Robust Control, Informational Frictions, and International Consumption Correlations*

Yulei Luo†
University of Hong Kong

Jun Nie‡
Federal Reserve Bank of Kansas City

Eric R. Young§
University of Virginia

December 7, 2011

Abstract

In this paper we examine the effects of model misspecification (robustness or RB) on international consumption correlations in two otherwise standard small open economy models: one with perfect state observation and the other with imperfect state observation. We show that in the presence of capital mobility in financial markets, RB lowers the international consumption correlations by generating heterogeneous responses of consumption to income shocks across countries facing different macroeconomic uncertainty. However, the calibrated RB model with perfect state observation cannot explain the observed consumption correlations quantitatively. We then show that the RB model with imperfect state observation is capable of matching the behavior of international consumption quantitatively via two channels: (i) the gradual response to income shocks that increases the correlations and (ii) the presence of the common noise shocks that reduce the correlations.

*We would like to thank Paul Bergin, Riccardo Colacito, David Cook, Richard Dennis, Reuven Glick, Christopher House, Tasos Karantounias, Fred Kwan, Charles Leung, Zheng Liu, Yang Lu, Monika Piazzesi, Kevin Salyer, Ulf Söderström, Lars Svensson, Yong Wang, Yu Zheng, and seminar and conference participants at North Carolina State University, UC Davis, University of Iowa, Hong Kong University of Science and Technology, City University of Hong Kong, Federal Reserve Banks of Atlanta, Kansas City, and San Francisco, Sveriges Riksbank, the 2011 Midwest Macro Meeting, NYU Alumni Economic Conference, the 2011 NASM of the Econometric Society, and the 2011 SED meeting for helpful comments and suggestions. Luo thanks the Hong Kong General Research Fund and HKU seed funding program for basic research for financial support. Nie thanks Li Yi for the research assistance. Young thanks the University of Virginia Faculty Summer Grant Program for financial support. All errors are the responsibility of the authors. The views expressed here are the opinions of the authors only and do not necessarily represent those of the Federal Reserve Bank of Kansas City or the Federal Reserve System.

†School of Economics and Finance, The University of Hong Kong, Hong Kong. E-mail: yluo@econ.hku.hk.

‡Economic Research Department, Federal Reserve Bank of Kansas City. E-mail: jun.nie@kc.frb.org.

§Department of Economics, University of Virginia, Charlottesville, VA 22904. E-mail: ey2d@virginia.edu.

Keywords: Robustness, Imperfect State Observation, International Consumption Correlations.
JEL Classification Numbers: D83, E21, F41, G15.

1 Introduction

A common assumption in international business cycles models is that world financial markets are complete in the sense that individuals in different countries are able to fully insure country-specific income risks using international financial markets. Under this assumption, the models predict that consumption (or consumption growth) is highly correlated across countries, and in some cases the international consumption correlation is equal to 1 regardless of income or output correlations. The intuition is that since consumers are risk averse they will choose to smooth consumption over time by trading in international financial markets. However, in the data cross-country consumption correlations are very low and are even lower than corresponding income correlations in many countries. For example, Backus, Kehoe, and Kydland (1992) solve a two-country real business cycles model and argue that the puzzle that empirical consumption correlations are actually lower than output correlations is the most striking discrepancy between theory and data.[1] In the literature, the empirical low international consumption correlations have been interpreted as indicating international financial markets imperfections – for examples, see Kollman (1996), Baxter and Crucini (1995), Lewis (1996), and Kehoe and Perri (2002).

Other extensions have been proposed to make the models better fit the data. For example, Devereux, Gregory, and Smith (1992) show that in the perfect risk-sharing model nonseparability between consumption and leisure has the potential to reduce the cross-country consumption correlation. Stockman and Tesar (1995) show that the presence of nontraded goods in the complete-market model can also improve the model's prediction. Kollman (1996) shows that asset market incompleteness can generate significantly lower cross-country consumption correlations. Wen (2007) shows that adding country-specific demand shocks can also help explain the cross-country business cycle comovements within a complete-market framework. In addition, Fuhrer and Klein (2006) show that habit formation has important implications for international consumption correlations. In particular, they show that with a shock to the interest rate habit formation *by itself* can generate positive consumption correlations across countries even in the absence of international risk sharing and common income shocks. They then argue that if habit is a good characterization of consumers' behavior, the absence of international risk sharing is *even more striking* than standard tests suggest; that is, existing studies may overstate the extent to which common consumption movements across countries reflect international risk sharing because some of them are due to habit.[2]

All of the above papers have assumed that agents in the economy fully trust the probability

[1] Pakko (1996) shows that, in the presence of complete asset markets, consumption should be more highly correlated with total world income than with domestic income, while the data shows the opposite. This result provides an alternative standard for evaluating models of international business cycles.

[2] Baxter and Jermann (1997) also argue that the international diversification puzzle is *"worse than you think"* due to nontraded labor income being correlated with the return to domestic assets.

model they use to make decisions. However, in reality, agents may not be able to know exactly the model generating the data, and they are concerned about whether their model is somehow misspecified. In this paper, we examine how introducing the preference for robustness (RB, a concern for model misspecification) into two otherwise standard small open economy (SOE) models can improve the models' predictions on the international consumption correlations puzzle we discussed above.[3]

The first model, our benchmark model, is based on Hansen, Sargent, and Tallarini (1999, henceforth, HST) and Hansen and Sargent (2007a) in which we assume that consumers can observe the state perfectly. In the second model, in addition to the concern for model misspecification, we assume that consumers have imperfect state observation, i.e., they face state uncertainty. Hansen and Sargent (1995, 2007) first introduce robustness into economic models. In robust control problems, agents are concerned about the possibility that their model is misspecified in a manner that is difficult to detect statistically; consequently, they make their decisions as if the subjective distribution over shocks was chosen by a malevolent nature in order to minimize their expected utility (the solution to a robust decision-maker's problem is the equilibrium of a max-min game between the decision-maker and nature).[4] Robustness models produce precautionary savings but remain within the class of LQ-Gaussian models, which leads to analytical simplicity.[5] After introducing RB into the standard full-information SOE model, we solve it explicitly and find that RB can help improve the model's consistency with the empirical evidence on international consumption correlations.[6] Specifically, we show that in the presence of capital mobility in international financial markets, RB lowers the international consumption correlations by introducing heterogenous responses of consumption to income shocks across countries which are facing different macroeconomic uncertainty. That is, we have uncovered a novel channel through which the fundamental economic shocks in different countries can interact with agents' concerns about model misspecification (i.e., model uncertainty), which, in turn, reduces consumption correlations across these countries.

[3]We adopt a small-open economy setting with quadratic utility and linear state transition equation rather than two-country general equilibrium setting with CRRA utility and stochastic interest rates in this paper for two reasons. First, as argued in Hansen and Sargent (2007), if the objective function is not quadratic or the state transition equation is not linear, worst possible distributions due to RB are generally non-Gaussian, which significantly complicates the computational task. Second, there does not exist a two-country general equilibrium in which the equilibrium interest rate is constant. See Section 3.1 for a detailed discussion.

[4]We interpret fear of model misspecification as an information imperfection because it implies that the true data-generating process is unknown.

[5]A second class of models that produces precautionary savings but remains within the class of LQ-Gaussian models is the risk-sensitive model of Hansen, Sargent, and Tallarini (1999). See Hansen and Sargent (2007a) and Luo and Young (2010) for detailed comparisons of the two models.

[6]Recently, some papers have incorporated model uncertainty into small open economy models and examined the effects of robustness on business cycles and monetary policy. See Cook (2002), Leitemo and Söderström (2008), Dennis, Leitemo, and Söderström (2009), and Justiniano and Preston (2010),.

After calibrating the RB parameter using the detection error probabilities in our benchmark model, we find that it can better match the data on international consumption correlations, but quantitatively, it is still not enough to explain the observed consumption correlations in the data. We then consider our extended model in which (in addition to model uncertainty) consumers cannot observe the state perfectly, i.e., they face state uncertainty (i.e., SU). The key assumption of SU in this paper is that consumers only observe noisy signals about the true state when making optimal decisions and thus need to extract the true signals using imperfect observations. This assumption, therefore, captures the situation where consumers have imperfect knowledge of the underlying common shocks.[7] Specifically, we assume that the noisy signal is the sum of the true state and an iid noise, which is standard in the signal extraction literature.[8] As we will show, the state uncertainty that agents face can significantly amplify the effects of RB on reducing international consumption correlations across countries.

Specifically, we find that the interactions between RB and SU can generate two competing forces affecting international consumption correlations. First, the model with SU generates gradual responses of consumption growth to income shocks due to imperfect state observation. Just like the habit formation or sticky expectations hypotheses (infrequent updating as in Bacchetta and van Wincoop 2010), this channel increases cross-country consumption correlations. Second, the noise due to imperfect observation can reduce consumption correlations across countries because it increases consumption volatility but has no impact on the covariance of consumption across countries. It is worth noting that these effects will disappear if there is no model uncertainty due to RB. The intuition is that the two forces have the same impact on the consumption adjustment processes across countries in the absence of heterogenous degrees of income uncertainty and the preference for RB, which thus does not affect international consumption correlations. Using the same calibration procedure, we find that the interaction between RB and SU can further improve the model's quantitative predictions on the observed consumption correlations. Specifically, we show that consumption correlations can be quite small if the agents' ability of observing the true state is low and their noise shocks are highly correlated across individuals. Both conditions seem

[7]In contrast, the full-information rational expectations (RE) hypothesis assumes that ordinary households can observe all available information without errors.

[8]For example, Muth (1960), Lucas (1972), Morris and Shin (2002), and Angeletos and La'O (2009). It is worth noting that this assumption can also be rationalized by Sims' rational inattention (RI) hypothesis (Sims, 2003). RI assumes that ordinary people only devote finite information-processing capacity to processing economic and financial information because they face many competing demands for their total capacity every day. (For example, Luo and Young (2011) showed that within the univariate linear-quadratic framework, if the signal-to-noise ratio is given in the signal extraction problem, signal extraction and rational inattention are observationally equivalent in the sense that they lead to the same decision rules.) However, modeling RI explicitly may lead to a potential problem when inattentive agents face model uncertainty: agents with finite capacity may face the attention problem about how to optimallly allocate limited attention to reduce model uncertainty and state uncertainty as both lead to larger welfare losses. Therefore, using state uncertainty can signficantly simplify the model to better focus on the key intuitions.

ex ante plausible: households would allocate very little of their scarce attention to aggregate movements in income (as they are not costly) and often would use common sources to obtain information (such as newspapers or TV).

The remainder of the paper is organized as follows. Section 2 reviews the standard full-information rational expectations small open economy (SOE) model and discuss the puzzling implications for international consumption correlations in the model. Section 3 presents the RB model, calibrates the model misspecification parameter, and show to what extent RB can improve the model's performance. Section 4 introduces RB into the SOE model with SU and shows that SU can further improve the model's predictions. Section 5 concludes.

2 A Full-information Rational Expectations Small Open Economy Model

2.1 Model Setup

In this section we present a full-information rational expectations (RE) version of a small open economy (SOE) model and will discuss how to incorporate robustness (RB) into this stylized model in the next sections. Following the literature, we assume that the model economy is populated by a continuum of identical infinitely-lived consumers, and the only asset that is traded internationally is a risk-free bond.

The full-information RE-SOE model, the small-open economy version of Hall's permanent income model, can be formulated as

$$\max_{\{c_t\}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \tag{1}$$

subject to the flow budget constraint

$$b_{t+1} = Rb_t - c_t + y_t, \tag{2}$$

where $u(c_t) = -\frac{1}{2}(\bar{c} - c_t)^2$ is the utility function, $\bar{c}$ is the bliss point, c_t is consumption, R is the exogenous and constant gross world interest rate, b_t is the amount of the risk-free world bond held at the beginning of period t, and y_t is net income in period t and is defined as output less than investment and government spending. The model assumes perfect capital mobility in that domestic consumers have access to the bond offered by the rest of the world and that the real return on this bond is the same across countries. In other words, the world risk-free bond provides a mechanism for risk sharing of domestic households who can use the international capital market to smooth consumption. Finally we assume that the no-Ponzi-scheme condition is satisfied.

A similar problem can be formulated for the rest of the world (ROW). We use an asterisk ("$*$") to represent the rest of world variables. For example, we assume that y_t^* is the average endowment (net income) of the rest of the world (G-7, OECD, or EU). Furthermore, we assume that domestic endowment and the ROW endowment are correlated. We will specify the structure of the income processes later.

Let $\beta R = 1$; optimal consumption is then determined by permanent income:

$$c_t = (R - 1)\, s_t \tag{3}$$

where

$$s_t = b_t + \frac{1}{R} \sum_{j=0}^{\infty} R^{-j} E_t \left[y_{t+j} \right] \tag{4}$$

is the expected present value of lifetime resources, consisting of financial wealth (the risk free foreign bond) plus human wealth. In order to facilitate the introduction of robustness and rational inattention we follow Luo (2008) and Luo and Young (2010) and reduce the multivariate model with a general income process to a univariate model with iid innovations to permanent income s_t that can be solved in analytically.[9] Letting s_t be defined as a new state variable, we can reformulate the PIH model as

$$v(s_0) = \max_{\{c_t, s_{t+1}\}_{t=0}^{\infty}} \left\{ E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \right\} \tag{5}$$

subject to

$$s_{t+1} = R s_t - c_t + \zeta_{t+1}, \tag{6}$$

where the time $(t+1)$ innovation to permanent income can be written as

$$\zeta_{t+1} = \frac{1}{R} \sum_{j=t+1}^{\infty} \left(\frac{1}{R} \right)^{j-(t+1)} \left(E_{t+1} - E_t \right) \left[y_j \right]; \tag{7}$$

$v(s_0)$ is the consumer's value function under RE. Under the RE hypothesis, this model with quadratic utility leads to the well-known random walk result of Hall (1978),

$$\begin{aligned} \Delta c_t &= \frac{R-1}{R} \left(E_t - E_{t-1} \right) \left[\sum_{j=0}^{\infty} \left(\frac{1}{R} \right)^j (y_{t+j}) \right] \\ &= (R-1)\, \zeta_t, \end{aligned} \tag{8}$$

which relates the innovations to consumption to income shocks.[10] In this case, the change in consumption depends neither on the past history of labor income nor on anticipated changes in

[9] See Luo (2008) for a formal proof of this reduction. Multivariate versions of the RI model are numerically, but not analytically, tractable, as the variance-covariance matrix of the states cannot generally be obtained in closed form.

[10] Under RE the expression of the change in individual consumption is the same as that of the change in aggregate consumption.

labor income. In addition, certainty equivalence holds, and thus uncertainty has no impact on optimal consumption.

We close the model by specifying domestic and foreign income processes as follows:

$$y_{t+1} = \rho y_t + \varepsilon_{t+1}, \tag{9}$$

$$y^*_{t+1} = \rho^* y^*_t + \varepsilon^*_{t+1}, \tag{10}$$

where ε_{t+1} and ε^*_{t+1} are Gaussian innovations to domestic and ROW income with mean 0 and variance ω^2 and ω^{*2}, respectively. To model the observed income correlations across countries, we assume that the correlation between ε_{t+1} and ε^*_{t+1},

$$\text{corr}\left(\varepsilon_{t+1}, \varepsilon^*_{t+1}\right) = \eta. \tag{11}$$

Given the income processes, (9) and (10), the income correlation can be written as

$$\text{corr}\left(y_{t+1}, y^*_{t+1}\right) = \Pi_y \eta, \tag{12}$$

where $\Pi_y = \frac{\sqrt{(1-\rho^2)(1-\rho^{*2})}}{1-\rho\rho^*}$. Note that $\Pi_y = 1$ when $\rho = \rho^*$ and $\Pi_y < 1$ when $\rho \neq \rho^*$. In this case, permanent income and the innovation to it are

$$s_t = b_t + \frac{y_t}{R-\rho}; \tag{13}$$

$$\zeta_t = \frac{\varepsilon_t}{R-\rho}, \tag{14}$$

respectively. Similarly, for the average country in the international organization, $s^*_t = b^*_t + \frac{y^*_t}{R-\rho^*}$ and $\zeta^*_t = \frac{\varepsilon^*_t}{R-\rho^*}$.

2.2 Implications for Consumption Correlations

In the full-information RE model proposed in Section 2.1, consumption growth can be written as

$$\Delta c_t = \frac{R-1}{R-\rho}\varepsilon_t, \tag{15}$$

which means that consumption growth is white noise and the impulse response of consumption to the income shock is *flat* with an immediate upward jump in the initial period that persists indefinitely. (See the solid line in Figure 1.) However, as well documented in the consumption literature, the impulse response of aggregate consumption to aggregate income takes a *hump-shaped* form, which means that aggregate consumption growth reacts to income shocks gradually. Similarly, for the rest of the world,

$$\Delta c^*_t = \frac{R-1}{R-\rho^*}\varepsilon^*_t.$$

The international consumption correlation can thus be written as

$$\operatorname{corr}(\Delta c_t, \Delta c_t^*) = \operatorname{corr}(\varepsilon_t, \varepsilon_t^*) = \frac{1}{\Pi_y} \operatorname{corr}(y_t, y_t^*). \tag{16}$$

Note that when income processes in both countries are unit roots, i.e., $\rho = \rho^* = 1$, the correlation reduces to $\operatorname{corr}(\Delta c_t, \Delta c_t^*) = \operatorname{corr}(\Delta y_t, \Delta y_t^*) = \eta$. In this case the consumption correlation across countries is the same as the corresponding income correlation.

It is clear that if the estimated income persistence parameters, ρ and ρ^*, are different and less than 1, $\Pi_y < 1$ and $\operatorname{corr}(c_t, c_t^*) > \operatorname{corr}(y_t, y_t^*)$. This prediction contradicts the empirical evidence, as international consumption correlations are lower than income correlations for most pairs of countries.

3 The Effects of RB on Consumption Correlations

3.1 The RB Version of the SOE Model

Robust control emerged in the engineering literature in the 1970s and was introduced into economics and further developed by Hansen, Sargent, and others. A simple version of robust optimal control considers the question of how to make decisions when the agent does not know the probability model that generates the data. Specifically, an agent with a preference for robustness considers a range of models surrounding the given approximating model, (6), and makes decisions that maximize lifetime expected utility given the worst possible model. Following Hansen and Sargent (2007a), a simple robustness version of the SOE model proposed in Section 2.1 can be written as

$$v(s_t) = \max_{c_t} \min_{\nu_t} \left\{ -\frac{1}{2} (\bar{c} - c_t)^2 + \beta \left[\vartheta \nu_t^2 + E_t \left[v(s_{t+1}) \right] \right] \right\} \tag{17}$$

subject to the distorted transition equation (i.e., the worst-case model):

$$s_{t+1} = R s_t - c_t + \zeta_{t+1} + \omega_\zeta \nu_t, \tag{18}$$

where ν_t distorts the mean of the innovation and $\vartheta > 0$ controls how bad the error can be.[11] As shown in Hansen, Sargent, and Tallarini (1999) and Hansen and Sargent (2007a), this class of models produces precautionary behavior while maintaining tractability within the LQ-Gaussian framework.

When domestic income follows an AR(1) process, (9), solving this robust control problem yields the following proposition:

[11] Formally, this setup is a game between the decision-maker and a malevolent nature that chooses the distortion process ν_t. $\vartheta \geq 0$ is a penalty parameter that restricts attention to a limited class of distortion processes; it can be mapped into an entropy condition that implies agents choose rules that are robust against processes which are close to the trusted one. In a later section we will apply an error detection approach to estimate ϑ in small open economies.

Proposition 1 *Under RB, the consumption function is*

$$c_t = \frac{R-1}{1-\Sigma} s_t - \frac{\Sigma \bar{c}}{1-\Sigma}, \tag{19}$$

the mean of the worst-case shock is

$$\nu_t \omega_\zeta = \frac{(R-1)\Sigma}{1-\Sigma} s_t - \frac{\Sigma}{1-\Sigma} \bar{c}, \tag{20}$$

and $s_t \left(= b_t + \frac{y_t}{R-\rho} \right)$ *is governed by*

$$s_{t+1} = \rho_s s_t + \frac{\Sigma}{1-\Sigma} \bar{c} + \zeta_{t+1}, \tag{21}$$

where $\zeta_{t+1} = \frac{\varepsilon_{t+1}}{R-\rho}$, $\Sigma = R\omega_\zeta^2 / (2\vartheta) > 0$ *measures the effect of robustness on consumption, and* $\rho_s = \frac{1-R\Sigma}{1-\Sigma} \in (0,1)$.

Proof. See Appendix 6.1. ■

It is worth noting that our univariate RB model with a unique state variable s leads to the same consumption function as the corresponding multivariate RB model in which the state variables are b_t and y_t. The key difference between these two models is that in our univariate RB model the evil agent distorts the transition equation of permanent income s_t, whereas in the multivariate RB model the evil agent distorts the income process y_t. Theoretically, the preference of robustness, ϑ, affects both the coefficients attached to b_t and y_t in the consumption function of the multivariate model. That is, in the multivariate model RB may affect the relative importance of the two state variables in the consumption function, whereas in the univariate model the relative importance of the two effects are *fixed* by reducing the state space. However, after solving the two-state model numerically using the standard procedure proposed in Hansen and Sargent (2007a), we can see that the two models lead to the same decision rule. (See Luo, Nie, and Young 2011 for a detailed proof.) The key reason is that in our univariate model the evil agent is not permitted to distort the law of motion for b_t as it is an accounting equation and has been used to obtain the transition equation of s, whereas in the multivariate RB model we also only need to consider the distortion to y_t as there is no innovation to b_t in the resource constraint.

The effect of the preference for robustness, Σ, is jointly determined by the RB parameter, ϑ, and the volatility of the permanent income, ω_ζ. This interaction provides a novel channel that the income shock can affect optimal consumption adjustments for different countries. That is, when there is a preference for robustness (i.e., $\vartheta < \infty$), the different volatilities for the income processes in two countries can lead to different consumption adjustments. This effect will disappear (i.e., $\Sigma = 0$) if there is no preference for robustness (i.e., $\vartheta \to \infty$).

The consumption function under RB, (19), shows that the preference for robustness, ϑ, affects the precautionary savings increment, $-\frac{\Sigma}{1-\Sigma}\bar{c}$. The smaller the value of ϑ the larger the precautionary saving increment, provided $\Sigma < 1$.

Proposition 2 $\Sigma < 1$.

Proof. The second-order condition for a minimization by nature can be rearranged into

$$\vartheta > \frac{1}{2} R^2 \omega_\zeta^2. \tag{22}$$

Using the definition of Σ we obtain

$$1 > R\Sigma.$$

Since $R > 1$, we must have $\Sigma < 1$. ■

The consumption function also implies that the stronger the preference for robustness, the larger the marginal propensity to consume out of permanent income, the more consumption responds initially to changes in permanent income; that is, under RB consumption is more sensitive to unanticipated income shocks. This response is referred to as "making hay while the sun shines" in van der Ploeg (1993), and reflects the precautionary aspect of these preferences. Note that for the average country in the international organization, we have analogous results: we can just replace s_t and ζ_t with s_t^* and ζ_t^*, respectively.

As mentioned before, we adopt the small-open economy model with the constant interest rate and quadratic utility rather than a two-country general equilibrium model with CRRA utility (e.g., Kollman 1996 and Daniel 1997) for two reasons. First, most existing RB models assume that the objective functions are quadratic and the state transition equations are linear, consequently, worst-case distributions are Gaussian. However, if the objective functions are not quadratic or the transition equations are not linear, worst-case distributions are generally non-Gaussian. As argued in Hansen and Sargent (2007a), the most difficult part in solving such non-LQ RB models is computational: representing the worst-case distribution parsimoniously enough that the model is tractable. Second, there does not exist a two-country general equilibrium in which the general equilibrium interest rate is constant. Specifically, consider a simple RE full-information two-country general equilibrium model in which the home country's budget constraint and consumption decision are characterized by (2) and (3), respectively,[12] and the agents in the foreign country solve the same problem in which its variables are denoted with an asterisk. In general equilibrium, the bond market-clearing condition is

$$b_t + b_t^* = 0 \text{ for all } t. \tag{23}$$

By Walras' law, (23) implies that the global resource constraint should also hold for all t:

$$c_t + c_t^* = y_t + y_t^* \equiv y_t^w, \tag{24}$$

where y_t^w denotes exogeneously given current world output. Using the expected resource constraint,

$$E_{t-1}\left[y_t^w\right] = E_{t-1}\left[c_t\right] + E_{t-1}\left[c_t^*\right] = \frac{1}{R}\left(\frac{1}{\beta} c_{t-1} + \frac{1}{\beta^*} c_{t-1}^*\right), \tag{25}$$

[12] Note that here we relax the assumption that $\beta R = 1$ such that β could be different in the two countries.

we can easily obtain the expression for the general equilibrium interest rate:

$$R=\frac{1}{E_{t-1}\left[y_t^w\right]}\left(\frac{1}{\beta}c_{t-1}+\frac{1}{\beta^*}c_{t-1}^*\right). \tag{26}$$

However, given that $c_{t-1}=(R-1)\left(b_{t-1}+\frac{y_{t-1}}{R-\rho}\right)$ and $c_{t-1}^*=(R-1)\left(b_{t-1}^*+\frac{y_{t-1}^*}{R-\rho}\right)$, the right-hand side of there (26) is a time-$(t-1)$ random variable, i.e., there does not exist a constant R such that (26) holds. It is obvious that this argument also holds for the RB model as both c_{t-1} and c_{t-1}^* are random variables at $t-1$.

3.2 Implications of RB for Consumption Correlations

Combining (19) with (21), consumption dynamics under RB in the domestic and average country are

$$c_t=\rho_s c_{t-1}+\frac{(R-1)\,\Sigma\overline{c}}{1-\Sigma}+\frac{R-1}{1-\Sigma}\zeta_t, \tag{27}$$

and

$$c_t^*=\rho_s^* c_{t-1}^*+\frac{(R-1)\,\Sigma^*\overline{c}}{1-\Sigma^*}+\frac{R-1}{1-\Sigma^*}\zeta_t^*, \tag{28}$$

respectively. Figure 1 also illustrates the response of aggregate consumption to an income shock ε_{t+1} in the domestic country; comparing the solid line (RE) with the dash-dotted line (RB), it is clear that RB raises the sensitivity of consumption to unanticipated changes in income. Given these two expressions, we have the following proposition about the cross-country consumption correlation:

Proposition 3 *Under RB, the consumption correlation between the home country and the ROW can be written as*

$$\operatorname{corr}\left(c_t,c_t^*\right)=\frac{\Pi_s}{\Pi_y}\operatorname{corr}\left(y_t,y_t^*\right); \tag{29}$$

where

$$\Pi_s=\frac{\sqrt{\left(1-\rho_s^2\right)\left(1-\rho_s^{*2}\right)}}{1-\rho_s\rho_s^*}, \tag{30}$$

$\Pi_y=\frac{\sqrt{(1-\rho^2)(1-\rho^{*2})}}{1-\rho\rho^*}$, $\rho_s=\frac{1-R\Sigma}{1-\Sigma}$, $\rho_s^*=\frac{1-R\Sigma^*}{1-\Sigma^*}$, *and we use the facts that* $\operatorname{corr}\left(\zeta_t,\zeta_t^*\right)=\operatorname{corr}\left(\varepsilon_t,\varepsilon_t^*\right)=\eta$ *and* $\operatorname{corr}\left(y_t,y_t^*\right)=\Pi_y\eta$.

Proof. See Appendix 6.3. ■

Expression (29) clearly shows that the degrees of preference for robustness (RB), ρ_s and ρ_s^* (Σ and Σ^*) , affect the consumption correlation across countries. The value of Π_s defined in (30) measures to what extent RB changes consumption correlations across countries. It is straightforward to show that when the effects of RB, Σ, are the same in the two economies,

($\rho_s = \rho_s^*$ or $\Sigma = \Sigma^*$), $\Pi_s = 1$. In this case, RB has *no* impact on the consumption correlation: $\operatorname{corr}(c_t, c_t^*) = \operatorname{corr}(y_t, y_t^*)/\Pi_y$, which is just the correlation obtained in the standard RE-SOE model, (16). Note that Σ can be written as $\Sigma^* + \Delta_\Sigma$ where Δ_Σ is defined as the difference between the domestic country and the ROW. Therefore, both the degree of RB in the ROW and the difference between the degrees of RB in the two economies affects the consumption correlation across countries.[13]

Figure 2 shows that the impact of RB on consumption correlations, Π_s, is increasing with Σ^*, the degree of RB in the ROW, for different values of Δ_Σ. (Here we set $R = 1.04$.) Note that holding Δ_Σ constant, Π_s is also increasing with Σ. Consumption is more sensitive to income shocks when Σ is larger, which by itself increases consumption correlations across countries when the difference in RB is fixed. Figure 2 also shows that Π_s is decreasing in the difference between Σ and Σ^*, Δ_Σ, for given values of Σ^*. For example, when $\Sigma^* = 0.05$, $\Pi_s = 0.84$ if $\Sigma - \Sigma^* = 0.1$, and $\Pi_s = 0.69$ if $\Sigma - \Sigma^* = 0.2$. After calibrating the model we will examine the net effect of RB on the consumption correlations.

3.3 Calibrating the RB Parameter

In this subsection we follow Hansen, Sargent, and Wang (2002) and Hansen and Sargent (2007a) to calibrate the RB parameter (ϑ and Σ). Specifically, we calibrate the model by using the model detection error probability that is based on a statistical theory of model selection (the approach will be precisely defined below). We can then infer what values of the RB parameter ϑ imply reasonable fears of model misspecification for empirically-plausible approximating models. In other words, the model detection error probability is a measure of how far the distorted model can deviate from the approximating model without being discarded; standard significance levels for testing are then used to determine what reasonable fears entail.

3.3.1 The Definition of the Model Detection Error Probability

Let model A denote the approximating model and model B be the distorted model. Define p_A as

$$p_A = \operatorname{Prob}\left(\log\left(\frac{L_A}{L_B}\right) < 0 \middle| A\right), \tag{31}$$

where $\log\left(\frac{L_A}{L_B}\right)$ is the log-likelihood ratio. When model A generates the data, p_A measures the probability that a likelihood ratio test selects model B. In this case, we call p_A the probability of the model detection error. Similarly, when model B generates the data, we can define p_B as

$$p_B = \operatorname{Prob}\left(\log\left(\frac{L_A}{L_B}\right) > 0 \middle| B\right). \tag{32}$$

[13] In the next section, we will calibrate the RB parameter Σ and Σ^* using the detection error probabilities and show that the values of Σ and Σ^* are different.

Following Hansen, Sargent, and Wang (2002) and Hansen and Sargent (2007a), the detection error probability, p, is defined as the average of p_A and p_B:

$$p(\vartheta) = \frac{1}{2}(p_A + p_B), \tag{33}$$

where ϑ is the robustness parameter used to generate model B. Given this definition, we can see that $1-p$ measures the probability that econometricians can distinguish the approximating model from the distorted model. Now we show how to compute the model detection error probability in the RB model.

3.3.2 Calibrating the RB Parameter in the SOE Model

Let's first consider the model with the robustness preference. In the domestic country, under RB, assuming that the approximating model generates the data, the state, s_t, evolves according to the transition law

$$\begin{aligned} s_{t+1} &= Rs_t - c_t + \zeta_{t+1}, \\ &= \frac{1-R\Sigma}{1-\Sigma}s_t + \frac{\Sigma}{1-\Sigma}\bar{c} + \zeta_{t+1}. \end{aligned} \tag{34}$$

In contrast, assuming that the distorted model generates the data, s_t evolves according to

$$\begin{aligned} s_{t+1} &= Rs_t - c_t + \zeta_{t+1} + \omega_\zeta \nu_t, \\ &= s_t + \zeta_{t+1}. \end{aligned} \tag{35}$$

In order to compute p_A and p_B, we use the following procedure:

1. Simulate $\{s_t\}_{t=0}^T$ using (34) and (35) a large number of times. The number of periods used in the simulation, T, is set to be the actual length of the data for each individual country.
2. Count the number of times that $\log\left(\frac{L_A}{L_B}\right) < 0 \Big| A$ and $\log\left(\frac{L_A}{L_B}\right) > 0 \Big| B$ are each satisfied.
3. Determine p_A and p_B as the fractions of realizations for which $\log\left(\frac{L_A}{L_B}\right) < 0 \Big| A$ and $\log\left(\frac{L_A}{L_B}\right) > 0 \Big| B$, respectively.

In practice, given Σ, to simulate the $\{s_t\}_{t=0}^T$ we need to know a) the volatility of ζ_t in (34) and (35), and b) the value of $\bar{c}$. For a), we can compute it from $\mathrm{sd}(\zeta) = \frac{\sqrt{1-\rho^2}}{R-\rho}\mathrm{sd}(y)$, where $\mathrm{sd}(y)$ is the standard deviation of the net output.[14] For b), we use the local coefficient for relative risk aversion $\gamma = -\frac{u''(c)c}{u'(c)} = \frac{c}{\bar{c}-c}$ to recover $\bar{c} = \left(1+\frac{1}{\gamma}\right)c$ where c is mean consumption. Here we set $\gamma = 2$ as the benchmark case. The gross interest rate R is set to be 1.04 in all calibrations. For the average country in the international organization, we can use the same procedure to calibrate the RB parameter.

[14]Net output is defined as $GDP - I - G$ where I and G are investment and government expenditure, respectively.

3.4 Data

To implement the calibration procedure we describe in the previous section, we have to first define our empirical counterpart of the domestic country and the rest of the world (ROW) in the model. Then we can compute the required input statistics from the data used in the calibration process.

First, we define ROW as the average of G-7.[15] Second, we choose one of the relatively small countries in G-7 as the domestic country. In practice, we have chosen 5 different countries, Canada, Italy, UK, France and Germany as the domestic country and report the results in the next section.[16] For example, if we choose Canada as the domestic country, the domestic income, y_t, is the net output of Canada at time t, and the average income of the risk sharing group, y_t^*, corresponds to the average level of net output for G-7 countries at time t.

The annual data we use come from World Development Indicators.[17] The data cover the period from year 1970 to year 2006 which are the common periods of the variables that we use in the data set for different countries. Net output (y) is constructed as $GDP - I - G$, where I is Gross Fixed Capital Formation and G is General Government Final Consumption Expenditure. Consumption (c) in defined as Household Final Consumption Expenditure, and the risk-free bond in the model (b) corresponds to Net Foreign Assets. All the variables are measured in the US currency of year 2000. We apply the HP filter to the time series before computing the statistics.

3.5 Main Findings

Table 1 reports the calibration results and some key data statistics (the stochastic properties of net output and consumption and net output correlations across countries) for the five countries we set as the domestic country in turn. In the calibration exercise, following Hansen and Sargent (2007), we set the detection error probability, p, to be a plausible value, 10% (i.e., with 90% probability consumers can distinguish the approximating model from the distorted model). The last column in Table 1 reports the model misspecification parameter ϑ relative to the one estimated for Canada.[18]

[15] In this paper we follow Crucini (1999) to use G-7 countries. Pakko (1998) used 15 OECD countries to study cross-country correlations. In one of our robustness checks, we also use an alternative definition which defines ROW as the average of the G-7 excluding the country we choose as the domestic country. For instance, when we choose Canada as the domestic country, we use the other six countries in G-7 to define ROW. However, we find this alternative definition does not alter our main findings.

[16] We do not model the two largest countries in G-7, US and Japan, as the domestic country because they are too large to be modeled as small open economies.

[17] For the US data, since the consumption variable (Household Final Consumption Expenditure) contains missing values, we replace it with the Personal Consumption Expenditure (CEA) from the FAME economic databases maintained by Board of Governors of Federal Reserve System.

[18] As shown in section 2.2, consumption in RE models follows a random walk process which does not allow us to explicitly write down the expression for $\mathrm{corr}\,(c, c^*)$ in that case. Thus, we approximate the value of $\mathrm{corr}\,(c, c^*)$ by choosing a value of Σ extremely close to zero to approximate the predictions of the RE model.

Given these estimation and calibration results, Table 2 compares the implications for the consumption correlations between the RE and RB models. Our key result here is that RB improves the performance of the model in terms of the cross-correlations of consumption; at the estimated Σ each country displays a lower correlation with the foreign aggregate. Furthermore, the cross-correlations of consumption are uniformly below the income correlations under RB, whereas the opposite holds for RE. Quantitatively, however, the improvements are relatively small for most of the countries we consider here. One exception is the United Kingdom; however, since the UK had the largest deviation under RE, even the relatively large improvement still leaves the RB model with a lot of ground to cover to reconcile theory and data. There are two clear patterns apparent in Tables 1 and 2. The reduction in the consumption correlation is decreasing in Π_s and increasing in Σ. The large improvement in the UK correlation is therefore due to the unusually low value of Π_s and unusually high value of Σ. (We will provide more explanations on this in the next paragraph.) These findings are consistent with our theoretical results obtained in Section 3.2.

`[Insert Tables 1-2 Here]`

To examine whether the findings are robust, we do sensitivity analysis by using different values of the detection error probabilities (p). Specifically, when p is set to 5% and 15%, Tables 3 and 4 clearly show that our main findings that the presence of RB reduces consumption correlations across countries and the prediction that consumption correlations under RB are consistently lower than the corresponding income correlations is robust to different values of the detection error probability. In addition, it is also clear from the tables that as p varies from 10% to 5% or 15%, RB only has a small impact on $\operatorname{corr}(c_t, c_t^*)$. For example, when p falls from 10% to 5%, $\operatorname{corr}(c_t, c_t^*)$ decreases from 0.323 to 0.322 in Canada; from 0.502 to 0.501 in Italy; and from 0.454 to 0.452 in the UK.

As p decreases, it generally leads to a lower calibrated ϑ (and ϑ^*) and a higher calibrated Σ (and Σ^*); this combination generates two competing effects on consumption correlations. First, the increase of Σ^* will increase Π_s (as we can see from Figure 2), so consumption correlations will increase. Second, reducing p not only increases Σ^* but also increases the difference $\Sigma - \Sigma^*$ (which means the increase of Σ is more than that of Σ^* as shown in Table 1). This increase of $\Sigma - \Sigma^*$ decreases the consumption correlation, as we can see from Figure 2 as well. Hence, these two offsetting effects imply that consumption correlations do not change much as p varies.

`[Insert Tables 3-4 Here]`

4 The RB Model with Imperfect State Observation

Since the full-information model with RB failed to produce a sufficiently-large decrease in the consumption correlations across the board, we modify the model to consider state uncertainty. We now turn to investigating the impact of state uncertainty due to imperfect observations (SU) on the consumption correlations, and will show that it can further reduce them by amplifying the effect of RB on model uncertainty.

4.1 State Uncertainty due to Imperfect Observations

We assume that consumers in the model economy cannot observe the true state s_t perfectly and only observes the noisy signal

$$s_t^* = s_t + \xi_t, \tag{36}$$

when making decisions, where ξ_t is the iid Gaussian noise due to imperfect observations. The specification in (36) is standard in the signal extraction literature and captures the situation where agents happen or choose to have imperfect knowledge of the underlying shocks.[19] Since imperfect observations on the state lead to welfare losses, agents use the processed information to estimate the true state.[20] Specifically, we assume that households use the Kalman filter to update the perceived state $\widehat{s}_t = E_t[s_t]$ after observing new signals in the steady state in which the conditional variance of s_t, $\Sigma_t = \text{var}_t[s_t]$, has converged to a constant Σ:

$$\widehat{s}_{t+1} = (1-\theta)(R\widehat{s}_t - c_t) + \theta(s_{t+1} + \xi_{t+1}), \tag{37}$$

where θ is the Kalman gain (i.e., the observation weight).[21] Note that in the signal extraction problem, the Kalman gain can be written as

$$\theta = \Sigma\Lambda^{-1}, \tag{38}$$

where Σ is the steady state value of the conditional variance of a_{t+1}, $\text{var}_{t+1}[a_{t+1}]$, and $\Lambda = \text{var}_t[\xi_{t+1}]$ is the variance of the noise. Σ and Λ are linked by the following updating equation for the conditional variance in the steady state:

$$\Lambda^{-1} = \Sigma^{-1} - \Psi^{-1}, \tag{39}$$

[19] For example, Muth (1960), Lucas (1972), Morris and Shin (2002), and Angeletos and La'O (2009). It is worth noting that this assumption is also consistent with the rational inattention idea that ordinary people only devote finite information-processing capacity to processing financial information and thus cannot observe the states perfectly.

[20] See Luo (2008) for details about the welfare losses due to information imperfections within the partial equilibrium permanent income hypothesis framework.

[21] Note that θ measures how much uncertainty about the state can be removed upon receiving the new signals about the state.

where Ψ is the steady state value of the ex ante conditional variance of a_{t+1}, $\Psi_t = \text{var}_t\left[a_{t+1}\right]$. Multiplying ω_ζ^2 on both sides of (39) and using the fact that $\Psi = R^2\Sigma + \omega_\zeta^2$, we have

$$\omega_\zeta^2\Lambda^{-1} = \omega_\zeta^2\Sigma^{-1} - \left[R^2\left(\omega_\zeta^2\Sigma^{-1}\right)^{-1} + 1\right]^{-1}, \tag{40}$$

where $\omega_\zeta^2\Sigma^{-1} = \left(\omega_\zeta^2\Lambda^{-1}\right)\left(\Lambda\Sigma^{-1}\right)$. Define the signal-to-noise ratio (SNR) at $\pi = \omega_\zeta^2\Lambda^{-1}$. We obtain the following equality linking SNR (π) and the Kalman gain (θ):

$$\pi = \theta\left(\frac{1}{1-\theta} - R^2\right). \tag{41}$$

Solving for θ yields

$$\theta = \frac{-\left(1+\pi\right) + \sqrt{\left(1+\pi\right)^2 + 4R^2\left(\pi + R^2\right)}}{2R^2}, \tag{42}$$

where we omit the negative values of θ because both Σ and Λ must be positive. Note that given π, we can pin down Λ using $\pi = \omega_\zeta^2\Lambda^{-1}$ and Σ using (38) and (42).

Combining (6) with (37), we obtain the following equation governing the perceived state $\widehat{s}_t$:

$$\widehat{s}_{t+1} = R\widehat{s}_t - c_t + \eta_{t+1}, \tag{43}$$

where

$$\eta_{t+1} = \theta R\left(s_t - \widehat{s}_t\right) + \theta\left(\zeta_{t+1} + \xi_{t+1}\right) \tag{44}$$

is the innovation to the mean of the distribution of perceived permanent income,

$$s_t - \widehat{s}_t = \frac{\left(1-\theta\right)\zeta_t}{1-(1-\theta)R\cdot L} - \frac{\theta\xi_t}{1-(1-\theta)R\cdot L} \tag{45}$$

is the estimation error, and $E_t\left[\eta_{t+1}\right] = 0$. Note that η_{t+1} can be rewritten as

$$\eta_{t+1} = \theta\left[\left(\frac{\zeta_{t+1}}{1-(1-\theta)R\cdot L}\right) + \left(\xi_{t+1} - \frac{\theta R\xi_t}{1-(1-\theta)R\cdot L}\right)\right], \tag{46}$$

where $\omega_\xi^2 = \text{var}\left[\xi_{t+1}\right] = \frac{1}{\theta}\frac{1}{1/(1-\theta)-R^2}\omega_\zeta^2$. Expression (46) clearly shows that the estimation error reacts to the fundamental shock positively, while it reacts to the noise shock negatively. In addition, the importance of the estimation error is decreasing with θ. More specifically, as θ increases, the first term in (46) becomes less important because $\left(1-\theta\right)\zeta_t$ in the numerator decreases, and the second term also becomes less important because the importance of ξ_t decreases as θ increases.[22]

[22] Note that when $\theta = 1$, $var\left[\xi_{t+1}\right] = 0$.

4.2 The RB-SU Version of the SOE Model

To introduce robustness into this model, we assume that the agent thinks that (43) is the approximating model for the true model that governs the data but that he cannot specify. Following Hansen and Sargent (2007a), we surround (43) with a set of alternative models to represent his preference for robustness:

$$\widehat{s}_{t+1} = R\widehat{s}_t - c_t + \omega_\eta \nu_t + \eta_{t+1}. \tag{47}$$

Under SU the innovation η_{t+1} that the agent distrusts is composed of two MA(∞) processes and includes the entire history of the exogenous income shock and the endogenous noise, $\{\zeta_{t+1}, \zeta_t, \cdots, \zeta_0; \xi_{t+1}, \xi_t, \cdots, \xi$
Following Hansen and Sargent (2007a) and Luo and Young (2010), the robust PIH problem with imperfect state observation can be written as

$$\widehat{v}\left(\widehat{s}_t\right) = \max_{c_t} \min_{\nu_t} \left\{ -\frac{1}{2}\left(c_t - \overline{c}\right)^2 + \beta E_t\left[\vartheta \nu_t^2 + \widehat{v}\left(\widehat{s}_{t+1}\right)\right] \right\}, \tag{48}$$

subject to (47) and (46), and $s_0 \sim N\left(\widehat{s}_0, \sigma^2\right)$ is fixed. (48) is a standard dynamic programming problem. The following proposition summarizes the solution to the RB model with imperfect state observation.

Proposition 4 *Given ϑ and θ, the consumption function under RB and SU is*

$$c_t = \frac{R-1}{1-\widetilde{\Sigma}}\widehat{s}_t - \frac{\widetilde{\Sigma}\overline{c}}{1-\widetilde{\Sigma}}, \tag{49}$$

the mean of the worst-case shock is

$$\omega_\eta \nu_t = \frac{(R-1)\widetilde{\Sigma}}{1-\widetilde{\Sigma}}\widehat{s}_t - \frac{\widetilde{\Sigma}}{1-\widetilde{\Sigma}}\overline{c}, \tag{50}$$

and $\widehat{s}_t$ is governed by

$$\widehat{s}_{t+1} = \rho_s \widehat{s}_t + \eta_{t+1}. \tag{51}$$

where $\rho_s = \frac{1-R\widetilde{\Sigma}}{1-\widetilde{\Sigma}} \in (0,1)$,

$$\widetilde{\Sigma} = R\omega_\eta^2/\left(2\vartheta\right) = \frac{\theta}{1-\left(1-\theta\right)R^2}\Sigma > \Sigma, \tag{52}$$

$$\omega_\eta^2 = \mathrm{var}\left[\eta_{t+1}\right] = \frac{\theta}{1-\left(1-\theta\right)R^2}\omega_\zeta^2 > \omega_\zeta^2, \text{ for } \theta < 1. \tag{53}$$

[23]The RB-SU model proposed in this paper encompasses the hidden state model discussed in Hansen, Sargent, and Wang (2002), Hansen and Sargent (2007b), and Hansen, Mayer, and Sargent (2010); the main difference is that none of the states in the RB-SU model are perfectly observable (or controllable).

Proof. See Appendix 6.2. ■

It is clear from (49)-(53) that RB and SU affect the consumption function via two channels in the model: (1) the marginal propensity to consume (MPC) out of the perceived state $\left(\frac{R-1}{1-\widetilde{\Sigma}}\right)$ and (2) the dynamics of the perceived state $(\widehat{s}_t)$. Given $\widehat{s}_t$, stronger degrees of SU and RB increase the value of $\widetilde{\Sigma}$, which increases the MPC. Furthermore, from (52) and (53), we can see that imperfect state observation can amplify the importance of model uncertainty measured by $\widetilde{\Sigma}$ in determining consumption and precautionary savings.

In the RB-SU model individual dynamics are not identical to aggregate dynamics. Combining (43) with (49) yields the change in individual consumption in the RB-SU economy:

$$\Delta c_t = \frac{(1-R)\,\widetilde{\Sigma}}{1-\widetilde{\Sigma}}\left(c_{t-1}-\overline{c}\right) + \frac{R-1}{1-\widetilde{\Sigma}}\left(\frac{\theta\zeta_t}{1-(1-\theta)R\cdot L} + \theta\left(\xi_t - \frac{\theta R\xi_{t-1}}{1-(1-\theta)R\cdot L}\right)\right), \tag{54}$$

where L is the lag operator and we assume that $(1-\theta)\,R < 1$.[24] This expression shows that consumption growth is a weighted average of all past permanent income and noise shocks. Since it permits exact aggregation, we can obtain the change in aggregate consumption as

$$\Delta c_t = \frac{(1-R)\,\widetilde{\Sigma}}{1-\widetilde{\Sigma}}\left(c_{t-1}-\overline{c}\right) + \frac{R-1}{1-\widetilde{\Sigma}}\left(\frac{\theta\zeta_t}{1-(1-\theta)R\cdot L} + \theta\left(\overline{\xi}_t E^i\left[\xi_t\right] - \frac{\theta R\overline{\xi}_{t-1}}{1-(1-\theta)R\cdot L}\right)\right), \tag{55}$$

where i denotes a particular individual, $E^i\left[\cdot\right]$ is the population average, and $\overline{\xi}_t = E^i\left[\xi_t\right]$ is the common component of the noise.[25] This expression shows that even if every consumer only faces the common shock ζ, the economy with imperfect state observation still has heterogeneity since each consumer may face the idiosyncratic noise.[26] Assume that ξ_t consists of two independent noises: $\xi_t = \overline{\xi}_t + \xi_t^i$, where $\overline{\xi}_t = E^i\left[\xi_t\right]$ and ξ_t^i are the common and idiosyncratic components of the error generated by ζ_t, respectively. A single parameter,

$$\lambda = \frac{\text{var}\left[\overline{\xi}_t\right]}{\text{var}\left[\xi_t\right]} \in [0,1],$$

can be used to measure the common source of noisy information on the aggregate component (or the relative importance of $\overline{\xi}_t$ vs. ξ_t).[27] Note that (55) can be written in the following AR(1) process:

[24] This assumption is innocuous, since it is weaker than the condition needed for convergence of the filter (it requires that $\kappa > \frac{1}{2}\log(R) \approx \frac{R-1}{2}$). The condition implies that consumption is responsive enough to the state to 'zero out' the effect of the explosive root in the Euler equation; see Sims (2003).

[25] For simplicity, here we use the same notation c for aggregate consumption.

[26] It can also be rationalized by the fact that the randomness in an individual's response to aggregate shocks will be idiosyncratic because it arises from the individual's own information-processing constraint, see Sims (2003) for a discussion.

[27] The special case that $\lambda = 1$ can be viewed as a representative agent model in which the aggregation issue does not arise.

$$c_t = \rho_s c_{t-1} + \frac{(R-1)\,\widetilde{\Sigma}\overline{c}}{1-\widetilde{\Sigma}} + \frac{R-1}{1-\widetilde{\Sigma}}\left(\frac{\theta\zeta_t}{1-(1-\theta)R\cdot L} + \theta\left(\overline{\xi}_t - \frac{\theta R\overline{\xi}_{t-1}}{1-(1-\theta)R\cdot L}\right)\right), \tag{56}$$

which can be written as the following ARMA$(2,1)$ process:

$$\left(1-\phi_1\cdot L-\phi_2\cdot L^2\right)c_t = \theta\frac{R-1}{1-\widetilde{\Sigma}}\left(\zeta_t+\overline{\xi}_t - R\overline{\xi}_{t-1}\right), \tag{57}$$

where

$$\phi_1 = \rho_s + \rho_\theta = \rho_s + (1-\theta)R,$$
$$\phi_2 = -\rho_s\rho_\theta = -\rho_s(1-\theta)R.$$

Figure 1 also shows how SU can help generate the smooth and hump-shaped impulse response of consumption to the income shock, which, as argued in Sims (2003) and Reis (2006), fits the VAR evidence better. Similarly, in the rest of the world, we have

$$\left(1-\phi_1^*\cdot L-\phi_2^*\cdot L^2\right)c_t^* = \theta^*\frac{R-1}{1-\widetilde{\Sigma}^*}\left(\zeta_t^*+\overline{\xi_t^*} - R\overline{\xi_t^*}\right), \tag{58}$$

where

$$\phi_1^* = \rho_s^* + \rho_\theta^* = \rho_s^* + (1-\theta^*)R,$$
$$\phi_2^* = -\rho_s^*\rho_\theta^* = -\rho_s^*(1-\theta^*)R.$$

4.3 Robust Consumption Correlations under SU

Given (57) and (58), we have the following proposition about the cross-country consumption correlation under RB and SU:

Proposition 5 *The consumption correlation between the two economies under RB and SU is*

$$\operatorname{corr}\left(c_t,c_t^*\right) = \frac{\operatorname{cov}\left(c_t,c_t^*\right)}{\sqrt{\operatorname{var}\left(c_t\right)\operatorname{var}\left(c_t^*\right)}} = \frac{\Pi}{\Pi_y}\operatorname{corr}\left(y_t,y_t^*\right), \tag{59}$$

where

$$\Pi = \frac{\sum_{k=0}^{\infty}\left\{\left[\sum_{j=0,j\le k}^{k}\left(\rho_\theta^j\rho_s^{k-j}\right)\right]\left[\sum_{j=0,j\le k}^{k}\left(\rho_\theta^{*j}\rho_s^{*k-j}\right)\right]\right\}}{\sqrt{\Xi_1\Xi_2}}, \tag{60}$$

$$\Xi_1 = \frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}+$$

$$\frac{\lambda^2}{(1/(1-\theta)-R^2)\,\theta}\left\{\frac{1}{1-\rho_s^2}+(\theta R)^2\frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\right\},$$

$$\Xi_2 = \frac{(1+\rho_s^*\rho_\theta^*)}{(1-\rho_s^*\rho_\theta^*)\left[(1+\rho_s^*\rho_\theta^*)^2-(\rho_s^*+\rho_\theta^*)^2\right]}+$$

$$\frac{\lambda^{*2}}{(1/(1-\theta^*)-R^2)\,\theta^*}\left\{\frac{1}{1-\rho_s^{*2}}+(\theta^* R)^2\frac{(1+\rho_s^*\rho_\theta^*)}{(1-\rho_s^*\rho_\theta^*)\left[(1+\rho_s^*\rho_\theta^*)^2-(\rho_s^*+\rho_\theta^*)^2\right]}\right\}.$$

Proof. See Appendix 6.4. ■

We assume that the SU parameters are the same in the domestic country and the rest of the world: $\theta^* = \theta$, $\rho_\theta^* = \rho_\theta$, and $\lambda^* = \lambda$.[28] Π will converge to Π_s, (30) defined in Section 3.2, as θ converges to 1. It is worth emphasizing that the presence of endogenous noises, $\overline{\xi}_{t-j}$ and $\overline{\xi}_{t-j}^*$ ($j \geq 0$), in the expressions for the dynamics of aggregate consumption does not affect the covariance between under RB and SU, $\text{cov}(c_t, c_t^*)$, as all noises are iid and are also independent of the exogenous income shocks ($\zeta_{t-j},\ j \geq 0$). Therefore, the presence of the common noise shocks will further reduce the consumption correlations across countries as they increase the variances of both c_t and c_t^*.

4.3.1 A Special Case: No Common Noise or $\lambda = 0$

In the case without common noises ($\lambda = 0$), (59) can thus be reduced to

$$\text{corr}(c_t, c_t^*) = \frac{\Pi}{\Pi_y}\text{corr}(y_t, y_t^*), \tag{61}$$

where

$$\Pi = \frac{\sum_{k=0}^{\infty}\left\{\left[\sum_{j=0,j\leq k}^{k}\left(\rho_\theta^j\rho_s^{k-j}\right)\right]\left[\sum_{j=0,j\leq k}^{k}\left(\rho_\theta^j\rho_s^{*k-j}\right)\right]\right\}}{\sqrt{\frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\frac{(1+\rho_s^*\rho_\theta)}{(1-\rho_s^*\rho_\theta)\left[(1+\rho_s^*\rho_\theta)^2-(\rho_s^*+\rho_\theta)^2\right]}}}. \tag{62}$$

Note that here we have assumed that $\theta^* = \theta$ and $\rho_\theta^* = \rho_\theta$. Figures 3 and 4 illustrate how the interaction between RB and SU

affects consumption correlations across countries in this special case when $\Sigma = 0.7$ and $R = 1.04$. Note that here we use Π to measure to what extent RB and SU can affect the correlation because Π converges to 1 as Σ^* and Σ reduces to 0 and θ increases to 1. They show that given the

[28] It is straightforward to show that allowing for the heterogeneity in θ will further reduce the cross-country correlations by a factor, $\Xi = \sqrt{\frac{[1-((1-\theta^*)R)^2][1-((1-\theta)R)^2]}{[1-(1-\theta)(1-\theta^*)R^2]^2}}$. This case may be of interest, however, since Luo and Young (2009) show that it can imply infrequent updating as in Bacchetta and van Wincoop (2010). Since it lies beyond our purposes here and poses calibration challenges, we leave it for future work.

level of finite capacity measured by θ, the consumption correlation is increasing with the degree of Σ^* (RB in the ROW) and is decreasing with the difference of RB in the two economies, $\Sigma - \Sigma^*$. These results are the same as those obtain the RB model in which $\theta = 1$ (channel capacity, κ, is infinite). In addition, it is also clear that given the Σ^* or $\Sigma - \Sigma^*$, the correlation is decreasing with the degree of observation imperfection (θ). That is, the gradual response of consumption to income shocks due to imperfect observations by itself increases the consumption correlation.

The effect of SU on cross-country consumption correlations in this special case is similar to that of habit formation, because habit formation also leads to slow adjustments in consumption.[29] As shown in Fuhrer and Klein (2006), the presence of habit formation increases the correlation of consumption across countries and the empirical evidence of high consumption correlations might reflect habit persistence rather than common income risks or risk sharing. In addition, this special case can also be compared to the sticky expectations (SE) model. The idea of SE is to relax the assumption that all consumers' expectations are completely updated at every period and assume that only a fraction of the population update their expectations on permanent income and re-optimize in any given period.[30] As shown in Carroll and Slacalek (2006), SE also generates the same predictions for aggregate consumption dynamics as habit formation. Consequently, it is straightforward to show that SE generates the same predictions on international consumption correlations as habit formation and the special case of $\lambda = 0$ do.

4.3.2 General Cases ($\lambda > 0$)

In the general cases in which $\lambda > 0$, the aggregate endogenous noise due to finite capacity plays an important role in determining the consumption correlations. Some recent papers have shown the importance of noise shocks for aggregate fluctuations. For example, Angeletos and La'O (2009) show how dispersed information about the underlying aggregate productivity shock contributes to significant noise in the business cycle and helps explain cyclical variations in observed Solow residuals and labor wedges in the RBC setting. Lorenzoni (2009) examines how demand shocks (noisy news about future aggregate productivity) contribute to business cycles fluctuations in a new Keynesian model. Here we will show that an aggregate noise component can improve the model's predictions on consumption correlation across countries.

To examine the effects of the aggregate noise, λ, on the consumption correlation in the RB-SU model, we set the degree of SU, θ, to be 60% and the strength of RB in the domestic country to be 0.7 in this subsection. Figure 5 illustrates how the consumption correlation is increasing with Σ^* for every given λ, and is decreasing with λ for every given Σ^*. As in the previous section, Figure 6 illustrates how the consumption correlation is decreasing with $\Sigma - \Sigma^*$. (It is also clear that the

[29]See Luo (2008) for a detailed proof for the observational equivalence between this special SU case and habit formation.

[30]Reis (2006) uses "inattentiveness" to characterize the infrequent adjustment behavior of consumers.

correlation is decreasing with λ for any given $\Sigma - \Sigma^*$.) Note that given the SU parameters, the effects of Σ^* and $\Sigma - \Sigma^*$ on the correlation are the same as in the RB model.

The intuition that the consumption correlation is decreasing with λ is as follows. Given the degree of SU (θ), λ has *no* impact on the covariance between the two consumption processes but increases the variance of consumption, which in turn reduces the consumption correlation. It is obvious that SU and RB have the most significant impacts on the cross-country consumption correlation in the representative agent case ($\lambda = 1$) because the impact of the noises due to SU on the variances of consumption that appear in the denominator of (60) is largest in this case.

The effect of θ on the consumption correlation differs in sign for $\lambda = 0$ and λ (sufficiently) positive; Figure 3 shows that the first case yields the correlation as an increasing function of θ, while Table 5 (to be discussed below) shows that the second case has the correlation as a decreasing function of θ. The intuition is that when $\lambda = 0$ there is a missing effect. Aggregate consumption in the model is affected only by the common noise shock $\overline{\xi}$; if $\lambda = 0$ this shock has zero variance, and the variance is increasing in λ. Thus, the effect of increasing θ when $\lambda > 0$ decreases the variances of c and c^* and thus reduces the correlation; if λ is large enough, this effect dominates the positive effect identified above and the correlation is increasing in θ.

4.4 Main Findings

To illustrate the quantitative implications of the RB-SU model on the consumption correlation, we fix the the RB parameter at the same levels we obtain in Section 3.5 and vary the two SU parameters, λ and θ. As in Section 3.5, we set the detection error probability, p, to be a plausible value, 10%. Table 5 reports the implied consumption correlations (between the domestic country and ROW) between the RE, RB, and RB-SU models. There are two interesting observations in the table. First, given the degrees of RB and SU (θ), $\mathrm{corr}\left(c_t, c_t^*\right)$ decreases with the aggregation factor (λ). Second, when λ is positive (even if it is very small, e.g., 0.1 in the table), $\mathrm{corr}\left(c_t, c_t^*\right)$ is decreasing with the degree of inattention (i.e., increasing with θ). The intuition is that when there are common noises, the effect of the noises could dominate the effect of gradual consumption adjustments on cross-country consumption correlations. This contrasts with the results in Section 4.3.1. That is, when there is no common noise, $\mathrm{corr}\left(c_t, c_t^*\right)$ is decreasing with the degree of SU (θ), and the effect of SU on $\mathrm{corr}\left(c_t, c_t^*\right)$ is similar to that of habit formation or sticky expectations.

As we can see from Table 5, for all the countries we consider here, introducing SU into the RB model can make the model better fit the data on consumption correlations at many combinations of the parameter values. For example, for Italy, when $\theta = 60\%$ (60% of the uncertainty is removed upon receiving a new signal about the innovation to permanent income) and $\lambda = 1$, the RB-SU model predicts that $\mathrm{corr}\left(c_t, c_t^*\right) = 0.27$, which is very close to the empirical counterpart, 0.25.[31]

[31] For example, Adam (2005) found $\theta = 40\%$ based on the response of aggregate output to monetary policy shocks. Luo (2008) found that if $\theta = 50\%$, the otherwise standard permanent income model can generate realistic

For France, when $\theta = 90\%$ and $\lambda = 0.5$, the RB-SU model predicts that corr $(c_t, c_t^*) = 0.46$, which exactly matches the empirical counterpart. Note that a small value of θ can be rationalized by examining the welfare effects of finite channel capacity.[32]

To examine whether the findings are robust, we do sensitivity analysis using different values of the detection error probabilities (p) in the calibration. As in the last section, here we also set p to be 5% and 15%. Tables 6 and 7 show that our main findings in the benchmark RB-SU model are very robust; varying the value of p only has tiny effects on the consumption correlations.

`[Insert Tables 5-7 Here]`

5 Conclusion

In this paper we provide further evidence that movements in consumption across countries can be understood easily when viewed through the lens of the permanent income model that incorporates robust decision-making; combined with the results in Luo, Nie, and Young (2011) on the model's ability to capture the dynamics of the current account, we can safely say that the interaction of robustness and imperfect state observation has a role in future open-economy macro studies. The model used here has many virtues – it is analytically tractable (leaving nothing hidden behind numerical computations), it displays precautionary savings, and it resolves the classic *excess sensitivity* and *excess smoothness* puzzles in aggregate consumption. However, it does have some shortcomings, such as reliance on a constant return to savings, linear-quadratic functional forms, and a univariate source of risk.[33] The absence of shocks to the interest rate may be of particular importance, given the results in Neumeyer and Perri (2005) regarding the importance of such disturbances. We are working to relax these limitations currently in order to confront the model with more aspects of small open economy behavior.

6 Appendix (Not for Publication)

6.1 Solving the Robust Model

To solve the Bellman equation (17), we conjecture that

$$v(s_t) = -As_t^2 - Bs_t - C, \tag{63}$$

relative volatility of consumption to labor income.

[32]See Luo and Young (2010) for details about the welfare losses due to imperfect observations in the RB model; they are uniformly small.

[33]We are currently pursuing an extension with capital to verify the robustness of our results.

where A, B, and C are undetermined coefficients. Substituting this guessed value function into the Bellman equation gives

$$-As_t^2 - Bs_t - C = \max_{c_t} \min_{\nu_t} \left\{ -\frac{1}{2} (\bar{c} - c_t)^2 + \beta E_t \left[\vartheta \nu_t^2 - As_{t+1}^2 - Bs_{t+1} - C \right] \right\}. \tag{64}$$

We can do the min and max operations in any order, so we choose to do the minimization first. The first-order condition for ν_t is

$$2\vartheta \nu_t - 2AE_t \left[\omega_\zeta \nu_t + Rs_t - c_t \right] \omega_\zeta - B\omega_\zeta = 0,$$

which means that

$$\nu_t = \frac{B + 2A (Rs_t - c_t)}{2 \left(\vartheta - A\omega_\zeta^2 \right)} \omega_\zeta. \tag{65}$$

Substituting (65) back into (64) gives

$$-As_t^2 - Bs_t - C = \max_{c_t} \left\{ -\frac{1}{2} (\bar{c} - c_t)^2 + \beta E_t \left[\vartheta \left[\frac{B + 2A (Rs_t - c_t)}{2 \left(\vartheta - A\omega_\zeta^2 \right)} \omega_\zeta \right]^2 - As_{t+1}^2 - Bs_{t+1} - C \right] \right\}, \tag{66}$$

where

$$s_{t+1} = Rs_t - c_t + \zeta_{t+1} + \omega_\zeta \nu_t.$$

The first-order condition for c_t is

$$(\bar{c} - c_t) - 2\beta\vartheta \frac{A\omega_\zeta}{\vartheta - A\omega_\zeta^2} \nu_t + 2\beta A \left(1 + \frac{A\omega_\zeta^2}{\vartheta - A\omega_\zeta^2} \right) (Rs_t - c_t + \omega_\zeta \nu_t) + \beta B \left(1 + \frac{A\omega_\zeta^2}{\vartheta - A\omega_\zeta^2} \right) = 0.$$

Using the solution for ν_t the solution for consumption is

$$c_t = \frac{2A\beta R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A} s_t + \frac{\bar{c} \left(1 - A\omega_\zeta^2/\vartheta \right) + \beta B}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}. \tag{67}$$

Substituting the above expressions into the Bellman equation gives

$$
\begin{aligned}
& -As_t^2 - Bs_t - C \\
&= -\frac{1}{2}\left(\frac{2A\beta R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}s_t + \frac{-2\beta A\bar{c} + \beta B}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}\right)^2 \\
&+ \frac{\beta\vartheta\omega_\zeta^2}{\left(2\left(\vartheta - A\omega_\zeta^2\right)\right)^2}\left(\frac{2AR\left(1 - A\omega_\zeta^2/\vartheta\right)}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}s_t + B - \frac{2\bar{c}\left(1 - A\omega_\zeta^2/\vartheta\right)A + 2\beta AB}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}\right)^2 \\
&- \beta A\left(\left(\frac{R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}s_t - \frac{-B\omega_\zeta^2/\vartheta + 2c + 2B\beta}{2\left(1 - A\omega_\zeta^2/\vartheta + 2\beta A\right)}\right)^2 + \omega_\zeta^2\right) \\
&- \beta B\left(\frac{R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}s_t - \frac{-B\omega_\zeta^2/\vartheta + 2c + 2B\beta}{2\left(1 - A\omega_\zeta^2/\vartheta + 2\beta A\right)}\right) - \beta C.
\end{aligned}
$$

Given $\beta R = 1$, collecting and matching terms, the constant coefficients turn out to be

$$A = \frac{R(R-1)}{2 - R\omega_\zeta^2/\vartheta}, \tag{68}$$

$$B = -\frac{R\bar{c}}{1 - R\omega_\zeta^2/(2\vartheta)}, \tag{69}$$

$$C = \frac{R\omega_\zeta^2}{2\left(1 - R\omega_\zeta^2/2\vartheta\right)} + \frac{R\bar{c}^2}{2\left(1 - R\omega_\zeta^2/2\vartheta\right)(R-1)}. \tag{70}$$

Substituting (68) and (69) into (67) yields the consumption function (19) in the text.

We impose parameter restrictions so that $A > 0$, implying the value function is concave; these restrictions amount to requiring that ϑ not be too small and are shown in the text to imply $\Sigma < 1$.

6.2 Solving the Robust SU Model

To solve the Bellman equation (48) subject to 47, we conjecture that

$$v(\widehat{s}_t) = -C - B\widehat{s}_t - A\widehat{s}_t^2, \tag{71}$$

where A, B, and C are undetermined coefficients. The detailed procedure is similar to that in Appendix 6.1. Here we only need to replace ω_ζ^2 with ω_η^2 in the constant terms obtained in (68), (69), and (70):

$$A = \frac{R(R-1)}{2 - R\omega_\eta^2/\vartheta}, \tag{72}$$

$$B = -\frac{R\bar{c}}{1 - R\omega_\eta^2/(2\vartheta)}, \tag{73}$$

$$C = \frac{R\omega_\eta^2}{2\left(1 - R\omega_\eta^2/2\vartheta\right)} + \frac{R\bar{c}^2}{2\left(1 - R\omega_\eta^2/2\vartheta\right)(R-1)}. \tag{74}$$

Using these coefficients, we can obtained the consumption function (49) and the worst possible rule (50) in the text.

6.3 Deriving International Consumption Correlations under RB

Given the AR(1) expressions for c_t and c_t^*, (27) and (28), the consumption correlation between the home country and the rest of the world (ROW) can be written as:

$$c_t = \rho_s c_{t-1} + \frac{(R-1)\Sigma\bar{c}}{1-\Sigma} + \frac{R-1}{1-\Sigma}\zeta_t, \tag{75}$$

$$c_t^* = \rho_s^* c_{t-1}^* + \frac{(R-1)\Sigma^*\bar{c}}{1-\Sigma^*} + \frac{R-1}{1-\Sigma^*}\zeta_t^*, \tag{76}$$

$$\begin{aligned}
&\operatorname{corr}(c_t, c_t^*) \\
&= \frac{\operatorname{cov}(c_t, c_t^*)}{\sqrt{\operatorname{var}(c_t)\operatorname{var}(c_t^*)}}, \\
&= \frac{\sqrt{(1-\rho_s^2)(1-\rho_s^{*2})}}{1-\rho_s\rho_s^*}\frac{\operatorname{cov}(\zeta_t, \zeta_t^*)}{\omega_\zeta\omega_{\zeta^*}},
\end{aligned}$$

which is just (29) in the main text. Note that here we use the following facts:

$$\begin{aligned}
\operatorname{var}(c_t) &= \left(\frac{R-1}{1-\Sigma}\right)^2 \frac{\omega_\zeta^2}{1-\rho_s^2}; \\
\operatorname{var}(c_t^*) &= \left(\frac{R-1}{1-\Sigma^*}\right)^2 \frac{\omega_\zeta^2}{1-\rho_s^{*2}}; \\
\operatorname{cov}(c_t, c_t^*) &= \operatorname{cov}\left(\frac{R-1}{1-\Sigma}\frac{\zeta_t}{1-\rho_s\cdot L}, \frac{R-1}{1-\Sigma^*}\frac{\zeta_t^*}{1-\rho_s^*\cdot L}\right) \\
&= \frac{R-1}{1-\Sigma}\frac{R-1}{1-\Sigma^*}\frac{1}{1-\rho_s\rho_s^*}\operatorname{cov}(\zeta_t, \zeta_t^*).
\end{aligned}$$

6.4 Deriving International Consumption Correlations under RB and SU

Given the AR(2) expressions for c_t and c_t^*, (57) and (58), the consumption correlation between the home country and the rest of the world can be written as:

$$
\begin{aligned}
&\operatorname{corr}(c_t, c_t^*) \\
&= \frac{\operatorname{cov}(c_t, c_t^*)}{\sqrt{\operatorname{var}(c_t)\operatorname{var}(c_t^*)}} \\
&= \frac{1 + (\rho_\theta + \rho_s)(\rho_\theta^* + \rho_s^*) + \left(\rho_\theta^2 + \rho_s\rho_\theta + \rho_s^2\right)\left(\rho_\theta^{*2} + \rho_s^*\rho_\theta^* + \rho_s^{*2}\right) + \cdots}{\sqrt{\left(\begin{array}{c} \frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]} \\ +\frac{\lambda^2}{(1/(1-\theta)-R^2)\theta}\left[\frac{1}{1-\rho_s^2} + (\theta R)^2\frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\right]\end{array}\right) \times \left(\begin{array}{c} \frac{\left(1+\rho_s^*\rho_\theta^*\right)}{\left(1-\rho_s^*\rho_\theta^*\right)\left[\left(1+\rho_s^*\rho_\theta^*\right)^2-\left(\rho_s^*+\rho_\theta^*\right)^2\right]} \\ +\frac{\lambda^{*2}}{(1/(1-\theta^*)-R^2)\theta^*}\left[\frac{1}{1-\rho_s^{*2}} + (\theta^* R)^2\frac{\left(1+\rho_s^*\rho_\theta^*\right)}{\left(1-\rho_s^*\rho_\theta^*\right)\left[\left(1+\rho_s^*\rho_\theta^*\right)^2-\left(\rho_s^*+\rho_\theta^*\right)^2\right]}\right]\end{array}\right)}} \frac{\operatorname{cov}(\zeta_t, \zeta_t^*)}{\omega_\zeta \omega_{\zeta^*}} \\
&= \frac{\sum_{k=0}^{\infty}\left\{\left[\sum_{j=0,j\le k}^{k}\left(\rho_\theta^j \rho_s^{k-j}\right)\right]\left[\sum_{j=0,j\le k}^{k}\left(\rho_\theta^{*j}\rho_s^{*k-j}\right)\right]\right\}}{\sqrt{\Xi_1\Xi_2}},
\end{aligned}
$$

which is just (59) in the text. Note that here we use the following facts:

$$
\begin{aligned}
\operatorname{var}(c_t) &= \left(\frac{R-1}{1-\Sigma}\right)^2 \theta^2 \left(\begin{array}{c} \frac{(1+\rho_s\rho_\theta)\omega_\zeta^2}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]} \\ +\frac{\operatorname{var}\left[\xi_t\right]}{1-\rho_s^2} + (\theta R)^2 \frac{(1+\rho_s\rho_\theta)\operatorname{var}\left[\xi_t\right]}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\end{array}\right) \\
&= \left(\frac{R-1}{1-\Sigma}\right)^2 \theta^2 \left(\begin{array}{c} \frac{(1+\rho_s\rho_\theta)\omega_\zeta^2}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]} \\ +\left[\frac{1}{1-\rho_s^2} + (\theta R)^2 \frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\right]\operatorname{var}\left[\xi_t\right]\end{array}\right) \\
&= \left(\frac{R-1}{1-\Sigma}\right)^2 \theta^2 \left(\begin{array}{c} \frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]} \\ +\left[\frac{1}{1-\rho_s^2} + (\theta R)^2 \frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\right]\frac{\lambda^2}{(1/(1-\theta)-R^2)\theta}\end{array}\right)\omega_\zeta^2,
\end{aligned}
$$

$$
\operatorname{var}(c_t^*) = \left(\frac{R-1}{1-\Sigma^*}\right)^2 \theta^{*2} \left(\begin{array}{c} \frac{\left(1+\rho_s^*\rho_\theta^*\right)}{\left(1-\rho_s^*\rho_\theta^*\right)\left[\left(1+\rho_s^*\rho_\theta^*\right)^2-\left(\rho_s^*+\rho_\theta^*\right)^2\right]} \\ +\frac{\lambda^2}{(1/(1-\theta^*)-R^2)\theta^*}\left[\frac{1}{1-\rho_s^{*2}} + (\theta^* R)^2 \frac{\left(1+\rho_s^*\rho_\theta^*\right)}{\left(1-\rho_s^*\rho_\theta^*\right)\left[\left(1+\rho_s^*\rho_\theta^*\right)^2-\left(\rho_s^*+\rho_\theta^*\right)^2\right]}\right]\end{array}\right)\omega_{\zeta^*}^2,
$$

and

$$
\begin{aligned}
\operatorname{cov}\left(c_t, c_t^*\right) &= \operatorname{cov}\left(\frac{R-1}{1-\Sigma}\frac{\theta\left(\zeta_t+\overline{\xi}_t-R\overline{\xi}_{t-1}\right)}{\left(1-\rho_s\cdot L\right)\left(1-(1-\theta)R\cdot L\right)}, \frac{R-1}{1-\Sigma^*}\frac{\theta^*\left(\zeta_t^*+\overline{\xi_t^*}-R\overline{\xi_t^*}\right)}{\left(1-\rho_s^*\cdot L\right)\left(1-(1-\theta^*)R\cdot L\right)}\right) \\
&= \theta\theta^*\frac{R-1}{1-\Sigma}\frac{R-1}{1-\Sigma^*}\operatorname{cov}\left(\frac{\zeta_t}{\left(1-\rho_s\cdot L\right)\left(1-(1-\theta)R\cdot L\right)}, \frac{\zeta_t^*}{\left(1-\rho_s^*\cdot L\right)\left(1-(1-\theta^*)R\cdot L\right)}\right) \\
&= \theta\theta^*\frac{R-1}{1-\Sigma}\frac{R-1}{1-\Sigma^*}\operatorname{cov}\begin{pmatrix} \zeta_t+\left(\rho_\theta+\rho_s\right)\zeta_{t-1}+\left(\rho_\theta^2+\rho_s\rho_\theta+\rho_s^2\right)\zeta_{t-2} \\ +\left(\rho_\theta^3+\rho_s\rho_\theta^2+\rho_s^2\rho_\theta+\rho_s^3\right)\zeta_{t-3}+\cdots, \\ \zeta_t^*+\left(\rho_\theta^*+\rho_s^*\right)\zeta_{t-1}^*+\left(\rho_\theta^{*2}+\rho_s^*\rho_\theta^*+\rho_s^{*2}\right)\zeta_{t-2}^* \\ +\left(\rho_\theta^{*3}+\rho_s^*\rho_\theta^{*2}+\rho_s^{*2}\rho_\theta^*+\rho_s^{*3}\right)\zeta_{t-3}^*+\cdots \end{pmatrix} \\
&= \theta\theta^*\frac{R-1}{1-\Sigma}\frac{R-1}{1-\Sigma^*}\left(1+\left(\rho_\theta+\rho_s\right)\left(\rho_\theta^*+\rho_s^*\right)+\left(\rho_\theta^2+\rho_s\rho_\theta+\rho_s^2\right)\left(\rho_\theta^{*2}+\rho_s^*\rho_\theta^*+\rho_s^{*2}\right)+\cdots\right)\operatorname{cov}\left(\zeta_t, \zeta_t^*\right)
\end{aligned}
$$

References

[1] Angeletos, George-Marios and Jennifer La'O (2009), "Noisy Business Cycles," *NBER Macroeconomics Annual 2009*, 319-378.

[2] Bacchetta, Phillippe and Eric van Wincoop (2010), "Infrequent Portfolio Decisions: A Solution to the Forward Discount Puzzle," *American Economic Review* 100(3), 870-904.

[3] Backus, David K., Patrick J. Kehoe, and Finn E. Kydland (1992), "International Real Business Cycles," *Journal of Political Economy* 84(1), 84-103.

[4] Baxter, Marianne and Mario J. Crucini (1993), "Explaining Saving-Investment Correlations," *American Economic Review* 83(3), 416-436.

[5] Baxter, Marianne and Urban J. Jermann (1997), "The International Diversification Puzzle Is Worse Than You Think," *American Economic Review* 87(1), 170-180.

[6] Canova, Fabio and Ravn, Morten O (1996), "International Consumption Risk Sharing," *International Economic Review* 37(3), 574-601.

[7] Carroll, Christopher D. and Jiri Slacalek (2006), "Sticky Expectations and Consumption Dynamics," mimeo, Johns Hopkins University.

[8] Cook, David (2002), "Robust Control and the Small Open Economy," mimeo, The Hong Kong University of Science and Technology.

[9] Crucini, Mario J. (1999), "On International and National Dimensions of Risk Sharing," *Review of Economics and Statistics* 81(1), 73-84.

[10] Daniel, Betty C. (1997), “Precautionary Saving and Persistent Current Account Imbalance, *Journal of International Economics* 42, 179-193.

[11] Devereux, Michael, Allan Gregory, and Gregor Smith (1992), “Realistic Cross-Country Consumption Correlations in a Two-country, Equilibrium, Business Cycle Model,” *Journal of International Money and Finance* 11(1), 3-16.

[12] Dennis, Richard, Kai Leitemo, and Ulf Söderström (2009), “Monetary Policy in a Small Open Economy with a Preference for Robustness,” Federal Reserve Bank of San Francisco Working Paper 07-04.

[13] Fuhrer, Jeffrey C. and Michael W. Klein (2006), “Risky Habits: On Risk Sharing, Habit Formation, and the Interpretation of International Consumption Correlations,” *Review of International Economics* 14(4), 722-740.

[14] Hansen, Lars Peter and Thomas J. Sargent (1995), “Discounted Linear Exponential Quadratic Gaussian Control,” *IEEE Transactions on Automatic Control* 40, pp. 968-971.

[15] Hansen, Lars Peter and Thomas J. Sargent (2007a), *Robustness*, Princeton University Press.

[16] Hansen, Lars Peter and Thomas J. Sargent (2007b), “Robust Estimation and Control without Commitment,” *Journal of Economic Theory* 136(1), 1-27.

[17] Hansen, Lars Peter, Ricardo Mayer, and Thomas Sargent (2010), “Robust hidden Markov LQG problems,” *Journal of Economic Dynamics and Control*, Elsevier, vol. 34(10), pages 1951-1966

[18] Hansen, Lars Peter, Thomas J. Sargent, and Thomas D. Tallarini, Jr. (1999), “Robust Permanent Income and Pricing,” *Review of Economic Studies* 66(4), 873-907.

[19] Hansen, Lars Peter, Thomas J. Sargent, and Neng Wang (2002), “Robust Permanent Income and Pricing with Filtering,” *Macroeconomic Dynamics* 6(1), 40-84.

[20] Justiniano, Alejandro and Bruce Preston (2010), “Monetary Policy and Uncertainty in an Empirical Small Open Economy Model,” *Journal of Applied Econometrics* 25(1), 93-128.

[21] Kehoe, Patrick and Fabrizio Perri (2002), “International Business Cycles with Endogenous Incomplete Markets,” *Econometrica,* 907-928.

[22] Kollmann, Robert (1996), “Incomplete Asset Markets and the Cross-Country Consumption Correlation Puzzle,” *Journal of Economic Dynamics and Control* 20, 945-962.

[23] Leitemo, Kai and Ulf Söderström (2008), “Robust Monetary Policy in a Small Open Economy,” *Journal of Economic Dynamics and Control* 32 (10), 3218-3252.

[24] Lewis, Karen K. (1996), “What Can Explain the Apparent Lack of International Consumption Risk Sharing?” *Journal of Political Economy* 104, 267-297.

[25] Lorenzoni, Guido (2009), “A Theory of Demand Shocks,” *American Economic Review* 99(5), 2050-2084.

[26] Lucas, Robert E. Jr. (1972), “Expectations and the Neutrality of Money,” *Journal of Economic Theory* 4, 103-24.

[27] Luo, Yulei (2008), “Consumption Dynamics under Information Processing Constraints,” *Review of Economic Dynamics* 11(2), 366-385.

[28] Luo, Yulei and Eric R. Young (2010), “Risk-sensitive Consumption and Savings under Rational Inattention,” *American Economic Journal: Macroeconomics* 2(4), 281-325.

[29] Luo, Yulei and Eric R. Young (2011), “Signal Extraction and Rational Inattention,” manuscript.

[30] Luo, Yulei, Jun Nie, and Eric R. Young (2011), “Robustness, Information-processing Constraints, and the Current Account in Small Open Economies,” manuscript.

[31] Morris, Stephen and Hyun Song Shin (2002), “The Social Value of Public Information,” *American Economic Review* 92, 1521-34.

[32] Muth, John F. (1960), “Optimal Properties of Exponentially Weighted Forecasts,” *Journal of the American Statistical Association* 55: 299-306.

[33] Neumeyer, Pablo A. and Fabrizio Perri (2005), “Business Cycles in Emerging Economies: The Role of Interest Rates, ”*Journal of Monetary Economics* 52(2), 345-380.

[34] Obstfeld, Maurice (1994), “Are Industrial-Country Consumption Risks Globally Diversified?” In Leonardo Leiderman and Assaf Razin, eds., *Capital Mobility: The Impact on Consumption, Investment and Growth.* Cambridge University Press, Cambridge.

[35] Obstfeld, Maurice, and Rogoff, Kenneth (2001), “The Six Major Puzzles in International Macroeconomics: Is There A Common Cause?” *NBER Macroeconomics Annual 2000*, 339-90.

[36] Pakko, Michael R. (1998), “Characterizing Cross-Country Consumption Correlations,” *Review of Economics and Statistics* 80(1), 169-174.

[37] Reis, Ricardo (2006), “Inattentive Consumers,” *Journal of Monetary Economics* 53, 1761-1800.

[38] Sims, Christopher A. (2003), "Implications of Rational Inattention," *Journal of Monetary Economics,* 50, 665-690.

[39] Stockman, Alan and Linda Tesar (1995), "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements," *American Economic Review* 85(1), 168-185.

[40] Tesar, Linda (1993), "International Risk-Sharing and Non-Traded Goods," *Journal of International Economics* 35, 69-89.

[41] Wen, Yi (2007), "By force of demand: Explaining International Comovements," *Journal of Economic Dynamics and Control* 31(1),1-23.

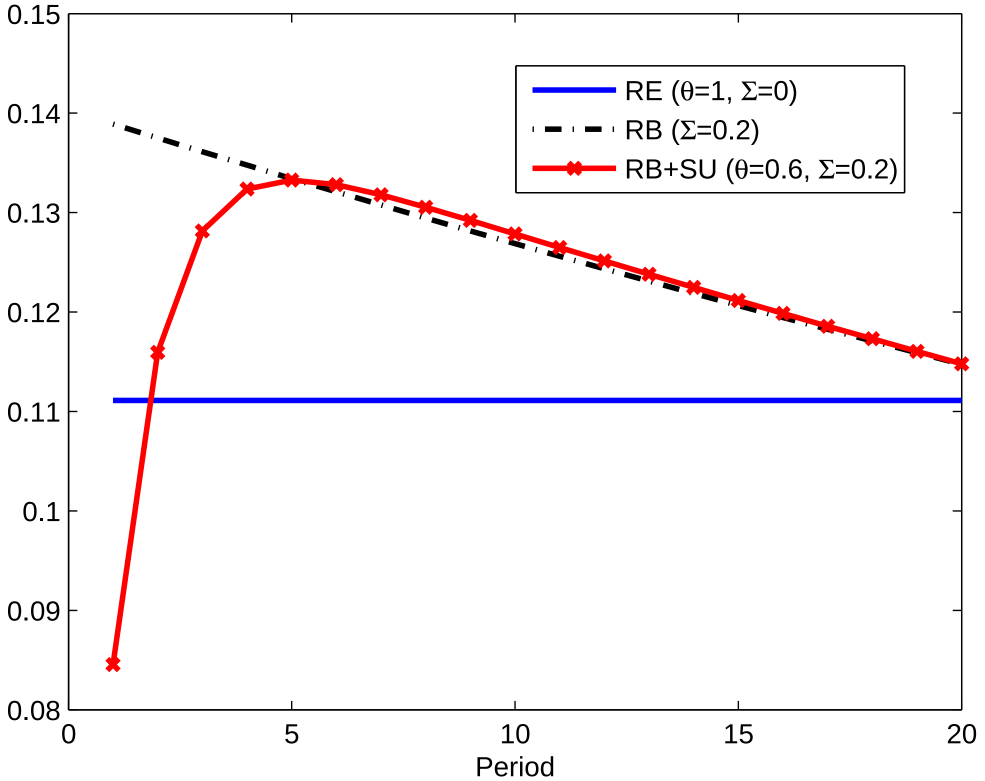

Figure 1: Impulse Responses of Consumption to Income Shocks

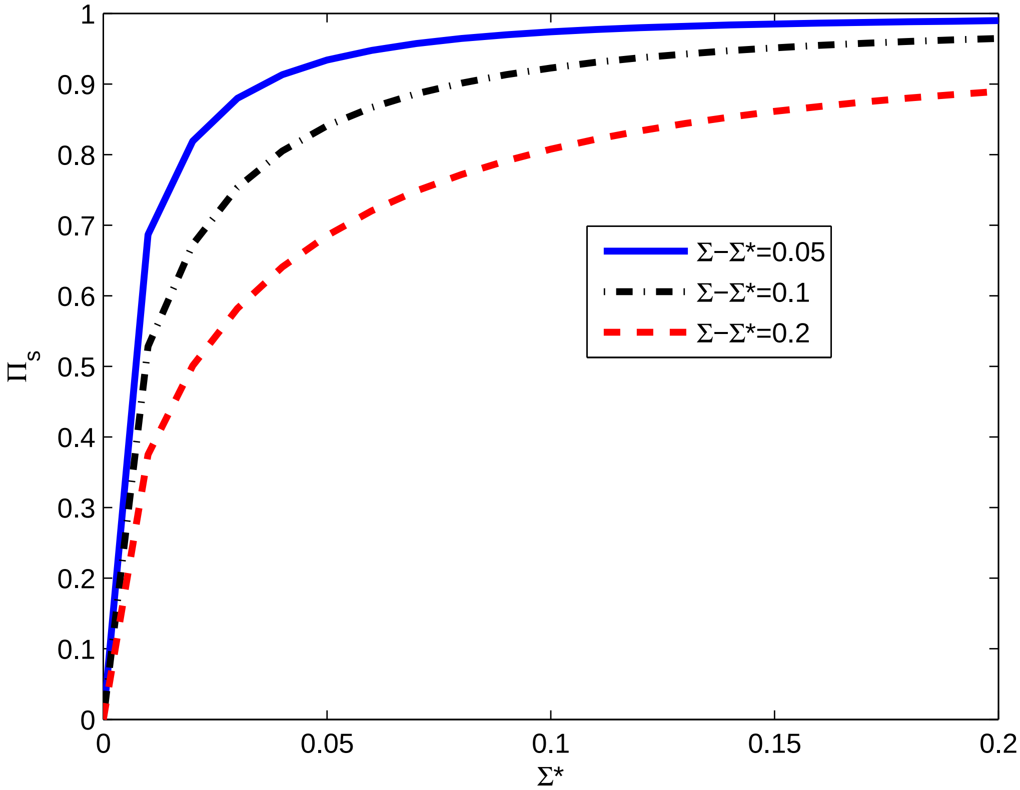

Figure 2: International Consumption Correlation under RB

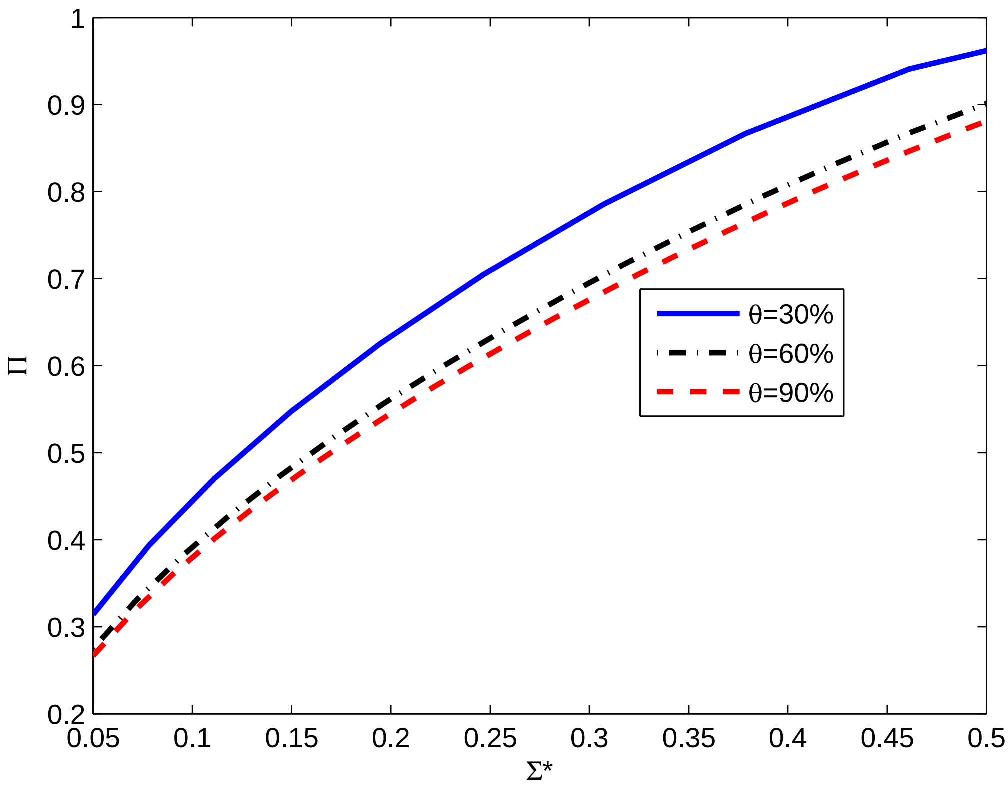

Figure 3: International Consumption Correlation under RB and SU when $\lambda = 0$.

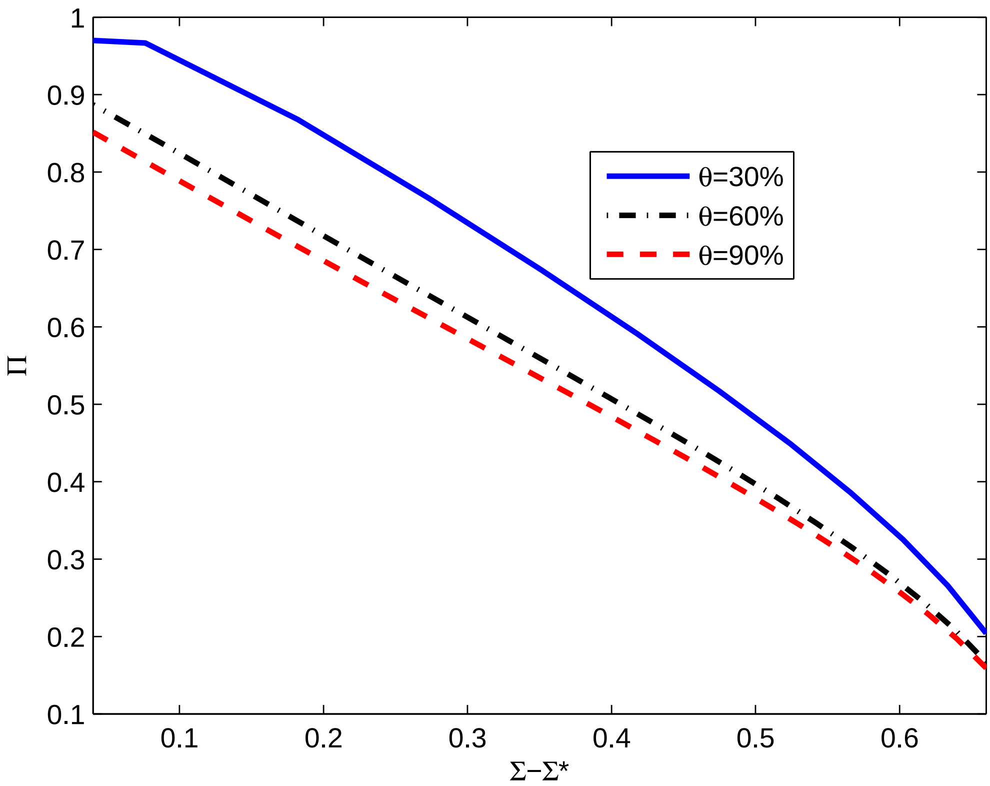

Figure 4: International Consumption Correlation under RB and SU when $\lambda = 0$.

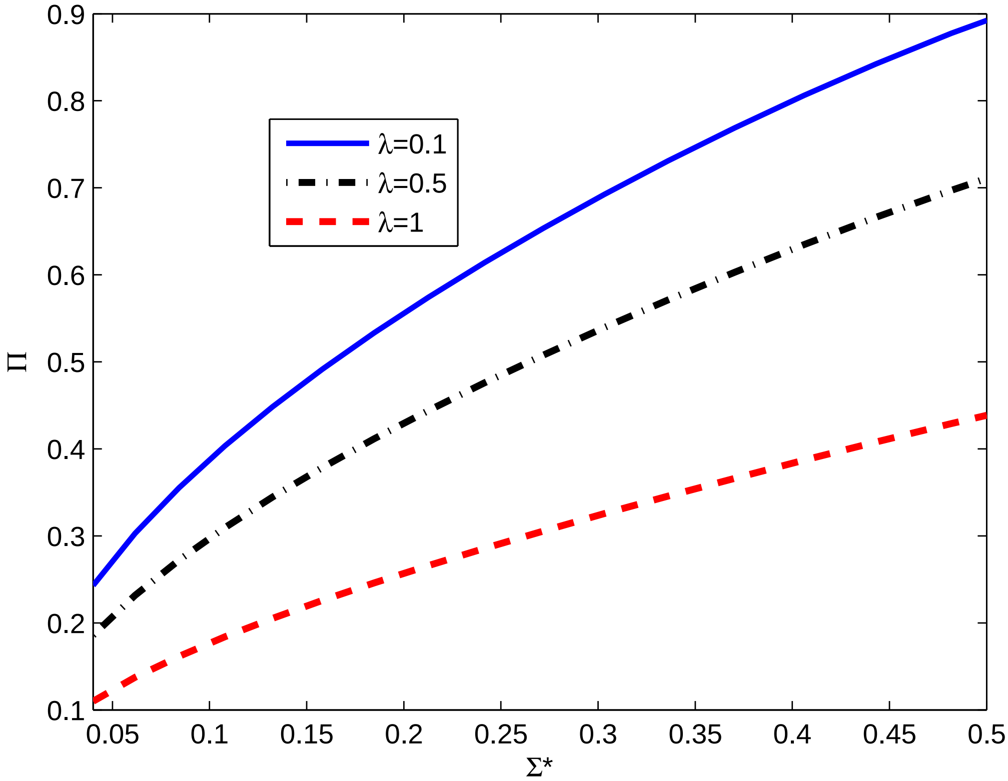

Figure 5: International Consumption Correlation under RB and SU when $\lambda \neq 0$.

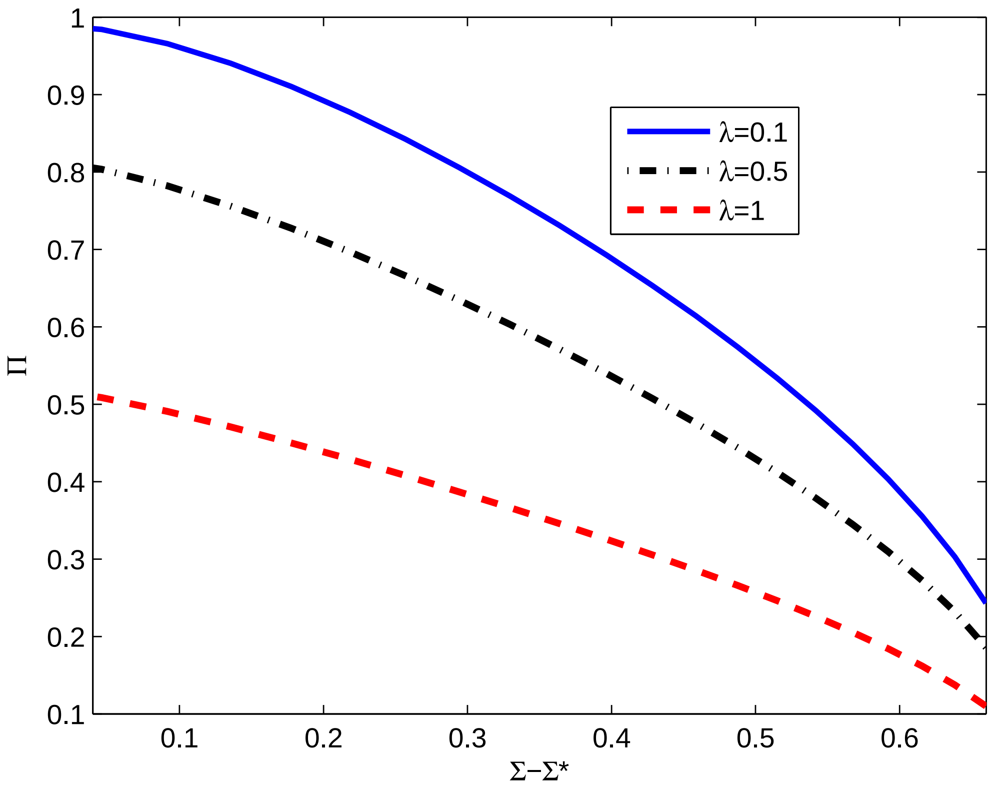

Figure 6: International Consumption Correlation under RB and SU when $\lambda \neq 0$.

Table 1: Estimation and Calibration Results for Different Countries ($p = 0.1$)

	ρ	ρ^*	$\sigma(y)/\sigma(y^*)$	Π_y	Π_s	Σ	Σ^*	$corr(y, y^*)$	$corr(c, c^*)$	ϑ
Canada	0.674	0.581	0.621	0.988	0.785	0.093	0.024	0.403	0.382	1.000
Italy	0.499	0.581	0.582	0.993	0.935	0.050	0.025	0.534	0.246	1.029
UK	0.683	0.581	1.062	0.986	0.666	0.150	0.025	0.683	0.207	1.871
France	0.666	0.581	0.588	0.990	0.950	0.045	0.024	0.509	0.460	1.821
Germany	0.629	0.581	1.031	0.997	0.890	0.061	0.024	0.451	0.040	3.681

Table 2: Comparing the Model Performance ($p = 0.1$)

	Data (corr (y, y^*))	Data (corr (c, c^*))	RE (corr (c, c^*))	RB (corr (c, c^*))
Canada	0.403	0.382	0.408	0.323
Italy	0.534	0.246	0.538	0.502
UK	0.683	0.207	0.693	0.454
France	0.509	0.460	0.514	0.490
Germany	0.451	0.040	0.453	0.402

Table 3: Calibration Results and Model Comparison ($p = 0.05$)

	Π_s	Σ	Σ^*	Data corr (y, y^*)	Data corr (c, c^*)	ϑ	RE corr (c, c^*)	RB corr (c, c^*)
Canada	0.788	0.118	0.031	0.403	0.382	1.000	0.408	0.322
Italy	0.931	0.066	0.031	0.534	0.246	0.995	0.538	0.501
UK	0.652	0.189	0.031	0.683	0.207	1.880	0.693	0.452
France	0.953	0.057	0.031	0.509	0.460	1.823	0.514	0.489
Germany	0.884	0.081	0.031	0.451	0.040	3.534	0.453	0.400

Table 4: Calibration Results and Model Comparison ($p = 0.15$)

	Π_s	Σ	Σ^*	corr (y, y^*)	corr (c, c^*)	ϑ	RE	RB
Canada	0.796	0.077	0.020	0.403	0.382	1.000	0.408	0.325
Italy	0.929	0.042	0.020	0.534	0.246	1.010	0.538	0.500
UK	0.657	0.126	0.020	0.683	0.207	1.829	0.693	0.455
France	0.954	0.036	0.020	0.509	0.460	1.864	0.514	0.490
Germany	0.892	0.051	0.020	0.451	0.040	3.632	0.453	0.404

Table 5: Theoretical corr (c, c^*) from Different Models $(p = 0.1)$

	Data	RE	RB	RB+SU ($\theta = 0.9$)	RB+SU ($\theta = 0.6$)	RB+SU ($\theta = 0.3$)
Canada						
($\lambda = 1$)	0.38	0.41	0.33	0.27	0.17	0.12
($\lambda = 0.5$)	0.38	0.41	0.33	0.31	0.26	0.23
($\lambda = 0.1$)	0.38	0.41	0.33	0.32	0.32	0.32
Italy						
($\lambda = 1$)	0.25	0.54	0.50	0.42	0.27	0.19
($\lambda = 0.5$)	0.25	0.54	0.50	0.48	0.41	0.36
($\lambda = 0.1$)	0.25	0.54	0.50	0.50	0.50	0.49
UK						
($\lambda = 1$)	0.21	0.69	0.45	0.38	0.25	0.17
($\lambda = 0.5$)	0.21	0.69	0.45	0.44	0.38	0.32
($\lambda = 0.1$)	0.21	0.69	0.45	0.46	0.46	0.45
France						
($\lambda = 1$)	0.46	0.51	0.49	0.40	0.26	0.18
($\lambda = 0.5$)	0.46	0.51	0.49	0.46	0.40	0.34
($\lambda = 0.1$)	0.46	0.51	0.49	0.49	0.48	0.48
Germany						
($\lambda = 1$)	0.04	0.45	0.40	0.33	0.22	0.15
($\lambda = 0.5$)	0.04	0.45	0.40	0.38	0.33	0.29
($\lambda = 0.1$)	0.04	0.45	0.40	0.40	0.40	0.40

Table 6: Theoretical corr (c, c^*) from Different Models ($p = 0.05$)

	Data	RE	RB	RB+SU ($\theta = 0.9$)	RB+SU ($\theta = 0.6$)	RB+SU ($\theta = 0.3$)
Canada						
($\lambda = 1$)	0.38	0.41	0.32	0.27	0.17	0.12
($\lambda = 0.5$)	0.38	0.41	0.32	0.31	0.26	0.23
($\lambda = 0.1$)	0.38	0.41	0.32	0.32	0.32	0.31
Italy						
($\lambda = 1$)	0.25	0.54	0.50	0.41	0.27	0.19
($\lambda = 0.5$)	0.25	0.54	0.50	0.47	0.41	0.35
($\lambda = 0.1$)	0.25	0.54	0.50	0.50	0.49	0.49
UK						
($\lambda = 1$)	0.21	0.69	0.45	0.37	0.24	0.17
($\lambda = 0.5$)	0.21	0.69	0.45	0.43	0.37	0.31
($\lambda = 0.1$)	0.21	0.69	0.45	0.45	0.45	0.44
France						
($\lambda = 1$)	0.46	0.51	0.49	0.40	0.26	0.18
($\lambda = 0.5$)	0.46	0.51	0.49	0.46	0.40	0.35
($\lambda = 0.1$)	0.46	0.51	0.49	0.49	0.48	0.48
Germany						
($\lambda = 1$)	0.04	0.45	0.40	0.33	0.22	0.15
($\lambda = 0.5$)	0.04	0.45	0.40	0.38	0.33	0.28
($\lambda = 0.1$)	0.04	0.45	0.40	0.40	0.40	0.39

Table 7: Theoretical corr (c, c^*) from Different Models ($p = 0.15$)

	Data	RE	RB	RB+SU ($\theta = 0.9$)	RB+SU ($\theta = 0.6$)	RB+SU ($\theta = 0.3$)
Canada						
($\lambda = 1$)	0.38	0.41	0.33	0.27	0.17	0.12
($\lambda = 0.5$)	0.38	0.41	0.33	0.31	0.27	0.23
($\lambda = 0.1$)	0.38	0.41	0.33	0.32	0.32	0.32
Italy						
($\lambda = 1$)	0.25	0.54	0.50	0.41	0.27	0.19
($\lambda = 0.5$)	0.25	0.54	0.50	0.47	0.41	0.35
($\lambda = 0.1$)	0.25	0.54	0.50	0.50	0.49	0.49
UK						
($\lambda = 1$)	0.21	0.69	0.45	0.38	0.24	0.17
($\lambda = 0.5$)	0.21	0.69	0.45	0.43	0.37	0.32
($\lambda = 0.1$)	0.21	0.69	0.45	0.46	0.45	0.44
France						
($\lambda = 1$)	0.46	0.51	0.49	0.40	0.26	0.18
($\lambda = 0.5$)	0.46	0.51	0.49	0.46	0.40	0.35
($\lambda = 0.1$)	0.46	0.51	0.49	0.49	0.48	0.48
Germany						
($\lambda = 1$)	0.04	0.45	0.40	0.33	0.22	0.15
($\lambda = 0.5$)	0.04	0.45	0.40	0.38	0.33	0.29
($\lambda = 0.1$)	0.04	0.45	0.40	0.40	0.40	0.40