Jointly optimal monetary and fiscal policy rules under liquidity constraints

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Abstract

We study the welfare properties of an economy where both monetary and fiscal policies follow simple rules, and where a subset of agents is liquidity constrained. The welfare benefits of optimizing the fiscal rule are far larger than those of optimizing the monetary rule. The optimized fiscal rule implements strong automatic stabilizers that primarily stabilize the income of liquidity-constrained agents, rather than output. Transfers targeted to liquidity-constrained agents are the preferred fiscal instrument. The optimized monetary rule exhibits super-inertia and a weak inflation response. Optimized simple rules perform as well as the optimal policy under the timeless perspective.

1. Introduction

In the wake of the recent financial crisis we have seen a lively debate concerning the merits of activist fiscal policy. Given the perceived urgency of preventing a deep recession, the initial attention was mostly focused on the pros and cons of fiscal stimulus measures, but some attention is now shifting to the need for longer run sustainability. Remarkably though, the economics profession entered this period of turmoil with few analytical tools to think about the systematic use of fiscal policy in response to the business cycle. Specifically, and unlike the monetary policy literature since Taylor (1993), there was little work on rules-based fiscal policy. This is where our paper attempts to make a contribution, by proposing and evaluating a novel and simple policy rule whereby the fiscal surplus to GDP ratio responds to a tax revenue gap. The paper studies the welfare consequences of jointly optimizing this fiscal rule and a conventional monetary rule, in an economy where a subset of households is liquidity constrained. We show that, despite its simplicity, our rule is able to match the welfare performance of the optimal policy from the timeless perspective.

Taylor (2000) discusses a fiscal rule in which the budget surplus responds to the output gap. He advises against it, and argues that the role of fiscal policy should be limited to ”letting automatic stabilizers work”. But this raises the very...
important question of how strong automatic stabilizers should be to maximize policy objectives. This question can be mapped into the problem of finding the optimal countercyclicality of a fiscal rule, which is at the heart of this paper.

Taylor (2000) makes two exceptions to his assessment, fixed exchange rate regimes and a situation where nominal interest rates approach their zero lower bound. There is however a third exception that has so far been largely neglected by the literature, the presence of a significant share of liquidity-constrained households, meaning households who can neither borrow nor save, as in Gali et al. (2007). There are several important reasons for considering such an environment. First, pure monetary business cycle models with nominal rigidities have been criticized for not adequately replicating the empirically observed short-run effects of fiscal policy, and non-Ricardian features such as liquidity-constrained households can help overcome some of these difficulties. Second, the assumption of liquidity constraints is supported by recent empirical evidence that we will discuss when calibrating our model.

The relaxation of the liquidity constraint, by way of the government substituting its ability to borrow for that of constrained households, partially offsets the effects of a market imperfection and is therefore critical for aggregate welfare. The real activity gap in the fiscal rule should therefore move closely with the tightness of the liquidity constraint, which ends up arguing strongly against choosing the output gap, and in favor of choosing an appropriate tax revenue gap. When this gap is present, the inclusion of an additional debt gap turns out to be redundant. Stabilization of the interest-inclusive surplus to GDP ratio at a long-run target value does stabilize the debt to GDP ratio, but with a near unit root on debt.

We show that tax revenue gap rules can be used to represent a continuum of rules from balanced budget rules to highly countercyclical rules. We find that the welfare gains available by moving from the former to the latter are very large compared to what is typically found in the monetary policy literature, and also compared to the welfare gains from optimizing monetary policy in our own model. Furthermore, these welfare increases have only modest costs in terms of additional fiscal instrument volatility. This argues in favor of implementing powerful automatic stabilizers such as unemployment insurance, as long as the associated incentive problems can be addressed. The best fiscal instrument is transfers targeted to liquidity-constrained households.

The optimal monetary policy rule exhibits super-inertia and a very small coefficient on inflation. The reason is that a gradual, non-aggressive interest rate response stabilizes the real wage, which improves welfare both by stabilizing the income of liquidity-constrained households, and by reducing the labor supply volatility of unconstrained households. This part of our results is similar to Stehn (2009), who uses a linear-quadratic model without capital.

From the point of view of a policymaker the optimality of a policy rule encompasses not only the maximization of household welfare, but also the avoidance of excessive instrument volatility. An analysis of fiscal and monetary instrument volatility will therefore accompany our welfare analysis. For welfare analysis, we perform a full second-order approximation of the model, and we numerically optimize the coefficients of the policy rules by way of grid searches. To complete the analysis we present a comparison of optimal simple rules with the optimal policy from the timeless perspective.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 discusses calibration of a baseline model. Section 4 presents impulse responses and welfare results for the baseline model. Section 5 compares optimal simple rules and the optimal policy from the timeless perspective. Section 6 concludes. The model’s first-order optimality conditions and some technical details are contained in a separate Technical Appendix.

2. The model

We consider a closed economy that is populated by two types of households, both of which consume output and supply labor. Infinitely lived households, identified by the superscript $INF$, have full access to financial markets, while liquidity-constrained households, identified by $LIQ$, are limited to consuming their after-tax wage income, augmented by government net transfers, in every period. The share of $LIQ$ households in the population equals $\psi$. Technology grows at the constant rate $g = A_t / A_{t-1}$, where $A_t$ is the level of labor augmenting technology. The model’s real variables, say $x_t$, therefore have to be re-scaled by $A_t$, where we will use the notation $\bar{x}_t = x_t / A_t$. The steady state of $\bar{x}_t$ is denoted by $\bar{x}$.

2.1. Infinitely lived (INF) households

The utility of a representative $INF$ household at time $t$ depends on consumption $c^{INF}_t$, labor supply $\ell_t^{INF}$ and government consumption spending $c_t$. Lifetime expected utility has the form

$$U_{0}^{INF} = E_0 \sum_{t=0}^{\infty} \beta^t \left( \left( 1 - \frac{b}{g} \right) e_t^{INF} \ln \left( c_t^{INF} - \nu c_t^{INF} \right) - \frac{K}{1 + \gamma} (\ell_t^{INF})^{1+\gamma} + \chi_t^{INF} \ln (c_t) \right). \quad (1)$$

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2 For recent exceptions see Kumhof and Laxton (2009) and Stehn (2009).
3 Alternative terminologies used in the literature include rule-of-thumb or hand-to-mouth consumers, and limited asset markets participation.
5 This has been found to be optimal in the theoretical literature. For a prominent example see Aiyagari et al. (2002).
where $\beta$ is the discount factor, $\nu$ indexes the degree of (external) habit persistence, $\gamma$ is the labor supply elasticity, $c_{i}^{t}$ is a shock to the marginal utility of consumption, and the scale factor $(1 - s_{g}^{t}/g)$ ensures that the marginal utility of consumption is independent of the degree of habit persistence in steady state. Consumption $c_{i}^{t}$, which is taxed at the rate $\tau_{c}$, is given by a CES aggregate over consumption goods varieties $c_{i}^{t}$, with elasticity of substitution $\sigma$. Lagged consumption $c_{t-1}$ is in average per capita terms.

INF households can hold nominal domestic government debt $b_{t}^{INF}$, with real debt given by $b_{t}^{INF} - R_{t}^{INF}/P_{t}$, and where $P_{t}$ is the consumer price index. The time subscript $t$ denotes financial claims held from period $t$ to period $t + 1$. The gross nominal interest rate on government debt held from $t$ to $t + 1$ is $i_{t}$. We denote gross inflation by $\pi_{t} = P_{t+1}/P_{t}$, and the gross real interest rate by $r_{t} = i_{t}/\pi_{t+1}$. In addition to interest income INF households receive after tax labor income, capital income and dividends. Real after-tax labor income equals $LIQ$, where the adjustment cost term gives rise to inertia in investment, $b_{t}$, where

$$LIQ = \frac{c_{t}^{t}}{b_{t+1}^{t}}$$

and consumption demand shocks:

$$\text{2.2. Liquidity-constrained (LIQ) households}$$

There is a continuum of firms indexed by $j \in [0,1]$. Firms are perfectly competitive in their input markets and monopolistically competitive in their output market. Their price setting is subject to nominal rigidities. Each firm operates a Cobb–Douglas technology in private capital $k_{t-1}(j)$ and labor $\ell_{t}(j)$. The technology’s productivity is augmented by a public capital stock $k_{t-1}$.

We have

$$y_{t}(j) = \alpha^{t}(k_{t-1}(j))^{\alpha_{k}}(A_{t} c_{t}^{t}(j))^{1-\alpha_{k}} (\frac{k_{t-1}}{A_{t-1}})^{\alpha_{k}}$$

where $\alpha$ is the production function, $\alpha_{k}$ is the share of capital in the production function, $A_{t}$ is the capital stock in each period, $c_{t}^{t}$ is a shock to labor augmenting productivity. The stock of public infrastructure $k_{t-1}$ is external to the firm’s decision, and is identical for all firms. The advantage of this formulation is that it retains constant returns to scale at the level of each firm.$^{7}$

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$^{6}$ We will use $L_{t}$ to calibrate the steady state tax structure, while $Y_{t}$ equals zero in steady state and will be used to model countercyclical fiscal policy.

$^{7}$ We have also briefly investigated the case of constant returns in accumulating factors, $s_{k} = s_{a} = 1$, but were not able to obtain dynamically stable solutions.
Firms’ optimization problem consists of maximizing the present discounted value of dividends $d_t(j)$, where the latter equal real revenue minus real marginal costs $mc_r(y_t(j))$, price adjustment costs $C^p_t(j)$ and a fixed cost $A_t \Psi'$ that will be used to calibrate the model’s steady state income shares. Price adjustment costs $C^p_t(j)$ follow Rotemberg (1982). They allow for a nonzero steady state rate of inflation $\pi$, which equals the inflation target of the central bank, and with costs scaled by the aggregate level of output $y_t$:

$$C^p_t(j) = \frac{\phi_p}{2} P_t(j) \left( \frac{P_{t-1}(j) - \pi}{P_t(j)} \right)^2. \quad (7)$$

Firms discount future nominal cash flows using the intertemporal marginal rate of substitution of their owners, INF household, which equals $\beta(\pi)/(\pi_j P_{t+1})$.

Their optimization problem is therefore

$$\max \ E_0 \sum_{t=0}^{\infty} \frac{r^t}{T^t} (P_t(j) y_t(j) - P_t mc_r(y_t(j)) - P_t C^p_t(j) - P_t A_t \Psi'). \quad (8)$$

First-order conditions, including a New Keynesian Phillips curve and cost minimization conditions, are presented in the Technical Appendix.

2.4. Government

Monetary policy follows a conventional inflation targeting rule for the nominal interest rate. Fiscal policy follows a rule that depends on a real activity gap and a debt gap, whose nature will be discussed in more detail below.

2.4.1. Monetary policy

The interest rate rule allows for smoothing of the nominal interest rate, and for responses to an inflation gap. It is given by

$$\ln i_t = \delta^1 \ln i_{t-1} + \delta^2 \ln \frac{\pi_t}{\pi}, \quad (9)$$

where $i_t$ is the product of the long-run or target real interest rate $\tilde{r}$ and the inflation target $\tilde{\pi}$. We have simplified our exposition of the rule by anticipating two results of our welfare analysis. Specifically, the optimal coefficients on the output gap and on a forward-looking inflation term in the inflation gap are very close to zero, so that these terms can be omitted.

2.4.2. Budget constraint

Government consumption spending $c^G_t$ adds to household utility, while government investment spending $k^G_t$ augments the stock of publicly provided infrastructure capital $k^G_t$, the evolution of which is given, in normalized form, by

$$k^G_t = (1 - \delta^c) \frac{k^G_{t-1}}{g} + k^G_t. \quad (10)$$

The government budget constraint, in real normalized terms, takes the form

$$b_t = i_{t-1} \tilde{b}_{t-1} - (\tilde{c}_t - \tilde{c}_r^G - \tilde{r}^G - b_{t-1} - \tilde{\tau}_t), \quad (11)$$

where $\tilde{c}_t - \tilde{c}_r^G - \tilde{r}^G - i_{t-1} - \tilde{\tau}_t = \tilde{b}_t^G$ is the primary surplus and $\tilde{\tau}_t$ is total tax revenue

$$\tilde{\tau}_t = \tau_{tax} \tilde{w} t + \tau_{ess} \tilde{c} t + \tau_{int}(r^G_t - \tilde{d}_t) k^G_{t-1}/g + \tau_{inv} \tilde{d} t. \quad (12)$$

Finally, the interest-inclusive fiscal surplus, which plays a key role in our specification of fiscal rules, is given by

$$\tilde{s}^G_t = \tilde{s}^G_t - ((i_{t-1} - 1)/(\pi_{t-1} g)) \tilde{b}_{t-1}. \quad (13)$$

2.4.3. Fiscal policy

We assume that the government targets the interest-inclusive fiscal surplus to GDP ratio through the rule

$$\tilde{s}^G_t / y_t = \gamma + d^r \left( \frac{\tilde{r}^r}{\tilde{y}_t} - \frac{\tilde{r}^p}{\tilde{y}_t} \right) + d^b \left( \frac{\tilde{b}_t}{\tilde{y}_t} - \frac{\tilde{b}}{\tilde{y}_t} \right). \quad (14)$$

where $\gamma$ and $\gamma^p$ are the right-hand side terms are a tax revenue gap and a debt gap. Our choice of this class of rules was motivated by the fact that structural surplus rules, which are a special case of (14) with $d^r = 1$ and $d^b = 0$, have become more popular in practice. A key contribution of this paper is its exploration of much more general variants of structural surplus rules that can be calibrated continuously, mainly by varying $d^r$, between balanced budget rules and highly countercyclical rules. With $d^r = 1$ and $d^b = 0$, the rule (14) states that when the economy is hit with a shock that produces additional tax revenue at

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8 Countries operating variants of such rules include Chile, Germany, Sweden and Switzerland.
given tax rates, all of that excess revenue should go towards repaying debt, while only the interest savings on debt that accrue over time should be used to gradually lower tax rates or increase spending. While this is superior to a balanced budget rule \( d^T = 0 \) and \( d^g = 0 \), which is procyclical by calling for lower taxes during a boom, it does not have business cycle stabilization or welfare objectives as its prime concern. On the other hand, with \( d^T > 1 \) taxes are raised during a boom and therefore act in a more countercyclical fashion.

Our specification of (14) also deviates from conventional structural surplus rules in that the tax revenue gap excludes capital income tax revenue:

\[
\begin{align*}
\dot{t}_t^\text{rule} &= \tau_{Ld} W_t \ell_t + \tau_{Ls} c_t, \\
\dot{t}_t^\text{opt} &= \tau_{Ld} W_t \hat{\ell} + \tau_{Ls} \hat{c}.
\end{align*}
\]

The reason is that in a model with liquidity-constrained households the most important role of fiscal policy is to alleviate the market imperfection of the liquidity constraint. The preferred tax revenue gap should therefore reflect the tightness of that constraint, and this means that it should exclude terms related to capital income, which is received exclusively by \( \text{INF} \) households.

It is important to emphasize that a structural surplus rule does not require a debt feedback term in order to stabilize government debt. Eqs. (11) and (13) show that the rule (14) anchors the long-run debt to GDP ratio \( \overline{b^T} \) at \( \overline{b^T_2} = -\pi g/(\pi g - 1)\overline{sm} \). Our calibrated economy features a 5% annual nominal growth rate. This implies a quarterly autoregressive coefficient on debt in Eq. (13) of 0.988, so that debt takes a very long time to return to its long-run value following a shock. A debt feedback term needs to be included in the rule only if this speed of debt stabilization should be considered insufficient.

Eq. (14) is a targeting rule and leaves open which instrument is to be used to move the government surplus in the desired direction. The default instrument for our baseline results is targeted transfers \( T_t \). Following our discussion of the baseline, we will also look at other five possible instruments, including three tax rates (\( \tau_{c,c} \), \( \tau_{d,c} \), \( \tau_{k,c} \)) and two spending items (\( c^I_t \), \( I^T_t \)).

### 2.5. Competitive equilibrium

In a competitive equilibrium \( \text{INF} \) and \( \text{LIQ} \) households maximize utility and firms maximize the present discounted value of their cash flows, taking as given the government’s policy rules. The following market clearing condition holds for the final goods market:

\[
\dot{y}_t = \hat{c}_t + \hat{c}_t^I + \hat{I}_t + \hat{I}_t^T + \Psi.
\]

The three shocks of the model are given by

\[
z_t = (1 - \rho_z) z + \rho_z z_{t-1} + \tilde{z} u_t,
\]

where \( z_t \in (\epsilon_t, \epsilon_t^I, \epsilon_t^T) \).

### 2.6. Aggregate welfare

Expected welfare of \( \text{INF} \) households is given by

\[
\omega^\text{INF}_t = u^\text{INF}_t + \beta E_t \omega^\text{INF}_{t+1},
\]

where \( u^\text{INF}_t \) is the utility of a representative \( \text{INF} \) household at time \( t \). We define the Lucas (1987) compensating consumption variation \( \eta^\text{INF} \) (in percent) as the percentage reduction of average consumption that households would be willing to tolerate while remaining indifferent between the expectations of welfare under the optimal and suboptimal combinations of fiscal and monetary rule coefficients, say \( E \omega^\text{INF opt} \) and \( E \omega^\text{INF sub} \). Then \( \eta^\text{INF} \) equals

\[
\eta^\text{INF} = 100 \left( 1 - \exp \left( \frac{(\beta - 1)}{(1 - \lambda)} \left( E \omega^\text{INF sub} - E \omega^\text{INF opt} \right) \right) \right) < 0.
\]

The formula for \( \eta^\text{LIQ} \) is identical, except for the absence of \( (1 - \psi/g) \). We will analyze these group specific welfare measures as well as aggregate welfare, which we quantify by way of the population-weighted average of compensating variations:

\[
\eta = (1 - \psi) \eta^\text{INF} + \psi \eta^\text{LIQ}.
\]

We use DYNARE++ to obtain the second order approximations of the model’s competitive equilibrium in order to compute unconditional welfare and compensating consumption variations. We perform a multi-dimensional grid search over all fiscal and monetary rule coefficients (\( d^T \), \( d^g \), \( d^s \) and \( d^p \)).
3. Calibration

We use US data for the period 1984Q1–2007Q4 to calibrate key national accounts ratios and, after removing a log-linear trend, the dynamics of the shock processes. We rely on the literature for a number of other parameters. The real growth rate is calibrated at 2% per annum, the steady state real interest rate at 3% per annum, and steady-state inflation at 3% per annum. For the share of liquidity-constrained households, recent theoretical studies such as Gali et al. (2007) and Erceg et al. (2005) have assumed $\psi = 0.5$. The empirical literature has not yet converged on a consensus estimate, but $\psi = 0.5$ is generally held to be at the high end, with some studies having found estimates below $\psi = 0.2$. We adopt an intermediate value of $\psi = 0.3$.

Following Smets and Wouters (2003), the habit parameter $\gamma$ is set to 0.7. The labor supply elasticity $\eta_l$ is fixed at 1, a common assumption in the monetary business cycle literature. The utility weights on government consumption $\gamma^{\text{INF}}$ and $\gamma^{\text{LIQ}}$ are fixed to obtain steady state marginal rates of substitution between private consumption and government consumption of 1.

The depreciation rate of private capital equals 10% per annum, and the investment adjustment cost parameter, at $\phi_I = 2.5$, follows Christiano et al. (2005). The price adjustment cost parameter is set to $\phi_P = 100$. Together with the assumption that the gross markup equals $\mu = 1.2$, this is equivalent to assuming that the average duration of price contracts equals roughly four quarters in a model with Calvo (1983) pricing and Yun (1996) indexation. The cost share of private capital $\delta^A$ and the fixed cost $\delta^F$ are calibrated to obtain a labor income share of 64% and a private investment to GDP ratio of 17%.

In the United States, public infrastructure investment represents one sixth of all government spending, but this assumes a zero productivity of public education and health spending. We therefore raise that share, to one fifth, by fixing the government consumption to GDP ratio at 16% and the government investment to GDP ratio at 4%. The lump-sum transfers to GDP ratio is set to 10%. Following the method in Jones (2002), the steady state labor income, capital income and consumption tax rates are computed as 19.18%, 39.49% and 8.6% respectively. We follow Kamps (2006) for the evidence on the depreciation rate of public capital at 4% per annum. The productivity of public capital is determined by the parameter $\alpha_E$. Lighthart and Suárez (2005) present a meta analysis that finds an elasticity of aggregate output with respect to public capital of 0.14, which we can replicate by setting $\alpha_E = 0.1$. On the basis of recent historical data, we set the steady state government debt to GDP ratio to 50%, with the corresponding surplus ratio determined by the steady state nominal growth rate. Our calibration of the fiscal rule followed by the United States during this period is based on an estimated output gap rule similar to Taylor (2000) by Girouard and André (2005). They posit the rule $s_I = \frac{\gamma}{\gamma-I} - \frac{\gamma}{\gamma-I} = \phi d^I \ln(y_t)/y_t$, and find $d^I = 0.34$.

The autocorrelation coefficients and standard deviations of the model’s three shocks, together with the coefficients of the monetary rule, are calibrated to reproduce the standard deviations and correlations of U.S. investment, consumption and inflation. Table 1 shows the moments of the data and the model. The implied shock autocorrelations are $\rho_\tau = 0.68$, $\rho_c = 0.6$ and $\rho_I = 0.4$, and shock standard deviations are $\sigma_c = 0.018$, $\sigma_c = 0.015$ and $\sigma_I = 0.018$. For the monetary rule the implied coefficients are $\delta_I = 0.7$ and $\delta_c = 2.0$.

4. Results for the baseline model

Using targeted transfers $Y_t$ as the fiscal instrument, the coefficients of the optimal fiscal and monetary rules in our baseline model are given by $d^c = 3$, $d^I = 0$, $\delta^c = 1.2$ and $\delta^I = 0.2$. Fiscal policy therefore responds in an aggressively countercyclical fashion to the tax revenue gap but does not respond at all to the debt gap. Monetary policy is characterized by super-inertia and a non-aggressive response to inflation. To build intuition for these results we begin by analyzing impulse responses, before turning to welfare analysis. Our exposition focuses mostly on technology shocks, because these are the main driver of our welfare results. But we start with a brief discussion of investment demand shocks, to highlight that our main results are very similar across supply and demand shocks.

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Table 1

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<th>Model</th>
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<td>Consumption</td>
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<td>Inflation</td>
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<td>1.318</td>
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<table>
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<tr>
<th>Correlations</th>
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<th>Model</th>
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<td>Investment and inflation</td>
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</tr>
<tr>
<td>Consumption and inflation</td>
<td>0.148</td>
<td>0.144</td>
</tr>
</tbody>
</table>

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9 The share has been estimated at 0.37 and 0.34 for the euro area by Coenen and Straub (2005) and Forni et al. (2009), at 0.248 for Japan by Iwata (2009), and at 0.18 for the United States by Traum and Yang (2009).

10 Similar to Lighthart and Suárez (2005) and Leeper et al. (2009) discuss that the empirical literature has a wide range of values for this elasticity. At one extreme, Holtz-Eakin (1994) and Evans and Karras (1994) use state-level data and find that public-sector capital has negative or no effect on private sector productivity. At the other extreme, Aschauer (1989) and Pereira and de Frustos (1999) obtain significant productivity effects from public capital, with elasticities in the range of 0.24 to 0.39.

11 The correlation between consumption and investment is smaller than in the data. As explained by Christiano et al. (2009), this is due to the extreme simplicity of the model on the financing side, and could be remedied by introducing financial accelerator-type features.
4.1. Impulse responses

We compare impulse responses that vary one of the four coefficients of our rules at a time, while holding the three other coefficients at their values corresponding to the overall welfare optimum.\textsuperscript{12} The size of shocks equals one standard deviation.

4.1.1. Investment demand shock: fiscal policy rule coefficients

Fig. 1 shows impulse responses for a contractionary investment demand shock, for different $\delta^s$. Monetary policy supports demand in response to this shock, by lowering the real interest rate during the contraction, except for a one-quarter delay due to interest rate smoothing. While the tax revenue to GDP ratio increases on impact, partly because of higher corporate profits taxes due to lower marginal costs combined with sticky product prices, the government spending to GDP ratio increases even more, because the level of spending is held constant while GDP declines significantly. Under a balanced budget rule there is therefore a reduction in transfers to LIQ households, which leads them to increase their labor supply and to reduce their consumption. The latter further reduces aggregate demand and thus the real wage.

Under demand shocks such as this one, LIQ households benefit greatly from stronger automatic stabilizers, meaning from higher $\delta^s$. The reason is that the targeted tax revenue, principally labor income taxes, declines during a demand contraction and thus calls for higher transfer payments, thereby allowing LIQ households to reduce the drop in their consumption and to reduce the increase in their labor supply, or even to reduce their labor supply. The latter helps to stabilize the real wage and therefore the labor supply volatility of INF households, who therefore always benefit from a higher $\delta^s$. But it can be shown that beyond $\delta^s = 4$ the losses due to an increase in the volatility of LIQ households’ labor supply start to exceed the gains due to a decrease in their consumption volatility. Impulse responses for consumption demand shocks are not shown, as their logic is very similar to that of investment shocks.

4.1.2. Technology shock: fiscal policy rule coefficients

Fig. 2 shows the impulse responses for an expansionary technology shock, again for different $\delta^s$. The shock increases output and, because this is a supply shock that reduces marginal costs, reduces inflation. The latter leads to a drop in nominal and real interest rates, which results in an increase in INF households’ consumption and investment. The shock also initially reduces labor demand and, unless fiscal policy is aggressively countercyclical, the real wage, which significantly reduces the labor income of LIQ households. However, the design of the fiscal rule ensures that the drop in LIQ households’ consumption on impact is kept small even under a balanced budget rule ($\delta^s = 0$, dotted line). The reason is that an expansionary technology shock increases capital income tax revenue by more than the simultaneous drop in labor income tax revenue, with consumption tax revenue remaining fairly flat. A balanced budget rule mandates distribution of this excess tax revenue to LIQ households by way of transfers, thereby supporting their consumption while helping them to reduce their labor supply to benefit from the higher productivity. Under a more countercyclical rule ($\delta^s = 1$, broken line, $\delta^s = 3$, solid line and $\delta^s = 5$, faint dotted line) the government deficit to GDP ratio is allowed to increase in response to the observed drop in the targeted tax revenue, which, similar to the investment shock, is principally due to a drop in labor income taxes. More resources are therefore transferred to LIQ households, thereby allowing them to consume more but also to work less. The latter reduces the drop in the real wage, which encourages INF households to supply additional labor. LIQ households can be shown to benefit from smoother consumption as $\delta^s$ increases from 0 to around 1, but beyond that higher $\delta^s$ becomes undesirable because it is too procyclical, excessively boosting consumption and making labor supply more volatile. INF households on the other hand benefit from a larger $\delta^s$ almost without limit, as the reduction in LIQ households’ labor supply accommodates the reduction in aggregate labor demand, thereby reducing the volatility of the real wage and thus the volatility of INF labor supply. INF households’ consumption is not much affected by $\delta^s$ as they can intertemporally smooth the consumption effects of the productivity shock.

4.1.3. Technology shock: monetary policy rule coefficients

Fig. 3 shows impulse responses for an expansionary technology shock, for different inflation coefficients $\delta^\pi$. A large $\delta^\pi$ implies that the central bank cuts the nominal interest rate aggressively in response to lower inflation. This lowers the drop in inflation but increases the drop in the real interest rate, thereby initially boosting the consumption and investment demand of INF households. Higher demand raises labor demand and the real wage. LIQ households can therefore also increase their consumption, which further raises the demand for goods and the real wage. We will show that the model calls for the optimal $\delta^\pi$ to be very low, at close to 0.2, and that this is entirely due to the presence of LIQ households. As in conventional infinite horizon models, a larger $\delta^\pi$ improves the welfare of INF households. But it reduces the welfare of LIQ households much more significantly, because more volatile real interest rates increase the volatility of real wages, and therefore of LIQ households’ income and consumption.

Fig. 4 again shows impulse responses for an expansionary technology shock, this time for different interest rate smoothing coefficients $\delta^i$. A large $\delta^i$ keeps the real interest rate low for a prolonged period and thereby supports a persistent expansion that largely offsets the disinflationary effects of the technology shock. We will show that the model calls for the optimal $\delta^i$.

\textsuperscript{12} This experiment allows us to understand the ceteris paribus effects of each individual coefficient. Reoptimizing the remaining coefficients when changing the coefficient of interest would make this much harder.
\[ di \] to be very high, at 1.2, and that this deviation from more conventional results is again entirely due to the presence of \( LIQ \) households. As in most conventional infinite horizon models, for \( INF \) households the persistent increase in demand observed under very high \( di \) increases their consumption and labor supply volatility and therefore reduces their welfare. But it also reduces the drop in the targeted tax aggregate, which leads to less reliance on fiscal transfers to \( LIQ \) households for stabilization purposes, with monetary policy instead causing the real wage to rise more strongly. This reduces the consumption and labor supply volatility of \( LIQ \) households, and therefore raises their welfare.

The fact that an "explosive" monetary rule \( (di > 1) \) does not produce explosive equilibria is related to the expectations of the private sector. The real interest rate would drop exponentially under \( di > 1 \) unless the price level accommodates. The expectation of a drop in the real interest rate however implies a substantial increase in future prices over the baseline. This expectation stabilizes the path of inflation, leading it to increase after the first period, and thereby causing the real rate to...
return to neutral following the initial drop. Optimality of super-inertial interest rates has also been found, albeit in other environments, by Rotemberg and Woodford (1999) and Giannoni and Woodford (2003).

4.2. Welfare and individual rule coefficients

Fig. 5 illustrates the welfare effects of varying one fiscal or monetary coefficient at a time around the overall optimum, under the combined three shocks. The left, middle and right column show welfare results for all households (weighted), INF households and LIQ households. The rows show the results of varying the coefficients $d^s$, $db^s$, $d^i$ and $dp$. In each case welfare is normalized at zero at the overall optimum values for the respective coefficients.
For $d_s$ welfare increases quite steeply until it reaches a maximum at around $d_s = 3$. Welfare moves little in response to the debt coefficient, with an optimum at $d_b = 0.13$.

For the interest rate smoothing coefficient $d_i$, welfare is increasing, with a maximum at $d_i = 1.2$, but again the magnitudes are very small. For $d_p$ welfare is decreasing over almost the entire range, with an optimum at $d_p = 0.2$.

The most striking result is that the welfare differences associated with varying the fiscal coefficients are far larger than the differences due to monetary coefficients, due to the ability of fiscal policy to directly stabilize the income of LIQ households. The weak monetary policy response to inflation is however also quantitatively important, in this case because it stabilizes the income of LIQ households by stabilizing their real wage.

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13 This result is robust to varying the share $\phi$ of LIQ households, whose baseline welfare with respect to this coefficient is hump-shaped with a maximum at around $d_s = 0.1$, within the empirically relevant range of around 0.2–0.5. The reason is that INF households’ losses from a positive coefficient are always larger.
4.3. Overall welfare and volatility

To judge the attractiveness of policy rules to policymakers it is essential to evaluate not only their welfare outcomes but also their implied fiscal instrument volatility. Fig. 6 addresses both questions by looking at three-dimensional surface plots and contour plots. In each case we use the overall optimal point as our zero welfare gain baseline. The left column looks at fiscal policy and shows the two fiscal policy coefficients on the axes of the plots. The right column looks at monetary policy and shows the two monetary policy coefficients on the axes of the plots. The first and second rows show surface and contour plots for overall welfare under all three shocks, the third row shows the associated standard deviation of the policy instruments, that is of the transfers to GDP ratio and of the nominal interest rate, and the fourth row shows the combinations of weighted welfare and of fiscal instrument volatility as the tax coefficient $d^s$ changes from 0 to 5, holding all other coefficients at their overall optimum values $d^b = 0$, $d^i = 1.2$ and $d^p = 0.2$. We refer to these combinations as an efficiency frontier.

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**Fig. 4.** Expansionary technology shock, $d^i = 0.1 \ldots , d^i = 1.2/C_0/C_0 , d^i = 2\ldots$. 

4.3. Overall welfare and volatility

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The welfare results are consistent with the information in the 2-dimensional welfare plots above. But they also contain information on the regions of the coefficient space that are consistent with dynamic stability, by excluding unstable regions from the surface or contour.

The volatility of the monetary instrument decreases with $d_i$, and it increases with $dp$, except at the lowest possible values consistent with dynamic stability. Interest rate volatility therefore does not contradict the welfare result, whereby the optimal monetary rule is super-inertial and does not respond to inflation aggressively. The volatility of the fiscal instrument increases with both $d_s$ and $db$. The figure therefore implies that the surplus target rule should not respond to the debt gap, as doing so both lowers welfare and raises volatility. A response to the tax revenue gap is optimal on welfare grounds up to $d_s = 3$. The efficiency frontier shows that the volatility of lump-sum transfers increases as welfare improves with $d_s$, but a very substantial (relative to gains typically found in the monetary policy literature) consumption equivalent gain of 0.028 can be achieved by going from the balanced budget rule to the best possible rule. The cost in additional volatility is small, amounting to an increase of roughly 0.45 in the standard deviation of the targeted transfers to GDP ratio. A more aggressive rule might therefore well be judged attractive by policymakers.
4.4. Welfare under alternative fiscal instruments

Transfers targeted to LIQ households have been our default instrument up to this point. However, our model allows for five alternatives. Fig. 7 considers four of them, consumption taxes, capital income taxes, government consumption, and government investment. The figure shows how welfare changes with respect to $d^\tau$ for each of the instruments, again holding $d^b$, $d^i$ and $\delta^\pi$ at their overall optimum values. To have a common point of comparison, we use the maximum welfare in the case of targeted transfers as the zero welfare benchmark.

We find that under consumption taxes welfare increases with $d^\tau$ in a similar pattern to transfers, but with much lower absolute welfare levels. The reason is that these taxes are not targeted, with far more than half of a tax cut going to households who are perfectly able to smooth the effects of shocks without help from the government. Capital income taxes and government investment are even less desirable, because the former has distortional effects on private capital accumulation, and the latter on public capital accumulation. Given that both capital stocks are state variables, their effects on output last...
very much longer than the shock that they are being used to offset, which adds to volatility and reduces welfare. Finally, government consumption performs worst of all, especially for high $d^s$. The reason is that, for the technology shocks that dominate our results, but not for demand shocks, higher government consumption arrives precisely at the time when private consumption is already elevated. Furthermore, government consumption is likely to be a problematic instrument in practice, because it is hard to increase and decrease it at will in significant amounts, and in a timely manner, in response to transitory economic shocks. Targeted transfers on the other hand are a more realistic option. They can be implemented as automatic stabilizers, for example through well-designed welfare programs that avoid work disincentive effects as much as possible.

We note that labor income taxes have been absent from this discussion. The reason is that they would destabilize rather than stabilize the economy in the presence of technology shocks. Following an expansionary technology shock the targeted tax revenue falls. If the government in response were to cut the labor income tax rate, the result would be a lower real wage due to lower LIQ labor supply. This combination would further reduce the targeted tax revenue and therefore the required labor tax rate. This induces instability if $d^s$ exceeds a fairly low upper limit.

4.5. Welfare under alternative fiscal rules

We have selected the rule (14) carefully, that is after testing a number of realistic alternatives to the tax revenue gap (15). An overall tax revenue gap that includes capital income tax revenue is far inferior in welfare terms, because it mandates tightening at times of high capital incomes that coincide with low labor incomes or vice versa, which is exactly the combination observed after technology shocks. An output gap rule cuts welfare gains in half, because in an economy with even a modest share of liquidity-constrained households the focus of the policymaker should be on stabilizing the income of those households, which is very different from stabilizing output.

5. Results under simpler fiscal environments

We have calibrated our baseline economy to be realistic enough to incorporate positive government debt, positive government spending, and multiple positive distortionary taxes. All our results on optimal fiscal rule coefficients are robust to removing some of these features. But we also consider much simpler model variants, for two reasons. First, they provide a useful sensitivity check for our results on optimal monetary rule coefficients. Second, we need a simpler model for computational reasons when comparing welfare under our best combination of simple rules with welfare under the optimal policy from the timeless perspective.

5.1. Monetary rule coefficients and the fiscal environment

Fig. 8 displays weighted welfare results $\eta$ for the two monetary rule coefficients, for four different model variants ranging between our baseline model and a much simpler model. The latter features zero government consumption and investment $c^g$.

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14 This result may therefore be different should the calibration of relative shock variances change.
and $Ig$, zero distortionary tax rates $sL$, $sc$ and $sk$, zero government debt in steady state, and an output subsidy that eliminates steady state markup distortions. All other aspects of the calibration are kept identical to the baseline. In each case the fiscal instrument is targeted transfers, the fiscal rule coefficients are set at their respective overall optimum values, and welfare is shown in deviations from zero.

We have already seen that $INF$ households prefer a more conventional monetary stance, with modest interest rate smoothing and an aggressive response to inflation, but with the opposite preferences of $LIQ$ households prevailing under the baseline, due to the relatively larger variability of their welfare with respect to changes in monetary coefficients. Here we show that when the baseline is modified by assuming zero government debt in steady state and zero distortionary taxes, the relative sizes of welfare gains are reversed and the preferences of $INF$ households prevail.

Under zero steady state government debt, as shown in the top right panel of Fig. 8, the optimal inflation coefficient becomes $\delta^i = 1.9$, while the interest rate smoothing coefficient remains above one at $\delta^i = 1.1$. Under positive government debt an aggressive inflation response gives rise to large wealth effects for $INF$ households due to revaluations of the debt stock, thereby increasing the volatility of their consumption and employment. Under zero government debt these wealth effects are absent, and $INF$ households therefore benefit much more strongly from a more aggressive interest rate response. Because a more aggressive monetary policy does a better job at controlling aggregate demand, real wages adjust more smoothly than under a more transfers-driven policy response. This in turn helps to reduce $LIQ$ households’ consumption and employment volatility, thereby reducing their welfare gains from a weak inflation response.

Zero distortionary taxes affect the optimal interest rate smoothing coefficient $\delta^i$, even though as mentioned above the welfare differences involved are extremely small. Distortionary taxes take a significant share of the benefits of expansionary shocks away from $INF$ households and transfer them to $LIQ$ households. Relative to the case of zero distortionary taxes, this reduces the consumption and employment volatilities of $INF$ households, including the extent to which these volatilities increase when interest rates become more persistent. At the same time, and again relative to the case of zero distortionary taxes, the presence of distortionary taxes increases the income volatility and therefore the consumption and employment volatilities of $LIQ$ households. Therefore, when distortionary taxes are removed, more persistent real interest rates have a larger negative effect on $INF$

15 In this case the coefficients on the tax bases in the fiscal rule are set equal to the tax rates in the baseline model, even though actual tax rates are zero.
welfare than before, and a smaller positive effect on LIQ welfare. This drives the optimal interest rate smoothing coefficient to $\delta^i = 0.1$, with the optimal inflation coefficient remaining high at $\delta^p = 1.6$.

The bottom left panel of Fig. 8 is quite close to the final optimum of the simple model, which also sets the steady state markup and government spending to zero and obtains $\delta^i = 0$ and $\delta^p = 1.7$.\(^{16}\)

5.2. Optimal policy from the timeless perspective

We finally present a comparison of the best simple rule under the simple model with the optimal policy from the timeless perspective. Because inflation stabilizing monetary rules and debt stabilizing fiscal rules are absent under the optimal policy setup, the long-run values of inflation and debt need to be pinned down by other means to obtain a dynamically stable equilibrium. We therefore impose, in the objective function of the social planner, extremely small penalties on deviations of debt and inflation from the steady state values of the simple model.\(^{17}\) Details are presented in the Technical Appendix. Fig. 9 presents the results, holding the monetary rule coefficients and $d_b$ at their values at the overall optimum, and plotting welfare as a function of $d^i$ against the welfare level implied by the optimal policy.

We first note that the overall welfare gains of moving from a balanced budget rule to the optimal rule are almost twice as large in the simple model as they are in the baseline model (compare Figs. 9 and 7). There are two reasons for this. First, in the baseline model, but not in the simple model, there is already a significant redistribution of cyclical capital income tax revenue under the balanced budget rule. As this improves the welfare of LIQ households, it limits additional welfare gains due to stronger automatic stabilizers. Second, in the simple model targeted transfers become far more effective at stabilizing the income of LIQ households, because there are no other distortionary taxes or interest charges on government debt that affect the surplus and therefore the timing of disbursement of targeted transfers. Monetary policy can then concentrate on stabilizing aggregate labor demand without worrying about excessive real wage volatility.

Finally, and most importantly, we find that the best simple rule performs almost exactly as well as the optimal policy from the timeless perspective.\(^{18}\) This, combined with the fact that the welfare gains available under this type of fiscal rule are very large relative to the gains from optimizing monetary policy, is a powerful argument in favor of designing fiscal systems with a view to creating strong automatic stabilizers.

6. Conclusion

We have studied the macroeconomic dynamics and welfare properties of an economy where both monetary and fiscal policies follow simple rules, and where a subset of households is liquidity constrained. We have found that the optimized fiscal rule implements strong automatic stabilizers, that this aggressive stance comes at a fairly modest cost in terms of fiscal instrument volatility, and that the preferred fiscal instrument is transfers targeted to liquidity-constrained households. The welfare gains of optimizing the fiscal rule, away from a balanced budget rule, are far larger than the welfare gains of optimizing the monetary rule.

\(^{16}\) The optimal $d^i$ remains in the range 2.8–3.5 for all simulations shown in Fig. 10.

\(^{17}\) We have verified that these are so small as to have no material effects on our results.

\(^{18}\) This is not as surprising as it may appear. As shown by Dennis (2010), the optimal policy under the timeless perspective can in principle even be inferior to discretion, depending on the model.
The optimized fiscal rule primarily stabilizes the income of liquidity-constrained households, rather than output. This is because the optimal course of action for the government is to offset the market imperfection of the missing capital market access of this group of households. Whether the optimal automatic stabilizers implied by our model are significantly stronger than what can be found in contemporary fiscal frameworks is an empirical question that we will attempt to answer in future work.

The optimized monetary rule in our baseline model features super-inertia and a very low coefficient on inflation. This result was shown to depend on the presence of a significant stock of government debt and of positive distortionary taxes. As these are present in almost any existing economy, we would argue that it is these baseline results that are practically most relevant. But we also show that, in a highly stylized economy without these features, one can obtain the opposite result, with no interest rate smoothing and an aggressive response to inflation.

Our final result puts the welfare effects of optimizing simple policy rules into perspective by comparing them to welfare under the timeless perspective. We find that the maximum welfare achieved in the two cases is nearly identical.

Much work remains to be done on the topic of rules-based fiscal policy. The question of implementation lags deserves particular attention, as a traditional objection of fiscal activism has been the perceived inability of fiscal policy, unlike monetary policy, to respond sufficiently fast to shocks, with delayed responses potentially being destabilizing. We aim to study this question in future work.

References