De-regulating Markets for Financial Information

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Abstract

The Dodd-Frank Act will eliminate the requirement that credit products must be rated before they can be sold to banks and pension funds. Supporters argue that if the information in ratings is valuable, issuers or investors will choose to buy the information, even without the requirement. But free-rider problems abound: investors might not buy ratings because asset prices partially reveal what others know and asset issuers might not pay for ratings if they believe investors will buy them anyway. This paper studies how removing ratings requirements affects provision of financial information, asset prices and welfare. It describes conditions under which de-regulated information markets could collapse. But it explains why, when an information market collapses, neither asset issuers nor investors prefer mandatory ratings. Furthermore, a calibration exercise suggests that information market collapse is unlikely. Instead, the repeal of ratings mandates will simply shift the cost of information production from asset issuers to investors.

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In July 2010, Congress passed the Dodd-Frank Act. One of the features of the act was its mandate for the SEC to remove ratings requirements for many credit products within a few years. Currently, some investors (pension funds for example) can only purchase credit products that achieve a minimum level of credit-worthiness, as determined by a nationally-recognized ratings agency. Eliminating such requirements would remove a major incentive for issuers of credit products to obtain ratings, allowing them to decide for themselves whether or not to pay a ratings agency to rate their asset. If such a rating is not provided by the asset issuer, investors themselves might want to purchase a rating, or some equivalent summary statistic about the quality of a credit product that they consider buying. This paper examines the effect of such a policy change on information provision, credit asset prices, the allocation of productive capital, and welfare. In doing so, it provides guidance about the wisdom of mandatory certification in many contexts.

The Dodd-Frank Act abolishes ratings requirements. The question this policy raises is not whether one ought to tinker with the ratings system in order to ameliorate a conflict of interest, but instead whether ratings should be required at all. Therefore, we abstract from the potential biases or conflicts of interest in the ratings process and simply model ratings as noisy signals about the future value of a risky asset. If the result is that, even if the ratings system functions perfectly, ratings should not be required, then that is a strong result. If the result is that ratings should be required, then it suggests that trying to mitigate the conflicts of interest is a worthwhile endeavor.

In our model, competitive rating agencies produce unbiased signals, at a cost. Agencies can either sell the rating service to the issuer, and disclose the rating free of charge to all investors, or can sell the rating to each investor individually. In this latter case, they must take into account that investors can free-ride by using equilibrium asset prices to partially infer what others have learned. Section 2 derives conditions under which an unregulated market for information will follow one business model or the other, or not exist at all.

Signals affect asset prices in two ways: First, a positive signal will push the price of the asset up, while a lower-than-expected signal will reduce the price investors are willing to pay for an asset. In expectation, signals are neutral. The second effect is that the rating makes the asset's payoff less uncertain. In doing so, it makes the asset less risky. Lowering risk lowers the equilibrium return and systematically raises the asset's price.

Given that information raises asset prices by lowering risk, the question of which assets would be most affected by the repeal of ratings requirements would appear to be straightforward: the assets for which the ratings convey the most information or those which investors have the least prior information about. Section 3 shows that this intuition is incorrect. When prior beliefs are very uncertain or signals are very informative, either asset issuers or investors will opt to purchase information, even without regulatory incentives. Conversely, when signals are uninformative or prior beliefs are very precise, ratings will indeed disappear but their disappearance has little price impact. Thus, the assets mostly likely to be affected by the policy change are neither the highest- nor the lowest-information securities, but the ones in between.

We also investigate whether the repeal of ratings requirements would have a greater effect on assets with a smaller or larger investor base. Again, we find that assets in the middle are most affected, but the reason is different. When the investor base is large, each investor needs to bear only a small amount of risk, so prices are close to expected values regardless of whether ratings are provided. Conversely, when the investor base is small, each investor must hold many shares of the asset, in order for the market to clear. Since they are bearing lots of risk from this asset, the price they are willing to pay for the asset is very sensitive to changes in risk. Since providing information reduces the risk the asset poses to an informed investor, the incentive for an asset issuer to acquire a rating and provide it to all investors is high because providing that rating will greatly increase the price the asset sells for. Thus, for assets with medium-sized investor bases, the Dodd-Frank reform may reduce their average price, as information becomes more scarce.

One potential reason to regulate financial information provision is because financial markets should facilitate efficient real investment decisions. We model this effect in the following way. At time 1, an entrepreneur can choose how much to invest. His payoff depends on the price the asset sells for in the time-2 financial market. If financial asset prices are very sensitive to changes in the value of the capital stock (they are informationally efficient), then the entrepreneur has incentives to invest the optimal amount. This force points towards a social benefit of providing information.

The efficiency benefits of information provision do not imply that requiring ratings is always good policy. Regulation might result in information over-provision in situations where the social benefit is less than the cost. Conversely, since information externalities invalidate the welfare theorems, the information market also might not get the tradeoff right. To understand the relationship between information and welfare, we compare investors' and issuers' expected utilities with and without the mandate, both theoretically and numerically.

Our theoretical welfare results (section 4) suggest that, in many cases, neither asset issuers nor investors prefer ratings requirements. Asset issuers can always choose to purchase and disclose ratings. Whenever they choose not to purchase a rating, they are better off without the rating, and otherwise they are indifferent. Surprisingly, even though mandatory ratings produce more efficient capital investment and a higher expected asset payoff, and even though the asset issuer pays for the ratings, investors typically prefer not to have ratings. Of course, the risk-averse investors like the fact that ratings make assets less risky. But less risk also implies a lower expected equilibrium asset return. The net effect is to make investors worse off. They prefer not to have a low-risk, low-return security, which in the limit, becomes redundant with the risk-free asset they already have access to. Each investor individually prefers more information. But all investors are better off when everyone is less informed. Thus, when the market for ratings collapses, investors benefit.

Ironically, investors prefer for asset issuers to provide ratings when ratings are cheap. When the cost of discovering ratings information is low, many investors will buy the information. The result will either be severe asymmetric information or full information. If a large fraction of investors become informed and few others remain uninformed, there is a severe asymmetric information problem that results in welfare losses. If all investors choose to purchase information on their own, then the situation is identical to one with a ratings mandate, except that the cost of the rating is borne by investors instead of by the issuer. In both cases, investors can benefit from ratings mandates.

The recent public debate on ratings policy suggests that ratings are costly and not very informative. Section 5 uses data on ratings and on prices and performance of corporate bonds issued between 2004 and 2005 to estimate the model parameters and uses those estimates to compare the costs and benefits of ratings. The resulting numerical predictions tell us that ratings costs are low, compared to the benefit of information, for the typical security. The costs are sufficiently low that without the ratings mandate, issuers would cease to buy ratings and all investors would buy ratings for themselves. Thus, the repeal of ratings mandates in Dodd-Frank will have no effect on the amount of information available about the average security. It would simply transfer the cost of providing the information from the asset issuers to investors.

Markets for information, and the question of whether to mandate information provision, matter beyond just the credit-ratings industry. For instance, buying consumer goods or services with uncertain benefits is similar to investing in a risky asset. Thus our main qualitative conclusions about the effects of information regulation carry over: mandating information provision about goods makes the most difference when the value of the information is neither so high that the private market will supply it anyway nor so low that it is inconsequential. Similarly, it would also have most effect when customers neither buy so much of this good that they would demand information privately, nor so little of the good that the information has negligible effect on their welfare. While financial information helps to allocate real productive capital, consumer goods information encourages high-value goods to be supplied and low-value goods to be withdrawn. In both cases, mandatory information improves allocative efficiency. But this efficiency gain may not benefit consumers because, in equilibrium, the price of goods with less-uncertain quality is higher.

Related literature The paper is most closely related to a recent literature on the welfare consequences of information disclosure. For example, Amador and Weill (2006), Kondor (2011) and Gorton and Ordonez (2011) also show how the release of financial information can be welfare-reducing. But none considers the incentives to purchase information. Goldstein, Ozdenoren, and Yuan (2011), Albagli, Hellwig, and Tsyvinski (2009) and Angeletos, Lorenzoni, and Pavan (2010) are similar because they model an interaction between information in financial markets and the real economy. But their financial investors can manipulate real investment through their asset purchasing decisions. This feedback creates complementarities in demand among investors and the potential for multiple equilibria. Our model shuts down this channel by having real investment take place first.

Our analysis is also related to work on costly information acquisition, such as Grossman and Stiglitz (1980), Verrecchia (1982), Peress (2010) and Fishman and Parker (2011). But it extends this work by considering the trade-offs between issuer- and investor-purchased information. If the issuer does not provide the signal, investors themselves can choose to purchase the information from an information market. We model the market for information in a richer way than most of the previous literature by considering the non-rival nature of information and solving for its endogenous market price (as in Wiederholt (2011)). This allows us to consider whether, in the absence of ratings regulation, either issuer-provided or investor-purchased information markets will fill in the void. Finally, the model connects financial information choices to real investment choices, output and welfare.

Previous models of rating agencies, while about the same institutions, have different tools and objectives. Sangiorgi, Sokobin, and Spatt (2008), Bolton, Freixas, and Shapiro (2007), Bolton, Freixas, and Shapiro (2008), Damiano, Li, and Suen (2008), Harris, Opp, and Opp (2011), Becker and Milbourn (2008) and Skreta and Veldkamp (2009) consider ratings inflation and conflicts of interest in the ratings system. Manso (2011) is more similar because it examines how ratings affect real firm performance and vice-versa. This paper abstracts from these incentive and performance issues and instead focuses on whether even unbiased ratings should be required at all.

Finally, this work is also related to a microeconomics literature on welfare and information disclosure (e.g. Shavell (1994), Diamond (1985) and Jovanovic (1982)). Our model differs because it features a continuum of investors in a market that has an equilibrium price. Many of our results come from equilibrium effects. Furthermore, informed trade in asset markets results in a more efficient allocation of productive capital. The fact that information creates economic value makes our finding different from a Hirshleifer (1971) effect. Likewise, we contribute to the literature on third-party certification (e.g. Lizzeri (1999)) by predicting whether a private market for certification will arise when public mandates are not present.

1 Model

The entrepreneur and real investment An risk-neutral entrepreneur chooses $k \geq 0$, how much real capital to invest in period 1, and whether to have his asset rated ($D = 1$) or not ($D = 0$) at a price C. If the entrepreneur has his asset rated, that rating is disclosed to all investors. The level of investment k is the entrepreneur's private information.

In period 2, the entrepreneur auctions off his firm. Its equilibrium price is p. The entrepreneur's expected utility is

$$\mathbb{E}\left(p|k, D\right) - k - CD. \tag{1}$$

The investment will produce output

$$y = f(k) + u$$

where $f(k)$ is a concave production function, $f(0) = 0$ and $u \sim N\left(0, \frac{1}{h_u}\right)$. Ratings are noisy signals about output: $\theta = y + \eta$ where $\eta \sim N(0, \frac{1}{h_\theta})$.

When making his rating decisions, the entrepreneur knows the function f and the distribution of u, but does not know what the realization of u will be or what the rating θ will be. Likewise, when making his investment choice, the entrepreneur knows his rating decision D, but does not know u or θ.

Investors and financial markets There is a continuum of ex-ante identical investors with measure Q. They have CARA expected utility with coefficient of risk aversion ρ:[1]

$$EU = E\left[-e^{-\rho W}\right], \tag{2}$$

[1] Since the model has a single asset, any risk is systematic and will be priced as such. More generally, since corporate defaults are correlated, the default risk that credit ratings measure has a systematic component, which justifies modeling investors in any given asset as risk-averse.

where W is their realized wealth. They have an initial endowment of wealth w_0.

Investors can purchase fractional shares in the entrepreneur's project. They can also store their initial endowment with zero net return.[2] If an investor purchases a fraction q of the firm, he pays qp for a claim to the payoff qy.

The price of the risky asset p is determined in an auction. Each investor submits a bidding function $b_i(q)$ that specifies the maximum amount that he is willing to pay for a fraction q of the risky asset as a function of his information. These bid functions determine the aggregate demand. The auctioneer specifies a market-clearing price p that equates aggregate demand and supply, and each trader pays this price for each unit purchased (a Walrasian auction).[3]

Each investor i also chooses whether to purchase a rating ($d_i = 1$) or not ($d_i = 0$) at a price c. If p is the market clearing price and the share of the firm investor i demands at price p is $q_i = b_i^{-1}(p)$, the budget constraint is

$$W = w_0 + q_i(y - p) - d_i c. \tag{3}$$

When making their ratings decisions, investors know the entrepreneur's rating decision D and they have rational expectations about k and therefore can infer the equilibrium $f(k^*)$. But they do not know the output shock u or what the realized rating will be. When making their bids, investors know the rating θ if the issuer pays for the asset to be rated ($D = 1$) or if they themselves have purchased the rating ($d_i = 1$). Since investors have rational expectations, when they determine the quantity of the risky asset they demand at each price, they consider what information would be conveyed if that were the realized price. It is as if the realized market price is in the information set of every investor when they form their asset demand. Let this information set at the time when investor i invests be denoted $\mathcal{I}_i$, where $\mathcal{I}_i = \{p, f(k^*)\}$ if i has not observed a rating and $\mathcal{I}_i = \{p, f(k^*), \theta\}$ if the issuer has disclosed the rating or i has chosen to purchase it.

Asset supply noise There is a set of agents who are subject to random shocks that force them to buy or sell the asset, at any current price. The demand of this group of agents is normally distributed with mean zero: $\xi \sim N(0, \frac{1}{h_x})$. Let x denote the net supply of the asset, after accounting for the noise trader demand: $x \equiv 1 - \xi$. Thus, $x \sim N(1, \frac{1}{h_x})$. This noise ensures that the price investors condition on is not perfectly informative about information that others may know.

[2] In a model with a gross riskless return $r > 1$, none of the result change qualitatively. We have also worked out an extensive appendix that analyzes a problem where the entrepreneur can choose how many shares in his project to issue. Both sets of results are available upon request.

[3] As shown by Reny and Perry (2006), this formulation of the financial market is equivalent to proposing a Walrasian rational-expectations equilibrium.

Rating agencies and information markets Credit-rating agencies produce noisy, unbiased signals about the risky asset payoff y: $\theta = y + \eta$ where $\eta \sim N(0, \frac{1}{h_\theta})$. We call these signals "ratings." θ can be discovered at a fixed cost χ. This can be interpreted as the cost of hiring staff to interview the firm managers, analyze financial information, etc. The information, once discovered, can be distributed at zero marginal cost.

Rating agencies may sell the rating service to the entrepreneur for a fee C, in which case we assume both parties commit to publishing the result for free to all investors. Alternatively, they can sell it to individual investors, at a price c. For the latter case, we assume that the information is protected by intellectual property law and reselling it is forbidden.[4]

In either setup, we assume that the market is perfectly contestable, so that ratings agencies make zero profits.[5] This implies that, if the entrepreneur buys the rating, $C = \chi$, whereas if individual investors are the ones paying for it, and a measure λ of them choose to purchase it $c = \frac{\chi}{\lambda}$.

That information markets are competitive is crucial. The exact market structure is not. Veldkamp (2006) analyzes a Cournot and a monopolistic competition market as well. All three markets produce information prices that decrease in demand.

Order of Events

1. The ratings agency chooses a price C to charge the entrepreneur
2. The entrepreneur decides whether or not he will pay for the rating
3. The entrepreneur chooses capital investment k.
4. (a) If the entrepreneur pays for the rating, the agency finds out θ and publishes it
   (b) If the entrepreneur does not pay for the rating, the ratings agency decides whether to find out θ and, if it does, chooses its price c. Investors then simultaneously decide whether or not to buy the signal. Those who do observe θ.
5. Investors submit menus of prices and quantities of assets they are willing to purchase at each price $b_i(q)$.
6. Asset auction takes place. The auctioneer sets a market-clearing price.

[4] This prohibition may be difficult to enforce. We analyze the consequences of this difficulty in section 5.1.

[5] One way to ensure that the market is contestable is to force agents to choose prices in a first stage and choose entry in a second stage.

7. y is realized and all payoffs are received.

Equilibrium An equilibrium is a rating decision D by the entrepreneur, a capital choice $k(D)$, investor's beliefs about that capital choice $k^*(D)$ given the entrepreneur's rating decision, a rating demand d_i by each investor, ratings prices for the entrepreneur and investors C and c, bidding functions $b(q|\mathcal{I}_i)$ for each possible information set and an asset price $p(\theta, D, \{d_i\}, \xi)$ such that: entrepreneurs choose a rating demand D to maximize (1); taking D as given, the entrepreneur chooses $k^*(D)$ to maximize (1); investors choose d_i and bidding functions to maximize (2) subject to (3); ratings agencies make zero profits, the asset market clears: $\int_0^Q q_i di = x$ and investors' belief about investment is correct: $k = k^*(D)$.

2 Solving the model

To solve the model, we start with the second-period financial market equilibrium for given real investment and information choices. Then we determine the outcome of information markets and finally we solve for real investment.

2.1 Equilibrium asset prices

We begin by deriving the investors' optimal bid function for risky assets and verifying that it constitutes an equilibrium. Since the asset payoff y is normally distributed, expected utility (2) takes the form $EU = -e^{-\rho(w_0+q_i(E(y|\mathcal{I}_i)-p)-d_ic)+(\rho^2 q_i^2/2)Var(y|\mathcal{I}_i)}$, where $E(y|\mathcal{I}_i)$ and $Var(y|\mathcal{I}_i)$ are the mean and variance of the risky asset's payoff, conditional on the investor's information. This investor maximizes EU subject to the budget constraint (3). The objective function of this constrained maximization problem is concave in q, so that the first-order condition describes the optimal portfolio:

$$q_i = \frac{1}{\rho} Var[y|\mathcal{I}_i]^{-1}(E[y|\mathcal{I}_i] - p). \tag{4}$$

To implement this optimal portfolio, the investor submits a bidding function. Each bidder is infinitesimal, which implies that he takes the market-clearing price as given. Thus, the bidding function $b(q|\mathcal{I}_i)$ is the inverse demand function of a trader who seeks to maximize (2) subject to (3), taking p as given. Note that bids depend on each investor's information set $\mathcal{I}_i$, which includes information inferred from b being the price paid per unit: $b_i(q) = E(y|\mathcal{I}_i) - q\rho Var(y|\mathcal{I}_i)$. Because $b(q|\mathcal{I}_i)$ is an inverse of (4), it is a best response given everyone else's bid function.

The expectation and variance in (4) are conditional on an information set that includes beliefs about the entrepreneur's capital investment $k^*(D)$ and knowledge of the distribution of u. Thus, prior to observing any signals, $E[y] = f(k^*(D))$ and $Var[y] = \frac{1}{h_u}$. The information set of investors who have observed the rating (either because it was provided by the issuer or because they bought it) also includes θ. For these informed investors, Bayes' law says that

$$E[y|\theta] = \frac{f(k^*(D))h_u + \theta h_\theta}{h_u + h_\theta} \tag{5}$$

$$Var[y|\theta] = \frac{1}{h_u + h_\theta}. \tag{6}$$

Thus, informed traders' inverse bid function (demand) is

$$q_I = \frac{1}{\rho}\left(f(k^*(D))h_u + \theta h_\theta - p\left(h_u + h_\theta\right)\right) \tag{7}$$

For investors who have not observed the rating, the market-clearing auction price of the risky asset partially reveals the rating that others (if any) have observed. Since the price depends on asset demand and demand depends on information in the price, there is a fixed point problem. We solve by guessing a linear price rule

$$p = \alpha + \beta\xi + \gamma(\theta - f(k^*(D))), \tag{8}$$

and solving for the coefficients α, β and γ. A linear transformation of the price $f(k^*(D)) + \frac{1}{\gamma}(p - \alpha)$ is an unbiased signal about the project output y, with variance $h_p^{-1} = \frac{1}{h_\theta} + \left(\frac{\beta}{\gamma}\right)^2 \frac{1}{h_x}$. Thus, h_p is a measure of the informativeness of prices.

The posteriors of the uninformed investors will be:

$$E[y|p] = \frac{f(k^*(D))h_u + \left[f(k^*(D)) + \frac{1}{\gamma}(p - \alpha)\right] h_p}{h_u + h_p} \tag{9}$$

$$Var[y|p] = \frac{1}{h_u + h_p} \tag{10}$$

Therefore, the menu of prices and quantities bid by each uninformed trader will be:

$$q_U = \frac{1}{\rho}\left(f(k^*(D))h_u + \left[f(k^*(D)) + \frac{1}{\gamma}(p - \alpha)\right] h_p - p\left(h_u + h_p\right)\right)$$

If a measure λ of traders chooses to become informed, total demand will be $\lambda q^I + (Q - \lambda)\, q^U$.

Equating this total demand to asset supply x yields coefficients for the linear price rule and confirms the conjecture of a linear price. The following price coefficients are derived in appendix A.1:

$$\alpha = f(k^*(D)) - \frac{\rho}{\lambda(h_u + h_\theta) + (Q-\lambda)(h_u + h_p)} \tag{11}$$

$$\beta = \frac{\rho}{\lambda h_\theta} \frac{\lambda h_\theta + (Q-\lambda)\, h_p}{\lambda\,(h_u + h_\theta) + (Q-\lambda)\,(h_u + h_p)} \tag{12}$$

$$\gamma = \frac{\lambda h_\theta + (Q-\lambda)\, h_p}{\lambda(h_u + h_\theta) + (Q-\lambda)(h_u + h_p)} \tag{13}$$

Substituting in for β and γ in the previous formula for h_p tells us that price informativeness is

$$h_p = \frac{\lambda^2 h_\theta^2 h_x}{\lambda^2 h_\theta h_x + \rho^2}. \tag{14}$$

The average price is α, and it consists of the expected payoff $f(k^*(D))$ less a term that accounts for investors' risk aversion ρ and the amount of information they have, which depends on the precision of the rating, the informativeness of prices and how many investors buy the rating. The sensitivity of the price to the rating is given by γ. γ takes values between 0 and 1, and is greater when ratings are very precise relative to the prior and a large fraction of investors buy them. The sensitivity of the price to noise in demand is given by β. Prices will tend to be relatively sensitive to demand noise when investors are risk averse, when few have bought the rating or when the ratings are not very informative.

For the case where the entrepreneur provides the rating, formulas (11) - (14) still apply, setting $\lambda = Q$, while for the case where no one buys the rating, the formulas apply taking the limit as $\lambda \to 0$.

2.2 Investor-based information market

Suppose that the entrepreneur decides not to provide a rating for the market. Investors must individually choose whether or not to acquire the rating at the price c. Since investors are ex-ante identical, they will only make different choices when those choices yield identical expected utility. Appendix A.2 computes the expected utility of an informed investor and the expected utility of an uninformed investor when a measure λ of the population of investors is informed. If there exists a $\lambda \in [0, Q]$ that equates the two expected utilities, then this is an equilibrium. Appendix A.2 shows that the equilibrium measure of informed investors is

$$\lambda = \frac{\rho}{\sqrt{h_x h_\theta}} \sqrt{\frac{h_\theta}{(h_u + h_\theta)(1 - \exp(-2\rho c))} - 1} \tag{15}$$

If this λ is not between 0 and Q, then there is a corner solution. If expected utility for uninformed is higher, the corner solution is $\lambda = 0$, otherwise, the solution is $\lambda = Q$. If the right side produces an imaginary number, it signifies that there is no positive measure of informed investors that equates expected utility for the informed and uninformed. In these instances, the only solution is for all investors to remain uninformed.

Equation (15) implies that demand for the rating is decreasing in the price c, decreasing in the precision of the prior h_u and increasing in the variability of noise trader demand $\frac{1}{h_x}$, which makes prices less informative. The effect of rating precision h_θ is ambiguous. On the one hand, more precise information is more valuable; on the other, it induces informed traders to take larger positions in the asset, which makes equilibrium prices more informative as well.

Equilibrium implies that, if the issuer does not provide the rating, (15) and the zero-profit condition

$$c = \frac{\chi}{\lambda} \tag{16}$$

must hold.

Proposition 1 ***(Investors do not buy low-precision ratings)*** *If*

$$\frac{h_\theta}{h_u} < \exp\left(\frac{2\rho\chi}{Q}\right) - 1 \tag{17}$$

investors will not buy a rating

Proof in appendix A.3. Proposition 1 implies that an investor-based information market will not exist if:

- the information content of the rating h_θ is small relative to the precision of the prior h_u, since this makes information less valuable
- either the fixed cost of information discovery χ is high or the investor base Q is small (which makes the price c that the ratings agency needs to charge high, or
- investors are very risk averse, which makes them take small positions in the asset and therefore profit little from better information.

Proposition 2 ***(Investors do not buy high-precision ratings)*** *Investors will not buy a rating if h_θ is sufficiently high.*

Proof in appendix A.4. Proposition 2 reveals a subtlety about the investor-driven information market. If the ratings contain very precise information, informed investors will take large positions, which makes prices highly informative. With a fixed price c for the rating, this would imply that as precision increases, only a vanishing measure of investors choose to become informed, as is the case in the model of Grossman and Stiglitz (1980). However, because the ratings agency must cover the fixed cost χ, low demand means it must raise prices. For sufficiently high precision, there is simply no price at which this market is viable.

Propositions 1 and 2 jointly imply that an investor-led market for ratings can only function if the information is of some intermediate level of precision.

2.3 Real investment decision

Replacing the equilibrium price into the entrepreneur's objective function in (2) and noting that $\theta = f(k) + u + \eta$, the entrepreneur solves

$$\max_k E\left[\alpha + \beta\xi + \gamma\left(f(k) + u + \eta - f(k^*(D))\right)\right] - k$$

Note that, because investment is unobserved, the entrepreneur cannot affect beliefs about $k^*(D)$ through the investment decision. The reason for the entrepreneur to undertake investment is to affect the rating and therefore to indirectly affect the selling price.

The first order condition for investment is

$$f'(k) = \frac{1}{\gamma}$$

The value of γ depends on whether the entrepreneur has provided a rating and, if he has not, on how many investors have purchased it. Since by equation (13) $\gamma < 1$, investment always falls below its first-best level, which is defined by $f'(k) = 1$. Furthermore, since γ is increasing in λ, investment will be higher when more investors are informed. Therefore whenever the equilibrium value of λ in an investor-driven market is less than Q, investment will be higher under issuer-provided ratings. Note further that if the rating is not produced at all then $\gamma = 0$ and therefore $k = 0$.

Information is socially valuable in this model because it allows entrepreneurs to appropriate part of the marginal product of additional investment even though the investment itself is unobserved.

Thus it promotes a level of investment that is closer to the efficient level.

2.4 Entrepreneur's rating decision

The entrepreneur will provide a rating iff expected payoffs net of the information cost χ exceed expected payoffs without information. He takes into account that his decision to rate the asset will affect his decision of how much to invest and will affect the price the asset sells for in the financial market. Let p_1 be the price of an asset when investment $k^*(1)$ is undertaken and all investors observe the asset's rating. Let p_0 be the price of the asset when investment $k^*(0)$ is undertaken and the investor-led ratings market determines how many investors observe the rating. Then, the entrepreneur will rate the asset when $E[p_1] - k^*(1) - \chi > E[p_0] - k^*(0)$.

Proposition 3 ***(Ratings provision by entrepreneur)***

1. If
$$f(k^*(1)) - k^*(1) - f(0) + \frac{\rho}{Q} \frac{h_\theta}{h_u (h_\theta + h_u)} > \chi, \tag{18}$$
then either the issuer will provide a rating or at least *some* investors will buy it

2. If condition (18) does not hold, the entrepreneur will not provide a rating.

Proof in appendix A.5. When the entrepreneur considers whether or not to provide a rating, the entrepreneur takes into account both the equilibrium measure of investors that will buy the rating if he doesn't provide it (λ) and how his own incentives to invest will change with the information structure. In case providing the rating results in more information (which will be the case unless $\lambda = Q$) this brings about two sources of gains. First, better information will result in closer-to-efficient investment. By equation (11), the average price moves one for one with expected output, so the entrepreneur appropriates the entire efficiency gain $f(k^*(1)) - k^*(1) - [f(k^*(0)) - k^*(0)]$. Second, by providing investors with information, the entrepreneur reduces the risk they have to bear, which increases average prices. The entrepreneur trades off these two sources of gains against the cost χ of the rating.

Condition (18) says that the gains from providing information outweigh the cost, assuming that if the entrepreneur does not provide information, the investors will not buy it either. If the condition holds, then either the entrepreneur expects a sufficient number of investors to buy the rating on their own, or will buy the rating himself. If the condition doesn't hold, then the entrepreneur prefers not to buy the rating even if he expects investors to remain uninformed.

Proposition 3 implies that entrepreneurs will not provide ratings if

- the precision h_θ is too low, because the value they add (both directly through reducing risk for investors and indirectly by providing incentives for investment) is too little
- the cost χ is too high
- investors are sufficiently risk tolerant (low ρ) or numerous (high Q) that the discount from bearing risk is small
- the precision of investors' prior is high enough that the additional information from the rating makes little difference

In summary, combining the results from propositions (1), (2) and (3) reveals when no ratings will be produced. Ratings will not be produced at all if signal precision h_θ is sufficiently low, the information fixed cost χ is sufficiently high, or if prior belief precision h_u is sufficiently high. These are the instances where de-regulation will have the strongest effects on information provision. Instead of all investors being informed with credit rating mandates, the market will not provide information for anyone. These are situations that we refer to as "information market collapse."

3 Effects of De-regulation on Asset Prices

What will happen to the prices of assets after de-regulation? In order to build intuition and focus on the pure asset-pricing effects, we consider the case where there are no effects on the real investment decision. We hold k and * fixed. The next section re-introduces the real economic effects of information provision.

The following proposition shows that on average, the price of credit assets will fall as information becomes less abundant.

Proposition 4 *For an issuer that does not provide a rating, the average asset price is increasing in λ.*

Proof in appendix A.6. To see why this is true, note that ratings affect asset prices in two ways: First, a positive signal will push the price of the asset up, while a lower-than-expected signal will reduce the price investors are willing to pay for an asset. In expectation, signals are neutral. Thus on average, the positive and negative effects of the signal cancel out. The second effect is that the rating makes the asset's payoff less uncertain. In doing so, it makes the asset less risky. Lowering risk lowers the equilibrium return and systematically raises the asset's price.

The next two results describe which assets are likely to be most affected by de-regulation. These assets are neither the largest or smallest investor base securities, but the ones in between. Likewise, they are neither the assets for which prior beliefs are most or least precise. To formalize these ideas, we consider the difference between the price of an asset with mandatory ratings and the price of an asset without mandatory ratings, but in an environment where either the entrepreneur or the investor can choose to purchase a rating.

Proposition 5 ***(De-regulation reduces the price of a medium-investor-base asset)*** *Let p^M be the price of the asset if ratings are mandatory and p^E be the price of the asset if the ratings decision is an equilibrium outcome. Then*

1. *If Q is sufficiently low, $p^M = p^E$.*
2. *If Q is sufficiently high, $E[p^M] > E[p^E]$.*
3. $\lim_{Q\to\infty}\left(E[p^M] - E[p^E]\right) = 0$.

Proof in appendix A.7. The first part of this result says that when the size of the investor base Q is sufficiently low, the entrepreneur will pay to have his own asset rated. The reason is as follows: When the measure of investors is small, each investor must hold more of the asset for the market to clear. If the investor is bearing lots of risk by holding lots of the asset, then reducing that risk by giving the investor information has a large effect on the price the investor is willing to pay for the asset. The fact that the auction price for the asset is sensitive to the amount of information investors have means that entrepreneurs get much higher profits from selling a rated asset versus an unrated asset. Furthermore, because the investor base is small, the per-copy price that ratings agencies would need to charge investors would be high, making the investor-driven market nonviable (as shown in Proposition 1). Knowing this, the entrepreneur has a strong incentive to pay for his asset to be rated.

The second part of the result says that when the base of investors is relatively large, each investor bears a small amont of risk and therefore the risk premium is not sufficiently large to persuade the issuer to provide the ratings. Moreover, the measure of investors exceeds the number who are willing to buy the rating, so not all investors become informed, meaning that prices are on average lower than if all investor became informed through mandatory ratings.

The last part of the result says that in the limit as $Q \to \infty$, it is still the case that not all investors are informed but this makes no difference for average prices because, since each investor bears almost no risk, average prices converge to expected dividends.

Proposition 6 ***(De-regulation reduces the price of a medium-precision asset)*** *Let p^M be the average price of the asset if ratings are mandatory and p^E be the average price of the asset if the ratings decision is an equilibrium outcome. Suppose*

$$\frac{\rho}{\sqrt{h_x h_\theta \frac{1-\exp(-2\rho\chi/Q)}{\exp(-2\rho\chi/Q)}}} > Q \tag{19}$$

Then

1. *If h_u is sufficiently low, $E[p^M] = E[p^E]$*
2. *There is an interval $(\underline{h}_u, \overline{h}_u)$ such that $E[p^M] > E[p^E]$ for all $h_u \in (\underline{h}_u, \overline{h}_u)$,*
3. $\lim_{h_u \to \infty} \left(E[p^M] - E[p^E]\right) = 0$

Proof in appendix A.8. This result considers what happens as prior beliefs become more or less precise. When priors are very imprecise, signals are valuable to individual investors and will be acquired by all of them, as long as they are not too numerous, which is guaranteed by condition (19). When priors are very precise, no information will be acquired. But any information acquired would have a tiny effect of already precise prior beliefs. Since ratings affect beliefs (mean and variance) very little, they affect asset prices very little. In the limit as the prior precision tends to infinity, the difference between the asset's price with mandatory ratings and without disappears. In between these extremes, there exists a region where not all investors are informed and where the asset price is strictly less than it would be under the mandatory ratings regime.

4 Ratings Regulation and Welfare

Ultimately, the most important question is whether government mandated information disclosure helps or hurts economic welfare. There are a few different ways we might think about a policy maker's objective in this model. We examine each in turn.

4.1 Maximizing output

One possible objective a government might have is to simply maximize the production of real goods. This is obviously a simplification, but it makes for a good starting point. The relevant question becomes: Which ratings policies maximize output $f(k)$?

The primary friction in the model is that investors' imperfect information about capital investment decisions of the firm reduces the entrepreneur's return to investing in capital. In other words, if investors don't know that the entrepreneur invested more, he won't be compensated for that investment when he sells his firm. Efficiency requires that the marginal return to investment be equal to its unit marginal cost: $f'(k) = 1$. Therefore if we somehow manage to ensure that the private return to a marginal unit of investment is equal to its social return, $\frac{\partial \mathbb{E}(p|k)}{k} = f'(k)$, then investment will be efficient. With imperfect information, the left side is typically smaller than the right because prices can only respond to changes in k to the extent that investors know k. The following analysis shows that mandatory information provision to financial markets helps to remedy this friction because it makes p more responsive to k.

Since the production function is concave, a higher $f(k)$ corresponds to a lower marginal product of capital $f'(k)$. The entrepreneur's first-order condition tells him to set $f'(k) = 1/\gamma$. The pricing coefficient γ (equation 13) is increasing in the measure of informed investors λ, as long as $h_\theta \geq h_p$. Inspecting equation (14) reveals that $h_\theta \geq h_p$. This makes sense because prices cannot reveal more information that what is contained in the signals they are revealing.

If ratings are mandated by the government, $\lambda = Q$, this maximizes γ, minimizes $f'(k)$ and thus maximizes $f(k)$ over all feasible values ($\lambda \in [0, Q]$). Thus, mandating ratings provides the maximum possible information, which maximizes output of real economic goods. Since information facilitates the efficient allocation of capital, mandatory information disclosure maximizes output.

4.2 Maximizing output net of costs

One obvious objection to the policy objective in the previous subsection is that it does not take into account the cost of investment or information production. In particular, it treats information as if it were free. More information might always be better. But if information is costly, it must be sufficiently valuable to justify its cost. Thus, another possible objective is to maximize $f(k) - k - \delta\chi$, where $\delta = 1$ if any agent (entrepreneur or investor) discovers information and $\delta = 0$ otherwise.

Since prices are uninformative when no agents observe a rating ($h_p = 0$ when $\lambda = 0$), the required $f'(k)$ is infinite, meaning that no investment takes place when the project is not rated:

$k^*(0) = 0$.

Next, note that since $f(k) - k$ is maximized when $\lambda = Q$, this means that if anyone incurs the cost χ to discover information, the output-maximizing outcome is for all investors to observe that information. Any $\lambda \neq \{0, Q\}$ does not maximize output net of costs. That leaves the question: In what circumstances is the higher output associated with $\lambda = Q$ large enough to compensate for the cost of information? In other words, what are the parameters of the problem for which $f(k^*(1)) - k^*(1) - \chi > 0$? Substituting in k from the first-order condition in this inequality yields

$$f\left((f')^{-1}\left(1 + \frac{h_u}{h_\theta}\right)\right) - (f')^{-1}\left(1 + \frac{h_u}{h_\theta}\right) > \chi.$$

For example, if production is $f(k) = k^\alpha$, then the high-information level of capital is $k^*(1) = ((1 + h_u/h_\theta)/\alpha)^{1/(\alpha-1)}$. This level of investment produces more output, net of investment and information costs when $k^*(1)((k^*(1))^{(\alpha-1)} - 1) > \chi$.

For a general, concave production function f, we know that $f'(k^*(1)) > 1$, so that anything that increases $k^*(1)$ also increases $f'(k^*(1)) - k^*(1)$ and therefore makes the inequality more likely to hold. A higher ratio of the signal precision to prior precision (h_θ/h_u) makes $k^*(1)$ higher, making it more likely that the high-information level of capital is the one that maximizes output net of investment and information costs.

4.3 Maximizing a weighted sum of utilities

This is the most commonly used social welfare criterion. In this setting, the objective this produces depends on how one weights the issuer (a single entity) versus the investors (a continuum of agents). The question of how one models the noise traders then also comes into play. Since we have no guidance on how to weight these various constituencies, we simply examine their utilities separately in order to answer the question of who gains and who loses from reform.

A simple revealed preference argument establishes that the asset issuer is always weakly better off without the ratings mandate. Without the mandate, the asset issuer can always choose to pay for and disclose the rating. But with the mandate, he cannot choose to forgo a rating.

Thus, the question becomes: How does the ratings mandate affect investors? On the one hand, information produces more efficient investment decisions that increase the total production and therefore the total payoffs to all risky assets. On the other hand, proposition 4 tells us that information increases the price investors must pay issuers for the asset, which makes them worse off.

Proposition 7 ***(Investors prefer information market collapse)*** *Investors have higher ex-ante expected utility when no information is provided ($\lambda = 0$) than when ratings are mandatory ($\lambda = Q$).*

Proof in appendix A.9. Investors benefit from access to a high-risk, high-return asset. They are indifferent between holding the last, marginal share of a risky asset, but earn a utility benefit from holding all the inframarginal shares. When ratings are issued, it is as if the asset is replaced by a lower-risk, lower return asset. Investors earn less of a utility benefit from holding this asset at the new, higher equilibrium price.

To see why investors prefer high return and high risk, note that when information is symmetric, ex-ante expected utility is a positive constant times

$$EU \propto -\exp\left\{\frac{-1}{2}\frac{(E(y|\mathcal{I}) - p)^2}{Var(y|\mathcal{I})}\right\}. \tag{20}$$

(See A.2 for derivation.) The fact that variance appears in the denominator of the fraction tells us that each investor individually would prefer more information. But when all investors acquire more information, the expected return falls. Recall (from equation 11) that expected return is proportional to the conditional variance: $E[y|\mathcal{I}] - p = \rho Var(y|\mathcal{I})$. Since expected return, and therefore variance enters squared in the numerator and only linearly in the denominator, the ratio is increasing in variance: $(E(y|\mathcal{I})-p)^2/Var(y|\mathcal{I}) = \rho^2 Var(y|\mathcal{I})$. Thus, expected utility is increasing in the conditional variance of the asset payoff.[6] Acquiring information is like a prisoner's dilemma. Each investor wants to observe more information. But investors would like to collectively commit to observe less.

Proposition (7) implies that if the choice were between mandating ratings and banning them, investors would collectively benefit from a ban. However, this does not immediately imply that they would benefit from removing the mandate. Without a mandate, each individual investor has an incentive to acquire information and, given the resulting equilibrium, may or may not be better off than with the mandate. In fact, investors prefer mandatory ratings when the alternative involves asymmetric information. If issuers will not provide the rating and only some investors are willing to buy the rating at the equilibrium information price, then there will be asymmetric information,

[6] In a Merton (1987)-style model with CRRA preferences, similar relationships hold: Holding all else constant, log expected returns are proportional to variance. Conditional on observed signals, interim expected utility also depends on (log expected return)^2/variance. But ex-ante utility has an additional term that comes from wealth effects, whose partial derivative with respect to signal precision depends on parameter values. Details and numerical results available on request.

with some investors knowing θ and others not. The informed and uninformed investors will hold different quantities of risky and riskless assets. But since all investors are identical ex-ante, holding different portfolios entails sharing risk inefficiently. Inefficient risk sharing reduces investor welfare. If this welfare effect is strong enough, investors prefer that a mandatory ratings statute restore information symmetry. The next result characterizes this information asymmetry region where mandatory ratings are preferable using threshold values of the information fixed cost χ.

Proposition 8 ***(Investors prefer mandatory ratings when information is cheap.)*** *There exists a cutoff χ^* such that for $\chi < \chi^*$, investor welfare with mandatory ratings is higher than with investor-purchased ratings.*

Proof in appendix A.9. This result is surprising because one might think that it is when information is very expensive that investors would prefer for asset issuers to pay for it and provide it to them for free. Instead, when information is expensive, investors know that few among them will buy ratings, so there will be few informed investors to drive up asset prices and excess returns will be available. Instead, when information is cheap, most investors will buy it. This leaves the individual investor with the alternative of either paying for the rating or trading with a large pool of better-informed investors. In this scenario, they will prefer that ratings be provided for free.

Noise traders' welfare Finally, there is the issue of how (whether) to include noise traders in the welfare calculation. One possible interpretation of noise traders is that they are merely a modeling convenience to capture the idea of imperfection in the information aggregation process and thus one can safely ignore them in the welfare calculation. Another is to assume that noise traders are either trading for liquidity reasons or are making mistakes. Their welfare is still affected by the profits or losses they make from trading in this market. The aggregate profits they make are given by

$$\pi = (y - p)\xi$$

and, using (8), expected profits are given by

$$\mathbb{E}\pi = -\frac{\beta}{h_x} \tag{21}$$

where β, given by equation (12), is the sensitivity of the asset price to noise trader demand. Noise traders are hurt by the fact that when they trade they move the price against themselves.

Proposition 9 ***(Noise traders benefit from mandates)*** *The profits of noise traders are maximized when ratings are mandatory ($\lambda = Q$).*

Proof in Appendix A.10. When all investors are informed, the asset is less risky for them, which makes their demand more elastic and thus more able to absorb noise with little change in price. Furthermore, the fact that investors are informed means they don't infer anything from prices, so noise traders do not adversely affect investors estimates of the value of the asset. For this reason, noise traders are always better off when $\lambda = Q$, which the mandate brings about.

5 A Quantitative Evaluation of Welfare

The theory can provide a set of parameter values for which investors prefer ratings mandates and set of parameters for which the investors prefer their repeal. So ultimately, the question of whether ratings enhance investor welfare or not is a quantitative one. This section proposes some rough estimates for the model parameters and then uses those estimates to predict welfare outcomes once the repeal of mandatory ratings, required by the Dodd-Frank Act, is implemented.

Data description We select parameters to match features of corporate bonds. Our data comes from Datastream and includes all corporate bonds issued in 2004 and 2005, with maturities of not more than 30 years, whose prices are tracked by Datastream. In total, this amounts to 770 different bonds. The bond ratings are the Standard and Poor's rating, prior to issuance.

For each bond, we know the price at the time when it was issued and the rating at the time of issue. It is this initial rating that we compare to the model rating θ. We also know the promised annual coupon (interest) payments on the bond, its face value and its market price 1 year later.[7] In our sample, the average coupon rate (annual interest promised) is 5.7%.

To make the data comparable to the objects in the model, we make two transformations. First, we adjust prices for fluctuations in the risk-free rate. The problem is that if a bond is issued in 2004 and then in 2005 the risk-free interest rises, the 2005 price of the bond will fall for reasons that are outside our model. Second, the contractual terms (e.g. the coupon rate) differ across bonds. To adjust for this, we construct a variable y^p that is the present value of all the promised payments – coupons plus face value at redemption. Then, we normalize the issue price $\tilde{p}$ and the bond payoff

[7] Ideally, one would follow each bond all they way up to maturity or default but data limitations prevented this. Thus, our measure of the output from the asset is the value an investor would have realized by selling the bond one year after issue, when at least some uncertainty has been realized. As a robustness check, we re-did the analysis using the bond's market price 2 years later and found very little difference in the result.

$\tilde{y}$ by y^p so that $p = \tilde{p}/y^p$ and $y = \tilde{y}/y^p$. These normalized prices and payoffs are what we compare to p and y in the model. The details of these transformations are laid out in appendix B.

Parameter selection In order to estimate parameters we assume that the data has been generated by the model under the current regime of issuer-provided ratings, which implies $\lambda = Q$. We set the values of the five key model parameters to match five moments of the data whose dependence on the parameters is fairly straightforward.

We do this in a slightly extended version of the model where, in addition to the rating, all investors observe a public signal $w = y + \nu$ where $\nu \sim N(0, h_w^{-1})$. Details of this extension are in Appendix C. The extension makes no difference for the theoretical results above since this public signal enters the model in exactly the same way as the prior. However, this extension allows the model to better fit the data since the public signal, though unobserved to the econometrician, is allowed to be different for each bond in the sample and get incorporated into prices. This allows the model to account for the fact that prices, even though they have noise, are slightly more informative about bond payoffs than are ratings.

The appendix derives the following five moments that are functions of the parameters: h_u, h_w, h_θ, h_x and ρ/Q:

1. The unconditional variance of bond payoffs. It pins down the parameter h_u.

$$Var\,(y) = \frac{1}{h_u} \tag{22}$$

2. Informativeness of the rating. This is the R^2 of a regression of bond payoffs y on ratings θ. Given that the first moment pinned down h_u, this one determines the noise in ratings h_θ.

$$R^2_{y|\theta} = \frac{1}{1 + \frac{h_u}{h_\theta}} \tag{23}$$

   Since ratings are discrete, when we estimate this R^2, we use a dummy variable for each possible rating.

3. Average returns. The average bond return is particularly sensitive to, and therefore particularly informative about risk aversion and the measure of investors ρ/Q.

$$\mathbb{E}\left[y - p\right] = \frac{\rho}{Q\left(h_u + h_w + h_\theta\right)} \tag{24}$$

This measure of return is an absolute amount, not a percentage return, as typically computed in the data. To convert this absolute return into an average percentage return, simply divide by the average (nomalized) bond price, which is 0.914.

4. Informativeness of the price. This is the R^2 of a regression of bond payoffs y on bond prices p. It is sensitive to the amount of public information h_w and how much noise the noise trading introduces h_x.

$$R^2_{y|p} = \frac{1}{\left(\frac{\rho}{(h_\theta+h_w)Q}\right)^2 \frac{h_u}{h_x} + 1 + \frac{h_u}{h_\theta+h_w}} \tag{25}$$

If h_w is very high, then this R^2 approaches 1. Instead if $h_w = 0$, this $R^2 = [\left(\frac{\rho}{h_\theta Q}\right)^2 \frac{h_u}{h_x} + 1 + \frac{h_u}{h_\theta}]^{-1}$, which means the informativeness of prices is necessarily lower than the informativeness of ratings. In the data, prices are slightly more informative than ratings, which means that w must contain at least some information. In other words, investors know more than just "this is a bond," even before they observe any bond ratings.

Similarly, if noise trader demand is very predictable (high h_x) then prices reveal most of the information in ratings and public signals. This makes the R^2 high. If noise trading is very volatile, then prices will reflect more noise and less information. The effect of noise trading also depends on risk aversion and signal precision. If investors have low risk aversion or very precise information, then noise traders have less effect on prices.

5. Price variance. The unconditional variance of the bond price also reflects how much noise trading causes the price to vary and how much public information moves price around.

$$Var\left(p\right) = \left(\frac{1}{h_u + h_w + h_\theta}\right)^2 \left[\left(\frac{\rho}{Q}\right)^2 \frac{1}{h_x} + \frac{(h_\theta + h_w)^2}{h_u} + h_\theta + h_w\right] \tag{26}$$

Notice that ρ and Q always enter as a ratio, implying that they are not separately identified in the model when $\lambda = Q$.

The one other parameter we need to calibrate is the fixed cost of information discovery. In the model, when issuers provide ratings, this is equal to the price that ratings agencies charge issuers. Treacy and Carey (2000) report that the average cost of rating an asset is 0.0325% of the value of the issue, so we set the χ equal to 0.0325% times the average price of 0.91. Table 1 summarizes our parameter estimates.

Note that ratings are about as informative as prior beliefs. But public information is more

Table 1: Parameter values for numerical results.

parameter	value	target
$\frac{\rho}{Q}$	12.4	average returns
h_u	142	bond payoff variance
h_w	266	informativeness of prices
h_θ	128	informativeness of ratings
h_x	0.330	price variance
χ	0.00029	Treacy and Carey (2000)

informative than either. The variance of noise trader demand is quite high (low h_x) to account for the relatively high variance of prices conditional on ratings, which the model interprets as resulting from noise.

Numerical results Given these parameters values, the optimal strategy for an asset issuer is not to pay to rate the asset. The reason is that the issuer knows that all investors will buy the rating anyway. Thus, with or without mandatory ratings, all investors are informed. The repeal of the ratings mandate simply transfers the amount of the ratings fee c from investors to issuers. These findings suggest that Dodd-Frank ratings provisions benefit asset issuers, at the expense of investors. But they also tell us that the reform is not likely to adversely affect market information or liquidity.

To see why all investors would choose to purchase the rating, consider the indifference condition for the marginal investor who decides whether or not to buy the rating. It tells us that the investor will buy the rating as long as the utility benefit (left-hand side) exceeds the utility cost (right hand side):

$$\sqrt{\frac{Var\,(u|p)}{Var\,(u|\theta)}} - 1 > \exp(\rho c) - 1 \tag{27}$$

Consider the case where all investors buy the rating and examine the incentive of the last infinitesimal investor to buy the rating as well. Given our estimated parameters, which imply $h_p = 27.58$, the conditional variances of payoffs are

$$\sqrt{\frac{Var\,(u|p)}{Var\,(u|\theta)}} = \sqrt{\frac{h_u + h_w + h_\theta}{h_u + h_w + h_p}} = \sqrt{\frac{536}{435.58}} = 1.109.$$

If all investors buy the signal, the ratings agencies charge each investor $c = \chi/Q$. Thus, $\exp(\rho c)$

$= \exp(\chi\rho/Q) = \exp(12.4 \cdot .00029) = 1.004$. Subtracting one and comparing these terms, we find that the utility benefit of the rating is 0.109, while the utility cost is 0.004. This means that, even when the value of information is at its lowest, when all other investors also have the information, the value of that information exceeds its cost by more than a factor of 25.

5.1 Copyright and Information Leakage

A maintained assumption in the model is that, unlike partial revelation through prices, direct leakage of information, for instance by investors who bought the rating sharing it with those who have not, can be effectively prevented by intellectual property laws. However, this might be hard to enforce due to technologies that make it easy to disseminate information. If information leakage cannot be prevented, rating agencies might not be able to sell enough copies of the information at a high enough price to pay for the fixed cost of information discovery. This would render the investor-pay market inviable through a far more direct channel than the model examines.

The degree to which information leakage is an insurmountable concern is a matter of debate. Ratings agencies did mainly follow an investor-pay model until around the mid-twentieth century, and historical accounts differ on the relative roles played by regulation and technological progress (in particular, photocopying machines) in driving the shift towards an inssuer-pay market (White, 2010). Ratings agencies could try to take measures to prevent easy retransmission of information, such as delivering their reports in non-recorded oral communications, but whether these attempts would be successful remains an open question.

If anything, if the threat of information leakage undermines the investor pay market, this would strengthen the welfare implications of the model. Asset issuers would still prefer deregulation because then they can choose to provide the rating or not. Investors' preference for deregulation would now be unambiguous because if it leads to any change at all, it is to the disappearance of ratings, which investors strictly prefer, and never to asymmetric information.

6 Conclusions

The paper investigated the likely consequences of repealing ratings mandates. It characterizes the types of assets for which a free market for information will provide ratings to investors. Information could be purchased by an entrepreneur who wants to provide the information to investors to make his project less risky and therefore more valuable to them so that it can fetch a higher price at auction. Alternatively, it could be purchased by investors who want to know how much of the risky asset to buy.

When the private market provides information to most investors, repealing the ratings mandate will have little effect on most assets' prices or on welfare. But in some instances, that private market does not provide information. In these cases, entrepreneurs are always better off without the ratings mandate. Surprisingly, investors are often better off without the mandate as well. Investors' welfare is maximized when no information about the asset payoff is available to anyone.

There are obvious limitations to interpreting these welfare results. This model included only a couple of potential benefits of ratings: facilitating the allocation of productive capital and preventing the inefficient risk-sharing that comes with asymmetrically informed investors. These benefits must be weighed against the cost of information discovery and the loss of investors surplus when an asset becomes less risky. There are other possible benefits of ratings, such as the ability to limit risk-taking by banks or portfolio managers or the ability to effectively summarize the average credit quality of large pools of assets. There are also other possible problems with credit ratings such as ratings inflation, the possibility that ratings crowd out some richer more nuanced sources of information, or outright investor deception. None of these are incorporated in the model. Yet, the ability of ratings to ameliorate asymmetric information problems and to improve the efficiency of asset prices are certainly two of the most widely-acknowledged benefits of ratings. And some of the weaknesses of the ratings system might be addressed by reforms that are less drastic than eliminating the ratings requirement system altogether. Thus, the conclusions provide some insight by weighing some of the most important advantages and disadvantages of credit ratings.

The results could also be re-interpreted more broadly in the context of a consumer goods market. We typically assume that when a seller provides customers with more complete information, efficiency improves and customers benefit. Just like financial asset prices direct the allocation of real capital, goods prices influence the quantities of goods that are produced. Mandatory information disclosure encourages high-value goods to be supplied and low-value goods to be withdrawn. But, when the supplier has some monopoly power, this efficiency gain may not ultimately benefit consumers because the equilibrium price of goods with better and less-uncertain quality is higher. Services that rate products, like Consumer Reports, benefit the buyers that obtain their information, but may harm the other buyers who are left with a market for lemons. The resulting inefficiency in the allocation of goods could be severe enough that buyers prefer sellers to always disclose information. In a partial equilibrium model with fixed prices, this argument for mandatory provision of information is straightforward. But in an equilibrium model, consumers could also benefit from repealing information regulations.

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A Mathematical Appendix

A.1 Solving for the financial market equilibrium

This appendix solves for the equilibrium price in the risky asset market. It verifies the conjecture of the existence of a price that is linear in signals and asset supply and it derives the formulas for the linear weights.

Beginning with the market clearing condition $\lambda q^I + (Q-\lambda) q^U = x$ we use the formulas for q^I and q^U and to solve for p:

$$\begin{aligned} Qf(k^*(D))h_u + \lambda[\theta h_\theta - p(h_u+h_\theta)] + (Q-\lambda)\Big[\Big(f(k^*(D)) + \frac{p-\alpha}{\gamma}\Big)h_p - p(h_u+h_p)\Big] &= \rho x \\ Qf(k^*(D))h_u + (Q-\lambda)\Big[f(k^*(D)) - \frac{\alpha}{\gamma}\Big]h_p + \lambda\theta h_\theta - p\Big[\lambda(h_u+h_\theta) + (Q-\lambda)(h_u+h_p) - (Q-\lambda)\frac{h_p}{\gamma}\Big] &= \rho x \end{aligned}$$

$$p = \frac{Qf(k^*(D))h_u + (Q-\lambda)\Big[f(k^*(D))) - \frac{\alpha}{\gamma}\Big]h_p + \lambda\theta h_\theta - \rho x}{\lambda(h_u+h_\theta) + (Q-\lambda)(h_u+h_p) - (Q-\lambda)\frac{h_p}{\gamma}} \tag{28}$$

which has a linear form as conjectured. Equating coefficients:

$$\begin{aligned} \alpha &= \frac{f(k^*(D))[\lambda(h_u+h_\theta) + (Q-\lambda)(h_u+h_p)] - (Q-\lambda)\frac{a}{\gamma}h_p - \rho}{\lambda(h_u+h_\theta) + (Q-\lambda)(h_u+h_p) - (Q-\lambda)\frac{h_p}{\gamma}} \\ \beta &= \frac{\rho}{\lambda\left(h_u+h_\theta\right) + \left(Q-\lambda\right)\left(h_u+h_p\right) - \left(Q-\lambda\right)\frac{h_p}{\gamma}} \\ \gamma &= \frac{\lambda h_\theta}{\lambda\left(h_u+h_\theta\right) + \left(Q-\lambda\right)\left(h_u+h_p\right) - \left(Q-\lambda\right)\frac{h_p}{\gamma}} \end{aligned} \tag{29}$$

Computing price informativeness yields

$$h_p = \frac{1}{\frac{1}{h_\theta} + \left(\frac{\beta}{\gamma}\right)^2 \frac{1}{h_x}}. \tag{30}$$

Substituting in expressions for β and γ yields (14) and replacing h_p in (29) yields (11)-(13).

A.2 Solving for the equilibrium measure of informed investors

Recall the utility function:

$$V = -E\left[\exp\left\{-\rho W\right\}\right]$$

where

$$W_i = (w_0 - cd) + q_i\left[y - p\right]$$

where c is the price of the rating and $d = 1$ if the investor bought it and zero otherwise.

Because of the CARA-Normal structure, given an information set for investor i, utility is

$$V_i = -\exp\left\{-\rho\left[E_i\left(W_i^I\right) - \frac{\rho}{2}Var_i\left(W_i^I\right)\right]\right\} \tag{31}$$

Use that $q_i = \frac{E[y|I_i]-p}{\rho Var[y|I_i]}$ so that

$$W_i^I = w_0 - cd + \frac{E\left[y|I_i\right] - p}{\rho Var\left[y|I_i\right]}\left[y-p\right]$$

Denote $E_i(y) \equiv E\left[y|I_i\right]$ and $Var_i(y) \equiv Var\left[y|I_i\right]$ and conclude that

$$E_i\left(W_i^I\right) = (w_0 - cd) + \frac{\left[E_i\left(y\right) - p\right]^2}{\rho Var_i\left(y\right)} \tag{32}$$

and

$$Var_i\left(W_i^I\right) = \frac{\left[E_i\left(y\right) - p\right]^2}{\rho^2 Var_i\left(y\right)} \tag{33}$$

Replacing (32) and (33) in (31):

$$V_i = -\exp\left(-\rho\left(w_0 - cd\right)\right)\exp\left\{-\frac{1}{2}\frac{\left[E_i\left(y\right) - p\right]^2}{Var_i\left(y\right)}\right\} \tag{34}$$

Utility of the informed investor The information set of an informed investor includes θ and p. Let

$$\Sigma_I \equiv Var\left[E_I(y) - p\right] \tag{35}$$

$$Z_I \equiv \frac{E_I(y) - p}{\sqrt{\Sigma_I}} \tag{36}$$

Replacing (35) and (36) into (34):

$$V_I = -\exp\left(-\rho\left(w_0 - c\right)\right)\exp\left\{-\frac{\Sigma_I}{2Var_I(y)}Z_I^2\right\} \tag{37}$$

Conditional on p, Z_I follows a Normal distribution with mean $A_I = \frac{E(y|p)-p}{\sqrt{\Sigma_I}}$ and standard deviation 1. Using that, by the law of total variance

$$Var\left(y|p\right) = \Sigma_I + Var_I\left(y\right)$$

and the MGF of a noncentral χ^2 distribution to take conditional expectations of (37), we conclude that

$$E\left[V_I|p\right] = -\exp\left(-\rho\left(w_0 - c\right)\right)\sqrt{\frac{Var_I(y)}{Var(y|p)}}\exp\left(-\frac{\left(E(y|p) - p\right)^2}{2Var(y|p)}.\right) \tag{38}$$

Utility of the uninformed investor Equation (34) directly implies

$$E\left[V_U|p\right] = -\exp\left(-\rho w_0\right)\exp\left(-\frac{\left(E(y|p) - p\right)^2}{2Var(y|p)}\right) \tag{39}$$

Utility comparison From (38) and (39) and noting that $Var_I(y) = Var(y|\theta, p) = Var(y|\theta)$:

$$E\left[V_I|p\right] - E\left[V_U|p\right] = \left[\exp\left(\rho c\right)\sqrt{\frac{Var(y|\theta)}{Var(y|p)}} - 1\right]E\left[V_U|p\right]$$

Taking expectations over p, ex-ante indifference requires:

$$\exp\left(\rho c\right)\sqrt{\frac{Var(y|\theta)}{Var(y|p)}} = 1 \tag{40}$$

Using

$$Var(y|\theta) = \frac{1}{h_u + h_\theta} \tag{41}$$

$$Var(y|p) = \frac{1}{h_u + h_p} \tag{42}$$

and equation (14) to solve for λ yields equation (15).

A.3 Proof of proposition 1

From (15), a positive solution for λ requires

$$\frac{h_\theta}{(h_u + h_\theta)(1 - \exp(-2\rho c))} - 1 > 0 \tag{43}$$

which reduces to

$$h_\theta \exp\left(-2\rho c\right) - h_u\left(1 - \exp\left(-2\rho c\right)\right) > 0 \tag{44}$$

Since the ratings agency must make nonnegative profits and at most a measure Q of investors purchase the rating, this means that $c \geq \frac{\chi}{Q}$. Therefore (44) cannot hold if (17) holds.

A.4 Proof of proposition 2

Rewrite (15) as

$$\lambda = \frac{\rho}{\sqrt{h_\theta h_x}} \sqrt{\frac{\frac{h_\theta + h_u}{h_\theta} \exp\left(-2\rho c\right) - \frac{h_u}{h_\theta}}{\frac{h_\theta + h_u}{h_\theta}\left(1 - \exp\left(-2\rho c\right)\right)}} \tag{45}$$

Fixing c, (45) implies $\lim_{h_\theta \to \infty} \lambda = 0$. Letting $c = \frac{\chi}{\lambda}$ does not alter this conclusion because λ is decreasing in c. Therefore, with an endogenous information price, the right side approaches zero even faster.

Even though $\lambda = 0$ in the limit, it could still be that for any finite h_θ, $\lambda > 0$. The following shows that this is not the case.

Suppose not. This means that for every h_θ (45) has a solution $\lambda \in (0, Q]$ with $c = \frac{\chi}{\lambda}$. Rearrange (45) and use $c = \frac{\chi}{\lambda}$:

$$\sqrt{h_\theta} = \frac{1}{\lambda} \frac{\rho}{\sqrt{h_x}} \sqrt{\frac{\left(1 + \frac{h_u}{h_\theta}\right) \exp\left(-2\rho \frac{\chi}{\lambda}\right) - \frac{h_u}{h_\theta}}{\left(1 + \frac{h_u}{h_\theta}\right)\left(1 - \exp\left(-2\rho \frac{\chi}{\lambda}\right)\right)}}.$$

Since the previous expression holds for every h_θ, by continuity it should also hold in the limit as $h_\theta \to \infty$. On the LHS we have that $\lim_{h_\theta \to \infty} \sqrt{h_\theta} = \infty$. On the RHS, we have that:

$$\lim_{h_\theta \to \infty} \frac{1}{\lambda} \frac{\rho}{\sqrt{h_x}} \sqrt{\frac{\exp\left(-2\rho \frac{\chi}{\lambda}\right)}{\left(1 - \exp\left(-2\rho \frac{\chi}{\lambda}\right)\right)}} = \frac{\rho}{\sqrt{h_x}} \lim_{\lambda \to 0} \sqrt{\frac{1}{\lambda^2 \exp\left(2\rho \frac{\chi}{\lambda}\right) - 1}}$$

where the right hand side considers λ a function of h_θ ($\lim_{h_\theta \to \infty} \lambda(h_\theta) = 0$). Finally, L'Hopital's rule tells us that $lim_{\lambda \to 0} \lambda^2 \exp\left(2\rho \frac{\chi}{\lambda}\right) = \infty$, and therefore (46) is zero in the limit.

Therefore, we have two sequences that must be equal for all finite values but are different in the limit. Since these two sequences come from continuous functions, this is a contradiction.

A.5 Proof of proposition 3

1. Suppose to the contrary that the issuer does not provide information, and investors do not buy it either. Expected profits for the issuer will be:

$$\Pi^0 = f(k^*(0)) - \frac{\rho}{Q h_u} - k^*(0)$$

   If instead the issuer paid for a rating, expected profits would be:

$$\Pi^i = f(k^*(1)) - \frac{\rho}{Q(h_\theta + h_u)} - k^*(1) - \chi$$

   Rearranging the inequality $\Pi^I - \Pi^0 > 0$ and using the result that $k^*(0) = 0$ yields condition (18). If the condition holds, it contradicts the assumption that the issuer does not provide information.
2. If condition (18) does not hold, then $\Pi^1 \leq \Pi^0$, so an issuer will not provide a rating even if he expects investors not to buy it either. By Proposition 4 below, this implies that the issuer will not provide a rating regardless of what he expects investors to do.

A.6 Proof of proposition 4

From equation (11), using the fact that we have abstracted from the real investment decision:

$$\frac{\partial \alpha}{\partial \lambda} = \frac{h_\theta - h_p + (Q - \lambda)\frac{\partial h_p}{\partial \lambda}}{\left(Q h_u + \lambda h_\theta + (Q - \lambda) h_p\right)^2} \rho > 0$$

because $\frac{\partial h_p}{\partial \lambda} > 0$ and $h_\theta > h_p$.

A.7 Proof of proposition 5

1. For any given set of other parameters, there is a Q sufficiently low such that (18) holds. From proposition 3, this implies that either the issuer will provide a rating or some investors will buy it. But from proposition 1, for Q sufficiently low, no investors will buy a rating. Therefore, $\exists \tilde{Q}$ such that for all $Q < \tilde{Q}$, (18) and (17) both hold. For any such Q, the issuer will provide the rating. If the issuer chooses to provides the rating, the information sets of all agents and therefore the asset prices are the same as if the issuer were required to buy the rating. Therefore $p^M = p^E$, for every realization of θ or x.
2. Having abstracted from the choice of k, the must exist a cutoff Q_1 such that for any $Q > Q_1$, condition (18) fails so the issuer will not provide the rating. λ will then be given by the minimum of Q or the solution to equations (15)-(16). Therefore there exists a cutoff $\bar{Q} = \max\{Q_1, \lambda\}$ such than whenever $Q > \bar{Q}$, $Q > \lambda$. Using (11), this implies $E[p^E] < E[p^M]$.
3. Note that $E[p] = \alpha$ and, from equation (11), $\lim_{Q\to\infty} \alpha = f(k^*)$ no matter what is the value of λ. Since we have fixed k^*, the result follows.

A.8 Proof of proposition 6

1. Condition (19) implies that for sufficiently low h_u, the solution to equations (15)-(16) is greater than Q, which implies that all investors would buy the rating (anticipating this, the entrepreneur does not provide it), so $p^E = p^M$.
2. Let $\underline{h}_u$ be the maximum value of h_u such that $\lambda = Q$. This value must exist since $\lambda = Q$ for h_u small enough and, from equation (15), $\lambda = 0$ for h_u large enough. Let Π^1 be the issuer's expected profits if he provides a rating and Π^0 be his profits if he does not. It follows that $\Pi^1 - \Pi^0 = -\chi$ for $h_u < \underline{h}_u$. Furthermore, because both Π^1 and Π^0 are continuous in h_u and λ is continuous in h_u at $\underline{h}_u$, $\Pi^1 - \Pi^0$ is continuous in h_u at $\underline{h}_u$. This implies there is a $\bar{h}_u > \underline{h}_u$such that if $h_u \in (\underline{h}_u, \bar{h}_u)$, the issuer prefers not to provide a rating even though $\lambda < Q$. Using (11), this implies $E[p^E] < E[p^M]$.
3. This follows because as $h_u \to \infty$, $p \to f(k^*(D))$ no matter whether there is a rating or not.

A.9 Welfare of investors - proof of propositions 7 and 8

Expected utility conditional on an information set is given by (34). Let

$$\begin{aligned} A_i &\equiv E\left[E_i\left(y\right) - p\right] \\ \Sigma_i &\equiv Var\left[E_i\left(y\right) - p\right] \\ Z_i &\equiv \frac{E_i\left(y\right) - p}{\sqrt{\Sigma_i}} \end{aligned}$$

Ex-ante, $Z_i \sim N\left(\frac{A_i}{\sqrt{\Sigma_i}}, 1\right)$.

Rewrite (34) as

$$V_i = -\exp\left(-\rho(w_0 - cd)\right)\exp\left\{-\frac{1}{2}\frac{1}{Var_i\left(y\right)}\Sigma_i Z_i^2\right\}$$

Using the formula for the moment-generating function of a chi-square distribution, the ex-ante expected utility is

$$E(V_i) = -\exp\left(-\rho(w_0 - cd)\right)\frac{\exp\left\{\frac{-\frac{1}{2}{A_i}^2\frac{1}{Var_i(y)}}{1+\frac{1}{Var_i(y)}\Sigma_i}\right\}}{\sqrt{1+\frac{1}{Var_i(y)}\Sigma_i}}$$

or, re-normalizing:

$$W_i \equiv -2\log\left[\frac{-E(V_i)}{\exp\left(-\rho w_0\right)}\right] = \frac{{A_i}^2}{Var_i\left(y\right) + \Sigma_i} + \log\left(Var_i\left(y\right) + \Sigma_i\right) - \log\left(Var_i\left(y\right)\right) - 2\rho cd \tag{46}$$

1. In case the issuer supplies the rating, then, using (11) - (14):

$$E_I(y) - p = \frac{\rho x}{Q(h_u + h_\theta)}$$
$$Var_I(y) = \frac{1}{h_u + h_\theta}$$

Therefore

$$\Sigma_I = \left[\frac{\rho}{Q(h_u + h_\theta)}\right]^2 \frac{1}{h_x} \tag{47}$$
$$A_I = \frac{1}{Q}\frac{\rho}{h_u + h_\theta} \tag{48}$$

2. In case the issuer does not supply the rating and $\lambda \in (0, Q)$, there are two expected utilities to consider, that of the informed agent and that of the uninformed. But in an interior equilibrium, the two must be equal. So, it suffices to look only at the expected utility of the uninformed agent. Using (11) - (14):

$$E_U(y) - p = A_U + B_U(1 - x) + C_U(\theta - f(k^*(D)))$$
$$Var_U(y) = \frac{1}{h_u + h_p}$$

where

$$A_U = \beta = \frac{\rho}{\lambda h_\theta}\frac{\lambda h_\theta + (Q-\lambda)h_p}{\lambda(h_u + h_\theta) + (Q-\lambda)(h_u + h_p)} \tag{49}$$
$$B_U = \left[\frac{h_p}{h_u + h_p} - \gamma\right]\frac{\beta}{\gamma} = \left[\frac{h_p}{h_u + h_p} - \frac{\lambda h_\theta + (Q-\lambda)h_p}{\lambda(h_u + h_\theta) + (Q-\lambda)(h_u + h_p)}\right]\frac{\rho}{\lambda h_\theta}$$
$$C_U = \left[\frac{h_p}{h_u + h_p} - \gamma\right] = \left[\frac{h_p}{h_u + h_p} - \frac{\lambda h_\theta + (Q-\lambda)h_p}{\lambda(h_u + h_\theta) + (Q-\lambda)(h_u + h_p)}\right]$$

so

$$\Sigma_U \equiv {B_U}^2\frac{1}{h_x} + {C_U}^2\left(\frac{1}{h_\theta} + \frac{1}{h_u}\right) = \left[\left(\frac{\rho}{\lambda h_\theta}\right)^2\frac{1}{h_x} + \left(\frac{1}{h_u} + \frac{1}{h_\theta}\right)\right]\left[\frac{h_p}{h_u + h_p} - \frac{\lambda h_\theta + (Q-\lambda)h_p}{\lambda(h_u + h_\theta) + (Q-\lambda)(h_u + h_p)}\right]^2 \tag{50}$$

3. In case the issuer does not supply the rating but in equilibrium $\lambda = 0$, utility can be found by setting $h_\theta = 0$ in (47) and (48):

$$\Sigma_0 = \left[\frac{\rho}{Qh_u}\right]^2\frac{1}{h_x} \tag{51}$$
$$A_0 = \frac{1}{Q}\frac{\rho}{h_u} \tag{52}$$

4. Finally, for the case where the issuer does not provide a rating but in equilibrium $\lambda = Q$, utility for each is as in the issuer-provided rating, subtracting the fixed cost $c = \frac{\chi}{Q}$, so that

$$W_Q = W_I - 2\rho\frac{\chi}{Q}$$

Replacing (51) , (52), (47) and (48) respectively into (46)

$$W_0 - W_I = \rho^2 h_x\Big[\frac{1}{Q^2 h_u h_x + \rho^2} - \frac{1}{Q^2(h_u + h_\theta)h_x + \rho^2}\Big] + \log\left(\frac{1 + \frac{1}{h_u}\left(\frac{\rho}{Q}\right)^2\frac{1}{h_x}}{1 + \frac{1}{h_u + h_\theta}\left(\frac{\rho}{Q}\right)^2\frac{1}{h_x}}\right) > 0$$

that is positive because $h_\theta > 0$. This proves Proposition 7.

Now we prove Proposition 8. First, from (49) and (48), it follows that $\lim_{\lambda\to Q} A_U = A_I$. Second, we use (50), (47) and (14) to establish the following two claims.

Claim 1 *1)* $\frac{\bar{\Sigma}_U}{\Sigma_I} = \frac{h_\theta - h_p}{h_u + h_p}\frac{h_u}{h_\theta}$ *and 2)* $\Sigma_I - \bar{\Sigma}_U = \frac{h_p}{h_\theta}\frac{h_u + h_\theta}{h_u + h_p}\Sigma_I$

Proof. Let $\bar{\Sigma}_U \equiv \lim_{\lambda \to Q} \Sigma_U = \left[\left(\frac{\rho}{Qh_\theta}\right)^2 \frac{1}{h_x} + \left(\frac{1}{h_u} + \frac{1}{h_\theta}\right)\right]\left[\frac{h_p}{h_u+h_p} - \frac{h_\theta}{h_u+h_\theta}\right]^2$. Then

$$\begin{aligned}
\frac{\bar{\Sigma}_U}{\Sigma_I} &= \frac{\left[\left(\frac{\rho}{Qh_\theta}\right)^2 \frac{1}{h_x} + \left(\frac{1}{h_u} + \frac{1}{h_\theta}\right)\right]\left[\frac{h_p}{h_u+h_p} - \frac{h_\theta}{h_u+h_\theta}\right]^2}{\left[\frac{\rho}{Q(h_u+h_\theta)}\right]^2 \frac{1}{h_x}} \\
&= \underbrace{[\rho^2 h_u + Q^2 h_\theta^2 h_x + Q^2 h_\theta h_x h_u]}_{(h_u+h_p)(\rho^2+Q^2h_\theta h_x)} \underbrace{(h_\theta - h_p)^2}_{\frac{\rho^4 h_\theta^2}{(\rho^2+Q^2h_\theta h_x)^2}} h_u \frac{1}{\rho^2 h_\theta^2 (h_u+h_p)^2} \\
&= \frac{\rho^2 h_u}{(\rho^2 + Q^2 h_\theta h_x)(h_u + h_p)} \\
&= \frac{h_\theta - h_p}{h_u + h_p}\frac{h_u}{h_\theta}
\end{aligned}$$

and

$$\begin{aligned}
\Sigma_I - \bar{\Sigma}_U &= \left[1 - \frac{h_\theta - h_p}{h_u + h_p}\frac{h_u}{h_\theta}\right]\Sigma_I \\
&= \frac{h_p}{h_\theta}\frac{h_u + h_\theta}{h_u + h_p}\Sigma_I
\end{aligned}$$

Claim 2 $\lim_{\lambda \to Q}\left[\frac{1}{h_u+h_p} + \Sigma_U\right] = \frac{1}{h_u+h_\theta} + \Sigma_I$

Proof. Observe that $\lim_{\lambda \to Q} h_p = \frac{Q^2 h_\theta^2 h_x}{\rho^2 + Q^2 h_\theta h_x}$. Then:

$$\begin{aligned}
\lim_{\lambda \to Q}\left[\frac{1}{h_u+h_p} + \Sigma_U\right] &= \frac{1}{h_u+h_\theta} + \Sigma_I \Leftrightarrow \\
\lim_{\lambda \to Q} \frac{1}{h_u+h_p} - \frac{1}{h_u+h_\theta} &= \Sigma_I - \bar{\Sigma}_U \qquad \Leftrightarrow \text{(By Claim 1)} \\
\lim_{\lambda \to Q} \frac{h_\theta - h_p}{(h_u+h_p)(h_u+h_\theta)} &= \frac{h_p}{h_\theta}\frac{h_u+h_\theta}{h_u+h_p}\Sigma_I \Leftrightarrow \\
\lim_{\lambda \to Q} \frac{h_p}{h_\theta}\left[\frac{\rho}{Q(h_u+h_\theta)}\right]^2 &= \frac{h_p}{h_\theta}\Sigma_I \qquad \Leftrightarrow \\
\left[\frac{\rho}{Q(h_u+h_\theta)}\right]^2 &= \Sigma_I
\end{aligned}$$

Now we establish the result:

$$\begin{aligned}
W_I - \lim_{\lambda \to Q} W_U &= \left(\frac{\rho}{Q(h_u+h_\theta)}\right)^2\left[\frac{1}{\frac{1}{h_u+h_\theta} + \Sigma_I} - \frac{1}{\frac{1}{h_u+h_p} + \Sigma_U}\right] \\
&+ \log\left(\frac{\frac{1}{h_u+h_\theta} + \Sigma_I}{\frac{1}{h_u+h_p} + \Sigma_U}\right) \\
&+ \log\left(\frac{h_u+h_\theta}{h_u+h_p}\right)
\end{aligned}$$

By Claim 2, the first two terms are equal to zero, and since $h_\theta > h_p$, we have that:

$$W_I - \lim_{\lambda \to Q} W_U = \log\left(\frac{h_u+h_\theta}{h_u+h_p}\right) > 0$$

Therefore, for λ sufficiently close to Q, $W_I > W_U$. Proposition 8 then follows from the fact that for a sufficiently small χ, the equilibrium value of λ will be Q.

A.10 Proof of proposition 9

Equation (12) and the fact that $h_p < h_\theta$ imply that β is minimized when $\lambda = Q$. The result then follows from equation (21).

B Data

Adjusting for fluctuations in the risk-free rate. We compute the spread as follows: By definition, the yield of the bond at the issue date, r_0^b satisfies

$$p_0 = \sum_{t=0}^{T} \frac{c_t}{\left(1 + r_0^b\right)^t}$$

where c_t is the bond's t-dated coupon (or coupon-plus-principal). The spread on the bond is

$$s_0 = r_0 - r_0^T$$

(where r_0^T is the T-maturity risk-free rate as of $t = 0$). At $t = 1$, instead of looking directly at the price of the bond, we look at a corrected price defined by

$$\tilde{p}_1 = \sum_{t=0}^{T} \frac{c_t}{\left(1 + r_0^T + s_1\right)^t}$$

where s_1 is the spread calculated on the basis of the $t = 1$ price. If $r_0^T = r_1^T$, the corrected price coincides with the pure price, but if risk-free interest rates have changed in the meantime, the corrected price filters out the effect.

Normalizing by the promised value. In order to account for the different contractual terms of different bonds, we normalize the price of bonds by the contractually-promised net present value y^p, defined by

$$y^p = \sum_{t=0}^{T} \frac{c_t}{\left(1 + r_0^T\right)^t}$$

For bonds with low probability of default (for instance, highly rated bonds), their price as a proportion of the contractually promised net present value (p/y^p) will be close to one. In our data, the average p/y^p is 0.91.

C Model with Public Signal

Suppose there was a public signal w that everyone could see in addition to the rating.

$$w = y + \upsilon \quad \text{with} \quad \upsilon \sim N\left(0, \frac{1}{h_w}\right).$$

The equilibrium price will have the form:

$$p = \alpha + \beta\xi + \gamma(\theta - f) + \delta(w - f)$$

Solving for the coefficients:

$$\begin{aligned}
\alpha &= f - \frac{\rho}{\lambda(h_u + h_w + h_\theta) + (Q - \lambda)(h_u + h_w + h_p)} \\
\beta &= \frac{\rho}{\lambda h_\theta} \frac{\lambda h_\theta + (Q - \lambda) h_p}{\lambda(h_u + h_w + h_\theta) + (Q - \lambda)(h_u + h_w + h_p)} \\
\gamma &= \frac{\lambda h_\theta + (Q - \lambda) h_p}{\lambda(h_u + h_w + h_\theta) + (Q - \lambda)(h_u + h_w + h_p)} \\
\delta &= \frac{h_w}{\lambda(h_u + h_w + h_\theta) + (Q - \lambda)(h_u + h_w + h_p)}
\end{aligned} \tag{53}$$

Assuming that the data comes from the model with publicly observable ratings ($\lambda = Q$), this reduces to

$$\begin{aligned}
\alpha &= f - \frac{\rho}{Q\left(h_u + h_w + h_\theta\right)} \\
\beta &= \frac{\rho}{Q\left(h_u + h_w + h_\theta\right)} \\
\gamma &= \frac{h_\theta}{h_u + h_w + h_\theta} \\
\delta &= \frac{h_w}{h_u + h_w + h_\theta}
\end{aligned}$$

Deriving five moments Next, we derive each of the five moments that we match to data.

1. Unconditional variance of bond payoff. This is the variance of output, which is, by assumption,

$$Var\left(y\right) = \frac{1}{h_u}. \tag{54}$$

2. Price variance. The variance of the price can be computed using the equilibrium price equation

$$\begin{aligned}
p &= \alpha + \beta\xi + \gamma\left(\theta - f\right) + \delta\left(w - f\right) \\
&= \alpha + \beta\xi + \left(\gamma + \delta\right)u + \gamma\eta + \delta\upsilon
\end{aligned}$$

$$\begin{aligned}
Var\left(p\right) &= \beta^2 \frac{1}{h_x} + \left(\gamma + \delta\right)^2 \frac{1}{h_u} + \gamma^2 \frac{1}{h_\theta} + \delta^2 \frac{1}{h_w} \\
&= \left(\frac{1}{h_u + h_w + h_\theta}\right)^2 \left[\left(\frac{\rho}{Q}\right)^2 \frac{1}{h_x} + \frac{\left(h_\theta + h_w\right)^2}{h_u} + h_\theta + h_w\right]
\end{aligned} \tag{55}$$

3. Average excess return. The excess return in the model is

$$\begin{aligned}
y - p &= y - \alpha - \beta\xi - \gamma\left(\theta - f\right) - \delta\left(w - f\right) \\
&= f + u - \alpha - \beta\xi - \gamma\left(u + \eta\right) - \delta\left(u + \upsilon\right)
\end{aligned}$$

so

$$\begin{aligned}
y - p &= u + \frac{\rho}{Q\left(h_u + h_w + h_\theta\right)} - \frac{\rho}{Q}\frac{1}{h_u + h_w + h_\theta}\xi - \frac{h_\theta}{h_u + h_w + h_\theta}\left(u + \eta\right) - \frac{h_w}{h_u + h_w + h_\theta}\left(u + \upsilon\right) \\
&= \frac{\rho}{Q\left(h_u + h_w + h_\theta\right)} - \frac{\rho}{Q}\frac{1}{h_u + h_w + h_\theta}\xi - \frac{h_\theta}{h_u + h_w + h_\theta}\eta - \frac{h_w}{h_u + h_w + h_\theta}\upsilon + \frac{h_u}{h_u + h_w + h_\theta}u
\end{aligned}$$

and therefore the average excess return is

$$\mathbb{E}\left[y - p\right] = \frac{\rho}{Q\left(h_u + h_w + h_\theta\right)} \tag{56}$$

4. Informativeness of prices. The standard formula for the R^2 in a regression of y on p is

$$R^2 = \frac{Cov\left(y, p\right)^2}{Var\left(y\right)Var\left(p\right)}$$

We can compute this covariance by rewriting price p as a function of the unexpected component of the bond payoff u:

$$\begin{aligned}
p &= \alpha + \beta\xi + \gamma\left(\theta - f\right) + \delta\left(w - f\right) \\
&= \alpha + \beta\xi + \left(\gamma + \delta\right)u + \gamma\eta + \delta\upsilon.
\end{aligned}$$

Since $y = f(k) + u$ and $f(k)$ is a known constant,

$$Cov\left(y, p\right) = \left(\gamma + \delta\right)\frac{1}{h_u}.$$

Using this covariance formula and the formulae for the unconditional variances (54) and (55),

$$
\begin{aligned}
R^2 &= \frac{(\gamma+\delta)^2 \left(\frac{1}{h_u}\right)^2}{\frac{1}{h_u}\left(\frac{1}{h_u+h_w+h_\theta}\right)^2 \left[\left(\frac{\rho}{Q}\right)^2 \frac{1}{h_x} + \frac{(h_\theta+h_w)^2}{h_u} + h_\theta + h_w\right]} \\
&= \frac{1}{\left(\frac{\rho}{(h_\theta+h_w)Q}\right)^2 \frac{h_u}{h_x} + 1 + \frac{h_u}{h_\theta+h_w}} \qquad (57)
\end{aligned}
$$

5. Informativeness of ratings. The standard formula for the R^2 in a regression of y on θ is

$$
\begin{aligned}
R^2 &= \frac{Cov(\theta, y)^2}{Var(y)\, Var(\theta)} \\
&= \frac{Var(y)^2}{Var(y)\left[Var(y) + Var(\eta)\right]} \\
&= \frac{1}{1+\frac{h_u}{h_\theta}} \qquad (58)
\end{aligned}
$$