Credit Default Swaps and Sovereign Debt with Moral Hazard and Debt Renegotiation

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Abstract

This paper studies how a lender's credit insurance activities affect a sovereign borrower in an environment with moral hazard and debt renegotiation. The moral hazard problem arises from the assumption of private information where the lender cannot observe if the sovereign invested or consumed the borrowed funds. We show that insurance serves as a commitment device for the lender. An insured lender has more bargaining power during debt reduction renegotiations and this enables him to extract more from the borrower. Thus, the existence of an insurance market alleviates the moral hazard problem by better aligning the incentives of the lender and the borrower. We also analyze the effect of naked buyers who do not lend directly to the sovereign. Our analysis shows that the market structure of the insurance market matters: if the market is imperfectly competitive, the existence of naked buyers can impede the alleviation of moral hazard.

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1 Introduction

With the recent European debt crises, sovereign credit default swaps (CDSs) have been blamed by politicians and regulators for increasing borrowing costs and exacerbating the debt crisis.[1] In May 2010, regulators in Germany temporarily banned the purchase of CDSs on euro zone government bonds by those who do not own the underlying bond, and in October 2011 the ban was made permanent and applicable across the European Union. An implicit argument in these criticisms and policy actions is that CDSs can somehow affect the underlying borrower. Most of the existing literature on credit derivatives, however, focuses on incentive problems between the lender and the insurer while abstracting from the effect on the borrower. But why might CDSs matter for the borrower? How might the lender's insurance activity affect the incentives and the welfare of the sovereign borrower?

This paper's answer is straightforward. The existence of CDSs can give lenders more leverage in ex-post renegotiation. This then alleviates ex-ante borrowing constraints, provides more external capital to the debtor country, increases investment and, therefore, welfare. CDSs can be beneficial for the borrower.

We derive this result using a framework with an agency problem between the lender and the sovereign borrower. The types of frictions we consider are motivated by two characteristics of sovereign debt. First, due to the lack of international law for sovereign bankruptcy, sovereign governments often manage to get away without fully repaying their debt[2] and this debt reduction is achieved through renegotiations with lenders. Second, due to asymmetric information about the sovereign's actions - such as investment - which in turn affect the repayment ability of the sovereign, the incentives of the borrower and the lender are mis-

[1]Credit default swaps are over-the-counter derivative contracts where the seller of the contract pays the buyer of the contract a pre-specified amount (called notional) when a credit event occurs (such as default by a firm or a government). In return, the buyer of the contract pays a periodic fee until either the contract matures or a credit event occurs. The contract specifies, among other things, the reference entity, the contract maturity date, the notional amount, and the events that constitute as a credit event.

[2]Benjamin and Wright (2009) constructed a database covering 90 defaults and renegotiations by 73 countries that occurred over the period of 1989-2006. Argentina, for example, defaulted in 2001 on its $94 billion international bonds, which, according to Benjamin and Wright (2009) estimates, led to creditor losses of 63% of the value of the debt.

aligned.[3] The asymmetric information, together with the inability of the lender to credibly deny any debt reduction, gives rise to a moral hazard problem on the part of the borrower: knowing that in a low-output state a debt reduction will be reached, the borrower does not invest enough to avoid a low future output. We model the asymmetric information friction similar to Atkeson (1991) and Gertler and Rogoff (1990), while the debt renegotiation framework is similar to Yue (2010).

In this setting, we find that insurance serves as a commitment device for the lender. It improves the lender's bargaining power during debt renegotiations enabling him to make the borrower pay more in the bad states of the world. The increase in the lender's bargaining power is due to insurance giving credibility to the lender's threat to deny a debt reduction. Because an insured lender is able to extract more repayment from the borrower, an insured lender can ex-ante offer better loan contracts (e.g. with a lower borrowing cost) than an uninsured lender. As the bad state is even less attractive to the borrower, the incentives of the borrower are better aligned with that of the lender, inducing the borrower to invest more efficiently. Thus, insurance has a disciplining effect on the borrower. The increased investment lowers the probability of default and the cost of borrowing. CDSs, in the end, are welfare improving as they alleviate the moral hazard problem.

These results are based on the assumption that lenders are the only agents purchasing insurance. In reality, there are investors, so called 'naked buyers', who purchase insurance but do not own the underlying bond. How robust are our findings if we allow for investors that purchase insurance but do not lend to the sovereign? We find that the existence of naked buyers impacts the debtor country only if the CDS market is concentrated (e.g. a monopoly) but not if it is perfectly competitive as we had assumed up to this point. If the insurer is a monopolist, he can indirectly affect the borrower's investment through the insurance contract offered to the lender. The insurer not only earns a profit from insuring

[3]If the borrower's investment and consumption are unobservable by the lender, then the borrower - who has secured a loan for an investment - can cheat and consume a part of what he was supposed to invest. This comes at a cost for the lender as less investment translates into a lower repayment ability.

the lender but also from insuring the naked buyer where, by assumption, the latter is affected by the borrower's investment.[4] The insurer designs the insurance contracts so as to induce the level of investment that maximizes his total profit. Thus, if there are naked buyers, the sovereign's investment is necessarily not the same as without the naked buyers: it is either lower or higher depending on the parameter values. Although these findings are based on assumptions that result in the insurer's ability to affect the borrower's investment, our analysis points to the importance of the market structure of the insurance market for whether the existence of naked buyers matters for the borrower.

What are the testable implications of our model? The model suggests that any mechanism that increases a lender's bargaining power during a debt restructuring process should discipline the borrower and result in a better macroeconomic performance and a lower probability of default. These predictions of our model shed light on the effects of a particular sovereign debt event: the 1989 Brady plan that ended the 1980's emerging markets debt crises. In the 1980's, emerging market economies, mostly in Latin America, repeatedly faced an inability to meet their debt obligations despite multiple rounds of debt reschedulings.[5] However, under the Brady plan, bank loans to these countries were converted into tradable bonds. A key feature of Brady bonds was that, to attract investors, they were collateralized by U.S. treasuries. Thus, if a sovereign defaulted on its Brady bonds, the bondholders would become owners of U.S. Treasuries instead.[6] The collateralization of Brady bonds is similar in spirit to the insurance contracts in our model. While the success of the Brady plan could be due to various factors, from the perspective of our model, one possible explanation for the improved macroeconomic performance and the low level of default that was observed after

[4]The naked buyer, we assume, has cash flows positively correlated to that of the sovereign's: his cash flow is high in the state where the sovereign's output is high and low in the sovereign's low output state. Then, by assumption, the probability that the naked buyer has a high cash flow increases with the borrower's investment; in other words, the borrower's investment affects the utility of the naked buyer. Thus, the naked buyer's utility (and hence the profit extracted by the insurer) depends not only on the naked buyer's own insurance but also on the insurance purchased by the lender since the latter affects the borrower's investment.

[5]One between 1982-1985 and one in 1985.

[6]The U.S. Treasuries were purchased with funding from official agencies (IMF, the World Bank), the Japanese government, and the debtor countries' own foreign reserves.

the Brady plan is the collateralization of Brady bonds: it might have changed the incentives of the sovereign.[7]

The framework of our model is similar to the standard moral hazard model in corporate finance with a lender and an entrepreneur who needs financing for a project (see, for example, Repullo and Suarez (1998), Tirole (2006)).[8] If the entrepreneur exerts effort, the project is more likely to succeed, but the entrepreneur's effort is noncontractible. There is an intermediate date at which the lender can choose to either liquidate the project, if he suspects that the entrepreneur shirked, or continue the project. If the lender can credibly commit to liquidate the project, the entrepreneur will exert more effort. But the lender's threat to liquidate is not credible ex-post, thus creating a moral hazard. Various mechanisms have been suggested that can make the lender's threat credible and discipline the borrower. Hart and Moore (1995), for example, suggest making the original lenders senior to new lenders who might come in and provide funding to continue the project. Dewatripont and Tirole (1994) argue for a diversity of tough and soft claimholders and Berglof and von Thadden (1994) show that short term lending can have a disciplining effect. Our paper suggests that insurance against default is another form of a such disciplining mechanism.

To summarize the literature on CDSs, most of the corporate CDS literature addresses asymmetric information between lenders and insurers about the loans on which the lenders purchase insurance.[9] However, sovereign debt is less prone to this type of asymmetric information since if information about a country is available to international lenders, then it is likely to be available to insurers. In contrast, our paper focuses on the effect of CDSs on the borrower-lender relationship. As our paper sheds light on CDS's effect on the probability

[7] Compared to the previous attempts at resolving debt crises where countries repeatedly defaulted, there was only one default on Brady bonds (by Ecuador). In our model, investment plays the same role as effort in standard principal agent models. Thus, an increase in investment would be analogous to an increase in policy effort by the sovereign which would lead to improved macroeconomic performance.

[8] See the discussion in Tirole (2006), section 5.5.

[9] See, for example, Duffee and Zhou (2001), Morrison (2005), Thompson (2007), Parlour and Winton (2008). See Acharya and Johnson (2007) for empirical support for asymmetric information problems between lenders and insurers. For an overview of the CDS market, see Stulz (2009) and Weistroffer (2009); and for an overview more specific to sovereign CDS, see Ranciere (2001), Packer and Suthiphongchai (2003) and Verdier (2004).

of default, it is also related to papers that study its effect on financial stability. Instefjord (2005) finds that CDSs can lead to banks taking on more risk. Allen and Carletti (2006) show that if banks face the same liquidity demand, credit risk transfer is beneficial, but if banks face an idiosyncratic risk, then credit risk transfer can increase the risk of financial crises. This paper, however, is most closely related to Arping (2004) and Bolton and Oehmke (2010) who both show the disciplining effect of CDSs in a framework with firm agency problems. However, these models are not applicable to sovereign debt as they are specific to corporate debt and bankruptcy and, hence, cannot capture the features of sovereign debt that distinguish it from corporate debt.

The next section lays out, first, the frictionless benchmark economy, followed by private information environment where we characterize the moral hazard problem. Then we introduce the insurance market and give our main result that demonstrates CDS's disciplining role. In appendix A.1, we show that insurance is inconsequential unless both private information and bargaining are present. Section 3 relaxes the perfectly competitive environment by considering a monopolistic insurer and looks at how the existence of naked buyers affects the debtor country. The last section concludes while the proofs of the results are relegated to the appendix.

2 The Model

Consider a small open economy representing the debtor country. There are two dates: t={0,1}. The borrower is risk neutral and does not discount, $U(c_0, c_1) = c_0 + E_0 c_1$, and can invest I at date 0 to earn a random output at date 1. The distribution of the output depends on the amount invested at date 0: output is high, $y_1 = y_h$, with probability $\pi(I)$ and low, $y_1 = y_l$, with probability $(1 - \pi(I))$ where $\pi'(I) > 0$, $\pi''(I) < 0$, $\pi(0) = 0$, and $\pi'(0)(y_h - y_l) > 1$.[10] Thus, the probability of the high state is increasing in investment. The borrower has zero endowment at date 0, but can borrow by trading one-period zero-coupon

[10]The last condition implies that positive investment is efficient.

bonds with risk neutral competitive foreign lenders. We denote the face value and the price of the bond as B and q respectively. $B > 0$ means the sovereign country is a net borrower: he receives $qB \geq 0$ consumption goods at date 0 and has to repay B at date 1 regardless of the state realized. The raised funds, qB, can be used for either consumption or investment, thus his date 0 consumption is: $c_0 = qB - I$.

The borrower has an incentive to repay at date 1 because if he defaults in full, he loses a fraction of his output, py_l. This cost of default can be interpreted as a reduced form for costs associated with temporary exclusion from credit markets, trade disruptions, or foreign investors' lack of confidence. If the borrower repays the loan, his consumption is:

$$c_1^{nd} = y_1 - B$$

While if he defaults in full, his consumption is:

$$c_1^d = y_1(1 - p)$$

We assume that the cost of default in the good output state is high enough to deter any default, while too low in the bad state. Thus a default can occur only in the bad state, which is an assumption consistent with the sovereign debt stylized fact that debtor countries typically default when they are in a recession.[11] In particular, the borrower defaults in the bad state by negotiating a debt reduction with the lender. If such a debt reduction agreement is reached, the borrower is able to avert the default cost. An interpretation of this assumption is that the ability of the sovereign to reach an agreement with the lenders is perceived positively by the market and prevents further loss of confidence by investors

[11] This fact can be reproduced in a dynamic model with a risk averse borrower. For a risk averse borrower, when output is already low, it hurts more to further lower consumption by paying off his debt. As our model is a two-period model with a risk neutral borrower, we resort to assuming this result. Nevertheless, we show in appendix A.4 that the results of this section hold in a more general setting where the borrower defaults and bargains both in the high and low states.

or trade partners.[12] From here on, we refer to defaults to mean partial defaults that occur through debt renegotiation when y_l is realized and the ex-ante probability of default is $1 - \pi(I)$. If an agreement is reached, $1 - \alpha$ and α are the shares of y_l going to the borrower and the lender respectively.[13]

Figure 1 shows the timeline of the model as well as the subgame that gets played out at date 1 once the borrower has borrowed and invested at date 0. When y_l is realized, a debt reduction agreement will be reached (as pointed out by the arrow) since the lender will prefer getting αy_l over nothing and for the borrower the bargaining outcome will be at least better than suffering the output loss py_l. While, if y_h is realized, the borrower will repay in full as indicated by the arrow.

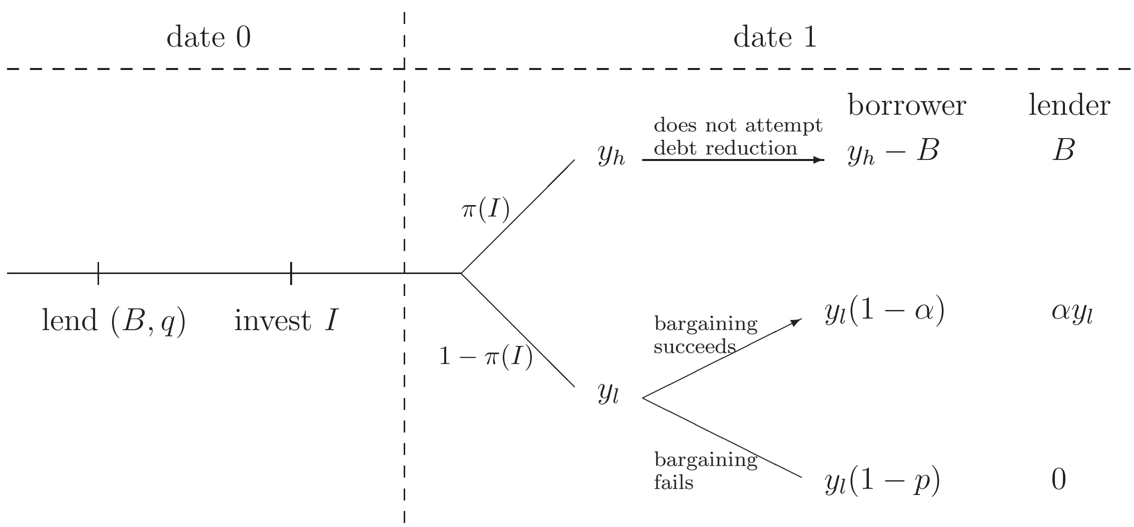


Figure 1: Model timeline

The loan contract signed by the borrower and the lender at date 0 will take into account the bargaining outcome of the subgame when y_l is realized at date 1. The borrower's expected

[12] A more general approach would be to have two types of default costs: one for partial default and a more costly one for full default, but for our purposes only the relative difference in default costs matters. Thus, we can assume the cost of partial default (i.e. for reaching a renegotiation agreement) is zero.

[13] This assumption is made for simplicity since usually debt renegotiations are over reductions of the actual debt, i.e. the borrower pays back α share of the original debt, B. Thus, more general approach would be for them to bargain over $min(B, y_l)$.

utility is:

$$c_0 + \beta E_0 c_1 = (qB - I) + \pi(I)(y_h - B) + (1 - \pi(I))y_l(1 - \alpha)$$

And the lender's zero-profit condition is:

$$\pi(I)B + (1 - \pi(I))\alpha y_l = qB$$

Next, we proceed by characterizing the outcome of the ex-post bargaining problem which will then be used to solve for the equilibrium loan contract and investment.

2.0.1 The Nash Bargaining Problem

Following the general approach of Yue (2010), we model the partial default (i.e. debt renegotiation or restructuring) that occurs when y_l is realized as an outcome from a Nash bargaining problem. We assume that the lender and the borrower have an equal bargaining power. If they fail to reach an agreement, the threat point for the borrower is being penalized by py_l and for the lender it is getting nothing. Thus, their respective surpluses from bargaining are:

$$\begin{aligned}
\Delta_B(a) &= (1 - a)y_l - (1 - p)y_l \\
\Delta_L(a) &= ay_l
\end{aligned}$$

The Nash bargaining solution is given by:

$$\begin{aligned}
\alpha = &\arg\max_{a \in [0,1]} \Delta_B(a)\Delta_L(a) \\
\text{s.t.} \qquad &\Delta_B(a) \geq 0 \qquad \text{i.e.} \qquad ay_l \leq py_l \\
&\Delta_L(a) \geq 0 \qquad \text{i.e.} \qquad ay_l \geq 0
\end{aligned}$$

Solving the above problem we get:

$$\alpha = \frac{1}{2}p$$

To interpret this result, since the lender's threat to the borrower is that the borrower's endowment will decrease by py_l, they are really bargaining over how to split py_l. Hence the outcome from bargaining is to split py_l in half because we have assumed that they have an equal bargaining power.

Now let us characterize the equilibrium debt level, bond price, and investment under the full information setting which we call the first best allocation. Because the lenders are competitive they compete to maximize the borrower's utility.

Definition: *The first best contract* (q^{FB}, B^{FB}, I^{FB}) is the optimal contract achieved under full information, bargaining setting. It maximizes the borrower's payoff subject to the lender's zero-profit condition:

Program $\mathcal{P}^{\mathcal{FB}}$:

$$\max_{I,q,B} \quad (qB - I) + \pi(I)(y_h - B) + (1 - \pi(I))y_l(1 - \alpha) \tag{$\mathcal{P}^{\mathcal{FB}}$}$$

$$\text{s.t.} \quad \pi(I)(B) + (1 - \pi(I))\alpha y_l = qB \tag{1}$$

$$qB - I \geq 0$$

where $\alpha = \frac{1}{2}p$. Solving this, I^{FB} and B^{FB} are given by:

$$\pi'(I^{FB})(y_h - y_l) = 1 \tag{2}$$

$$B^{FB} = \frac{I^{FB} - (1 - \pi(I^{FB}))\alpha y_l}{\pi(I^{FB})}$$

$$c_0 = q^{FB}B^{FB} - I^{FB} = 0$$

Equation (2) says that the borrower invests such that the marginal benefit of increasing investment (an increase in date 1 consumption) equals the marginal cost (decrease in date

0 consumption). If I was lower than I^{FB}, additional investment would yield extra date 1 consumption that more than compensates for the decrease in date 0 consumption.[14][15] Next let us introduce private information and characterize the moral hazard problem it creates.

2.1 Private Information

Under private information, the borrower's output is observable, but not his investment or consumption decision, and hence investment or consumption cannot be contracted upon. In this case, if the borrower is offered the first best loan contract (B^{FB}, q^{FB}), the borrower will not invest I^{FB} since his optimization problem is:

$$\max_{I} \quad (q^{FB}B^{FB} - I) + \pi(I)(y_h - B^{FB}) + (1 - \pi(I))y_l(1 - \alpha)$$

where FOC with respect to I gives:

$$\pi'(I)(y_h - y_l - (B^{FB} - \alpha y_l)) = 1 \tag{3}$$

Comparing (3) with (2), we see that as long as $B^{FB} \geq \alpha y_l$, due to the concavity of $\pi(I)$, the borrower will invest less than I^{FB} and consume the rest.

Intuition. With the loan contract (q^{FB}, B^{FB}) the borrower has secured himself the consumption profile of $y_h - B^{FB}$ in the good state and $y_l(1 - \alpha)$ in the bad state regardless of his investment, thus he can cheat and consume, unobserved by the lender, some of the

[14]In this setup, there is no point in borrowing more than I^{FB}, investing I^{FB}, and consuming the rest, $qB - I^{FB}$, since the borrower will have to repay this amount back in full in the high state. Thus, we can safely restrict our attention to the case where he will borrow $qB = I^{FB}$ and invest all of it without consuming any.

[15]Note that the lender is ex-ante competitive but ex-post non-competitive and hence bargain with the borrower. This is a feature common in sovereign debt bargaining models. A motivation for this is that potential new lenders, afraid that a troubled borrower is going to default on somebody (including themselves), do not lend until they see the borrower repay the incumbent lender. Thus, until some repayment is made to the incumbent lender, the borrower cannot access the competitive loan market and the incumbent lender has some market power over the borrower. Kovrijnykh and Szentes (2007) show formally how this lender's switch from competitive in the pre-default stage to noncompetitive ex-post can arise endogenously when old debt is senior to new debt. Nevertheless, we have checked that the main intuition of our model that CDS can alleviate moral hazard would still hold if the lender have as much bargaining power ex-ante as ex-post.

$q^{FB}B^{FB}$ that he was supposed to invest. This is the moral hazard problem.

Before characterizing the second best contract, let us first specify the particular functional form that we assumed for $\pi(I)$:

Assumption 1. $\pi(I) = \sqrt{I}$ where I is investment as a fraction of the steady state output.[16] [17]

Using assumption 1, we can express the condition under which the borrower has an incentive to invest less than the first best, $B^{FB} \geq \alpha y_l$, in terms of the model parameters:[18]

$$(y_h - y_l)^2 \geq 2py_l \tag{4}$$

We assume that condition (4) holds, hence, the borrower has an incentive to cheat and there is a moral hazard problem. Since the lender will not break even with the first best contract, the lender has to offer a contract that accounts for such behavior of the borrower (i.e. it has to be incentive compatible) while still maximizing the borrower's utility and earning a zero profit for the lender.

Definition: *The second best (SB)* loan contract (q^{SB}, B^{SB}) and investment, I^{SB}, under private information and bargaining is an incentive compatible contract given by the solution to Program $\mathcal{P}^{SB}$.[19]

[16] All the other variables are in steady state output units as well.

[17] For general $\pi(I) = I^\gamma$, where $\gamma < 1$, there is no analytical solution, but we have computationally checked that the main results of our paper still hold. For example, figure 4 looks qualitatively the same for $\gamma \neq \frac{1}{2}$.

[18]

$$B^{FB} \geq \alpha y_l \quad \Leftrightarrow \quad \frac{I^{FB} - (1-\pi(I^{FB})\alpha y_l}{\pi(I^{FB})} \geq \alpha y_l \quad \Leftrightarrow \quad I^{FB} - \alpha y_l + \pi(I^{FB})\alpha y_l \geq \pi(I^{FB})\alpha y_l$$

$$\Leftrightarrow \quad I^{FB} \geq \alpha y_l. \quad \text{For } \pi(I) = I^\gamma \text{ and } \gamma = 0.5 \quad \sqrt{I^{FB}} = \frac{1}{2}(y_h - y_l) \Rightarrow \quad (y_h - y_l)^2 \geq 2py_l$$

[19] The appendix shows that the constraint $c_0 \geq 0$ will hold with equality.

Program $\mathcal{P}^{SB}$:

$$\max_{q,B,I} \quad qB - I + \pi(I)(y_h - B) + (1-\pi(I))(y_l - \alpha y_l) \tag{$\mathcal{P}^{SB}$}$$

$$\text{s.t.} \quad \pi'(I)(y_h - B - (1-\alpha)y_l) = 1 \tag{IC_B}$$

$$\pi(I)B + (1-\pi(I))\alpha y_l = qB \tag{IR_L}$$

$$c_0 = qB - I \geq 0$$

where, as before, $\alpha = \frac{1}{2}p$. (IR_L) is the lender's zero profit condition and (IC_B) is the incentive compatibility constraint. Comparing (2) with (IC_B), we see that since $\pi(I)$ is concave, the second best investment will be less than the first best as long as: $B^{SB} \geq \frac{1}{2}py_l$. Thus, the moral hazard problem constrains borrowing and results in an investment less than the first best. Also, the utility achieved under private information will always be less than under full information because of the extra (IC) constraint.

2.2 Credit insurance

Up to now, the results on moral hazard are standard.[20] Now let us introduce an insurance market where the lender can buy a protection against default from risk neutral competitive insurers. An insurance contract with notional, i, insures the lender up to the amount i in case of a credit event. A credit event here is defined as a full default by the borrower, in other words the lender has not agreed to any debt reduction. We assume the lender's insurance activity is observable by the borrower.[21]

When negotiations fail, the lender now receives i instead of getting nothing; thus, an

[20] See, for instance, Atkeson (1991) and Gertler and Rogoff (1990).

[21] We implicitly rule out the lender and the borrower together falsifying the credit event so that the lender still gets paid by the borrower some amount while lying to the insurer about the credit event and getting an insurance payment. Thus, we assume a credit event is contractible. A similar situation that we rule out is the lender and the insurer negotiating ex-post so that the insurer pays less than what was contracted; in return, the lender does not reject debt restructuring and gets paid by the borrower also.

insurance improves his outside option and the lender's surplus from bargaining is now:

$$\Delta_L(a) = ay_l - i$$

Figure 2 reflects this change.

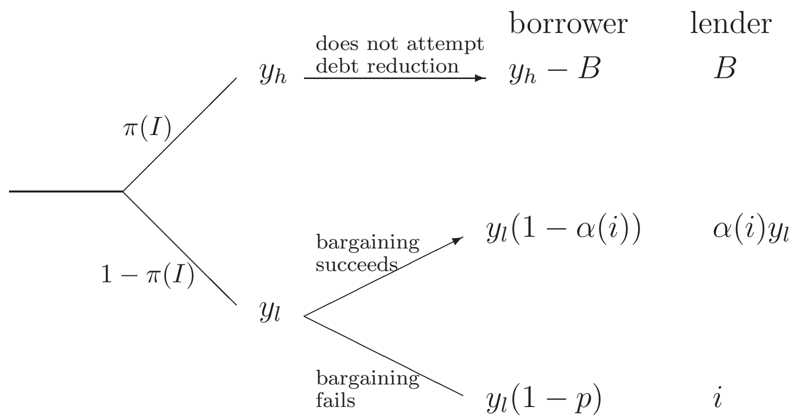


Figure 2: Date 1 subgame with insurance

Now the bargaining outcome when y_l is realized depends on i. Specifically, solving the bargaining problem as before, the share the lender gets from the borrower is:

$$\alpha(i) = \frac{1}{2}p + \frac{1}{2y_l}i \quad \text{if } i \leq py_l \tag{5}$$

Figure 3 shows the solution graphically. If $i = 0$, we are back to the no insurance case. On the interval $[0, py_l)$, as the amount of insurance purchased increases, the lender's share paid by the borrower increases and will be strictly larger than the insurance itself i (i.e. the 45° line). The lender will strictly prefer restructuring over insurance. When $i = py_l$, $\alpha(i)y_l$ crosses the 45° line and the payment from the borrower (in case of restructuring) will exactly equal the insurance (in case of default). Thus, the lender will be indifferent between receiving the py_l from the borrower (by accepting restructuring) or from the insurer (by rejecting restructuring) and we model the lender's decision as choosing the best mixed

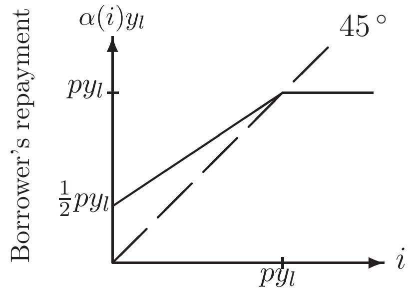


Figure 3

strategy. Whether the lender accepts or rejects, the borrower's endowment decreases by the same amount (because the borrower either has repaid py_l or defaulted and was penalized by py_l). If $i > py_l$, the most the lender can get from the borrower is py_l, thus the lender will reject any restructuring and trigger insurance, and the borrower will suffer output loss of py_l.

To characterize the zero-profit conditions of the lender and the insurer, we first subdivide i into three intervals based on what the lender will do on each interval as discussed above: 1) $i \leq py_l$, the lender always accepts restructuring, 2) $i = py_l$, the lender is indifferent between accepting or rejecting and getting insurance, and 3) $i > py_l$, the lender claims insurance. The lender's zero-profit condition when he purchases insurance is:

$$\pi(I)B + (1 - \pi(I))c^L = qB + m \tag{6}$$

where c^L is the lender's consumption in the low state. On the first interval, since the lender restructures and does not claim insurance, the payment from the insurer is zero. Thus, $m = (1 - \pi(I))0 = 0$, $c^L = \alpha(i)y_l$, and $c^B = (1 - \alpha(i))y_l$ where c^B is the borrower's consumption in the low output state.[22] When indifferent, the lender uses a mixed strategy $(\omega,\ (1 - \omega))$ where ω is the probability he will accept restructuring and $(1 - \omega)$ is the probability he will claim insurance. Insurance premium then is $m = (1 - \pi(I))(1 - \omega)i$ while

[22] See the end of section 2.1 for more discussion about insurance premium equalling zero.

$c^L = \omega\alpha(i)y_l + (1-\omega)i$ and $c^B = \big(\omega(1-\alpha(i)) + (1-\omega)(1-p)\big)y_l = (1-p)y_l$. On the third interval, since the borrower defaults the insurer pays the lender the full i ($c^L = i$) and $m = (1-\pi(I))i$ while $c^B = (1-p)y_l$.

We are now ready to characterize the equilibrium when an insurance market exits.

Definition: The optimal loan contract (q, B), I, and i when there exists an insurance market is such that it is incentive compatible for the borrower and maximizes the borrower's utility subject to the zero profit conditions of the lender and the insurer.

Program $\mathcal{P}^{\mathcal{SB},\text{ins}}$:

$$\begin{aligned} \max_{q,B,I,i,\omega} \quad & (qB - I) + \pi(I)(y_h - B) + (1-\pi(I))c^B \qquad (\mathcal{P}^{\mathcal{SB},\text{ins}}) \\ \text{s.t.} \quad & \pi'(I)(y_h - B - c^B) = 1 \\ & \pi(I)B + (1-\pi(I))c^L = qB + m \\ & c_0 = qB - I \geq 0 \end{aligned}$$

where m, c^B, and c^L in each of the three intervals of i are:

$$m = \begin{cases} (1-\pi(I))0 = 0 \\ (1-\pi(I))(1-\omega)i \\ (1-\pi(I))i \end{cases} \quad c^B = \begin{cases} (1-\alpha(i))y_l \\ (1-p)y_l \\ (1-p)y_l \end{cases} \quad c^L = \begin{cases} \alpha(i)y_l & \text{if } i < py_l \\ \omega\alpha(i)y_l + (1-\omega)i & \text{if } i = py_l \\ i & \text{if } i > py_l \end{cases}$$

and $\alpha(i)$ is given by (5). From solving the above problem, we arrive at the main result of our paper, which compares the effects of insurance to the second best with no insurance:

Proposition 1.

(i) The optimal insurance is: $i^* = min\{\frac{1}{2}(y_h - y_l)^2 - py_l, py_l\}$. *Specifically:*

$$\textit{if } (y_h - y_l)^2 < 4py_l, \ \textit{then it's an 'interior' solution: } i^* = \frac{1}{2}(y_h - y_l)^2 - py_l \qquad (7)$$

$$\textit{if } (y_h - y_l)^2 \geq 4py_l, \ \textit{then it's a 'corner' solution: } i^* = py_l \qquad (8)$$

(ii) The borrower is better off: $U^{ins} \geq U^{SB}$*.*

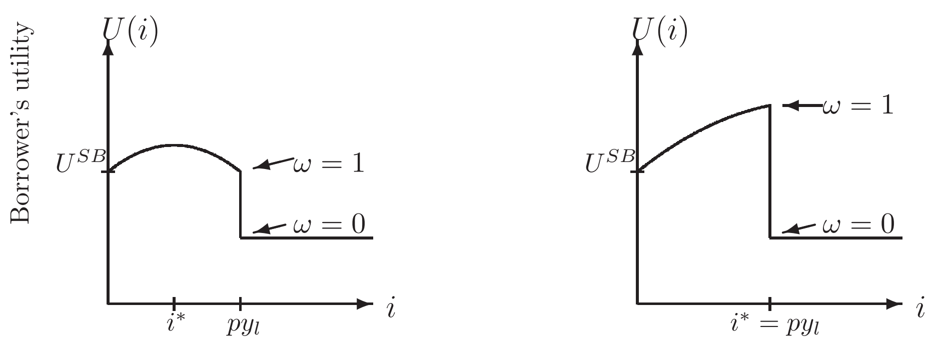
(iii) Investment increases: $I^{ins} \geq I^{SB}$*, and the probability of default* $(1 - \pi(I))$ *decreases.*

(iv) The borrower is more indebted: $B^{ins} \geq B^{SB}$*.*

(v) The bond price increases: $q^{ins} \geq q^{SB}$*, or equivalently, the borrowing cost decreases.*

Proof. See appendix □

Figure 4 demonstrates the result by showing the borrower's utility as a function of i. In figure 4(a), condition (7) holds, in which case i^* is given by an "interior" solution on the interval $[0, py_l]$. In this case, the constraint $i_l \leq p_l y_l$ does not bind, allowing i_l to be as high as it needs to be, and there is a complete alleviation of moral hazard: $I^{ins} = I^{FB}$. Whereas in figure 4(b), condition (8) holds and the borrower's utility is strictly increasing in i on the interval $[0, py_l]$; thus, the optimal insurance is given by the corner solution $i^* = py_l$, in which case moral hazard is only partially alleviated. When $i = py_l$, the lender plays a mixed strategy, but borrower's utility is increasing in ω, hence the best mixed strategy is the degenerate one: $\omega = 1$. When $i > py_l$, the borrower would be worse off than the second best. This is because py_l - the amount the borrower gets penalized by - is a deadweight cost that no one benefits from; it is better if it instead gets used to repay the loan.



4(a): 'interior' solution $(y_h - y_l)^2 < 4py_l$

4(b): 'corner' solution $(y_h - y_l)^2 \geq 4py_l$

Figure 4

Remark on Proposition 1(ii): As long as there is the moral hazard problem given by condition (4), the existence of insurance improves the borrower's welfare. This is because it turns out that (4) is also the condition for $U'(i = 0) > 0$, i.e. positive insurance is pareto optimal. Condition (4) is satisfied if, for instance, y_l is small compared to y_h or the volatility of output is high.

Remark on Proposition 1(iii): The lender's insurance activity disciplines the borrower. In the bad state, if the borrower's offer is not high enough compared to lender's insurance, debt reduction negotiation will fail and the borrower will be penalized. To avoid being penalized, the borrower will have to pay more than when insurance did not exist. Consequently, the low output state looks less attractive to the borrower, thus he will invest more (i.e. closer to the first best amount) to avoid it.

Remark on Proposition 1(iv): The borrower finds it optimal to invest more to avoid the bad outcome and that requires an increase in the face value of the bond (i.e. he has to pay back more).

Remark on Proposition 1(v): When investment increases due to CDS's disciplining effect, probability of the high state (hence full repayment) increases. Also the borrower pays more in the low state. These two effects lead to a higher bond price or, equivalently, to a lower borrowing cost.

Discussion of the corner solution. When $i^* = py_l$ (figure 4(b)), the lender buys just enough insurance to make him indifferent between accepting the debt restructuring $\alpha(i^*) = py_l$ and rejecting it and triggering an insurance payment. This amount of insurance lets him extract the maximum possible amount from the borrower. Although ex-post he will be indifferent between accepting or rejecting the debt restructuring, ex-ante it is optimal to always get repaid by the borrower and not file a claim with the insurer (i.e. play the degenerate mixed strategy: $\omega = 1$). That way the insurance premium is the cheapest possible (zero, to be specific), hence the borrowing cost is the lowest possible. In figure 4(b), we can see that any $\omega < 1$ would not be an equilibrium since for such ω, there is an ϵ such that

$U(i^* - \epsilon) > U(\omega | i = py_l)$. Thus, we implicitly assume that the lender can credibly commit to, ex-post, always accept the payment from the borrower and not the insurer although he is indifferent.[23] In the end, the reason an insurance makes a difference is that, before with no insurance as an outside option, the lender could not credibly reject a partial repayment and punish the borrower. But now he can credibly reject any restructuring offer less than the insurance purchased.

Remark on insurance premium. Insurance in this context becomes a costless mechanism to extract the maximum repayment possible from the borrower. The reason for the zero price for an insurance contract is due to the assumptions that the only credit event is a full default and that there are only two output realizations where in the low output state the borrower ends up, in equilibrium, paying partially. Thus, in equilibrium, there is never a full default and the insurer never has to make any payment. But the zero cost of insurance does not have to be taken literally. As shown in figure 5, if, for example, the support of the output has a state where output realized is zero and the sovereign does not have any means to pay, then there is an automatic default in that state. Thus, there is always a state in which insurance will be paid out to the lender so that the insurance premium will be positive. Although this kind of general setting may be desirable, the main results are likely to be the same.

2.3 Restructuring - a Credit Event

So far we have assumed that only a full default is a credit event. In this section, we consider what happens if a debt restructuring is also a credit event.[24]

[23]If the lender's credibility is an issue, a policy implication could be to limit $i \leq py_l - \epsilon$ where ϵ is a small number. Then, the insurance purchased would be $i = py_l - \epsilon$, which would be slightly less than the share achieved in negotiations with that amount of insurance as the outside option: $\alpha(i) = py_l - \frac{\epsilon}{2}$. The outcome in this case will still be better than the second best.

[24]Market participants follow credit event definitions developed by the International Swaps and Derivative Association (ISDA) as a legal framework. In ISDA definitions, restructuring is included as a credit event in sovereign CDSs as long as it was due to a deterioration in the creditworthiness of the sovereign. But a voluntary restructuring is not included and there is an ambiguity in terms of what constitutes as a voluntary. It is common now for sovereigns to offer voluntary exchange offers as a way to reduce debt payments. Even

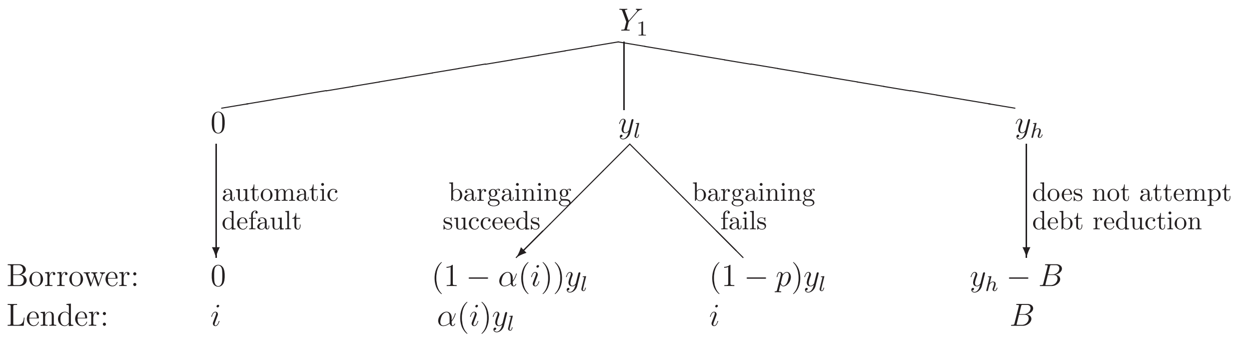


Figure 5: Date 1 subgame with insurance

If there is a debt restructuring and the lender files a claim with the insurer, the payment made from the insurer is the incured loss up to i which is the difference between i and the recovery value. In particular, if αy_l is the recovery value (what the lender receives from the borrower through debt restructuring), then the insurer pays the remaining $i - \alpha y_l$.[25] Thus, when a debt restructuring is a credit event, the lender is always indifferent between accepting the partial repayment αy_l and getting the remaining $i - \alpha y_l$ from the insurer versus completely rejecting a debt reduction (and having the borrower default in full) and getting the insured amount, i, in full from the insurer:

$$c^L = \begin{cases} \alpha y_l + (i - \alpha y_l) = i & \text{if accepts restructuring} \\ i & \text{if rejects restructuring} \end{cases}$$

Thus, α cannot be determined from the bargaining problem because the lender's surplus

though the lender often has no other option but to accept the offer and the exchange entails a deterioration in the creditworthiness of the sovereign (e.g. an extension in the maturity or a reduction in the interest payments), the creditor could be considered to have agreed to the restructuring. There have not been many cases of restructuring and subsequent CDS payment triggers to help us draw conclusions. One exception is the New York court case, Eternity Global Master Fund vs. Morgan Guaranty, regarding Argentina's debt restructuring in November of 2001 before its actual default in December of 2001. The november debt exchange entailed both lower yields and longer maturity. Following the debt exchange, Eternity demanded payments from Morgan Guaranty, the protection seller, arguing that there was a debt restructuring but Morgan Guaranty argued that it was not a credit event because it was voluntary. Eventually, the court ruled on Morgan's side. See Verdier (2004) for more discussion.

[25] This is analogous to cash settlement.

from bargaining is always zero.

One way to go about this is to find the borrower's optimal repayment in the bad state from the perspective of date 0. Let us denote the repayment in the bad state as $\tilde{\alpha}$, where the tilde is to notationally distinguish it from α, which was determined from the bargaining problem. Since the lender is always indifferent, suppose he accepts restructuring with a probability ω and rejects it with a probability $1-\omega$. Then we can solve solve $\tilde{\alpha}$ along with the optimal loan contract, investment, ω, and insurance that maximizes the borrower's utility subject to the incentive compatibility constraint and the zero-profit conditions of the lender and the insurer:

$$\max_{q,B,I,\tilde{\alpha},\omega,i} (qB-I)+\pi(I)(y_h-B)+(1-\pi(I))\{\omega(1-\tilde{\alpha})y_l+(1-\omega)(1-p)y_l\}$$

$$\text{s.t.} \quad \pi'(I)\big(y_h-B-\{\omega(1-\tilde{\alpha})y_l+(1-\omega)(1-p)y_l\}\big)=1$$

$$\pi(I)B+(1-\pi(I))\big(\omega(\alpha y_l+(i-\tilde{\alpha}y_l))+(1-\omega)i\big)=qB+m$$

$$m=(1-\pi(I))\big(\omega(i-\tilde{\alpha}y_l)+(1-\omega)i\big)$$

$$c_0=qB-I\geq 0$$

$$i\geq \tilde{\alpha}y_l$$

Solving this problem, the optimal $\tilde{\alpha}$ and ω are:

$$\omega^*=1 \quad \text{and} \quad \tilde{\alpha}^*=min\bigg(\frac{(y_h-y_l)^2}{4y_l},p\bigg)$$

Remember that when restructuring was not a credit event, the optimal insurance was $i^*=min\{\frac{1}{2}(y_h-y_l)^2-py_l,py_l\}$, so that $\alpha(i^*)=\frac{1}{2}p+\frac{i^*}{2y}=min\big(\frac{(y_h-y_l)^2}{4y_l},p\big)$. Thus, the optimal repayment when y_l is realized is exactly the same as when restructuring was not a credit event and there is as much alleviation of moral hazard as before. Thus, the optimal investment and utility achieved does not depend on whether restructuring is a credit event or not.

Although ex-post, at date 1, the lender would be indifferent between accepting any restructuring $\tilde{\alpha} \leq \tilde{\alpha}^*$ (since he will be compensated by the insurer up to i on the remaining $(i - \tilde{\alpha} y_l)$), ex-ante it is optimal to not accept any $\tilde{\alpha} < \tilde{\alpha}^*$ so that there is as much alleviation of moral hazard as possible. This outcome hinges upon the lender's credibility to ex-post do what was ex-ante optimal. Nevertheless, how does insurance make a difference in this case? Before, a lender's threat that he will reject any $\tilde{\alpha} < \tilde{\alpha}^*$ was not credible because without insurance he strictly prefered accepting restructuring over rejecting it. But now, insurance makes the lender's threat credible precisely because the lender is now indifferent.

The only difference in results from allowing restructuring to be a credit event is that there is no unique equilibrium insurance level as long as $i \geq \tilde{\alpha}^* y$. Suppose $i > \tilde{\alpha}^* y$ and consider the zero-profit conditions of the lender and the insurer after substituting in $\omega^* = 1$:

$$\pi(I)B + (1 - \pi(I))(\tilde{\alpha}^* y_l + (i - \tilde{\alpha}^* y_l)) = qB + m \tag{9}$$

$$m = (1 - \pi(I))(i - \tilde{\alpha}^* y_l) \tag{10}$$

From (10), the higher the payment from the insurer, $i - \tilde{\alpha}^* y$, the higher the insurance price, m. But, from (9), the increase in the insurance price gets exactly offset by the increase in the lender's consumption when y_l is realized. So there would be no point in purchasing $i > \tilde{\alpha}^* y$.

3 Monopolist Insurer and Naked Buyers

We have assumed so far that the lenders are the only ones that purchase insurance when in reality there are 'naked buyers' who purchase insurance but do not own the underlying bond. In this section we consider how the results of the previous section change with the existence of naked buyers.

We assume that the naked buyer has cash flows correlated to the output of the sovereign: endowment is high in state H and low in state L. He is risk averse and buys sovereign CDS to insure against the low output state. These assumptions are motivated by the following example: suppose that an investor has an investment in a Greek firm but CDSs on the firm itself do not exist or are relatively illiquid. He could instead purchase CDS on a Greek government bond because during the states of the world where the government is struggling, the private sector is likely to be struggling as well.[26]

If the insurer's market is perfectly competitive, as in the previous section, then the existence of naked buyers does not have any impact on the lender-borrower contract. Also even if the insurer is a monopolist but the naked buyer is risk neutral, then again the existence of naked buyers does not make any difference. Thus, we assume that the insurer's market is imperfectly competitive (the insurer is a monopolist, to be specific) and that the naked buyer is risk averse. We first show how the problem changes when the insurer is a monopolist instead of perfectly competitive and this will be our new "benchmark." Then we introduce naked buyers and compare the result with that of the benchmark scenario without the naked buyers.

3.1 Benchmark: Monopolist Insurer

The lender's problem is the same as before: he is choosing a loan contract that maximizes the borrower's utility and is incentive compatible for the borrower. But now the monopolist insurer makes the lender take-it-or-leave-it offer for insurance contract so that the lender takes the price and the quantity of insurance as given. Let (i_l, m_l) denote the insurance contract bought by the lender. Then, the lender's problem is:

[26] There could be other reasons for naked buying. One reason could be due to heterogenous beliefs about the likelihood of default: an investor might think that a particular government or company is more or less likely to default than is suggested by CDS prices. Another reason could be due to the fact that CDS trading is often done through dealers who buy and sell CDS without holding the underlying security. For example, suppose bank X with Greek government bonds wants to decrease its exposure to default risk and purchases CDS from bank A. Bank A wants to hedge its increased exposure so it purchases CDS on Greek government bond from another party, Bank B. Bank B does the same as Bank A and purchases CDS from Bank Y. Bank Y, on the other hand, is willing to bear the risk of Greek default and hence does not purchase CDS. In this example, banks X and Y were the end users of CDS while banks A and B acted as the dealers and would be considered 'naked buyers' as they purchased CDS without actually holding the underlying security. Dealers contribute to the liquidity of CDS market as they eliminate the need for Bank X, in our example, to directly find the end user, Bank Y, who is willing to take the opposite position of Bank X.

$$
\begin{aligned}
U(i_l, m_l) \equiv \max_{q,B,I} \quad & (qB - I) + \pi(I)(y_h - B) + (1 - \pi(I))(1 - \alpha(i_l))y_l \\
\text{s.t.} \quad & \pi'(I)(y_h - B - (1 - \alpha(i_l))y_l) = 1 \\
& \pi(I)B + (1 - \pi(I))\alpha(i_l)y_l = qB + m_l \\
& c_0 = qB - I \geq 0 \\
& i_l \leq p y_l
\end{aligned}
$$

Note that the resulting investment, debt level, and bond price will be functions of insurance level i_l and price m_l: $I(i_l, m_l)$, $B(i_l, m_l)$, $q(i_l, m_l)$.

The equilibrium insurance level and insurance price will be determined by the monopolist's profit maximization problem. The insurer maximizes his profit subject to the lender's individual rationality constraint: if the lender purchases insurance, it has to make him (and hence the borrower) at least better off than without the insurance. The insurer's profit is just the price charged for the insurance as we have previously explained at the end of section **??**.

Definition: The equilibrium insurance bought, i_l^{mon}, and premium charged, m_l^{mon}, when the insurer is a monopolist will be the solution to Program $\mathcal{P}^{Benchmark}$.

Program $\mathcal{P}^{Benchmark}$:

$$
\begin{aligned}
\{i_l^{mon}, m_l^{mon}\} \equiv & \arg\max_{i_l, m_l} m_l \qquad (\mathcal{P}^{Benchmark}) \\
& \text{s.t.} \quad U(i_l, m_l) \geq U^{SB}
\end{aligned}
$$

where U^{SB} is the borrower's utility achieved when the lender does not purchase any insurance, $U^{SB} = U(i_l = 0, m_l = 0)$, and is the same as the second best of the previous section.

From the equilibrium insurance level and insurance price we can derive the equilibrium investment level $I^{mon} \equiv I(i_l^{mon}, m_l^{mon})$. Proposition 2 compares the equilibrium investment level when the insurer is a monopolist, I^{mon}, with the equilibrium investment level when the insurer is perfectly competitive, I^{ins}:

Proposition 2.

$$
\begin{aligned}
&I^{SB} \leq I^{mon} = I^{ins} = I^{FB} && \textit{if 'interior' solution:} \quad 2p_l y_l \leq (y_h - y_l)^2 \leq 4p_l y_l \\
&I^{SB} \leq I^{mon} = I^{ins} < I^{FB} && \textit{if 'corner' solution:} \quad (y_h - y_l)^2 > 4p_l y_l
\end{aligned}
$$

Proof. See Appendix □

Thus the equilibrium investment level is exactly the same as when the insurer was competitive ($I^{mon} = I^{ins}$). Consequently, $I^{mon} \geq I^{SB}$ which means insurance still alleviates the moral hazard problem and increases the social welfare. The increase in social welfare is exactly the same as before but, in contrast to the previous section, the increased welfare only goes to the insurer and none to the borrower. The insurer achieves this through a higher insurance price while keeping the borrower's payoff the same as the second best. Because the lender is now paying a higher price for the insurance, the lender accounts for this in the bond price he charges the borrower; thus compared to the last section borrowing cost is higher when the insurer is a monopolist.

3.2 Naked Buyers

We now introduce naked buyers who are risk averse and have cash flows that are correlated with that of the sovereign's: its cash flow is likely to be high when the sovereign has a high output state realization and vice versa. The naked buyer's cash flow is c_h with probability $\pi(I)$ and c_l with probability $1-\pi(I)$. Thus, implicit in this assumption is that the sovereign's action (i.e. investment) affects the naked buyer's cash flow: the probability of the high cash flow state is increasing in the sovereign's investment. We assume the naked buyer buys a type of CDS where a restructuring is considered a credit event. If the naked buyer buys a

CDS where only full default is a credit event, then it will not provide him with any insurance since a full default never occurs in equilibrium. Let (i_n, m_n) denote the insurance contract sold to the naked buyer. Thus, when y_l is realized and there is a debt restructuring, the insurance pays i_n minus the recovery value $\alpha(i_l)y_l$ to the naked buyer. When y_h is realized there is no credit event and hence no payments from the insurer. The insurance premium, m_n, is paid in both states. The naked buyer's consumption is:

$$c = \begin{cases} c_h - m_n & \text{if } y_h \text{ is realized which occurs with probability } \pi(I) \\ c_l + i_n - \alpha(i_l)y_l - m_n & \text{if } y_l \text{ is realized which occurs with probability } 1 - \pi(I) \end{cases}$$

Therefore, the naked buyer has the following expected utility:

$$U^N(i_n, m_n) = \pi(I)u(c_h - m_n) + (1 - \pi(I))u(c_l + i_n - \alpha(i_l)y_l - m_n)$$

The insurer now maximizes his profit over two sets of insurance contracts: one for the lender (i_l, m_l) and one for the naked buyer (i_n, m_n). Each insurance contract has to be individually rational: the lender is at least better off with (i_l, m_l) than without it, and the naked buyer is also at least better off with insurance (i_n, m_n) than without it. We assume that the insurer can tell apart between the lender and the naked buyer such that incentive compatibility constraints (that the naked buyer will prefer the contract designed for him rather than the one for the lender and vice versa) are not imposed.[27]

Definition: The equilibrium insurance contracts (i_l^*, m_l^*) and (i_n^*, m_n^*) and hence the equilibrium investment level when there is a naked buyer who buys insurance are given by the solution to Program $\mathcal{P}^{spec}$.

[27]This is because the problem becomes analytically intractable with the lender's incentive compatibility constraint. By design the naked buyers insurance contract is incentive compatible: the naked buyer will prefer the contract designed for him rather than the one for the lender. In equilibrium there is only a debt restructuring and never a full default and the naked buyers insurance contract includes restructuring as a credit event while the lenders does not. We have checked numerically that imposing lender's incentive compatibility constraints do not qualitatively change our results.

Program $\mathcal{P}^{spec}$:

$$\max_{i_l,m_l,i_n,m_n} m_l + m_n - (1-\pi(I))(i_n - \alpha(i_l)y_l) \tag{$\mathcal{P}^{spec}$}$$

$$\text{s.t.} \quad U(i_l, m_l) \geq U^{SB} \tag{IR^L}$$

$$U^N(i_n, m_n) \geq U^N(0,0) \tag{IR^N}$$

The main result of this section is given next.

Proposition 3. *Depending on the parameter conditions, in equilibrium, there is either an over-investment, ($I^* \geq I^{mon}$) or an under-investment ($I^* \leq I^{mon}$) compared to the benchmark case ($\mathcal{P}^{Benchmark}$) without the naked buyers. When there is an over-investment, the cost of borrowing is lower ($q^* \geq q^{mon}$), while if there is an under-investment, the cost of borrowing is higher $q^* \leq q^{mon}$ than without the naked buyers.*

Proof. See appendix. □

Intuition:

Let us first simplify Program $\mathcal{P}^{spec}$. The appendix shows that both of the individual rationality constraints of Program $\mathcal{P}^{spec}$ bind. Moreover, $i_n - \alpha(i_l)y_l = c_h - c_l$. Denote $\sigma_c \equiv c_h - c_l$. Then after some algebra, Program $\mathcal{P}^{spec}$ becomes:

$$\max_{i_l,m_n} \quad m_l(i_l) + m_n - (1-\pi(I(i_l)))\sigma_c$$

$$\text{s.t.} \quad u(c_h - m_n) = \pi(I(i_l))u(c_h) + (1-\pi(I(i_l)))u(c_l) \tag{11}$$

We can further simplify by solving for m_n from eq. (11) and substituting it into the objective function. Program $\mathcal{P}_{spec}$ boils down to maximizing over only one variable which is the lender's insurance level:

$$\max_{i_l} \quad m_l(i_l) + m_n(i_l) - (1-\pi(I(i_l)))\sigma_c$$

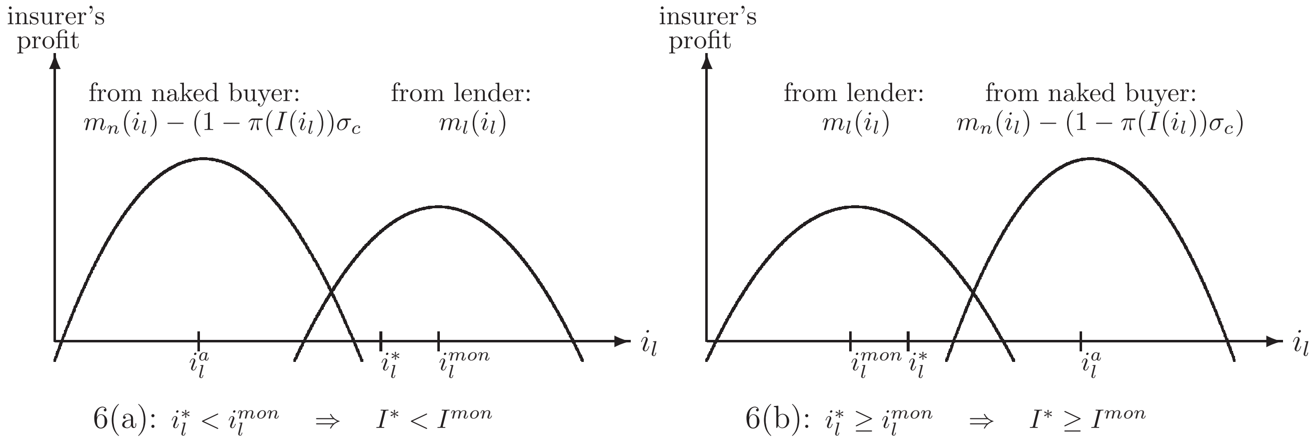


Figure 6

The objective function of the insurer (i.e. his total profit) is comprised of profits from insuring both the lender, $m_l(i_l)$, and the naked buyer, $m_n(i_l) - (1 - \pi(I(i_l)))\sigma_c$. Figure 6 shows a stylized representation of the insurer's problem: it plots the insurer's profit as a function of the insurance sold to the lender (i_l). In panel (a), the parameter conditions are such that the profit function from insuring the naked buyer is to the left of the lender's meaning that the maximum profit received from the naked buyer occurs at a lower i_l than the profit from the lender. To determine the optimal insurance level for the lender, i_l^*, we know from looking at the graph that i_l^* will not be less than the level that maximizes the profit from insuring the naked buyer (denoted by i^a) because if it is, then by increasing i_l the insurer can increase the profit he receives both from the naked buyer and the lender. Similarly, i_l^* cannot be greater than i_l^{mon}, the level that maximizes the profit from the lender. Thus, i_l^* will be between i^a and i_l^{mon}. In fact, i_l^* will be where the marginal benefit of an extra i_l (increase in profit received from the lender) equals the marginal cost (decrease in profit from the naked buyer). More importantly, i_l^* will be less than i_l^{mon} which was the equilibrium insurance in the no-naked-buyers case: $m_l'(i_l^{mon}) = 0$. Since investment is an increasing function of i_l, the equilibrium investment is lower than without the naked buyer: $I^* < I^{mon}$. The equilibrium investment of the no-naked-buyers case resulted in as much alleviation of moral hazard as possible (e.g. $I^{mon} = I^{FB}$ if corner solution); thus when

$I^* < I^{mon}$ the existence of a naked buyer impedes the alleviation of moral hazard. In panel (b), the parameter conditions are the opposite of panel (a). By the same arguments, we have that $i_l^* \geq i_l^{mon}$ and hence an over-investment.

What is going on is that due to insuring the naked buyer, the insurer profits from insuring not only the lender but also the naked buyer where the profit from the latter is affected by the borrower's investment. Since the insurer can indirectly control the borrower's investment through the insurance contract offered to the lender, (i_l, m_l), the insurer chooses (i_l, m_l) and hence the borrower's investment to maximize the sum of the two profits. Depending on the naked buyer's preferences and cash flows c_h and c_l, it can be more profitable to induce a higher borrower investment as in panel (b) or a lower borrower investment as in panel (a). For example, when the cash flow of the naked buyer in the low state (c_l) is very low, the insurer will have to pay out a larger claim, σ_c, to the naked buyer in the low state since the optimal insurance for the naked buyer completely smoothes his consumption across states. In this case, it is more profitable to the insurer to induce a higher borrower investment so that the state in which he has to make a large net transfer occurs with a lower probability.

4 Conclusion

Motivated by the concerns raised over the use of credit default swaps during the recent European sovereign debt crises, we ask: could credit default swaps be beneficial for the debtor country, and if so, why? We find that CDSs can be beneficial for the borrower because they can serve as a disciplining mechanism in an environment with debtor moral hazard and debt renegotiation. Specifically, the moral hazard problem arises from the assumption of private information about the borrower's investment, and, as a consequence, results in credit rationing and suboptimal investment level. In this framework, we find that insurance serves as a commitment device for a lender to credibly reject low levels of repayment from the borrower, thereby, increasing the lender's bargaining power in ex-post debt renegotiations. The increased bargaining power of the lender, in turn, alleviates ex-ante borrowing constraints, provides more external capital to the debtor country, and increases investment and, hence, welfare. Thus, the existence of an insurance market alleviates the moral hazard problem by better aligning the lender and borrower's incentives.

Using this framework, we also analyze the effect of naked buyers who do not lend directly to the sovereign. If there are naked buyers who purchase insurance, our analysis shows that the market structure of the insurer's market could be important. If the insurer's market is relatively competitive, our model suggests that the existence of naked buyers should have no impact on the debtor country. While if the insurer's market is concentrated, the existence of naked buyers could either lead to an over-investment or impede the alleviation of moral hazard. Nevertheless, this paper raises some issues in favor of CDS, thus putting a larger onus on those who call for regulation to come up with serious analyses of the detrimental aspects of CDSs.

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A Appendix

Claim 1. *$c_0 = 0$ in Program $\mathcal{P}^{SB}$ of section 2.1.*[28]

Proof. The Lagrangian is given by:

$$\begin{aligned}\mathcal{L} = {} & qB - I + \pi(I)(y_h - B) + (1-\pi(I))(y_l - \alpha y_l) \\ & + \mu[\pi'(I)(y_h - B - (1-\alpha)y_l) - 1] \\ & + \psi[\pi(I)B + (1-\pi(I))\alpha y_l - qB] \\ & + \lambda[qB - I]\end{aligned}$$

First order conditions with respect to I, B, and q are:

$$-1+\pi'(I)(y_h - B-(1-\alpha)y_l)+\mu\pi''(I)(y_h - B-(1-\alpha)y_l)+\psi\pi'(I)(B-\alpha y_l)-\lambda = 0 \quad (FOC_I)$$

$$q - \pi(I) - \mu\pi'(I) + \psi[\pi(I) - q] + \lambda q = 0 \quad (FOC_B)$$

$$B - \psi B + \lambda B = 0 \quad (FOC_q)$$

$$\lambda(qB - I) = 0$$

Suppose $\lambda = 0$, then

$$(FOC_q) \Rightarrow \quad \psi = 1$$

$$(FOC_I) \quad \Rightarrow \quad \mu\pi''(I)/\pi'(I) = -\psi\pi'(I)(B - \alpha y_l) < 0 \quad \Rightarrow \quad \mu > 0 \quad \text{since } \pi''(I) < 0$$

$$(FOC_B) \quad \Rightarrow \quad \mu\pi'(I) = 0 \quad \text{which is a contradiction since } \pi'(I) > 0 \text{ and } \mu > 0$$

□

A.1 Demonstrating the necessity of both bargaining and private information

A.1.1 Bargaining but no private information

The equilibrium under full information, bargaining setting without the insurance market is already given by Program $\mathcal{P}^{\mathcal{FB}}$. The equilibrium when there exists an insurance maximizes the borrower's utility subject to the zero-profit conditions of the lender and the insurer.
Program $\mathcal{P}^{\mathcal{FB},\text{ins}}$:

$$\begin{aligned}\max_{q,B,I,i,\omega} \quad & (qB - I) + \pi(I)(y_h - B) + (1-\pi(I))c^B && (\mathcal{P}^{\mathcal{FB},\text{ins}}) \\ \text{s.t.} \quad & \pi(I)B + (1-\pi(I))c^L = qB + m \\ & c_0 = qB - I \geq 0\end{aligned}$$

[28]The same argument holds for Program $\mathcal{P}^{SB,ins}$.

where m, c^B, and c^L in each of the three intervals of i are:

$$m = \begin{cases} (1-\pi(I))0 = 0 \\ (1-\pi(I))(1-\omega)i \\ (1-\pi(I))i \end{cases} \quad c^B = \begin{cases} (1-\alpha(i))y_l \\ (1-p)y_l \\ (1-p)y_l \end{cases} \quad c^L = \begin{cases} \alpha(i)y_l & \text{if } i < py_l \\ \omega\alpha(i)y_l + (1-\omega)i & \text{if } i = py_l \\ i & \text{if } i > py_l \end{cases}$$

and $\alpha(i)$ is given by (5).

Solving this problem, the optimal investment is given by:

$$\pi'(I)(y_h - y_l) = 1 \tag{12}$$

which is exactly the same as when the insurance market did not exist. Moreover, the utility of the borrower, given by $U = -I + \pi(I)y_h + (1-\pi(I))y_l$, does not depend on insurance and is also exactly as it was before without the insurance. Thus, the lender's credit insurance activity does not matter.

Intuition. First, the lender will not buy insurance that is more than py_l. Since the borrower will at most pay py_l, when $i > py_l$, the lender will prefer full default so that he can get $i > py_l$ from the insurer. However, this increases the cost of insurance and, due to the lender's break-even condition, the lender passes down to the borrower the insurance cost through higher borrowing cost.[29] By the same argument, $\omega = 1$. Thus, we can narrow down i to $i \leq py_l$ in which case $m = 0$, $c^L = \alpha(i)y_l$, and $c^B = (1-\alpha(i))y_l$. Substituting them in, Program $\mathcal{P}^{\mathcal{FB},\text{ins}}$ boils down to:

$$\begin{aligned} \max_{q,B,I,i} \quad & (qB - I) + \pi(I)(y_h - B) + (1-\pi(I))(1-\alpha(i))y_l \\ \text{s.t.} \quad & \pi(I)B + (1-\pi(I))\alpha(i)y_l = qB \\ & c_0 = qB - I \geq 0 \end{aligned}$$

From here, it is straight forward to see that investment is given by (12). However, B and q will be functions of i:[30]

$$B(i) = \frac{1}{\pi(I)}\big(I - (1-\pi(I))\alpha(i)y_l\big) \qquad q = \frac{I}{B(i)}$$

For $i = 0$, we are back to Program $\mathcal{P}^{\mathcal{FB}}$, while for $0 < i \leq py_l$ all insurance does is make the borrower pay more in the low-output state. Paying more in the low-output state lowers the borrowing cost, or, equivalently, increases the bond price. So without borrowing as much (i.e. B is lower) he is able to raise the same funds, qB, such that: $qB = I^{FB} = I^{FB^*}$. But the borrower's investment and utility does not change with i and, hence, there is no unique optimal insurance level as long as $i \leq py_l$. In the end, insurance does not matter because we still do not have a friction that constrains borrowing and results in a suboptimal investment.

[29] The py_l that the borrower gets penalized by is a deadweight cost that no one benets from; it is better if it instead gets used to repay back the loan.

[30] We can again safely assume $qB - I = 0$ as before.

A.1.2 Private information but no bargaining

What happens if we shut off bargaining and consider if an insurance market makes a difference in a private information but full default setting? The optimal contract without the existence of an insurance market is given by the solution to:

$$\begin{aligned} \max_{q,B,I} \quad & qB - I + \pi(I)(y_h - B) + (1-\pi(I))y_l(1-p) \\ \text{s.t.} \quad & \pi'(I)(y_h - B - y_l(1-p)) = 1 \\ & \pi(I)B + (1-\pi(I))0 = qB \qquad (13) \\ & c_0 = qB - I \geq 0 \end{aligned}$$

Now with an insurance market:

$$\begin{aligned} \max_{q,B,I,i} \quad & qB - I + \pi(I)(y_h - B) + (1-\pi(I))y_l(1-p) \\ \text{s.t.} \quad & \pi'(I)(y_h - B - y_l(1-p)) = 1 \\ & \pi(I)B + (1-\pi(I))i = qB + m \qquad (14) \\ & m = (1-\pi(I))i \qquad (15) \\ & c_0 = qB - I \geq 0 \end{aligned}$$

Note that (14) and (15) together gives you exactly (13). Thus, the two problems are the same and the lender's insurance activity does not affect the borrower's behavior. This is because without bargaining, the lender's credibility to penalize is no longer in question: the borrower automatically gets penalized when y_l is realized.

A.2 Proof of Proposition 1

Proof of Proposition 1(i): We first solve for the optimal investment and utility achieved without insurance (i.e. the second best) which will be used to compare to the case with insurance. Then Proposition 1(i) will follow from Lemmas 1.1 and 1.2 shown below. □

Solution of the second best contract:

$$\begin{cases} \pi'(I)(y_h - B - (1-\alpha^{SB})y_l) = 1 \\ \pi(I)B + (1-\pi(I))\alpha^{SB}y_l = I \end{cases} \Rightarrow \begin{cases} B = -\frac{1}{\pi'(I)} + y_h - (1-\alpha^{SB})y_l \\ B = \frac{I-(1-\pi(I))\alpha^{SB}y_l}{\pi(I)} \end{cases}$$

$$\Rightarrow \qquad \sqrt{I^{SB}} = \frac{y_h - y_l + \sqrt{(y_h-y_l)^2 + 12\alpha^{SB}y_l}}{6}$$

$$U^{SB} = \frac{(y_h-y_l)^2}{9} + \frac{(y_h-y_l)\sqrt{(y_h-y_l)^2+12\alpha^{SB}y_l}}{9} - \frac{\alpha^{SB}y_l}{3} + y_l$$

Lemma 1.1: Let i_1 be the optimal insurance on the interval $i \leq py_l$. If $(y_h - y_l)^2 \geq 4py_l$, then $\alpha = p$ and $i_1 = py_l$. If $(y_h-y_l)^2 < 4py_l$, then $\alpha = \frac{(y_h-y_l)^2}{4y_l}$, and $i_1 = \frac{1}{2}(y_h-y_l)^2 - py_l$

Proof. We first find the optimal investment, loan, and utility amounts for a fixed insurance i on the interval $[0, py_l]$ and denote them as $I_1(i)$, $B_1(i)$, $U_1(i)$ respectively. Then we find i that maximizes $U_1(i)$ on the interval $[0, py_l]$, i.e. $i_1 \equiv \arg\max U_1(i)$.

$I_1(i)$, $B_1(i)$ for specific i on $[0, py_l]$ are given by the solution to:

$$\begin{cases} \pi'(I)(y_h - B - (1-\alpha(i))y_l) = 1 \\ \pi(I)B + (1-\pi(I))\alpha(i)y_l = I \end{cases} \Rightarrow \begin{cases} B = -\frac{1}{\pi'(I)} + y_h - (1-\alpha(i))y_l \\ B = \frac{I-(1-\pi(I))\alpha(i)y_l}{\pi(I)} \end{cases}$$

$$\Rightarrow \qquad \sqrt{I_1(i)} = \frac{1}{6}(y_h - y_l + \sqrt{(y_h-y_l)^2 + 12\alpha(i)y_l}) \tag{16}$$

The utility achieved for a given i is then:

$$\begin{aligned} U_1(i) =& \pi(I)y_h - \pi(I)B + (1-\pi(I))(y_l - \alpha(i)y_l) \\ =& \frac{1}{9}(y_h-y_l)^2 + \frac{1}{9}(y_h-y_l)\sqrt{(y_h-y_l)^2 + 12\alpha(i)y_l} - \frac{1}{3}\alpha(i)y_l + y_l \end{aligned}$$

We now find i where $\frac{\partial U_1}{\partial i} = \frac{\partial U_1}{\partial \alpha}\frac{\partial \alpha}{\partial i} = 0$. Since $\alpha(i) = \frac{1}{2}p + \frac{1}{2y_l}i$, $\frac{\partial \alpha}{\partial i} > 0$. So we just need to find α where $\frac{\partial U_1}{\partial \alpha} = 0$:

$$\Rightarrow \quad \alpha = \frac{(y_h-y_l)^2}{4y_l}$$

Thus:

$$\alpha = \begin{cases} \frac{(y_h-y_l)^2}{4y_l} & \text{if} \quad (y_h-y_l)^2 < 4py_l \\ p & \text{if} \quad (y_h-y_l)^2 \geq 4py_l \end{cases}$$

$$i_1 = \begin{cases} \frac{1}{2}(y_h-y_l)^2 - py_l & \text{if} \quad (y_h-y_l)^2 < 4py_l \\ py_l & \text{if} \quad (y_h-y_l)^2 \geq 4py_l \end{cases}$$

□

Lemma 1.2: Let $\bar{U}_1$ and $\bar{U}_2$ be the utilities achieved with the optimal insurance on the intervals $i \leq py_l$ and $i \geq py_l$ respectively. Then $\bar{U}_1 \geq \bar{U}_2$.

Proof. Let $\bar{I}_1$, $\bar{B}_1$, and $\bar{U}_1$ be the values achieved with the optimal insurance i_1, i.e. $\bar{I}_1 \equiv I_1(i_1)$, $\bar{B}_1 \equiv B_1(i_1)$, and $\bar{U}_1 \equiv U_1(i_1)$. Likewise, let $\bar{I}_2$, $\bar{B}_2$, and $\bar{U}_2$ be the optimal values on $i \geq py_l$. We show the proof by the following four steps: (i) solve for $\bar{I}_2$, (ii) show $\bar{B}_1 \leq \bar{B}_2$, (iii) show $\bar{I}_1 \geq \bar{I}_2$, and then as a consequence: (iv) $\bar{U}_1 \geq \bar{U}_2$.

Step (i). Solve for $\bar{I}_2$

We do the same as in Lemma 1.1 but constraining insurance to be at least greater than py_l. We first solve for the optimal investment $I_2(i)$ for a fixed insurance i when $i \geq py_l$:

$$\begin{cases} \pi'(I)(y_h - B - (1-p)y_l) = 1 \\ \pi(I)B = I \end{cases} \Rightarrow \begin{cases} B = -\frac{1}{\pi'(I)} + y_h - (1-p)y_l \\ B = \frac{I}{\pi(I)} \end{cases}$$

$$\Rightarrow \qquad \sqrt{I_2(i)} = \frac{y_h - y_l + py_l}{3} = \frac{y_h - y_l}{3} + \frac{py_l}{3} \tag{17}$$

Note that investment in this case does not depend on i. Thus, $\bar{I}_2$ is given by (17).

Step (ii). Proof of $\bar{B}_1 \le \bar{B}_2$

$$\bar{B}_2 = \frac{\pi(I)}{I} = \frac{y_h - y_l + py_l}{3}$$

For the interior case, where $\frac{1}{4}(y_h - y_l)^2 \le py_l$*:*

$$\bar{B}_1 = -2\sqrt{I_1} + y_h - (y_l - \alpha(i_1)y_l) = \alpha y_l = \frac{1}{4}(y_h - y_l)^2 \le py_l$$

$$\bar{B}_2 \ge \frac{(y_h - y_l)^2 + py_l}{3} \ge \frac{3py_l}{3} = py_l$$

where the last inequality is due to the moral hazard condition, thus, $\bar{B}_1 \le \bar{B}_2$.
For the corner case:

$$\bar{B}_1 = -\frac{(y_h - y_l) + \sqrt{(y_h - y_l)^2 + 12py_l}}{3} + y_h - y_l + py_l$$

$$\Rightarrow \quad \bar{B}_1 - \bar{B}_2 = \frac{1}{3}(y_h - y_l) - \frac{\sqrt{(y_h - y_l)^2 + 12py_l}}{3} + \frac{2}{3}py_l$$

Suppose $\bar{B}_1 - \bar{B}_2 > 0$, then:

$$(y_h - y_l) + 2py_l > \sqrt{(y_h - y_l)^2 + 12py_l}$$

$$\Rightarrow \quad py_l(y_h - y_l) + (py_l)^2 > 3py_l$$

But the LHS: $\quad py_l(y_h - y_l) + (py_l)^2 \le py_l(y_h - y_l) + py_l = py_l(y_h - y_l + 1) \le 2py_l$

which is a contradiction, thus, $\bar{B}_1 \le \bar{B}_2$. □

Step (iii). Proof of $\bar{I}_1 \ge \bar{I}_2$

For the interior solution case:

$$\alpha(i_1)y_l = \frac{1}{4}(y_h - y_l)^2$$

$$\sqrt{\bar{I}_1} = \sqrt{I_1(i_1)} = \frac{1}{6}(y_h - y_l + \sqrt{4(y_h - y_l)^2}) = \frac{y_h - y_l}{2} = \frac{y_h - y_l}{3} + \frac{y_h - y_l}{6}$$

From the moral hazard $(y_h - y_l)^2 \ge 2py_l$ and using the fact that $y_h - y_l \le 1$:

$$\frac{y_h - y_l}{6} \ge \frac{py_l}{3}$$

For the corner solution case where $(y_h - y_l)^2 \ge 4py_l$*:*

$$\alpha(i_1)y_l = py_l$$

$$\pi'(\bar{I}_1)(y_h - \bar{B}_1 - (1-p)y_l) = 1$$

$$\pi'(\bar{I}_2)(y_h - \bar{B}_2 - (1-p)y_l) = 1$$

Thus, due to the concavity of $\pi(I)$, when $\bar{B}_1 \leq \bar{B}_2$, we have $\bar{I}_1 \geq \bar{I}_2$. $\square$

Step (iv). Proof of $\bar{U}_1 \geq \bar{U}_2$

Using (i), (ii), and (iii) it is straightforward to see the result from:

$$\bar{U}_1 = \pi(\bar{I}_1)(y_h - \bar{B}_1) + (1 - \pi(\bar{I}_1))(1-p)y_l = \pi(\bar{I}_1)(y_h - \bar{B}_1 - (1-p)y_l) + (1-p)y_l$$
$$\bar{U}_2 = \pi(\bar{I}_2)(y_h - \bar{B}_2) + (1 - \pi(\bar{I}_2))(1-p)y_l = \pi(\bar{I}_2)(y_h - \bar{B}_2 - (1-p)y_l) + (1-p)y_l \quad \square$$

Proof of Proposition 1(ii). U^{ins} is the maximum utility achieved when an insurance market exits. Thus, $U^{ins} = \bar{U}_1$ since we have shown that $\bar{U}_1 \geq \bar{U}_2$. But using the solution of the second best contract we have: $U^{SB} = U_1(i)|_{i=0}$, but $U_1(i)|_{i=0} \leq \max_i U_1(i) \equiv \bar{U}_1$ and the result follows. $\square$

Proof of Proposition 1(iii). From the solution to the second best contract: $I^{SB} = I_1(i = 0)$. But $I_1(i)$ is increasing in insurance on $[0, py_l]$ from eq. (16) in Lemma 1.1. $\square$

Proof of Proposition 1(iv).

$$B(i) = -\frac{1}{\pi'(I(i))} + y_h - (1-\alpha(i))y_l = -2\sqrt{I(i)} + y_h - y_l + \frac{1}{2}py_l + \frac{1}{2}i$$
$$= -\frac{1}{3}(y_h - y_l + \sqrt{(y_h - y_l)^2 + 6py_l + 6i}) + y_h - y_l + \frac{1}{2}py_l + \frac{1}{2}i$$

$$B''(i) = 3((y_h - y_l)^2 + 6py_l + 6i)^{-\frac{3}{2}} > 0 \quad \Rightarrow \quad B(i) \text{ is convex}$$

We will that show i^* is less than i^{min} where $B'(i^{min}) = 0$

$$B'(i) = -\frac{1}{\sqrt{(y_h - y_l)^2 + 6py_l + 6i}} + \frac{1}{2} = 0 \quad \Rightarrow \quad i^{min} = \frac{2}{3} - \frac{y_h - y_l}{6} - py_l$$

For the interior solution case where $i^* = \frac{y_h - y_l}{2} - py_l$:

$$(y_h - y_l)^2 \leq 1 \quad \Rightarrow \quad \frac{y_h - y_l}{2} \leq \frac{2}{3} - \frac{y_h - y_l}{6}$$

For the corner solution case: since in this case $i^* = py_l \leq \frac{y_h - y_l}{2} - py_l$, the same argument as for the interior solution holds. $\square$

Proof of Proposition 1(v). Since $q = \frac{I}{B}$, the proof is a corollary of Propositions 1(iii) and 1(iv). $\square$

A.3 Proofs for the Section with Speculators

Proof of Proposition 2. First, we want to show that the lender's IR constraint binds in Program $\mathcal{P}^{Benchmark}$:

$$\begin{aligned} \max_{i_l, m_l} \quad & m_l \\ \text{s.t.} \quad & U(i_l, m_l) \geq U^{SB} \end{aligned}$$

where $U^{SB} = (y_h - y_l)^2/9 + (y_h - y_l)\sqrt{(y_h - y_l)^2 + 6p_l y_l}/9 - p_l y_l/6 + y_l$.

Proof. In the lender's problem, the constraint $qB - I \geq 0$ binds as before, so B and I can be solved as functions of i_l and m_l from:

$$\begin{cases} \pi'(I)(y_h - B - (1 - \alpha(i_l))y_l) = 1 \\ \pi(I)B + (1 - \pi(I))\alpha(i_l)y_l = I + m_l \end{cases}$$

To simplify the complexity of notation, let us define:

$$\sigma_y \equiv y_h - y_l$$

$$u \equiv U^{SB} - y_l$$

$$x(i_l, m_l) \equiv \sqrt{I(i_l, m_l)}$$

I and U are given by:

$$\sqrt{I(i_l, m_l)} = \frac{\sigma_y + \sqrt{\sigma_y^2 + 12(\frac{1}{2}p_l y_l + \frac{1}{2}i_l - m_l)}}{6}$$

$$\begin{aligned} U(i_l, m_l) &= -m_l - I(i_l, m_l) + \sqrt{I(i_l, m_l)}\sigma_y + y_l \\ &= -m_l - x(i_l, m_l)^2 + x(i_l, m_l)\sigma_y + y_l \end{aligned}$$

Then Program $\mathcal{P}^{Benchmark}$ becomes:

$$\begin{aligned} \max_{i_l, m_l} \quad & m_l \\ \text{s.t.} \quad & -m_l - x(i_l, m_l)^2 + x(i_l, m_l)\sigma_y + y_l \geq U^{SB} \\ & i_l \leq p_l y_l \end{aligned}$$

The first order conditions are:

$$FOC_{m_l}: \quad 1 + \lambda_1(-1 - 2x x_m + x_m \sigma_y) = 0$$

$$FOC_{i_l}: \quad \lambda_1(-2xx_{i_l} + x_{i_l}\sigma_y) + \lambda_2 = 0$$

where λ_1 and λ_2 are the lagrange multipliers on the first and second constraints respectively. We see from FOC_{m_l} that if $\lambda_1 = 0$, we get a contradiction. Thus, the constraint binds. $\square$

Next, we see from FOC_{i_l} that $x = \frac{\sigma_y}{2}$, that is $I = I^{FB}$. To solve for the optimal insurance, we first solve for m_l as a function of i_l. It can be shown that:

$$m_l(i_l) = -\frac{3}{2}\left(u + \frac{1}{6}(p_l y_l + i_l)\right) + \frac{\sigma_y}{2}\sqrt{2u + p_l y_l + i_l}$$

Then Program $\mathcal{P}^{Benchmark}$ boils down to just:

$$\max_{i_l} \quad m_l(i_l)$$
$$i_l \leq p_l y_l$$

Taking the first order condition with respect to i_l, optimal i_l^{mon} is given by:

$$1 = \sigma_y \frac{1}{\sqrt{p_l y_l + i_l^{mon} + 2u}} \quad \Rightarrow$$

$$i_l^{mon} = \min\{-2u + \sigma_y^2 - p_l y_l, p_l y_l\}$$

To check again that $\sqrt{I^{mon}} = \frac{1}{2}\sigma_y$ for the interior case:

$$m(i_l^{mon}) = -\frac{3}{2}\left(\frac{1}{6}\sigma_y^2 + \frac{2}{3}u\right) + \frac{1}{2}\sigma_y^2 = \frac{1}{4}\sigma_y^2 - u$$

$$\frac{1}{2}(p_l y_l + i_l^{mon}) - m_l(i_l^{mon}) = -u + \frac{1}{2}\sigma_y^2 - \frac{1}{4}\sigma_y^2 + u = \frac{1}{4}\sigma_y^2$$

plug this in the expression for I:

$$\sqrt{I(i_l^{mon}, m_l)} = \frac{\sigma_y\sqrt{\sigma_y^2 + 3\sigma_y^2}}{6} = \frac{1}{2}\sigma_y$$

For the corner solution case.

$$U(i_l, m_l) \geq U^{SB}$$
$$-I^{mon} + (y_h - y_l)\sqrt{I^{mon}} - m_l - y_l \geq -I^{SB} + (y_h - y_l)\sqrt{I^{SB}} - y_l$$
$$-I^{mon} + (y_h - y_l)\sqrt{I^{mon}} \geq m_l - I^{SB} + (y_h - y_l)\sqrt{I^{SB}}$$
$$\Rightarrow I^{mon} \geq I^{SB}$$

$\square$

Proof of Proposition 3. The proof consists of four steps. To simplify notation, define:

$$\sigma_c \equiv c_h - c_l$$

$$x_m \equiv \frac{\partial x(i_l, m_l)}{\partial m_l}$$

$$x_i \equiv \frac{\partial x(i_l, m_l)}{\partial i_l}$$

Step 1:

We show that in Program $\mathcal{P}_{spec}$ both of the individual rationality constraints bind and that $i_n - \alpha(i_l)y_l = c_h - c_l$:

$$\begin{aligned}
\max_{i_l,m_l,i_n,m_n} \quad & m_l + m_n - (1 - x(i_l, m_l))(i_n - (p_l y_l + i_l)/2 \\
\text{s.t.} \quad & -m_l - x(i_l, m_l)^2 + x(i_l, m_l)\sigma_y + y_l \geq U^{SB} \\
& x(i_l, m_l)\sqrt{c_h - m_n} + (1 - x(i_l, m_l))\sqrt{c_l + i_n - (p_l y_l + i_l)/2 - m_n} \geq \\
& \qquad x(i_l, m_l)u(c_h) + (1 - x(i_l, m_l))u(c_l) \\
& i_l \leq p_l y_l
\end{aligned}$$

Let λ_1, λ_2, and λ_3 be the lagrange multipliers of the above three constraints respectively. The first order conditions are:
FOC_{m_l} :

$$1 + x_m(i_n - (p_l y_l + i_l)/2) + \lambda_1(-1 - 2xx_m + x_m\sigma_y) + \lambda_2 x_m\big(\sqrt{c_h - m_n} - \sqrt{c_l + i_n - (p_l y_l + i_l)/2 - m_n} - (\sqrt{c_h} - \sqrt{c_l})\big) = 0$$

FOC_{i_l} :

$$x_i\big(i_n - (p_l y_l + i_l)/2\big) + (1 - x)/2 + \lambda_1(-2xx_i + x_i\sigma_y) + \lambda_2\Bigg(x_i\big(\sqrt{c_h - m_n} - \sqrt{c_l + i_n - (p_l y_l + i_l)/2 - m_n} - (\sqrt{c_h} - \sqrt{c_l})\big) - \frac{(1 - x)/2}{2\sqrt{c_l + i_n - (p_l y_l + i_l)/2 - m_n}}\Bigg) + \lambda_3 = 0$$

FOC_{m_n} :

$$1 - \frac{1}{2}\lambda_2\Big(\frac{x}{\sqrt{c_h - m_n}} + \frac{1 - x}{\sqrt{c_l + i_n - (p_l y_l + i_l)/2 - m_n}}\Big) = 0$$

FOC_{i_n} :

$$(1 - x) = \lambda_2 \frac{1 - x}{2\sqrt{c_l + i_n - (p_l y_l + i_l)/2 - m_n}}$$

Then from either FOC_{m_n} or FOC_{i_n}, $\lambda_2 \neq 0$. And from both FOC_{m_n} and FOC_{i_n} we have that:

$$\sqrt{c_h - m_n} = \sqrt{c_l + i_n - (p_l y_l + i_l)/2 - m_n}$$

$$\Rightarrow \quad i_n - (p_l y_l + i_l)/2 = c_h - c_l \quad \text{and} \quad \lambda_2 = 2\sqrt{c_h - m_n}$$

Suppose $\lambda_1 = 0$, then using $\lambda_2 = 2\sqrt{c_h - m_n}$, FOC_{m_l} becomes:

$$1 + x_m\sigma_y - 2x_m\sqrt{c_h - m_n}(\sqrt{c_h} - \sqrt{c_l}) = 0 \tag{18}$$

$$\Rightarrow \quad \sqrt{c_h - m_n} = \frac{x_i\sigma_c - \frac{1}{2}}{2x_i(\sqrt{c_h} - \sqrt{c_l})} \tag{19}$$

And since $\lambda_2 \neq 0$, $\sqrt{c_h - m_n} = x\sqrt{c_h} + (1-x)\sqrt{c_l}$. Plug this in to (19):

$$2(x(\sqrt{c_h} - \sqrt{c_l})^2 + \sqrt{c_h}(\sqrt{c_h} - \sqrt{c_l})) = \sigma_c - \sqrt{\sigma_y^2 + 12(\frac{1}{2}p_l y_l + \frac{1}{2}i_l - m_l)} \tag{20}$$

Define:

$$a \equiv \sqrt{\sigma_y^2 + 12(\frac{1}{2}p_l y_l + \frac{1}{2}i_l - m_l)}$$

then,

$$(20) \Rightarrow \qquad (\frac{\sigma_y}{3} + \frac{a}{3})(\sqrt{c_h} - \sqrt{c_l})^2 + 2\sqrt{c_h}(\sqrt{c_h} - \sqrt{c_l}) = \sigma_c - a$$

$$\Rightarrow \quad a = \frac{\sigma_c - 2\sqrt{c_h}(\sqrt{c_h} - \sqrt{c_l}) - \frac{\sigma_y}{3}(\sqrt{c_h} - \sqrt{c_l})^2}{\frac{1}{3}(\sqrt{c_h} - \sqrt{c_l})^2 + 1} = \frac{-(\sqrt{c_h} - \sqrt{c_l})^2 - \frac{\sigma_y}{3}(\sqrt{c_h} - \sqrt{c_l})^2}{\frac{1}{3}(\sqrt{c_h} - \sqrt{c_l})^2 + 1} \leq 0$$

which is a contraction. □

Step 2:

After some simplifications allowed from step 1, we derive the optimal insurance contracts $(i_l,\, m_l)$, $(i_n,\, m_n)$ and λ_1 which are given by the following five equations:

$$\begin{aligned}
FOC_{m_l}: &\quad 1 + x_{m_l}\sigma_c + \lambda_1(-1 - 2xx_m + x_m\sigma_y) - 2\sqrt{c_h - m_n}x_m(\sqrt{c_h} - \sqrt{c_l}) = 0 \\
FOC_{i_l}: &\quad \sigma_c + \lambda_1(-2x + \sigma_y) - 2\sqrt{c_h - m_n}(\sqrt{c_h} - \sqrt{c_l}) = 0 \\
FOC_{i_n} \;\&\; FOC_{m_n}: &\quad i_n = \frac{1}{2}(py + i_l) + \sigma_c \\
&\quad - m_l - x^2 + x\sigma_y - u = 0 \\
&\quad \sqrt{c_h - m_n} = x(\sqrt{c_h} - \sqrt{c_l}) + \sqrt{c_l}
\end{aligned} \tag{21}$$

From FOC_{m_l} and FOC_{i_l}:

$$1 + x_m\sigma_c + \frac{2\sqrt{c_h - m_n}(\sqrt{c_h} - \sqrt{c_l}) - \sigma_c}{-2x + \sigma_y}(-1) + (2\sqrt{c_h - m}(\sqrt{c_h} - \sqrt{c_l}) - \sigma_c)x_m - 2\sqrt{c_h - m_n}x_m(\sqrt{c_h} - \sqrt{c_l}) = 0$$

$$\Rightarrow \quad 1 = \frac{2\sqrt{c_h - m_n}(\sqrt{c_h} - \sqrt{c_l}) - \sigma_c}{-2x + \sigma_y} \quad \Rightarrow \quad 2\sqrt{c_h - m_n}(\sqrt{c_h} - \sqrt{c_l}) - \sigma_c = -2x + \sigma_y$$

Then the above together with (21):

$$2x(\sqrt{c_h} - \sqrt{c_l})^2 + 2\sqrt{c_l}(\sqrt{c_h} - \sqrt{c_l}) - \sigma_c = -2x + \sigma_y$$

$$\Rightarrow \quad x = \frac{\frac{1}{2}(\sigma_y\sigma_c) - \sqrt{c_l}(\sqrt{c_h} - \sqrt{c_l})}{1 + (\sqrt{c_h} - \sqrt{c_l})^2}$$

Define:

$$g \equiv \frac{\frac{1}{2}(\sigma_y \sigma_c) - \sqrt{c_l}(\sqrt{c_h} - \sqrt{c_l})}{1 + (\sqrt{c_h} - \sqrt{c_l})^2}$$

$$\begin{aligned} x(i_l) = x(i_l, m_l(i_l)) &= \left(\sigma_y \sqrt{\sigma_y^2 + 6(p_l y_l + i_l) - 12 m_l(i_l)}\right)/6 \\ &= \left(\sigma_y \sqrt{\sigma_y^2 + 9(p_l y_l + i_l) + 18u - 2\sigma_y \sqrt{9(p_l y_l + i_l) + 18u}}\right)/6 \end{aligned}$$

Setting $x(i_l)$ equal to g, i_l can be solved as:

$$i_l = 4g^2 - 2u - p_l y_l$$

Thus, the optimal insurance contracts are given by:

$$\begin{aligned} i_l^* =& \min\{4g^2 - 2u - p y_l, p_l y_l\} \\ m_l^* =& \begin{cases} -u - g^2 + \sigma_y g & \text{if} \quad i_l \leq p_l y_l \\ -\frac{3}{2}\left(u + \frac{1}{3} p_l y_l\right) + \frac{\sigma_y}{2}\sqrt{2u + 2p_l y_l} & \text{if} \quad i_l = p_l y_l \end{cases} \\ i_n^* =& \frac{1}{2}(p_l y_l + i) + \sigma_c = \min\{2g^2 - u + \sigma_c, p_l y_l + \sigma_c\} \\ m_n^* =& c_h - (x(\sqrt{c_h} - \sqrt{c_l}) + \sqrt{c_l})^2 \end{aligned}$$

Step 3: **Proof of the result on over and under-investment.**

Consider the following four cases:

$$\begin{aligned} &\text{Case 1:} \quad \max\{\sigma_y^2 - 2u - p y_l, 4g^2 - 2u - p y_l\} \leq p y_l \\ &\text{Case 2:} \quad 4g^2 - 2u - p y_l \leq p y_l \leq \sigma_y^2 - 2u - p y_l \\ &\text{Case 3:} \quad \sigma_y^2 - 2u - p y_l \leq p y_l \leq 4g^2 - 2u - p y_l \\ &\text{Case 4:} \quad p y_l \leq \min\{\sigma_y^2 - 2u - p y_l, 4g^2 - 2u - p y_l\} \end{aligned}$$

Case 1. In this case, both i^{mon} and i_l^* are given by the respective interior solutions. There is an over-investment if $I^{mon} = \frac{1}{2}\sigma_y \leq g = I^*$, an under-investment if otherwise. For example, if $c_h = y_h$ and $c_l = y_l$, the condition $g \geq \frac{1}{2}\sigma_y$ boils down to whether $1 \geq \sigma_y$.

Case 2. Here, $i^{mon} = p_l y_l$, i.e. it is a corner solution while i_l^* is an interior solution. Then:

$$I^{mon} = \frac{\sigma_y}{6} + \frac{1}{6}\sqrt{\sigma_y^2 + 18(p_l y_l + u) - 2\sigma_y \sqrt{18(p_l y_l + u)}} \qquad \text{and} \qquad I^* = g$$

Thus, there is an over-investment if $g \geq \frac{\sigma_y}{6} + \frac{1}{6}\sqrt{\sigma_y^2 + 18(p_l y_l + u) - 2\sigma_y\sqrt{18(p_l y_l + u)}}$ and an under-investment if otherwise.

Case 3. In this case, $\sigma_y \leq 4g^2$, thus $i^{mon} \leq i^*$. Also $m_l(i_l)$ attains maximum at $i^{mon}(i_l)$ and is concave, thus $m_l(i^*) \leq m_l(i^{mon})$. Hence, $I^{mon} \leq I^*$

Case 4. $i^{mon} = i^* = p_l y_l$, thus again $m_l(i^{mon}) = m_l(i^*)$, hence $I^{mon} = I^*$. $\square$

Step 4: **Proof of the result on cost of borrowing.**

Cases 1 and 3 where $i_l^{mon} = \sigma_y^2 - 2u - p_l y_l$:

$$\pi'(I^{mon})(y_h - B^{mon} - (1-\alpha(i^{mon}))y_l) = 1 \quad \Rightarrow \quad \pi'(I^{mon})(y_h - y_l) = 1 \quad \Rightarrow \quad B^{mon} = \alpha(i^{mon})y_l$$

$$B^{mon} = \alpha(i^{mon})y_l = \frac{1}{2}py_l + \frac{1}{2}i^{mon}$$

$$B^* = -2\sqrt{I^*} + y_h - y_l + \frac{1}{2}py_l + \frac{1}{2}i^*$$

$$B^{mon} - B^* = \frac{1}{2}(i^{mon} - i^*) + 2\sqrt{I} - (y_h - y_l)$$

Thus, $B^{mon} \geq B^*$ if an over-investment and vice versa and since $q = \frac{I}{B}$ the result about the cost of borrowing follows.

Case 2.

$$B^* = -2\sqrt{I^*} + (y_h - y_l) + \frac{1}{2}p_l y_l + \frac{1}{2}i^*$$

$$B^{mon} = -2\sqrt{I^{mon}} + (y_h - y_l) + \frac{1}{2}p_l y_l + \frac{1}{2}i^{mon}$$

$$B^{mon} - B^* = -2(\sqrt{I^{mon}} - \sqrt{I^*}) + \frac{1}{2}(p_l y_l - i^*)$$

If an over-investment: $-(\sqrt{I^{mon}} - \sqrt{I^*}) \geq 0 \quad \Rightarrow \quad B^{mon} - B^* \geq 0$. Thus, the result about cost of borrowing follows since $q = \frac{I}{B}$. If an under-investment: it is analytically intractable in this case to show the result about cost of borrowing, so we resort to checking this computationally.

Case 4:
$I^{mon} = I^*$, thus $q^{mon} = q^*$. $\square$

A.4 Default and Bargaining in Both States {H, L}

In this subsection, we relax the assumption that the borrower and the lender do not bargain when y_h is realized.

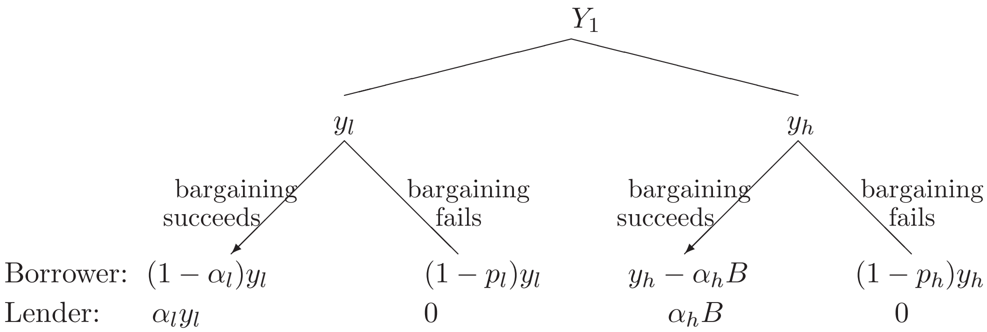

Figure 1: Date 1 subgame without insurance

Solving the bargaining problem:[31]

$$\alpha_h = \frac{p_h y_h}{2B} \qquad \Rightarrow \qquad \alpha_h B = \frac{p_h y_h}{2} \tag{22}$$

The first best:

$$\begin{aligned}
\max_{q,B,I} \quad & qB - I + \pi(I)(y_h - \alpha_h B) + (1-\pi(I))(1-\alpha_l)y_l \\
s.t. \quad & \pi(I)\alpha_h B + (1-\pi(I))\alpha_l y_l = qB \\
& qB - I \geq 0
\end{aligned}$$

The second best:

$$\begin{aligned}
\max_{q,B,I} \quad & qB - I + \pi(I)(y_h - \alpha_h B) + (1-\pi(I))(1-\alpha_l)y_l \\
s.t. \quad & \pi'(I)(y_h - \alpha_h B - (1-\alpha_l)y_l) = 1 \\
& \pi(I)\alpha_h B + (1-\pi(I))\alpha_l y_l = qB \\
& qB - I \geq 0
\end{aligned}$$

[31] The product of the bargaining surpluses are:

$$\Delta_B \Delta_L = \big(y_h - \alpha_h B - (1-p_h)y_h\big)\alpha_h B = (-\alpha_h B + p_h y_h)\alpha_h B = -B^2\alpha_h^2 + p_h y_h B \alpha_h$$

Maximizing this w.r.t α_h, we get (22).

Substituting in the solutions to the bargaining problem:

$$
\begin{aligned}
\max_{q,B,I} \quad & qB - I + \pi(I)(y_h - \frac{1}{2}p_h y_h) + (1-\pi(I))(y_l - \frac{1}{2}p_l y_l) \\
s.t. \quad & \pi'(I)\big(y_h - \frac{1}{2}p_h y_h - (y_l - \frac{1}{2}p_l y_l)\big) = 1 \\
& \pi(I)\alpha_h B + (1-\pi(I))\alpha_l y_l = qB \\
& qB - I \geq 0
\end{aligned} \tag{23}
$$

A.4.1 With CDS

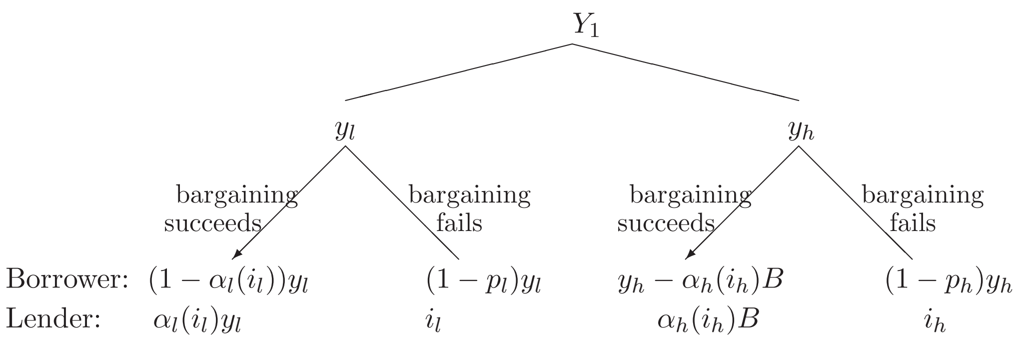

Figure 2: Date 1 subgame with insurance

Solving the bargaining problem:[32]

$$
\alpha_h(i_h) = \frac{p_h y_h + i_h}{2B} \qquad \Rightarrow \qquad \alpha_h(i_h)B = \frac{p_h y_h}{2} + \frac{i_h}{2} \tag{24}
$$

Equilibrium

$$
\begin{aligned}
\max_{q,B,I,i_h,i_l} \; & qB - I + \pi(I)(y_h - \frac{1}{2}(p_h y_h + i_h)) + (1-\pi(I))(y_l - \frac{1}{2}(p_l y_l + i_l)) \\
s.t. \quad & \pi'(I)(y_h - \alpha_h B - (1-\alpha_l)y_l) = 1 \\
& \pi(I)\alpha_h B + (1-\pi(I))\alpha_l y_l = qB \\
& qB - I \geq 0 \\
& i_l \leq p_l y_l \\
& i_h \leq p_h y_h
\end{aligned}
$$

[32]The product of the bargaining surpluses are:

$\Delta_B \Delta_L = \big(y_h - \alpha_h B - (1-p_h)y_h\big)\big(\alpha_h B - i_h\big) = (-\alpha_h B + p_h y_h)(\alpha_h B - i_h) = -B^2\alpha_h^2 + (p_h y_h B + i_h B)\alpha_h - p_h y_h i$

Maximizing this w.r.t α_h, we get (24).

Substituting in the solutions to the bargaining problem:

$$\max_{q,B,I,i_h,i_l} qB - I + \pi(I)(y_h - \frac{1}{2}(p_h y_h + i_h)) + (1-\pi(I))(y_l - \frac{1}{2}(p_l y_l + i_l))$$

$$s.t. \quad \pi'(I)\big(y_h - \frac{1}{2}(p_h y_h + i_h) - (y_l - \frac{1}{2}(p_l y_l + i_l)\big) = 1 \tag{25}$$

$$\pi(I)\big(\frac{1}{2}p_h y_h + \frac{1}{2}i_h\big) + (1-\pi(I))\big(\frac{1}{2}p_l y_l + \frac{1}{2}i_l\big) = qB \tag{26}$$

$$qB - I \geq 0$$

$$i_l \leq p_l y_l$$

$$i_h \leq p_h y_h$$

Note that if $i_h = i_l$, comparing (25) with the second best equivalent (23), the borrower will choose the same investment level as in the second best; in other words, the lender's insurance activity will not matter. This is because the borrower's consumption in both states goes down by exactly same amount ($\frac{1}{2}i_h$ or $\frac{1}{2}i_l$). Thus, the optimal i_l and i_h will have to be different to induce the borrower to invest an amount other than the second best.

Solving for the optimal insurance

Comparing (25) with the second best equivalent (23) rewritten here:

$$\pi'(I)\big(y_h - y_l - \frac{1}{2}(p_h y_h - p_l y_l) - \frac{1}{2}(i_h - i_l)\big) = 1$$

$$\pi'(I)\big(y_h - y_l - \frac{1}{2}(p_h y_h - p_l y_l)\big) = 1$$

we see that due to the concavity of $\pi(I)$, CDS increases investment and thereby alleviates moral hazard only if $-\frac{1}{2}(i_h - i_l) \geq 0$ or $i_l > i_h$. In fact the bigger the difference $i_l - i_h$ is, the bigger the investment. I is increasing in i_l and we can set $i_h = 0$.

Substituting (26) into the objective function and cancelling terms, we get:

$$\max_{I,i_l} - I + \pi(I)y_h + (1-\pi(I))(1-y_l) \tag{27}$$

$$s.t. \quad \pi'(I)\big(y_h - \frac{1}{2}p_h y_h - (y_l - \frac{1}{2}(p_l y_l + i_l)\big) = 1 \tag{28}$$

$$i_l \leq p_l y_l$$

Since we have assumed that $\pi(I) = \sqrt{I}$, then from (28):

$$\sqrt{I} = \frac{1}{2}\left(y_h - y_l - \frac{1}{2}(p_h y_h - p_l y_l) + \frac{1}{2}i_l\right)$$

Substituting the above equation into the objective function (27) and maximizing with respect to i_l we get:

$$i_l = min\big\{p_h y_h - p_l y_l, p_l y_l\big\}$$

Thus, the borrower's utility is increasing in i_l up until $i_l = p_h y_h - p_l y_l$. When $i_l = p_h y_h - p_l y_l$, the moral hazard is completely alleviated since $I(i_l) = I^{FB}$. However, we have the constraint $i_l \leq p_l y_l$ and if the parameters are such that $p_l y_l \leq p_h y_h - p_l y_l$, then the constraint will bind and the optimal i_l equals $p_l y_l$ and $U^{FB} \geq U^{ins} \geq U^{SB}$. Nevertheless, $U^{ins} \geq U^{SB}$ and the main result of the paper that the lender's insurance activity has a disciplining effect holds in this slightly more general setting. An issue here is the fact that q and B are not identified separately because B is not a control variable anymore. Because of the bargaining in both of the states, how much the borrower ends up repaying is fixed: $\frac{1}{2}p_h y_h$ in the high state and $\frac{1}{2}p_l y_l$ in the low state regardless of the investment level or how much was borrowed initially qB.

A.5 Uncompetitive Lender

$$\alpha = (1-\beta)p$$

[33] **First best**

$$\max_{q,B,I,i} \quad U^\beta L^{1-\beta} \tag{29}$$
$$\text{st:} \quad qB - I \geq 0 \tag{30}$$
$$L \geq 0 \tag{31}$$
$$U \geq 0 \tag{32}$$

where

$$U = qB - I + \pi(I)(y_h - B) + (1-\pi(I))(1-\alpha)y_l - y_l$$
$$L = \pi(I)B + (1-\pi(I))\alpha y_l - qB$$

Throughout I don't explicitly the conditions $U \geq 0$ and $L \geq 0$, and just assume these are satisfied. Also I let $qB = I$:

$$\max_{B,I} \quad \Big(\pi(I)(y_h - B) + (1-\pi(I))(1-\alpha(i))y_l - y_l\Big)^\beta \Big(\pi(I)B + (1-\pi(I))\alpha(i)y_l - I\Big)^{1-\beta} \tag{33}$$

$$\max_{B,x} \quad \Big(x(y_h - B) + (1-x)(1-\alpha)y_l - y_l\Big)^\beta \Big(xB + (1-x)\alpha y_l - x^2\Big)^{1-\beta} \tag{34}$$

$$FOC_B: \quad \beta U^{\beta-1}L^{1-\beta}(-x) + (1-\beta)U^\beta L^{-\beta}x = 0 \qquad \Rightarrow \qquad \beta U^{-1}L = (1-\beta) \tag{35}$$

$$FOC_x: \quad \beta U^{\beta-1}L^{1-\beta}(y_h - B - (1-\alpha)y_l) + (1-\beta)U^\beta L^{-\beta}(B - \alpha y_l - 2x) = 0 \tag{36}$$

[33]

$$\max_\alpha \Big((1-\alpha)y_l - (1-p)y_l\Big)^\beta \Big(\alpha y\Big)^{1-\beta}$$

$$\beta(-\alpha y + py)^{\beta-1}(\alpha y)^\beta(-y) + (1-\beta)(-\alpha y + py)^\beta(\alpha y)^{-\beta}y = 0$$

$$\beta(-\alpha y + py)^{-1}\alpha y = (1-\beta) \quad \Rightarrow \quad \beta\alpha = (1-\beta)(-\alpha + p) \quad \Rightarrow \quad \alpha = (1-\beta)p$$

These two FOCs combine to get: $x = \frac{\sigma}{2}$

Second best without insurance

$$\max_{B,I} \quad \left(\pi(I)(y_h - B) + (1-\pi(I))(1-\alpha)y_l - y_l\right)^{\beta}\left(\pi(I)B + (1-\pi(I))\alpha y_l - I\right)^{1-\beta} \tag{37}$$
$$\text{st:} \quad \pi'(I)(y_H - B - (1-\alpha)y_L) = 1 \tag{38}$$

$$\max_{B,x} \quad \left(x(y_h - B) + (1-x)(1-\alpha)y_l - y_l\right)^{\beta}\left(xB + (1-x)\alpha y_l - x^2\right)^{1-\beta} \tag{39}$$
$$\text{st:} \quad B = -2x + \sigma + \alpha y_l \tag{40}$$

$$U = x(2x + (1-\alpha)y_l) + (1-\alpha)y - x(1-\alpha)y_l - yl = 2x^2 - \alpha y_l$$

$$L = -2x^2 + \sigma x + \alpha y_l x + (1-x)\alpha y_l - x^2 = -3x^2 + \sigma x + \alpha y_l \tag{41}$$
$$= -3\left(x - \frac{\sigma + \sqrt{\sigma^2 + 12\alpha y_l}}{6}\right)\left(x - \frac{\sigma - \sqrt{\sigma^2 + 12\alpha y_l}}{6}\right) \tag{42}$$

Suppose $x > \frac{1}{2}\sigma$

The moral hazard condition boils down to:

$$\sigma^2 \geq 4\alpha y_l \quad \Rightarrow \quad 12\alpha y_l \leq 3\sigma^2 \quad \Rightarrow \quad \frac{\sigma + \sqrt{\sigma^2 + 12\alpha y_l}}{6} \leq \frac{1}{2}\sigma$$

Then $L < 0$, hence it's a contradiction that $x > \frac{1}{2}\sigma$. Thus, $I^{SB} \leq I^{FB}$.

With insurance:

$$\max_{q,B,I,i} \quad U^{\beta}L^{1-\beta} \tag{43}$$
$$\pi'(I)(y_H - B - (1-\alpha(i))y_L) = 1 \tag{44}$$
$$qB - I \geq 0 \tag{45}$$
$$i \leq py \tag{46}$$

$$\max_{B,I,i} \quad \left(\pi(I)(y_h - B) + (1-\pi(I))(1-\alpha(i))y_l\right)^{\beta}\left(\pi(I)B + (1-\pi(I))\alpha(i)y_l - I\right)^{1-\beta} \tag{47}$$
$$\pi'(I)(y_H - B - (1-\alpha(i))y_L) = 1 \tag{48}$$
$$i \leq py \tag{49}$$

$$\max_{B,x,i} \quad \Big(x(y_h - B) + (1-x)(1-\alpha(i))y_l\Big)^{\beta}\Big(xB + (1-x)\alpha(i)y_l - I\Big)^{1-\beta} \tag{50}$$

$$\text{st:} \quad B = -2x + \sigma + \alpha y_l \tag{51}$$

$$i \leq py \tag{52}$$

$$\max_{x,i} \quad \Big(2x^2 - \alpha(i)y_l\Big)^{\beta}\Big(-3x^2 + \sigma x + \alpha(i)y_l\Big)^{1-\beta} \tag{53}$$

$$\text{st:} \quad i \leq py \tag{54}$$

$$\alpha(i)y_l = \beta i + (1-\beta)py_l$$

[34]

$$FOC_x: \quad \beta U^{\beta-1}L^{1-\beta}4x + (1-\beta)U^{\beta}L^{-\beta}(-6x+\sigma) = 0 \quad \Rightarrow \quad \beta U^{-1}L4x = (1-\beta)(6x-\sigma)$$

$$FOC_i: \quad \beta U^{\beta-1}L^{1-\beta}(-\beta) + (1-\beta)U^{\beta}L^{-\beta}(\beta) = 0 \quad \Rightarrow \quad \beta U^{-1}L = (1-\beta)$$

Combining these two, we get: $x = \frac{1}{2}\sigma$

[34]

$$\max_{\alpha} \Big(-\alpha y + py\Big)^{\beta}\Big(\alpha y - i\Big)^{1-\beta}$$

$$\beta(-\alpha y + py)^{\beta-1}(\alpha y - i)^{\beta}(-y) + (1-\beta)(-\alpha y + py)^{\beta}(\alpha y - i)^{-\beta}y = 0$$

$$\beta(-\alpha y + py)^{-1}(\alpha y - i) = 1 - \beta \quad \Rightarrow \quad \beta(\alpha y - i) = (1-\beta)(-\alpha + p)y \quad \Rightarrow \quad \alpha y_l = \beta i + (1-\beta)py_l$$