On the nature of the financial system in the Euro Area:  
a Bayesian DSGE approach *

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Abstract

This paper builds and compares from a Bayesian perspective two dynamic stochastic general equilibrium (DSGE) models with different sources of financial frictions. The two DSGE models, which are both extensions of Smets and Wouters (2007), are: (i) the SWBGG model, with asymmetric information originating in the demand side of the credit market à la Bernanke et al. (1999); and (ii) the SWGK model with financial frictions originating in the supply side of the credit market, à la Gertler and Karadi (2011). This paper estimates the two models with Euro Area data. The analysis of the Bayes Factor and the forecasting performance provides evidence in favour of the SWGK model. Since the financial sectors differ among the two models, so do the financial shocks and, therefore, the propagation mechanisms. Finally, this paper investigates the predictive power of the two models in forecasting inflationary pressures in the Euro Area.

Keywords: Financial frictions, DSGE models, Bayesian estimation.

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1 Introduction

In the aftermath of the 2007-2009 crisis the structure of the financial system has received an increasing attention in the dynamic stochastic general equilibrium (DSGE) literature. The features of external financing are particularly important because of the substantial effects of lending on the real activity. Since the onset of the crisis both quantities and prices in credit markets have been affected: total financing to non-financial corporations have declined both in the Euro Area (EA) and in US. In the EA loans of financial institutions to non-financial corporations (index of notional stock) decreased by more than 10 percent between 2008Q4 and 2010Q4 and the ratio of non-financial corporation loans to GDP fell as well (ECB, 2011). At their peak following the collapse of the Lehman Brothers, the credit spreads skyrocketed. Lenza et al. (2010) reported that the spread between unsecured deposit rates (EURIBOR) and overnight indexed swap (OIS) rates at the three-month maturity approached 200 basis points in the EA. Equivalent spreads were even higher in the US and UK.

While equilibrium lending has clearly decreased and the credit spreads have increased, it is important to distinguish between demand and supply-side influences on loan developments. The distinction between credit demand and credit supply factors could be difficult. Generally speaking, the supply of credit can be determined by the ability and willingness of banks to lend, while the demand of credit can be determined by borrowers-specific situation, such as balance-sheets, and by the possibility of alternative financing sources. In the EA, loan supply shocks played a non-negligible role in shaping the pattern of private sector loan growth since the onset of the financial crisis. However, other forces, many of which can be associated with demand factors (such as shocks to consumption, investment, fiscal policy and monetary policy) mainly affected loan growth (ECB, 2011). The overall equilibrium in the credit market is also affected by the presence of asymmetric information problems which can stem from the supply side or demand side of the credit market (e.g. Stiglitz and Weiss, 1981).

This paper distinguishes the source of the financial frictions between informational asymmetries originating in the demand-side versus the supply-side of the credit market. It builds two DSGE models with financial frictions in a Smets and Wouters (2007)(SW) economy. The first model, labelled as the SWBGG model, incorporates financial frictions originating in the demand side of the credit market, à la Bernanke et al. (1999) (BGG). In this model financial frictions arise because monitoring the loan applicant is costly and this drives an endogenous wedge between the cost of external and internal funds, the external finance premium (i.e. the credit spread). Carrillo and Poilly (2010) and Gelain (2010) present a SW economy with the BGG type of financial frictions to investigate the effects of a fiscal stimulus in a zero lower bound setting and to obtain a time series for the external finance premium respectively. The second model, labelled as the SWGK model, incorporates in a SW economy financial frictions originating in the supply side of the credit market, à la Gertler and Karadi (2011). The
source of financial frictions is the financial intermediary, facing endogenously determined balance sheet constraints and an endogenously determined leverage (i.e. the ratio between total assets and net worth). Therefore, the SWBGG model and the SWGK model are similar in all aspects, but the financial sector and the borrowing/lending relationships.

The two models are estimated with EA data for the period 1996Q1-2008Q3 using as observables output, consumption, investment, wage, employment, inflation and the credit spread (spread, henceforth). The comparison between the two estimated models is made along different dimensions: (i) the estimated parameters and the Bayes Factor; (ii) the forecasting performance of estimated models; (iii) impulse response function analysis; and (iv) variance decomposition. Robustness analysis of main result stemming from the Bayes Factor is also presented by examining: (i) different calibrations of the leverage ratio of the SWBGG and SWGK models; (ii) a different measure of the spread used as observable in the estimation procedure; and (iii) different model specifications.

The presence of asymmetric information amplifies the propagation mechanism of the contractionary shocks hitting the economy. Since the financial sectors differ across the two models, so do the financial shocks as well as the internal propagation mechanisms. In the SWBGG model, two “financial shocks” are analysed: a shock to the net worth of firms and an investment-specific shock. When the former hits the economy, firms have less wealth to contribute to the project financing. This implies that the potential divergence of interests between firms and lenders (the suppliers of external funds) is greater and, therefore, agency costs increase. In equilibrium lenders must be compensated for higher agency costs by a larger spread. Hence, the spread depends inversely on borrowers’ net worth. A rise in the spread causes a fall in investment and, therefore, output. An investment-efficiency shock attenuates the financial accelerator effect embedded in the model, due to the rise in the price of capital which, on one hand, leads to a fall in investment and, on the other, implies an increase in the net worth of firms. As a result, the spread decreases mitigating the impact of the contractionary shock. The monetary policy shock causes the standard transmission mechanism (Smets and Wouters, 2007), plus the financial accelerator effect stemming from the decline in the net worth of firms. This effect further reinforces the simulated contraction. In the SWGK, there are two “financial shocks”: a shock to the net worth of financial intermediaries and a shock to the quality of capital. They both affect the financial intermediaries balance sheet. Financial intermediaries cannot be over-leveraged because of the incentive compatibility constraint arising from the presence of asymmetric information. They are forced to cut back lending and, to restore profits, they increase the lending rate more than the increase in the policy rate. As a result, the spread increases; this causes a further decline in loans and investment. The monetary policy shock determines a reduction in investment and, therefore, in the demand for loans. This implies a deterioration in the balance sheet of financial intermediaries which leads to a rise in the spread. The increase in financing costs makes lending more expensive and reduces
the demand for loans, further squeezing investment. Financial frictions, therefore, exacerbate
the simulated crisis.

Finally, the two models are compared in their ability to forecast inflation in terms of mean
squared forecast error in a Phillips curve specification with either the flexible-price measure
of output gap or the measure of the spread.

The structure of the paper is as follows. Section 2 illustrates the two models. Section 3
describes the data and discusses the estimation strategy. Section 4 compares the two estimated
models. Section 5 presents robustness checks. Section 6 investigates the predictive power of
the two models in gauging inflationary pressures in the EA. Finally, Section 7 concludes.

2 The Models

This section presents the two DSGE models. Compared to the SW economy, the different
features are: (i) the presence of financial frictions; (ii) non-separability over consumption
and leisure in a standard utility function (e.g. Smets and Wouters, 2003; Gertler and Karadi,
2011); (iii) internal habits in consumption; (iv) the Dixit-Stiglitz aggregator for final output
and composite labour, as in Galí et al. (2011); (v) the price mark-up, wage mark-up and
government shocks are modelled as in Smets and Wouters (2003), the risk-premium shock is
absent whereas financial shocks are added in the models. The presence of financial frictions
changes the production side of the SW economy since intermediate goods firms are involved
in the decision of borrowing in addition to the standard profit maximisation activity. In order
to simplify the optimisation problems of intermediate goods firms, in both models retailers
are the source of price stickiness similarly to Bernanke et al. (1999) and Gertler and Karadi
(2011). For the rest the two models follow SW.

In both models the economy is populated by: households; labour unions; labour packers;
retailers; final good firms; the policy maker; intermediate goods firms; and capital producers.
In the SWGK model the economy is also populated by financial intermediaries (FI, henceforth).
While the set-up of households, labour unions, labour packers, retailers, final goods firms, and
the policy maker is the same among the two models, the rest of the production sector and the
financial sector differ.

Households consume, save, and supply labour. A labour union differentiates labour and
sets wages in a monopolistically competitive market. Competitive labour packers buy labour
service from the union, package and sell it to intermediate goods firms. The good market has
a similar structure: retailers buy goods from intermediate goods firms, differentiate them and
sell them in a monopolistically competitive market. The aggregate final good is produced by
perfectly competitive firms assembling a continuum of intermediate goods. The policy maker
sets the nominal interest rate following a Taylor rule.

In the SWBGG model, intermediate goods firms maximize the flow of discounted profits
by choosing the quantity of factors for production and stipulate a financial contracts to ob-
tain funds from lenders. For the latter decision there is a costly state verification problem
(Townsend, 1979) and lenders must pay a fixed auditing cost to observe an individual bor-
rower’s return. FI are just a “veil” in the model. Capital producers purchase investment and
depreciated capital to transform them into capital sold to intermediate goods firms and used
for production. They face adjustment costs for investment.

In the SWGK model, the production sector is also made of intermediate goods firms and
capital producers. The intermediate goods firms finance their capital acquisitions each period
by obtaining funds from the FI; there are no financial frictions in this activity. They maximise
profits by choosing the quantity of factors for production. Capital producers buy capital from
intermediate goods firms and then repair depreciated capital and builds new capital. They
then sell both the new and re-furbished capital. In addition, the supply-side of the credit
market, i.e. FI, is explicitly modelled. In particular, FI lend funds to intermediate goods
firms earning a stochastic return and pay to households a non-contingent real gross return on
liabilities. There is imperfect information between depositors/households and FI.

2.1 Households

The economy is populated by a continuum of households indexed by \( j \in (0, 1) \). Each house-
hold’s preferences are represented by the following intertemporal utility function:

\[
U_0(j) = E_t \sum_{t=0}^{\infty} \beta^t [U(C_t(j), 1 - L_t(j))]
\]  

(1)

where \( \beta \in (0, 1) \) is the discount factor and \( L_t(j) \) is labour supply in terms of hours worked.
Total time available to households is normalized to unity, thus \( 1 - L_t(j) \) represents leisure
time. The SWGK model assumes a specific structure for the households briefly presented in
Subsection 2.6.

Each period the representative household enters period \( t \) with real deposits in the FI and
real government bonds. Both intermediary deposits and government debt are one period real
bonds that pay the gross real interest rate, \( R_t \), between \( t - 1 \) and \( t \). As in Gertler and Karadi
(2011), both instruments are riskless and are thus perfect substitutes. During period \( t \), each
household chooses to consume \( C_t(j) \); supplies \( L_t(j) \) hours of work; and allocates savings in
deposits at the FI and in government bonds \( B_{t+1}(j) \). Each household gains an hourly real
wage, \( W^h_t(j)/P_t \); and dividend payments, \( \int_0^1 \Omega_{bd} \, db \), from bankers. In addition, the government
grants transfers \( TR_t \) and imposes real lump-sum taxes \( T_t \). The household’s intertemporal
budget constraint can thus be expressed as:

\[
C_t(j) + B_{t+1}(j) \leq \frac{W^h_t(j)}{P_t} L_t(j) + R_t B_t(j) + \int_0^1 \Omega_{bd} \, db + TR_t - T_t
\]  

(2)
Maximization yields the following first-order conditions with respect to \( C(j), B_{t+1}(j) \) and \( L_t(j) \):

\[
U_{Ct}(j) = m u_t(j) \quad (3)
\]

\[
\beta E_t[R_{t+1}m u_{t+1}(j)] = m u_t(j) \quad (4)
\]

\[
-U_{L_t}(j) = m u_t(j) \frac{W_t^h(j)}{P_t} \Leftrightarrow \frac{U_{L_t}(j)}{U_{Ct}(j)} = -\frac{M R S_t}{P_t} \quad (5)
\]

where \( m u_t(j) \) is the Lagrange multiplier associated to the budget constraint and let \( \Lambda_{t,t+1} \equiv \frac{m u_{t+1}}{m u_t} \).

### 2.2 Wage stickiness

Households supply homogeneous labour to monopolistic labour unions which differentiate it. Labour service used by intermediate goods firms is a composite of differentiated types of labour indexed by \( l \in (0, 1) \):

\[
L_t = \left[ \int_0^1 L_t(l) \frac{\varepsilon_w}{\varepsilon_w - 1} dl \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \quad (6)
\]

where \( \varepsilon_w \) is the elasticity of substitution across different types of labour. Labour packers solve the problem of choosing the varieties of labour to minimise the cost of producing a given amount of the aggregate labour index, taking each nominal wage rate \( W_t(l) \) as given:

\[
\min_{L_t(l)} \int_0^1 W_t(l) L_t(l) dl \quad (7)
\]

s.t. \( \int_0^1 L_t(l) \frac{\varepsilon_w}{\varepsilon_w - 1} dl \frac{\varepsilon_w}{\varepsilon_w - 1} \geq \bar{L} \quad (8) \)

The demand for labour is given by:

\[
L_t(l) = \left( \frac{W_t(l)}{W_t} \right)^{-\varepsilon_w} L_t \quad (9)
\]

where \( W_t \) is the aggregate wage index. Equations (9) and (6) imply:

\[
W_t = \left[ \int_0^1 W_t(l)^{1-\varepsilon_w} dl \right]^{\frac{1}{1-\varepsilon_w}} \quad (10)
\]

Labour unions adjust wages infrequently following the Calvo scheme. Let \( \sigma_w \) be the probability of keeping wages constant and \( (1 - \sigma_w) \) the probability of changing wages. In other words, each period there is a constant probability \( (1 - \sigma_w) \) that the union is able to adjust the wage, independently of past history. This implies that the fraction of unions setting wages at \( t \) is \( (1 - \sigma_w) \). For the other fraction that cannot adjust, the wage is automatically in-
creased at the aggregate inflation rate. As explained by Cantore et al. (2010), the wage for non-optimising unions evolves according to the following trajectory $W^*_t(l)$, $W^*_t(l) \left( \frac{P_t}{P_{t-1}} \right)^{\sigma_{wi}}$, $W^*_t(l) \left( \frac{P_{t+1}}{P_{t-1}} \right)^{\sigma_{wi}}$, ..., where $\sigma_{wi}$ denotes the degree of wage indexation.

The union chooses $W^*_t$ to maximise:

$$E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}(\beta \sigma_w)^s \left[ L_{t+s}(l) W^*_t(l) \left( \frac{P_{t+s-1}}{P_{t-1}} \right)^{\sigma_{wi}} - L_{t+s}(l) W^*_{t+s} \right]$$

subject to the labour demand (9), and the indexation scheme so that $L_{t+s}(l) = \left[ W^*_t(l) \left( \frac{P_{t+s-1}}{P_{t-1}} \right)^{\sigma_{wi}} \right]^{-\varepsilon_w} L_{t+s}$. The first order condition is:

$$E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}(\beta \sigma_w)^s L_{t+s}(l) \left[ W^*_t \left( \frac{P_{t+s-1}}{P_{t-1}} \right)^{\sigma_{wi}} - W^*_{t+s} M_{w,t} \right] = 0$$

where $M_{w,t} = \frac{\varepsilon_w}{\varepsilon_w + u_t^w}$ is the time varying gross wage mark-up and $u_t^w$ is the wage mark-up shock which follows an autoregressive (AR) (1) process, $\rho_w$ is an autoregressive coefficient and $\varepsilon_t^{w_{no}}$ is a serially uncorrelated, normally distributed shock with zero mean and standard deviation $\sigma_{w_{wm}}$. The dynamics of the aggregate wage index is:

$$W_{t+1} = \left[ (1 - \sigma_w) (W^*_{t+1}(l))^{1-\varepsilon_w} + \sigma_w \left( W_t \left( \frac{P_t}{P_{t+1}} \right)^{\sigma_{wi}} \right)^{1-\varepsilon_w} \right]^{1\over 1-\varepsilon_w}$$

2.3 Price stickiness

Competitive final goods firms buy intermediate goods from the retailers and assemble them. Final output is a composite of intermediate goods indexed by $f \in (0,1)$ differentiated by retailers:

$$Y_t = \left[ \int_0^1 Y_t(f) \frac{\varepsilon-1}{\varepsilon} df \right]^{\varepsilon-1\over \varepsilon-1}$$

where $\varepsilon$ is the elasticity of substitution across varieties of goods. Final goods firms solve the problem of choosing $Y_t(f)$ to minimise the cost of production:

$$\min_{Y_t(f)} \int_0^1 P_t(f) Y_t(f) df$$

$$\text{st} \left[ \int_0^1 Y_t(f) \frac{\varepsilon-1}{\varepsilon} df \right]^{\varepsilon-1\over \varepsilon} \geq \bar{Y}$$

The demand function for intermediate good $f$ is given by:

$$Y_t(f) = \left( \frac{P_t(f)}{P_t} \right)^{-\varepsilon} Y_t$$
where $P_t$ is the aggregate wage index. Equations (17) and (14) imply:

$$P_t = \left[ \int_0^1 P_t(f)^{1-\varepsilon} \, df \right]^{1/\varepsilon}$$  \hspace{1cm} (18)

Retailers simply purchase intermediate goods at a price equal to the marginal cost and differentiate them in a monopolistically competitive market, similarly to labour unions in the labour market. Retailers set nominal prices in a staggered fashion à la Calvo (1983). Each retailer resets its price with probability $1 - \sigma_p$. For the fraction of retailers that cannot adjust, the price is automatically increased at the aggregate inflation rate. As explained by Cantore et al. (2010) the price for non-optimising retailers evolves according to the following trajectory $P_t^* (f), P_t^* (f) \left( \frac{P_t}{P_{t-1}} \right)^{\sigma_{pi}}, P_t^* (f) \left( \frac{P_{t+1}}{P_{t-1}} \right)^{\sigma_{pi}}, ...$, where $\sigma_{pi}$ denotes the degree of price indexation. The real price $\Phi_t$ charged by intermediate goods firms in the competitive market represents also the real marginal cost common to all final good firms, i.e. $MC_t = \Phi_t$.

A retailer resetting its price in period $t$ maximises the following flow of discounted profits with respect to $P_t^*$:

$$E_t \sum_{s=0}^{\infty} (\sigma_{p/\beta})^s \Lambda_{t+s} Y_{t+s}(f) P_t^*(f) \left( \frac{P_{t+s-1}}{P_{t-1}} \right)^{\sigma_{pi}} - Y_{t+s}(f) P_{t+s} MC_{t+s}$$  \hspace{1cm} (19)

subject to the demand function (17), and the indexation scheme so that $Y_{t+s}(f) = \left[ \frac{P_t^*(f)}{P_{t+s} \left( \frac{P_{t+s}}{P_{t-1}} \right)^{\sigma_{pi}}} \right]^{-\varepsilon}$ $Y_{t+s}$. Let $MC^n_t$ denote the nominal marginal cost. The gross mark-up charged by final good firm $f$ can be defined as $M_t(f) \equiv P_t(f)/MC^n_t = \frac{P_t(f)}{P_t} / \frac{MC^n_t}{P_t} = p_t(f)/MC_t$. In the symmetric equilibrium all final good firms charge the same price, $P_{t}(f) = P_t$, hence the relative price is unity. It follows that, in the symmetric equilibrium, the mark-up is simply the inverse of the marginal cost.

The first order condition for this problem is:

$$E_t \sum_{s=0}^{\infty} (\sigma_{p/\beta})^s \Lambda_{t+s} Y_{t+s}(f) \left[ P_t^*(f) \left( \frac{P_{t+s-1}}{P_{t-1}} \right)^{\sigma_{pi}} - M_{p,t} P_{t+s} MC_{t+s} \right] = 0$$  \hspace{1cm} (20)

Similarly to the labour market, the gross time varying price mark up is $M_{p,t} = \frac{\varepsilon_{t}}{\varepsilon - 1} u_{t}^p$ and $u_{t}^p$ is the price mark-up shock, which follows an AR(1) process, $\rho_p$ is an autoregressive coefficient and $\varepsilon_{t}^{pm}$ is a serially uncorrelated, normally distributed shock with zero mean and standard deviation $\sigma_{pm}$.

The equation describing the dynamics for the aggregate price level is given by:

$$P_{t+1} = \left[ (1 - \sigma_p)(P_{t+1}^*(f))^{1-\varepsilon} + \sigma_{p} \left( \frac{P_t}{P_{t-1}} \right)^{\sigma_{pi}} \right]^{1/(1-\varepsilon)}$$  \hspace{1cm} (21)
### 2.4 Policymaker

The policymaker sets the nominal interest rate according to the following Taylor rule:

$$
\frac{R_n^t}{R_n^{t-1}} = \left( \frac{\Pi_t}{\Pi_t^{t-1}} \right)^{\rho_i} \left[ \left( \frac{\Pi_t}{\Pi_t^{t-1}} \right) \left( \frac{Y_t}{Y_t^*} \right)^{\rho_y} \right]^{1-\rho_i} \left( \frac{\Pi_t}{\Pi_t^{t-1}} \right)^{\rho_y \Delta_y} \left( \frac{Y_t/\Pi_t}{Y_t^*/\Pi_t^{t-1}} \right) \exp(\varepsilon_t^r) \tag{22}
$$

and:

$$
R_{t+1} = E_t \left[ \frac{R_n^t}{\Pi_t} \right], \tag{23}
$$

where $R_n^t$ is the nominal gross interest rate, $Y_t^*$ is the level of output that would prevail under flexible prices and wages without the two mark-up shocks, and $\varepsilon_t^r$ is the monetary policy shock. The meanings of the parameters of equation (22) are standard (Smets and Wouters, 2003).

### 2.5 Production and financial sector in the SWBGG model

The presence of financial frictions in the demand side of the credit market alters the set-up of intermediate goods firms compared to the SW economy. This section also presents the set-up of capital producers.

#### 2.5.1 Capital producers

Following Gelain (2010), capital producers purchase at time $t$ investment and depreciated capital to transform them into capital sold to firms and used for production at time $t+1$. Capital producers face adjustment costs for investment as in Christiano et al. (2005). The law of motion of capital is then equal to:

$$
K_{t+1} = (1 - \delta)K_t + x_t \left[ 1 - F \left( \frac{I_t}{I_{t-1}} \right) \right] I_t \tag{24}
$$

where $\delta$ stands for depreciation. The adjustment cost function $F$ satisfies the following properties: $F(1) = F'(1) = 0$, and $F''(1) = \xi > 0$. The exogenous shock $x_t$ follows an AR(1) process, $\rho_x$ is an autoregressive coefficient and $\varepsilon_t^x$ is a serially uncorrelated, normally distributed shock with zero mean and standard deviation $\sigma_x$. The shock to the marginal efficiency of investment varies the efficiency with which the final good can be transformed into physical capital and it affects net worth of intermediate goods firms through changes in the price of capital.

The profits are given by the difference between the revenue from selling capital at the relative price $Q_t$ and the costs of buying capital from intermediate goods firms and the investment needed to build new capital. The optimality condition is a Tobin’s $Q$ equation, which relates...
the price of capital to the marginal adjustment cost:

\[ 1 = Q_t x_t \left[ 1 - F \left( \frac{I_t}{I_{t-1}} \right) - F' \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{I_{t-1}}{I_t} \right) \right] + \beta E_t \left[ \Lambda_{t,t+1} Q_{t+1} x_{t+1} F' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right] \]  
(25)

### 2.5.2 Intermediate goods firms

Intermediate goods firms produce goods in a perfectly competitive market and they borrow in order to finance the acquisition of capital. They solve two optimization problems: (i) they maximize the flow of discounted profits by choosing the quantity of factors for production; and (ii) they stipulate a financial contract to obtain funds from lenders. The first problem consists in maximising the following flow of profits:

\[ E_t \beta \Lambda_{t,t+1} \left[ \Phi_t Y_{t+1} - Z^k_{t+1} K_{t+1} - \frac{W_{t+1}}{P_t} L_{t+1} \right] \]  
(26)

where \( \Phi_t \) is the competitive real price at which intermediate good is sold and \( Z^k_t \) is the real rental price of capital. Maximisation yields the following first order conditions with respect to capital and labour:

\[ Z^k_t = MC_t M^K_t \]  
(27)

\[ W_t \frac{P_t}{P_t} = MC_t M^L_t \]  
(28)

where \( M^K_t \) is the marginal product of capital and \( M^L_t \) is the marginal product of labour. The real price \( \Phi_t \) represents the shadow value of output and hence, given perfect competition in the market, it also represents its real marginal cost, \( MC_t \).

Following Gelain (2010), firms also decide the optimal capital utilization rate solving the following maximisation problem:

\[ \max_{U_t} Z^k_t U_t K_{t-1} - \Psi(U_t) K_{t-1} \]  
(29)

where \( \Psi(U_t) \) represent the costs of changing capital utilization, with \( \zeta = \Psi''(U_t)/\Psi'(U_t) \). This optimization problem is summarized by the following equilibrium condition:

\[ Z^k_t = \Psi'(U_t) \]  
(30)

Intermediate goods firms face also the problem of stipulating the financial contract. In order to ensure that entrepreneurial net worth will never be enough to fully finance capital acquisitions, it is assumed that each firm survives until the next period with probability \( \theta \) and her expected lifetime is consequently \( 1/(1 - \theta) \). At the same time, the new firms entering
receive a transfer, $N_t^e$, from firms who die and depart from the scene.\footnote{Following Christensen and Dib (2008) consumption of exiting firms, a small fraction of total consumption, is ignored in the general equilibrium.} This transfer ensures that new firms have at least a small but positive amount of net worth so that they can buy capital. At the end of period $t$, firms buy capital $K_{t+1}$ that will be used throughout time $t+1$ at the real price $Q_t$. The cost of purchased capital is then $Q_t K_{t+1}$. A fraction of capital acquisition is financed by their net worth, $N_{t+1}$, and the remainder by borrowing from a financial intermediary that obtains funds from household deposits and faces an opportunity cost equal to the risk-free rate, $R_t$. Total amount of lending is:

$$loan_{t+1} = Q_t K_{t+1} - N_{t+1}$$ (31)

In equilibrium the optimal capital demand is:

$$E_t \left[ R^k_{t+1} \right] = E_t \left[ \frac{Z^k_{t+1} + (1 - \delta) Q_{t+1}}{Q_t} \right]$$ (32)

The expected marginal external financing cost, $E_t \left[ R^k_{t+1} \right]$, is equal to the expected marginal return on capital given by the marginal productivity of capital and the value of one unit of capital used in time $t+1$.

BGG assume that an agency problem makes external finance more expensive than internal funds and solve a financial contract that maximises the payoff to the firms subject to the lender earning the required rate of return. Firms are risk neutral while households are risk averse; according to the financial contract the firms absorb any aggregate risk. Following Townsend (1979), lenders must pay a fixed auditing cost to observe an individual borrower’s return. The monitoring cost is a proportion of the realized gross payoff to the firm’s capital. The financial contract implies an external finance premium $EP(\cdot)$, i.e. the difference between the cost of external and internal funds, that depends on the inverse of the firm’s leverage ratio.\footnote{See BGG for the derivation of the financial contract and for the aggregation.} Hence, in equilibrium, the marginal external financing cost must equate the external finance premium gross of the riskless real interest rate:

$$R^k_{t+1} = \left[ EP \left( \frac{N_{t+1}}{Q_t K_{t+1}} \right) \right] R_{t+1}$$ (33)

with $EP(\cdot) < 0$ and $EP'(1) = 1$. As the borrower’s equity stake in a project $N_{t+1}/Q_t K_{t+1}$ falls, i.e. the leverage ratio rises, the loan becomes riskier and the cost of borrowing rises. Linearisation of equation (33) yields:\footnote{A variable with a `hat’ denotes a percentage deviation from steady state.}

$$R^k_{t+1} = \left[ EP \left( \frac{\hat{N}_{t+1}}{Q_t \hat{K}_{t+1}} \right) \right] R_{t+1}$$
where $\kappa \equiv -\frac{\partial R_k}{\partial N} \frac{N}{K} R$ measures the elasticity of the external finance premium with respect to the leverage position of intermediate goods firms.

Aggregate entrepreneurial net worth evolves according to the following law of motion:

$$N_{t+1} = \left\{ \theta \left[ R_k^t Q_{t-1} K_t - E_{t-1} \left[ R_k^t (Q_{t-1} K_t - N_t) \right] \right] + (1 - \theta) N_t^e \right\} \exp(\varepsilon_{t}^n) \quad (35)$$

where the first component of the right-hand-side represents the net worth of the $\theta$ fraction of surviving entrepreneurs net of borrowing costs carried over from the previous period, and $N_t^e$ is the transfer that newly entering entrepreneurs receive. The term $\varepsilon_{t}^n$ represents a shock to the firm’s net worth with zero mean and standard deviation $\sigma_n$.

The resource constraint completes the model:

$$Y_t = C_t + I_t + G_t + \Psi(U_t) K_{t-1} \quad (36)$$

Following BGG and Gabriel et al. (2010), monitoring costs are ignored since, under reasonable parameterisations, they have negligible impact on model’s dynamics.

2.6 Production and financial sector in the SWGK model

The presence of financial frictions à la Gertler and Karadi affects the optimisation problem of intermediates goods firms. In addition this section presents the set up of financial intermediaries and capital producers.

2.7 Financial intermediaries

Within each household there are two types of members at any point in time: the fraction $g$ of the household members are workers and the fraction $(1 - g)$ are bankers. The turnover between bankers and workers is as follows: every banker stays banker next period with a probability $\theta$, which is independent of history. Therefore, every period $(1 - \theta)$ bankers exit and become workers. Similarly, a number of workers become bankers, keeping the relative proportion of each type of agents constant. The household provides her new banker with a start-up transfer, which is a small fraction of total assets, $\chi$. Each banker manages a financial intermediary. The FI have a finite horizon in order to avoid the possibility that they can reach the point where they can fund all investment from their own capital. The set up of financial intermediaries follows Gertler and Karadi (2011), but some equilibrium conditions are reported here to facilitate the interpretation of parameters in Subsection 3.1 and of the impulse responses in Subsection 4.3.
The FI’s balance sheet is simply:

\[ Q_t S_t = N_t + B_{t+1} \] (37)

where \( B_t \) stands for deposits, \( N_t \) is FI capital (or net worth), \( S_t \) is the quantity of financial claims on intermediate goods firms and \( Q_t \) is the price of each claim.

The problem of moral hazard consists in the fact that the banker can choose to divert the fraction \( \lambda \) of available funds from the project and transfer them back to her household. The depositors require to be willing to supply funds to the banker that the gains from diverting assets should be less or equal than the costs of doing so:

\[ \Upsilon_t \geq \lambda Q_t S_t \] (38)

where \( \Upsilon_t \) is the expected terminal wealth, defined as:

\[ \Upsilon_t = E_t \sum_{s=0}^{\infty} (1 - \theta)^s \beta^{s+1} \Lambda_{t,t+1+s} \left[ \left( \left( R_{t+1+s}^k - R_{t+1+s} \right) Q_{t+s} S_{t+s} + R_{t+1+s} N_{t+s} \right) \right] \] (39)

Equation (38) translates in the following constraint for the FI:

\[ Q_t S_t = \text{lev}_t N_t \] (40)

where \( \text{lev}_t \) stands for the FI leverage ratio. The agency problem introduces an endogenous capital constraint on the FI’s ability to expand assets.

Total net worth is the sum of net worth of existing bankers, \( N^e_t \), and net worth of new bankers, \( N^n_t \). As far as the first is concerned, net worth evolves as:

\[ N_{t+1}^e = \theta \left( \left( R_{t+1}^k - R_{t+1} \right) \text{lev}_t + R_{t+1} \right) N_t \exp(\varepsilon_{t}^n) \] (41)

where \( \varepsilon_{t}^n \) is a shock to FI net worth. Net worth of new bankers is a small fraction of total assets:

\[ N_{t}^n = \chi Q_t S_t \] (42)

2.7.1 Capital producers

At the end of period \( t \), competitive capital producing firms buy capital from intermediate goods firms and then repair depreciated capital and builds new capital. They then sell both the new and re-furbished capital. The value of a unit of new capital is \( Q_t \). Their profit is the difference between the revenue from selling the net capital and the costs of buying capital from intermediate firms and the investment needed to build new capital. The law of motion
of capital is equal to:

\[ K_{t+1} = [1 - \delta(U_t)] \psi_t K_t + \left[ 1 - F \left( \frac{I_t}{I_{t-1}} \right) \right] I_t \] (43)

where the depreciation function \( \delta(U_t) \) has the following properties: \( \delta'(U_t) > 0 \), and \( \delta''(U_t) > 0 \) and \( \zeta = \delta''(U_t) / \delta'(U_t) \). The shock to the quality of capital, \( \psi_t \), follows an AR(1) process \( \rho_k \) is an autoregressive coefficient and \( \varepsilon^k_t \) is a serially uncorrelated, normally distributed shock with zero mean and standard deviation \( \sigma_k \). The solution to this optimisation problem yields the following equation:

\[ 1 = Q_t \left[ 1 - F \left( \frac{I_t}{I_{t-1}} \right) - F' \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{I_t}{I_{t-1}} \right) \right] + \beta E_t \left[ \Lambda_{t,t+1} Q_{t+1} F' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right] \] (44)

### 2.7.2 Intermediate goods firms

Intermediate goods firms maximize profits in a perfectly competitive market and borrow from FI. At the end of period \( t \) they acquire capital from capital producers for use in production in subsequent period. After production in period \( t + 1 \), they can sell capital to capital producers without adjustment costs at the firm level. Total revenue is given by the output produced and the capital sold to the capital producers (which is subject to a shock). After production, the number of unit of capital left over is \( (Q_{t+1} - \delta(U_{t+1})) \psi_{t+1} K_{t+1} \). Total cost is the expenditure for factors for production. The profit function is given by:

\[ E_t \sum_{s=0}^{\infty} \beta^s A_{t,t+s} \left[ \Phi_{t+s} Y_{t+s} + (Q_{t+s} - \delta(U_{t+s})) \psi_{t+s} K_{t+s} - R^k_{t+s} Q_{t-1+s} K_{t+s} - \frac{W_{t+s}}{P_{t+s}} L_{t+s} \right] \] (45)

Profit maximization yields the following first-order conditions with respect to capital, labour and utilization rate:

\[ R^k_t Q_{t-1} = MC_t MP^K_t + [Q_t - \delta(U_t)] \psi_t \] (46)

\[ MC_t MP^L_t = \frac{W_t}{P_t} \] (47)

\[ \delta'(U_t) \psi_t K_t = MC_t MP^U_t \] (48)

where \( MP^K_t \) is the derivative of the production function with respect to capital. Each intermediate goods firm finances the acquisition of capital, \( K_{t+1} \), by obtaining funds from the FI. The firm issues \( S_t \) state-contingent claims equal to the number of units of capital acquired and prices each claim at the price of a unit of capital \( Q_t \):

\[ Q_t K_{t+1} = Q_t S_t \] (49)
2.8 Functional forms

Following Gertler and Karadi (2011), the utility function specialises as:

\[ U_t(\cdot) = \ln (C_t - hC_{t-1}) - \frac{L_t^{1+\phi}}{1+\phi} \]  

(50)

where \( h \) measures the degree of superficial internal habits in consumption and \( \phi \) is the inverse of the Frisch elasticity of labour supply.

The production function slightly differs among the two models because of the presence of the shock to the quality of capital, \( \psi_t \), in the SWGK model as shown in equation (52), absent in the SWBGG model, equation (51):

\[ Y_t = A_t(U_tK_t)^{\alpha}L_t^{1-\alpha} - \Theta \]  

(51)

\[ Y_t = A_t(U_tK_t\psi_t)^{\alpha}L_t^{1-\alpha} - \Theta \]  

(52)

As in Smets and Wouters (2007), the parameter \( \Theta \) represents fixed costs in production. \( A_t \) is the transitory technology shock following an AR(1) process, \( \rho_a \) is an autoregressive coefficient and \( \varepsilon^{\phi}_t \) is a serially uncorrelated, normally distributed shock with zero mean and standard deviation \( \sigma_a \).

3 Data and estimation strategy

In the both models there are seven orthogonal structural shocks. The following shocks are common to both models: the monetary policy, \( \varepsilon^r_t \); the technology, \( \varepsilon^n_t \); the government, \( \varepsilon^n_g \); the price mark-up, \( \varepsilon^{pm}_t \); and the wage mark-up, \( \varepsilon^{wm}_t \), shocks. Since the financial sectors differ across the two models, the two financial shocks differ as well. In the SWBGG model there are an investment-specific shock, \( \varepsilon^x_t \), and a shock to borrowers’ net worth, \( \varepsilon^n_t \). In the SWGK models there are a shock to the quality of capital, \( \varepsilon^{\psi}_t \), and a shock to FT’ net worth, also labelled \( \varepsilon^{\phi}_t \). The addition of “financial shocks” can be justified by the results of Jermann and Quadrini (2009), who find that this addition brings the model much closer to the data. In each model, the shocks follow an AR(1) process, but the shocks to the monetary policy and to net worth.

The two models, SWBGG and SWGK, are estimated with quarterly EA data for the period 1996Q1-2008Q3 using as observables real GDP, real investment, real private consumption, employment, GDP deflator inflation, real wage and the spread. The short sample is dictated by the availability of the data for the spread series. The final quarter corresponds to the pre-crisis period: the collapse of the Lehman Brothers in September 2008 has been used as characterizing the crisis period, e.g. Lenza et al. (2010) and Giannone et al. (2011).
come from the Area Wide Model database (Fagan et al., 2005, see) but the spread, which is taken from the ECB database. The spread is computed as the difference between the yield on BBB corporate bonds and government AAA bonds. Section 5 explores the robustness of the results to a different measure of the spread. Following Smets and Wouters (2003), all real variables are detrended by a linear trend. GDP, investment, consumption and wage are logged and first-differenced. The inflation rate is measured as a quarterly log-difference of GDP deflator and demeaned. Data on the spread are demeaned and then divided by 100 to make the units compatible with the log-first-differenced data. Data on employment are used since there are no data available for hours worked in the Euro Area. As in Smets and Wouters (2003) a Calvo-type of adjustment is assumed for employment and hours worked:

\[
\hat{E}_t = \frac{1}{1 + \beta} \hat{E}_{t-1} + \frac{\beta}{1 + \beta} E_t \left[ \hat{E}_{t+1} - \frac{(1 - \beta \sigma_E)(1 - \sigma_E)}{(1 + \beta)\sigma_E} \left( \hat{L}_t - \hat{E}_t \right) \right]
\]

where \( E_t \) is employment and \( 1 - \sigma_E \) represents the fraction of firms that can adjust the level of employment to the preferred amount of total labor input. Data on employment are logged and detrended since there is an upward trend in the employment series for the Euro area and hours worked and employment are stationary variables in the model. Transformed data are shown in Figure 1.

The solution of the rational expectations system takes the form:

\[
s_t = A s_{t-1} + B \eta_t \tag{53}
\]

\[
o_t = C s_t + D u_t \tag{54}
\]

\[\eta_t \sim N(0, \Omega) \quad \text{and} \quad u_t \sim N(0, \Phi)\]

where \( s_t \) is a vector containing the model’s variables expressed as log-deviation from their steady-state values. It includes not only endogenous variables but also the exogenous processes. Vector \( \eta_t \) contains white noise innovations to the shocks. Matrices \( A \) and \( B \) are functions of the structural parameters of the DSGE model; \( o_t \) is the vector of observables and \( u_t \) is a set of shocks to the observables (like measurement errors).

As far as the Bayesian estimation procedure is concerned, the likelihood function and the prior distributions are combined to approximate a posterior mode, which is used as the starting value of a Random Walk Metropolis algorithm (RWMA).\(^4\) This Markov Chain Monte Carlo (MCMC) method generates draws from the posterior density and updates the candidate parameter after each draw (see An and Schorfheide, 2007; Fernández-Villaverde, 2009, for details).

\(^4\)Version 4.2.1 of the Dynare toolbox for Matlab is used for the computations.
3.1 Calibration and priors

The parameters which cannot be identified in the dataset and/or are related to steady state values of the variables are calibrated, following a standard procedure (Christiano et al., 2010; Fragetta and Kirsanova, 2010, among others). The time period in the model corresponds to one quarter in the data.

Table 1 shows the calibration of the parameters common to both models. The discount factor, $\beta$, is equal to $0.99$, implying a quarterly steady state real interest rate of $1\%$; the capital income share, $\alpha$, is equal to $0.33$, implying a steady state labour income share of two third. The depreciation rate is equal to $0.025$, corresponding to an annual depreciation rate of $10\%$. The ratio of government spending to GDP is equal to $0.22$. The elasticities of substitution in goods and labour markets are equal to $6$ in order to target a gross steady state mark up of $1.20$, as in Christiano et al. (2010) and Gelain (2010), among others. The parameter $\theta$ represents the survival rate of intermediate goods firms in the SWBGG model and of FI in the SWGK model. This parameter is set equal to $0.9715$ implying an expected working life for bankers and firms of almost a decade; this value is consistent with both BGG and Gertler and Karadi (2011).

The calibration of the financial parameters is shown in Table 2. In the SWBGG model,
Table 1: Calibration of parameters common to both models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$, discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\alpha$, capital income share</td>
<td>0.3</td>
</tr>
<tr>
<td>$\delta$, depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\gamma$, government spending to GDP ratio</td>
<td>0.22</td>
</tr>
<tr>
<td>$\varepsilon$, elasticity of substitution in good market</td>
<td>set to target $M = 1.20$</td>
</tr>
<tr>
<td>$\varepsilon_w$, elasticity of substitution in labour market</td>
<td>set to target $M^w = 1.20$</td>
</tr>
<tr>
<td>$\theta$, survival rate</td>
<td>0.9715</td>
</tr>
</tbody>
</table>

Table 2: Calibration of model-specific parameters

<table>
<thead>
<tr>
<th>Financial Parameters</th>
<th>SWBGG Model</th>
<th>SWGK Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$, steady state spread</td>
<td>1.003125</td>
<td>–</td>
</tr>
<tr>
<td>$\frac{\kappa}{N}$, leverage ratio</td>
<td>2</td>
<td>5 (implied by $\chi$ and $\lambda$)</td>
</tr>
<tr>
<td>$\chi$, fraction of assets given to the new bankers</td>
<td>–</td>
<td>0.0007</td>
</tr>
<tr>
<td>$\lambda$, fraction of divertable assets</td>
<td>–</td>
<td>0.4255</td>
</tr>
</tbody>
</table>

The parameter pinning down the steady state spread, $S$, is set equal to 1.003125 to match the steady state spread in the dataset of 125 basis points. Following BGG, Christensen and Dib (2008) and Gelain (2010), the ratio of capital to net worth is set to 2, implying that 50% of firm’s capital expenditures are externally financed. As long as the calibration of the SWGK model is concerned, the fraction of assets given to new bankers, $\chi$, and the fraction of assets that can be diverted, $\lambda$, are equal to 0.0007 and 0.4255, respectively, to target the steady state spread in the dataset and a steady state leverage ratio of 5, a value comparable to that used by Gertler and Karadi (2011). Section 5 investigates the robustness of the main results to the calibration of the financial parameters.

Table 3 shows the assumptions for the prior distributions of the estimated parameters for both models. The choice of the functional forms of parameters and the location of the prior mean correspond to a large extent to those in Smets and Wouters (2003, 2007) where applicable. In general, the Beta distribution is used for all parameters bounded between 0 and 1, the Normal distribution is used for the unbounded parameters and the Inverse Gamma (IG) distribution for the standard deviation of the shocks. The prior of some model-specific parameters are as follows. The parameter measuring the inverse of the Frisch elasticity of labour supply follows a Normal distribution with a prior mean of 0.33, the value used by Gertler and Karadi (2011). Following De Graeve (2008), the elasticity of external finance premium with respect to leverage is assumed to follow a Uniform distribution, with values in the interval (0, 0.3).
4 Model comparison

The comparison between the two models is made first by looking at the estimated parameters and the Bayes Factor. Second, the forecasting performance is discussed. Finally, impulse response functions and variance decomposition are presented.

4.1 Estimated parameters and the Bayes Factor

The mean of the estimated parameters for each model are computed with the Metropolis-Hastings algorithm with a sample of 250,000 draws (see Smets and Wouters, 2003 for further details). For each model Table 3 reports the posterior mean with 95% probability intervals in parentheses. Most parameters are remarkably similar across the two models. As in Smets and Wouters (2005), the fact that in almost all the cases the posterior estimate of a parameter in one model falls in the estimated confidence band for the same parameter of the other model can be considered as a rough measure of similarity. Nevertheless, the posterior mean of few parameters differs.

Concerning the set of parameters similar across the two models, the main findings are as follows. The degree of price stickiness reveals that firms adjust prices almost every two-and-a-half years. This value is consistent with the evidence reported by Smets and Wouters (2003). The Calvo parameter for wage stickiness reveals that the average duration of wage contracts is almost a year, considerably lower than the degree of price stickiness, as in Smets and Wouters (2003). There is a moderate degree of wage and price indexation. The elasticity of the cost of changing investment is in the region of the estimates by Smets and Wouters (2003), with a higher value in the SWGK model. There is evidence of habit in consumption, with a mean of 0.6 in both models, close to the value found by Gelain (2010). The estimates of the parameter measuring fixed costs in production and the Taylor rule parameter to inflation are also in line with previous estimates for the EA. There is strong evidence of short-term reaction to the current change in inflation, with a mean value of about 0.29. The response to the output gap level is low in both models, similarly to Gelain (2010). There is also evidence of short-term reaction to the current change in the output gap. Turning to the exogenous shock processes, all shocks are quite persistent. The mean of the standard errors of the shocks is lower than the studies of Smets and Wouters (2003) and Gelain (2010) who use data over the period 1980Q2-1999Q4 and 1980Q1-2008Q3 respectively. The more recent period used here is characterised by lower volatility, similarly to the findings of Gerali et al. (2010), who use data in the period 1998Q1-2008Q4.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior distribution</th>
<th>Posterior mean</th>
<th>Posterior mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_p$, Calvo prices</td>
<td>Beta 0.75, 0.05</td>
<td>0.903 [0.879;0.928]</td>
<td>0.887 [0.860;0.915]</td>
</tr>
<tr>
<td>$\sigma_w$, Calvo wages</td>
<td>Beta 0.75, 0.05</td>
<td>0.743 [0.687;0.801]</td>
<td>0.733 [0.674;0.794]</td>
</tr>
<tr>
<td>$\sigma_{E}$, Calvo employment</td>
<td>Beta 0.5, 0.2</td>
<td>0.797 [0.761;0.835]</td>
<td>0.814 [0.780;0.849]</td>
</tr>
<tr>
<td>$\sigma_{pi}$, price indexation</td>
<td>Beta 0.5, 0.2</td>
<td>0.083 [0.011;0.149]</td>
<td>0.147 [0.012;0.272]</td>
</tr>
<tr>
<td>$\sigma_{wi}$, wage indexation</td>
<td>Beta 0.5, 0.2</td>
<td>0.127 [0.014;0.233]</td>
<td>0.116 [0.015;0.213]</td>
</tr>
<tr>
<td>$\xi$, inv. adj. costs</td>
<td>Normal 4, 1.5</td>
<td>6.760 [5.104;8.393]</td>
<td>8.374 [6.571;10.126]</td>
</tr>
<tr>
<td>$\zeta$, elasticity of capital util</td>
<td>Normal 0.25, 0.1</td>
<td>0.779 [0.707;0.854]</td>
<td>0.244 [0.146;0.334]</td>
</tr>
<tr>
<td>$h$, habit parameter</td>
<td>Beta 0.7, 0.1</td>
<td>0.604 [0.555;0.655]</td>
<td>0.623 [0.574;0.673]</td>
</tr>
<tr>
<td>$\Theta$, fixed costs in production</td>
<td>Normal 1.25, 0.125</td>
<td>1.558 [1.407;1.706]</td>
<td>1.359 [1.154;1.519]</td>
</tr>
<tr>
<td>$\phi$, inverse of Frisch elasticity</td>
<td>Normal 0.33, 0.1</td>
<td>0.162 [0.050;0.256]</td>
<td>0.353 [0.202;0.506]</td>
</tr>
<tr>
<td>$\kappa$, elast. of external finance</td>
<td>Uniform 0, 0.3</td>
<td>0.044 [0.035;0.052]</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_{\pi}$, Taylor rule</td>
<td>Normal 1.7, 0.1</td>
<td>1.646 [1.485;1.810]</td>
<td>1.751 [1.604;1.899]</td>
</tr>
<tr>
<td>$\rho_{\Delta \pi}$, Taylor rule changes in $\pi$</td>
<td>Normal 0.3, 0.1</td>
<td>0.293 [0.139;0.452]</td>
<td>0.281 [0.127;0.439]</td>
</tr>
<tr>
<td>$\rho_{y}$, Taylor rule</td>
<td>Normal 0.125, 0.05</td>
<td>0.007 [0.005;0.010]</td>
<td>0.033 [0.015;0.059]</td>
</tr>
<tr>
<td>$\rho_{\Delta y}$, Taylor rule changes in $y$</td>
<td>Normal 0.0625, 0.05</td>
<td>0.111 [0.061;0.160]</td>
<td>0.205 [0.082;0.322]</td>
</tr>
<tr>
<td>$\rho_{x}$, Taylor rule smoothing</td>
<td>Beta 0.80, 0.2</td>
<td>0.105 [0.018;0.187]</td>
<td>0.332 [0.157;0.504]</td>
</tr>
<tr>
<td>$\rho_{a}$, persistence of tech shock</td>
<td>Beta 0.85, 0.1</td>
<td>0.804 [0.739;0.869]</td>
<td>0.771 [0.655;0.896]</td>
</tr>
<tr>
<td>$\rho_{x}$, persistence of investment shock</td>
<td>Beta 0.85, 0.1</td>
<td>0.977 [0.958;0.997]</td>
<td>-</td>
</tr>
<tr>
<td>$\rho_{k}$, persistence of capital shock</td>
<td>Beta 0.85, 0.1</td>
<td>-</td>
<td>0.452 [0.343;0.560]</td>
</tr>
<tr>
<td>$\rho_{g}$, persistence of gov shock</td>
<td>Beta 0.85, 0.1</td>
<td>0.767 [0.669;0.870]</td>
<td>0.841 [0.730;0.961]</td>
</tr>
<tr>
<td>$\rho_{p}$, persistence of price mark-up shock</td>
<td>Beta 0.85, 0.1</td>
<td>0.002 [0.448;0.767]</td>
<td>0.846 [0.771;0.924]</td>
</tr>
<tr>
<td>$\rho_{w}$, persistence of wage mark-up shock</td>
<td>Beta 0.85, 0.1</td>
<td>0.467 [0.293;0.633]</td>
<td>0.470 [0.284;0.648]</td>
</tr>
<tr>
<td>$\sigma_{a}$, std of tech shock</td>
<td>IG 0.1, 2</td>
<td>0.016 [0.013;0.019]</td>
<td>0.016 [0.013;0.019]</td>
</tr>
<tr>
<td>$\sigma_{x}$, std of investment shock</td>
<td>IG 0.1, 2</td>
<td>0.060 [0.047;0.072]</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_{k}$, std of capital quality shock</td>
<td>IG 0.1, 2</td>
<td>-</td>
<td>0.021 [0.018;0.026]</td>
</tr>
<tr>
<td>$\sigma_{i}$, std of monetary shock</td>
<td>IG 0.1, 2</td>
<td>0.018 [0.014;0.022]</td>
<td>0.020 [0.015;0.024]</td>
</tr>
<tr>
<td>$\sigma_{n}$, std of net worth shock</td>
<td>IG 0.1, 2</td>
<td>0.027 [0.020;0.033]</td>
<td>0.077 [0.058;0.095]</td>
</tr>
<tr>
<td>$\sigma_{g}$, std of gov shock</td>
<td>IG 0.1, 2</td>
<td>0.040 [0.031;0.048]</td>
<td>0.015 [0.013;0.018]</td>
</tr>
<tr>
<td>$\sigma_{pm}$, std of price mark-up shock</td>
<td>IG 0.1, 2</td>
<td>0.089 [0.032;0.146]</td>
<td>0.033 [0.021;0.044]</td>
</tr>
<tr>
<td>$\sigma_{wm}$, std of wage mark-up shock</td>
<td>IG 0.1, 2</td>
<td>0.050 [0.029;0.071]</td>
<td>0.068 [0.034;0.100]</td>
</tr>
</tbody>
</table>
The second set of parameters is made of those for which the posterior means differ. The mean of the parameter measuring the elasticity of capital utilisation is higher in the SWBGG model. This result is not surprising since the way in which this elasticity is modelled differs among the two models. While in the SWGK model the cost of capital utilisation is directly related with the increased depreciation à la King and Rebelo (2000), in the SWBGG model the capital utilisation function is modelled as in Christiano et al. (2005), who use a more general function. The SWBGG model suggests a lower response of capital utilization. The degree of interest rate smoothing is considerably low compared to the similar studies for the Euro Area, since its mean is equal to 0.105 in the SWBGG model and to 0.332 in the SWGK model versus 0.956 in Smets and Wouters (2003) and 0.845 in Gelain (2010). This result is in line with the literature dealing with the problem of weak instruments to identify the degree of monetary policy inertia. According to Consolo and Favero (2009) and Rudebusch (2002), a high degree of interest rate smoothing hardly reconciles with the low predictability of monetary policy rates. The value in the SWBGG model, however, is very low even compared to the results of Consolo and Favero (2009) for the US.

The AR coefficients of the shock processes are generally similar among the models, but for the price mark up shock which is more persistent in the SWGK model. The standard deviations of the shocks are generally lower in the SWGK model, but for the standard deviation of the FI net worth shock with a higher posterior mean in the SWGK model. Both models feature two “financial shocks”: the net worth shock is common to both models but it originates in the demand side of the credit market in the SWBGG model and in the supply-side in the SWGK model; the shock to the marginal efficiency of investment in the SWBGG model; and to the quality of capital in the SWGK model.

A third set of parameters includes those parameters which differ among the two models, such as the shock to the marginal efficiency of investment in the SWBGG model and to the quality of capital in the SWGK model, the first being more persistent. The elasticity of the external finance premium with respect to the leverage position of intermediate goods firms has a posterior mean of 0.044, similar to the value found by Gelain (2010), revealing an external premium reactive to the firms’ leverage position.

Another dimension along which the two estimated models is compared is the Bayes Factor, as in An and Schorfheide (2007) and Levine et al. (2010), among many others. Such a comparison is based on the marginal likelihood of alternative models. Let \( m_i \) be a given model, with \( m_i \in M \), \( \theta \) the parameter vector and \( p_i(\theta|m_i) \) the prior density for model \( m_i \). The marginal likelihood for a given model \( m_i \) and common dataset \( Y \) is:

\[
L(Y|m_i) = \int_\theta L(Y|\theta,m_i)p_i(\theta|m_i)d\theta
\]

where \( L(Y|\theta,m_i) \) is the likelihood function for the observed data \( Y \) conditional on the param-
eter vector and on the model; and \( L(Y|m_i) \) is the marginal data density. The Bayes Factor is calculated as follows:

\[
BF = \frac{L(Y|m_i)}{L(Y|m_j)} = \frac{\exp(LL(Y|m_i))}{\exp(LL(Y|m_j))}
\]  

(55)

where \( LL \) stands for log-likelihood. The log data density of the two models is computed with the Geweke (1999)’s modified harmonic mean estimator.

The Bayes Factor between the SWGK model and the SWBGG model is:

\[
BF = \frac{\exp(LL(Y|m_{SWGK}))}{\exp(LL(Y|m_{SWBGG}))} = \frac{\exp(1421.09)}{\exp(1377.29)} = 1.05 \times 10^{19}
\]  

(56)

The large value of the Bayes Factor provides clear evidence in favour of the SWGK model.

4.2 Forecasting performance

One-step ahead forecasts are computed in order to evaluate the forecasting performance of alternative models, as Kirchner and Rieth (2010) and In’t Veld et al. (2011) among others. The forecasts are the estimates of the observed variables, \( \hat{o}_t \), conditional on period \( t \) information: \( o_{t+1|t} = C s_{t+1|t} \), where \( s_{t+1|t} \), containing the model’s variables, is computed as \( s_{t+1|t} = A s_{t|t} \) and \( s_{t|t} \) is the updated variables obtained from the application of the Kalman filter. Following Kirchner and Rieth (2010), Table 4 reports the root mean squared forecast error (RMSFE) computed with one-sided Kalman filtered estimates of the observed variables at the posterior mean of the estimated parameters in each model.\(^5\)

Table 4 shows that the two models produce the same RMSFE for inflation, equal to 0.0006. The RMSFE for the spread is equal to 0.0004 in the SWGK model and 0.0025 in the SWBGG model. The value of the RMSFE for the spread reveals that the introduction of financial frictions shows reasonable empirical properties in forecasting the financial variable included in the dataset. The comparison between the two models reveals that RMSFE for output, investment, employment and the spread are lower in the SWGK model. RMSFE for consumption and wage are lower in the SWBGG model compared to those of the SWGK model, though quantitatively speaking for these two variables the difference between the two models is negligible.

RMSFE for output and investment are larger than RMSFE for the other variables in both models. The relatively inferior forecasting performance for output and investment in both models is not surprising, since both models do not include factors such as the external sector which might contribute to provide a better forecast for these variables.

Overall, the SWGK model performs better in terms of forecasting some selected variables of the Euro Area.

\(^5\)The formula is: \( \text{RMSFE} = \sqrt{T^{-1} \sum_{t=1}^{T} (y_t - \hat{y}_t^f)^2} \), where \( y_t \) stands for the observable and \( \hat{y}_t^f \) is the one-step ahead forecast.
Table 4: Forecasting performance

<table>
<thead>
<tr>
<th>Variable</th>
<th>Root mean squared forecast error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model SWBGG</td>
</tr>
<tr>
<td>Output</td>
<td>0.0064</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.0019</td>
</tr>
<tr>
<td>Investment</td>
<td>0.0082</td>
</tr>
<tr>
<td>Employment</td>
<td>0.0017</td>
</tr>
<tr>
<td>Wage</td>
<td>0.0020</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.0006</td>
</tr>
<tr>
<td>Spread</td>
<td>0.0025</td>
</tr>
</tbody>
</table>

4.3 Impulse response functions

This section presents the impulse response functions analysis. Figures 2, 3 and 4 examine three shocks, the two “financial” and the monetary policy shocks, since they highlight the different transmission mechanisms among the two models. All the shocks are set to produce a downturn. The first four charts of each figure show the responses of output, investment, inflation and the spread in the SWBGG model, while the other four charts show the responses of the same variables in the SWGK model. In all the figures, the solid lines represent the estimated median and the dotted lines represent the 95% highest posterior density confidence intervals.

Figure 2 shows the consequences of a shock to net worth. In the SWBGG model this shock affects net worth of intermediate goods firms, while in the SWGK model it affects net worth of FI. In the former model, a reduction in net worth of firms has two direct effects: (i) the amount of borrowing should increase since firms have less internal funds to acquire the same amount capital, equation (31); and (ii) the leverage ratio increases by definition. This last effect implies an increase in the agency costs, equation (33), and loans become riskier. As a result, the spread rises, as evident from the chart. The higher costs of purchasing capital depress the demand for it and investment falls. Output should increase due to the first effect which is then more than offset by the second effect, implying a significant reduction in output. Inflation increases though not significantly; its response is negative and significant after 10 quarters. In the SWGK model, the shock to FI net worth shown in Figure 2 leads to an immediate increase in the leverage, as evident from equation (40). Since the endogenous balance sheet constraint is always binding, financial intermediaries are obliged to curtail their supply of lending. As a result, investment is falling as well as output. The reduction in loans causes a fall in bank profits. As explained in Villa and Yang (2011), three factors affect the profits of financial intermediaries: the amount of loans, the lending rate and the leverage. The fall in profits caused by a reduction of loans makes financial intermediaries willing to increase the lending rate more than the increase in the deposit rate, in order to restore profits. Hence
the spread rises as shown in the chart. The increase in financing costs makes lending more expensive; firms reduce their demand for loans, further squeezing investment. It should also be noted that in this model there is an identity between loans and capital, equation (49). The reduction of loans, i.e. capital, also immediately affect the production function. As a result, in addition to the change in the aggregate demand, the aggregate supply is also clearly affected. On the nominal side the contraction in aggregate demand leads to a decline in inflation.

Figure 3 shows the effects of the investment-efficiency shock in the SWBGG model and of the shock to the quality of capital in the SWGK model. In the former case, the shock implies a rise in the price of capital, $Q_t$. This leads to two effects: (i) investment falls as well as output; and (ii) net worth of firms increases due to the higher return on capital, equation (35). The latter effect explains the fall in the spread shown in the chart. This should cause an increase in investment. However, the first effect dominates and investment decreases. The presence of financial frictions, therefore, attenuates the fall in investment and output, as also shown by Christensen and Dib (2008) and Gelain (2010). In the SWGK model, the shock to the quality of capital, also present in the model by Gertler and Kiyotaki (2010), is meant to capture economic obsolescence. If capital is good-specific, when the shock hits the economy, a random fraction of goods become obsolete and the capital used to produce the obsolete
Figure 3: Investment-efficiency/quality of capital shock. Solid lines represent mean IRF and dashed line represent the 95% highest posterior density confidence intervals.

goods becomes worthless. Therefore, given a standard production function in capital and labour, this shock implies a contraction in output. However, this is only part of the story. The shock to the quality of capital directly translates into a shock to the bank balance sheet because of the identity between capital and assets, equation (49). The loans provided by the financial intermediaries to firms are used by the latter to fully finance their acquisition of capital. Therefore, this shock implies a reduction in the "quality of intermediary assets". The reduction in total assets leads to a fall in banks profits. The same mechanism of the shock to FI net worth is at work. Financial intermediaries increase the lending rate to increase profits and this causes a rise in the spread and a further decline in lending and investment, as shown in the chart. Both aggregate demand and aggregate supply are affected; the change in aggregate demand is stronger, leading to a fall in inflation similarly to Gertler and Karadi (2011).

Contractionary monetary policy shock is shown in Figure 4. While the impact responses are similar between the two models, the dynamics is different. In both models an increase in the nominal interest rate reduces investment and, therefore, output. Demand downward pressures feed through changes in the output gap to inflation. This causes a downward shift in aggregate demand, which reduces inflation on impact. This is the standard interest rate
Figure 4: Monetary policy shock. Solid lines represent mean IRF and dashed line represent the 95% highest posterior density confidence intervals.

channel of monetary policy transmission. In these two models, the transmission mechanism of the policy shock is enhanced through its impact on credit markets. In the SWBGG model the tightening of monetary policy leads to a decline in the price of capital. This causes a fall in net worth of intermediate goods firms, and the spread rises, as shown in the chart. This mechanism further reinforces the contraction in capital and investment. In the SWGK model, due to the retrenchment in investment, the demand for loans decreases as well. At the same time the fall in asset prices worsens FI’s balance sheet. In order to restore profits, FI increase the lending rate more than the increase in the policy rate. As a result, the spread increases; this causes a further decline in loans and investment as shown in Figure 4.

4.4 Variance decomposition

Movements in GDP, investment, inflation and spread are now decomposed into parts caused by each shock at different time horizons, based on the mean of the model’s posterior distribution. The model economy is driven by seven shocks: productivity, monetary policy, government spending, net worth, shock to the investment/quality of capital, price mark-up and wage mark-up shocks. In Figure 5 the two “financial shocks” have been merged and are represented
by the gray bar; the two mark-ups shocks are also merged and represented by the white bar.

The first chart of Figure 5 shows that in the SWBGG model short run fluctuations in output are mainly driven by the demand shocks, in this model interest rate (dark gray bar) and government (the white gray bar) shocks. Similarly to the results of SW for the US economy, the contribution of the monetary policy shock declines as the time horizon increases. Differently from SW, the contribution of productivity shock (the black bar) declines as well, while financial shocks (the gray bar) account for an increasing proportion of long-run movements in real GDP. This result is consistent with the findings of other models featuring imperfect financial markets, such as Jermann and Quadrini (2009) and Brzoza-Brzezina et al. (2011). Fiscal policy shocks explain a decreasing proportion of fluctuations in output. In the long run the mark-ups shocks play a minor contribution in explaining output fluctuations. Movements in investment are mainly accounted by “financial” shocks; the role of monetary policy shocks is decreasing over time. Similarly to SW, mark-up shocks are the most important drivers of inflation. Productivity and monetary policy shocks account for a smaller fraction of inflation variability at every horizon. “Financial” and government shocks play a minor role. In the chart showing the variance decomposition for the spread, the contribution of each shock is not substantially different between the short, medium and long run. Not surprisingly financial shocks are the main drivers of this variable. The two financial shocks contribute to more than 70 percent of fluctuations in the spread, while interest rate shocks play a minor role.

As Figure 5 shows, in the SWGK model the contribution of the monetary policy shock in explaining output variations is constant over time and its impact is greater than in the SWBGG model. The contribution of financial shocks increases over time, while the role of productivity and government shocks is negligible. The introduction of a banking sector playing an active role in the transmission mechanism of the shocks affects both short and long run movements in output. As far as investment is concerned, short and long-run fluctuations are mainly driven by monetary and financial shocks. Not surprisingly, results are similar for the spread, defined as the difference between lending rate and risk free rate. The latter component is mainly explained by interest rate shocks, while the two financial shocks contribute in the explanations in the movements of the lending rate, which originates within the banking sector. Inflation is mainly driven by productivity and mark-up shocks with financial shocks explaining an increasing fraction over time.

As a term of comparison, the contributions of the structural shocks in explaining business cycle fluctuations differ among the two models, in particular for output: the SWGK model tends to emphasize the role of the financial sector in explaining business cycle fluctuations compared to the SWBGG model.
Figure 5: Forecast error variance decomposition computed at the mean of the posterior distribution in the SWBGG model and in the SWGK model.

5 Robustness analysis

This section illustrates a series of modifications to: (i) the calibration of the steady state leverage ratio of the two models; (ii) the series of the spread used as observable; and (iii) the models’ specification.

The importance of the value of the leverage ratio is stressed by several studies, such as Peersman and Smets (2005) and Carlstrom et al. (2011). In the SWBGG model a change in
the steady state leverage ratio has a direct impact on equation (35). This equation specifies the evolution of the net worth of intermediate goods firms: any change in the leverage ratio clearly influences the financial accelerator effect. In the SWGK model a change in the steady state leverage ratio affects the evolution of net worth of financial intermediaries, equation (41). Similarly to the SWBGG model, a change in the steady state leverage ratio of banks affects the spread, and therefore total output, as explained in Subsection 4.3.

The leverage ratio is equal to 2 in the SWBGG model and 5 in the SWGK model as shown in the baseline calibration shown, Table 2. Table 5 shows how the Bayes Factor is affected by changes in the leverage ratio of the two models one at a time. In the first two columns of the table the Bayes Factor is computed using the baseline value of the log data density of the SWGK model and the values of the log data density of the SWBGG model derived from the different values of the firms’ leverage. For sake of brevity, parameters estimates are not reported. The leverage ratio of firms in the SWBGG model changes from 1.2 to 3, implying that from 16% to 66% of firms’ capital expenditure are externally financed. The Bayes Factor shows clear evidence in favour of the SWGK model, for any value of the leverage ratio in the SWBGG model. The last two columns of Table 5 shows the sensitivity of the Bayes Factor to different steady state leverage of the SWGK model, while the leverage ratio of the SWBGG model is equal to its baseline value, i.e. 2. Financial intermediaries are generally more leveraged than firms; in this experiment their leverage ratio changes from a value of 4 to a value of 8. The result of Subsection 4.1 is confirmed: there is clear evidence in favour of the SWGK model for different values of the steady state leverage ratio of financial intermediaries.

As an additional sensitivity check, a different series of the spread is used as observable. The series is computed as the difference between a yield on A corporate bonds and government AAA bonds. The average spread is now 82 basis points, lower than the baseline calibration. The parameter $S$ in the SWBGG model and the parameter $\chi$ in the SWGK model are now changed to match the different value of the steady state spread; the other calibrated parameters are unchanged. The log data densities deriving from these estimations are used to compute the Bayes Factor, which is:

$$BF = \frac{\exp(LL(Y|m_{SWGK}))}{\exp(LL(Y|m_{SWBGG}))} = \frac{\exp(1401.65)}{\exp(1356.52)} = 4.0 \times 10^{19}$$

(57)

The comparison between the two models is again in favour of the SWGK model, even with

---

6 Two methods can be used to evaluate the log data density: the modified harmonic mean and the Laplace approximation. As in Smets and Wouters (2007), the results of both approximations are very close in the baseline specification of Subsection 4.1: 1377.3 with the modified harmonic mean and 1376.6 with the Laplace approximation in the SWBGG model; and 1421.1 with the modified harmonic mean and 1421.0 with the Laplace approximation in the SWGK model. Since the former method is computationally costly, the latter method is used in these experiments.

7 It would be interesting to use a series with a higher steady state average spread; however, this experiment is not possible due to data limitations.
a different series of the spread.

The two models embed the following same types of frictions: price stickiness, price indexation, wage stickiness, wage indexation, investment adjustment costs, variable capital utilization, habit in consumption and fixed costs in production. And the frictions originating in the financial sector are different. As a further robustness check, each of the common frictions is turned off one at a time in the spirit of SW. Then, the Bayes Factor is computed and reported in Table 6. This experiment makes also it possible to analyse which frictions are important to account for the dynamics of each model. As shown in the second column of Table 6, the Bayes Factor is always in favour of the SWGK model, no matter which friction is turned off. The first row reports the Bayes Factor of the baseline estimates in Subsection 4.1. On the side of nominal frictions, removing price stickiness implies a considerable deterioration in terms of the log data density. Similarly to SW, price or wage indexation do not play an important role in explaining the model dynamics. On the side of real frictions, the most important in terms of the log data density is investment adjustment costs. Reducing habit formation in consumption and fixed costs in production is also costly in terms of the log data density. A larger value of the capital utilization elasticity implies higher marginal depreciation cost, and therefore less variation in capital utilization. Removing this friction does not imply a deterioration of the log-likelihood; its value is even higher in the SWBGG model. In the SWGK model, instead, variable capital utilization plays a role in explaining model’s performance.

| Leverage = $\frac{K}{N}$ | SWBGG model | | SWGK model |
|--------------------------|-------------|--------------------------|
| Bayes Factor = $\exp(LL(Y|m_{SWGK}))$ | $\exp(LL(Y|m_{SWBGG}))$ | $\exp(LL(Y|m_{SWGK}))$ | $\exp(LL(Y|m_{SWBGG}))$ |
| 1.2 | $1.6 \times 10^{34}$ | 4 | $6.2 \times 10^{19}$ |
| 1.5 | $1.1 \times 10^{19}$ | 4.5 | $6.1 \times 10^{19}$ |
| 1.8 | $7.4 \times 10^{18}$ | 5 | $1.9 \times 10^{19}$ |
| 2 | $1.9 \times 10^{19}$ | 5.5 | $4.8 \times 10^{18}$ |
| 2.2 | $6.1 \times 10^{19}$ | 6 | $9.7 \times 10^{18}$ |
| 2.4 | $2.6 \times 10^{20}$ | 6.5 | $5.0 \times 10^{18}$ |
| 2.6 | $7.6 \times 10^{20}$ | 7 | $2.1 \times 10^{17}$ |
| 2.8 | $2.0 \times 10^{21}$ | 7.5 | $3.1 \times 10^{17}$ |
| 3 | $3.5 \times 10^{35}$ | 8 | $5.3 \times 10^{17}$ |

Table 5: Sensitivity of the Bayes Factor to the steady state leverage ratio.
Table 6: Bayes Factor for different models’ specifications

<table>
<thead>
<tr>
<th>Friction</th>
<th>Bayes Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>exp(1421.00) / exp(1376.60) = 1.9 × 10^{19}</td>
</tr>
<tr>
<td>$\sigma_p = 0.1$, Calvo prices</td>
<td>exp(1362.12) / exp(1299.97) = 9.3 × 10^{48}</td>
</tr>
<tr>
<td>$\sigma_w = 0.1$, Calvo wages</td>
<td>exp(1364.44) / exp(1258.94) = 2.0 × 10^{49}</td>
</tr>
<tr>
<td>$\sigma_{pi} = 0$, price indexation</td>
<td>exp(1424.99) / exp(1382.94) = 1.8 × 10^{18}</td>
</tr>
<tr>
<td>$\sigma_{wi} = 0$, wage indexation</td>
<td>exp(1425.93) / exp(1381.67) = 1.7 × 10^{19}</td>
</tr>
<tr>
<td>$\xi = 0.1$, inv. adj. costs</td>
<td>exp(1341.79) / exp(1247.73) = 7.1 × 10^{40}</td>
</tr>
<tr>
<td>$\zeta = 2$, elasticity of capital util</td>
<td>exp(1411.11) / exp(1377.24) = 5.1 × 10^{14}</td>
</tr>
<tr>
<td>$h = 0.1$, habit parameter</td>
<td>exp(1392.51) / exp(1339.15) = 1.4 × 10^{23}</td>
</tr>
<tr>
<td>$\Theta = 1.1$, fixed costs in production</td>
<td>exp(1413.09) / exp(1365.02) = 6.9 × 10^{20}</td>
</tr>
</tbody>
</table>

6 Forecasting evaluation

The estimation results can be used to analyse the predictive power of some series such as the output gap and credit spreads in the two models. Coenen et al. (2009) find that flexible-price output gap performs relatively well in predicting EA inflation over medium-term horizons. Gilchrist and Zakrajšek (2011) find that the credit spread is a more powerful predictor of economic activity compared to the standard default-risk indicators.

This section examines the predictive power of the SWBGG model versus the SWGK model in forecasting inflation in the EA. Following Coenen et al. (2009) and Gelain (2010), inflation forecasts are based on the basis of a traditional Phillips curve with the flexible-price output gap (output gap henceforth) generated by the two estimated models. Then a modified version of the Phillips curve (PC) replaces the output gap with the spread. As benchmarks, two control models, a random walk and an autoregressive process, are used.

The forecast of inflation is made using several vintages of data, i.e., for rolling samples in pseudo-real time, as described in Fischer et al. (2006). In particular, 19 vintages are considered, with the initial sample spanning 1996Q1-2003Q4 and the final sample covering 1996Q1-2008Q3. Similarly to Gelain (2010), the 4-quarter change in the private consumption deflator, $\pi_{t+4}$, is forecast:

$$\pi_{t+4} = 100 \left( \frac{P_{t+4}}{P_t} - 1 \right)$$ (58)

First, the following equation is estimated by OLS:

$$\pi_{t+4} = a_v + b_v(L)\pi_{t+4} + c_v(L)x_{vt+4} + \epsilon_{t+4}$$ (59)
where $\pi_{v,t} = 400 \left( \frac{P_{t+4}}{P_t} - 1 \right)$ is the annualised one-period change in the private consumption deflator, $x_{v,t}$ is either the output gap or the credit spreads generated by the estimated models, $b_v(L)$ and $c_v(L)$ are finite polynomials. The optimal number of lags is selected using the Schwartz information criterion.

Then, for each vintage a forecast of inflation is obtained:

$$\tilde{\pi}_{t+4}^4 = a_v^{OLS} + b_v(L)^{OLS} \pi_{v,t} + c_v(L)^{OLS} x_{v,t} + \varepsilon_{v,t+4}^4$$

(60)

The autoregressive model of inflation is obtained following the same procedure described above. In the random walk model inflation forecast is given by the average rate of inflation over the previous four quarters available for a given data vintage:

$$\tilde{\pi}_{t+4}^{4,RW} = 100 \left( \frac{P_t}{P_{t-4}} - 1 \right)$$

(61)

For each model $M$, forecast errors, $f_{t+4}^M$, are defined as:

$$f_{t+4}^M = \tilde{\pi}_{v,t+4}^M - \pi_{t+4}$$

(62)

where $\pi_{t+4}$ is the realized inflation rate in the last available vintage of data.

Alternative models $M$ are compared on the basis of the mean squared forecast error (MSFE), which is given by:

$$MSFE^M = (bias^M)^2 + (\sigma^M)^2$$

(63)

where

$$bias^M = \frac{1}{T} \sum_{t=1}^{T} f_{t+4}^M$$

$$\sigma^M = \frac{1}{T} \sum_{t=1}^{T} \left( f_{t+4}^M - \frac{1}{T} \sum_{t=1}^{T} f_{t+4}^M \right)^2$$

(64)

Table 7 shows the MSFE for 6 different models: the SWBGG model with the output gap in the PC, the SWBGG model with the spread in the PC, the SWGK model with output gap in the PC, the SWGK model with the spread in the PC, the random walk, and the AR model. The third column of Table 7 shows the ratio between the MSFE of model $M$ and the MSFE of the random walk model; the last two columns shows the bias and the variance of each model. The results of this forecasting exercise are as follows. First, the AR model outperforms all models in terms of the MSFE criterion and shows the lowest forecast error variance. Second, the RW model shows the worst performance in terms of the MSFE criterion, as shown by the third column, and the highest forecast error variance. Third, the comparison between the SWBGG model and the SWGK model provides evidence in favour of the SWGK model no matter whether the PC is estimated with the output gap or the spread. Finally, the flexible-price output gap adds more predictive power compared to the spread in both models.
Table 7: MSFE for 4 steps ahead inflation forecast

<table>
<thead>
<tr>
<th>Model</th>
<th>MSFE</th>
<th>MSFE$^{\text{MS}}$</th>
<th>bias$^2$</th>
<th>$\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWBGG with output gap</td>
<td>0.049</td>
<td>0.202</td>
<td>0.033</td>
<td>0.016</td>
</tr>
<tr>
<td>SWBGG with spread</td>
<td>0.067</td>
<td>0.277</td>
<td>0.049</td>
<td>0.018</td>
</tr>
<tr>
<td>SWGK with output gap</td>
<td>0.043</td>
<td>0.177</td>
<td>0.026</td>
<td>0.016</td>
</tr>
<tr>
<td>SWGK with spread</td>
<td>0.052</td>
<td>0.214</td>
<td>0.037</td>
<td>0.015</td>
</tr>
<tr>
<td>RW</td>
<td>0.241</td>
<td>1.000</td>
<td>0.028</td>
<td>0.212</td>
</tr>
<tr>
<td>AR</td>
<td>0.041</td>
<td>0.169</td>
<td>0.027</td>
<td>0.013</td>
</tr>
</tbody>
</table>

These findings need to be interpreted cautiously given the short forecast interval used for this exercise.

7 Conclusion

This paper builds and compares two DSGE models which have a Smets and Wouters (2007) economy in common but feature different types of financial frictions: (i) the SWBGG model incorporates frictions originating in the demand side of the credit market à la Bernanke et al. (1999); and (ii) the SWGK model incorporates frictions embedded in the supply side of the credit market à la Gertler and Karadi (2011). The two models are estimated with Bayesian techniques for the period 1996Q1-2008Q3 with Euro Area data. The SWGK model provides better results according to the analysis of the Bayes Factor and the forecasting performance. This result is robust to a series of models’ calibration and specification. None of the models generates counter-intuitive impulse response functions (IRFs) and/or variance decomposition. The internal propagation mechanism and the financial accelerator effect of the shocks differ between the two models. Finally, the SWGK model outperforms the SWBGG model in forecasting Euro Area inflation.
References


Rudebusch, G. (2002). Term structure evidence on interest rate smoothing and monetary


