Market Timing, Investment, and Risk Management*

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December 22, 2011

Abstract

Firms face uncertain financing conditions and in particular the risk of a sudden rise in financing costs during financial crises. We capture the firm's precautionary and market timing motives in a tractable model of dynamic corporate financial management when external financing conditions are stochastic. While firms value financial slack and build cash reserves to mitigate financial constraints, uncertainty about future financing opportunities induce them to rationally time the equity market. The market timing motive can cause investment to be decreasing (and the marginal value of cash to be increasing) in financial slack, and can lead a financially constrained firm to gamble. Quantitatively, we find that firms' optimal responses to the threat of a financial crisis can significantly smooth out the impact of financing shocks on investments, marginal values of cash, and the risk premium over time. As a result, financially constrained firms might appear to be unconstrained even though the level of their investments has dropped significantly in response to the threat of the crisis. Finally, we highlight the differences in how firms respond to financing shocks and productivity shocks.

*We are grateful to Viral Acharya, Michael Adler, Nittai Bergman, Charles Calomiris, Andrea Eisfeldt, Xavier Gabaix, Zhiguo He, Jennifer Huang, David Hirshleifer, Stewart Myers, Emi Nakamura, Paul Povel, Adriano Rampini, Doriana Ruffino, Antoinette Schoar, Alp Simsek, Jeremy Stein, Jeffrey Wurgler and seminar participants at Columbia, Duke Fuqua, Fordham, LBS, LSE, SUNY Buffalo, Berkeley, UNC-Chapel Hill, CKGSB, UCLA, Global Association of Risk Professionals (GARP), Theory Workshop on Corporate Finance and Financial Markets (at NYU), and Minnesota Corporate Finance Conference for their comments.

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1 Introduction

The financial crisis of 2008 and the European debt crisis of 2011 are fresh reminders of the substantial uncertainties that corporations face at times about their financing conditions. Recent studies have documented dramatic changes in firms' financing and investment behaviors during the crisis. For example, Ivashina and Scharfstein (2010) document aggressive credit line drawdowns by firms for precautionary reasons. Campello, Graham, and Harvey (2010) and Campello, Giambona, Graham, and Harvey (2010) show that the financially constrained firms planned deeper cuts in investment, spending, burned more cash, drew more credit from banks, and also engaged in more asset sales in the crisis.

Rational firms will adapt to fluctuations in financing conditions by hoarding cash, postponing or bringing forward investments, and by timing favorable market conditions to raise more funds than they need in good times, or by hedging against unfavorable market conditions. However, there is little theoretical research that tries to answer the following questions. How should firms change their financing, investment, and risk management policies during a period of severe financial constraints? And how should firms behave when facing the threat of financial crisis in the future?

We address these questions in this paper by proposing a dynamic model of investment, financing, and risk management for firms facing stochastic financing conditions. Our model builds on the dynamic framework of a firm facing external financing costs considered by Decamps, Mariotti, Rochet, and Villeneuve (2011) and Bolton, Chen, and Wang (2011) by adding stochastic financing opportunities. The five main building blocks of the model are: 1) a constant-returns-to-scale production function with independently and identically distributed (i.i.d.) productivity shocks and convex capital adjustment costs as in Hayashi (1982); 2) stochastic external financing costs; 3) constant cash carry costs; 4) risk premia for productivity and financing shocks, and 5) dynamic hedging opportunities. The firm optimally manages its cash reserves, financing, and payout decisions, by following a state-dependent double-barrier (issuance and payout) policy intertwined with continuous adjustments of investment, cash accumulation, and hedging in between the issuance and payout barriers.

The main results of our analysis are as follows. First, during a financial crisis (e.g., a period of high external financing costs), the firm cuts investment aggressively (and engages in asset sales), even if the productivity of its capital remains unaffected, and especially so if it enters the crisis with low cash reserves. These predictions are in line with the findings of Duchin, Ozbas, and Sensoy (2010).

Second, during favorable market conditions (a period of low external financing costs), the firm may *time the market* depending on the level of its cash holdings. For sufficiently high cash holdings, the precautionary motive for holding cash dominates, so that firm value is concave in cash holdings, and investment is increasing in financial slack. However, when the firm's cash holding is low, the market timing option the firm faces in good times becomes sufficiently attractive that it chooses to tap equity markets even though it is not in immediate need of cash. In that case, the firm makes lumpy equity issues in order to economize on its fixed cost of tapping equity markets. The firm's timing of equity markets behavior in good times is consistent with the findings in Baker and Wurgler (2002), DeAngelo, DeAngelo, and Stulz (2010), Fama and French (2005), and Huang and Ritter (2009).

Third, the timing behavior in good times implies that firm value can be convex in financial slack, and that investment can be decreasing in financial slack. Indeed, when the firm's timing motive dominates its precautionary saving motive, firm value turns convex and risk seeking behavior surfaces. Rather than hedging, the firm then speculates in order to increase the market timing option value. This counter-intuitive prediction contradicts the predictions of most models of investment with financial slack.[1]

Fourth, we show quantitatively that anticipation of deteriorating financing conditions in the future provides strong cash hoarding incentives. Firms invest more conservatively in good times in anticipation of adverse financing shocks in the future. Consequently, the impact of financing shocks on investment ex post is much weaker if shocks are well anticipated.

[1] See, e.g., Fazzari, Hubbard, and Petersen (1988), Kaplan and Zingales (1997), Riddick and Whited (2009).

Following financing shocks, the firm piles up cash into its war chest as a defense for prolonged adverse financing conditions. These predictions are consistent with the findings of Ivashina and Scharfstein (2010), and Campello et al. (2010a, 2010b).

Fifth, the firm's risk premium has two components when there are aggregate stochastic financing opportunities: a productivity risk premium and a financing risk premium. Both risk premia change substantially with the firm's cash holding, especially when financing conditions are poor. Quantitatively, the financing risk premium is significant for firms with low cash holdings, especially in a financial crisis or when the probability of a financial crisis is high. While the productivity risk premium generally decreases with the firm's cash holdings, the market timing motive induces it to increase with cash due to the firm's risk-seeking motive.

Sixth, as the expected duration of the state with favorable financing conditions shortens, the firm issues equity sooner (and delays payouts to shareholders) because the window of opportunity is smaller. Overall, the firm's cash inventory rises in anticipation of a significant worsening of equity financing opportunities. Bates, Kahle, and Stulz (2009) find that the average cash-to-asset ratio of US firms has nearly doubled in the past quarter century, and attribute this rise in cash holdings to firms' perceived increase in idiosyncratic risk. Our model provides an alternative explanation to their empirical findings. Our model also helps explain the investment and financing policies of many US non-financial firms in the years prior to the financial crisis of 2007-2008, to the extent that these firms had anticipated a potential worsening of financing conditions.

Our results highlight the sophisticated dynamic interactions between firm savings and investment. Typically, we expect that higher cash holdings or lower expected future financing costs will relax a firm's financial constraint. Hence, investment should increase with cash (and other financial slack measures such as credit) and decrease with expected financing costs. This is generally true and holds in dynamic corporate finance models and also optimal dynamic contracting models in the absence of stochastic financing conditions. However, we show that with stochastic financing opportunities, investment is no longer monotonically increasing in cash, nor is it monotonically decreasing with expected financing costs. The key to these relations lies in the optionality of market timing and the time-varying behavior of the marginal value of cash.

Our results also show that first-generation static models of financial constraints and corporate investment are inadequate to explain corporate investment policy solely on the basis of a comparative statics analysis. Static models, such as Fazzari, Hubbard, and Petersen (1988), Froot, Scharfstein, and Stein (1993) and Kaplan and Zingales (1997), are unsuited to explain the effects of market timing on corporate investment, since these effects cannot be captured by an *exogenous* change in the cost of external equity financing or an *exogenous* change in the firm's cash holdings. Importantly, market timing can only matter when there is a *finitely-lived window of opportunity* of getting access to cheaper equity financing. Moreover, market timing interacts in a complex way with the firm's precautionary cash management and investment policies: When cash is tight and dwindling, the firm accelerates its investment, times the equity market, and speculates, but not otherwise. More recent dynamic models on investment with financial constraints include Gomes (2001), Almeida, Campello, and Weisbach (2004), Hennessy and Whited (2005, 2007), Riddick and Whited (2009), Decamps, Mariotti, Rochet, and Villeneuve (2011), and Bolton, Chen, and Wang (2011), among others. All these models, however, study the firm's dynamic investment and financing behavior, taking the financing conditions as time-invariant.

Our work also relates to two other sets of dynamic models. First, dynamic limited commitment contracting models of corporate investment and financing, which build on Bolton and Scharfstein (1990), by DeMarzo and Fishman (2007) and DeMarzo, Fishman, He, and Wang (2010). These models derive optimal dynamic contracts and corporate investment with capital adjustment costs. Second, Rampini and Viswanathan (2010, 2011) develop a dynamic model of collateralized financing, in which the firm has access to complete markets, but is subject to endogenous collateral constraints induced by limited enforcement.

By construction, the productivity shocks in our model are *i.i.d.* Thus, firms that time equity markets are ones with low cash holdings as opposed to ones having better investment opportunities.[2] DeAngelo, DeAngelo, and Stulz (2009) find that many firms issuing equity look as if they are cash constrained. Our model shows that their finding is consistent with both market timing and precautionary hoarding motives. Testing of our market timing hypothesis would ideally look for firm behavior not only in equity issuance, but also in investment, and hedging decisions. For cash-strapped firms, corporate investment may increase, and speculation may arise as the firm's cash dwindles and gets closer to the issuance boundary under favorable equity market conditions.

Our paper is one of the first dynamic models of corporate investment with stochastic financing conditions. We echo the view expressed in Baker (2010) that supply effects (of equity in favorable equity markets) are important for corporate finance. While we treat changes in financing conditions as exogenous in this paper, the cause of these variations could be changes in market sentiment, the frictions of financial intermediation, investors' risk aversion, or aggregate uncertainty and adverse selection. In contemporaneous work, Hugonnier, Malamud, and Morellec (2011) also develop a dynamic model with stochastic financing conditions. They model investment as growth options, while we model investment as in Hayashi's q-theory framework. Also, they model stochastic financing opportunities via Poisson arrival, where the firm decides whether to undertake the financing opportunity or not. The duration of the financing opportunity is instantaneous in their model, while the duration of cheap financing in our model can be calibrated to the data. The two papers share the same overall focus but differ significantly in their modeling approach, and thus complement each other.

2 The Model

We consider a financially constrained firm facing stochastic investment and external financing conditions. Specifically, we assume that the firm can be in one of two states of the

[2] Time-varying investment opportunities may also play a significant role on cash accumulation and external financing. Eisfeldt and Muir (2011) empirically document that liquidity accumulation and external financing are positively correlated, and argue that a cash holding based precautionary saving model to some extent accounts for the empirical evidence.

world, denoted by $s_t = 1, 2$. In each state, the firm faces potentially different financing and investment opportunities. The state switches from 1 to 2 (or from 2 to 1) over a short time interval Δ with a constant probability $\zeta_1\Delta$ (or $\zeta_2\Delta$).[3]

2.1 Production technology

The firm employs capital and cash as the only factors of production. We normalize the price of capital to one and denote by K and I respectively the firm's capital stock and gross investment. As is standard in capital accumulation models, the capital stock K evolves according to:

$$dK_t = (I_t - \delta K_t)\, dt, \quad t \geq 0, \tag{1}$$

where $\delta \geq 0$ is the rate of depreciation for capital stock.

The firm's operating revenue is proportional to its capital stock K_t, and is given by $K_t dA_t$, where dA_t is the firm's productivity shock over time increment dt. We assume that

$$dA_t = \mu\,(s_t)\, dt + \sigma\,(s_t)\, dZ_t^A, \tag{2}$$

where Z_t^A is a standard Brownian motion and $\mu(s_t)$ and $\sigma(s_t)$ denote the drift and volatility in state s_t. The firm's operating profit dY_t over time increment dt is then given by:

$$dY_t = K_t dA_t - I_t dt - \Gamma(I_t, K_t, s_t) dt, \quad t \geq 0, \tag{3}$$

where $K_t dA_t$ is the firm's operating revenue, $I_t dt$ is the investment cost over time dt and $\Gamma(I_t, K_t, s_t)dt$ is the additional adjustment cost that the firm incurs in the investment process.[4]

Following the neoclassical investment literature (Hayashi (1982)), we assume that the firm's adjustment cost is homogeneous of degree one in I and K. In other words, the

[3]The model can be generalized to $s_t > 2$ states in a straightforward way. For an illustration of the more general setup, see the appendix.

[4]Note that we allow the adjustment costs to be state dependent.

adjustment cost takes the homogeneous form $\Gamma(I, K, s) = g_s(i)K$, where i is the firm's investment capital ratio ($i = I/K$), and $g_s(i)$ is a state-dependent function that is increasing and convex in i.[5] We also assume that $g_s(i)$ is quadratic:

$$g_s(i) = \frac{\theta_s(i - \nu_s)^2}{2}, \tag{4}$$

where θ_s is the adjustment cost parameter and ν_s is a constant parameter.[6] These assumptions make the analysis more tractable and our main results also hold for other functional forms of $g_s(i)$.

Finally, the firm can liquidate its assets at any time and obtain a liquidation value L_t that is also proportional to the firm's capital stock K_t. We let the liquidation value $L_t = l_s K_t$ depend on the state s_t (where l_s denotes the recovery value per unit of capital in state s).

2.2 Stochastic Financing Opportunities

The firm may choose to raise external equity financing at any point in time.[7] When doing so it incurs a fixed as well as a variable cost of issuing stock. The fixed cost is given by $\phi_s K$, where ϕ_s is the fixed cost parameter in state s. We take the fixed cost to be proportional to the firm's capital stock K, as this ensures that the firm does not *grow out of its fixed costs of issuing equity.* This assumption is also analytically convenient, as it preserves the homogeneity of the model in the firm's capital stock K. After paying the fixed cost $\phi_s K$ the firm also pays a variable (state dependent) cost $\gamma_s > 0$ for each incremental dollar it raises.

We denote by:

1. H – the process for the firm's cumulative external financing (so that dH_t denotes the

[5] For notational convenience we use the notation x_s to denote a state dependent variable $x(s)$ whenever there is no ambiguity.

[6] In the literature, common choices of ν_s are either zero or the rate of deprecation δ. While the former choice implies zero adjustment cost for zero gross investment, the latter choice implies a zero adjustment cost when net investment is zero.

[7] For simplicity, we only consider external equity financing as the source of external funds for the firm. We leave the generalization of allowing the firm to also issue debt for future research.

incremental external funds over time dt);

2. X – the firm's cumulative issuance costs;

3. W – the process for the firm's cash stock;

4. U – the firm's cumulative non-decreasing payout process to shareholders (so that dU_t is the incremental payout over time dt).

Distributing cash to shareholders may take the form of a special dividend or a share repurchase.[8] The benefit of a payout is that shareholders can invest the proceeds at the market rate of return and avoid paying a *carry cost* on the firm's retained cash holdings. We denote the unit cost of carrying cash inside the firm by $\lambda dt \geq 0$.[9]

If the firm runs out of cash ($W_t = 0$), it needs to raise external funds to continue operating or its assets will be liquidated. If the firm chooses to raise new external funds to continue operating, it must pay the financing costs specified above. The firm may prefer liquidation if the cost of financing is too high relative to the continuation value (e.g. when μ is low). We denote by τ the firm's stochastic liquidation time.[10]

We can then write the dynamics for the firm's cash W as follows:

$$dW_t = [K_t dA_t - I_t dt - \Gamma(I_t, K_t, s_t)]dt + (r(s_t) - \lambda)\, W_t dt + dH_t - dU_t\,, \tag{5}$$

where the firm term is the firm's cash flows from operations dY_t given in (3), the second term is the return on W_t (net of the carry cost λ), the third term dH_t is the cash inflow from

[8]We cannot distinguish between a special dividend and a share repurchase, as we do not model taxes. Note, however, that a commitment to regular dividend payments is suboptimal in our model. We also exclude any fixed or variable payout costs so as not to overburden the model.

[9]The cost of carrying cash may arise from an agency problem or from tax distortions. Cash retentions are tax disadvantaged because the associated tax rates generally exceed those on interest income (Graham (2000)). Since there is a cost λ of hoarding cash the firm may find it optimal to distribute cash back to shareholders when its cash inventory grows too large. If $\lambda = 0$ the firm has no incentives to pay out cash since keeping cash inside the firm does not have any disadvantages, but still has the benefit of relaxing financial constraints. We could also imagine that there are settings in which $\lambda \leq 0$. For example, if the firm may have better investment opportunities than investors. We do not explore this case in this paper as we are interested in a trade-off model for cash holdings.

[10]Note that $\tau = \infty$ means that the firm never chooses to liquidate.

external financing, and the last term dU_t is the cash outflow to investors.[11] Note that this is a completely general financial accounting equation, where dH_t and dU_t are endogenously determined by the firm.

The homogeneity assumptions embedded in the production technology, the adjustment cost, and the financing costs allow us to deliver our key results in a parsimonious and analytically tractable homogeneous model. Adjustment costs may not always be convex and the production technology may exhibit long-run decreasing returns to scale in practice, but these functional forms substantially complicate the formal analysis.[12] As will become clear below, the homogeneity of our model in W and K allows us to reduce the dynamics to a one-dimensional equation, which is relatively straightforward to solve.

2.3 Systematic Risk and the Pricing of Risk

There are two different sources of systematic risk in our model: a) a small and continuous diffusion shock, and b) a large discrete shock when the economy switches from one state to another. The diffusion shock in any given state s may be correlated with the firm's productivity shock, and we denote the correlation coefficient by ρ. The discrete shock affects both the firm's productivity and its external financing costs, as we have highlighted.

How are these sources of systematic risk priced? Our model can allow for either risk-neutral or risk-averse investors. If investors are risk neutral, then the pricing of risk is zero and the physical probability distribution coincides with the risk-neutral probability distribution. If investors are risk averse, we need to distinguish between physical and risk-neutral measures. We can do so as follows.

For the diffusion risk, we assume that there is a constant market price of risk η_s in each state s. The firm's *risk adjusted* productivity shock (under the risk-neutral probability

[11] Thus $(dH_t - dU_t)$ denotes the net cash flow from financing.
[12] See Hennessy and Whited (2005, 2007) for an analysis of a non-homogenous model.

measure $\mathbb{Q}$) is then given by

$$dA_t = \hat{\mu}(s_t)\,dt + \sigma(s_t)\,d\hat{Z}_t^A, \tag{6}$$

where the mean for the productivity shock accounts for the firm's exposure to the diffusion risk as follows:

$$\hat{\mu}(s_t) \equiv \hat{\mu}_s = \mu_s - \rho\eta_s\sigma_s, \tag{7}$$

and $\hat{Z}_t^A$ is a standard Brownian motion under the risk-neutral probability measure $\mathbb{Q}$.[13]

A risk-averse investor also requires a risk premium to compensate for the discrete risk of the economy switching states. We capture this risk premium through the wedge between the transition intensity under the physical probability measure and the transition intensity under the risk-neutral probability measure $\mathbb{Q}$. Let $\hat{\zeta}_1$ and $\hat{\zeta}_2$ denote the transition intensities under the risk-neutral measure from state 1 to state 2 and from state 2 to state 1, respectively. The risk-neutral intensities are then related to their physical intensities ζ_1 and ζ_2 as follows:

$$\hat{\zeta}_1 = e^{\kappa_1}\zeta_1\,, \text{ and } \; \hat{\zeta}_2 = e^{\kappa_2}\zeta_2, \tag{8}$$

where the parameters κ_1 and κ_2 capture the risk premium required by a risk-averse investor for the exposure to the discrete risk of state switching. A positive κ_s implies that $\hat{\zeta}_s > \zeta_s$, so that when κ_s is positive it is as if a risk-averse investor perceived a higher transition intensity under the risk-neutral probability measure than under the physical measure. Vice versa, a negative κ_s implies that $\hat{\zeta}_s < \zeta_s$. In other words, the perceived transition intensity for a risk-averse investor under the risk-neutral measure is lower. As we show in the appendix, κ_s is positive in one state and negative in the other. Intuitively, this reflects the idea that a risk-averse investor makes an upward adjustment of the transition intensity from the good to the bad state (with $\kappa_s > 0$) and a downward adjustment of the transition intensity from the bad to the good state (with $\kappa_s < 0$). In sum, it is as if a risk-averse investor were

[13] In the appendix, we provide a more detailed discussion of systematic risk premia. The key observation is that the adjustment from the physical to the risk-neutral probability measure reflects a representative risk-averse investor's stochastic discount factor (SDF) in a dynamic asset-pricing model.

uniformly more ‘pessimistic’ than a risk-neutral investor: she thinks ‘good times’ are likely to be shorter and ‘bad times’ longer.

2.4 Firm optimality

The firm chooses its investment I, cumulative payout policy U, cumulative external financing H, and liquidation time τ to maximize firm value defined as follows (under the risk-neutral measure):

$$\mathbb{E}_0^{\mathbb{Q}}\left[\int_0^{\tau} e^{-\int_0^t r_u du}\left(dU_t - dH_t - dX_t\right) + e^{-\int_0^{\tau} r_u du}\left(L_\tau + W_\tau\right)\right], \tag{9}$$

where r_u denotes the interest rate at time u. The first term is the discounted value of payouts to shareholders, and the second term is the discounted value upon liquidation. Note that optimality may imply that the firm never liquidates. In that case, we simply have $\tau = \infty$.

3 Model Solution

Given that the firm faces external financing costs ($\phi_s > 0$, $\gamma_s \geq 0$), its value depends on both its capital stock K and its cash holdings W. Thus, let $P(K, W, s)$ denote the value of the firm in state s. Given that the firm incurs a carry cost λ on its stock of cash one would expect that it will choose to pay out some of its cash once its stock grows large. Accordingly, let $\overline{W}_s$ denote the (upper) payout boundary. Similarly, if the firm's cash holding is low, it may choose to issue equity. We therefore let $\underline{W}_s$ denote the (lower) issuance boundary.

The interior regions: $W \in (\underline{W}_s, \overline{W}_s)$ **for** $s = 1, 2$**.** When a firm's cash holding W is in the interior regions, $P(K, W, s)$ satisfies the following system of Hamilton-Jacobi-Bellman (HJB) equations:

$$\begin{aligned} r_s P(K, W, s) = \max_I \left[(r_s - \lambda_s) W + \hat{\mu}_s K - I - \Gamma(I, K, s)\right] P_W(K, W, s) + \frac{\sigma_s^2 K^2}{2} P_{WW}(K, W, s) \\ + (I - \delta K) P_K(K, W, s) + \hat{\zeta}_s \left(P(K, W, s^-) - P(K, W, s)\right). \end{aligned} \tag{10}$$

The first and the second terms on the right side of the HJB equation (10) represent the effects of the expected change in the firm's cash holding W and volatility of W on firm value. Note first that the firm's cash grows at the net return $(r_s - \lambda_s)$ and is augmented by the firm's expected cash flow from operations (under the risk-neutral measure) $\hat{\mu}_s K$ minus the firm's capital expenditure $(I + \Gamma(I, K, s))$. Second, the firm's cash stock is volatile only to the extent that cash flows from operations are volatile, and the volatility of the firm's revenues is proportional to the firm's size as measured by its capital stock K. The third term represents the effect of capital stock changes on firm value, and the last term the expected change of firm value when the state changes from s to s^-.

Since firm value is homogeneous of degree one in W and K in each state, we can write $P(K, W, s) = p_s(w)K$. Substituting for this expression into (10) and simplifying, we obtain the following system of ordinary differential equations (ODE) for $p_s(w)$:

$$
\begin{aligned}
r_s p_s(w) = \max_{i_s} & \left[(r_s - \lambda_s) w + \hat{\mu}_s - i_s - g_s(i_s) \right] p_s'(w) + \frac{\sigma_s^2}{2} p_s''(w) \\
& + (i_s - \delta)\left(p_s(w) - w p_s'(w)\right) + \hat{\zeta}_s \left(p_{s^-}(w) - p_s(w)\right).
\end{aligned} \tag{11}
$$

The first-order condition (FOC) for the investment-capital ratio $i_s(w)$ is then given by:

$$
i_s(w) = \frac{1}{\theta_s} \left(\frac{p_s(w)}{p_s'(w)} - w - 1 \right) + \nu_s, \tag{12}
$$

where

$$
p_s'(w) = P_W(K, W, s)
$$

is the *marginal value of cash* in state s.

The payout boundary $\overline{W}_s$ and the payout region ($W \geq \overline{W}_s$). The firm starts paying out cash when the marginal value of cash held by the firm is less than the marginal value of cash held by shareholders. The payout boundary $\overline{w}_s = \overline{W}_s / K$ thus satisfies the following value matching condition:

$$
p_s'(\overline{w}_s) = 1. \tag{13}
$$

When the firm chooses to pay out, the marginal value of cash $p'(w)$ must be one. Otherwise, the firm can always do better by changing $\overline{w}_s$. Moreover, payout optimality implies that the following super contact condition (Dumas (1991)) holds:

$$p_s''(\overline{w}_s) = 0. \tag{14}$$

We specify next the value function outside the payout boundary. If the marginal value of cash in state s is such that $p_s'(w) < 1$ the firm is better off reducing its cash holding to $\overline{w}_s$ by making a lump-sum payout. Therefore, we have

$$p_s(w) = p_s(\overline{w}_s) + (w - \overline{w}_s), \quad w > \overline{w}_s. \tag{15}$$

This situation could arise when the firm starts off with too much cash or when the firm's cash holding in state s is such that $w_s > \overline{w}_{s-}$ and the firm suddenly moves from state s into state s^-.

The equity issuance boundary $\underline{W}_s$ and region $(W \leq \underline{W}_s)$. Similarly, we must specify the value function outside the issuance boundary. Indeed, it is possible that the firm could suddenly transit from the state $s-$ with the financing boundary $\underline{w}_{s-}$ into the other state s with a higher financing boundary ($\underline{w}_s > \underline{w}_{s-}$) and that its cash holding lies between the two lower financing boundaries ($\underline{w}_{s-} < w < \underline{w}_s$). What happens then?

Basically, if the firm is sufficiently valuable it then chooses to raise external funds through an equity issue, so as to bring its cash stock back into the interior region. But how much should the firm raise in this situation? Let M_s denote the firm's cash level after equity issuance, which we refer to the "target" return level, and let $m_s = M_s/K_s$. Similarly, let $\underline{W}_s$ denote the boundary for equity issuance in state s and $\underline{w}_s = \underline{W}_s/K$. Firm value per unit of capital in state s, $p_s(w)$, when $w \leq \underline{w}_s$ then satisfies

$$p_s(w) = p_s(m_s) - \phi_s - (1 + \gamma_s)(m_s - w), \quad w \leq \underline{w}_s. \tag{16}$$

We thus have the following value matching and smooth pasting conditions for $\underline{w}_s$:

$$p_s(\underline{w}_s) = p_s(m_s) - \phi_s - (1+\gamma_s)(m_s - \underline{w}_s), \tag{17}$$

$$p_s'(m_s) = 1+\gamma_s. \tag{18}$$

With fixed issuance costs ($\phi_s > 0$), equity issuance will thus be lumpy. The firm first pays the issuance cost ϕ_s per unit of capital and then incurs the marginal cost γ_s for each unit raised. Equation (17) states that firm value is continuous around issuance time. Additionally, the firm optimally selects the return target m_s so that the marginal benefit of issuance $p_s'(m_s)$ is equal to the marginal cost $1+\gamma_s$, which yields (18).

How does the firm determine its equity issuance boundary $\underline{w}_s$? We use the following two-step procedure. First, suppose that the issuance boundary $\underline{w}_s$ is interior ($\underline{w}_s > 0$). Then, the standard optimality condition implies that:

$$p_s'(\underline{w}_s) = 1+\gamma_s. \tag{19}$$

Intuitively, if the firm chooses to issue equity before it runs out of cash, it must be the case that the marginal value of cash at the issuance boundary $\underline{w}_s > 0$ is equal to the marginal issuance cost $1+\gamma_s$. If (19) fails to hold, the firm will not issue equity until it exhausts its cash holding, i.e. $\underline{w}_s = 0$. In that case, the option value to tap equity markets earlier than absolutely necessary is valued at zero. Using the above two-step procedure, we characterize the optimal issuance boundary $\underline{w}_s \geq 0$.

We also need to determine whether equity issuance or liquidation is optimal, as the firm always has the option to liquidate. Under our assumptions, the firm's capital is productive and thus its going-concern value is higher than its liquidation value. Therefore, the firm never voluntarily liquidates itself before it runs out cash.

However, when it runs out of cash, liquidation may be preferred if the alternative of accessing external financing is too costly. If the firm liquidates, we have

$$p_s(0) = l_s. \tag{20}$$

The firm will prefer equity issuance to liquidation as long as the equilibrium value $p_s(0)$ under external financing arrangement is greater than the complete liquidation value l_s.

Finally, for our later discussion it is helpful to introduce the following notions. First, *Enterprise value* is often defined as firm value net of short-term liquid asset. This measure is meant to capture the value created from productive illiquid capital. In our model, it equals $P(K, W, s) - W$. Second, we define *average* q as the ratio between enterprise value and its capital stock,

$$q_s(w) = \frac{P(K, W, s) - W}{K} = p_s(w) - w. \tag{21}$$

Third, the *sensitivity of average* q *to changes in cash holdings* measures how much enterprise value increases with an extra dollar of cash, and is given by

$$q_s'(w) = p_s'(w) - 1\,. \tag{22}$$

We also refer to $q_s'(w)$ as the *(net) marginal value of cash.* As w approaches the optimal payout boundary $\overline{w}$, $w \to \overline{w}$, $q_s(w) \to 0$.

4 Quantitative results

Having characterized the conditions that the solution to the firm's dynamic optimization problem must satisfy, we can now illustrate the numerical solutions for given parameter choices of the model. We begin by motivating our choice of parameters and then illustrate the model's solutions in respectively the good and bad states of the world.

4.1 Parameter choice and calibration

In our choice of parameters, we select plausible numbers based on existing empirical evidence to the extent that it is available. For those parameters on which there is no empirical evidence we make an educated guess to reflect the situation we are seeking to capture in our model. Finally there are a few parameters we do not allow to vary across the two states so as to better isolate the effects of changes in external financing conditions.

The capital liquidation value is set to $l_G = 1.0$ in state G, in line with estimates provided by Hennessy and Whited (2007).[14] In the bad state the capital liquidation value is set to $l_B = 0.3$ to reflect the severe costs of asset fire sales during a financial crisis, when few investors have either sufficiently deep pockets or the risk appetite to acquire assets.[15] The model solution will depends on these liquidation values only when the firm finds it optimal to liquidate instead of raising external funds.

We set the marginal cost of issuance in both states to be $\gamma = 6\%$ based on estimates reported in Altinkihc and Hansen (2000). We keep this parameter constant across the two states for simplicity and focus only on changes in the fixed cost of equity issuance to capture changes in the firm's financing opportunities. The fixed cost of equity issuance in the good state is set at $\phi_G = 0.5\%$ of the firm's capital stock. In the benchmark model, this value implies that the average cost per unit of external financing raised in state G is around 10%. This is somewhat on the low side compared with cost estimates for IPOs in Ritter (2003), but is in the ballpark with estimates for seasoned offers in Eckbo, Masulis and Norli (2007).[16] As for the issuance costs in state B, we chose $\phi_B = 50\%$. There is no empirical study we can rely on for the estimates of issuance costs in a financial crisis state, for the obvious reason that there are virtually no IPOs or SEOs in a crisis. Our choice for the parameter of

[14]They suggest an average value for l of 0.9, so that the liquidation value in the good state should be somewhat higher.

[15]See Shleifer and Vishny (1992), Acharya and Viswanathan (2011), and Campello, Graham, and Harvey (2010).

[16]Eckbo, Masulis and Norli (2007) report total costs of 6.09% for firm commitment offers, excluding the cost of the offer price discount and the value of Green Shoe options. They also report a negative average price reaction to an SEO announcement of -2.22%.

ϕ_B is meant to reflect the fact that raising external financing becomes extremely costly in a financial crisis, and only firms that are desperate for cash are forced to raise new funds. We show that even with $\phi_B = 50\%$ firms that run out of cash in the crisis state still prefer raising equity to liquidation.

The transition intensity out of state G is set at $\zeta_G = 0.1$, which implies an average duration of 10 years for good times. The transition intensity out of state B is $\zeta_B = 0.5$, with an implied average length of a financial crisis being 2 years. We choose the price of risk with respect to financing shocks in state G to be $\kappa_G = \ln 3$, which implies that the risk-adjusted transition intensity out of state G is $\hat{\zeta}_G = e^{\kappa_G}\zeta_G = 0.3$. Due to symmetry, the risk-adjusted transition intensity out of state B is then $\hat{\zeta}_B = e^{-\kappa_G}\zeta_B = 0.167$. These risk adjustments are clearly significant. While we take these risk adjustments as exogenous in this paper, they can be generated in general equilibrium when the same financing shocks also affect aggregate investment and output (see e.g., Chen 2010).

The other parameters remain the same in the two states: the riskfree rate is $r = 5\%$, the volatility of the productivity shock is $\sigma = 12\%$, the rate of depreciation of capital is $\delta = 15\%$, and the adjustment cost parameter ν is set to equal the depreciation rate, so that $\nu = \delta = 15\%$. We rely on the technology parameters estimated by Eberly, Rebelo, and Vincent (2010) for these parameter choices. The cash-carrying cost is set to $\lambda = 1.5\%$. While we do not take a firm stand on the precise interpretation of the cash-carrying cost, it can be due to either a tax disadvantage of cash or to agency frictions. Following the tax interpretation, with a 30% marginal tax rate and a 5% interest rate, we are in the ball park for a 1.5% cash carrying cost. Although in reality these parameter values clearly change with the state of nature, we keep them fixed in this model so as to isolate the effects of changes in external financing conditions. All the parameter values are annualized whenever applicable and summarized in Table 4.

Finally, we calibrate the expected productivity μ and the adjustment cost parameter θ to match the median cash-capital ratio and investment-capital ratio for U.S. public firms during the period of 1981-2010.[17] We then obtain $\mu = 22.7\%$ and $\theta = 1.8$, both of which are within the range of empirical estimates documented in previous studies.[18]

4.2 Market timing in good times

When the firm is in state G, it may enter the crisis state B with probability $\zeta_G = 0.1$ per unit time. As the firm faces substantially higher external financing costs in state B, we show that the option to time the equity market in good times has significant value and generates rich implications for investment dynamics.

Figure 1 plots average q and investment-capital ratio for state G as well as their sensitivities with respect to cash-capital ratio w. Panel A shows as expected that the average q increases with w and is relatively stable in state G. The optimal external financing boundary is $\underline{w}_G = 0.027$. At this point the firm still has sufficient cash to continue operating. Further deferring external financing would help the firm save on the time value of money for financing costs and also on subsequent cash carry costs. However, doing so would mean taking the risk that the favorable financing opportunities disappear in the mean time and that the state of nature switches to the bad state when financing costs are much higher. The firm trades off these two margins and optimally exercises the equity issuance option by tapping securities markets when w hits the lower barrier $\underline{w}_G$.

Once reaching the financing boundary, the firm issues an amount $(m_G - \underline{w}_G) = 0.128$ per unit of its capital stock. The size of issuance reflects the need to economize on the fixed cost of issuance ϕ_G. When the firm's cash holding reaches $\overline{w}_G = 0.371$, it pays out incremental cash since the net marginal value of cash (that is the difference between the value of a dollar in the hands of the firm and the value of a dollar in the hands of investors) is zero at that point: $q'_G(\overline{w}) = 0$. See Panel B.

When firms face external financing costs, it is optimal for them to hoard cash for precau-

[17] For the median firm, the average cash-capital ratio is 0.29, and the average investment-capital ratio is 0.17. The details of the data construction are given in Appendix B.

[18] See for example Eberly, Rebelo, and Vincent (2010) and Whited (1992).

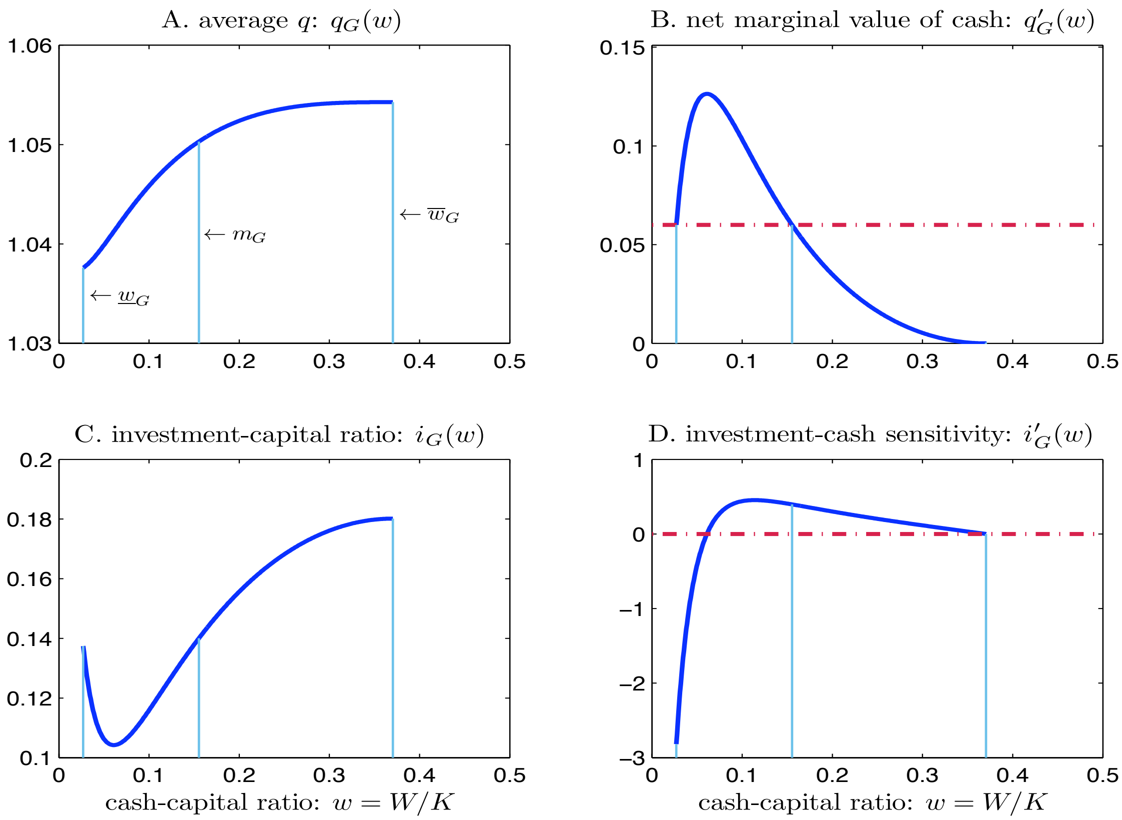

Figure 1: **Firm value and investment in the normal state, state G.**

tionary reasons. Firm value is thus increasing and concave in financial slack in most models of financially constrained firms, causing these firms *endogenously risk-averse*. In our model, while the precautionary motive for hoarding cash is still a key factor, stochastic financing opportunities introduce an additional motive for the firm to issue equity: timing equity markets in good times. This market timing option is more in the money near the equity issuance boundary, which causes firm value to become *convex* in w so that the firm actually becomes *endogenously risk-loving* as its cash holdings approach the lower barrier $\underline{w}_G$.

Panel B clearly shows that firm value is not globally concave in w. For sufficiently high w, $w \geq 0.061$, $q_G(w)$ is concave. When the firm has sufficient cash, the firm's equity issuance need is quite distant so that the financing timing option is out of the money. The precautionary motive to hoard cash then dominates causing the firm to behave in an effectively risk-averse manner. In contrast, for low values of w, $w \leq 0.061$, the firm is more concerned about the risk that financing costs may increase in the future, when the state switches to B. A firm with low cash holdings may want to issue equity while it still has access to relatively cheap financing opportunities, even before it runs out of cash.

Given that firms face fixed equity issuance costs it is optimal for them to raise lumpy amounts of cash when they choose to tap equity markets. Since the issuance boundary $\underline{w}_G$ and the return target cash balance m_G are optimally chosen, the marginal values of cash at these two points must be equal:

$$q_G'(\underline{w}_G) = q_G'(m_G) = \gamma_G \,. \tag{23}$$

The dash-dotted line in Panel B gives the (net) marginal cost of equity issuance at $\underline{w}_G$ and m_G: $\gamma_G = 0.06$. One immediate implication of (23) is that Tobin's $q_G(w)$ is not globally concave in w.

The optimal investment rule (12) implies that investment sensitivity $i_s'(w)$ satisfies the condition

$$i_s'(w) = -\frac{1}{\theta_s}\frac{p_s(w)p_s''(w)}{p_s'(w)^2}. \tag{24}$$

Therefore, investment increases with w if and only if firm value is concave in w. For $0.061 \leq w \leq 0.371$, $q_G(w)$ is concave and corporate investment increases with w. In contrast, in the region where $w \leq 0.061$, $q_G(w)$ is convex in w, which implies that investment *decreases* with w, contrary to conventional wisdom. This surprising result is due to the *interaction effect* between stochastic external financing conditions and the fixed equity issuance costs.

As the firm approaches the financing boundary $\underline{w}_G$ it may choose to accelerate investment (and thus reduce under-investment) to take advantage of the equity issuance timing option in good times, which becomes increasingly valuable as w decreases. In other words, anticipating equity issuance, the firm finds it undesirable to significantly cut investment to preserve cash. Simply put, the firm is more willing to invest when it expects to issue a lumpy amount of equity soon. Panels C and D of Figure 1 highlight this non-monotonicity of investment in cash. Our model is thus able to account for the seemingly paradoxical behavior that the prospect of higher financing costs in the future can cause investment to respond negatively to an *increase* in cash today.

Although there is considerable disagreement and debate in the literature on corporate investment about the sensitivity of investment with respect to cash flow (see e.g. the discussions in Kaplan and Zingales (1997) and FHP on this issue), there is a general consensus that investment is obviously monotonically increasing with cash, *ceteris paribus*. In this context, our result that, when firms face market-timing options, the monotonic relation between investment and cash only holds in the precautionary saving region is remarkable, for it points to the fragility of seemingly plausible but misleading predictions about how corporate investment is affected by cash holdings derived from simple static models. Next, we turn to investment and firm value in bad times (state B) and compare these with those in good times (state G).

4.3 High financing costs in bad times

Figure 2 plots average q and investment for both states, and their sensitivities with respect to w. As expected, average q in state G is higher than in state B. More remarkable is the fact that the difference between the average q in the two states is so large especially for lower levels of cash holdings. In our example, at $w = 0$, $q_G(0) = 1.036$ and $q_B(0) = 0.537$. This difference in average q is purely due to differences in *financing* opportunities. An important implication of this observation for a favored approach in the empirical literature on corporate investment is that using average q to control for *investment opportunities* and then testing for the presence of *financing constraints* by using variables such as cash flows or cash is not generally valid. Panel C shows that investment in state G is higher than in state B for a given w, and again the difference is especially large when w is low. Also, investment is much less variable with respect to w in state G. It is almost as if investment were independent of w, which might lead one to misleaingly conclude that the firm is essentially unconstrained in state G, if one focuses only on the investment sensitivity to cash in state G.

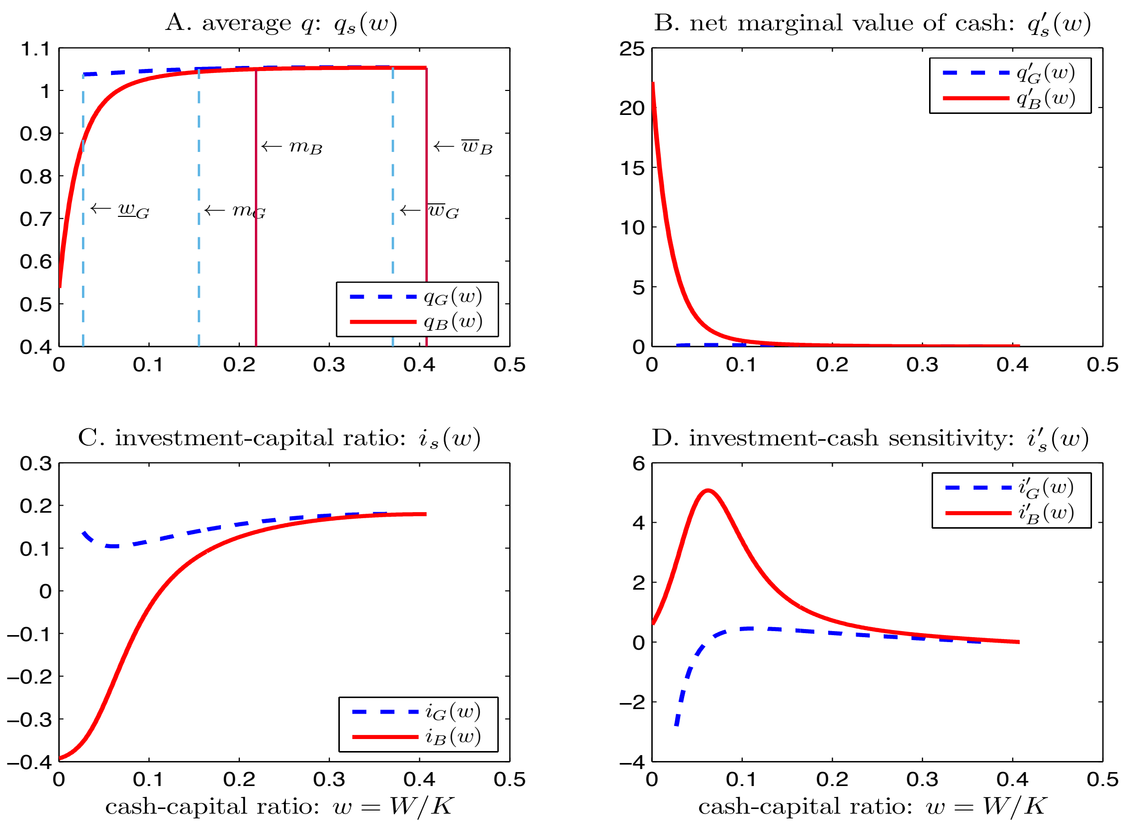

Figure 2: **Firm value and investment: Comparing states B vs G.**

In state B there is no market timing and hence the firm only issues equity when it runs out of cash, $\underline{w}_B = 0$. The amount of equity issuance is then $m_B = 0.219$, which is much larger than $m_G - \underline{w}_G = 0.128$, the amount of issuance in good times. The significant fixed issuance cost $\phi_B = 0.5$ in bad times causes the firm to be more aggressive should it decide to tap equity markets. The amount of issuance would of course be significantly decreased in bad times if we were to specify a proportional issuance cost γ_B that is much higher than the cost γ_G in good times. Note also that, since there is no market timing opportunity in state B firm value is globally concave in bad times. The firm's precautionary motive is stronger in bad times, so that we should expect to see the firm hoarding more cash. This is indeed reflected in the lower levels of investment and the higher payout boundary $\overline{w}_B = 0.408$, which is significantly larger than $\overline{w}_G = 0.371$.

Panel B underscores the significant impact of financing constraints on the marginal value of cash in bad times, even though state B is not permanent. In our model, as the firm runs out of cash (w approaches 0) the net marginal value of cash $q_B'(w)$ reaches 23! Strikingly, the firm also engages in large asset sales and divestment to avoid incurring the very costly external financing in bad times. Despite the fact that there is a steeply increasing marginal cost of asset sales, the firm chooses to sell up to 40% of its capital near $w = 0$ in bad times ($i_B(0) = -0.4$). Finally, unlike in good times, investment is monotonic in w because the firm behaves in a risk-averse manner and $q_B(w)$ is globally concave in w.

Conceptually, the firm's investment behavior and firm value are thus quite different in bad and good times. Quantitatively, the variation of the firm's behavior in bad times dwarfs the variation of its behavior in good times. In particular, firm value at low levels of cash holdings is much more volatile in state B than in state G, as can be seen from Panel A. This may be one reason why stock price volatility tends to rise sharply in downturns.

4.4 The stationary distribution

Table 1 reports the conditional stationary distributions for w, $i(w)$, and $q'(w)$ in both states G and B. Panel A shows that the average cash holding in state B (0.312) is higher than the average cash holding in state G (0.283) by about 10%. Understandably, firms on average hold more cash for precautionary reasons under unfavorable financial market conditions in order to mitigate the under-investment problem. Additionally, for a given percentile in the distribution, the cutoff wealth level is higher in state B than in state G, meaning that the precautionary motive is unambiguously stronger in state B than state G. Finally, it is striking that even at the bottom 1% of the cash holding, the firm's cash-capital ratio is still reasonably high, 0.088 for state G and 0.114 for state B, which reflects the firm's strong incentive to avoid running out of cash.

Panel B illustrates the conditional distribution of investment in states G and B. The average investment-capital ratio $i(w)$ is lower in state B (16.1%) than in state G (17%), as cash is more valuable on average in state B than in state G. Naturally, the under-

Table 1: CONDITIONAL DISTRIBUTIONS OF CASH-CAPITAL RATIO, INVESTMENT-CAPITAL RATIO, AND NET MARGINAL VALUE OF CASH

	mean	1%	25%	50%	75%	99%
	A. cash-capital ratio: $w = W/K$					
G	0.283	0.088	0.240	0.300	0.341	0.370
B	0.312	0.114	0.266	0.325	0.371	0.408
	B. investment-capital ratio: $i_s(w)$					
G	0.170	0.112	0.166	0.176	0.179	0.180
B	0.161	0.003	0.159	0.173	0.178	0.180
	C. net marginal value of cash: $q_s'(w)$					
G	0.015	0.000	0.001	0.005	0.019	0.111
B	0.031	0.000	0.001	0.008	0.028	0.351

investment problem is more significant for firms with low cash holdings (at the bottom of the distribution) in state B compared to state G. For example, the firm that ranks at the bottom 1% in state B invests only 0.3% of its capital stock, while the firm that ranks at the bottom 1% in state G invests 11.2%, which is about 38 times the investment level for the firm that ranks at the bottom 1% in state B. Thus, firms substantially cut investment in order to decrease the likelihood of expensive external equity issuance in bad times. As soon as the firm piles up a moderate amount of cash, the under-investment wedge between the two states disappears. In fact, the top half of the distributions of investments in the two states are almost identical. This result is in sharp contrast to the large gap between the investment-capital ratios $i_G(w)$ and $i_B(w)$ in Panel C of Figure 2. It again illustrates the firm's ability to smooth out the impact of financing constraint on real activities.

Panel C reports the net marginal value of cash $q'(w)$ in states G and B. As one might expect, the marginal value of cash is higher in state B than in state G on average. However, quantitatively, the difference is small (0.015 versus 0.031). Firms optimally manage their cash reserves in anticipation of unfavorable market conditions and therefore firms spend little

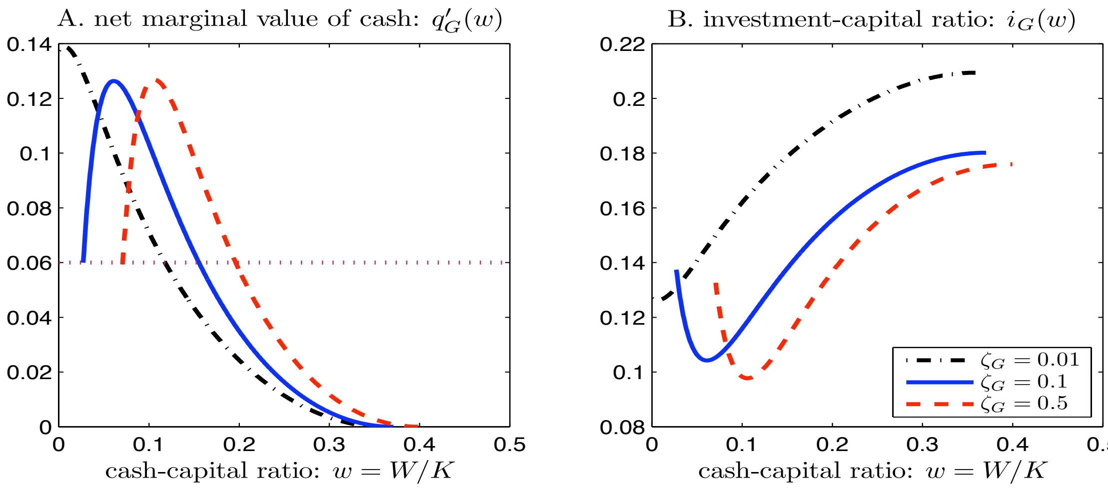

Figure 3: **The effect of duration in state G on $q'_G(w)$ and $i_G(w)$.** This figure plots the net marginal value of cash $q'_G(w)$ and investment-capital ratio $i_G(w)$ for three values of transition intensity, $\zeta_G = 0.01, 0.1, 0.5$. All other parameter values are given in Table 4.

time around the issuance boundary. The difference between the net marginal value of cash in states G and B is indeed substantial for the 99th percentile firms (0.11 versus 0.35), but still much less than the extreme cases we observe in Panel B of Figure 2.

4.5 Comparative analysis

The effect of changes in the duration of state G. How does the transition intensity ζ_G out of state G, which implies a duration of $1/\zeta_G$ for state G, affect firms' market timing behavior? Consider first the case when state G has a very high average duration of 100 years ($\zeta_G = 0.01$). In this case, the firm taps equity markets only when it runs out of cash ($\underline{w}_G = 0$), and to economize the fixed cost of issuance, the firm issues a lumpy amount $m_G = 0.118$. Firm value $q_G(w)$ is then globally concave in w and $i_G(w)$ increases with w everywhere. Essentially, the expected duration of favorable market conditions is so long that the market timing option has no value for the firm.

With a sufficiently high transition intensity ζ_G, however, the firm may time the market by selecting an interior equity issuance boundary $\underline{w}_G > 0$. In that case the firm also equates the net marginal value of cash at $\underline{w}_G$ with the proportional financing cost γ, $q'_G(\underline{w}_G) = \gamma = 6\%$, as can be seen from Panel A. Since the net marginal value of cash at the return cash-capital ratio, m_G, also satisfies $q'_G(m_G) = 6\%$, it must be the case that for $\underline{w}_G \leq w \leq m_G$, the net marginal value of cash $q'_G(w)$ first increases with w and then decreases, as Panel A again illustrates.

When ζ_G increases from 0.1 to 0.5, the firm taps the equity market even earlier ($\underline{w}_G$ increases from 0.027 to 0.071) and holds onto cash longer (the payout boundary $\overline{w}_G$ increases from 0.370 to 0.400) for fear that favorable financial market conditions may be disappearing faster. For sufficiently high w, the firm facing a shorter duration of favorable market conditions (higher ζ_G) values cash more at the margin (higher $q_G(w)$) and invests less (lower $i_G(w)$). However, for sufficiently low w, the opposite holds because the firm with a shorter lived market timing option taps equity markets sooner, so that the net marginal value of cash is lower. Consequently and somewhat surprisingly, the incentive to under-invest is smaller for a firm with shorter-lived timing options and its investment is actually higher, as Panel B illustrates.

We note that both investment and the net marginal value of cash are highly nonlinear and non-monotonic in cash despite the fact that the real side of our model is time invariant. Our model, thus, suggests that the typical empirical practice of detecting financial constraints is conceptually flawed. Using average q to control for investment opportunities and then testing for the presence of financing constraints by using variables such as cash flows or cash (which is often done in the empirical literature) would be misleading and miss the rich dynamic adjustment that the firm engages in order to balance the firm's market timing and precautionary saving motives.

The effect of changes in issuance cost ϕ_G. Table 2 reports the effects of changing the issuance cost parameter ϕ_G on the issuance boundary $\underline{w}_G$, the issuance amount $m_G - \underline{w}_G$, the average issuance cost, and the payout boundary $\overline{w}_G$.

As the issuance cost ϕ_G increases, the issuance boundary $\underline{w}_G$ decreases, the payout bound-

Table 2: FIXED COST OF EQUITY ISSUANCE

ϕ_G	$m_G - \underline{w}_G$	average cost	$\underline{w}_G$	$\overline{w}_G$
0	0.000	0.060	0.092	0.357
0.5%	0.128	0.099	0.027	0.370
1.0%	0.153	0.126	0.013	0.375
2.0%	0.176	0.174	0	0.380
5.0%	0.189	0.324	0	0.388
10.0%	0.199	0.563	0	0.394

ary $\overline{w}_G$ increases, the amount of issuance $m_G - \underline{w}_G$ increases, and the average cost of issuance increases. As expected, the more costly it is to issue equity, the less willing the firm is to issue and hence the lower the issuance boundary $\underline{w}_G$, the longer the firm holds onto cash (higher payout boundary $\overline{w}_G$), and the more the firm issues (larger $m_G - \underline{w}_G$) when it taps the equity market. While a firm with a larger fixed cost issues more, the average issuance cost is still is higher. Without the fixed cost ($\phi_G = 0$), the firm issues just enough equity to stay away from its optimally chosen financing boundary $\underline{w}_G = 0.092$, and the net marginal value of cash at issuance equals $q'_G(w) = \gamma = 6\%$, so that the average issuance cost is precisely 6%. In this extreme case, the marginal value of cash $q'_G(w)$ is monotonically decreasing in w, and firm value is globally concave in w even under market timing.

When the fixed cost of issuing equity is positive but not very high (consider $\phi_G = 1\%$), the firm times equity markets at the optimally chosen issuance boundary of $\underline{w}_G = 0.013$, and issues the amount $m_G - \underline{w}_G = 0.153$. Neither the marginal value of cash nor investment is then monotonic in w in the region $\underline{w}_G \leq w \leq m_G$, as we have discussed earlier. Moreover, higher fixed costs lead firms to choose larger issuance sizes $(m_G - \underline{w}_G)$. Notice also that $\underline{w}_G = 0$ when $\phi_G = 2\%$. This result shows that market timing does not necessarily lead to a violation of the pecking order between internal cash and external equity financing, and importantly that $\underline{w}_G > 0$ is not necessary for the convexity of the value function. Finally, when the fixed cost of issuing equity is very high (not shown in the graph), the market timing effect is so weak that the precautionary motive dominates again, so that the net marginal value of cash is monotonically decreasing in w.

Having determined why the value function may be locally convex, we next explore the implications of convexity for investment. Recall from equation (24) that the sign of the investment-cash sensitivity $i'_s(w)$ depends on $p''_s(w)$. Thus, in the region where $p_G(w)$ is convex, *investment is decreasing in cash holdings* w.

There may be other ways of generating a negative correlation between changes in investment and cash holdings. First, when the firm moves from state G to B, this not only results in a drop in investment, especially when w is low (see Panel C in Figure 2), but also in an increase in the payout boundary, which may explain why firms during the recent financial crisis have increased their cash reserves and cut back on capital expenditures, as Acharya, Almeida, and Campello (2010) have documented. Second, in a model with persistent productivity shocks (as in Riddick and Whited (2009)), when expected future productivity falls, the firm will cut investment and the cash saved could also result in a rise in its cash holding.[19]

Is it possible to distinguish empirically between these two mechanisms? In the case of a negative productivity shock, the firm has no incentive to significantly raise its payout boundary, as lower productivity lowers the costs of underinvestment, hence reducing the precautionary motive for holding cash. This prediction is opposite to the prediction related to a negative financing shock. Thus, following negative technology shocks we will not see firms aggressively increasing cash reserves. In fact, firms that already have high cash holdings will likely pay out cash after a negative productivity shock, but hold on to even more cash after a negative financing shock.

Another empirical prediction which differentiates our model from other market timing models concerns the link between equity issuance and corporate investment. Our model predicts that underinvestment is substantially mitigated when the firm is close to the equity financing boundary. Moreover, the positive correlation between investment and equity

[19]This mechanism is captured in our model with the two states corresponding to two different values for the return on capital μ_s.

issuance in our model is not driven by better investment opportunities (as the real side of the economy is held constant across the two states); it is driven solely by the market timing and precautionary demand for cash.

5 Real Effects of Financing Shocks

Several empirical studies have tried to measure the impact of financing shocks on real activities (see e.g., Campello et al., 2010a, 2010b, Duchin et al., 2011, and Paravisini et al., 2011). To determine the impact of financing shocks on investment and value, a key question is to what extent these shocks come as surprises, or whether they have been anticipated. Our model is naturally suited to compare the impact of "anticipated" versus "unanticipated" financing shocks on investment and value.

We capture the difference between "anticipated" and "unanticipated" shocks as follows. In our benchmark model, in the good state, the firm solves the value maximization problem with $\zeta_G = 0.1$. We refer to this case as where the firm "anticipates" the negative financing shock. Anticipated shocks does not mean that the firm knows for sure whether and when a financial crisis occurs. It simply means that the firm knows the true probability of a crisis and the associated risk premia. In contrast, an "unanticipated shock" is a situation where the firm is effectively blind to the impending risk of a crisis. In other words, the firm takes the probability of a change in financing condition to be close to zero. We approximate this case using $\zeta_G = 0.01$.[20]

As one might expect, a fully anticipated shock would lead firms to endogenously respond to the threat of a crisis by adjusting their real and financial policies. For instance, firms might choose to hold more cash, adopt more conservative investment policies, and raise external financing sooner, etc. As a result, the *ex-post* impact of "anticipated" financing shock on investment and other real decisions can appear to be small due to the fact that the shocks

[20] To have the crisis be fully unanticipated, we would choose $\zeta_G = 0$. This is arguably less realistic, although it has no significant effects for the substance of our analysis. In both the "anticipated" and "unanticipated" cases, we fix the transition intensity in the bad state at $\zeta_B = 0.5$.

have already been partially "smoothed out" through risk management.

Figure 4 illustrates this idea. The two solid lines plot the investment-capital ratio in the case of "anticipated shocks," with $\zeta_G = 0.1$. Under these parameter choices the firm factors in a 10% probability that a financial crisis will occur within a year. We contrast this to the case of "unanticipated shocks," where $\zeta_G = 0.01$ (dotted lines).

A comparison of these two scenarios demonstrates that risk management smoothes out financing shocks in two ways. First, a heightened concern about the incidence of a financial crisis pushes firms to invest more conservatively in state G most of the time. Second, a firm anticipating a higher probability of crisis also holds more cash on average, which further reduces the impact of financial shocks on investment.[21] Figure 4 illustrates the size of the investment response to a financing shock *at the average cash holding in state G.* With "unanticipated" shocks ($\zeta_G = 0.01$), the average cash holding in state G is 0.224, at which point investment drops by 4.03% following the shock. In contrast, when financing shocks are anticipated ($\zeta_G = 10\%$), average cash holding in state G rises to 0.283, and the drop in investment reduces to 0.96% at this level of cash holding.

Importantly, a small observed response to a financing shock does not imply that financing shocks are unimportant for the real economy. As Figure 4 illustrates, when a higher risk of a crisis is anticipated the firm responds by taking actions ahead of the realization of the shock. Thus, the firm substantially scales back its investment in state G when the transition intensity ζ_G rises from 1% to 10%, which is a main contributor to the overall reduction in the firm's investment response. The *ex-ante* responses of the firm in state G, such as lower levels of investment, higher (costly) cash holdings, earlier use of costly external financing, are all reflections of the impending threat of a negative financing shock and are all costs incurred as a result of the deterioration in financing opportunities.

Panel A of Table 3 provides information about the entire distribution of investment responses to financing shocks in the two cases. The average investment reduction following a severe financing shock is 1.78% when $\zeta_G = 10\%$, compared to 6.59% when shocks are

[21] Note that the investment response in state B is similar when shocks are anticipated and when not.

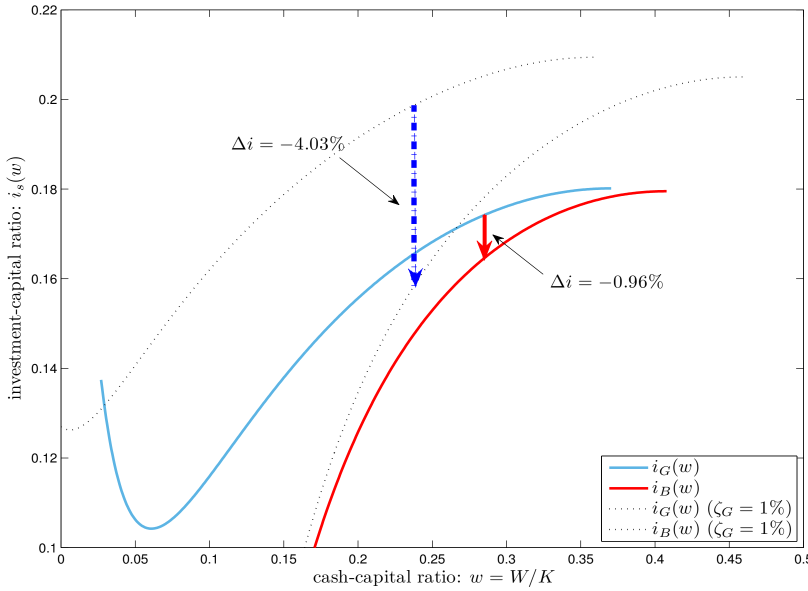

Figure 4: **Impact of Financing Shocks on Investment.** This figure plots the response in investment when the financing shock occurs.

effectively unanticipated ($\zeta_G = 1\%$). The median investment decline is 0.76% for $\zeta_G = 10\%$, which is much lower than 3.66%, the median investment drop when $\zeta_G = 1\%$. Moreover, in the scenario where the financing shock comes as a big surprise, the distribution of investment responses also has significantly fatter left tails. For example, at the 5th percentile, the investment decline is 23.66% when financing shocks are unanticipated ($\zeta_G = 1\%$), which is much larger than the drop of 6.49% when shocks are anticipated ($\zeta_G = 10\%$). Intuitively, when a financial crisis strikes, firms that happen to have low cash holdings will have to cut investment dramatically. This result is consistent with the findings of Campello, Graham, and Harvey (2010). By conducting CFO surveys, they report that during the financial crisis in 2008, while constrained U.S. firms plan to cut capital expenditures by 9.1% on average, the average cuts for unconstrained firms are only 0.6%. Our results further demonstrate that the fraction of firms that have to significantly cut investment (e.g., by over 5%) following a severe financing shock decreases significantly as the probability that firms assign to a financial crisis shock rises.

Table 3: Distribution of Investment Responses

	mean	1%	5%	25%	50%	75%
	A. financing shock					
$\zeta_G = 1\%$	-6.59	-43.17	-23.66	-7.06	-3.66	-2.23
$\zeta_G = 10\%$	-1.78	-18.11	-6.49	-1.67	-0.76	-0.39
	B. shock to expected productivity					
$\zeta_G = 1\%$	-6.59	-6.84	-6.84	-6.81	-6.67	-6.40
$\zeta_G = 10\%$	-3.15	-3.17	-3.17	-3.17	-3.16	-3.15

While firms can effectively shield investment from financing shocks by hoarding more cash, there is little they can do to avoid cuts in investment when they are hit by shocks to expected productivity. To illustrate and contrast the effects of shocks to expected productivity with the effects of financing shocks, we carry out the following experiment. Holding the financing cost constant ($\phi_G = \phi_B = 0.5\%$ and $\gamma_G = \gamma_B = 6\%$), we instead assume that the conditional mean return on capital (productivity) is higher in state G than B. Specifically, we hold μ_G at 22.7% as in the benchmark model but calibrate $\mu_B = 19.25\%$ such that the average drop in investment following a productivity shock is 6.59% when $\zeta_G = 1\%$, the same as in the scenario with financing shocks. Again, we consider the two scenarios with $\zeta_G = 1\%$ and $\zeta_G = 10\%$ respectively, while holding ζ_B at 0.5. The results for the distribution of investment responses in the wake of a productivity shock are reported in Panel B of Table 3.

A higher transition intensity ζ_G means that the high-productivity state is expected to end sooner on average, and that the firm will invest less aggressively in state G as a result. This is why the average decline in investment following a productivity shock is smaller when $\zeta_G = 10\%$ than when $\zeta_G = 1\%$, although the impact of the rise in transition intensity ζ_G for productivity shocks is smaller than that for financing shocks. Even more striking is the finding that, unlike the effects of financing shocks, for which there is significant heterogeneity in investment responses across different levels of cash-capital ratios, the investment responses following a productivity shock are essentially the same across all levels of cash holdings. The contrast of investment responses to financing shocks and shocks to expected productivity in our calibrated model suggests that financing and productivity shocks have significantly different implications for investment responses.

In summary, the effects of financing shocks on a firm's investment policy crucially depend on two variables: (1) the probability that the firm attaches to the financing shock, and (2) the firm's cash holding. A relatively small rise in the probability of a financing shock can already cause firms to behave more conservatively in good times. The result is that the average impact of financing shocks *ex post* is small. However, it will vary significantly across firms with different cash holdings. In contrast, there is little risk management that a firm can do in anticipation of a sudden shock to expected productivity, which is why there will be little heterogeneity in how firms adjust their investment policies following such shocks.

6 Financial Constraints and the Risk Premium

In this section, we explore how *aggregate* financing shocks affect the risk premium for a financially constrained firm. Without financing constraint, the firm in our model will have a constant risk premium. When the firm's financing conditions remain the same over time, a conditional CAPM (capital asset pricing model) holds in our model, where the conditional Beta is monotonically decreasing in the firm's cash-capital ratio. In the presence of aggregate financing shocks, however, the volatility in stock returns tends to rise sharply in state B, as can be inferred from Figure 2. This suggests that the conditional risk premium will be determined by a two-factor model, which prices both the aggregate shocks to profitability and the shocks to financing conditions.[22][23]

[22]Livdan, Sapriza, and Zhang (2009) also study the effect of financing constraints on stock returns. Their model, however, does not allow for stochastic financing conditions or cash accumulation.

[23]One interpretation of the pricing model in this section is that all investors are rational risk-averse investors who anticipate shocks to a firm's financing opportunities, which may be driven by (unmodeled) shocks to financial intermediation costs or changes in the opaqueness of the firm's balance sheets. An alternative interpretation is that the firm's external financing costs are driven by (unmodeled) changes in market sentiment. This behavioral interpretation is still consistent with the view that investors require

A heuristic derivation of the firm's (risk-adjusted) expected return involves a comparison of the HJB equations under the physical and risk-neutral measures $\mathbb{P}$ and $\mathbb{Q}$. Let the firm's *conditional risk premium* in state s be $\mu_s^R(w)$. We may then write the HJB equation under the physical measure as follows:

$$\begin{aligned}\left(r_s + \mu_s^R(w)\right) p_s(w) = \quad & \max_{i_s} \left[(r_s - \lambda_s) w + \mu_s - i_s - g_s(i_s) \right] p_s'(w) + \frac{\sigma_s^2}{2} p_s''(w) \\ & + (i_s - \delta)\left(p_s(w) - w p_s'(w)\right) + \zeta_s \left(p_{s^-}(w) - p_s(w)\right), \qquad (25)\end{aligned}$$

where μ_s and ζ_s respectively denote the expected return on capital and the transition intensity from state s to s^- under the physical probability measure. By matching terms in the HJB equations (11) and (25), and using the risk adjustments specified in (7) and (8), we then obtain the following expression for the *conditional risk premium*:[24]

$$\mu_s^R(w) = \eta_s \rho_s \sigma_s \frac{p_s'(w)}{p_s(w)} - \left(e^{\kappa_s} - 1\right) \zeta_s \frac{\left(p_{s^-}(w) - p_s(w)\right)}{p_s(w)}. \qquad (26)$$

The first term in (26) is the productivity risk premium, which is the product of the firm's exposure to aggregate (Brownian) productivity shocks $\rho_s \sigma_s p_s'(w)/p_s(w)$ and the price of Brownian risk η_s (where ρ_s is the conditional correlation between the firm's productivity shock dA and the stochastic discount factor in state s). This term is positive for firms whose productivity shocks are positively correlated with the aggregate market.

The second term is the financing risk premium, which compensates risk-averse investors for the firm's exposure to aggregate financing shocks. Financing shocks will be priced when their arrival corresponds to changes in the stochastic discount factor. As seems empirically

compensation for the risk with respect to changes in the firm's financing opportunities if one takes the approach based on differences of opinion à la Scheinkman and Xiong (2003). The point is that in the differences of opinion model investors are aware that at any moment in time there may be more optimistic or pessimistic other investors. Each investor is not always the marginal investor and to the extent that each investor is aware of this (as is assumed in the differences of opinion model) he faces risk with respect to other investors' optimism (which here takes the form or risk with respect to the firm's financing opportunities) for which he requires compensation.

[24]The same expression can also be obtained via the standard covariance between return and stochastic discount factor derivation. See Appendix B for details.

plausible, we suppose that the stochastic discount factor jumps up when aggregate financing conditions deteriorate, that is, $\kappa_G = -\kappa_B > 0$ in our two state model. In other words, investors will demand an extra premium for investing in firms whose values drop (rise) during times when external financing condition worsens (improves) ($p_G(w) > p_B(w)$).

In the *first-best* setting where a firm has free access to external financing, its risk premium is constant and can be recovered from (26) by setting η, ρ, and σ to constants and dropping the second term. We then obtain the standard CAPM formula:

$$\mu^{FB} = \eta\rho\sigma\frac{1}{q^{FB}}\,. \tag{27}$$

The comparison between $\mu_s^R(w)$ and μ^{FB} highlights the impact of external financing frictions on the firm's cost of capital.

Constant equity issuance costs: When financing opportunities are constant over time, financial constraints only affect the cost of capital by amplifying (or dampening) a firm's exposure to productivity shocks. This effect is captured by the productivity (diffusion) risk premium in (26). As the cash-capital ratio w increases, the firm tends to become less risky for two reasons. First, if a greater fraction of its assets is cash, the firm *beta* is automatically lower due to a simple portfolio composition effect. As a financially constrained firm hoards more cash to reduce its dependence on costly external financing, the firm beta becomes a weighted average of its asset beta and the beta of cash, which is equal to zero.[25] Second, an increase in w effectively relaxes the firm's financing constraint and therefore reduces the sensitivity of firm value to cash flow, which also tends to reduce the risk of holding the firm.

Time-varying equity issuance costs: Time-varying equity issuance costs affect the cost of capital for a financially constrained firm in two ways. First, the firm's exposure to productivity shocks changes as financing conditions change, as the marginal value of

[25] In particular, with a large enough buffer stock of cash relative to its assets, this firm may be even safer than a firm facing no external financing costs and therefore holding no cash.

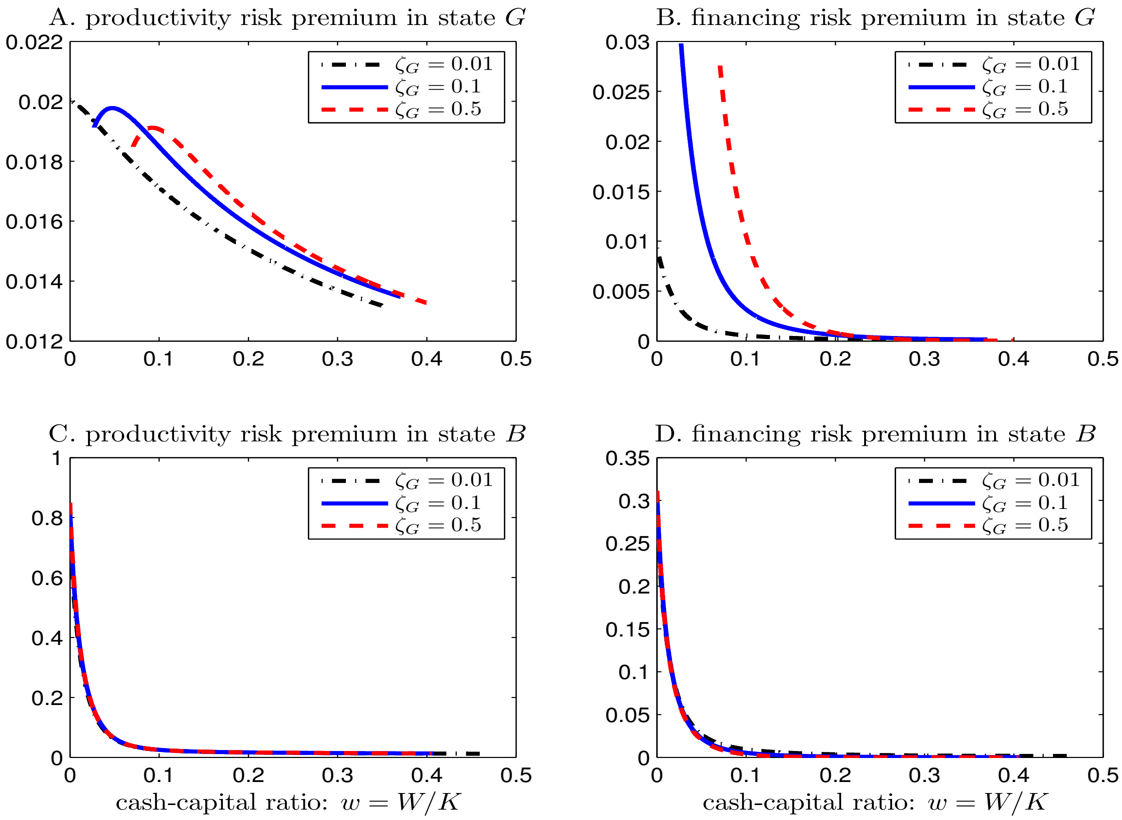

Figure 5: **The effects of stochastic financing conditions on the cost of capital.** This figure plots firm risk premium in state G and B for the case where G is transitory. We consider three levels of transition intensity from G to B: $\zeta_G = 0.01, 0.1, 0.5$.

cash $p_s'(w)$ and firm value $p_s(w)$ both depend on the state s. Second, when external financing shocks are priced, investors demand an extra premium for investing in firms that do poorly when financing conditions worsen. These firms expose investors to higher stock-return volatility in state B. This effect is captured by the second term in (26).Note that $\left(p_{s^-}(w) - p_s(w)\right)/p_s(w)$ gives the percentage change of firm value if financing conditions change, and this term measures the sensitivity of firm value with respect to changes in w. Intuitively, the financing risk premium is larger the bigger the relative change in firm value due to a change in external financing conditions.

Figure 5 Panel A plots the productivity risk premium (the first term in 26) in state G as a function of the cash-capital ratio w. This premium is generally decreasing in the cash-capital ratio, except near the financing boundary. In the benchmark case ($\zeta_G = 0.1$), the risk with respect to higher future financing costs generates market timing behavior and non-monotonicity in the marginal value of cash (Figure 1, Panel B), which in turn may cause the productivity risk premium to be locally increasing in w for low levels of w. As the non-monotonicity in the marginal value of cash is partially offset by the asset composition effect, the non-monotonicity in the productivity risk premium is relatively weak. Similarly, holding w fixed at a low level, market timing can lower $p'_G(w)$ as the transition intensity ζ_G increases. This explains why the productivity risk premium may be decreasing in the transition intensity for low w. When the transition intensity is sufficiently low (e.g., $\zeta_G = 0.01$), the non-monotonicity in the productivity risk premium disappears.

Second, Panel B plots the financing risk premium. The size of this premium depends on the relative change in firm value when external financing conditions change. It is increasing in the transition intensity ζ_G, but decreasing in w. Intuitively, when cash holdings are low, a sudden worsening in external financing conditions is particularly costly; but when cash holdings are high, the firm is able to avoid costly external financing by cutting investment, engaging in asset sales, and deferring payout, all of which mitigate the impact of the financing shock.

In Panel C and D, both the productivity risk premium and financing risk premium in state B are monotonically and rapidly decreasing in the firm's cash holding. When w is close to 0, the annualized conditional productivity risk premium can exceed 80%. The high premium and sharp decline with w mirror the rapid decline in the marginal value of cash (see Figure 2, Panel B): high marginal value of cash in the low w region can dramatically amplify the firm's sensitivity to productivity shocks. The productivity risk premium eventually falls below 2% when the firm is near the payout boundary. Similarly, the conditional financing premium can exceed 30% when w is close to 0; this is due to the large jump in firm value when the financing state changes (see Figure 2, Panel A).

Quantitatively, the level and variation of the conditional risk premium generated by financing constraint should be interpreted in conjunction with the stationary distributions of cash holdings in Section 4.4. Because the firm's cash holding will rarely drop to very low levels, its risk premium will be small and smooth most of the time in our model.

Our model has several implications for expected returns of financially constrained firms. Controlling for productivity and financing costs, the model predicts an inverse relation between returns and corporate cash holdings, which has been documented by Dittmar and Mahrt-Smith (2007) among others. Our analysis points out that this negative relation may not be due to agency problems, as they emphasize, but may be driven by relaxed financing constraints and a changing asset composition of the firm.[26]

A related prediction is that firms that are more financially constrained are not necessarily more risky. The risk premium for a relatively more constrained firm can be lower than that for a less constrained firm if the more constrained firm also holds more cash. This observation may shed light on the recent studies by Ang, Hodrick, Xing, and Zhang (2006, 2009) documenting that stocks with high idiosyncratic volatility have low average returns. In our model, firms that face higher idiosyncratic risk will optimally hold more cash on average, which could explain their lower risk premium.

With time-varying financing conditions, our model can be seen as a conditional two-factor model to explain the cross section of returns (we provide details of the derivation in the Appendix). A firm's risk premium is determined by its productivity beta and its financing beta. Other things equal, a firm whose financing costs move closely with aggregate financing conditions will have a larger financing beta and earn higher returns than one with financing costs independent of aggregate conditions. Empirically, this two-factor model can be implemented using the standard market beta plus a beta with respect to a portfolio that is sensitive to financing shocks (e.g. a banking portfolio). This model, in particular, shows how a firm's conditional beta depends on the firm's cash holdings.

[26]When heterogeneity in productivity and financing costs is difficult to measure, it is important to take into account the endogeneity of cash holdings when comparing firms with different cash holdings empirically. A firm with higher external financing costs will tend to hold more cash, however its risk premium may still be higher than for a firm with lower financing costs and consequently lower cash holdings. Thus, a positive relation between returns and corporate cash holdings across firms may still be consistent with our model (see Palazzo (2008) for a related model and supporting empirical evidence).

7 Market Timing and Dynamic Hedging

We have thus far restricted the firm's financing choices to only internal funds and external equity financing. In this section, we extend the model to allow the firm to engage in dynamic hedging via derivatives such as market-index futures. How does market timing behavior interact with dynamic hedging? And, how does the firm's dynamic hedging strategy affect its market timing behavior? These are the questions we address in this section. We denote by F the index futures price for a market portfolio that is already completely hedged against financing shocks. Under the risk-neutral probability measure, the future prices F then evolves according to:

$$dF_t = \sigma_m F_t d\hat{Z}_t^M \, , \tag{28}$$

where σ_m is the volatility of the market index portfolio, and $\{\hat{Z}_t^M : t \geq 0\}$ is a standard Brownian motion that is correlated with the firm's productivity shock $\{Z_t^A : t \geq 0\}$ with a constant correlation coefficient ρ.[27]

We denote by ψ_t the fraction of the firm's total cash W_t that it invests in the futures contract. Futures contracts require that investors hold cash in a margin account. Thus, let $\alpha_t \in [0,1]$ denote the fraction of the firm's total cash W_t held in the margin account. Cash held in this margin account incurs a flow unit cost $\epsilon \geq 0$. Futures market regulations typically require that an investor's futures position (in absolute value) cannot exceed a multiple π of the amount of cash $\alpha_t W_t$ held in the margin account. We let this multiple be state dependent and denote it by $\pi(s_t)$. The margin requirement in state s then imposes the following limit on the firm's futures position: $|\psi_t| \leq \pi(s_t)\alpha_t$. As the firm can costlessly reallocate cash between the margin account and its regular interest-bearing account, it optimally holds the minimum amount of cash necessary in the margin account when $\epsilon > 0$. Without much loss of generality, we shall ignore this haircut on the margin account and assume that $\epsilon = 0$. Under this assumption, we do not need to keep track of cash allocations in the margin account and outside the account. We can then simply set $\alpha_t = 1$. We provide a detailed derivation of

[27]Note that the futures price F follows a martingale after risk adjustment. The interesting case to consider is when the index futures is imperfectly correlated with the firm's productivity shock.

firm value and optimal hedging policy in the Appendix D, where we establish that:

1. in state B, the optimal futures position is given by

$$\psi_B^*(w) = \max\left\{\frac{-\rho\sigma_B}{w\sigma_m},\ -\pi_B\right\}.$$

2. in state G, the optimal futures position is given by

$$\psi_G^*(w) = \begin{cases} \max\left\{-\rho\sigma_G{\sigma_m}^{-1}/w,\ -\pi_G\right\}, & \text{for} \quad w \geq \widehat{w}_G\,, \\ \pi_G\,, & \text{for} \quad \underline{w}_G \leq w \leq \widehat{w}_G\,. \end{cases}$$

We choose the correlation between index futures and the firm's productivity shock to be $\rho = 0.4$ and a market return volatility of $\sigma_m = 20\%$. The margin requirements in states G and B are set at $\pi_G = 5$ and $\pi_B = 2$, respectively. All other parameter values are the same as in the previous sections.

Optimal hedge ratios $\psi_s^*(w)$. Figure 6 plots the optimal hedge ratios in both states: $\psi_G^*(w)$ and $\psi_B^*(w)$. First, we note that for sufficiently high w, the firm hedges in the same way in both states. Hedging is then unconstrained by the firm's cash holdings and costless. The firm then chooses its hedge ratio to be equal to $-\rho\sigma\sigma_m^{-1}/w$ so as to eliminate its exposure to *systematic volatility* of the productivity shock. This explains the concave and overlapping parts of the hedging policies in Figure 6.

Second, for low w hedging strategies differ in the two states as follows: in state B the hedge ratio hits the constraint $\psi_B^*(w) = -\pi_B = -2$ for $w \leq 0.12$. In state G on the other hand, the firm issues equity at $\underline{w}_G = 0.0219$ and firm value is convex in w (due to market timing) for $w \leq \tilde{w}_G = 0.0593$ (where $p''(\tilde{w}_G) = 0$). In other words, for $w \in (\underline{w}_G, \tilde{w}_G)$ *firm value is convex* in w and the firm does the opposite of hedging and engages in maximally allowed *risk taking* by setting $\psi_G^*(w) = \pi_G = 5$ for $w \in (0.0219, 0.0593)$.

When cash holdings w are sufficiently high, hedging lowers the firm's precautionary holdings of cash and hence lowers its payout boundary $\overline{w}_G$ from 0.3703 (no hedging benchmark)

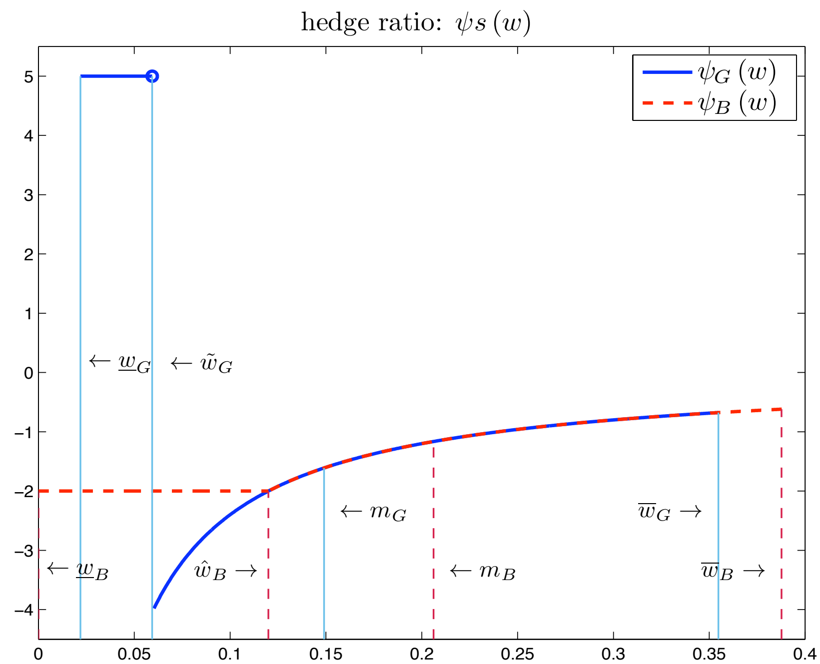

Figure 6: **Optimal hedge ratios $\psi^*(w)$ in states G and B when state B is absorbing.** The parameter values are: market volatility $\sigma_m = 20\%$, correlation coefficient $\rho = 0.4$, margin requirements $\pi_G = 5$ and $\pi_B = 2$. All other parameter values are given in Table 4.

to0.3548. For sufficiently low cash holdings, the ability to speculate lowers the firm's issuance boundary $\underline{w}_G$ because the marginal value of cash for a firm with speculation/hedging opportunity is higher. The ability to increase the volatility of the cash accumulation process makes the equity issuance option more valuable and hence causes the issuance boundary $\underline{w}_G$ to be lowered from 0.0268 (no hedging/speculation benchmark) to 0.0219.

Froot, Scharfstein, and Stein (1993) argue that hedging increases firm value by mitigating its underinvestment problem. However, we show that this result does not hold generally in a dynamic setting. For sufficiently high cash holding, hedging indeed mitigates the firm's under-investment problem by reducing exposure to systemic volatility. However, when the firm's cash holding is sufficiently low, the firm may engage in speculation to take advantage of its market timing option.

8 Conclusion

There is mounting evidence that stock prices may occasionally deviate substantially from fundamental value, whether it is on the up side or the down side (see Baker and Wurgler, 2011, for a survey of the empirical literature). What is more, in rare episodes of financial crises primary markets essentially shut down. Firms have become increasingly aware of the risks and opportunities they face with respect to external financing costs, and they do appear to 'time equity markets' as Baker and Wurgler (2002). Yet, very few theoretical analyses are available on the implications of changing external financing costs for the dynamics of corporate investment and financing. This study aims to close this gap in the finance literature, by taking the perspective of a rational firm manager seeking to maximize shareholder value by timing favorable equity market conditions and shielding the firm against crisis episodes by holding a precautionary cash buffer.

We have proposed a simple integrated analytical framework based on the classical q-theory of investment, but for a financially constrained firm facing stochastic financial market conditions. We have shown that firms optimally hold cash buffer stocks and issue equity in favorable market conditions even when they do not have immediate funding needs. This dynamic behavior is broadly in line with existing empirical evidence. As simple as this market timing behavior by the firm appears to be, we have shown that it has subtle implications for the dynamics of corporate investment and for the dynamics of stock returns. Thus, for example, firms anticipating an equity issuance under favorable market conditions may exhibit investment behavior that is decreasing in the firm's cash buffer stock. The reason is that when firms get closer to equity issuance their investment policy is effectively less constrained by the availability of internal funds, as the firm anticipates that more cash will be raised through an equity issue in the near future. We have also shown that market timing is consistent with optimal risk-seeking behavior by the firm. The key driver of these surprising dynamic implications is the finite duration of favorable financing conditions combined with the fixed issuance costs firms incur when they tap equity markets. Finally, we have highlighted how much a firm that optimally times equity markets and holds optimal precautionary cash buffers is able to shield itself against large external financing costs. A firm entering a crisis state with an optimally replenished cash buffer in good times is able to maintain its investment policy almost unaltered, and thus substantially smooth out adverse external financing shocks.

Table 4: Summary of Key Variables and Parameters

This table summarizes the symbols for the key variables used in the model and the parameter values in the benchmark case. For each upper-case variable in the left column (except K, A, and F), we use its lower case to denote the ratio of this variable to capital. Whenever a variable or parameter depends on the state s, we denote the dependence with a subscript s. All the boundary variables are in terms of the cash-capital ratio w_t.

Variable	Symbol	Parameters	Symbol	state G	state B
A. Baseline model					
Capital stock	K	Riskfree rate	r	5.0%	
Cash holding	W	Rate of depreciation	δ	15%	
Investment	I	Mean productivity shock	μ	22.7%	
Cumulative productivity shock	A	Volatility of productivity shock	σ	12%	
Investment adjustment cost	G	Adjustment cost parameter	θ	1.8	
Cumulative operating profit	Y	Center of adjustment cost parameter	ν	15%	
Cumulative external financing	H	Proportional cash-carrying cost	λ	1.5%	
Cumulative external financing cost	X	Proportional financing cost	γ	6%	
Cumulative payout	U	Correlation between Z_t^A and Z_t^M	ρ	0.4	
Firm value	P	Price of risk for technology shocks	η	0.4	
Average q	q				
Net marginal value of cash	q'	State transition intensity	ζ_s	0.1	0.5
Payout boundary	$\overline{w}$	Capital liquidation value	l_s	1.0	0.3
Financing boundary	$\underline{w}$	Fixed financing cost	ϕ_s	0.5%	50%
Return cash-capital ratio	m	Price of risk for financing shocks	κ_s	$\ln(3)$	$-\ln(3)$
Conditional risk premium	μ^R				
B. Hedging					
Hedge ratio	ψ	Market volatility	σ_m	20%	
Fraction of cash in margin account	α				
Futures price	F	Margin requirement	π_s	5	2
Maximum-hedging boundary	$\hat{w}$				
Speculation boundary	$\tilde{w}$				

Appendix

A A more general formulation of the model

Our analysis in the text focuses on the special case of two states of the world. However, it is straightforward to generalize our model to a setting with more than two states, denoted by $s_t = 1, \cdots, n$. The transition matrix in the n-state Markov chain is then given by $\boldsymbol{\zeta} = [\zeta_{ij}]$. The n-state Markov chain can also capture both aggregate and firm-specific shocks, and also productivity and financing shocks. In sum, the firm's expected return on capital, volatility, and financing costs may all change when the state changes in the general formulation of the model.

A.1 Risk adjustments

To make the adjustments for systematic risk in the model, we assume that the economy is characterized by a stochastic discount factor (SDF) Λ_t, which evolves as follows:

$$\frac{d\Lambda_t}{\Lambda_{t^-}} = -r\left(s_{t^-}\right)dt - \eta\left(s_{t^-}\right)dZ_t^M + \sum_{s_t \neq s_{t^-}} \left(e^{\kappa\left(s_{t^-}, s_t\right)} - 1\right) dM_t^{\left(s_{t^-}, s_t\right)}, \tag{29}$$

where $r(s)$ is the risk-free rate in state s, $\eta(s)$ is the price of risk for systematic Brownian shocks Z_t^M, $\kappa\left(i,j\right)$ is the relative jump size of the discount factor when the Markov chain switches from state i to state j, and $M_t^{(i,j)}$ is a compensated Poisson process with intensity ζ_{ij},

$$dM_t^{(i,j)} = dN_t^{(i,j)} - \zeta_{ij}dt, \quad i \neq j. \tag{30}$$

In the equation (29) we have made use of the result that an n-state continuous-time Markov chain with generator $[\zeta_{ij}]$ can be equivalently expressed as a sum of independent Poisson processes $N_t^{(i,j)}$ $(i \neq j)$ with intensity parameters ζ_{ij} (see e.g., Chen (2010)).[28] The above SDF

[28]More specifically, the process s solves the following stochastic differential equation, $ds_t = \sum_{k \neq s_{t^-}} \delta_k\left(s_{t^-}\right) dN_t^{\left(s_{t^-}, k\right)}$, where $\delta_k\left(j\right) = j - i$.

captures two different types of risk in the market: small systematic shocks generated by the Brownian motion, and large systematic shocks from the Markov chain. We assume that dZ_t^M is partially correlated with the firm's productivity shock dZ_t^A, with instantaneous correlation ρdt. Chen (2010) shows that the SDF in (29) can be generated from a consumption-based asset pricing model.

The SDF defines a risk neutral probability measure $\mathbb{Q}$, under which the process for the firm's productivity shocks becomes (6). In addition, if a change of state in the Markov chain corresponds to a jump in the SDF, then the corresponding large shock also carries a risk premium, which leads to an adjustment of the transition intensity under $\mathbb{Q}$ as folows:

$$\hat{\zeta}_{ij} = e^{\kappa(i,j)}\zeta_{ij}\,, \quad i \neq j.$$

A.2 Solution of the n-state model

Under the first best, the HJB equation for the n-state model is as follows,

$$r_s q_s^{FB} = \hat{\mu}_s - i_s^{FB} - \frac{1}{2}\theta_s\left(i_s^{FB} - \nu_s\right)^2 + q_s^{FB}\left(i_s^{FB} - \delta\right) + \sum_{s' \neq s}\hat{\zeta}_{ss'}\left(q_{s'}^{FB} - q_s^{FB}\right),$$

where for each state $s = 1, \cdots, n$ the average q is given by:

$$q_s^{FB} = 1 + \theta_s\left(i_s^{FB} - \nu_s\right). \tag{31}$$

While there are no closed form solutions for $n > 2$, it is straightforward to solve the system of nonlinear equations numerically.

With financial frictions, the HJB equation is generalized from (11) as follows:

$$\begin{aligned} r_s P(K,W,s) = \max_I \left[(r_s - \lambda_s)W + \hat{\mu}_s K - I - \Gamma(I,K,s)\right] P_W(K,W,s) + \frac{\sigma_s^2 K^2}{2} P_{WW}(K,W,s) \\ + (I - \delta K) P_K(K,W,s) + \sum_{s' \neq s}\hat{\zeta}_{ss'}\left(P(K,W,s') - P(K,W,s)\right), \end{aligned} \tag{32}$$

for each state $s = 1, \cdots, n$, and $\underline{W}_s \leq W \leq \overline{W}_s$. As before firm value is homogeneous of degree one in W and K in each state, so that

$$P(K, W, s) = p_s(w)K, \tag{33}$$

where $p_s(w)$ solves the following system of ODE:

$$\begin{aligned} r_s p_s(w) = \max_{i_s} \left[(r_s - \lambda_s) w + \hat{\mu}_s - i_s - g_s(i_s) \right] p_s'(w) + \frac{\sigma_s^2}{2} p_s''(w) \\ + (i_s - \delta)(p_s(w) - w p_s'(w)) + \sum_{s' \neq s} \hat{\zeta}_{ss'} (p_{s'}(w) - p_s(w)) . \end{aligned} \tag{34}$$

The boundary conditions in each state s are then defined in similar ways as in Equation (13-16).

B Calibration

We use annual data from COMPUSTAT to calculate the moments of the investment-capital ratio and cash-capital ratio for our model calibration. The sample is from 1981 to 2010 and excludes utilities (SIC codes 4900-4999) and financial firms (SIC codes 6000-6999). We require firms to be incorporated in the United States and have positive assets and positive net PPE (property, plant, and equipment). In addition, since our model does not allow for lumpy investment, mergers and acquisitions, or dramatic changes in profitability, we eliminate firm-years for which total assets or sales grew by more than 100%, or investments exceeded 50% of capital stock from the previous year.

Capital investment is measured using capital expenditure ($CAPX_t$). Since our calibrated model does not allow for short term debt, we measure cash holding as the difference between cash and short-term investments (CHE_t) and average short-term borrowing ($BAST_t$). Capital stock is the total net Property, Plant and Equipment ($PPENT_t$). Then, the cash-capital ratio for year t is defined as $\frac{CHE_t - BAST_t}{PPENT_t}$, while the investment-capital ratio for year t is

$\frac{CPAX_t}{PPENT_{t-1}}$. We first compute moments for the cash-capital ratio and the investment-capital ratio at the firm level, and then calibrate the model parameters to match the moments of the median across firms.

C Beta Representation

As indicated by the SDF Λ_t in (29) with $n = 2$, in state s, the price of risk for the technology shock (risk premium for a unit exposure to the shocks) is $\lambda_s^T = \eta_s$, whereas the price of risk for the financing shock is $\lambda_s^F = -\left(e^{\kappa_s} - 1\right)$. Thus, we can rewrite the risk premium using the Beta representation:

$$\mu_s^R(w) = \beta_s^T(w)\lambda_s^T + \beta_s^F(w)\lambda_s^F,$$

where

$$\beta_s^T(w) = \rho_s \sigma_s \frac{p_s'(w)}{p_s(w)} \tag{35}$$

$$\beta_s^F(w) = \zeta_s \frac{p_{s^-}(w) - p_s(w)}{p_s(w)} \tag{36}$$

are the *technology Beta* and *financing Beta* respectively for the firm in state s. The technology Beta will be large when the marginal value of cash relative to firm value is high; the financing Beta will be large when the probability that financing conditions will change is high, or when the change in financing conditions has a large impact on the firm value.

Since there are two sources of aggregate shocks in this model, the CAPM does not hold. Instead, expected returns reflect aggregate risk driven by a two-factor model. We thus assume that there are two diversified portfolios T and F, each only subject to one type of aggregate shock, a technology or a financing shock. Suppose their return dynamics are given as follows:

$$dR_t^T = (r_s + \mu_s^T)dt + \sigma_s^T dB_t, \tag{37}$$

$$dR_t^F = (r_s + \mu_s^F)dt + \left(e^{\kappa_1^F} - 1\right) dM_t^1 + \left(e^{\kappa_2^F} - 1\right) dM_t^2. \tag{38}$$

Then, the stochastic discount factor (29) implies that

$$\mu_s^T = \sigma_s^T \eta_s,$$

$$\mu_s^F = \zeta_s (e^{\kappa_s^F} - 1) \left(e^{\kappa_s} - 1\right). \tag{39}$$

We can now rewrite the risk premium in (37) and (38) using Betas as follows:

$$\mu_s^R(w) = \beta_s^T(w)\mu_s^T + \beta_s^F(w)\mu_s^F,$$

where

$$\beta_s^T(w) = \frac{\rho_s \sigma_s}{\sigma_s^T} \frac{p_s'(w)}{p_s(w)} \tag{40}$$

$$\beta_s^F(w) = \frac{p_{s^-}(w) - p_s(w)}{p_s(w)\left(e^{\kappa_s^F} - 1\right)} \tag{41}$$

are the technology Beta (Beta with respect to Portfolio T) and financing Beta (Beta with respect to Portfolio F) for the firm in state s. The technology Beta will be large when the marginal value of cash relative to firm value is high; the financing Beta will be large when the probability that financing conditions will change is high, or when the change in financing conditions has a large impact on firm value.

D Dynamic Hedging

We now derive the optimal hedging policy in detail for Section 7. The firm's cash holding thus evolves as follows:

$$dW_t = K_t \left[\mu(s_t)dA_t + \sigma(s_t)dZ_t\right] - \left(I_t + \Gamma_t\right) dt + dH_t - dU_t + \left[r(s_t)) - \lambda(s_t)\right] W_t dt + \psi_t W_t \sigma_m dB_t \,, \quad (42)$$

where $|\psi_t| \leq \pi(s_t)$. To avoid unnecessary repetition, we only consider the case with positive correlation, i.e., $\rho > 0$. We consider first the crisis state.

In state B. Given that firm value is always concave in cash in state B ($P_{WW}(K, W, G) < 0$), the firm in state B faces the same decision problem as the firm in Bolton, Chen and Wang (2011) (BCW). BCW show that the optimal hedge ratio (with time-invariant opportunities) is given by

$$\psi_B^*(w) = \max\left\{\frac{-\rho\sigma_B}{w\sigma_m}, -\pi_B\right\}. \quad (43)$$

Intuitively, the firm chooses the hedge ratio ψ so that the firm only faces idiosyncratic volatility after hedging. The hedge ratio that achieves this objective is $-\rho\sigma_B\sigma_m^{-1}/w$. However, this hedge ratio may not be attainable due to the margin requirement. In that case, the firm chooses the maximally admissible hedge ratio $\psi_B^*(w) = -\pi_B$. Equation (43) captures the effect of margin constraints on hedging. Because there is no hair cut (i.e., $\epsilon = 0$), the hedge ratio ψ given in (43) is independent of firm value and only depends on w. We next turn to the focus of this section: hedging in state G.

In state G. Before entering the crisis state, the firm has external financing opportunities. Moreover, the margin requirement may be different (i.e., $\pi_G > \pi_B$). In state G, the firm chooses its investment policy I and its index futures position ψW to maximize firm value

$P(K,W,G)$ by solving the following HJB equation:

$$
\begin{aligned}
r_G P(K,W,G) = \max_{I,\psi} & \left[(r_G - \lambda_G) W + \mu_G K - I - \Gamma(I,K,G)\right] P_W + (I - \delta K) P_K \\
& + \frac{1}{2}\left(\sigma_G^2 K^2 + \psi^2 \sigma_m^2 W^2 + 2\rho\sigma_m\sigma_G \psi W K\right) P_{WW} + \zeta\left[P(K,W,G) - P(K,W,B)\right],
\end{aligned} \tag{44}
$$

subject to $|\psi| \leq \pi_G$.

When firm value is concave in cash, we have the same solution as in state B, but with margin π_G. That is,

$$
\psi_G^*(w) = \max\left\{-\rho\sigma_G{\sigma_m}^{-1}/w,\ -\pi_G\right\}.
$$

However, market timing opportunities combined with fixed costs of equity issuance imply that firm value may be convex in cash, i.e., $P_{WW}(K,W,G) > 0$ for certain regions of $w = W/K$. With convexity, the firm naturally *speculates* in derivatives markets. Given the margin requirement, the firm takes the maximally allowed futures position, i.e. the corner solution $\psi_G(w) = \pi_G$. Note that the firm is long in futures despite positive correlation between its productivity shock and the index futures. Let $\widehat{w}_G$ denote the endogenously chosen point at which $P_{WW}(K,W,G) = 0$, or $p_G''(\widehat{w}_G) = 0$. We now summarize the firm's futures position in state G as follows:

$$
\psi_G^*(w) = \begin{cases} \max\left\{-\rho\sigma_G{\sigma_m}^{-1}/w,\ -\pi_G\right\}, & \text{for} \quad w \geq \widehat{w}_G\,, \\ \pi_G\,, & \text{for} \quad \underline{w}_G \leq w \leq \widehat{w}_G\,. \end{cases} \tag{45}
$$

Note the discontinuity of the hedge ratio $\psi_G^*(w)$ in w. The firm switches from a hedger to a speculator when its cash-capital ratio w falls below $\widehat{w}_G$.

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