthermore, it is unclear whether consumers can predict cognitive decline, particularly when it comes to memory, and self-reported memory does not change with age in the HRS.25

We also allow for mortality risk differences across education groups, estimated using Gompertz hazard regressions in HRS data for people age 50+, allowing for proportional education effects.26 We assume the same proportionality by education prior to age 50, but we use age/mortality profiles taken from population life tables.

Upon finding optimal consumption, financial knowledge investment, and technology participation at each point in the state-space and at each age, we then use our decision rules to simulate 5,000 individuals moving through their life cycles. We draw income shocks, out-of-pocket medical expenditure surprises, and rate of return shocks, and we then simulate the life cycle paths of all consumers. These consumers are given the initial conditions for education, earnings, and assets derived from the PSID for individuals age 25-30. We initialize financial knowledge at the lowest level (0), because we lack baseline information on financial knowledge. This also makes clear how endogenous accumulation of financial knowledge affects wealth outcomes, and it abstracts from differences in initial conditions.

6 Simulations

Our discussion of the simulation results focuses on outcomes around the time of retirement, since this is when heterogeneity in net assets is most evident. Table 2 reports statistics for each education group at the time of retirement, where we see that wealth patterns are quite unequal. As of age 65, the median high school dropout has accumulated less than half

25Objectively-measured memory scores do fall after about age 65, but self-assessed perceptions are more likely to affect individual behavior.
26These regressions are available upon request. Life expectancy at age 25 is five years higher for the college-educated compared to high school dropouts.
as much wealth compared to high school graduates ($61,500 versus $180,300), and college graduates have accumulated twice as much in retirement assets ($370,200) compared to high school graduates. In fact, compared to the outcomes reported in Figure 2, the model somewhat overpredicts wealth inequality, due mainly to very low predicted assets among dropouts. The ratio of median wealth to income (average lifetime income for each group) is 1.91 for dropouts and 7.8 for college graduates. Accordingly, the ratio of wealth to income for these two groups stands at 4.08. In other words, our model generates a strongly positive relationship between accumulated wealth (normalized by income) and income. We proxy the percent of individuals who will be poor in old age, proxied by the fraction of consumers reaching retirement age with assets below their current income levels. As indicated in Figure 2, close to 40% of high school dropouts will be poor according to this metric, versus 17% for college graduates.

[Table 2 here]

At retirement, the fraction of consumers investing in the sophisticated technology also varies by education group, with 35% of the dropouts, 54% of the high school graduates, and 69% of the college graduates doing so. This pattern nicely matches the patterns shown in Table 1 for participation in sophisticated financial products. There we found that 32% of dropouts, 53% of high school graduates, and 76% of college graduates held what we defined to be sophisticated saving technologies.

Finally, we compute the fraction of consumers with low financial knowledge at the time of retirement. Given the production function, a threshold of 25 units implies that such households could expect an excess return of only one percentage point or less. In our model, such a low level of financial knowledge turns out to be optimal for many, given the constraints and shocks that individuals face. These “optimally ignorant” individuals include 67% of the dropouts, 47% of the high school graduates, and 33% of college graduates. Since financial knowledge strongly influences participation in the sophisticated technology, it is perhaps not surprising that almost all of those with a financial knowledge level of over 25 do use the technology. In this way, financial knowledge can be seen as a type of entry cost, allowing
users to deploy the technology effectively. This entry cost varies by education groups, since incentives to invest in financial knowledge also differ.

Figure 4 illustrates the life cycle paths of average financial knowledge by education groups, all of which which prove to be hump-shaped. Financial knowledge peaks around the age of 65 and declines thereafter. In the accumulation phase, better-educated consumers invest more because they have more to gain from higher returns that help them smooth lifetime marginal utilities. At some point, the opportunity cost of investing becomes too large in terms of foregone consumption and depreciation, and the marginal benefit decreases due to the shorter horizon over which consumers have to enjoy the investments. Raising the depreciation rate of knowledge with age would only make this decline more marked.

[Figure 4 here]

6.1 Quantitative Importance of Endogenous Financial Knowledge

Our model embodies several differences across education groups that generate differential wealth accumulation patterns. First, the consumption floor acts as a tax on saving for those most likely to experience a substantial negative income shock, since subsistence benefits are means-tested (HSZ 1995). Second, differences in replacement rates, demographics, and mortality patterns can create differential incentives to save. Finally, there is the mechanism we propose: financial knowledge, which creates a positive relationship between normalized wealth and income. To clarify the relative contribution of each mechanism in the life cycle model, we next undertake a decomposition exercise which is depicted in Figure 5.

[Figure 5 here]

To this end, we recall that the ratio of median wealth-to-income for college graduates to dropouts is 4.08 at retirement in our baseline simulation. To offer a contrast, we eliminate the possibility of accumulating knowledge along with all differences across education groups other than income while working and medical expenditure differences. For this alternative case, we fix all constraints to those of high school graduates and eliminate the consumption floor. The top bar of Figure 5 shows in the setup with only income and medical expenditure
uncertainty that the wealth-to-income ratio of college graduates is virtually the same as that of dropouts, at 0.96. In other words, confirming what we noted at the outset, the basic life cycle model predicts that all groups accumulate wealth in the same proportion to income.

Next we reintroduce the consumption floor, which reduces precautionary saving of high school dropouts by more than that of college graduates. As illustrated, this does raise the wealth-to-income ratio for college versus high school dropouts, but the impact is small, raising it only to 1.08 (the second bar in Figure 5). Thus in our model, the consumption floor plays a relatively inconsequential role in generating wealth inequality. This finding differs from HSZ (1995) because our precautionary saving motive is much smaller than theirs due to lower risk aversion. Reintroducing differences in old-age income replacement rates is important since college graduates have much lower replacement rates than do dropouts. Moreover, this change alters both wealth accumulation and lifetime income patterns; the net effect, of course, depends on the substitutability of retirement wealth and private wealth. The third bar in Figure 5 represents this simulation, which does increase inequality by 50% (from 1.08 to 1.5). Introducing differences in demographics (the 4th bar down) contributes another increase of roughly 0.5 in the ratio. What this means is that, in the model, differences in household composition are as important as differences in replacement rates. Accounting for mortality differences (the 5th bar) again increases the ratio, now to 2.49. This is because college-educated households must finance consumption over a longer horizon, while high school dropouts face a shorter horizon. Consequently, this is the amount of inequality generated using a life cycle model that lacks endogenous financial knowledge.

The bottom bar in Figure 5 shows how outcomes change when we reintroduce the possibility of consumer investments in financial knowledge, in addition to the other factors mentioned above. The impact of allowing consumers to access the sophisticated technology and earn higher expected returns is striking. Now the wealth-to-income ratio across education groups rises from 2.49 to 4.09, an increase in the ratio of 1.6 (or 65%). Thus of all the explanations examined here for heterogeneity in wealth outcomes, financial knowledge accounts for more than half the cross-group wealth inequality \( \frac{0.51=1.6/(4.09-1)}{ } \).
To more fully illustrate the impact of having access to the higher returns as a result of investing in financial knowledge, we undertake a simple counterfactual exercise. For each schooling group, we first compute average simulated consumption, investment, and medical expenditures by age. Then we evaluate the average return factor for each educational group by age using its accumulated financial knowledge, and we compare this to the average wealth path assuming all groups could only earn the average return earned by high school dropouts. We find that wealth would have been 39% lower for college graduates at the time of retirement if they had experienced the returns paid to dropouts; for high school graduates, the decline would have been just over 30% compared to the paths using their actual average rates of return. Since rates of return differ by roughly 1% between education groups, over many years these differences compounded produce substantial differences in wealth. Moreover, our model generates this wealth inequality endogenously, building only on differences in marginal utilities of consumption over the life cycle.

6.2 Policy Simulations

In the real world, several institutional factors can help shape the process of financial knowledge accumulation. For instance, means-tested benefits protect consumers against bad states of nature: when consumers seek financial knowledge to create a buffer stock of saving, having such programs may provide a disincentive to invest in financial knowledge. Similarly, Social Security benefits may crowd out household saving and also discourage the accumulation of financial knowledge. And the educational system can be influential in boosting initial levels of financial knowledge, as demonstrated by Bernheim (1998); Behrman, Mitchell, Soo, and Bravo (2012) and Lusardi, Mitchell, and Curto (2010).

To explore the relative importance of each type of policy, we next undertake three policy simulations. First, we reduce expected retirement benefits by 20%, which might mimic what Social Security can pay future retirees unless revenues to the program are increased (Cogan and Mitchell 2003). Second, we examine the impact of a reduction in means-tested benefits by half, which could mean either that generosity is decreased or that eligibility is restricted.
Finally, our last scenario is aimed at understanding what would happen if all consumers starting their life cycle had a financial knowledge level of 25; this implies that they would start their life cycle with a possible excess rate of return of 1%. Results are reported in Table 3.

Panel A reports the impact of a 20% reduction in the retirement income benefit. Across all educational groups, median assets rise compared to Table 2, which is not surprising because retirement income crowds out private wealth accumulation in a life cycle setting. Median assets more than double for dropouts, and they rise by roughly one-third for college graduates. Lowering retirement income generosity thus reduces wealth inequality, instead of increasing it.

We can also compute the change in the present value of retirement income by education group in this scenario. Expressing the change as a fraction of the change in the expected present value of retirement income yields an estimate of the displacement or crowd-out effect of retirement income. A naive life cycle model would predict a complete offset, once adjustment is made for the fact that wealth is measured at the time of retirement, so the reduction in lifetime income is only partially offset by that age. By contrast, the unadjusted displacement effects in our simulations range from -0.79 to -1.03. The percentage of dropouts who face shortfalls due to having assets below their current income falls by 30%, with not much change among the best educated. All groups boost their holdings of the sophisticated technology, and even more interestingly, the fraction of optimally ignorant respondents falls. In other words, since all consumers must now save for retirement, investment in financial knowledge rises across the board. Of course, this comes at a cost: the present value of investment expenditures rises by about $8,000 for dropouts, $4,000 for high school graduates, and $6,000 for college graduates.

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27 These numbers are computed as the change in wealth at retirement divided by the change in the present value of retirement income when we reduce the generosity of pensions. We use a discount rate of 3% for these calculations.

28 These present values are computed using simulated data on financial knowledge investment over the life cycle, the cost function, and a discount rate of 3%.
The next scenario, in Panel B, halves the means-tested consumption floor from $10,000 to $5,000 per year. As before, this boosts incentives to save for precautionary reasons. But because our precautionary saving motive is less important than in other studies, such a policy change would produce few differential effects for wealth accumulation and/or financial knowledge. That is, both wealth and knowledge rise following the benefit reduction, but the impact is relatively similar across education groups. Accordingly, in this model, means-tested benefits do not appear to be an important factor shaping saving and investment in financial knowledge.

The final policy scenario, reported in Panel C, considers the possibility that consumers could start on their life cycle paths already having a positive endowment of financial knowledge. This could happen, for instance, if financial education were included in high school curricula. To explore how results change, we chose a level of 25; this implies consumers can earn an initial excess return of 1% on their investments. Interestingly, this policy changes retirement outcomes only slightly. Since financial knowledge is endogenous in the model, people who do not need the knowledge will let it depreciate to their target optimal levels. Although wealth is slightly greater at retirement and financial ignorance less prevalent, these effects are small in comparison to the initial change in financial knowledge.

The fact that outcomes at retirement are relatively insensitive to this policy change does not, however, mean that financial education is unimportant to welfare. In fact, the last row of Panel C in Table 3 reports the change in initial wealth at age 25 that a consumer would require to make him as well off as with the initial endowment of financial knowledge. It would take an additional $6,800 to make a dropout equally well off, $6,900 for a high school graduate, and $15,400 for a college graduate. In other words, high school dropouts would need 82% more initial wealth to make them as well off in expected utility terms, as with a higher starting value of literacy; the wealth equivalent value is 56% for college graduates. Such substantial wealth equivalence measures demonstrate that consumers do value financial knowledge, even when they make no new investments thereafter.

\footnote{Calvet, Campbell, and Sodini (2007) also report large welfare consequences of enhancing consumer knowledge, in a very different framework.}
is due to the fact that investment costs are reduced when people are endowed with a positive level of initial financial knowledge.

Interestingly, this last simulation suggests that a policy which exogenously raises financial knowledge early in life might not have measurable long-term effects, when consumers have both optimal financial knowledge and optimal target wealth levels in mind when solving their life cycle problems. In other words, it is possible that a financial education program could enhance saving in the short run, but it might have little enduring impact in terms of additional future wealth. Nonetheless, the training would still bring important welfare benefits, since additional short-term saving increases lifetime consumption and thus utility. Therefore any effort to evaluate the impact of financial knowledge programs should take these factors into account.

6.3 Sensitivity Analysis

6.3.1 Preference, Depreciation, and Cost Parameters

We have not estimated all parameters of the model because we lack data on individual rates of return. For this reason, we next provide a rich set of sensitivity analyses to help assess how results might change when important parameters are varied. To this end, we explore sensitivity of the asset ratio of college-educated individuals relative to dropouts, the fraction using the sophisticated technology, and the fraction optimally ignorant at retirement, for key model parameters. Table 4 permits a comparison of results for the baseline case, as well as the case where we vary one variable at a time. In all cases, we report the college to high school dropout ratio, with the first column showing the ratio of wealth values at retirement; the second providing the ratio of persons with the sophisticated technology; and the third reporting the ratio of the populations with low financial knowledge.

[Table 4 here]

The first two rows of Table 4 indicate how results change with different values of the relative risk aversion parameter $\sigma$. We vary this parameter from 1.1 to 3, spanning the baseline value of 1.6. This variation proves to have an important impact on wealth inequal-
ity as well as on the fraction holding sophisticated technology and with low knowledge. Specifically, when risk aversion is low ($\sigma = 1.1$), individuals save more for retirement. Since the intertemporal elasticity of substitution is large, better-educated people are also willing to invest in financial knowledge and hold equities. Such complementarity substantially increases the wealth dispersion across educational groups; the degree of inequality rises by more than half \[53\% = (6.25-4.08)/4.08\]. In contrast, setting $\sigma$ to 3 produces the opposite result: the precautionary saving motive is now more important, so all groups accumulate more wealth and financial knowledge. Accordingly, in a cross-sectional context, our model implies that more risk averse individuals would invest more in financial literacy, hold more assets, and invest in financial knowledge. Overall, this reduces wealth inequality and reduces the financially ignorant share of the population.

The next two lines illustrate that results change relatively little when we vary the depreciation rate for financial knowledge, $\delta$, from the baseline level of 0.06 to a low of 0.03 and a high of 0.09 (always holding other factors constant). Wealth inequality changes are small, from 0 to 10\% \[= (4.08-3.58)/4.08\]. The higher depreciation rate generates a bit less wealth heterogeneity because college graduates cut back on their financial knowledge investments. Relatively more of the college-educated group now fall in the low knowledge category than at baseline.

The next four rows indicate that the sensitivity of results depends on the production function for financial knowledge, $\pi(i)$. It will be recalled that the baseline representation of this function $\pi(i_t) = 100i_t^{1.75}$ had two parameters. The multiplicand affects the average cost of acquiring financial knowledge, while the exponent influences the function’s convexity. We vary both in Table 4, in turn. Varying the multiplicand from its baseline value of 100 to a low of 75 and a high of 125, has a relatively small impact on the results. By contrast, changing the function’s convexity has a non-linear effect. Increasing convexity gives larger incentives to spread investment over the life cycle and to avoid large investments. Hence, this should lower differences in financial knowledge. However, raising convexity increases the average cost of reaching a certain level of financial knowledge. Hence, this could amplify
differences as college-educated households have more resources. The effect we observe is not
monotonic: inequality first increases (from 3.81 to 4.08) and then decreases (from 4.08 to
3.42).

The final four rows of Table 4 change the fixed cost of participation in the sophisticated
technology, $c_d$, and the discount factor, $\beta$. Altering $c_d$ around the baseline value of $750
has relatively little impact on wealth-to-income ratios as well as the other two outcomes.
By contrast, the wealth inequality ratio is relatively sensitive to changes in the discount
factor, $\beta$, set initially to a value of 0.96. In the final row of Table 4, we see that setting $\beta =
0.98$ produces one-third less wealth inequality compared to the baseline $\frac{4.08-2.69}{4.08}$, less
dispersion in risky asset holdings, and far smaller differences in financial literacy across
groups.

In sum, our sensitivity analysis illustrates that, although results are somewhat dependent
on parameter values, the amount of wealth inequality generated by financial knowledge
remains high.

6.3.2 Production of Returns

In order to assess how the production function shapes the role of financial knowledge in
wealth dispersion, we next examine a different model from the one used in the baseline
setup. Consider the following function:

$$ r(f_{t+1}) = \alpha_0 f_{t+1}^{\alpha_1}. $$

Here $\alpha_1$ is the elasticity of risk-adjusted returns to financial knowledge. Lacking data on
rates of return at the household level, there is little known about this elasticity. We use
a plausible range from the human capital literature as reported in Browning, Hansen, and
Heckman (1999), namely 0.5 to 0.9, and we consider three elasticity values: low ($\alpha_1 = 0.5$),
medium ($\alpha_1 = 0.75$), and high ($\alpha_1 = 0.9$). Table 5 reports the ratio of college to less-than-
high school median wealth for each of these scenarios.
What we find is that when the production function is more concave, less dispersion in income-to-wealth ratios is generated. For example, an elasticity of 0.9 yields a ratio of 4.02, versus 4.09 in the base case. An elasticity of 0.75, at the mid-range of the human capital literature, yields a ratio of 4.0. The lower-bound elasticity of 0.5 yields a value of 3.35. It is worth noting that even the lowest value of 3.35 represents considerably more wealth inequality than results from a model without financial knowledge (there the ratio was 2.5). Accordingly, more concavity in the knowledge production function attenuates the wealth inequality created by financial knowledge. This is mostly due to the fact that, relative to when the technology is linear, less-educated households now have higher marginal returns to investing. Better-educated individuals have slightly flatter incremental returns to investing in knowledge, relative to the baseline, and thus they invest slightly less. In general, concavity of the function does therefore influence the degree of inequality generated, when financial knowledge is endogenous. Yet in any case, allowing for a concave production function with plausible elasticities instead of a linear function still implies a substantial role for financial knowledge.

7 Discussion and Conclusions

This paper has developed an augmented stochastic life cycle model that endogenizes the decision to acquire financial knowledge. Our goals were to explore the forces that shape financial knowledge accumulation over the lifetime and to evaluate how much wealth inequality might be attributable to differences in financial knowledge. Our formulation posits that financial knowledge offers higher expected returns though it is costly to acquire and depreciates with time. The profile of optimal financial knowledge is shown to be hump-shaped over the life cycle, and it also differs by educational groups because of differences in life cycle income paths. Most importantly, we demonstrate that allowing for endogenous financial knowledge creates large differences in wealth holdings, and that social insurance
influences the incentives to acquire financial knowledge. Thus our model can rationalize and account for a sizeable share of observed differences in wealth across education groups, while other authors have had to rely on social insurance and preference parameters to produce similar dispersion.

In generating wealth inequality above and beyond what traditional models of saving have delivered, we also can account for some of the large differences in wealth found in most empirical work on saving. The precise values depend on model parameters, but our estimates are strongly supportive of the conclusion that financial literacy plays a key role in explaining inequality and should not be ignored. The model relies on an important and intuitively sensible fact: individuals do not start their economic lives with full financial knowledge, so this knowledge may be acquired endogenously over the life cycle. Moreover, we show that some level of financial ignorance may actually be optimal: since it is expensive to acquire financial knowledge and not everyone benefits from greater financial sophistication, some consumers will rationally fail to invest. We also show that financial knowledge can be an important public policy tool. For example, we predict that growing reliance on individually managed 401(k) accounts or personal Social Security accounts would be accompanied by more financial knowledge and more wealth inequality. Additionally, an increase in labor income risk, as in the current macroeconomy, is likely to be accompanied by an increase not only in precautionary saving but also in financial knowledge.

Our theory is also helpful in explaining several recent empirical findings mentioned above, including the widespread low levels of measured financial knowledge in the U.S. and around the world. Our model has predictions about why some population sub-groups are ill-informed, particularly those anticipating larger old-age social insurance benefits. This also would imply that nations promising higher levels of old-age benefits will also be those with lower population levels of financial knowledge. And finally, our model helps explain why financial education programs may not appear to generate large behavioral changes, particularly for those finding it suboptimal to invest in financial knowledge.

In sum, we have shown that allowing for different levels of financial knowledge early in
life has important implications for how much people will save. Adding financial knowledge to life cycle models permits a more accurate rendering of a world where consumers must cope with complex financial markets and must save so as to provide for their own retirement. Incorporating these more realistic features in life cycle models will permit us to better understand empirical regularities, make predictions for the future, and provide more accurate policy recommendations. An important task for future research would be to obtain evidence on differential returns over the life cycle, to confirm the exact form of the financial knowledge production function.

References


Figure 1: Life Cycle Net Household Income Profiles by Educational Attainment. This figure shows median net household income by education group computed from the PSID for waves 1980-1999 (in $2004; see text). The term <HS refers to households where the head has less than a high school diploma, HS means the head completed high school, and College+ means the head had some college. The figure adjusts for cohort effects based on median regressions with age controls; predictions are for those born 1935-1945. Age profiles are smoothed using a lowess filter.
Figure 2: **Life Cycle Wealth by Educational Attainment**. This figure shows median wealth profiles by education group, from the PSID (in $2004; see text). The curves are predicted from median regressions with a correction for cohort effects (following French, 2005); wealth refers to the sum of assets minus debt. Wealth is predicted for all persons born 1935-1945; age profiles are smoothed using a lowess filter.
Figure 3: **Relationship of the Wealth-to-Income Ratio and Income in a Two-Period Model.** This figure shows how the wealth-to-income ratio increases with income in a two-period model with first-period consumption, $y$, and cost of investing, $\pi = 2.5$. Also $\sigma = 1.6$, and $\beta = 0.96$. See text and Figure 1 for definitions.
Figure 4: **Baseline Scenario: Simulated Life-cycle Profile of Financial Knowledge by Age.** This figure plots simulated average financial knowledge scores (0 to 100) by age and education level. See text and Figure 1 for definitions.
Figure 5: Decomposition of Wealth Inequality at Retirement by Education Group. This figure reports the simulated median wealth-to-lifetime income ratios expressed as a ratio of the College+ to those with no high school (<HS). The top bar accounts only for differences in uncertain lifetime income and medical expenditures; all other differences across education groups are suppressed and all education groups use the values for those who finished high school. Subsequent bars then progressively add mechanisms that can generate dispersion in asset ratios. The second bar adds a realistic consumption floor. For the third bar, we add differences in replacement rates by education group. The fourth bar includes differences in household size over the life cycle. The fifth bar incorporates mortality differences by education. The bottom bar adds the impact of financial knowledge accumulation.

<table>
<thead>
<tr>
<th>Age group</th>
<th>&lt;HS</th>
<th>HS</th>
<th>College+</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>25-35</td>
<td>21.8</td>
<td>24.8</td>
<td>51.5</td>
<td>38.6</td>
</tr>
<tr>
<td>35-45</td>
<td>24.6</td>
<td>39.8</td>
<td>58.3</td>
<td>48.7</td>
</tr>
<tr>
<td>45-55</td>
<td>24.1</td>
<td>42.3</td>
<td>65.5</td>
<td>53.4</td>
</tr>
<tr>
<td>55-65</td>
<td>32.1</td>
<td>53.3</td>
<td>75.6</td>
<td>59.5</td>
</tr>
<tr>
<td>Total</td>
<td>25.9</td>
<td>38.5</td>
<td>61.1</td>
<td>49.1</td>
</tr>
</tbody>
</table>

Table 1: Life Cycle Participation (%) in Sophisticated Financial Products (Stocks and IRAs) by Educational Attainment. This table reports participation percentages predicted from regressions in the PSID with controls for age categories and cohort dummies; cohort born 1935-1945. See text and Figure 1 for definitions.
<table>
<thead>
<tr>
<th>At retirement</th>
<th>&lt;HS</th>
<th>HS</th>
<th>College+</th>
<th>College/&lt;HS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Med. wealth ($ 000)</td>
<td>61.5</td>
<td>180.3</td>
<td>370.2</td>
<td>6.05</td>
</tr>
<tr>
<td>Med. wealth-to-income ratio</td>
<td>1.91</td>
<td>4.7</td>
<td>7.8</td>
<td>4.08</td>
</tr>
<tr>
<td>% Poor ($a_t &lt; y_t$)</td>
<td>0.39</td>
<td>0.22</td>
<td>0.17</td>
<td>0.44</td>
</tr>
<tr>
<td>% Sophis. Tech. ($\kappa_t = 1$)</td>
<td>0.35</td>
<td>0.54</td>
<td>0.69</td>
<td>1.95</td>
</tr>
<tr>
<td>% Low Knowledge ($f_t \leq 25$)</td>
<td>0.67</td>
<td>0.47</td>
<td>0.33</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Table 2: **Simulated Outcomes at Retirement (age 65)**. This table summarizes outcomes from baseline simulations at age 65. The last column shows the ratio of college to high school dropout values for each row ($2004$). Note: Assets ($a_t$), income ($y_t$), sophisticated technology ($\kappa_t$), and financial knowledge ($f_t$); poor is defined as having less net wealth than annual income. See text and Figure 1 for definitions.
Table 3: Simulation Results of Policy Experiments. This table summarizes outcomes from simulations at the time of retirement in three scenarios. The first lowers retirement income by 20%; the second lowers means-tested benefits from $10,000 to $5,000. The last scenario provides a boost of 25 units of financial knowledge at age 25 for all consumers; here, we compute the initial wealth equivalent at age 25 that would make the average consumer at baseline as well-off in terms of utility, compared to the scenario where he inherits 25 units of financial literacy. See Figure 1 and Table 2 for definitions.
Table 4: Sensitivity Analysis for Preference and Cost Parameters. Each row computes the ratio of college+ to <HS outcomes as of age 65, for the case when we vary the single parameter indicated versus the baseline value. Baseline values as reported in the text are: relative risk aversion ($\sigma = 1.1$), financial knowledge depreciation rate ($\delta = 0.06$), investment production function ($\pi(i) = 100i^{1.75}$), participation cost ($c_d = 750$), discount factor ($\beta = 0.96$). See Figure 1 and Table 2 for definitions.
### Table 5: Sensitivity Analysis for Production Function

Each row reports wealth-to-income ratios for College+ and <HS groups, as well as the ratio of the two, for the baseline as well as three scenarios where we reduce the elasticity of returns to financial knowledge ($\alpha_1$) in steps. See Figure 1 and Table 2 for definitions.

<table>
<thead>
<tr>
<th>At retirement:</th>
<th>&lt;HS</th>
<th>College+</th>
<th>College+/&lt;HS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline ($\alpha_1 = 1$)</td>
<td>1.91</td>
<td>7.83</td>
<td>4.09</td>
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<tr>
<td>$\alpha_1 = 0.9$</td>
<td>1.97</td>
<td>7.97</td>
<td>4.03</td>
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<tr>
<td>$\alpha_1 = 0.75$</td>
<td>2.04</td>
<td>8.16</td>
<td>4.00</td>
</tr>
<tr>
<td>$\alpha_1 = 0.5$</td>
<td>2.52</td>
<td>8.46</td>
<td>3.35</td>
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</table>