Wisdom of The Crowd

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Abstract

This paper studies a market for new projects with endogenous information acquisition by experts (venture capitalists). It finds that the crowd-like behavior of experts is characteristic of investments in new fields where prospects of new projects are unknown but correlated and experts independently collect small bits of information: each expert acquires a signal about a project and invests when the signal is favorable. Given that the projects are related many experts are likely to invest when the projects are promising. The paper shows that uninformed investors at IPOs trust the crowd of experts and follow it: booms with high IPO prices occur when many experts are selling their projects because this conveys positive information about the projects.

The analysis highlights that a critical mass of independent experts with the necessary expertise in a particular industry is essential for an active market for new projects to emerge: if experts are few each of them can manipulate the market by investing at random and driving the projects' prices up. Outside investors at IPOs anticipate this and do not follow the experts. As a result the IPO market for new projects ceases to exist which in turn discourages experts to start new projects.

Introduction

In environments where proceeds of new projects are difficult to assess for outsiders early investments in projects are often provided by expert investors (venture capitalists). Eventually the projects must be "sold" to outsiders for experts to cash in.

This paper studies the market for new projects, it analyzes experts' information acquisition and investment. The main finding of this paper is that in certain environments informed experts can afford keeping no skin in the game and, in spite of the full exit, experts have an incentive to acquire ex ante information about the projects.

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In our paper, starting a project at random or holding it for a long period is not profitable for experts as they need the cash in the short term. Thus ex ante the expert is willing to acquire information and invest in a project only if she expects the short-term price to react to her information. The analysis shows that when experts invest in projects belonging to a certain filed, some information about the projects' prospects is revealed to outsiders when experts bring projects to the market. Many experts' selling projects implies that many experts found the projects promising, and so the outsiders' valuations of the projects increase and the price goes up. This in turn induces experts to acquire information about the projects. Experts thus acquire information not because they anticipate keeping some skin in the game, but because they want to predict their peers' behavior.

We find that an active market for new projects emerges only if there is a critical number of independent experts. If experts are few, each of them can manipulate the market and, consequently, outsiders are willing to buy shares only if the expert keeps a stake in the firm. This highlights that the presence of numerous venture capitalists is a prerequisite for an active IPO market for firms operating in new industries. In accordance with this observation the most active IPO market for young firms is found in the US which has the most developed venture capital industry.

Our analysis is coherent with some observations about venture capital and initial public offerings (IPOs). Venture capitalists before investing in firms acquire information about and part of this information is about the prospects of the industry the firms operate in.[1] Investors at IPOs see the presence of venture capitalists and other expert investors which supported the firms prior the IPO as a positive signal about the firm.[2] When investors see many venture-capital-backed firms going through IPOs in some industry they become optimistic about the industry, share prices go up, and the market becomes hot.[3] This explains the waves of investment at initial public offerings (IPOs), which have been documented in the literature (see for instance Lowry (2003) and Ritter and Welch (2002)).

Related literature

Leland and Pyle (1977) analyzes the sale of shares by an informed entrepreneur to uninformed investors and finds that the entrepreneurs with favorable information about their projects retain a fraction of shares in order to signal their information. Our set up is different, the expert in order to be able to sell shares has to invest in the project and may acquire information. That is the expert's decision to become an informed seller is endogenous. Also we study the investment by many independent experts in the projects belonging to the same field and put forward the wisdom of the crowd: outsiders infer the projects' prospects from the number of experts investing

[1] Consistent with this idea Gompers et al. (2008) finds that the investment by experienced venture capitalists in an industry is high when the industry's Tobin's Q high, suggesting that the crowd of venture capitalists is wise.

[2] An indirect evidence for this is the low underpricing of the venture capital backed IPOs documented in Jain and Kini (1995).

[3] Lowry (2003) finds the investors' optimism to be one of the key drivers of the volume of investment at IPOs.

in them.

In this our paper relates to Scharfstein and Stein (1990) which shows that a manager who decides on investment at a late stage and is concerned about his reputation may ignore his information and mimic the managers who have already made their investment decision. In our paper outsiders do not ignore their private information but are uninformed by definition, also the research question of our paper is different from that of Scharfstein and Stein (1990) and of other papers on herd behavior that seek to explain why herding may occur. Our papers focuses on the experts' incentives to acquire information about projects given that their ultimate goal is to sell projects to outsiders.

Hirshleifer et al. (1994) models trades by informed investorsm market makers and liquidity traders in the stock market. The paper shows that the risk averse traders that get informed early can earn profits: when they get favorable information about the stock they anticipate the other traders to get favorable information and the stock price to go up, they first buy shares and then sell a fraction of them. The traders that get the information late appear to follow the early informed traders even though they act on their own private information. In our model the analogs of early and late traders are experts and outside investors. The difference is that in our set up outside investors never get any private information about the projects and yet they do follow informed experts and buy shares from them.

In Froot et al. (1992) speculators acquire information about a security and submit market orders that are executed either early or late with equal probabilities. In the presence of liquidity traders the speculators profit if their orders happen to be executed early: the orders which are executed late push the price in the same direction as the early orders since all orders are based of the speculators' information.[4] The authors show that speculators prefer to herd on the same information: if only few speculators acquire the same information each speculator is unlikely to be followed by other speculators and is unlikely to earn profits. We also argue that a critical number of experts must invest in similar projects for an active market for projects to emerge. The main difference between our paper and Froot et al. (1992), Hirshleifer et al. (1994) is that in our paper there are no liquidity traders that loose money, in fact experts (analogs of speculators) face a liquidity need and trade against rational uninformed investors. In other words experts are the liquidity traders in our set up. We show that experts acquire information and earn positive profits. Also differently from Froot et al. (1992) in our paper even if an expert is the only one to acquire information she can earn positive profits: when she invests in the project the market does not observe this and, unlike in Froot et al., the price does not react immediately. Later the expert signals her information by selling only a fraction of her project to outside investors.

The paper is organized as follows. Section 1 introduces the model, section 2 studies the single

[4]In some sense the speculators profit because their orders are executed not simultaneously but sequentially, this makes the price only partially informative at early stages and allows speculators to exploit their superior information as in Kyle (1985).

project case, section 3 analyzes many projects with correlated returns. Section 4 provides the discussion.

1 Environment

There is a field of similar potential projects each requiring an investment I: start-ups in a particular industry (Internet, bio-tech), research agendas in academia. Projects can be either promising (state θ_H) or not (state θ_L), with equal probabilities. If the field is not promising then each project succeeds and delivers R with probability θ_L, with complementary probability it delivers 0. If the field is promising the probability of success is $\theta_H = \theta_L + \Delta > \theta_L$. Note that the returns of individual projects are conditionally independent. A project if started at date 1, can be sold to outsiders at date 2, and delivers return at date 3 (the timing is shown on figure 1). The ex ante present value of a project without any additional information is negative.

Assumption 1. $(\theta_L + \Delta/2)R - I < 0$.

All agents are risk neutral and can hold cash which transfers value 1-for-1 between all dates.

There are N experts (researchers, venture capitalists) each of whom can execute one project. An expert's resources are just enough to start a project at date 1. Before starting a project each expert can investigate it and acquire a signal $s \in \{s_L, s_H\}$ at a private cost $c > 0$, or remain uniformed ($s = \varnothing$) and bear no cost. The signal is informative about the state of the world: $\Pr(s_H|\theta_H) = q(\theta_H) = q \in (\frac{1}{2}, 1)$ and $\Pr(s_H|\theta_L) = q(\theta_L) = 1 - q$, which implies $\Pr(s_H) = \frac{1}{2}$. Conditional on the high signal the project's net present value is positive $(\theta_L + \Delta q)R - I > 0$, moreover the chance of executing a high-signal project permits to recover the investigation cost c:

Assumption 2. $\frac{1}{2}((\theta_L + \Delta q)R - I) - c > 0$.

At date 2 the expert faces an investment opportunity λ, which delivers $1 + \lambda > 1$ per unit of investment at date 3. In order to take advantage of the investment opportunity, the expert may choose to sell a stake in her project at date 2. The investment opportunity λ is sufficiently attractive: if the expert knew she could not sell her project at date 2, she would prefer to keep cash at date 1 even when receving the high signal:

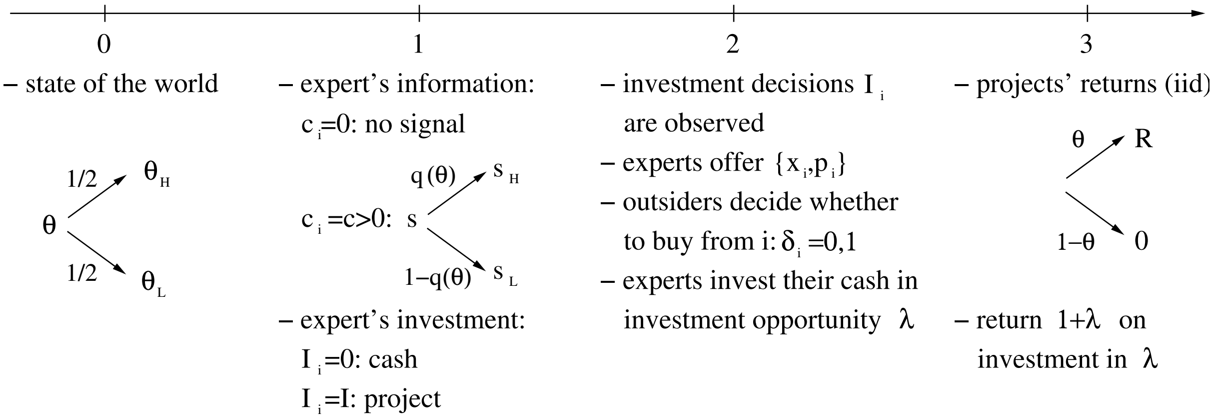
Assumption 3. $I(1 + \lambda) > (\theta_L + \Delta q)R$.

Competitive uninformed investors have no expertize to investigate or start projects. They can buy shares in projects at date 2, after observing the investment decisions and offers of experts and updating the expectations of the returns accordingly. The outsiders demand zero expected return and have enough resources to buy all projects sold by experts.

Assumptions 1-3 are essential for the analysis. Assumptions 1 and 2 ensure that without investigation by experts the projects are never started, hence experts can create value. Assumption

3 guarantees that the expert never invests in the project if she cannot sell a fraction of her stake in the market afterwards.[5]

Figure 1: Timing



Equilibrium

The environment described above induces a game between experts and outsiders. Expert i decides whether to spend $c_i \in \{0, c\}$ and acquire a signal $s_i \in S = \{s_L, s_H, \varnothing\}$; having observed the signal s_i she decides whether to invest in a project $I_i \in \{0, I\}$. At date 2 the expert observes the investments of all experts $(I_1, ..., I_N)$ and, if invested in the project at date 1, offers $x_i \in [0, 1]$ shares in her project at price $p_i \in R^+$ per share. Thus expert i's behavior strategy $\sigma_i(.|.) = (\sigma_c(c_i), \sigma_I(I_i|s), \sigma_{xp}(x_i, p_i|k, s))$ defines a probability distribution over the space of actions $\{0, c\} \times \{0, I\} \times [0, 1] \times R^+$. Without loss of generality let the expert invest the proceeds from the sale of shares in the investment opportunity λ.

At date 1 expert i's beliefs about the signals of other experts $s_{-i} \in S^{N-1}$ for each signal $s_i \in S$ specify a probability distribution $\nu_i(s_{-i}|s_i)$. At date 2 expert i's beliefs about $s_{-i} \in S^{N-1}$ are updated taking into account the investments $(I_1, ..., I_N)$.

At date 2 outsiders observe the investments of all experts $(I_1, ..., I_N)$, the shares offered by experts $x = (x_1, ..., x_N)$ and prices $p = (p_1, ..., p_N)$ (the experts who have not invested in projects do not offer any shares $x_i = 0$, $p_i = 0$). For every expert i the outsiders form a belief about her signal, which specifies a probability distribution $\mu_i(s|I_i, x_i, p_i)$. For each offer $\{x_i, p_i\}$ the outsiders' pure strategy specifies whether to accept it $\delta_i \in \{0, 1\}$, $i = 1, ..., N$.

The outsiders' payoff from dealing with expert i at date 2 is

$$u(x_i, p_i, \delta_i) = \delta_i x_i(\theta R - p_i),$$

[5] Venture capitalists sell their stakes in mature ventures at initial public offerings (IPOs) or in private deals with other investors. A typical venture capital fund has a prespecified life span of around 10 years after which the fund must pay its investors back. Parameter λ and assumption 3 capture the idea of the early exit by venture capitalist.

expert i's expected profit at date 2 is

$$\pi(c_i, I_i, x_i, p_i, \delta_i) = \mathbf{1}\{I_i = I\}((\delta_i x_i p_i(1+\lambda) + (1-\delta_i x_i)\theta R) + \mathbf{1}\{I_i = 0\}I(1+\lambda) - c_i.$$

We look for a symmetric Bayesian-Nash equilibrium of the game $(\sigma^*, \delta^*, \mu^*, \nu^*)$ where each expert maximizes her payoff given her beliefs, the strategies of other experts and those of outsiders; outsiders maximize their payoff given the beliefs and the strategies of experts; the beliefs of experts and outsiders are consistent with Bayes' rule.

Theorem 1. *The no-market (autarky) equilibrium exists: experts do not investigate and start no projects, there is no market for projects.*

Proof. Suppose outsiders believe that only uninformed experts sell projects at date 2, that is $\mu_i^*(s = \varnothing | I, x_i, p_i) = 1$ for any $\{x_i, p_i\}$, then the maximum price an expert can elicit at date 2 is $\frac{1}{2}(\theta_L + \theta_H)R < I$ (assumption 1), hence if the expert invests in the project she must prefer not to sell it. The uninformed expert who have not acquired the signal if invests can expect the return of $\frac{1}{2}(\theta_L + \theta_H)R < I$, hence she must not invest. The expert who received the low signal also must not invest. Suppose the expert investigates and invests in the case of the high signal, in the case of the low signal she waits and invests in the investment opportunity λ, her expected payoff is $\pi = \frac{1}{2}(\theta_L + \Delta q)R + \frac{1}{2}I(1+\lambda) - c$. The expert can attain the payoff of $I(1+\lambda)$ by keeping the cash at date 1 and pursuing the investment opportunity λ at date 2, which is higher than π due to assumption 3, $(\theta_L + \Delta q)R < I(1+\lambda)$. Thus no expert invests, projects are not traded at date 2 and the beliefs $\mu_i^*(.)$ can be set arbitrarily QED.

When the investment opportunity λ accruing at date 2 is very attractive the expert wants to have cash at this stage. If there is no active market for projects at date 2 the expert prefers not to invest her resources into the project at date 1 and keep the cash. When all experts do so there is no investment in new projects at date 1 and, hence, no active market at date 2.

2 Uncorrelated returns (single expert)

Consider the baseline version of the model where returns of individual projects are not correlated. Each expert and her project can be treated in isolation, which is equivalent to the general model with only one expert $N = 1$. The main difference with respect to $N \geq 2$ is that a single expert perfectly anticipates the number of projects on the market and, hence, does not learn anything from this number. At date 2 after the expert has invested in the project the signaling "sub-game" is played. We say the expert is of "type s" if she received signal $s \in \{s_H, s_L, \varnothing\}$. The expert can signal her type at date 2 by offering x shares in her project at price p, her strategy $\sigma_{xp}(.|s)$ specifies the probability distribution over possible pairs $\{x, p\} \in [0,1] \times R^+$. For each offer $\{x, p\}$ the outsiders form beliefs about the expert's type $\mu(s|x,p) = \Pr(s|x,p)$ and decide whether to buy shares $\delta(x,p) \in \{0,1\}$.

Definition 1. *Type $s \in \{s_L, \varnothing\}$ is **identified** if she invests in the project and offers a pair $\{x_s, p_s\}$ which is never offered by type s_H: if $\sigma_I^*(I|s) > 0$ and there exists $\{x_s, p_s\}$ such that $\sigma_{xp}^*(x_s, p_s|s) > 0$ and $\sigma_{xp}^*(x_s, p_s|s_H) = 0$.*

Definition 2. *The equilibrium is **informative** if the expert acquires the signal with positive probability: $\sigma_c^*(c) > 0$.*

Lemma 1. *For $N = 1$ an informative equilibrium where type $s \in \{s_L, \varnothing\}$ is identified does not exist.*

Proof. Suppose an equilibrium where type $\varnothing$ is identified exists. Take a pair $\{x_\varnothing, p_\varnothing\}$ such that $\sigma(x_\varnothing, p_\varnothing|\varnothing) > 0$ and $\sigma(x_\varnothing, p_\varnothing|s_H) = 0$, then Bayesian updating requires $\mu(s_H|x_\varnothing, p_\varnothing) = 0$. The offer can be accepted by outsiders if $p_\varnothing \leq E(\theta|\varnothing)R$. The expert invests the proceeds from the sale of shares in the investment opportunity λ, given that type $\varnothing$ chooses $\{x_\varnothing, p_\varnothing\}$ with positive probability her expected profit can be computed as $\pi(.|\varnothing) = x_\varnothing p_\varnothing(1+\lambda) + (1 - x_\varnothing)E(\theta|\varnothing)R$. Since $p_\varnothing \leq E(\theta|\varnothing)R$ and $x_\varnothing \in [0, 1]$ her profit is less or equal to $E(\theta|\varnothing)R(1+\lambda)$. Assumption 1 states $E(\theta|\varnothing)R = (\theta_L + \Delta/2)R < I$ which implies $\pi(.|\varnothing) < I(1+\lambda)$. Consequently type $\varnothing$ is better off not investing in the project which contradicts $\sigma_I^*(I|\varnothing) > 0$ and type $\varnothing$ being identified, hence the proposed equilibrium does not exist. Analogous reasoning holds for type s_L QED.

Intuitively when the uninformed expert (type $\varnothing$) is identified she cannot charge a price higher than the expected return of her project, but her project has a negative net present value. It follows that the uninformed expert is better off not investing in the project. Thus in an informative equilibrium if the uninformed expert invests in the project she must be selling the project at a price higher than the expected value, that is she must be pooled with the expert who got the high signal. Similarly if the expert who got the low signal invests she must be pooled with the high type on the market.

Lemma 2. *If an informative equilibrium exists for $N = 1$, then the expert strictly prefers to invest if the signal is high and not to invest if the signal is low.*

Proof. Suppose an expert who received the high signal weakly prefers not to invest. That is for any pair $\{x, p\} \in [0, 1] \times R^+$ possibly offered by the expert and accepted by outsiders the following inequality holds $xp(1+\lambda) + (1-x)E(\theta|s_H)R \leq I(1+\lambda)$. Provided that $E(\theta|s_L) < E(\theta|s_H)$ and $x \in [0, 1]$ it follows that for any such pair $xp(1+\lambda)+(1-x)E(\theta|s_L) \leq I(1+\lambda)$ and the expert who received the low signal also prefers not to invest. Consequently the expert's profit from acquiring the signal is at most $I(1+\lambda) - c$, so she can save on the signal cost and get a higher payoff of $I(1+\lambda)$ by not investing in the project, but the expert not acquiring the signal contradicts the informative equilibrium. Therefore if the informative equilibrium exists the expert strictly prefers to invest upon receiving the high signal. Denote the set of pairs $\{x, p\}$ proposed by the expert in this case with positive probability by $H = \{(x, p) : \sigma_{xp}^*(x, p|s_H) > 0\}$.

Suppose in an informative equilibrium the expert upon receiving the low signal weakly prefers to invest. By lemma 1 she must not be identified, that is any pair offered by the expert with the low signal must be also offered by the expert with the high signal, take such a pair $\{x', p'\} \in H$. Given that $\sigma^*_{xp}(x', p'|s_L) > 0$ we must have $\pi(.|s_L) = \int_{x,p}(xp(1+\lambda)+(1-x)E(\theta|s_L)R)\sigma^*_{xp}(x,p|s_L) - c = x'p'(1+\lambda) + (1-x')E(\theta|s_L)R - c$, similarly $\sigma^*_{xp}(x', p'|s_H) > 0$ implies $\pi(.|s_H) = x'p'(1+\lambda) + (1-x')E(\theta|s_H)R - c$. The expert's expected profit from acquiring the signal is $\Pr(s_L)\pi(.|s_L) + \Pr(s_H)\pi(.|s_H) = x'p'(1+\lambda)+(1-x')E(\theta)R-c$, but the expert can get a higher profit $x'p'(1+\lambda)+(1-x')E(\theta)R$ if she invests and offers $\{x', p'\}$ without acquiring the signal. The expert prefers not to acquire the signal which contradicts the informative equilibrium. Hence if the informative equilibrium exists the expert who received the low signal strictly prefers not to invest QED.

The lemma is very intuitive, the expert acquires the signal only if the signal matters for her subsequent actions. If for some realization of s the expert is indifferent between investing in the project or not, but for the other realization she strictly prefers (not) to invest, then she is better off (not) to invest and not to acquire the costly signal. The result can be generalized for a more general environment where the signal can take more than two values, in this case there must exist at least two realizations of the signal in which the expert strictly prefers different actions.

Informative equilibrium

In an informative equilibrium any pair $\{x, p\}$ offered by the expert at date 2 must be such that the expert prefers to sell shares at price p

$$p(1+\lambda) \geq E(\theta|s_H)R, \tag{1}$$

the outsiders are willing to buy shares at price p

$$p \leq \sum_\theta \sum_s \Pr(\theta|s)\mu(s|x,p)\theta R. \tag{2}$$

By lemma 2 the expert receiving the high signal strictly prefers to invest, while the expert receiving the low signal prefers not to invest. In an informative equilibrium the expert must choose to acquire the signal: $\frac{1}{2}[xp(1+\lambda) + (1-x)E(\theta|s_H)R] + \frac{1}{2}I(1+\lambda) - c \geq I(1+\lambda)$ and $\frac{1}{2}[xp(1+\lambda) + (1-x)E(\theta|s_H)R] + \frac{1}{2}I(1+\lambda) - c \geq xp(1+\lambda) + (1-x)E(\theta|\varnothing)R$. Hence using $\frac{1}{2}E(\theta|s_L)R + \frac{1}{2}E(\theta|s_H) = E(\theta|\varnothing)$ we obtain the incentive compatibility condition

$$\begin{aligned} xp(1+\lambda) + (1-x)E(\theta|s_H)R \geq I(1+\lambda) + 2c, \\ xp(1+\lambda) + (1-x)E(\theta|s_L)R \leq I(1+\lambda) - 2c. \end{aligned} \tag{3}$$

Note that (3) guarantees that the expert strictly prefers to invest in the case of the high signal and not to invest in the case of the low signal.

The outsider's beliefs should be consistent with the Bayes' rule on the equilibrium path. Given the equilibrium strategy $(\sigma^*_c(.), \sigma^*_I(.|.), \sigma^*_{xp}(.|.))$ the probability that the high type comes up with the offer $\{x, p\}$ is $\Pr(x, p, s_H) = \sigma^*_c(c)\Pr(s_H)\sigma^*_I(I|s_H)\sigma^*_{xp}(x, p|s_H)$, substituting $\Pr(s_H) = \frac{1}{2}$ and $\sigma^*_I(I|s_H) = 1$ we get $\Pr(x, p, s_H) = \frac{1}{2}\sigma^*_c(c)\sigma^*_{xp}(x, p|s_H)$. For the type $\varnothing$ we obtain $\Pr(x, p, \varnothing) =$

$(1-\sigma_c^*(c))\sigma_I^*(I|\varnothing)\sigma_{xp}^*(x,p|\varnothing)$. Bayesian updating requires

$$\mu(s_H|x,p) = \frac{\frac{1}{2}\sigma_c^*(c)\sigma_{xp}^*(x,p|s_H)}{\frac{1}{2}\sigma_c^*(c)\sigma_{xp}^*(x,p|s_H) + (1-\sigma_c^*(c))\sigma_I^*(I|\varnothing)\sigma_{xp}^*(x,p|\varnothing)}. \tag{4}$$

Finally if in equilibrium the expert does not acquire the signal with positive probability $\sigma_c^*(c) < 1$ there can be additional constraints on the behavior of the expert of type $s = \varnothing$.

Denote

$$\underline{x}(p) = \frac{I(1+\lambda) + 2c - E(\theta|s_H)R}{p(1+\lambda) - E(\theta|s_H)R}, \tag{5}$$

$$\overline{x}(p) = \frac{I(1+\lambda) - 2c - E(\theta|s_L)R}{p(1+\lambda) - E(\theta|s_L)R}, \tag{6}$$

$$\phi = \frac{4c}{(E(\theta|s_H) - E(\theta|s_L))R}.$$

Note that $\phi \in (0,1)$ because $E(\theta|s_H)R > I + 2c$ and $I > E(\theta)R = \frac{1}{2}(E(\theta|s_H) + E(\theta|s_L))R$.

Proposition 1. *The incentive compatibility condition (3) is equivalent to* $x \in [\underline{x}(p), \overline{x}(p)]$. *The interval* $[\underline{x}(p), \overline{x}(p)]$ *is not empty iff*

$$p \geq I + \frac{\phi}{1-\phi}\frac{(1+\lambda)I - E(\theta)R}{1+\lambda}. \tag{7}$$

Proof. From the first inequality in (3) we express the minimum fraction x of the project which the expert must sell in equilibrium $\underline{x}(p)$, from the second inequality in (3) we express $\overline{x}(p)$. Since $I(1+\lambda) > E(\theta|s_H)R$ (assumption 3) condition $xp(1+\lambda) + (1-x)E(\theta|s_H)R \geq I(1+\lambda) + 2c$ and the fact that $x \in [0,1]$ guarantee $p \geq I + \frac{2c}{1+\lambda} > \frac{E(\theta|s_H)R}{1+\lambda}$. Also $I(1+\lambda) > E(\theta|s_H)R$ implies $I(1+\lambda) - 2c - E(\theta|s_L)R > 0$. Hence $\{x,p\}$ satisfies (5) and (6) iff it satisfies (3) .

To show that the interval $[\underline{x}(p), \overline{x}(p)]$ is not empty iff $I + \frac{\phi}{1-\phi}\frac{(1+\lambda)I - E(\theta)R}{1+\lambda}$ substitute for $\overline{x}(p)$ and $\underline{x}(p)$ into $\underline{x}(p) \leq \overline{x}(p)$ and get a condition: $4c(p(1+\lambda) - \frac{1}{2}E(\theta|s_H)R - \frac{1}{2}E(\theta|s_L)R) \leq (p-I)(1+\lambda)(E(\theta|s_H) - E(\theta|s_L))R$. Substituting for ϕ we obtain $I + \frac{\phi}{1-\phi}\frac{(1+\lambda)I - E(\theta)R}{1+\lambda}$ QED.

The proposition is intuitive, at date 2 the expert can invest in the investment opportunity λ which delivers $1+\lambda$ per unit of investment. If the expert retains the shares in the project she misses this investment opportunity, hence the more shares she sells the higher is her expected profit in equilibrium. The maximum and the minimum number of shares the expert can sell is pinned down by the incentive compatibility constraint, which ensures that the expert is willing to acquire the signal. The expert acquires the signal and invest in the project in the case of the good signal only if she can sell at least fraction $\underline{x}(p)$ of shares at date 2, otherwise she prefers to keep the cash. At the same time the expert must not be willing to invest in the project without any signal, which requires her to sell not more than $\overline{x}(p)$ of shares at date 2. Both $\underline{x}(p)$ and $\overline{x}(p)$ decrease with the price. If the price is low $p < I + \frac{\phi}{1-\phi}\frac{(1+\lambda)I - E(\theta)R}{1+\lambda}$, then $\underline{x}(p) > \overline{x}(p)$ and the incentive compatibility is impaired.

Corollary 1. *Condition (7) implies (1) :* $I + \frac{\phi}{1-\phi}\frac{(1+\lambda)I - E(\theta)R}{1+\lambda} > \frac{E(\theta|s_H)R}{1+\lambda}$.

Proof. First, $I + \frac{2c}{1+\lambda} > \frac{E(\theta|s_H)R}{1+\lambda}$ since $I(1+\lambda) > E(\theta|s_H)R$ by assumption 3. Second, $\phi((1+\lambda)I - E(\theta)R) > \phi(E(\theta|s_H) - E(\theta))R = 2c$ and $\phi < 1$ imply $I + \frac{\phi}{1-\phi}\frac{(1+\lambda)I - E(\theta)R}{1+\lambda} > I + \frac{2c}{1+\lambda}$, hence $I + \frac{\phi}{1-\phi}\frac{(1+\lambda)I-E(\theta)R}{1+\lambda} > \frac{E(\theta|s_H)R}{1+\lambda}$ QED.

Assumption 3 states that the expert is willing to invest in the project only if she can sell a fraction of it at date 2. Naturally the minimum share price necessary to induce the expert to acquire the information and to invest is higher than the return the expert can get by keeping the shares in the project.

Corollary 2. *In the informative equilibrium the expert never sells the whole project:* $\overline{x}(p) < 1$.

Proof. From (7) we have $p \geq I$, hence $\overline{x}(p) = \frac{I(1+\lambda) - 2c - E(\theta|s_L)R}{p(1+\lambda) - E(\theta|s_L)R} < 1$ QED.

2.1 Fully informative equilibrium

Consider the fully informative equilibrium, where the expert acquires the signal with probability one $\sigma_c^*(c) = 1$. In this equilibrium Bayes' rule (4) gives $\mu(s_H|x,p) = 1$, and condition (2) becomes $p \leq E(\theta|s_H)R$. No additional constraint on the behavior of the expert of type $s = \varnothing$ is required because this type does not appear in equilibrium.

Theorem 2. *The fully informative equilibrium exists if and only if*

$$c \leq c^* = \frac{1}{4}\frac{(E(\theta|s_H) - I)(E(\theta|s_H) - E(\theta|s_L))R}{E(\theta|s_H) - E(\theta)/(1+\lambda)}. \tag{8}$$

Proof. The necessary condition $p \geq I + \frac{\phi}{1-\phi}\frac{(1+\lambda)I-E(\theta)R}{1+\lambda}$ is satisfied for the highest possible price $p = E(\theta|s_H)R$ iff $\phi \leq \frac{E(\theta|s_H) - I/R}{E(\theta|s_H) - E(\theta)/(1+\lambda)}$, which is equivalent to $c \leq c^* = \frac{1}{4}\frac{(E(\theta|s_H)R - I)(E(\theta|s_H) - E(\theta|s_L))}{E(\theta|s_H) - E(\theta)/(1+\lambda)}$.

If the necessary condition $p \geq I + \frac{\phi}{1-\phi}\frac{(1+\lambda)I-E(\theta)R}{1+\lambda}$ is satisfied for $p = E(\theta|s_H)R$ the fully informative equilibrium exists. Indeed in this case the interval $[\underline{x}(p), \overline{x}(p)]$ is not empty. Take $x \in [\underline{x}(p), \overline{x}(p)]$, $p = E(\theta|s_H)R$, let beliefs to be $\mu(s_H|x,p) = 1$ and $\mu(s_H|x',p') = 0$ for any $\{x',p'\} \neq \{x,p\}$, then the pair $\{x,p\}$ satisfies all equilibrium conditions (1), (2), (3) and (4). Indeed, conditions (4) and (2) are satisfied. By corollary 2 condition (1) follows from (7). Condition (3) is satisfied for $x \in [\underline{x}(p), \overline{x}(p)]$.

Conversely if $p \geq I + \frac{\phi}{1-\phi}\frac{(1+\lambda)I-E(\theta)R}{1+\lambda}$ is not satisfied for $p = E(\theta|s_H)R$ the fully informative equilibrium does not exist. Indeed due to (2) only prices $p \leq E(\theta|s_H)R$ are admissible in the fully informative equilibrium, hence the necessary condition $p \geq I + \frac{\phi}{1-\phi}\frac{(1+\lambda)I-E(\theta)R}{1+\lambda}$ does not hold QED.

Naturally the fully informative equilibrium exist when the cost of the signal c is small. The autarky equilibrium where the expert invests in the alternative investment opportunity always exist. Interestingly the necessary and sufficient condition for the informative equilibrium is more likely to be satisfied when the alternative investment opportunity is not too attractive (when λ is small), that is the expert's desire to exit is not too strong. One can think that λ is related to the interest rate in the economy: the higher the interest rate the higher the return the expert can get elsewhere, the less willing the expert is to invest in the new project. Alternatively high λ might correspond to an environment where the experts are likely to face a liquidity need: this could be the case if the economy is exposed to macroeconomic shocks or natural disasters. These observations suggest that in economies with high interest rates and high exposure to external shocks the experts are less likely to engage in new projects.

Taking into account all equilibrium conditions we can compute the highest expected profit the expert can achieve in the fully informative equilibrium if the latter exists.

Problem 1. $\max\limits_{\{x,p\}} \frac{1}{2}[xp(1+\lambda)+(1-x)E(\theta|s_H)R]+\frac{1}{2}I(1+\lambda)-c$
s.t. $x \in [\underline{x}(p), \overline{x}(p)]$, *(5), (6),* $p \in [I+\frac{\phi}{1-\phi}\frac{(1+\lambda)I-E(\theta)R}{1+\lambda}, E(\theta|s_H)R]$.

The demand for shares for a given price p is determined by the maximum number of shares which can be sold $\overline{x}(p)=\frac{I(1+\lambda)-2c-E(\theta|s_L)R}{p(1+\lambda)-E(\theta|s_L)R}$.

Proposition 2. *In the fully informative equilibrium with the highest expert's profit, the expert sets the highest possible price and sells the highest possible fraction of shares:* $p=E(\theta|s_H)R$ *and* $x=\overline{x}(p)$*. The expert's expected profit is*

$$\pi^*=\frac{1}{2}E(\theta|s_H)[R+\lambda\frac{I(1+\lambda)-2c-E(\theta|s_L)R}{E(\theta|s_H)(1+\lambda)-E(\theta|s_L)}]+\frac{1}{2}I(1+\lambda)-c.$$

Proof. First we prove that $x=\overline{x}(p)$. The expert's expected profit is continuous in x and p, if $c \leq c^*$ the optimization set is not empty and compact, hence the maximization problem has a solution. Take a solution x, p and suppose that constraint $x \leq \overline{x}(p)$ is not binding. Then a deviation to $x'=x+\varepsilon$ for ε small enough is feasible. This deviation is profitable because $p(1+\lambda)>I(1+\lambda)>E(\theta|s_H)R$ by assumption 3, a contradiction, therefore $x=\overline{x}(p)$.

We substitute $x=\overline{x}(p)$ in the expert's profit and drop the terms independent of x and p, we get a problem equivalent to the problem 1:
$\max\limits_{p}\frac{1}{2}\frac{I(1+\lambda)-2c-E(\theta|s_L)R}{p(1+\lambda)-E(\theta|s_L)R}[p(1+\lambda)-E(\theta|s_H)R]$ s.t. $p \in [I+\frac{\phi}{1-\phi}\frac{(1+\lambda)I-E(\theta)R}{1+\lambda}, E(\theta|s_H)R]$.

The objective function is increasing in p. First, $\frac{p(1+\lambda)-E(\theta|s_H)R}{p(1+\lambda)-E(\theta|s_L)R}=1-\frac{(E(\theta|s_H)-E(\theta|s_L))R}{p(1+\lambda)-E(\theta|s_L)R}$ increases with p. Second, $I(1+\lambda)-2c-E(\theta|s_L)R>0$ because $(1+\lambda)I \geq E(\theta|s_H)$ (assumption 3) and $E(\theta|s_H)-2c-I>0$ (assumption 2). Given that $\phi<1$ it follows that in optimum the constraint $p \leq E(\theta|s_H)R$ must bind. Using $p=E(\theta|s_H)R$ we get $x=\overline{x}(p)=\frac{I(1+\lambda)-2c-E(\theta|s_L)R}{E(\theta|s_H)R(1+\lambda)-E(\theta|s_L)R}$. Substituting for p and x we obtain π^* QED.

This result is not obvious, since demand for shares $\overline{x}(p)$ is downward sloping and the expert could underprice in order to sell more shares. For instance Allen and Faulhaber (1989) find that underpricing can happen in equilibrium. In their paper firms signal the high type by selling the shares at a low price in the initial public offering and then recoup this loss with the subsequent equity sales. In our model the expert prefers to exit at date 2, that is she cannot wait for subsequent equity sales, in equilibrium she does not underprice but signals the high type by selling only a fraction of the equity to outsiders.

From now on whenever we refer to a fully informative equilibrium we imply the equilibrium with the highest expert's profit.

Corollary 3. *If $c < c^*$ then the profit the expert can achieve in the fully informative equilibrium is strictly higher than the expert's profit in the autarky: $\pi^* > I(1+\lambda)$.*

Proof. We express $\pi^* - I(1+\lambda) = \frac{1}{2}(E(\theta|s_H)R - I)(1 - \lambda\frac{E(\theta|s_L)}{E(\theta|s_H)(1+\lambda)-E(\theta|s_L)}) - c(1 + \lambda\frac{E(\theta|s_H)}{E(\theta|s_H)(1+\lambda)-E(\theta|s_L)})$ and we obtain $\pi^* > I(1+\lambda)$ iff $c < c^{**} = \frac{1}{2}\frac{(E(\theta|s_H)R-I)(E(\theta|s_H)-E(\theta|s_L))}{E(\theta|s_H)(1+\frac{\lambda}{1+\lambda})-E(\theta|s_L)\frac{1}{1+\lambda}}$. The fully informative equilibrium exists for $c \leq c^* = \frac{1}{4}\frac{(E(\theta|s_H)R-I)(E(\theta|s_H)-E(\theta|s_L))}{E(\theta|s_H)-E(\theta)\frac{1}{1+\lambda}}$. Provided that $2(E(\theta|s_H)-E(\theta)\frac{1}{1+\lambda}) - (E(\theta|s_H)(1+\frac{\lambda}{1+\lambda}) - E(\theta|s_L)\frac{1}{1+\lambda}) = (E(\theta|s_H)+E(\theta|s_L)-2E(\theta))\frac{1}{1+\lambda} = 0$ we have $c^* = c^{**}$, hence for $c < c^*$ the highest expected profit the expert can get in the fully informative equilibrium is higher than the profit the expert obtains in the autarky QED.

2.2 Partially informative equilibrium

In a partially informative equilibrium the expert acquires the signal with probability $\sigma_c^*(c) < 1$. Two variations of the equilibrium are possible. In the first variation the expert who has not acquired the signal does not invest with a positive probability $\sigma_I^*(I|\varnothing) < 1$. The expert's expected profit in this equilibrium is the same as in the autarky $I(1+\lambda)$, and such an equilibrium is of a little interest. In the second variation the expert who has not acquired the signal invests in the project with probability one $\sigma_I^*(I|\varnothing) = 1$, we consider this equilibrium.

For the equilibrium to exist condition (1), (2), (3) and (4) must hold. On top of that the type $s = \varnothing$ must prefer to invest in the project

$$xp(1+\lambda) + (1-x)E(\theta|\varnothing)R \geq I(1+\lambda).$$

It turns out that this condition is implied by constraint (3): since the expert is indifferent about acquiring the signal the second inequality in (3) binds, summing up the two inequalities in (3) we obtain the above condition.

Theorem 3. *The partially informative equilibrium exists if and only if $c < c^*$.*

Proof. For any $c < c^*$ the fully informative equilibrium with price $p = E(\theta|s_H)$ exists, moreover (7) holds as a strict inequality. Let us construct a partially informative equilibrium. Given that $E(\theta|s_H) > I + 2c$ we can find $p' < p$ which respects (7), by corollary 2 condition (1) is satisfied. Take $x \in [\underline{x}(p'), \overline{x}(p')]$ so that by proposition 1 condition (3) is satisfied. Choose $\mu = \mu(s_H|x,p) < 1$ to satisfy (2): $p' = \mu E(\theta|s_H) + (1-\mu)E(\theta)$. Finally let both types of experts s_H and $\varnothing$ to propose p and x with probability 1, Bayes' rule becomes $\mu(s_H|x,p) = \frac{\frac{1}{2}\sigma_c^*(c)}{\frac{1}{2}\sigma_c^*(c)+1-\sigma_c^*(c)}$, it is satisfied with $\sigma_c^*(c) = \frac{2\mu}{1+\mu} < 1$. That is we have constructed the partially informative equilibrium, hence it exists.

Suppose $c^* \geq c$, then constraint (7) can be satisfied only for $p \geq E(\theta|s_H)$. Thus if the partially informative equilibrium exists it must have $p = E(\theta|s_H)$, this implies $\mu(s_H|x,p) =$

$\frac{\frac{1}{2}\sigma_c^*(c)}{\frac{1}{2}\sigma_c^*(c)+1-\sigma_c^*(c)} = 1$ and $\sigma_c^*(c) = 1$, which contradicts the definition of the partially informative equilibrium. Hence for $c^* \geq c$ the partially informative equilibrium does not exist QED.

This result is not surprising. Roughly it says that if the necessary and sufficient condition (8) for the existence of the fully informative equilibrium holds with some slack, one can construct a partially informative equilibrium which in its properties is very close to the fully informative equilibrium.

Proposition 3. *The expert's profit in the partially informative equilibrium is always lower than the expert's profit in the fully informative equilibrium.*

Proof. The set of pairs x, p feasible in the partially informative equilibrium is obtained from the set feasible in the fully informative equilibrium by replacing the constraint $p \leq E(\theta|s_H)$ with $p \leq \mu E(\theta|s_H) + (1-\mu)E(\theta) < E(\theta|s_H)$. Due to proposition 3 in the fully informative equilibrium the expert's expected profit is maximized for $p = E(\theta|s_H)$, from the proof of the proposition it follows that the profit is strictly lower than the maximum for any $p < E(\theta|s_H)$. Consequently the expert's profit in the partially informative equilibrium is lower than her profit in the fully informative equilibrium QED.

If the projects' returns are uncorrelated or if there is only one expert on the market the informative equilibrium exists only if the cost of the signal is not too high. Otherwise the no-market (autarky) equilibrium prevails. In the informative equilibrium the expert sells only a fraction $\overline{x}(p) < 1$ of shares at date 2 in order to signal that she got the high signal. The expert's expected profit is the highest in the fully informative equilibrium where the expert acquires the signal with probability one.

3 Correlated returns (multiple experts)

Consider the general model with N experts who invest in projects belonging to a field, t the returns of projects in the field are correlated as in section 1. Each expert $i = 1, ..., N$ can acquire a signal and invest in a project belonging to the field. In this section we show that with sufficiently many experts an active market at date 2 can emerge even if the expert sells all shares in the project ($x_i = 1$).

Definition 3. *The symmetric informative equilibrium in pure strategies, where each expert investigates a project; invests if gets the high signal; does not invest if gets the low signal, is **full-exit** if the expert sells all shares in the project at date 2.*

Consider the sale of projects at date 2.

Proposition 4. *In the full-exit equilibrium the project's price is*

$$p(k) = (\theta_L + \Delta \Pr(\theta_H|k))R, \tag{9}$$

$$\Pr(\theta_H|k) = \frac{q^k(1-q)^{N-k}}{q^k(1-q)^{N-k} + (1-q)^k q^{N-k}},$$

the price is an increasing function of the number of projects on the market k.

Proof. In the full-exit equilibrium every expert investigates and starts the project only in the case of the high signal, hence the number of projects pins down the number of high signals. Bayesian updating requires the outsiders' belief about the signal of expert i to be $\mu_i^*(s_H|I_i, x_i, p_i) = 1$ if the expert has invested ($I_i = I$) and $\mu_i^*(s_H|I_i, x_i, p_i) = 0$ if the expert has not invested ($I_i = 0$). We assume the beliefs not to depend on the offer $\{x_i, p_i\}$. If k experts invested in projects at date 2 the expected return of a project in the field is $E(\theta|k)R = (\theta_L + \Delta\Pr(\theta_H|k))R$, given that $\Pr(s_H|\theta_H) = q > 1/2$ and $\Pr(s_L|\theta_H) = 1 - q$ we have $\Pr(\theta_H|k) = \frac{q^k(1-q)^{N-k}}{q^k(1-q)^{N-k}+(1-q)^k q^{N-k}}$ (note that $\Pr(\theta_H|k) = \frac{1}{1+\left(\frac{q}{1-q}\right)^{N-2k}}$ increases with k). Outsiders are ready to pay $p \leq E(\theta|k)R$ independently of offers $\{x_i, p_i\}$. Expert i who invested in the project and received signal s expects the project to deliver $E(\theta|k, s)R \leq E(\theta|k)R$ (we have equality only if $s = s_H$), her expected profit at date 2 is $x_i p_i(1 + \lambda) + (1 - x)E(\theta|k, s)R$. She optimally sets $x_i = 1$, $p_i = E(\theta|k)$. Thus the price at stage 2 is $p(k) = (\theta_L + \Delta\Pr(\theta_H|k))R$, it increases with k since $\Pr(\theta_H|k)$ increases with k QED.

Proposition 5. *The expert's expected profit in the full-exit equilibrium is*

$$\pi = \frac{1}{2}(1+\lambda)(\theta_L + q\Delta)R + \frac{1}{2}(1+\lambda)I - c.$$

Proof. In the full-exit equilibrium each expert invests only if gets the high signal. Suppose expert i acquired signal s_H, then the probability that $N-1$ other experts bring k' projects to the market is $\Pr(k'|s_H, N-1) = \Pr(\theta_H|s_H)\Pr(k'|\theta_H, N-1) + \Pr(\theta_L|s_H)\Pr(k'|\theta_L, N-1)$. If expert i invests and other $N-1$ experts invest in k' projects there are $k'+1$ projects on the market, expert i gets $\pi(I_i = I|s_H) = (1+\lambda)\sum_{k'=0}^{N-1} p(k'+1)\Pr(k'|s_H, N-1)$. From proposition 4 $p(k) = (\theta_L + \Delta\Pr(\theta_H|k))R$, $\Pr(\theta_H|k) = \frac{q^k(1-q)^{N-k}}{q^k(1-q)^{N-k}+(1-q)^k q^{N-k}}$ hence $\sum_{k'=0}^{N-1} p(k'+1|N)\Pr(k'|s_H, N-1) = \theta_L R + \Delta R\sum_{k'=0}^{N-1}\Pr(\theta_H|k'+1)\Pr(k'|s_H, N-1)$, further $\sum_{k'=0}^{N-1}\Pr(\theta_H|k'+1)\Pr(k'|s_H, N-1) = \sum_{k'=0}^{N-1}\frac{q^{k'+1}(1-q)^{N-k'-1}[q^{k'+1}(1-q)^{N-1-k'}+(1-q)^{k'+1}q^{N-1-k'}]}{q^{k'+1}(1-q)^{N-k'-1}+(1-q)^{k'+1}q^{N-k'-1}}C_{N-1}^{k'} = q$ thus $\pi(I_i = I|s_H) = (1+\lambda)(\theta_L + q\Delta)R$.

If the expert receives the low signal she does not invest $\pi(I_i = 0|s_L) = (1+\lambda)I$. Taking into account the cost of the signal the expert's expected profit in the full-exit equilibrium is $\pi = \frac{1}{2}(1+\lambda)(\theta_L + q\Delta)R + \frac{1}{2}(1+\lambda)I - c$ QED.

Remark 1. The expert's expected profit in the full-exit equilibrium is higher than the expert's expected profit in the informative equilibrium with $N = 1$: $\pi > \pi^*$.[6]

[6]Indeed $\pi - \pi^* = \frac{1}{2}E(\theta|s_H)\lambda\frac{(E(\theta|s_H)R-I)(1+\lambda)+2c}{E(\theta|s_H)(1+\lambda)-E(\theta|s_L)} > 0$.

Theorem 4. *A critical number $N \geq 2$ of independent experts is required for the full-exit equilibrium to exist. There exist a number $N^* < \infty$ such that for any $N \geq N^*$ the full-exit equilibrium exists.*

The proof can be found in the appendix.

4 Discussion

Proposition 4 illustrates the idea of wisdom of the crowd and provides an explanation for the existence of hot IPO markets similar to the dot-com boom of late 90s. When young firms doing new business, as the Internet business was in the 90s, start to offer their shares at IPO many potential investors have insufficient capacity and expertise to evaluate the prospects of these firms and of the new field in general. The proposition suggests that one piece of information for investors is the number of young firms operating in this new field. If many firms in a particular field make it to the initial public offering it must be the case that venture capitalists and other early stage investors found these firms promising, as a result the potential investors become optimistic during the IPOs, the share prices go up and heat the market. As proposition 4 highlights the crowd of investors at IPOs follows the crowd of venture capitalists who invested in the new firms before IPOs. The model predicts the waves of IPOs to be foreshadowed by waves of venture capital investment in new fields (we stress that the model does not apply to conventional businesses).

Note that the crowd of investors buying new shares at high prices is wise because the firms are traded at their expected value. This, however, does not preclude the bust episodes for the whole industries as the actual realization of the share value may well be lower than its expected value. What makes the bust episodes dramatic and attracts a lot of public attention is the simultaneous drop in value of many firms operating in the new field. Our results suggest that this phenomenon is natural since the emergence of the market for young firms operating in a new field is facilitated by the correlation of firms' returns. Indeed if all firms are independent the full-exit equilibrium does not exist. In this case the best equilibrium from the expert's point of view is the fully informative equilibrium described in proposition 3, in which the expert sells only a fraction of her project at date 2 and, due to proposition 5, gets a lower profit then in the full-exit equilibrium .

According to theorem 4 experts can sell all shares in the projects at date 2 only when sufficiently many $N \geq 2$ experts investigate and invest, so that the number of projects started is a reliable signal for outsiders about the projects' prospects. Indeed if $N = 1$ there can be either one project on the market or no projects at all. If outsiders were to perceive the project being on the market as good news, then the expert would always start the project irrespectively of her signal (in fact she would choose not to acquire the signal at all). But then the project being on the market would convey no information to outsiders, and they would be ready to pay only its ex ante expected return. Provided that a project started at random generates a negative expected return it would not be profitable for the expert to start the project. More generally when N is small a single expert can manipulate the market and benefit by starting a project without any information. If such a speculation by a single expert can push the prices at the second stage high enough to make the speculation profitable, the market at date 2 ceases to exist, because the outsiders expect speculations and do not trust the market. The situation is different with many experts ($N \geq N^*$), then the marginal impact of the expert's investment on price is negligible, which makes speculations not profitable and motivates experts to acquire information.

A natural illustration of this result is the venture capital investment and initial public offerings of new technology firms.[7] The model predicts that an active IPO market for new projects originates when there are many independent venture capitalists capable of evaluating new projects in a field. If it happens that each venture capitalist independently investigates a particular start-up in the field and decides to invest in this start-up only if she finds it promising, then the number of start-ups that are brought to the IPO by venture capitalists reflects the overall prospects of the start-ups in the field. This creates a liquid IPO market for start-ups and permits venture capitalists to sell their stakes to outsiders at a fair price. Anticipation of the liquid IPO market in turn makes venture capitalists willing to investigate and invest in the start-ups.

Even if there are many independent venture capitalists capable of investigating and investing in start-ups each venture capitalist might not invest in a new start-up if others do not invest. This is due to the strategic complementarity among decisions of experts to investigate and invest. If no one investigates the market price at date 2 does not reveal the projects' prospects, that is the outsiders cannot tell from the aggregate supply of projects the underlying quality of the projects. In this case the price offered for a project is low, which makes investigation unprofitable for an expert and no expert starts a project in the first place. This corresponds to the no-market (autarky) equilibrium described in theorem 1.

Two extensions of the model seem natural. First, one can introduce competition among projects belonging to the same field. A way to accomplish this is to assume that the return of each project $R(k)$ is a decreasing function of the total number of projects k. If the expert's investment decision is based only on her signal, then such an extension would account for episodes of overinvestment in certain industries mentioned by DeMarzo et al. (2007).[8] Intuitively if an expert investigates a projects and finds out the project to be promising, she invests in it. With certain probability it can happen that many experts independently find similar projects promising, which is good news for the field the projects belong to, but might be bad news for individual projects because of the intensive competition. In such a situation excessive investment and entry into the industry is possible.

Second, one can let the experts to start projects in different fields. This would allow to see if the

[7]It has to be stressed that the model applies only to firms doing innovative business which an outsider finds difficult to evaluate, thus traditional businesses are not covered by the model.

[8]DeMarzo et al. (2007) speculate that there was an overinvestment by telecom companies from 1996 to 2000 in the fiber optic communication lines which resulted in at least six national communication networks in the US, it also argues that "rail mania" in Britain in the mid 19th century lead to a similar overinvestment.

need for a critical mass of experts working in a particular field pushes the experts to concentrate on one popular filed in stead of spreading the experts' resources more evenly among different fields. This analysis relates to the practices in academic research, where the concentration of research effort in certain fashionable areas seems to be common.

Appendix

Proof of theorem 4. In the full-exit equilibrium each expert invests only if gets the high signal, from proposition 5 we have$\pi(I_i = I|s_H) = (1+\lambda)(\theta_L + q\Delta)R$. If expert does not invest $\pi(I_i = 0|s_H) = (1+\lambda)I$, from assumption 2 follows $(\theta_L + q\Delta)R > I$, hence the expert invests if she gets the high signal.

If the expert invests when $s = \varnothing$ the probability of k' projects being on the market is $\Pr(k'|N-1) = \Pr(\theta_H)\Pr(k'|\theta_H, N-1) + \Pr(\theta_L)\Pr(k'|\theta_L, N-1) \leq \Pr(k'|s_H, N-1)$. If invests expert i gets $\pi(i = I|\varnothing) = (1+\lambda)\sum_{k'=0}^{N-1} p(k'+1)\Pr(k'|N-1)$. Using proposition 4 and substituting $k = k'+1$ we express $\sum_{k'=0}^{N-1} p(k'+1|N)\Pr(k'|N-1) = \theta_L R + \Delta R \sum_{k=1}^{N} \Pr(\theta_H|k)\Pr(k-1|N-1)$, further

$$\sum_{k=1}^{N} \Pr(\theta_H|k)\Pr(k-1|N-1) = \sum_{k=1}^{N} \frac{q^k(1-q)^{N-k}[\frac{1}{2}(1-q)^{k-1}q^{N-k} + \frac{1}{2}q^{k-1}(1-q)^{N-k}]}{q^k(1-q)^{N-k} + (1-q)^k q^{N-k}} C_{N-1}^{k-1}$$

which becomes

$$\frac{1}{2}\sum_{k=1}^{N}(1 + \frac{(2q-1)(1-q)^{k-1}q^{N-k}}{q^k(1-q)^{N-k} + (1-q)^k q^{N-k}})q^{k-1}(1-q)^{N-k}C_{N-1}^{k-1} = \frac{1}{2}(1 + (2q-1)Z(N,q)),$$

here

$$Z(N,q) = \sum_{k=1}^{N} \frac{q^{N-1}(1-q)^{N-1}C_{N-1}^{k-1}}{q^k(1-q)^{N-k} + (1-q)^k q^{N-k}}.$$

We obtain $\pi(I_i = I|\varnothing) = (\theta_L + \Delta(\frac{1}{2} + (q-\frac{1}{2})Z(N,q))R(1+\lambda)$.

The expert must be willing to acquire the signal for $c > 0$, if she does so she gets $E(\pi(c_i = c)) = \frac{1}{2}\pi(I_i = I|s_H) + \frac{1}{2}(1+\lambda)I - c$. Incentive compatibility requires $E(\pi(c_i = c)) \geq \max[\pi(I_i = I|\varnothing), (1+\lambda)I]$. Given that $\frac{1}{2}\pi(i = I|s_H) + \frac{1}{2}\pi(i = 0|s_L) - c - \frac{1}{2}(1+\lambda)I = \frac{1}{2}(1+\lambda)((\theta_L + q\Delta)R - I) - c \geq 0$ by assumption 2 we need to check $\frac{1}{2}\pi(i = I|s_H) + \frac{1}{2}I(1+\lambda) - c \geq \pi(I_i = I|\varnothing)$. After substitutions we get $\frac{1+\lambda}{2}((\theta_L + q\Delta)R + I - 2(\theta_L + \Delta(\frac{1}{2} + (q-\frac{1}{2})Z(N,q))R) \geq c$ which is equivalent to

$$Z(N,q) \leq Z^* = \frac{I - (\theta_L + \frac{\Delta}{2})R + \Delta(q-\frac{1}{2})R - \frac{2c}{1+\lambda}}{2\Delta(q-\frac{1}{2})R}.$$

The expert is willing to acquire the signal iff $Z(N,q) \leq Z^*$.

It remains to check that for $Z(N,q) \leq Z^*$ the expert does not invest after receiving the low signal: $\pi(I_i = I|s_L) < (1+\lambda)I$. Compute $\pi(I_i = I|s_L) = \theta_L R(1+\lambda) + \Delta R(1+\lambda)\sum_{k'=0}^{N-1} \Pr(\theta_H|k'+1)\Pr(k'|s_L, N-1)$, express

$\sum_{k'=0}^{N-1} \Pr(\theta_H|k'+1)\Pr(k'|s_L, N-1) = \sum_{k'=0}^{N-1} \frac{q^{k'+1}(1-q)^{N-k'-1}[q^{k'}(1-q)^{N-k'}+(1-q)^{k'}q^{N-k'}]}{q^{k'+1}(1-q)^{N-k'-1}+(1-q)^{k'+1}q^{N-k'-1}} C_{N-1}^{k'}$, further $\sum_{k'=0}^{N-1}(1-q)(1+\frac{(1-q)^{k'-1}q^{N-k'+1}-(1-q)^{k'+1}q^{N-k'-1}}{q^{k'+1}(1-q)^{N-k'-1}+(1-q)^{k'+1}q^{N-k'-1}})C_{N-1}^{k'}q^{k'}(1-q)^{N-k'-1} = (1-q)+(2q-1)Z(N,q)$. Condition $\pi(I_i = I|s_L) < (1+\lambda)I$ is equivalent to

$$Z(N,q) < Z_L = \frac{I-(\theta_L+(1-q)\Delta)R}{2\Delta(q-\frac{1}{2})R}.$$

Given that $Z_L - Z^* = \frac{c}{(1+\lambda)\Delta(q-\frac{1}{2})R} > 0$ condition $Z(N,q) < Z_L$ is guaranteed by $Z(N,q) \leq Z^*$.

Therefore the full-exit equilibrium exists iff $Z(N,p) \leq Z^*$.

For $N = 1$ we have $Z(1,q) = 1 > Z^*$ since $I - (\theta_L + \frac{\Delta}{2})R + \Delta(q-\frac{1}{2})R - 2\Delta(q-\frac{1}{2})R = I - (\theta_L + q\Delta)R < 0$, that is the full-exit equilibrium does not exists for $N = 1$, and the necessary condition is that $N \geq 2$.

Provided that $Z^* = \frac{I-(\theta_L+\frac{\Delta}{2})R+\Delta(q-\frac{1}{2})R-\frac{2c}{1+\lambda}}{2\Delta(q-\frac{1}{2})R} > 0$ lemma 3 implies that we can find a finite N^* such that $\overline{Z}(N,q) \leq Z^*$ for $N \geq N^*$. Given that $Z(N,q) \leq \overline{Z}(N,q)$ condition $N \geq N^*$ is sufficient for the full-exit equilibrium to exist QED.

Lemma 3. *$Z(N,q) \leq \overline{Z}(N,q)$ for any N and q, $\overline{Z}(N,q)$ decreases with N and $\overline{Z}(N,q) \underset{N\to\infty}{\to} 0$.*

Proof. Let $\overline{Z}(N,q) = 2(4q(1-q))^{\frac{N}{2}-1}$, given that $q^k(1-q)^{N-k} + (1-q)^k q^{N-k} \geq 2(q(1-q))^{N/2}$ we get $Z(N,q) \leq \frac{(q(1-q))^{N/2}}{2q(1-q)}\sum_{k=1}^{N} C_{N-1}^{k-1} = \overline{Z}(N,q)$. From $q > 1/2$ it follows that $4q(1-q) < 1$, hence $\overline{Z}(N,q)$ decreases with N and $\overline{Z}(N,q) \underset{N\to\infty}{\to} 0$ QED.

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