THE IMPACT OF INTERLINKED INDEX INSURANCE AND CREDIT CONTRACTS ON FINANCIAL MARKET DEEPENING AND SMALL FARM PRODUCTIVITY

Michael R. Carter [†]	Lan Cheng [†]	Alexander Sarris†††
University of California, Davis	University of California, Davis	University of Athens

ABSTRACT. This paper explores the relationship between credit and index insurance market development using a theoretical model in which small farm households have the option to either (i) adopt a capital-intensive technology that is risky but high-yielding, or (ii) self-insure by adopting a traditional low-input and low-yielding technology. We show that neither market is likely to develop in isolation from the other, and that uptake of improved technology will be low absent efforts to link credit and insurance. The failure of index insurance markets to independently develop is not per se due to the existence of basis risk or to its expense, as self-insurance strategies are similarly characterized by basis risk and are costly to the household as they reduce mean incomes. However, we show that the interlinkage of credit and index insurance contracts can allow both markets to develop because the interlinked contract is more likely to stochastically dominate self-insurance. The analysis also shows that the way interlinkage will work depends fundamentally on the nature of the credit market and the degree to which lenders are able to demand and seize collateral in the event of loan default. This interplay between collateral and the nature of credit-insurance interlinkage has direct and important implications for the design of programs to simultaneously boost small farm productivity and deepen rural financial markets.

Keywords: Index insurance, Credit rationing, Interlinkage, Technology adoption

[†] Department of Agricultural and Resource Economics, mrcarter@ucdavis.edu.

^{††} Department of Agricultural and Resource Economics, lancheng@ucdavis.edu.

^{† † †} Department of Economics, alekosar@otenet.gr.

1. INTRODUCTION

The correlated risks and information asymmetries that typify many low-income, small-holder agricultural economies can keep rural financial markets (credit and insurance market) thin or even absent.¹ The costs of these thin markets are obvious and well documented, but the solution is far less clear. An earlier generation of efforts to employ conventional agricultural insurance to address the risk needs of the small farm sector failed under the weight of transactions costs, adverse selection and moral hazard (Hazell (1992); Barrett et al. (2008); Rothschild and Stiglitz (1976)). While it is tempting to declare the problem unsolvable, the pernicious role that risk plays in the construction and perpetuation of rural poverty demands further efforts in this area.

Enabled by technological advances in remote sensing and meteorological data, novel forms of agricultural index insurance would appear to offer a solution to this problem of thin financial markets twinned with low small farm productivity. Unlike conventional insurance whose indemnities is determined by individual outcomes, index insurance indemnifies insured farmers based on an index that is correlated with individual outcomes but is not influenced by individual behavior. Despite their advantages in overcoming moral hazard and adverse selection, agricultural index insurance contracts have met with sometimes indifferent demand and low uptake by the intended beneficiary populations. While there can be multiple explanations for low uptake rates, this paper argues on theoretical grounds that uptake and impacts will be higher when index insurance is interlinked with credit contracts. Put simply, our argument is that either market, credit or insurance, in isolation is likely to be thin or slow to develop in small-holder agriculture. When contracts are interlinked, the gains in market deepening and productivity growth are likely to be higher. We show that impacts of interlinkage are somewhat subtle, and differ across different types of economic environments.

Interlinking index insurance with credit is far different from simply bundling the two contracts together. Giné and Yang (2009) empirically estimate the impact of bundled contracts on take-up of credit under borrowers' limited liability using a field experiment in Malawi. Farmers in the treatment group were offered bundled loan contracts, while those in control group were offered stand-alone loan contracts. The result shows that the take-up rate of bundled loan contracts was 13% lower than that of stand-alone loans. Miranda and Gonzalez-Vega (2010) build a model in a similar context with limited liability loans. Their simulation shows that loans bundled with index insurance raise loan default rates and reduce lenders' expected profits. They attribute the poor

¹See Binswanger and Rosenzweig (1986) for a persuasive, if somewhat informal discussion of these points.

performance of bundled index insurance to high basis risk or loading costs, concluding that index insurance does not have much value for individual farmers. However, these two studies overlook the positive externality generated by borrowers' purchases of index insurance on lenders and do not endogenize loan contracts terms. When borrowers purchase index insurance under limited liability, index insurance not only reduces borrowers' risks but also protects lenders by reducing default rates. Interlinked index insurance contracts internalize this externality by allowing loan contract terms to respond to insurance contract, thus increasing the value of index insurance to individual farmers. Our model suggests that interlinked contracts outperform both non-interlinked and stand-alone loan contracts, especially in a low collateral environment, which induce high take-up rates of financial products, high productivity technology and raise farmers' welfare level.

There is a tendency to explain low uptake rates of index insurance by inappropriate analogy to developed country experience, or by general statements that basis risk or loading costs are too high (Smith and Watts, 2009; Miranda and Gonzalez-Vega, 2010). However, the fact is that the self-insurance strategies employed by small farmers expose farmers to significant basis risk and are actuarially unfair, with high implicit loadings and premium. The question is thus not whether or not there is basis risk under index insurance, but whether farmers' welfare under index insurance can stochastically dominate that under self-insurance.

Asking the question this way motivates the search for ways to combine index insurance with adoption of higher-yielding, but riskier technologies. That is, index insurance will more likely stochastically dominate self-insurance if it is a non-zero sum proposition that simultaneously allows an increase in expected income even as it reduces risk exposure. We here explore ways in which this might happen through the interlinkage of index insurance with credit contracts.

To do this, Section 2 presents a stylized model of the technology and contracts potentially available to producer-consumer households in a low-income agricultural economy. We show that index insurance contracts can be represented as a mean preserving squeeze of the stochastic distribution determining output, and the agricultural credit supply is determined by lenders' exposure to covariant default risks. Section 3 then explores households' demand for technology, credit and insurance facing three insurance schemes associated with high-yielding technology: 1) no formal insurance, where loan contracts provide implicit insurance when loans are not fully collateralized, 2) noninterlinked or bundled index insurance and credit, 3) interlinked index insurance and credit. We analyze each scheme in two stylized environments: one characterized by high levels of collateral such as in Latin America and another characterized by low levels of collateral such as in Africa and China. The introduction of non-interlinked insurance substantially improves the demand for high-yielding technology and financial products when the collateral level is high, but has small positive or even negative impacts on households' demand when the collateral level decreases. This is consistent with studies by Giné and Yang (2009) and Miranda and Gonzalez-Vega (2010). In contrast, interlinked index insurance significantly boosts households' demand for high-yielding technology in both low and high collateral environments. Section 4 analyzes the impacts of different insurance schemes when the credit market reaches equilibrium. We show that the demand for new technology and financial products under no formal insurance and non-interlinked insurance schemes discussed in Section 3, will be choked off by the increased price of credit. However, when insurance is interlinked with credit, lenders are willing to provide any amount of agricultural loans at a fixed low price, so that any expansion of credit demand will be satisfied. Section 4.2 analyzes the heterogeneous impact of interlinked contracts. Section 5 concludes with the main findings.

2. Environment, Technology and Financial Contracts

This section lays out a stylized model of the risk-averse and small farm household. While highly simplified, the model captures the key elements of the small farm problem that are relevant to the problem at hand, including the self-insurance options available to the household. Agricultural production is influenced by both covariant and idiosyncratic shocks. Against this backdrop we define the set of potential financial contracts that could be offered by competitive lenders and insurance firms.

2.1. Risks and Household Production. Small farm households are assumed to have access to two technologies, a traditional technology with low, but stable returns, and a higher yielding, but riskier technology that requires substantial use of purchased inputs. Both technologies are subject to idiosyncratic (or specific) shocks, θ_s and covariant shocks, θ_c^2 . We assume a multiplicative structure and write the output of low-yielding technology as:

$$(2.1) y_\ell = \theta g_\ell$$

where $\theta = (\theta_c + \theta_s)$ with support $[0, \overline{\overline{\theta}}]$, probability distribution function denoted $f(\theta)$, cumulative distribution function denoted $F(\theta)$ and $E(\theta) = 1$. The net income from low yielding technology is

 $^{^2 \}mathrm{The}$ simulation uses 75% of covariant risk over the total risk.

denoted as $\rho_l = y_\ell$. Similarly, we write the output of the high-yielding technology as

(2.2)
$$y_h = \theta g_h(K)$$

where K is the amount of purchased inputs required. The superiority of high-yielding technology is the higher expected net return, $g_h(K) - K > g_\ell$. We further assume capital K can only be financed by borrowing from the rural credit market. Denote the loan contract as $\ell < K, r, \chi >$, where r is the contractual interest rate and χ is the collateral required (Section 2.3 gives details on the determination of contract terms). Net returns to the household under this loan contract are as follows:

(2.3)
$$\rho_h = \begin{cases} \theta g_h(K) - (1+r)K, \text{ if } \theta > \tilde{\theta} \\ -\chi, \text{ otherwise} \end{cases}$$

where $\tilde{\theta} = \frac{(1+r)K-\chi}{g_h(K)}$ is the threshold level of the shock such that the value of the collateral plus the output produced just equals the value required for full loan repayment. Note that this specification sharply assumes that the household retains no income (or collateral assets) until after the loan is repaid.

Assuming the separability between household's consumption and production, household consumption is given by $c_t = \rho_t + W + B$, $t = h, \ell$, where W is the household's consumable and collateralizable wealth and B is the risk-free income from off-farming activities. The lowest consumption can be under the high-yielding technology is $\underline{c} = W + B - \chi$. Figure 5 shows household consumption as a function of the stochastic factor under the two technologies. The dashed line represents the low technology. The solid and dotted lines represent the high technology in a high and low collateral environment respectively. As collateral level decreases, the consumption floor rises and lenders bear more down-side risk.

Assume the household is risk-averse and has a conventional concave utility function, u(c). For purposes of later numerical analysis that we will use to illustrate various propositions, we assume that the utility function exhibits Constant Relative Risk Aversion (CRRA). Households choose between low-yielding and high-yielding technology to maximize expected utility. The population of the economy is distributed following the joint probability distribution function $h(\psi, W)$, where ψ is the Arrow-Pratt measure of relative risk aversion. Figure 2 demonstrates the effectiveness of self-insurance by adopting low yielding technology, using the numerical specification detailed in Appendix A. The black solid line and the blue dashed line depict the CDFs of household consumption under high technology financed by fully-collateralized loans and low technology respectively. We see that low technology substantially reduces the probability of low outcomes. However, this self-insurance strategy is far from perfect. First, it is actuarially unfair, reducing expected household income (and consumption) by the difference in expected incomes between the two technologies (23% of expected income reduction in the numerical parameters used to generate the figure). Second, compared to an idealized contract that shielded the household against any consumption losses any time when the high yielding technology results in production less than its expected value (illustrated in Figure 2by the pink dash-dotted line)³, self-insurance exposes the household to residual or basis risk as there is still a substantial probability of consumption well below the expected level. As can be seen, this idealized contract stochastically dominates self-insurance. While the index insurance contracts discussed in the next section are clearly not going to dominate this type of idealized contract either, the relevant question is whether they can dominate self-insurance given the basis risk and implicit loadings associated with it.

2.2. Index Insurance Contracts. Unlike conventional agricultural insurance that pays off based on individual outcomes (y_t in our notation), an index insurance contract pays off based on direct observation of the covariant shock (θ_c) or on average yields ($\theta_c g_t$)⁴. To keep matters simple, we will assume that the index insurance contract is based directly on θ_c . We denote the insurance contract for technology t as $I_t < \hat{\theta}_c$, h_t , z_t , $\beta_t >$, where $\hat{\theta}_c$ is the strike point for the contract, h_t is indemnity normalized by g_t , z_t is the normalized actuarially fair premium and β_t is the normalized loading or markup of the insurance as a percentage of z_t . To simplify the mathematical analysis the following theoretical structure assumes the insurance contract is actuarially fair, but the simulation results are based on a 30% of loading costs. The indemnity is defined by $h_t(\theta_c) = 1(\hat{\theta}_c > \theta_c)(\hat{\theta}_c - \theta_c)^5$.

³The contract illustrated in Figure 2 emulates a multi-peril contract that restores farm income to its average level any time an idiosyncratic or covariant shock occurs. Subtracted from consumption is the actuarially fair premium for such coverage. Such contracts typically do not exist for the small farm sector because of moral hazard, adverse selection and high transaction costs.

⁴As discussed by many authors, index insurance avoids the moral hazard, transactions costs and adverse selection problems that render conventional agricultural insurance unsustainable.

⁵This implicitly assumes that the farmers can choose the optimal insurance coverage level to reduce the basis risk, so that insurance contract always reduces farmers' risks

Under the actuarially fair insurance contract, gross returns to the farm household are given by:

(2.4)
$$y_t^I = \begin{cases} (\theta_c + \theta_s)g_t + (\hat{\theta}_c - \theta_c)g_t - z_tg_t = (\hat{\theta}_c + \theta_s - z_t)g_t, \text{ if } \hat{\theta}_c > \theta_c \\ (\theta_c + \theta_s)g_t - z_tg_t, = (\theta_c + \theta_s - z_t)g_t & \text{otherwise} \end{cases}$$

where $z_t = E[1(\hat{\theta}_c > \theta_c)(\hat{\theta}_c - \theta_c)]$. Note that this specifications assumes that the household first receives indemnity and pays insurance premium, and then applies the net income to repaying outstanding debt before bolstering its consumption.

As a prelude to later analysis, define $\theta^I = \theta + s(\theta)$, where $s(\theta) = 1(\hat{\theta}_c > \theta_c)(\hat{\theta}_c - \theta_c) - z_t$. Then y_t^I can be written as $y_t^I = \theta^I g_t$. This indicates that index insurance contract essentially transforms the multiplicative shock from θ to θ^I , where θ^I represents the net output shock. Since z_t is the actuarially fair premium, $E[\theta^I] = E[\theta] = 1$. Denote the PDF and CDF of θ^I as $f^I(\theta)$ and $F^I(\theta)$. We proof that θ^I is a "mean preserving squeeze" of θ and the following two properties hold (see mathematical proof in Appendix B):

(2.5)
$$\int_0^{\bar{\theta}} [F(\theta) - F^I(\theta)] d\theta = 0$$

(2.6)
$$\int_0^y [F(\theta) - F^I(\theta)] d\theta \ge 0 \,\forall 0 \le y < \bar{\bar{\theta}}$$

. Figure 5 illustrates the PDF of θ and θ_I .

2.3. Credit Supply. We assume that the credit markets are competitive and that banks are willing (at the margin) to offer an agricultural loan that provides an expected return equal to an opportunity cost of funds, $\bar{\pi}_a$. In the first part of this section, we assume $\bar{\pi}_a$ is exogenously given and examine the nature of marginal loan offers and the impact of insurance on loan contract terms. In the second part, we endogenize $\bar{\pi}_a$ and explore the impact of index insurance on aggregate supply of agricultural credit.

The Iso-expected profit contract locus. Under a standard agricultural loan contract $\ell(K, r, \chi)$, lender's profits, π , are given by:

(2.7)
$$\pi = \begin{cases} (1+r)K, & \text{if } \theta > \tilde{\theta} \\ \chi + \theta g_h(K), & \text{otherwise} \end{cases}$$

, and lenders' expected profits are given by:

(2.8)
$$E(\pi) = [1 - F(\tilde{\theta})]rK + \int_{0}^{\tilde{\theta}} (\chi + \theta g_h(K) - K)f(\theta)d\theta$$

. Since lender's profits π are concave in the random variable θ , expected profits $E(\pi)$ decrease when the variance of θ increases holding others constant. Assume $\bar{\pi}_a$ is exogenously given. Using the implicit function theorem, we can characterize the iso-expected profits contract locus as those combinations of interest rates and collateral requirements that just yield expected returns equal to $\bar{\pi}_a$ ($E(\pi) = \bar{\pi}_a$). Figure 5 depicts the iso-expected profit lines with and without index insurance. When borrowers are not insured, the locus has a negative slope $\frac{\partial r}{\partial \chi} = \frac{F(\tilde{\theta})}{(1-F(\tilde{\theta}))K} < 0$. This is because as collateral level declines, lenders have to raise interest rates to compensate the loss from higher default rates. When borrowers are insured, lenders face net output shock θ^I and the locus becomes much less steeper with the slope satisfying $\frac{F(\tilde{\theta})}{(1-F(\tilde{\theta}))K} < \frac{F^I(\tilde{\theta})}{(1-F^I(\tilde{\theta}))K} < 0$. This is because even though low collateral level exposes lenders to higher risks, index insurance compensates the loss so that lenders can maintain the level of interest rates ⁶.

In general, we would not expect a lender to be genuinely indifferent between the different points on the iso-expected profit loci. As explored by a number of papers, higher collateral and lower interest rate contracts diminish incentives for morally hazardous behavior and adverse selection by lenders (for a recent treatment, see Boucher et al. (2008)). In the analysis here, we ignore borrower heterogeneity that might generate adverse selection (e.g., differences in borrower honesty or individual level heterogeneity in the structure of risk). We also ignore potential sources of morally hazardous behavior (e.g., credit diversion as in Carter (1988) or non-contractible effort as in Boucher et al. (2008)). Instead, we follow Stiglitz and Weiss (1981) and simply assume that lenders demand that borrowers present a minimum amount of collateral in order to leverage of a loan of size K.

While these assumption are somewhat artificial, they allows us to focus on the impact of index insurance in two distinctive, but empirically important environments. The first of these might be considered to be representative of areas of Latin America where agricultural land is individually titled and potentially can be seized in the event of loan default. In these environments, we will assume that lenders require a collateral of value χ_h , as shown in Figure 5. In other areas where agricultural land ownership is less individualized and less securely titled (e.g., in China and many

 $^{^{6}}$ The slope of the iso-profit locus under insurance is flatter than under no insurance but it is still downward sloping, since uninsured idiosyncratic risks can cause default

parts of Africa), we will assume that the loan package requires a lower amount of collateral, denoted χ_{ℓ} in Figure 5. While we could impose additional structure on the model to endogenize collateral levels, our goal here is to show that index insurance and its interaction with small farm productivity and financial markets will exhibit subtle differences across these two types of stylized agricultural economies.

Aggregate credit supply to the agricultural sector. The analysis in the prior section considered the conditions of competitive loan supply taking the lender's overall loan portfolio as given. When loan repayment is subject to purely idiosyncratic shocks, the lender's overall portfolio will be self-insuring. However, a portfolio of agricultural loans will not be purely self-insuring as a negative covariant shock (e.g., a drought) could trigger a large scale episode of default.

To explore this issue further, this section examines the lender's portfolio decisions on aggregate credit supply. We assume that in the short-run, the lender has sufficient loanable funds to extend nloans of size K. The lender can extend type a agricultural loans, or type b loans, which we assume to be risk free (or subject only to idiosyncratic shocks and therefore self-insuring). The lender's gross rate of return from the portfolio of n loans, G, can be written as:

(2.9)
$$G = \frac{\sum_{i=1}^{n_a} \pi_i(\bar{\pi}_a) + n_b \bar{\pi}}{n}$$

. If G falls below a critical threshold level $\tilde{\pi}$, the lender faces a penalty function, P(G), which reduces the lender's net portfolio returns. The net portfolio returns, N, is written as:

(2.10)
$$N = G - P(G) = \begin{cases} G \text{,if } G > \tilde{\pi} \\ G - \tilde{P}(G) \text{, otherwise, where } \tilde{P}', \tilde{P}'' \le 0 \end{cases}$$

The penalty for low gross portfolio return occurs for several reasons. First, when the lender realizes too low a gross return on the loan portfolio, it runs afoul of reserve and other regulatory requirements. Second, when the gross portfolio return is too low, the lender has to sell a large amount of collateral to repay depositors, which drives down the price of collateral and lenders' net return. Next, low gross return from the portfolio forces the lender to borrow from the money market and pay for high interest rates. Lastly, from a political economy perspective, the lender understands that a massive

default, driven by a drought or other unfavorable event, will likely trigger a political economy reaction with the government tempted to mandate at least partial default forgiveness.⁷

Now assuming the expected net portfolio return $\bar{\pi}$ is exogenous in a competitive market, lenders have to satisfy the participation constraint in which $E(N) \ge \bar{\pi}$. Given this constraint, lenders have to adjust $\bar{\pi}_a$ as the composition of the portfolio changes. Let F_G and f_G denote the CDF and PDF of G. Taking the expected value of N, the Lender's Participation Constraint (LPC) can be written as:

(2.11)
$$\bar{\pi} + \frac{n_a}{n}(\bar{\pi}_a - \bar{\pi}) - \int_0^{\bar{\pi}} P(G)f_G(G)dG \ge \bar{\pi} \qquad (LPC)$$

, where the integral term is the expected penalty. Using implicit function theorem, the expected return of agricultural loan can be written as a function of the quantity of agricultural loans, $\bar{\pi_a} =$ $\bar{\pi}_a(n_a)$. The function $\bar{\pi}_a(n_a)$ represents an aggregate supply curve of agricultural loans, which is shown in Figure 5. It shows that in absence of formal insurance and low collateral environment, $\bar{\pi}_a$ has to increase dramatically above $\bar{\pi}$ to maintain the participation constraint as the share of agricultural loans rises in the lending portfolio (the black solid line). This is because low collateral requirement exposes lenders to large covariant risk and increases the probability of paying penalty. The penalty policy P implies an increasing marginal cost of unit agricultural lending when lenders are exposed to covariant risks. The introduction of formal insurance reduces the cost of credit and thus flattens the supply curve (the black dashed line). Thus, when loans are insured, π_a keeps constant at a low level of $\bar{\pi}$ as the number of agricultural loans increases. As the collateral level increases, uninsured supply curves (the red and blue solid lines) shift down and insured supply curves (the red and blue dashed lines) keep equal to $\bar{\pi}$. In Figure 5 all insured supply curves and the uninsured supply curve in a low collateral environment overlap together on the flat straight line of $\pi_a = \bar{\pi}$. Insurance isolates the rate of return of agricultural loans from the impact of the collateral level. (Appendix C provides a mathematical proof of the shape of the function $\bar{\pi}_a = \bar{\pi}_a(n_a)$ and the determinant factors.)

Given a fixed level of χ , the aggregate supply function can also be written in terms of interest rates as $r = r(n_a \mid \chi, f^I(\theta), f(\theta), P)$, which is shown in Figure 6. Similar with Figure 5, when index insurance is not available, the lower the collateral level, the steeper the rise of interest rates. When

⁷Following the 1998 El Nino event, the Peruvian government instituted a "financial rescue" that instructed agricultural lenders to forgive outstanding debt (see Trivelli).

borrowers are insured, all the supply curves become straight flat lines and lenders would supply as many loans as they could at fixed interest rates. The level of interest rates under insurance is slightly different when collateral level changes. The interest rates under low collateral (the black dashed line) is higher than that under medium and high collateral (the red and blue dashed line) because low collateral exposes lenders to more idiosyncratic risks.

3. Demand for credit, insurance and technology Under Alternative Insurance Schemes

This section analyzes farmers' optimal choices of technology and financial contracts under alternative insurance schemes. In Section 3.1 we simulate the CDFs of household consumption under the four projects: the fallback project of low-yielding technology (self-insurance), high-yielding technology associated with stand-alone loan contracts (implicit insurance), high-yielding technology associated with non-interlinked and interlinked index insurance. Then we analyze farmers' choices between the low-yielding and high-yielding technology in each of the three insurance schemes by comparing the expected utility function. In Section 3.2, we show that when high-yielding technology is associated with stand-alone loan contract, farmers are likely to be risk-rationed, either because lenders charge high interest rates when collateral level is low or because farmers have to bear substantial default risks when collateral level is high. In both cases, farmers are likely to choose low technology rather than high technology. In Section 3.3 the high-yielding technology is associated with non-interlinked index insurance and credit contracts, where index insurance is introduced as an independent market. We show that in low collateral environments, the impact of non-interlinked index insurance on uptakes of high technology is adverse or minimal due to the existing implicit insurance provided by loan contracts. In contrast, in high collateral environments, non-interlinked insurance substantially improves household welfare by crowding in demand for credit and high technology. In Section 3.4, we examine the impact of contractual interlinkage in which loans and insurance are interlinked as a single contract (i.e., because loans are linked, lenders know when a loan is or is not secured by an insurance contract) and interest rates are endogenously determined by borrowers' purchase of insurance. We show that even in low collateral environment, interlinked insurance increases uptakes of credit and high technology by inducing lender to lower interest rates and increase credit supply. It should be stressed that the analysis is predicated on the simultaneous existence of an improved, capital-dependent technology.

3.1. Stochastic dominance of interlinked index insurance. Figure 5 draws the CDFs of consumption under the four projects that are discussed below when collateral level is low. The implicit insurance associated with high technology (the black solid line) is hard to compete with the selfinsurance associated with low technology (the blue dashed line), since implicit insurance has higher probability of low consumption than self-insurance. The introduction of the non-interlinked index insurance (the green dashed line) makes a small improvement over the implicit insurance. But highly risk-averse farmers would still prefer self-insurance to non-interlinked insurance, because limited liability reduces the value of non-interlinked index insurance. However, when index insurance is interlinked with loan contracts, the CDF shifts forward from the green dashed line to the red dashed line, so that the interlinked insurance is very likely to stochastically dominate self-insurance.

3.2. Absent formal insurance. This section compares farmers' expected utility between low technology and high technology when formal insurance is not available. Under the specification in Section 2, households' expected utility under low technology, V_{ℓ} , and high technology, V_h , are given by:

(3.1)
$$V_{\ell} = \int_{0}^{\overline{\theta}} u(\theta g_{\ell} + W + B) f(\theta) d\theta$$

(3.2)
$$V_{h} = F(\tilde{\theta})u(\underline{c}) + \int_{\tilde{\theta}}^{\bar{\theta}} u(\theta g_{h} - (1+r)K + W + B)f(\theta)d\theta.$$

. The household will choose high technology and demand for credit if $\Delta_l^h = V_h - V_\ell > 0$. Using the expressions above, we rewrite Δ_l^h as:

$$(3.3) \quad \Delta_{\ell}^{h} = \left[F(\tilde{\theta})u(W+B-\chi) - \int_{0}^{\tilde{\theta}} u(\theta g_{\ell}+W+B)f(\theta)d\theta \right] \\ + \left[\int_{\tilde{\theta}}^{\tilde{\theta}} [u(\theta g_{h}-(1+r)K+W+B) - u(\theta g_{\ell}+W+B)]f(\theta)d\theta \right] \\ + \left[\int_{\tilde{\theta}}^{\bar{\theta}} [u(\theta g_{h}-(1+r)K+W+B) - u(\theta g_{\ell}+W+B)]f(\theta)d\theta \right]$$

where $\check{\theta}$ satisfies $\check{\theta}g_h - (1+r)K = \check{\theta}g_l$, which is the threshold of θ when the net output of the high-yielding technology is equal to the output of the low-yielding technology. Since $\check{\theta} = \frac{(1+r)K}{g_h - g_l}$, $\check{\theta} > \tilde{\theta} (= \frac{(1+r)K - \chi}{g_h})$. The first and the second term in square brackets of equation 3.3 are both strictly negative (even when $\chi = 0$), representing the risks farmers have to bear when bad shocks occur. The first term represents the risks implicitly insured by loan contracts, while the second term represents the risks that are not covered by loan contracts. The third term is positive, representing the gain from the high-yielding technology when good shocks happen ⁸.

In the high collateral environment, the lower bound of income $\underline{c} = W + B - \chi$ is low and $\hat{\theta}$ is close to zero. This means the coverage of the implicit insurance is small and the sum of the first two terms, the total risks farmers have to bear, is negatively big. Risk-averse farmers are likely to have a negative Δ^h_ℓ and choose low technology. This is called risk rationing by Boucher et al. (2008). The risk-rationed are those households who eschewed the risk of borrowing and instead self-insured their livelihood by choosing the low income activity. In the low collateral environment, c rises and $\tilde{\theta}$ expands the implicit insurance. The risks farmers have to bear are reduced. However, the implicit insurance is far from being perfect. First, the sum of the first two terms is still negative even when $\chi = 0$, meaning farmers still have to bear risks when bad shock happens. Second and more importantly, since production risks, especially covariant risks, are passed to lenders under low collateral level, lenders ask for higher expected rate of return and raise interest rates as discussed in Section 2.3. As higher interest rates make the third term positively small, risk-averse farmers are still likely to be risk rationed. Therefore, in absence of formal insurance, the demand for high technology and credit are likely to be low in both low and high collateral environments. Figure 8 depicts certainty equivalent (CE) of the four projects for a representative farmer whose CRRA coefficient is equal to 2. Under the specification in Appendix A, the CE of high technology without formal insurance (the black solid line) is lower or only slightly higher than the CE of low technology (the blue dashed line). This is because high technology is too risky when collateral level is high and the implicit insurance is too costly that eats up the expected profit of high technology when collateral level is low. The rest of the paper focuses on such scenario where $\Delta^h_\ell < 0$ due to risk rationing. The next two subsections will analyze how index insurance reduces risk rationing in different collateral environments.

⁸The high-yielding technology is defined such that $g_h - (1 + r)K > g_l$ at any interest rates offered by lenders. Therefore, price-rationing is excluded.

3.3. Non-interlinked index insurance contracts . This section compares farmers' expected utility between low-yielding technology and high-yielding technology that is associated with noninterlinked index insurance and credit contract. The non-interlinked index insurance is merely bundled with loan contracts, where loan contracts terms are independent of index insurance. Before deriving expected utility, we first examine how actuarially fair non-interlinked contracts change farmers' expected consumption level. The difference of expected consumption from taking highyielding technology between with non-interlinked index insurance and without formal insurance is equal to

(3.4)
$$E(c_h^I) - E(c_h) = \chi[F(\tilde{\theta}) - F^I(\tilde{\theta})] - \int_{\tilde{\theta}}^{\bar{\theta}} [\theta g_h(K) - (1+r)K][f(\theta) - f^I(\theta)]d\theta$$

Integrating by parts and using the properties of mean-preserving spread (equation 2.5 and 2.6), the above expression reduces to:

(3.5)
$$E(c_h^I) - E(c_h) = g_h \int_0^{\tilde{\theta}} [F^I(\theta) - F(\theta)] d\theta \le 0$$

which is negative when loans are not fully collateralized and equal to zero when loans are fully collateralized. It indicates that the lower the collateral level, the lower the farmers' expected consumption under non-interlinked contracts.

Farmers' optimal choices over high and low yielding technology are based on V_h^I and V_l , which represent the value function of high-yielding technology under non-interlinked insurance and lowyielding technology respectively. Whether farmers will adopt high technology depends on the sign of $V_h^I - V_l$, which can be written as

(3.6)
$$\Delta_l^{h^I} = V_h^I - V_l = (V_h^I - V_h) + (V_h - V_l) = \Delta_h^{h^I} + \Delta_l^h$$

where $\Delta_l^h < 0$, and the sign of $\Delta_h^{h^I}$ is ambiguous. Since loan contract terms are isolated from index insurance under non-interlinked contracts, interest rates can be written as $r = r(\chi, n_a, f(\theta), P)$. Then we have

(3.7)
$$V_h^I = U(\underline{c})F^I(\tilde{\theta}) + \int_{\tilde{\theta}}^{\bar{\theta}} U[\theta g_h - (1+r)K + W + B]f^I(\theta)d\theta$$

and

(3.8)
$$\Delta_h^{h^I} = V_h^I - V_h = U(\underline{c})[F^I(\tilde{\theta}) - F(\tilde{\theta})] + \int_{\tilde{\theta}}^{\bar{\theta}} U[\theta g_h - (1+r)K + W + B](f^I(\theta) - f(\theta))d\theta$$

After integrating by part of $\Delta_h^{h^I}$ twice, we have

(3.9)
$$\Delta_h^{h^I} = U'(\underline{c})g_h \int_0^{\tilde{\theta}} [F^I(\theta) - F(\theta)]d\theta + \int_{\tilde{\theta}}^{\bar{\theta}} [\int_0^{\theta} (F^I(y) - F(y))dy]U''g_h^2d\theta$$

Since θ is a mean-preserving spread of θ^{I} , the first term of equation 3.9 is non-positive and the second term is non-negative. As can be seen from equation (3.5), the first term of $\Delta_{h}^{h^{I}}$ represents the change of expected utility due to the change in expected consumption. The second term represents the change of expected utility due to the change in consumption fluctuation. This indicates that risk neutral farmers for who the first term is non-positive and the second term is zero, would always prefer the implicit insurance rather than the non-interlinked insurance. This is consistent with the conclusion drawn from equation 3.5.

As for the risk-averse farmers, the sign and magnitude of $\Delta_h^{h^I}$ can be determined by collateral requirement. If fully collateralized with $\chi = (1+r)K$ and $\tilde{\theta} = 0$, the first term of equation 3.9 shrinks to zero and $\Delta_h^{h^I}$ is positive, indicating that farmers will be willing to buy non-interlinked insurance, since non-interlinked contracts maintain expected income level and reduce risks. Non-interlinked index insurance crowd in risk-rationed and raises uptakes of financial products and high-yielding technology. If $\chi = 0$ and $\tilde{\theta} > 0$, $\Delta_h^{h^I}$ decreases and is more likely to be negative, indicating farmers will not be willing to buy non-interlinked insurance, since they are already insured by loan contract and insurance premium lowers the expected income. Intuitively, under a low collateral environment, the lender bears most of the risk. Insurance is valuable to lenders by transferring the risk from the lender to the insurance provider, but yields no benefit to the household who nonetheless pays for insurance premium. In contrast, under a high collateral environment, the household who bears nearly all the risk, enjoys the gains from the insurance. The dashed line in Figure 8 illustrates the certainty equivalent of V_h^I as a percentage of that of low technology. We see that the CE under non-interlinked insurance is even lower than that under implicit insurance ($\Delta_h^{h^I}$ is negative) when collateral is low, which is consistent with the empirical evidence from Giné and Yang (2009). As collateral rises, the CE of the non-interlinked becomes higher than implicit insurance, and finally higher than self-insurance of low technology. This indicates that non-interlinked insurance can only solve risk rationing in a high collateral environment.

3.4. Interlinked insurance contracts. This section compares farmers' expected utility between low technology and high technology associated with interlinked index insurance and credit contracts. Farmers' choice of technology is based on the value functions of the two projects, denoted as V_h^{II} and V_l respectively. Farmers make decisions based on the sign of $V_h^{II} - V_l$, which can be disaggregated as

(3.10)
$$\Delta_l^{h^{II}} = V_h^{II} - V_l = (V_h^{II} - V_h^I) + (V_h^I - V_h) + (V_h - V_l) = \Delta_{h^{II}}^{h^{II}} + \Delta_h^{h^I} + \Delta_l^h$$

where $\Delta_l^h < 0$ and $\Delta_h^{h^I}$ increases in χ as shown in the above section. The rest of this section explores factors influencing the sign of $\Delta_{h^I}^{h^{II}}$.

As shown in Section 2.3, households' purchase of index insurance influences lenders' expected return. Under interlinked contracts, interest rates the lender offers are endogenously determined by index insurance contract, which can be written as $r^{I} = r_{m}(\chi, n_{a}, f^{I}(\theta))$. Because $\tilde{\theta}$ is a function of interest rates , the critical point of θ when default occurs is denoted as $\tilde{\theta}^{I} = \tilde{\theta}(r^{I})$ under interlinked insurance. Then V_{h}^{II} becomes

(3.11)
$$V_h^{II} = U(\underline{c})F^I(\tilde{\theta^I}) + \int_{\tilde{\theta^I}}^{\bar{\theta}} U[\theta g_h - (1+r^I)K + W + B]f^I(\theta)d\theta$$

The difference of expected utility between interlinked and non-interlinked insurance, $\Delta_{h^{I}}^{h^{II}}$ can be written as

$$(3.12)$$

$$\Delta_{h^{I}}^{h^{II}} = V_{h}^{II} - V_{h}^{I} = \int_{\tilde{\theta}}^{\bar{\theta}} (U[\theta g_{h} - (1+r^{I})K + W + B] - U[\theta g_{h} - (1+r)K + W + B])f^{I}(\theta)d\theta$$

$$+ \int_{\tilde{\theta}^{I}}^{\tilde{\theta}} (U[\theta g_{h} - (1+r^{I})K + W + B] - U(\underline{c}))f^{I}(\theta)d\theta$$

As Figure 6 shows, when $\chi < (1+r)K$, $r^I < r$, and $\tilde{\theta}^I < \tilde{\theta}$. Thus $\Delta_{h^I}^{h^{II}}$ is positive when loans are not fully collateralized, and is equal to zero when loans are fully collateralized. $\Delta_{h^I}^{h^{II}}$ is always non-negative and decreasing in χ . This means the interlinked insurance is always at least as good as non-interlinked insurance for farmers. Interlinkage will thus are able to crowd-in more credit demand by lowering interest rates for farmers who purchase index insurance. Since $\Delta_{h}^{h^{I}}$ is increasing in χ , interlinked insurance has advantages over non-interlinked insurance especially in a low collateral environment. The dotted line in Figure 8 denotes the CE of V_h^{II} , which always lies above the CE of V_h^I . The gap between the dashed and the dotted line (representing $\Delta_{h^I}^{h^{II}}$) increases as collateral decreases.

Combining $\Delta_{h^{I}}^{h^{II}}$, Δ_{h}^{h} , Δ_{l}^{h} together, the dotted solid line in Figure 8 demonstrates $\Delta_{l}^{h^{II}}$. The Certainty Equivalent (CE) of the interlinked high technology for a typical household is almost constant around 1.5% more than that of low technology regardless of the change in collateral level. This can be explained using the mean and variance of the income from interlinked high technology. Since interlinked insurance always brings the cost of unit credit back to a constant level $\bar{\pi}$ as shown in Figure 5, farmers' expected income from interlinked contracts is equal to

(3.13)
$$E(c_{h_{II}}) = E(y_h) - (1 + \bar{\pi})K + W + B$$

which is independent of collateral level. The income variance satisfies

(3.14)
$$Var(c_{h_{II}}) = \frac{Var(y_h)Var(\theta_I)}{Var(\theta)}$$

which is also independent of collateral level. Since the expected utility is mainly determined by the first and second order of income, the above two equations indicate that the CE under interlinked insurance mainly depends on the productivity of the technology and risk structure (basis risk), but is not influenced by the characteristics of the the credit market such as collateral level and numbers of agricultural loans, which have significant impacts on the performance of non-interlinked contracts.

4. Farm Productivity and the financial Market development in the Equilibrium of the the credit market

This section analyzes farm productivity and the credit market development when the credit market reaches an equilibrium⁹. In absence of interlinked insurance or fully collateralization, the aggregate supply curve of credit shown in Figure 6 is uprising and thus any increase in demand discussed in Section 3 will raise interest rates that choke off the increased expansion . When insurance and credit are interlinked, the credit supply curve is flattened at a constant level and thus the increased demand induced by insurance will not be choked off by interest rates. In other words, while non-interlinked contracts only shift credit demand curve, interlinked contracts shift both curves of credit demand and credit supply and thus induce a high uptake rate of high technology and financial products. The second part of this section analyzes the heterogeneous impact of index insurance and shows that interlinked contracts can crowd in highly risk-averse and poor smallholders, who are excluded from the credit market when interlinked contracts are not available.

4.1. the credit market Equilibrium. According to Section 2.3, we can write the aggregate supply of agricultural loans, n_a^s , as a function of the price r conditional on collateral level, the distribution function of θ , purchase of insurance and penalty function:

(4.1)
$$n_a^s = n_a^s(r \mid \chi, f(\theta), f^I(\theta), P)$$

According to Section 3, aggregate effective demand of agricultural loans, n_a^d , is a function of r conditional on collateral, the distribution of θ , purchase of insurance and the distribution of population on risk preference and wealth:

⁹Since the model does not consider imperfect information problems of moral hazard and adverse selection, interest rates do not affect the riskiness of lenders' return, and thus there exists an equilibrium interest rates that equates credit demand and supply.

(4.2)
$$n_a^d = n_a^d(r \mid \chi, f(\theta), f^I(\theta), h(\psi, W))$$

. Because the population is heterogeneous in ψ and W, the demand function is downward sloping in r. The the credit market reaches an equilibrium when demand equals supply,

$$(4.3) n_a^s = n_a^d = n_a$$

The quantity of agricultural loans and interest rates at the equilibrium, n_a^* and r^* , vary on the different insurance schemes associated with high technology: no formal insurance , non-interlinked and interlinked index insurance. Figure 9 shows the supply and demand curve of credit under the three insurance schemes in different collateral environments. Point A, B, C represent the equilibrium allocations under no formal insurance, non-interlinked and interlinked index insurance respectively. The horizontal axis, $n_a^*\%$, represents the percentage of farmers who obtain an agricultural loan and adopt high-yielding technology. The rest of population, $1 - n_a^*\%$, use traditional low-yielding technology.

In a low collateral environment (the first graph in Figure 9), demand and supply curve without formal insurance interact at a point with high price and relatively low quantity. When loans are insured with non-interlinked contract, supply curve keeps unchanged and demand curve shifts to the left due to the implicit insurance provided by low collateral. The lower demand curve drives down both r^* and n_a^* , as shown by the black arrow from point A to B. This coincides with the empirical observation that bundled loan contracts reduce uptake rates of loans by Giné and Yang (2009). When the two financial contracts are interlinked, the demand curve shifts in the same way as the one under the non-interlinked, and the supply curve shifts down and becomes flat. The shifting credit supply curve generates an equilibrium with lower r^* and higher n_a^* , as shown by the red arrow from point B to C. In a medium collateral environment, non-interlinked contracts increase credit demand and improves the equilibrium towards a higher n_a^* but also drives up the price. Interlinked contracts increase n_a^* further by lowering supply curve. Finally in the high collateral environment, non-interlinked contracts induce a big expansion of the demand so that almost the whole population obtain credit. Since the lender is fully insured by the high collateral, the interlinked contract performs as well as the non-interlinked. 4.2. The heterogeneous impact of index insurance. The impact of insurance on individuals varies as their risk preference and wealth change. The empirical evidence from Giné and Yang (2009) shows that wealth indicators have positive (although not significant) impact and risk aversion has negative impact on uptakes of non-interlinked contracts. Figure 10 shows simulated critical levels of risk aversion coefficient and wealth (ψ^*, W^*) when the credit market reaches an equilibrium, below which households adopt the high technology and above which they do not. In addition to risk rationing, this figure also considers the quantity-rationed who cannot borrow when their wealth level is lower than the collateral requirement. The solid lines represent no formal insurance , the dashed lines non-interlinked contracts, and the dash-dotted lines interlinked contracts. The red lines denote low collateral environment and the green lines denote high collateral environment .

In a low collateral environment, highly risk-averse and poor farmers in the northwest corner of Figure 10 are risk-rationed out of the the credit market when formal insurance is not available. Non-interlinked index insurance worsens the risk-rationing and expand the rationing area to the southeast. This is because the implicit insurance provided by loan contracts renders formal insurance "effectively an increases in the interest rate on the loan" (Giné and Yang, 2009). However, the introduction of interlinked index insurance reduces risk rationing, moving the boundary towards the northwest so much that highly risk-averse and poor farmers are willing to borrow from the the credit market and adopt high-yielding technology.

In a high collateral environment, two types of credit rationing occur when formal insurance is not available. First, poor farmers on the left side of the green vertical line are quantity-rationed out because of lack of wealth to put as collateral. Second, among those farmers who are eligible to apply for a loan (on the right side of the green vertical line), highly risk-averse farmers are risk-rationed because they fear the loss of collateral. The introduction of non-interlinked insurance contracts reduces risk rationing and crowd in all the eligible farmers into the the credit market. Because lenders are protected by high collateral, interlinked contracts benefit farmers and performs as well as non-interlinked contracts.

5. Conclusion

Covariant risks associated with agricultural activities can hamper development of the rural credit market and thus prevent poor smallholder farmers from escaping poverty. On the other hand, despite its advantage of addressing imperfect information problems (moral hazard and adverse selection), novel index insurance experiences low uptake rates in the field. Our model suggests that the two financial markets, credit and insurance, have to interlink with each other and associate with income-enhancing technologies, in order to achieve market deepening and productivity growth.

While the uptake of novel index insurance contracts has at times been disappointing, the simple explanation that uptake is slow because index insurance contracts are not actuarially fair and have basis risk overlooks the fact that small farmers in low income economies typically self-insure using mechanisms at that are costly (actuarially unfair) and expose the farmer to significant basis risk. These inefficient forms of self-insurance thus leave ample space in which they can be stochastically dominated by formal index insurance contracts. The analysis here shows that this kind of stochastic domination is most likely to occur when index insurance is combined with the introduction of improved technologies and credit contract.

The analysis has compared three insurance schemes associated with the high-yielding technology: no formal insurance (implicit insurance provided by loan contracts), bundled or non-interlinked index insurance where loan contract terms are independent of index insurance, and interlinked index insurance where loan contract terms are endogenously determined by index insurance. The model and the simulation results show that the impact of index insurance differs when collateral environment changes. In a low collateral environment, interlinked contracts crowd in poor and highly risk-averse farmers, who are excluded from the financial market when absent of formal insurance or under non-interlinked contracts. Non-interlinked contracts have small or even negative impacts on uptakes of financial products and high-yielding technology. In a high collateral environment, interlinked and non-interlinked contracts perform equally well in reducing risk-rationing. While subject to empirical confirmation, these theoretically grounded observations have significant implications for the design of efforts to promote both small farm productivity growth and rural financial market deepening.

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FIGURE 1. Technologies



FIGURE 2. CDF of high tech, low tech and high tech with conventional insurance



FIGURE 3. Index Insurance as Mean Preserving Squeeze



FIGURE 4. Iso-expected profit locus with $\bar{\pi_a}=20\%$



FIGURE 5. Aggregate Supply of Agricultural Credit–Rate of return



FIGURE 6. Aggregate supply of agricultural loans-Interest rates



FIGURE 7. Cumulative Distribution of Consumption



FIGURE 8. Certainty Equivalent of high tech as a percentage of low tech for a highly risk-averse household $\psi=3$





FIGURE 10. Equilibrium uptake when x=0/10 and theta c%=75% with quantity rationing

APPENDIX A. NUMERICAL SPECIFICATION

Utility function adopts CRRA utility function: $U(x) = \frac{x^{1-a}}{1-a}$ where x > 0, a > 0 and $a \neq 1$. Other parameters: A = 1, $g_{\ell} = 10$, $g_h = 25$, K = 10, $\bar{\pi} = 20\%$, $\chi_h = 10$, $\chi_{\ell} = 0$, $\chi_m = 5/2$, B = 10, n = 1001, T=200 (years).

 θ_c and θ_s are independent, following truncated normal distribution. High covariant risk : $\frac{Var(\theta_c)}{Var(\theta)} = 75\%$; low covariant risk: $\frac{Var(\theta_c)}{Var(\theta)} = 25\%$, $Var(\theta) = 0.27$. $E(\theta_c) = 1$, $\theta \in [0, 2]$, $E(\theta_s) = 0$, $\hat{\theta_c} = E(\theta_c)$. Small loading cost $\beta = 30\%$; big loading costs $\beta = 100\%$.

Penalty function $P(G) = \{ \begin{array}{cc} 0 & \text{if } G > \tilde{\pi} \\ p * \bar{\pi} - p * \bar{\pi} / \tilde{\pi} * G & \text{,otherwise} \end{array}$, where $\bar{\pi} = 2, \ \tilde{\pi} = 0.8, \ p = 4$.

Population distribution $\psi \in [0, 4]$, $W \in [1, 15]$ if quantity rationing exist, $W \in [11, 20]$ if no quantity rationing. ψ and W are independent and each, after normalization, follows a beta distribution (2,2).

APPENDIX B. PROOF OF MEAN-PRESERVING SPREAD

Property B.1 $\int_{0}^{\bar{\theta}} [F(\theta) - F^{I}(\theta)] d\theta = 0$. The mean of θ^{I} is expressed as $E(\theta^{I}) = \int_{0}^{\bar{\theta}} \theta f^{I}(\theta) d\theta$. Since θ^{I} is a function of θ , the mean can be also expressed as $E(\theta^{I}) = \int_{0}^{\bar{\theta}} [\theta + s(\theta)] f(\theta) d\theta = \int_{0}^{\bar{\theta}} \theta f(\theta) d\theta + \int_{0}^{\bar{\theta}} s(\theta) f(\theta) d\theta$. Since by the definition of z_{t} , $\int_{0}^{\bar{\theta}} s(\theta) f(\theta) d\theta = 0$, $E(\theta^{I}) = \int_{0}^{\bar{\theta}} \theta f^{I}(\theta) d\theta = \int_{0}^{\bar{\theta}} \theta f(\theta) d\theta = E(\theta)$. Then $\int_{0}^{\bar{\theta}} \theta [f^{I}(\theta) - f(\theta)] d\theta = 0$. Using integration by part, we have

(B.1)
$$\int_{0}^{\bar{\theta}} \theta[f^{I}(\theta) - f(\theta)] d\theta = \theta[F^{I}(\theta) - F(\theta)] \mid_{0}^{\bar{\theta}} - \int_{0}^{\bar{\theta}} [F^{I}(\theta) - F(\theta)] d\theta = -\int_{0}^{\bar{\theta}} [F^{I}(\theta) - F(\theta)] d\theta = 0$$

Therefore the first property is proofed.

Property B.2 $\int_{0}^{y} [F(\theta) - F^{I}(\theta)] d\theta > 0 \ \forall y < \overline{\theta}$. Since *s* is a function of θ_{c} , define a new variable θ_{c}^{I} as $\theta_{c}^{I}(\theta_{c}) = \theta_{c} + s(\theta_{c})$ where $s(\theta_{c}) = 1(\hat{\theta_{c}} > \theta_{c})(\hat{\theta_{c}} - \theta_{c}) - z_{t}$. Denote their CDF as $F_{c}(\theta)$ and $F_{c}^{I}(\theta)$. Let θ_{c}' denote θ_{c} that satisfies $s(\theta_{c}') = 0$. Since $z_{t} > 0$, then $\theta_{c}' = \hat{\theta_{c}} - z_{t} < \hat{\theta_{c}}$. Take an arbitrary point of θ_{c}^{I} at θ_{c}'' that satisfies $\theta_{c}'' > \theta_{c}'$, and the corresponding θ_{c}''' satisfying $\theta_{c}^{I}(\theta_{c}''') = \theta_{c}''$. Then the cumulative probabilities when $\theta_{c} \leq \theta_{c}'''$ and $\theta_{c}^{I} \leq \theta_{c}''$ are equal to $F_{c}(\theta_{c}''')$ and $F_{c}^{I}(\theta_{c}'')$. Since $\theta_{c}'' > \theta_{c}''$, $F_{c}(\theta_{c}'') < F_{c}(\theta_{c}''')$. When $\theta_{c} < \hat{\theta_{c}} - z_{t}$, $F_{c}^{I}(\theta_{c}) > 0$. So there exists a $\theta_{c}' = \hat{\theta_{c}} - z_{t}$ such that

(B.2)
$$\begin{cases} F_c(\theta_c) \le F_c^I(\theta_c), \ \forall \theta_c \ge \theta'_c \\ F_c(\theta_c) > F_c^I(\theta_c), \ \forall \theta_c < \theta'_c \end{cases}, \\ \end{cases}$$

in other word, the two distributions have a single crossing. Since θ_s is independent of θ_c , after adding an independent variable the above property still hold for θ and θ^I .

According to Diamond and Stigliz (1974), if two distributions satisfy Property B.1 and B.2, the two distributions have a relation of mean-preserving spread. Following Diamond and Stigliz (1974), Property B.2 can be derived using Property B.1 and the property of single crossing.

Appendix C. Proof of aggregate supply of credit

Integrating by parts, the expected penalty can be rewritten as:

(C.1)
$$E(P) = P(\tilde{\pi})F_G(\tilde{\pi}) - \int_0^{\tilde{\pi}} F_G(G)P'(G)dG$$

Using the implicit function theorem, we can derive the slope of the supply curve from the LPC as:

(C.2)
$$\frac{\partial \bar{\pi_a}}{\partial n_a} = \frac{\frac{1}{n}(\bar{\pi_a} - \pi) - \frac{\partial E(P)}{\partial n_a}}{\frac{\partial E(P)}{\partial \bar{\pi_a}} - \frac{n_a}{n}}$$

To sign $\frac{\partial \pi_a}{\partial n_a}$ as n_a rises, we will first informally show three partial derivatives, which describe how the distribution of G changes when n_a and π_a increases and θ changes to θ^I respectively. Then based on these relations we will try to disentangle the effect of n_a on π_a . Given the first and second moments of G which can be written as

(C.3)
$$E(G) = \frac{n_a \bar{\pi_a} + n_b \bar{\pi}}{n} = \bar{\pi} + \frac{n_a (\bar{\pi_a} - \bar{\pi})}{n}$$

(C.4)
$$Var(G) = \frac{n_a Var(\pi_i) + \frac{n_a(n_a-1)}{2} Cov(\pi_i \pi_j)}{n_a^2} = \frac{Var(\pi_i)}{n_a} + \frac{(n_a-1)Cov(\pi_i \pi_j)}{2n_a}$$

as n_a increases we have the followings for the first partial derivatives. Note that if the covariance between individual agricultural loans was zero, then the Var(g) would quickly drop towards zero as the number of agricultural loans increased (i.e., the portfolio would be self-insuring). However, when the covariance is positive, the variance of the portfolio approaches $Cov(\pi_i \pi_j)/2$ as the number of agricultural loans becomes large. When $\bar{\pi}_a = \bar{\pi}$, E(G) is constant and Var(G) increase, which indicates a mean-preserving spread of G. Therefore $\frac{\partial F_G(G)}{\partial n_a} > 0$ and $\frac{\partial E(P)}{\partial n_a} > 0$. When $\bar{\pi}_a > \bar{\pi}$, both E(G) and Var(G) increase, which gives an ambiguous change in F_G . When $\bar{\pi}_a < \bar{\pi}$, E(G) decreases and Var(G) increase, indicating that the original F_G stochastically dominates the new one. For the second partial derivative as $\bar{\pi}_a$ increases, it is more obvious that E(G) increase, Var(G) is constant, and thus the new F_G stochastically dominates the origin one. Since P' < 0, $\frac{\partial e(P)}{\partial \pi_a} < 0$.

Now we are ready to sign the equation of $\frac{\partial \bar{\pi}_a}{\partial n_a}$ when no insurance is available. When $\bar{\pi}_a = \bar{\pi}$, because $\frac{\partial E(P)}{\partial \bar{\pi}_a} < 0$ and $\frac{\partial E(P)}{\partial n_a} > 0$, $\frac{\partial \bar{\pi}_a}{\partial n_a} |_{\bar{\pi}_a = \bar{\pi}} > 0$. When $\bar{\pi}_a$ increases and $\bar{\pi}_a > \bar{\pi}$, $\frac{\partial \bar{\pi}_a}{\partial n_a}$ goes down, since $(\bar{\pi}_a - \pi)$ becomes positive and $\frac{\partial E(P)}{\partial n_a}$ may become smaller. Finally the supply curve turn flat since the gain from expected profit of agricultural loans offset the loss from covariant risk. Therefore, the shape of the function $\bar{\pi}_a(n_a)$ is concave. In other words, the penalty function makes the lenders maxim and concave in total portfolio return. As can be easily shown, for a given portfolio of n loans, each one of which has an expected return of $\bar{\pi}$, the distribution function of total portfolio returns becomes more dispersed through a mean-preserving spread as n_A increases. With its expected net

profits consequently decreasing in n_A , the lender will only offer a significant fraction of its total loans to agriculture if it receives a compensating increase in expected returns on agricultural loans.

Now let us see the impact of insurance on aggregate supply. When output is insured and θ changes to θ^{I} , π_{a} increases and $Var(\pi_{i})$ decreases, since θ is a mean-preserving spread of θ^{I} and π_{i} is a concave function of θ . Because $\frac{\partial Cov(\pi_{i}\pi_{j})}{\partial Var(\theta_{c})} < 0$, $Cov(\pi_{i}\pi_{j})$ also decreases with θ^{I} . Therefore, E(G) goes up and Var(G) goes down. Note that increases in E(G) represents the effect of insurance through partial equilibrium, and decrease in Var(G) represents effect of insurance through market equilibrium. Furthermore, the F_{G} with insurance stochastically dominates the one without insurance. Then given insurance $\frac{\partial E(P)}{\partial n_{a}}$ will decrease and $\frac{\partial E(P)}{\partial \pi_{a}}$ will increase. Therefore, $\frac{\partial \pi_{a}}{\partial n_{a}}|_{\theta} > \frac{\partial \pi_{a}}{\partial n_{a}}|_{\theta} I$. When θ_{c} is fully insured, $Cov(\pi_{i}\pi_{j}) = 0$ and Var(G) = 0. To satisfy $E(N) = \pi$, $\pi_{a} = \pi$ has to hold.