Credit Market Speculation and the Cost of Capital*

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Abstract

We examine the effects of speculation on the cost of debt and the likelihood of default, with focus on credit derivatives. Such contracts induce investors who are optimistic about borrower revenues to sell protection instead of buying bonds. This benefits borrowers if protection can only be bought with an insurable interest. However, if naked credit default swaps are permitted, speculation is damaging to borrowers when beliefs about worst-case outcomes are especially pessimistic. In this case availability of naked credit default swaps can expand the range of funding requirements for which multiple equilibria exist, thus exacerbating the problem of rollover risk.

Keywords: Speculation, credit derivatives, heterogeneous beliefs, cost of debt, rollover risk.

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1 Introduction

Financial markets provide individuals with means to reduce their overall risk exposure, as well as opportunities to take new risks by betting on the movement of asset prices. These two activities—hedging and speculation—are inextricably linked. Since mutually offsetting risk exposures are rare, the hedging of risk by one party usually requires speculation by another, and widespread hedging would not generally be possible in the absence of speculation. But speculation is entirely possible in the absence of hedging: two parties to a contract could take opposite sides of a bet on some future event without either of them having any offsetting exposures. In fact, the volume of speculation far exceeds the amount needed to accommodate the demand for hedging, and a variety of financial instruments such as options, futures, and swaps allow for the making of such two-sided directional bets with ease.

The consequences of unrestricted speculation on resource allocation have been debated for decades. From the efficient markets perspective, these effects are largely benign: speculation serves to rapidly correct departures of prices from fundamentals, thus ensuring that assets are valued in accordance with the best available information about future cash flows (Friedman 1953, Fama 1965). But it has also been argued, for instance by Keynes (1936) and Tobin (1984), that excessive speculation can result in price distortions and real resource costs.

In this paper we examine the effects of speculation in credit markets on the terms of lending and the likelihood of default. The focus is on a particular class of credit derivatives, **credit default swaps (CDS)**. These are contracts in which one party sells protection to another against a failure (by a third party) to make contractual debt repayments; they are said to be **naked** if the protection buyer does not also hold the underlying security. Naked credit default swaps are therefore two-sided directional bets with payoffs that net to zero: one party is betting on default while the other is betting against, and there is no requirement that either has an insurable interest or hedging motive. The notional value of such contracts on US corporate debt prior to the financial crisis of 2007-08 was estimated to be about ten times as great as that of the underlying bonds (Brunnermeier, 2009). Even with the netting out of multilateral positions, there is little doubt that much of this volume was speculative.[1]

In May 2010, Germany became the first major economy to prohibit such contracts out-

[1] Vause (2010) estimates that the elimination of offsetting positions is largely responsible for the decline in the notional amount of outstanding credit default swaps from a peak of \$60 trillion in late 2007 to about \$30 trillion in early 2010, but even this latter amount is consistent with significant speculative interest. Zuckerman (2010) and Lewis (2010) document some spectacular examples of directional bets using naked purchases of protection on securitized mortgage debt. Sheila Bair, former Chair of the FDIC, likens such contracts to “a game of fantasy football” with the magnitude of speculative trading amounting to “many multiples of the size of the underlying mortgage market” (Bair, 2012).

right when it announced a unilateral ban on naked credit default swaps on eurozone debt, and the European parliament has now followed suit. Although it is widely accepted that any such restrictions will have major economic repercussions, there is no consensus on whether these effects will be positive or negative on balance. Some have argued that naked credit default swaps should be banned outright on the grounds that they increase volatility, facilitate bear raids, and make default more likely to occur (Buiter, 2009; Portes, 2010). Others have countered that they result in more complete markets, better aggregation of information and beliefs, and increased bond market liquidity, making it easier for debt to be issued by distressed borrowers (Stulz, 2010; Jarrow 2011).

One argument for the benefits of credit derivatives to borrowers derives from the observation that they facilitate the separation of funding from exposure to credit risk. This allows borrowers to raise funds even from those who are relatively pessimistic about their ability to repay, since this group of investors can shed credit risk by purchasing protection. Meanwhile, those who are most optimistic about future borrower revenues can sell protection, and thereby expose themselves to credit risk on a scale that would not be possible without derivatives. These effects should shift the terms of financing in favor of borrowers while broadening the range of assets available to investors.[2] But when protection can be purchased by investors who do not hold the underlying bonds, those who are most pessimistic about future borrower revenues can also exploit the implicit leverage that derivatives provide. This can shift interest rates in a manner that is damaging to issuers of debt.[3]

Our main purpose in this paper is to explore the impact of credit derivatives on interest rates, debt capacity, the likelihood of default, and rollover risk. We begin with a simple model in which a borrower seeks to meet a fixed funding requirement by selling bonds to a group of investors with heterogeneous beliefs concerning the debtor's future income. This belief heterogeneity is due not simply to differences of information applied to a common prior, but to fundamental differences in the interpretation of common information.[4] Within this framework, we consider four regimes. The benchmark case is that in which no credit

[2]Geithner (2007) makes this point as follows: "For borrowers, credit market innovation offers the prospect of increased credit supply; better pricing; and a relaxation of financial constraints. For investors, new credit instruments bring the prospect of broader risk and return opportunities; the ability to diversify portfolios; and increased flexibility. And for lenders, innovations can help free up funding and capital for other uses; they can help improve credit risk and asset/liability management; and they can improve the return on capital and provide new and cheaper funding sources." Along similar lines, Jarrow (2011) argues that "trading of CDS increases the welfare of the traders in financial markets via the optimal allocation of risks, thereby lowering debt costs... which in turn increases aggregate investment."

[3]See, especially, Geanakoplos (2010) on this point: "CDS are, despite their names, not insurance at all, but a vehicle for pessimists to leverage their views. Conventional leverage allows optimists to push the price of assets unduly high; CDS allows pessimists to push asset prices unduly low."

[4]This appears to be a natural assumption in any analysis of speculation, since a zero sum directional bet requires that the parties to the contract *agree to disagree* in the sense of Aumann (1976).

derivatives exist. This is compared with a **covered CDS** regime in which bondholders can hedge their risk by purchasing protection against default, but investors cannot purchase protection if they do not also hold the underlying bonds. This is the regime that would presumably result in the sovereign debt market from the ban envisaged by the EU. The third regime, **naked CDS**, allows for unrestricted contracts, including naked credit default swaps. Finally, we consider a regime in which bonds may be sold short, but credit derivatives are not present. The terms of lending and the likelihood of default are compared across these four scenarios.

We show that the presence of credit derivatives is beneficial to borrowers if protection may be purchased only by those with an insurable interest. That is, the maximum amount that can be funded is greater and the terms of lending more favorable to the borrower relative to the case in which no credit derivatives exist. This is because some of the most optimistic investors switch from buying bonds to selling protection, thereby increasing the scale of their exposure to credit risk. Since each unit of protection sold corresponds to a unit of bonds purchased by some other investor, each dollar of collateral set aside by protection sellers corresponds to more than a dollar's worth of expenditure on bonds. In fact, the effects on interest rates and debt capacity of covered credit default swaps are precisely the same as those of collateralized lending, where optimists borrow from pessimists to take leveraged positions in bonds, subject to margin requirements that ensure full repayment. This leveraging effect results in a higher bond price and a smaller likelihood of default for any given funding requirement.

When protection can be purchased without an insurable interest, however, the effects of credit derivatives on the terms of lending are more ambiguous. Relative to the case where protection can be purchased only with an insurable interest, borrowers face an increased cost of debt. But relative to the case of no credit derivatives, allowing for naked credit default swaps can be beneficial to borrowers under some circumstances and harmful in others. The ambiguity arises because of two countervailing effects. On the one hand, such contracts allow for the separation of funding from credit risk exposure, which is beneficial to borrowers. On the other hand, the availability of these contracts diverts the capital of optimists away from bond purchases and towards collateral to support speculative positions against pessimists, who purchase naked protection only to bet on default (rather than to insure bonds against default). We show that the former effect dominates, so borrowers benefit, if beliefs about the worst case outcome for borrower revenues are sufficiently optimistic; otherwise the latter effect dominates and borrowers are made worse off by the existence of credit derivatives. This suggests that under crisis conditions, when beliefs about borrower revenues decline sharply, firms with traded credit derivatives will be hurt more than those that are insulated from such effects.[5]

Qualitatively similar effects arise if one allows for short sales instead of credit derivatives, but these effects are quantitatively different. As expected, allowing for short sales results in a lower bond price relative to the case in which neither short sales nor credit derivatives are permitted. But as long as the worst-case outcome for bondholders is nonzero recovery, short sales result in more depressed bond prices than would arise under unrestricted trading of credit derivatives. The reason for this is the following: betting on default via short sales forces pessimists to take a negative position on both the safe and the risky components of the cash flow promised by the bond, while credit derivatives allow for the shorting of just the risky component. As a result, bond issuers are damaged less by credit derivatives than by short sales.

While the baseline model sheds some light on the manner in which the terms of financing can be affected by the availability of credit derivatives, it does not deal with one of the major objections to such contracts: the possibility of *self-fulfilling bear raids* under conditions of financial distress. To address this issue, we extend the model to allow for a mismatch between the maturity of debt and the life of the borrower. This raises the possibility that a borrower who is unable to meet contractual obligations because of a revenue shortfall can roll over the residual debt, thereby deferring payment into the future. Multiple equilibria arise naturally in this setting. If investors are confident that debt can be rolled over in the future, they accept lower rates of interest on current lending, which in turn implies reduced future obligations and allows the debt to be rolled over if necessary. But if investors suspect that refinancing may not be possible, they demand greater interest rates on current debt, resulting in larger future obligations and an inability to refinance if the revenue shortfall is large. This, in turn, validates their demand for a higher risk premium.

As in the baseline model, we compare the case of no credit derivatives with that in which naked protection can be purchased, and uncover two effects. First, the equilibrium in which investors are pessimistic about the ability of the borrower to roll over debt involves higher interest rates when credit derivatives are in use than when they are not. That is, the terms of financing are worse (from the perspective of the borrower) conditional on the selection of the pessimistic equilibrium. Second, the pessimistic equilibrium exists for a larger range of initial borrowing requirements when credit derivatives exist than when they do not. In other words, there is a range of initial borrowing requirements such that fears about the ability of the borrower to repay debt can be self-fulfilling if and only if naked credit default swaps are permitted. It is in this precise sense that the possibility of self-fulfilling bear raids can be

[5] This finding is consistent with evidence presented in Shim and Zhu (2010), which we discuss below.

said to arise when the use of credit derivatives is unrestricted.[6]

The rest of the paper is organized as follows. Section 2 discusses empirical evidence and related literature. Section 3 sets up a model, beginning with the benchmark case of no credit derivatives (3.1), then proceeding to consider the effect of CDS under different regulatory regimes (3.2–3.3), and finally comparing these with short sales (3.4). Section 4 extends the model to consider rollover risk and self-fulfilling crises, and Section 5 concludes.

2 Empirical Evidence and Related Literature

The empirical evidence on the effects of derivatives on credit supply and the terms of lending is broadly consistent with the model presented here. The most comprehensive study of which we are aware is by Ashcraft and Santos (2009), who explore the effects on the cost of debt for borrowers by comparing firms with and without active credit derivative markets. They report that contrary to the conventional wisdom on the topic, the onset of trading in credit derivatives provides no benefit to the average firm in terms of lower bond spreads or lower rates on bank loans. In fact, for relatively risky and informationally opaque firms, they find a significant and robust negative effect. The authors interpret these findings in terms of reduced incentives for ex-post monitoring by lenders once credit risk has been shed, which would apply even if protection purchases were made only by those with exposure to default risk. Our interpretation is different, and relies crucially on the distinction between covered and naked credit default swaps: the presence of the latter raises the cost of debt for distressed firms relative to the benchmark case of no credit derivatives.

Shim and Zhu (2010) conduct a similar exercise using data on a sample of Asian firms and find that under normal credit market conditions, those with traded credit default swaps experience a lower cost of borrowing on new bond issues.[7] This effect is reversed, however, under the stressed credit market conditions following the failure of Lehman in September 2008. They argue that under normal credit market conditions borrowers benefit from the enhanced information flows and more efficient allocation of risk-bearing that credit derivatives facilitate, along lines suggested by Duffie (2008). The reversal of this effect under stressed

[6] We do not consider here the possibility that asset prices may be used to infer the beliefs of potentially informed speculators about the distribution of future borrower revenues, resulting in bear raids of a different kind as speculators enter short positions that lead to the abandonment of projects and loss of firm value (Goldstein and Guembel, 2008). Such feedback effects between prices and realized returns can make market manipulation using credit derivatives potentially profitable. But, as we show here, even in the absence of manipulation a maturity mismatch between loans and revenues can result in increased rollover risk when credit derivatives are unrestricted.

[7] Along similar lines, Hirtle (2009) finds that the possibility of hedging by banks results in lower spreads and longer maturities for loans to firms with traded credit derivatives in the US market.

conditions can be accounted for by our finding that credit derivatives result in more punitive terms for the borrower when investors are especially pessimistic, even as they benefit borrowers when investors are sufficiently optimistic. Furthermore, our finding that pessimistic equilibria exist for a broader range of funding requirements in the presence of unrestricted credit derivatives than in their absence also suggests that firms with traded credit derivatives could experience particularly adverse shifts in the terms of financing under stressed market conditions.

Bruneau et al. (2012) have recently argued that an abrupt and self-fulfilling change in expectations of default was a key factor in accounting for the sharp rise in interest rates paid by the countries of the eurozone periphery. They argue that the relationship between fundamentals and credit spreads was subject to a structural break around March 2010, indicative of a switch to a different equilibrium with greater pessimism about default. They also maintain that the coordinated change of expectations was facilitated by the sovereign CDS market. It is possible that the CDS market not only served to coordinate expectations, but also to alter the set of equilibria itself, affecting both the degree of multiplicity and the cost of debt, in keeping with the model developed here.

Our paper is related to a number of prior theoretical contributions. To begin with, we join a growing literature that assumes heterogenous priors in the analysis of financial markets (see Miller 1977, Harrison and Kreps 1978, Scheinkman and Xiong 2003, Hong et al. 2006, and Geanakoplos 2010 for example).[8] Heterogeneous priors provide a simple way of explaining equilibrium speculation, which is difficult to account for under standard common prior assumptions (Milgrom and Stokey, 1982). However, our goal here is not to explain speculation, but to examine its implications for the terms of lending and the likelihood of default. The mechanism by which this occurs is the collateral requirement: speculation affects prices because it requires traders to set aside cash or other forms of collateral that could otherwise be used to support asset purchases.

The collateral requirement also figures prominently in Geanakoplos (2010), where agents with heterogeneous beliefs enter into debt contracts with each other, allowing the leveraged purchase of an asset in fixed supply. Collateral requirements are endogenously determined and cover the worst-case loss on such loans. Adding credit derivatives to the model completes the market and causes the asset price to decline. We build on this framework but

[8] Miller argued that "the very concept of uncertainty implies that reasonable men may differ in their forecasts." Harris and Raviv (1993) and Kandel and Pearson (1995) suggest that divergence of opinions may result from investors having different economic models which lead them to interpret the same event in different ways. The usefulness of belief heterogeneity as a modeling platform has been recognized by Hong and Stein (2007), who point out that such models "uniquely hold the promise of being able to deliver a comprehensive joint account of stock prices and trading volume."

focus instead on the supply of the underlying asset by a third-party borrower and the endogenous likelihood of default under different regulatory regimes. In addition, our exploration of the role of credit derivatives in coordinating expectations on pessimistic outcomes and exacerbating rollover risk is novel.

The literature on multiple equilibria in currency and debt markets is large. For instance, Calvo (1988) and Cole and Kehoe (2000) interpret sovereign debt crises as multiple equilibrium phenomena arising from self-fulfilling pessimism about default risk.[9] In these models, the borrower's decision *ex post* to default strategically—and inability *ex ante* to commit not to—is what causes the crisis to arise endogenously. The pessimism in our analysis does not arise from concerns about strategic default, but rather from the impact of an endogenously larger debt burden on the borrower's ability to repay. When asset prices incorporate more pessimistic beliefs, borrowers are forced to issue larger debt obligations to meet any given funding requirement. This makes default more likely and justifies the pessimism that prices reflect. Our key concern is with the manner in which speculation affects this feedback loop. While credit derivatives have been implicated in coordinating expectations on pessimistic equilibria in the context of the eurozone periphery (Bruneau et al., 2012), their role in affecting the equilibrium set itself, and hence the possibility and severity of crises, has not been explored to date.

Finally, our research is related to the emerging literature on the corporate finance implications of credit derivatives. Bolton and Oehmke (2010) consider the economic consequences of covered (as opposed to naked) credit default swaps. Bondholders who have purchased protection against default have minimal incentives to work with a distressed borrower to restructure debt. While this has often been cited as a source of inefficiency, Bolton and Oehmke argue that the stronger bargaining position of protected creditors can improve the pledgeability of borrower income, making it easier to raise funds *ex ante*. Since our focus here is on naked credit default swaps, we disregard the issue of bankruptcy reorganization and the effects on fundamentals of covered protection.

3 Debt Contracts

Suppose a borrower faces a funding requirement of $b > 0$, and finances this by issuing a quantity $q > 0$ of one-period bonds, each with unit face value. The price of these bonds (to be determined endogenously) is p. Successful funding enables the borrower to generate income y, which is a random variable. Creditors are paid in full if this income meets the

[9]See also Giavaggi and Pagano (1990), Alesina et al. (1990), and Cohen and Portes (2006).

debt obligation; otherwise they receive a share of the income in proportion to their bond holdings. Since its obligation on the maturity date is q, the debtor will repay $\min\{y, q\}$ in the aggregate and each bond will accordingly pay $\min\{y/q, 1\}$.

There exists a unit mass of risk-neutral investors, each endowed with a single currency unit.[10] Investors maximize their expected returns, given heterogeneous beliefs about the distribution of y. An agent with belief $\theta \in [0, 1]$ perceives that the borrower's future revenue y is distributed according to $G(y|\theta)$ with support $[\eta, 1]$, where $\eta \in [0, 1)$. We adopt the convention that higher values of θ correspond to more optimistic expectations regarding y in the sense of first-order stochastic dominance. Investor beliefs θ are drawn from the interval $[0, 1]$ according to the distribution $F(\theta)$. Note that η is the lower bound for the borrower's future income for all belief types — a system-wide perception of the worst-case scenario. This parameter, which we interpret as a measure of overall investor sentiment, is liable to be high during booms and low under stressed conditions.

We assume throughout that $b > \eta$, which ensures that there is some perceived risk of default even if the borrower were able to meet the funding requirement at a zero interest rate.[11] Under this assumption, the worst case payoff per bond from the perspective of the lender when the issue size is $q \geq b$ is $\eta/q < 1$. Let

$$\psi(q; \theta) := \int_{\eta}^{1} \min\left\{\frac{y}{q}, 1\right\} dG(y|\theta) \tag{1}$$

denote the expected payoff per unit face value, as perceived by a bondholder of type θ. Note that ψ is decreasing in q, increasing in θ, and satisfies $\psi(q; \theta) < 1$ for any $q > \eta$. Let

$$\Psi(\theta) := \psi(1, \theta) = \int_{\eta}^{1} y dG(y|\theta)$$

denote the expected value of y as perceived by an investor of type θ. Clearly, $\Psi(0) > 0$ and $\Psi(1) < 1$. Note also that

$$q\psi(q; \theta) = \int_{\eta}^{1} \min\{y, q\} dG(y|\theta) \leq \int_{\eta}^{1} y dG(y|\theta) = \Psi(\theta), \tag{2}$$

with strict inequality for $q < 1$ and equality for $q = 1$.

[10] Given risk-neutrality, the assumption that all investors have the same cash endowment is without loss of generality since we allow for an arbitrary belief distribution: an investor with a large endowment can be interpreted as a large number of investors with the same belief.

[11] Were this not the case, then there would always exist an equilibrium with no possibility of default and no trading in credit derivatives.

Since all investors agree that the borrower's future income is at most equal to 1, this is also the maximum debt obligation that can be undertaken if there is to be any chance of full repayment. Accordingly, we assume $q \leq 1$. Obligations exceeding this imply certain default *ex ante*. Although it is conceivable that investors would purchase such bonds at a sufficiently low price, we rule it out on practical grounds.

Let $\theta_m \in (0, 1)$ denote a critical investor type such that

$$\Psi(\theta_m) = 1 - F(\theta_m).$$

The left side of the equation is simply the borrower's expected income—and thus the maximum amount that it can promise to repay—as perceived by type θ_m. The right side is the total cash endowment of agents who are more optimistic than type θ_m. Clearly, this critical value is well-defined, since $\Psi(\theta_m)$ is increasing in θ_m, while $1 - F(\theta_m)$ is decreasing in θ_m and varies from one when $\theta = 0$ to zero when $\theta = 1$. Given investor risk-neutrality, $\Psi(\theta_m)$ is the maximum amount that the borrower can raise in the benchmark case without credit derivatives.

Finally, consider the payoffs of the borrower, given any arbitrary belief $\theta_0 \in [0, 1]$. If the funding requirement is met ($pq \geq b$), then the borrower's payoff, given realized income y, is $y - b - (1 - p)q$ if this magnitude is non-negative and zero otherwise. Hence the expected payoff, given belief θ_0, is

$$E\,[\,\max\{y - b - (1 - p)q, 0\} \,|\, \theta_0\,] \tag{3}$$

provided that the funding requirement is met. As long as the bond price is nonincreasing in q (which will be the case throughout our analysis), the borrower will never issue more debt than is necessary to meet the funding requirement, regardless of θ_0. This simply reflects the fact that any borrowing in excess of b has no productive use but will be subject to positive interest payments. For reasons discussed below, we assume that if $pq < b$ then all funds are returned to investors, no bonds are issued, and no income is realized. In this case the borrower's payoff is zero and all investors retain their endowments.

We now examine the manner in which the terms and limits of borrowing are affected by restrictions on the use of credit derivatives.

3.1 Equilibrium without Credit Derivatives

First consider the case in which no credit derivatives are available, so investors must choose between bonds and cash. We consider the properties of an equilibrium in which the borrower is able to raise the needed funds, and then identify conditions under which such an equilibrium exists. We assume that agents can convert a unit of cash into one unit of a divisible consumption good in either period, and choose to maximize their (undiscounted) aggregate consumption.

Consider any equilibrium in which the borrower is able to meet its funding requirement, so that

$$pq \geq b. \tag{4}$$

is satisfied. Each investor can purchase $1/p$ units of the bond with her cash endowment. If the investor has belief θ, her expected payoff when the bond matures is $\psi(q;\theta)/p$. Such an individual will purchase bonds if and only if

$$\psi(q;\theta) \geq p.$$

This expected payoff is monotonic in θ, implying that each investor adopts a cutoff strategy such that she purchases the bond if and only if θ is no less than

$$\hat{\theta}(p,q) := \sup\{\theta \in [0,1] \,|\, \psi(q;\theta) \leq p\}, \tag{5}$$

with the convention that $\hat{\theta}(p,q) = 0$ if $\psi(q;\theta) > p$ for all θ. For notational simplicity we suppress the dependence of $\hat{\theta}$ on (p,q) where possible.

Whenever $\hat{\theta} \in (0,1)$, we must have

$$\psi(q,\hat{\theta}) = p. \tag{6}$$

Observe that $\hat{\theta}$ is continuous and nondecreasing in (p,q) and that $\hat{\theta}(1,q) = 1$ and $\hat{\theta}(0,q) = 0$ for any $q \in (0,1)$. Since q units of bond are sold, the bond market clearing condition is

$$1 - F(\hat{\theta}(p,q)) = pq. \tag{7}$$

That is, the market clears when the revenue from bond sale (the right side) equals the total cash endowment of those who are more optimistic than the marginal agent (the left side). Since $\hat{\theta}$ is nondecreasing in p, the left side of (7) is nonincreasing in p. The right side is clearly strictly increasing in p. Furthermore, the left side is continuous, close to one for p sufficiently close to zero, and close to zero for p sufficiently close to one. Hence, for any $q > 0$, there exists a unique price $\hat{p}(q) < 1$ that satisfies (7) and therefore clears the bond market. Moreover, since $\hat{\theta}$ is nondecreasing in (p,q), it must be the case that $\hat{p}(q)$ is decreasing: a larger bond issue results in a lower price per unit. Note also that $\hat{p}(q)$ is continuous.

Suppose that there exists an issue size q such that $p(q)q \geq b$. Then there is clearly an equilibrium in which the borrower meets the funding requirement: we call this a funding equilibrium. There is another equilibrium in which no bonds are sold, supported by an out-of-equilibrium belief that the borrower cannot meet the requirement and therefore earns no income, which validates the decision by investors not to purchase the bond. However, this latter equilibrium is simply an artifact of investor miscoordination, and can be formally eliminated by our assumption that if the borrower is unable to raise b, then it refunds its investors. This ensures that it is a dominant strategy for an investor to purchase at a price that yields a surplus, and allows us to focus on the funding equilibrium (when it exists) as the unique equilibrium in undominated strategies.

Since $\hat{p}(q) < 1$ and decreasing, it is clear from the objective function (3) that the borrower will select the smallest issue size consistent with a funding equilibrium, assuming that such an equilibrium exists, and will raise exactly b. That is, the borrower chooses

$$q^*(b) = \min\{q \in [0,1] \,|\, \hat{p}(q)q \geq b\}. \tag{8}$$

Let $p^*(b) := \hat{p}(q^*(b))$ denote the market clearing price corresponding to this optimally chosen issue size, and let $\theta^*(b) := \hat{\theta}(p^*(b), q^*(b))$ denote the belief of the marginal investor at this price-quantity pair.

The following result characterizes the feasible range of funding requirements and the equilibrium bond issue size and price as functions of the funding requirement.[12]

Proposition 1. *The maximum revenue that can be raised in equilibrium is $b^* = \Psi(\theta_m)$ at $q = 1$. If $b \leq b^*$, then there exists a unique equilibrium in which the borrower meets the funding requirement exactly by issuing $q^*(b)$ bonds, each of which is sold at price $p^*(b)$. The equilibrium issue size rises and the price falls as the funding requirement rises within the feasible range.*

The manner in which the equilibrium bond contract varies with the funding requirement is illustrated by the following example.

Example 1. *Suppose that $\eta = 0$, $G(y|\theta) = y^{\theta+1}$, and θ is uniformly distributed so $F(\theta) = \theta$ for $\theta \in [0,1]$. Then for any $q \leq 1$,*

$$\psi(q;\theta) = 1 - \frac{q^{\theta+1}}{\theta+2}$$

Figure 1 shows how the price and total revenue vary with q. The upper bound for total revenue

[12]All proofs, unless evident from the discussion in the text, are collected in the appendix.

is $b^* = 0.59$.

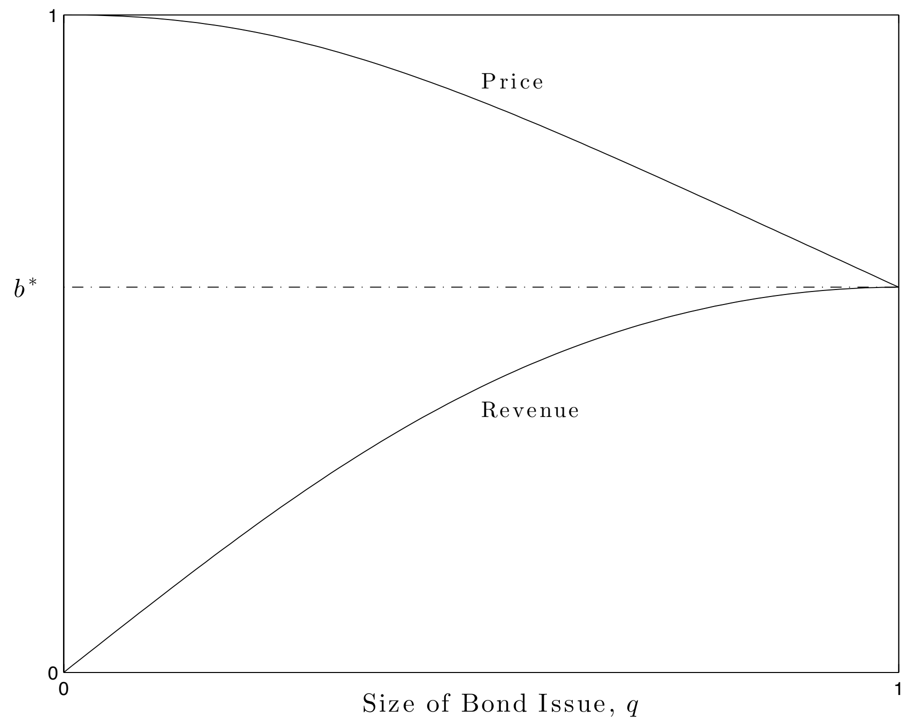

Figure 1: Bond Price and Total Revenue as Functions of Issue Size

3.2 Covered Credit Default Swaps

We now consider equilibrium in the market for debt under the assumption that protection against default can be purchased using credit derivatives, but only with a long position in the underlying bonds. Let r denote the (credit default) **swap spread**: the amount paid per unit face value to insure bonds for one period. As before, the bond price is p per unit face value. Now agents have four choices: they can sell protection (using their cash endowment as collateral), they can buy bonds with or without protection, or they can remain in cash. Clearly, an agent can combine choices, but for reasons that will be apparent below, no investor will simultaneously buy and sell protection.[13]

Consider an investor who buys $a \geq 0$ units of the bond and sells x units of protection, where $x < 0$ entails buying protection. For this to be feasible, three constraints must be

[13]We are abstracting here from dealers, who do in fact buy and sell protection in order to profit from the bid-ask spread, and are the main providers of liquidity to the market. While gross positions of dealers can have important effects on counterparty risk and systemic stability, our focus here is on the cost of capital and accordingly on the beliefs of those with *net* exposures to credit risk.

satisfied. First, the portfolio must satisfy the budget constraint:

$$pa \leq 1 + rx. \tag{9}$$

Next, since purchases of protection can only be used for hedging in this regime, the agent's purchase of protection cannot exceed her purchase of bonds:

$$a + x \geq 0. \tag{10}$$

This holds trivially if $x > 0$, since $a \geq 0$. Finally, if the agent sells protection ($x > 0$) then she is obliged to set aside enough collateral to cover the losses of the protection buyer, which requires a transfer of

$$\left(1 - \min\left\{\frac{y}{q}, 1\right\}\right) x$$

when the bonds mature. If $y \geq q$ then there is no transfer, and in the worst case, if $y = \eta$, she pays $(1 - \eta/q)x$. The value of her collateral in this worst case scenario is

$$1 + rx - pa + \frac{\eta}{q} a.$$

We are assuming here that the protection seller is required to hold enough collateral to cover this worst case loss, thus ruling out default by the protection seller. While this assumption is made primarily for simplicity, it can also be derived as an equilibrium phenomenon in a model with multiple feasible margin requirements.[14] Given this assumption, total collateral must be large enough to fully compensate bondholders in the worst case scenario:

$$\left(1 - \frac{\eta}{q}\right) x \leq 1 + rx - pa + \frac{\eta}{q} a,$$

or

$$\left(1 - r - \frac{\eta}{q}\right) x + \left(p - \frac{\eta}{q}\right) a \leq 1. \tag{11}$$

If $x \leq 0$, this condition is implied by (9). As we show below, the coefficients of a and x will be positive in equilibrium.

[14] Geanakoplos (2010) shows that when agents choose from a rich set of loan contracts differing in margin requirements, only the contract which precludes default is selected in equilibrium. The same result can be obtained if one strengthens the signal structure by assuming that $G(y|\theta)$ satisfies the monotone likelihood ratio property. In general, default on contracts with financial assets serving as collateral is rare under normal conditions, although it is clear that in the absence of government intervention such defaults would have occurred during the recent financial crisis. We are allowing for default by the bond issuer (which is common, since the bonds are backed by physical assets), but not the swap counterparty. It is important to note that the crowding out effect we identify below is robust to relaxing this no-default constraint, as long as the CDS seller must post collateral at some positive level.

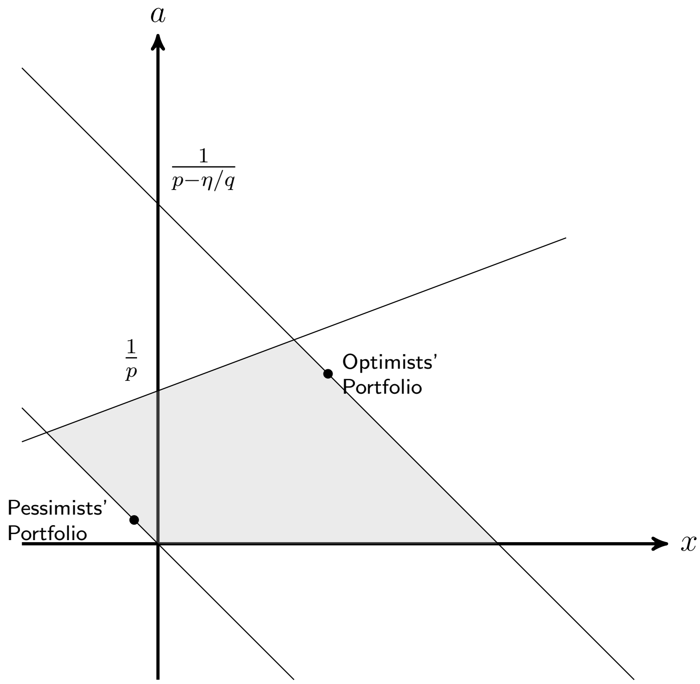

Figure 2: The Feasible Set of Choices for an Investor under Covered CDS

The collateral constraint (11) reveals a central tradeoff faced by investors. Any increase in the purchase of bonds necessitates a decline in the sale of protection. This is the case because the value of the bond as collateral is less than its purchase price. Hence an investor's total available collateral falls as bond purchases are scaled up. An investor wishing to increase the sale of protection must therefore cut back on bond purchases in order to set aside more cash collateral.

The three constraints and the set of feasible portfolios is shown in Figure 2. The budget constraint is the positively sloped line with vertical intercept $1/p$. The covering constraint is the negatively sloped line through the origin. And the collateral constraint is the negatively sloped line with intercept $1/(p - \eta/q)$. The feasible set is the shaded area defined by the three constraints, together with the requirement that bond purchases must be non-negative.

Given a portfolio (a, x), an investor of type θ has an expected payoff of

$$\begin{aligned} u(a, x \,|\, \theta) &:= 1 + rx - x \int_{\eta}^{1} \left(1 - \min\left\{\frac{y}{q}, 1\right\}\right) dG(y|\theta) + a \int_{\eta}^{1} \min\left\{\frac{y}{q}, 1\right\} dG(y|\theta) - pa \\ &= 1 + (\psi(q;\theta) - 1 + r)x + (\psi(q;\theta) - p)a. \end{aligned}$$

The problem facing an agent of type θ is therefore given by

$$\max_{(a,x)\in\mathbb{R}_+\times\mathbb{R}} u(a,x\,|\,\theta)$$

subject to the three constraints (9), (10), and (11), where the prices p and r are exogenously given from the perspective of any investor. Equilibrium prices must satisfy the following.

Lemma 1. *In any funding equilibrium, $p+r=1$.*

The proof of this claim is relegated to the appendix, and relies on the following reasoning. If $p+r>1$, any investor with a positive investment in bonds could increase her payoff by lowering her bond holdings and increasing the sale (or reducing the purchase) of protection by exactly the same amount without violating any of the three constraints. Since some investors must hold bonds in any funding equilibrium, there can be no such equilibrium with $p+r>1$. And if $p+r<1$, the budget constraint for all investors must be satisfied with equality, otherwise any investor could increase her payoff by increasing her bond holdings and reducing the sale (or increasing the purchase) of protection by exactly the same amount without violating any of the three constraints. As shown in the appendix, this in turn implies that either the bond market or the market for credit derivatives must fail to clear. Hence we must have $p+r=1$ in any funding equilibrium.

Given $p+r=1$, the payoff for a type θ investor may be written as:

$$u(a,x\,|\,\theta) = 1 + (\psi(q;\theta)-p)(a+x). \tag{12}$$

Since $\psi(q;\theta)-p$ is strictly increasing in θ, the equilibrium portfolio $(a(\theta),x(\theta))$ of a type θ agent must have the following threshold structure. Any belief type $\theta<\hat{\theta}(p,q)$ chooses $a(\theta)+x(\theta)=0$ since this is feasible and minimizes $a+x$. (A pessimist's choice is depicted by a point in the bottom constraint line corresponding to (10) in Figure 2.) Such types may buy the bond with protection, but their incentives for doing so are weak in the sense that they could do just as well be simply holding cash. Belief types satisfying $\theta>\hat{\theta}(p,q)$ maximize $a+x$ to a level that causes the collateral constraint (11) to be binding, so

$$a(\theta)+x(\theta) = \frac{1}{p-\eta/q}. \tag{13}$$

An optimist's choice is depicted in Figure 2 by the point on the top constraint line corresponding to (11). These investors are optimistic enough to sell protection and/or to buy the bond without protection. They are indifferent between these two choices, but their payoffs are pinned down by substituting (13) into the utility function:

$$u(a, x \mid \theta) = \frac{\psi(q; \theta) - \eta/q}{p - \eta/q}. \tag{14}$$

This expression may be interpreted as follows: the payoff equals the risky component of the return promised by the bond (the numerator) multiplied by $a + x$, the notional value of bonds purchased or insured. In effect, selling a unit of protection is equivalent to buying only the risky component of the bond return. This allows for the separation of funding from credit risk exposure.

Hence the set of agents can be partitioned into two groups: those who invest safely (in bonds with protection or cash), and those who sell protection and/or buy the bond without protection. The aggregate amount of protection this latter group sells must equal the aggregate quantity of bonds that the former group buys:

$$\int_0^{\hat{\theta}(p,q)} a(\theta) dF(\theta) = \int_{\hat{\theta}(p,q)}^1 x(\theta) dF(\theta).$$

Hence, the total purchase of bonds must equal

$$\int_0^1 a(\theta) dF(\theta) = \int_{\hat{\theta}(p,q)}^1 (a(\theta) + x(\theta)) dF(\theta) = (1 - F(\hat{\theta}(p, q))) \frac{1}{p - \eta/q}.$$

Since this must equal q for the bond market to clear, the following must hold in equilibrium:

$$1 - F(\hat{\theta}(p, q)) = pq - \eta. \tag{15}$$

There is a unique market clearing price $\tilde{p}(q)$ that solves (15), and hence a unique spread $\tilde{r}(q) = 1 - \tilde{p}(q)$ consistent with equilibrium. Note that $\tilde{p}(q) > \hat{p}(q)$. To see this, suppose to the contrary that $\tilde{p}(q) \leq \hat{p}(q)$. Then, $\hat{\theta}(\tilde{p}(q), q) \leq \hat{\theta}(\hat{p}(q), q)$, so the left side of (15) is weakly greater than that of the corresponding market clearing condition (7). However, the right side of (15) is strictly greater than that of (7), a contradiction. Hence $\tilde{p}(q) > \hat{p}(q)$. Note further that the right side of (15) falls with η. This implies that the equilibrium bond price $\tilde{p}(q)$ increases with η, and thus the equilibrium swap spread $\tilde{r}(q)$ declines as η rises.

At equilibrium prices, pessimists are indifferent between cash and bonds with protection, while optimists are indifferent between unprotected bonds and sales of protection. Nevertheless, in order for markets to clear, pessimists must purchase the risk-free portion η of the bond. We shall assume that they have the resources to do so; a sufficient condition for this is given below.

As before, the borrower selects the smallest issue size that allows for the funding requirement to be met:

$$q^c(b) := \min\{q \in [0,1] \,|\, \tilde{p}(q)q \geq b\},$$

provided that this set is nonempty. Define $p^c(b) := \tilde{p}(q^c(b))$ and note that $q^c(b) < q^*(b)$ for any $b < b_m$. In equilibrium, agents with type $\theta < \theta^c(b) := \hat{\theta}(p^c(b), q^c(b))$ either remain in cash or buy bonds together with protection from those with $\theta > \theta^c(b)$. The equilibrium cost of protection is $r^c(b) := \tilde{r}(q^c(b))$. The resulting equilibrium always exists, as long as $F(\theta^c(b)) \geq \eta$, which allows the pessimists to absorb the risk free portion of the borrower's income; we assume that this condition is satisfied. Since $\tilde{p}$ is increasing in η, $q^c(b)$ is decreasing in η. The equilibrium is thus characterized as follows.

Proposition 2. *There exists $b^c > b^*$ such that for any $b < b^c$, there exists a unique equilibrium in which the borrower meets the funding requirement b by issuing $q^c(b) < q^*(b)$ bonds, each of which is sold at price $p^c(b) > p^*(b)$. The borrower's default probability is lower when bonds can be insured than when they cannot. An increase in η causes b^c and $p^c(b)$ to rise, while $q^c(b)$, $r^c(b)$ and the probability of borrower default all decline.*

The availability of (covered) credit default swaps therefore benefits the borrower, allowing for more to be raised on better terms. This happens because the credit risk is held by sellers of protection, and the marginal protection seller is more optimistic than the marginal bond buyer in the case of no credit derivatives. In effect, those who are optimistic about the future income of the borrower can hold a larger number of units of credit risk, since some of the more pessimistic investors are financing the safe component of the loan. Credit derivatives allow optimists to leverage without actually borrowing to buy bonds. This leverage effect is amplified when η, the system-wide perception of the worst-case outcome, increases.

It can be shown that the outcome described here is identical to that which would arise if optimists could borrow from pessimists to take leveraged positions in bonds (as in the case of repo contracts), subject to a margin requirement that ensures full repayment even in the worst-case outcome. Specifically, this margin requirement limits an agent's leveraged position to at most z, where z is determined by the constraint that the worst case payout from the purchase, $1+\eta z/q$ (which is the investor's cash endowment plus the largest possible loan) is no less than the cost pz of bond purchase. It is easily verified that this implies the same demand for bonds, $z = 1/(p - \eta/q)$, that optimists would choose in the covered CDS regime. The same effect could be achieved if the issuer sells bonds of varying seniority, with claims to the first η of income senior to claims to any surplus beyond this amount. In other words, collateralized lending, tranching, and covered credit default swaps are essentially equivalent with respect to both the terms of lending in the aggregate and the credit risk exposure chosen by the different classes of investors.

3.3 Naked Credit Default Swaps

Now suppose that investors may purchase protection without an insurable interest. The problem facing an agent is similar to that studied in the previous section, except that the covering constraint (10) is no longer required. Specifically, an agent with belief θ solves

$$\max_{(a,x)\in\mathbb{R}_+\times\mathbb{R}} u(a,x\,|\,\theta)$$

subject to (9) and (11).

It is easily verified that Lemma 1 continues to hold in this regime; arbitrage preserves $p+r=1$ in any funding equilibrium. Given this, the utility for type θ is exactly as in (12), and since $\psi(q;\theta)-p$ is strictly increasing in θ, the equilibrium portfolios $(a(\theta),x(\theta))$ again have a cutoff structure. Investors with beliefs $\theta<\hat{\theta}(p,q)$ set $a(\theta)=0$ and minimize x to a level that causes the budget constraint (9) to be binding, thus choosing $(a(\theta),x(\theta))=(0,-1/r)$. In contrast with the covered CDS regime, the incentive for pessimists to purchase protection is no longer weak. They buy protection not for hedging exposure to credit risk but rather to actively speculate on default, and use their entire cash endowment to do so. Meanwhile, those with beliefs $\theta>\hat{\theta}(p,q)$ set $a+x$ to the highest level consistent with the collateral constraint (11), so

$$a(\theta)+x(\theta)=\frac{1}{p-\eta/q}. \tag{16}$$

As in the covered CDS regime, optimists are long credit risk through a combination of bond purchases and protection sales and, as before, are indifferent between these two choices. This does not mean, however, that prices are the same in the two two regimes. Since pessimists do not buy any bonds, the entire bond supply must be absorbed by optimists, even as they need to set aside more collateral to meet the increased demand for (naked) protection by pessimists. The set of optimists must therefore be larger, and the marginal belief type accordingly more pessimistic. This in turn means that the bond price must be lower.

This can be seen more formally. Given our characterization of equilibrium choices, the market for protection will clear if and only if

$$\frac{1}{r}F(\hat{\theta}(p,q))=\int_{\hat{\theta}(p,q)}^{1} x(\theta)dF(\theta),$$

and the bond market will clear if and only if

$$\int_{\hat{\theta}(p,q)}^{1} a(\theta)dF(\theta)=q.$$

Since

$$\int_{\hat{\theta}(p,q)}^{1} (a(\theta) + x(\theta))dF(\theta) = (1 - F(\hat{\theta}(p,q)))\frac{1}{p - \eta/q},$$

these two conditions can be collapsed into:

$$\frac{F(\hat{\theta}(p,q))}{r} = \frac{1 - F(\hat{\theta}(p,q))}{p - \eta/q} - q. \tag{17}$$

Rewriting (17) using $p + r = 1$, we get

$$1 - \left(\frac{1 - \eta/q}{1 - p}\right) F(\hat{\theta}(p,q)) = pq - \eta. \tag{18}$$

For any $q \in (0, 1]$, there exists a unique bond price $\overline{p}(q)$ that satisfies (18), and a corresponding swap spread $\overline{r}(q) = 1 - \overline{p}(q)$. As before, the borrower chooses the smallest feasible bond issue:

$$q^n(b) = \min\{q \in [0, 1] \,|\, \overline{p}(q)q \geq b\},$$

provided that this set is nonempty (that is, if the funding requirement b can be met in equilibrium). The associated bond price is $p^n(b) := \overline{p}(q^n(b))$, and the equilibrium swap spread is $r^n(b) := \overline{r}(q^n(b))$. In equilibrium, agents of type $\theta < \theta^n(b) := \hat{\theta}(p^n(b), q^n(b))$ buy naked protection from those with $\theta > \theta^n(b)$.

Comparing (18) with (15), the left side of the former is less than that of the latter (since $pq > \eta$ in equilibrium), while the expressions on the right side are identical. Hence $\overline{p}(q) < \tilde{p}(q)$. That is, the equilibrium bond price is lower in the regime with naked protection relative to that with covered protection only, for any given bond issue size. This effect can be seen in Figure 3, where the dotted lines correspond to the three constraints in the covered CDS regime, and the solid lines to the two constraints in the naked CDS regime, taking account of the changes across regimes in prices, spreads, and portfolio choices. The collateral constraint is shifted out since bonds are cheaper and revenues from the sale of protection are higher. For the same reason, the budget constraint has a larger slope and vertical intercept. And the covering constraint is no longer operative.

Since both old and new budget constraints must intersect the old covering constraint at the same point, the old feasible set is fully contained in the new feasible set. As a result, *all* investors benefit subjectively (based on their respective, mutually incompatible beliefs) from the change in regime. This is clear for optimists, since they can sell protection at a higher price and buy bonds more cheaply. But even pessimists perceive themselves to be better off, since their earlier optimal choice remains feasible. The borrower, of course, is worse off as a result of the lower bond price.

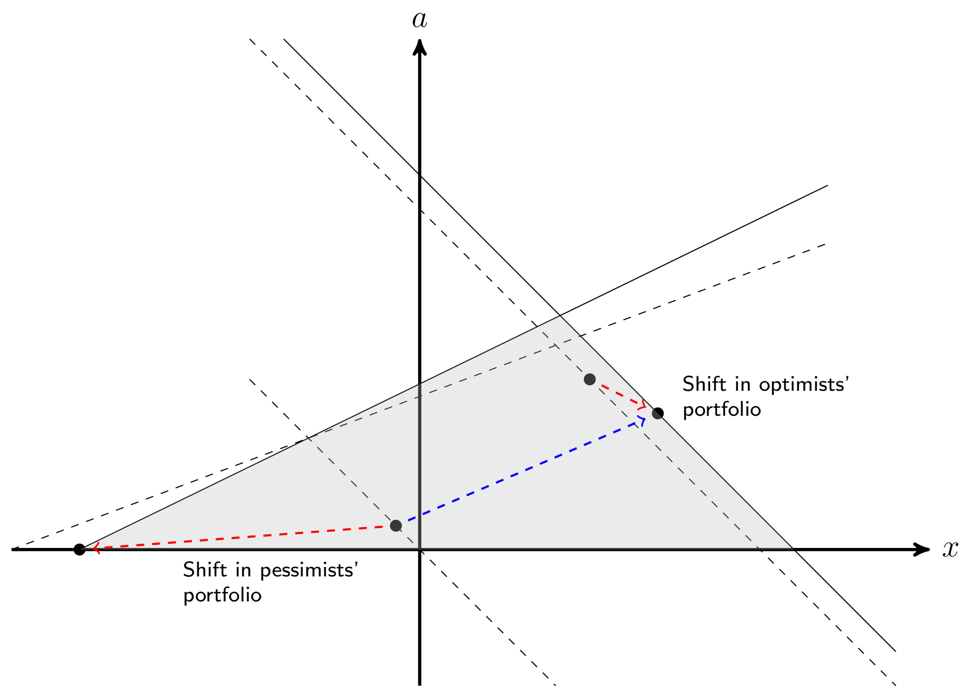

Figure 3: Feasible and Optimal Portfolios under Covered and Naked CDS

Note that the partitioning of individuals into optimists and pessimists is different across the two regimes. Under naked CDS the marginal type is less optimistic, so the set of optimists is itself larger. This allows for the possibility, depicted in Figure 3, that both pessimists and optimists reduce their holdings of bonds even as the bond supply remains unchanged. Some of those who previously held cash or bonds with protection switch to buying bonds and selling protection, increasing their holdings of bonds in the process.

Figure 3 contrasts regimes with covered and naked credit default swaps respectively. It is also important to consider how the regime with unrestricted credit derivatives compares with that in which credit derivatives are entirely absent. Comparing (18) with (7), we see that as η goes to zero, the right sides of the two equations converge, while the left side of the former remains strictly above that of the latter. For η sufficiently small, therefore, the bond price in the naked protection regime is lower than than that in the regime with no credit derivatives. In this case, the presence of naked credit default swaps raises the cost of borrowing. When η is large, however, this effect is reversed:

Proposition 3. *The maximum revenue that can be raised in equilibrium with naked credit default swaps is $b^n = \overline{p}(1)$. If $b \leq b^n$, then there is a unique equilibrium in which the borrower issues $q^n(b)$ bonds at price $p^n(b) < p^c(b)$. There exists $\hat{\eta} \in (0, b)$ such that $p^n(b) < p^*(b)$ if $\eta < \hat{\eta}$, and $p^n(b) > p^*(b)$ otherwise.*

This result may be interpreted in terms of two competing forces. On the one hand, allowing for naked protection purchases allows pessimists to bet on default, which induces optimists to divert resources away from bond purchases to support the sale of protection. This resource-diversion effect raises the cost of debt. But there is also a countervailing effect: optimists can use part of the revenue from sales of protection to invest in bonds, effectively doubling up on credit risk.[15] They can do this if η is large because the bonds themselves can serve as collateral. Even if the worst-case outcome were to materialize, the recovery from the bonds, together with the remaining cash collateral, can allow optimists to meet the obligations arising from protection sales. Since the receipt of the premium relaxes the budget constraint of the optimists, allowing them to buy more bonds, a smaller set of optimists can meet the funding requirement. Hence the marginal bond buyer is more optimistic relative to the case of no credit derivatives. Pessimists indirectly finance the purchase of bonds, and credit derivatives therefore facilitate the separation of funding from credit risk exposure as in the covered CDS regime. This latter effect becomes more pronounced as η increases, since optimists need to set aside less collateral per unit of protection sold. For small η, the resource-diversion effect swamps the separation effect. But for η sufficiently large, the separation effect starts to dominate, and the cost of debt is lower than in the absence of credit derivatives.

The finding that when η is small, bond prices (and hence also total revenues) are lower when credit derivatives are unrestricted than when they are absent or restricted to an insurance function is illustrated in Figure 4, which is based on the same specifications as in Example 1 (with $\eta = 0$). Not only are the terms of financing worse when protection can be purchased without an insurable interest, but the range of deficits that can be financed is itself smaller. The net effect is that any borrowing requirement that is feasible without credit derivatives either becomes infeasible, or requires a larger bond issue (and hence a higher interest rate).

Naked credit default swaps allow both optimists and pessimists to hold leveraged positions. The former can amplify their exposure to credit risk by selling protection instead of simply buying bonds. The latter can effectively hold short positions in bonds, which would not be possible if they were required to have an insurable interest in order to purchase protection. The extent to which each of these parties can leverage depends on η, the system-wide perception of the worst-case scenario. When overall investor sentiment is sufficiently optimistic, so that η is above the threshold $\hat{\eta}$, the presence of credit derivatives (even if unrestricted) can lower the cost of debt. On the other hand when investor sentiment is so pessimistic that η lies below the threshold, unrestricted credit derivatives raise the cost of borrowing. Hence allowing for naked credit default swaps could increase volatility in the cost

[15] As Stulz (2010) notes, this is precisely what AIG did, buying CDOs in addition to selling protection on their default.

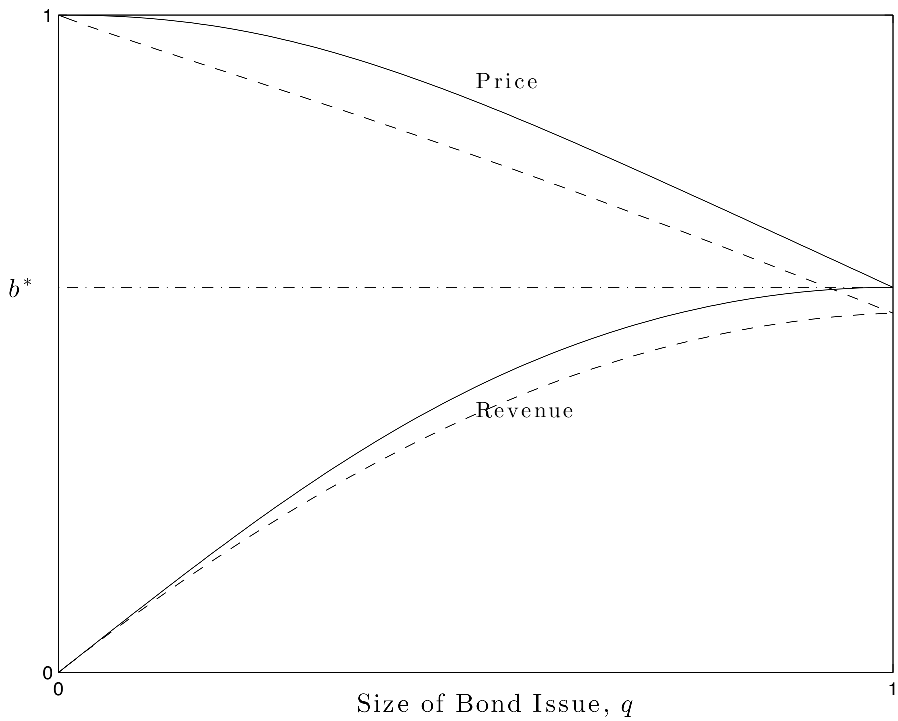

Figure 4: Comparison of the baseline model with unrestricted protection

of debt relative to the baseline with no credit derivatives. Furthermore, firms with traded credit derivatives could benefit from this under normal market conditions but suffer disproportionately (relative to firms without traded credit derivatives) under crisis conditions. This appears consistent with the empirical findings of Shim and Zhu (2010), as discussed above.

3.4 Short Sales

It is often argued that the availability of unrestricted credit derivatives acts as a substitute for short sales in the bond market, since short sales also constitute speculative side bets that enable investors with opposing views to enter positions as counterparties.[16] Speculators wishing to bet on default can do so by selling bonds short rather than buying naked protection. Although the qualitative effects of these two activities are similar, we now show

[16] A traditional short sale requires an investor to borrow (and replace upon demand) the securities that are sold. A naked short sale, in contrast, involves the creation of a synthetic bond that replicates the payments of the underlying security. The buyer pays the sale price of the bond and receives in exchange the promised stream of payments; the seller must post collateral to ensure contract fulfillment. These two alternatives are equivalent in the environment considered here, since margin requirements preclude default and investors do not discount future income.

that their quantitative effects are not the same.

In our framework, a short sale contract requires the seller to commit to replicate the payment of the bond on the maturity date, in exchange for a current payment equal to the price of the bond. Suppose that credit derivatives are not present, but that bonds can be sold short by investors using their cash endowment to satisfy margin requirements. In this case there are three options available: buying bonds (from issuers or short sellers), shorting bonds, or remaining in cash. The objective function of an investor of type θ is exactly as in (12), except that x is constrained to be zero and a is allowed to be negative. So an investor maximizes

$$u(a, 0 \,|\, \theta) = 1 + (\psi(q; \theta) - p)a,$$

subject to the budget constraint $pa \leq 1$ and a margin requirement that must be met by short sellers. To obtain this latter constraint, note that if $a < 0$, the investor's cash position becomes $1 - ap$. The worst case outcome for the short seller is that the bond pays its face value in full, in which case she will be required to pay out $-a$. Assuming, as before, that collateral requirements are such as to preclude default by the short seller, and taking account of the budget constraint for those who choose $a > 0$, investor portfolios must satisfy

$$-\frac{1}{1-p} \leq a \leq \frac{1}{p}.$$

As before, equilibrium has a threshold structure: investors with $\theta < \hat{\theta}(p, q)$ short bonds to the maximum extent allowed by the margin constraint, while those with $\theta > \hat{\theta}(p, q)$ use their entire cash endowment to buy bonds.

For the bond market to clear, the total supply (by the issuer and short sellers) must equal the demand from purchasers:

$$q + \frac{F(\hat{\theta}(p, q))}{1-p} = \frac{1 - F(\hat{\theta}(p, q))}{p},$$

which simplifies to

$$1 - \left(\frac{1}{1-p}\right) F(\hat{\theta}(p, q)) = pq. \tag{19}$$

There is a unique market clearing price $\check{p}(q)$ that solves (19). Given this, the borrower chooses the smallest feasible bond issue:

$$q^s(b) = \min\{q \in [0, 1] \,|\, \check{p}(q)q \geq b\},$$

provided that this set is nonempty. The resulting bond price is $p^s(b) := \check{p}(q^n(b))$. In equilibrium, agents of type $\theta < \theta^s(b) := \hat{\theta}(p^s(b), q^s(b))$ short the bond (i.e., they choose $a = -1/(1-p)$) while those with $\theta > \theta^s(b)$ buy the bond from the issuer or from short sellers (i.e., they choose $a = 1/p$).

A comparison of (19) with (7) reveals that $\check{p}(q) < \hat{p}(q)$, so allowing for short sales results in a lower bond price relative to the benchmark with no credit derivatives. To see this, suppose to the contrary that $\check{p}(q) \geq \hat{p}(q)$. Then $\hat{\theta}(\check{p}(q), q) \geq \hat{\theta}(\hat{p}(q), q)$. Since $\check{p}(q) < 1$, this implies that the left side of (19) is strictly greater than that of the corresponding market clearing condition (7), while the right sides of of the two equations are identical, a contradiction. Hence $\check{p}(q) < \hat{p}(q)$.

How does the price effect of short sales compare with that of naked credit default swaps? A comparison of (19) with (18) reveals that the bond prices in the two regimes are identical if $\eta = 0$. If $\eta > 0$, however, the cost of debt is greater under short sales than under unrestricted credit derivatives:

Proposition 4. *$p^s(b) \leq p^n(b)$, with strict inequality if and only if $\eta > 0$.*

Hence short sales and naked credit default swaps have similar qualitative effects, but are quantitatively equivalent only in the limiting case of $\eta = 0$, when the common investor belief about the worst-case outcome is at its most pessimistic. This is intuitive, because short sales force pessimists to enter a short position even on the *safe* portion of the bond's cash flow if they want to bet on default. Credit derivatives allow them to short credit risk without simultaneously shorting the safe portion of the cash flow. As long as the worst-case payoff from the bond is non-zero, the former regime results in a more depressed bond price than the latter.

4 Rollover Risk

The preceding analysis explored the manner in which the magnitude and sign of the effect of credit derivatives on the cost of borrowing depends on the distribution of investor beliefs. In particular, we saw that naked credit default swaps essentially replicate pure short sales when investor beliefs are at their most pessimistic. In this latter case, there is an additional concern that credit derivatives may give rise to self-fulfilling bear-raids. We now consider this possibility explicitly, by extending our baseline model to accommodate a multi-period setting with rollover risk.

One of the key features of debt contracts is that they frequently involve maturity transformation: the term of the loan is too short to enable full repayment without refinancing.

In this case the cost of current financing depend on expectations regarding the ability of the borrower to roll over debt when it comes due. Multiple equilibria arise naturally in this setting, and we are interested in the manner in which the use of credit derivatives affects the cost of debt and the set of equilibria.

Consider three periods $T = 0, 1, 2$. In period $T = 0$, the borrower faces a funding requirement $b_0 > 0$, and proposes to finance this by issuing q_0 bonds each with unit face value. The bond price p_0 is determined by a competitive market in period $T = 0$. In period $T = 1$, the borrower's revenue y_1 is realized. If $y_1 \geq q_0$, then all bonds are paid in full and no refinancing is necessary. If $y_1 < q_0$, then the borrower must issue a quantity q_1 of bonds with unit face value to cover the shortfall of $q_0 - y_1$. As before, a competitive market at period $T = 1$ sets the price p_1 of the bonds. In period $T = 2$, the revenue y_2 is realized, and the bond holders are paid $\min\{q_1, y_2\}$ in the aggregate.

To focus on the main idea, we make the simplifying assumption that the borrower's ability to repay is binary: $y_t \in \{0, 1\}$, for $t = 1, 2$. In particular, this means that $\eta = 0$; we are thus focusing on the case of zero recovery as a worst-case outcome. In period $T = 0$, a type θ-agent believes that $y_1 = 1$ with probability θ. As before, θ is drawn from the distribution $F(\theta)$. In period $T = 1$, there is no belief heterogeneity about the distribution of y_2; all investors believe that $y_2 = 1$ with probability λ (and $y_2 = 0$ with probability $1 - \lambda$). This common belief assumption plays no essential role; its only purpose is to simplify the analysis. In particular, it implies that in period $T = 1$, there will be no market for credit derivatives.

We assume, as before, that the borrower cannot take on greater debt obligations than could be honored even in the highest revenue state. That is, we assume $q_t \leq 1$, for $t = 1, 2$. As we show below, this constraint will not be binding in equilibrium as long as the initial borrowing requirement b_0 is not too large. We also assume that the borrower cannot complete the project if the outstanding debt cannot be rolled over when the low income state is realized at $T = 1$. This rules out a partial rollover of debt, in which earlier investors are not paid in full but the firm is nevertheless able to raise new funds. Such a partial rollover of debt is rare in practice, given that default entails high fixed costs, a loss of reputation, and severely restricted access to capital markets.

Before proceeding, it is important to consider why the borrower finances via a sequence of short-term obligations rather than a long-term bond that matures at $T = 2$ and therefore avoids rollover risk. There are a number of reasons why firms engage in such maturity transformation, among which is the inability to credibly pledge income that is realized in the interim stage $T = 1$. Income earned well in advance of the maturity date is difficult to monitor, and it is easier for the borrower to divert such resources away from creditors without raising suspicion.[17] This inability to pledge near-term income to service long-term debt implies that the terms available for long-term financing are not generally favorable relative to a sequence of shorter maturity debt. For instance, if the borrower can fully divert her income at $T = 1$ in the presence of a long term contract, the loan is effectively backed only by $T = 2$ income. Such a contract is dominated by the sequence of short term loans that we consider.[18]

We start by characterizing the set of equilibria without credit derivatives, beginning our analysis at period $T = 1$. If $y_1 = 1$ the initial debt is fully repaid. If $y_1 = 0$, the borrower owes q_0 and must borrow this amount to avoid default. Suppose this is done by issuing an amount q_1 of new one period bonds, each with unit face value. Recall that there is common belief on the part of investors that each such bond will have an expected payoff of precisely λ at $T = 2$. Hence the equilibrium bond price must satisfy $p_1 = \lambda$, and the borrower can therefore borrow $p_1 q_1 = \lambda q_1$. Since $q_1 \leq 1$, the debt can be rolled over if and only if $q_0 \leq \lambda$. In particular, if $q_0 > \lambda$, then no refinancing occurs at all in the low income state (i.e., $y_1 = 0$), and bondholders are paid nothing.

Now consider period $T = 0$. If $b_0 \leq \lambda$, then there exists a trivial equilibrium in which the borrower issues $q_0 = b_0$ at a price $p_0 = 1$. This bond is risk-free (since the debt is certain to be rolled over if necessary) and all investors are therefore willing to pay the face value for each unit regardless of their beliefs.

If $b_0 > \lambda$, then no such equilibrium exists since a debt this large cannot be refinanced if $y_1 = 0$. Hence any bonds sold in the initial period will be repaid if and only if $y_1 = 1$. If an agent of type θ spends her unit cash endowment on purchasing bonds, she will expect to earn θ/p_0. Since this strategy is optimal only when this payoff is no less than a dollar, the agent will purchase bonds if and only if

$$\theta > \hat{\theta} = p_0. \tag{20}$$

Given that the borrower needs to raise $p_0 q_0 = b_0$, the market clearing condition $1 - F(\hat{\theta}) = b_0$ may be written

$$1 - F\left(\frac{b_0}{q_0}\right) = b_0. \tag{21}$$

[17]For instance, such diversions are unlikely to be regarded as fraudulent by a bankruptcy court.

[18]More precisely, a long-term bond with unit face value will be paid in full with probability λ and will pay nothing with probability $1 - \lambda$. Thus the borrower can finance only $b \leq \lambda$, and must issue $q = b/\lambda$ bonds, each of which will be sold at price λ. As we show below, a sequence of short term loans can allow for better terms of financing for the borrower, and for a larger funding requirement to be met.

There is a unique bond issue size that satisfies this, given by

$$\hat{q}_0 \left(b_0 \right) = \frac{b_0}{F^{-1} \left(1 - b_0 \right)}.$$

Note that $\hat{q}_0(b_0) > b_0$. This means that even when $b_0 \leq \lambda$ (so an equilibrium with $q_0 = b_0$ exists), there can be a second equilibrium if $\hat{q}_0(b_0) > \lambda \geq b_0$ in which investors have pessimistic expectations regarding the borrower's ability to refinance in the low income state. This pessimistic equilibrium has a lower bond price and requires the borrower to incur a larger debt obligation in order to meet its borrowing requirement. Define $\hat{b}_0 := \hat{q}_0^{-1}\left(\lambda \right)$. That is, $\hat{b}_0$ is a critical borrowing requirement that satisfies

$$1 - F \left(\frac{\hat{b}_0}{\lambda} \right) = \hat{b}_0.$$

Clearly, $\hat{b}_0 < \lambda$. The following result identifies a range of values for the initial borrowing requirement such that a multiplicity of equilibria exists.

Proposition 5. *If $b_0 > \lambda$, there exists a unique equilibrium in which the borrower issues $\hat{q}_0 \left(b_0 \right)$ bonds with unit face value at price $p_0 = b_0 / \hat{q}_0 \left(b_0 \right)$. Default occurs if and only if $y_1 = 0$. If $b_0 < \hat{b}_0$, there is a unique equilibrium in which the borrower issues $q_0 = b_0$ bonds with unit face value and unit price, never defaults on these bonds, and rolls over the debt if $y_1 = 0$. If $\hat{b}_0 \leq b_0 \leq \lambda$, then both equilibria exist.*

If the initial borrowing requirement is sufficiently low, then investors fully expect that debt will be successfully rolled over in the low income state, and there is a unique equilibrium with zero interest. If the initial borrowing requirement is sufficiently high, there is again a unique equilibrium but one in which default is expected in the low income state, and the interest rate is correspondingly higher. For intermediate values of the initial borrowing requirement, both equilibria can co-exist. If investors believe that the borrower will be unable to roll over debt in the low income state, they will require higher interest rates as compensation for this risk, and the greater debt burden that results will cause these beliefs to be correct. On the other hand, if they expect that refinancing will be available at either state, this too will be self-fulfilling since the debt burden will be correspondingly lower.[19]

[19] The multiplicity of equilibria here differs in several respects from the rather trivial multiplicity caused by investor miscoordination on the borrower's ability to meet his funding requirement in the single period model. First, in the current context, the borrower meets his funding requirement in both equilibria; only the terms of financing differ. Second, whether multiplicity arises in the multi-period case depends on fundamentals such as λ and b, whereas the multiplicity in the single-period case does not. Finally, although we were able to eliminate the no-funding equilibrium in the single-period case by restricting attention to undominated strategies, dominance does not lead to equilibrium selection in the multi-period setting.

Now consider the effects of allowing for naked credit default swaps in this environment. The market for these contracts never materializes in period $T = 1$, and the same is true in period $T = 0$ if investors are confident that the borrower will be able to raise b_0 by issuing $q_0 \leq \lambda$ bonds. These bonds never default, since the debt can be rolled over even in the low income state, and this is known to all agents. But, as in the case without credit derivatives, there can be another equilibrium in which investors are not confident about the borrower's ability to roll over debt in the low income state.

If default protection can be purchased without holding the underlying bond, then, as in the one period model considered earlier, optimistic agents will sell protection or buy bonds without protection in equilibrium, while pessimistic agents will buy naked credit default swaps. As before, the swap spread and bond price must satisfy $p_0 + r_0 = 1$ (recall that the current model corresponds to the case of $\eta = 0$.) Our earlier analysis implies that agents with $\theta > \hat{\theta}$ buy bonds without protection or sell protection, while each agent with $\theta < \hat{\theta}$ purchases protection on bonds with face value $1/(1 - p_0)$. Here the threshold type $\hat{\theta} = p_0$ as in (20).

In equilibrium we must have

$$\frac{1}{p_0}\left(1 - F(\hat{\theta})\right) = q_0 + \left(\frac{1}{1 - p_0}\right) F(\hat{\theta}).$$

Collecting terms and using $\hat{\theta} = p_0$ and $p_0 q_0 = b_0$, we get

$$1 - \left(\frac{q_0}{q_0 - b_0}\right) F\left(\frac{b_0}{q_0}\right) = b_0. \tag{22}$$

One can check that the left side is increasing in q_0 for $q_0 > b_0$, so there is a unique value $\overline{q}_0(b_0) > b_0$ that satisfies the equation. Since the left side of (22) is smaller than that of (21), it also follows that $\overline{q}_0(b_0) > \hat{q}_0(b_0)$. The market clearing bond price is then $\overline{p}_0 = b_0/\overline{q}_0(b_0) < \hat{p}_0$.

Define $\overline{b}_0 := \overline{q}_0^{\,-1}(\lambda)$. Then, $\overline{b}_0 < \hat{b}_0$. The following result identifies the equilibrium set when naked credit default swaps are permitted.

Proposition 6. *If $b_0 > \lambda$, there exists a unique equilibrium in which the borrower issues $\overline{q}_0(b_0)$ bonds with unit face value at price $\overline{p}_0 < \hat{p}_0$. Default occurs if and only if $y_1 = 0$. If $b_0 < \overline{b}_0$, there is a unique equilibrium in which the borrower issues $q_0 = b_0$ bonds with unit face value and unit price, never defaults on these bonds, and rolls over its debt if $y_1 = 0$. If $\overline{b}_0 \leq b_0 \leq \lambda$, then both equilibria exist.*

The comparison with the case without credit derivatives is instructive. If $b_0 \in (\overline{b}_0, \hat{b}_0)$ then, in the absence of credit derivatives, the no default outcome is the unique equilibrium.

Allowing for naked credit default swaps introduces an additional equilibrium in which the borrower defaults in the low income state. Furthermore, even if multiple equilibria exist under both regimes, the terms of financing are worse for the borrower at the equilibrium with the higher interest rate in the presence of naked credit default swaps. The following example (depicted in Figure 5) illustrates.

Example 2. *Suppose that* $\lambda = 0.40$ *and* $F(\theta) = \theta^2$. *In this case* $\hat{b}_0 = 0.33$ *and* $\bar{b}_0 = 0.23$. *The range of initial debt levels for which multiple equilibria exist with naked credit default swaps is* $[0.23, 0.40]$, *but when no such contracts are allowed, this range is* $[0.33, 0.40]$. *Furthermore, when the more pessimistic equilibrium exists under both regimes, it is more punitive in the presence of naked credit default swaps.*

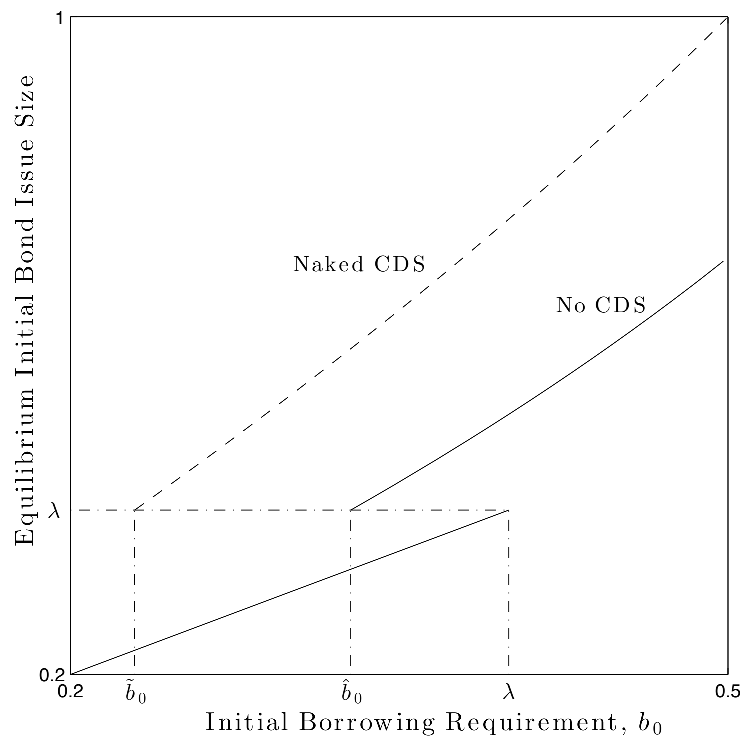

Figure 5: Equilibrium Bond Issues with and without Naked CDS

As is clear from the figure, the presence of naked credit default swaps has two effects. First, it expands the range of initial borrowing requirements for which an equilibrium with default in the low income state exists. And second, conditional on such an equilibrium being selected, interest rates are higher when naked credit default swaps are permitted than when they are not. The latter effect is similar to that identified in the one-period version of the model. And the former effect confirms that self-fulfilling pessimism about the borrower's ability to roll over debt is more likely to arise when naked credit default swaps are permitted than when they are not.

This result provides a new perspective on the empirical claim in Bruneau et al. (2012) that the relationship between fundamental factors and the cost of debt faced by the countries of the Eurozone periphery has undergone a structural shift, and that credit derivatives have facilitated coordination on a high interest equilibrium. The simple model considered here suggests that the effect of derivatives extends beyond the coordination of expectations. Holding constant fundamental factors, the presence of derivatives can alter the terms of lending at any given equilibrium, and can even increase the cardinality of the set of equilibria.

5 Conclusions

Since naked credit default swaps are speculative bets with payoffs that net to zero, it is not immediately apparent what (if any) effects their presence has on economic fundamentals. These effects can be nontrivial. The availability of such contracts can shift the terms of debt contracts against borrowers by inducing optimistic investors to divert their capital away from financing real investment and towards the support of collateralized speculative positions. This effect is strongest when beliefs about worst-case outcomes are at their most pessimistic. But derivatives also facilitate the separation of funding from credit risk exposure and can improve terms for borrowers when beliefs about worst-case recovery are sufficiently optimistic. Taken together, the net effect is greater cyclical variation in the cost of debt. In addition, the presence of credit derivatives can result in the emergence of equilibria in which borrowers are unable to rollover their debt, even when such equilibria would not exist in their absence.

Our focus has been on the effects of credit derivatives on the cost of capital and rollover risk without attention to the welfare implications of these effects. Welfare analysis when individuals have heterogeneous and incompatible beliefs gives rise to certain conceptual problems. For instance, those who willingly take opposite sides of a zero-sum bet do so because they each believe that their positions have positive expected returns, so the availability of such contracts appears to benefit both parties. But they cannot both benefit from a zero sum bet from the perspective of *any* third-party observer.[20]

One could consider welfare from the perspective of a planner with some given belief about the distribution from which the borrower's future revenues are drawn. Regardless of what this belief happens to be, the payoffs from the trading of naked credit default swaps sum

[20] Even if all investors consider themselves to be better off with unrestricted contracts conditional on their own respective beliefs, it is not possible for both optimists (who are bondholders in all regimes) and the borrower to be simultaneously better off. Hence the criterion of Pareto efficiency does not yield decisive welfare conclusions.

to zero, so (utilitarian) welfare depends only on the joint surplus generated by the borrower and the set of investors. That is, it depends on the efficiency of the funding decision. Since the presence of credit derivatives shifts the terms of financing and the range of borrowing requirements that can be met, it also affects the set of projects that are funded. The efficiency effects of this depend on whether or not these projects have positive net present value from the planner's perspective. If investors on the whole are too pessimistic (relative to the planner) then the regime that results in the largest levels of real investment will be favored. Similarly, if investors on the whole are euphoric then the regime that most constrains real investment will be favored. If we interpret pessimism as corresponding to the case of low η and optimism to high η then, in light of Proposition 3, both goals are accomplished by restricting the use of credit derivatives. Nevertheless, it is clear that there exist circumstances in which the presence of credit derivatives can prevent the funding of inefficient projects.

A more subtle efficiency effect can arise when the borrower is faced with a choice of projects with varying levels of risk and expected return. Under a debt contract, creditors bear the losses from low revenue realizations, but do not share in the gains from unusually high revenue realizations. The higher the debt burden, the greater is the incentive for the firm to choose projects with higher upside potential, even if they have lower expected returns (Adrian and Shin, 2008). Since the availability of unrestricted credit derivatives affects the total debt obligation that must be undertaken to meet any given funding requirement, such availability can raise the riskiness of projects selected despite lowering their expected returns.[21]

Finally, a natural extension would be to consider the implications of credit derivatives for capital structure when investor beliefs are heterogeneous. In this case the portfolio choices of investors and the shares of debt and equity in total financing would be determined jointly. Based on the results presented here, it seems likely that the presence of credit derivatives will affect the cost of debt relative to equity and hence the firm's capital structure, although the precise nature of this effect is as yet unclear. We leave this extension to future research.

[21] This just one of a broad range of possible effects of derivatives on project choice; see our working paper (Che and Sethi, 2010) for further details.

Appendix

Proof of Proposition 1. Deduce from (6) and (7) that

$$1 - F(\hat{\theta}(\hat{p}(q), q)) = \hat{p}(q)q = q\psi(q; \hat{\theta}(\hat{p}(q), q)) \leq \Psi(\hat{\theta}(\hat{p}(q), q)), \tag{23}$$

where the first equality follows from (7), the second from (6) and the inequality follows from (2). Since the left most term of (23) is decreasing in $\hat{\theta}$ and the right most term is increasing in $\hat{\theta}$, the middle term (total revenue) is bounded above by $\Psi(\theta_m)$. The upper bound is attained at $q = 1$, $\hat{p}(q) = \Psi(\theta_m) = \psi(1, \theta_m)$ and $\hat{\theta}(\hat{p}(1), 1) = \theta_m$. Hence the maximum revenue that can be raised in equilibrium is $b^* = \Psi(\theta_m)$. Any funding requirement $b \leq b^*$ can be met since total revenue varies continuously between 0 and b^* as q varies between 0 and 1. Equilibrium exists and is unique in this case since $q^*(b)$ is unique by definition. To prove the last statement, consider $b < b' \leq b_m$. By definition, $q^*(b) \leq q^*(b')$ and $p^*(b) = \hat{p}(q^*(b)) \geq \hat{p}(q^*(b')) = p^*(b')$. We must have $q^*(b) < q^*(b')$ and $p^*(b) > p^*(b')$, or else $b = p^*(b)q^*(b) = p^*(b')q^*(b') = b'$, a contradiction. *Q.E.D.*

Proof of Lemma 1. Suppose that $p + r > 1$ and consider any investor with $a > 0$. There must be at least one such investor in any funding equilibrium, otherwise the bond market would not clear. Now consider a change in this investor's portfolio, reducing a by $\epsilon > 0$ and increasing x by exactly the same amount. To see that this change is feasible, note from (9) that

$$p(a - \epsilon) \leq 1 + rx - p\epsilon = 1 + r(x + \epsilon) - (p + r)\epsilon < 1 + r(x + \epsilon)$$

so the budget constraint is satisfied. The covering constraint (10) is clearly unaffected by this change, and it is easily verified that the collateral constraint (11) also continues to be satisfied. Hence the change in portfolio is feasible. Furthermore, the change results in an increase in the investor's payoff regardless of her belief type θ, since

$$u(a - \epsilon, x + \epsilon \,|\, \theta) = u(a, x \,|\, \theta) + (p + r - 1)\epsilon > u(a, x \,|\, \theta).$$

Hence no investor with $a > 0$ can be optimizing if $p + r > 1$.

Now suppose that $p + r < 1$. In this case, the budget constraint (9) must hold with equality, otherwise an investor could increase a and reduce x by the same small amount ϵ without violating any of the three constraints, thus increasing her payoff to

$$u(a + \epsilon, x - \epsilon \,|\, \theta) = u(a, x \,|\, \theta) + (1 - p - r)\epsilon > u(a, x \,|\, \theta).$$

Substituting (9) with equality into the utility function, we get

$$1+(\psi(q;\theta)-1+r)x+(\psi(q;\theta)-p)a=\psi(q;\theta)\left(\frac{p+r}{p}\right)-1.$$

Define

$$\tilde{\theta}(p;q):=\sup\left\{\psi(q;\theta)\leq\frac{p}{p+r}\right\}.$$

An agent with belief $\theta<\tilde{\theta}(p;q)$ will choose

$$(a(\theta),x(\theta))=\left(\frac{1}{p+r},-\frac{1}{p+r}\right),$$

and one with belief $\theta>\tilde{\theta}(p;q)$ will choose

$$(a(\theta),x(\theta))=\left(\frac{q-\eta}{pq-(p+r)\eta},\frac{\eta}{pq-(p+r)\eta}\right).$$

For the protection market to clear, we must have

$$\frac{1}{p+r}F(\tilde{\theta})=\frac{\eta}{pq-(p+r)\eta}(1-F(\tilde{\theta}))\Leftrightarrow pqF(\tilde{\theta})=(p+r)\eta.$$

For the bond market to clear,

$$\frac{1}{p+r}F(\tilde{\theta})+\frac{q-\eta}{pq-(p+r)\eta}(1-F(\tilde{\theta}))=q.$$

Combining these two conditions yields $pq=1$, but this means $b=pq=1>b$, which cannot happen in equilibrium. *Q.E.D.*

Proof of Proposition 3. We first recall the equilibrium conditions for the case of no credit derivatives. For a given fixed b, recall that the marginal type is θ^* satisfying

$$1-F(\theta^*)=b. \tag{24}$$

Given this, the equilibrium price p^* satisfies $\psi(q^*,\theta^*)=p^*$. Since $p^*q^*=b$, this latter condition may be written as:

$$\int_\eta^1\min\left\{y,\frac{b}{p^*}\right\}dG(y\,|\,\theta^*)=b. \tag{25}$$

Since we shall vary η, denote the equilibrium price satisfying (25) as $p^*(\eta)$. Note that θ^* is pinned down by (24), so it does not vary with η.

Now consider the naked CDS regime. As argued in the text, the condition (25) defines the equilibrium price except that the marginal type differs from θ^*. Let $p^n(\eta)$ and $\theta^n(\eta)$ denote the equilibrium price and marginal type in the naked CDS regime. These must satisfy

$$\int_\eta^1 \min\left\{y, \frac{b}{p^n(\eta)}\right\} dG(y \,|\, \theta^n(\eta)) = b. \tag{26}$$

The market clearing condition (18) can be rewritten, using the fact that $q^n(\eta)p^n(\eta) = b$, as follows:

$$1 - \left(\frac{1 - \eta p^n(\eta)/b}{1 - p^n(\eta)}\right) F(\theta^n(\eta)) = b - \eta. \tag{27}$$

Since an increase in θ shifts G in the sense of first-order stochastic dominance, it suffices to show that $\theta^n(0) < \theta^* < \lim_{\eta \uparrow b} \theta^n(\eta)$ and $\theta^n(\eta)$ is increasing in η.

We have already observed (in the text) that $\theta^n(0) < \theta^*$. We next show that $\theta^n(\cdot)$ is strictly increasing. To prove this, suppose to the contrary that for some $\eta' > \eta$, $\theta^n(\eta') \le \theta^n(\eta)$. Then, it follows from (27) that

$$\frac{1 - \eta' p^n(\eta')/b}{1 - p^n(\eta')} > \frac{1 - \eta p^n(\eta)/b}{1 - p^n(\eta)},$$

which implies

$$\frac{1 - \eta p^n(\eta')/b}{1 - p^n(\eta')} > \frac{1 - \eta p^n(\eta)/b}{1 - p^n(\eta)}$$

and hence $p^n(\eta') > p^n(\eta)$. This, together with $\theta^n(\eta') \le \theta^n(\eta)$, contradicts (26).

Last, we show that $\lim_{\eta \uparrow b} \theta^n(\eta) = 1$. Suppose to the contrary that there exists $\overline{\theta} < 1$ such that $\lim_{\eta \uparrow b} \theta^n(\eta) \le \overline{\theta}$. Then, since $F(\overline{\theta}) < 1$, (27) cannot hold for all η unless $\lim_{\eta \uparrow b} p^n(\eta) = 1$. But then $\lim_{\eta \uparrow b} r^n(\eta) = \lim_{\eta \uparrow b}(1 - p^n(\eta)) = 0$, and also $\lim_{\eta \uparrow b} q^n(\eta) = \lim_{\eta \uparrow b} b/p^n(\eta) = b$. Using (16), each agent with $\theta > \overline{\theta}$ must therefore choose a portfolio that satisfies

$$a(\theta) + x(\theta) = \frac{1}{p^n(\eta) - \eta/q^n(\eta)} \to \infty \text{ as } \eta \uparrow b.$$

But if $\overline{\theta} < 1$, either the sum of bond purchases or the amount of protection sold will be unbounded, violating either the bond market clearing condition or the market clearing condition for credit derivatives.

Taken together, these results imply that that there exists a threshold $\hat{\eta} \in (0, b)$ with the properties claimed in the Proposition. *Q.E.D.*

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