Status, Intertemporal Choice and Risk-Taking

Debraj Ray
New York University

Arthur Robson
Simon Fraser University

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Abstract

This paper takes the position that an individual's concern for relative status is expressed in her preferences. We embed this hypothesis in an otherwise conventional model of economic growth, and examine its consequences. The model generates two kinds of equilibrium. In the first, all individuals always follow a deterministic consumption and investment strategy. Such a strategy must be linear in output, is independent of payoffs and technology, and *only* depends on the discount factor. This sort of equilibrium exists when the individual cares only about status and the individual production function is convex in investment. In contrast, the other kind of equilibrium involves convergence to a steady state in which there is persistent and endogenously generated gambling. This kind of equilibrium exists in more plausible circumstances. The individual has a utility function that may depend on consumption *per se* as well as implied status and individual production functions are concave. We characterize the unique steady state in such a situation, and prove that any dynamic equilibrium path must converge to it. Risk-taking arises not from the presumption of a utility function with strictly convex segments (as in Friedman-Savage), but naturally from a view of utility as depending on relative status. Our steady state is broadly consistent with the stylized facts that individuals both insure downside risk and to gamble over upside risk, and generates similar patterns of risk-taking and avoidance across environments with quite different overall wealth levels. Finally, in contrast to Friedman (1953), endogenous risk-taking here is generally Pareto-inefficient.

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1. Introduction

The idea that utility or happiness depends, at least in part, on comparing one's own consumption to that of others can be traced back to Veblen (1899) and Duesenberry (1949), and has found expression in the work of a number of prominent scholars.[1] Nevertheless, much of modern economic theory has somewhat stubbornly maintained the presumption that an agent's utility depends solely on her absolute level of consumption. Robert Frank, writing as recently as 2005 in The New York Times observed that

> "Despite Mr. Duesenberry's apparent success, many economists felt uncomfortable with his relative-income hypothesis, which to them seemed more like sociology or psychology than economics . . . In light of abundant evidence that context matters, it seems fair to say that Mr. Duesenberry's theory rests on a more realistic model of human nature than Mr. Friedman's. It has also been more successful in tracking actual spending. And yet, as noted, it is no longer even mentioned in leading textbooks."

This obdurate stance has been harder to maintain in the face of a large body of empirical evidence of an individual's tendency to compare her consumption to that of others (for example, Frank (1985), Clark and Oswald (1996), and Dynan and Ravina (2007)). It is therefore not surprising to see a relatively young but growing literature that includes relative concern as an argument in the utility function in addition to the absolute consumption level.[2]

In line with this literature, we embed relative payoff concerns into an otherwise conventional model of economic growth, and study the intertemporal equilibrium of such a model. The theory we develop provides an abstract account of endogenous risk-taking (even when there is no intrinsic uncertainty), and links such risk-taking to economic inequality.

In the model, there are many agents, each choosing consumption and investment policies in the setting of a growth model. There is no uncertainty, but the agent has unlimited access to (fair) risk-taking opportunities and can exercise them in order to affect her output and consumption. Each agent has a one-period utility function, defined on her rank in the economy-wide cumulative distribution function of consumption, as well as possibly on consumption itself. She maximizes a discounted sum of these one-period utilities.

Unlike the usual growth model, in which every man is an island, or the competitive variant of it, in which each agent interacts with society via market prices for labor and capital, our model exhibits a fundamental interaction across agents via the aggregate *distribution* of consumption at any date. This distribution determines each individual's optimizing choices, while (in equilibrium) these choices also generate the overall distribution.

The main idea we want to explore is simple. If the production function exhibits increasing returns, and so allows economic inequality to persist across agents, then an equilibrium takes

[1]See, e.g., Frank (1985, 1997), Easterlin (2002), Scitovsky (1976), Sen (1973), Schor (1992) among many others.

[2]See, for example, Robson (1992), Cooper, García-Peñalosa and Funk (1998), Corneo and Jeanne (1998), Kockesen, Ok and Sethi (2000), Becker, Murphy and Werning (2005), Hopkins and Kornienko (2004, 2006) and Ray (2006). The bibliographical notes in Section 5 discusses some of this literature.

on a particularly simple, deterministic form. If, however, there is a tendency towards society-wide convergence (as in the canonical Solow or Ramsey models) then that simple equilibrium can no longer exist. Because individuals are partly affected by relative status, the urge to create an environment of risk-taking becomes irresistibly strong when individual wealths bunch together. In this case, our equilibrium involves the periodic creation of consumption inequality via endogenous risk-taking. This observation — that the attainment of economic equality may be hampered by the endogenous taking of risk — is the central theme of the paper.

In the remainder of the Introduction, we describe our results and elaborate further on this general theme. Section 2 sets up the general model we use. Section 3 studies the case of a convex production function, and specializes to a "pure status" model, in which individual utility depends on relative status alone. Proposition 1 identifies an equilibrium in this model with no gambling at any date. The equilibrium is remarkably simple: independently of the form of the utility and production functions, it exhibits a common consumption strategy at every date which is linear in output and only depends on the discount factor. A potentially complex model is thereby reduced to one in which each individual behaves as if she is maximizing a logarithmic utility function, and faces a linear technology of accumulation.

This equilibrium always exists when the agent is concerned with status alone, and the individual production function is weakly convex in investment. Under mild initial conditions, the equilibrium precipitates a strictly *concave* optimization problem for each individual, no matter how convex the production function is. It is also the only deterministic equilibrium in a broad class of deterministic strategies, in which consumption functions are allowed to vary over time and across individuals but must be differentiable and a strict best response (Proposition 2).

This result sets the stage for the main part of the paper (Section 4), in which we assume that production functions are strictly concave. By Proposition 2, a deterministic equilibrium must induce convergence across individual wealths. That convergence dramatically raises the payoff to investment in terms of status, and invariably creates room for profitable deviations from the putative equilibrium. In short, no deterministic equilibrium can exist in this case (Proposition 3). This motivates our study of endogenous risk-taking. Our main argument is that concerns regarding status-seeking must endogenously generate risky behavior, even in a society with no exogenous risk.

To develop this argument, we presume that all fair gambles are freely available: there is a competitive fringe of firms, each equipped with a randomization technology, that stand ready to supply such demands for a zero profit. (A particularly topical interpretation is uninformed trading on the stock market.) Translating our intuition about gambling to a formal analysis is not straightforward, not least because the accumulation policies followed by every individual are themselves endogenous, as are the gambles actually taken. Nevertheless, we prove that when individual production functions are strictly concave, there exists a unique steady state (Propositions 4 and 5) that involves endogenous gambling. In this steady state, all individuals save the same amount, and hence start each period with identical wealth levels. However, individuals then all take the fair bets over consumption that induce a nontrivial distribution of consumption.[3]

A full intertemporal equilibrium from an arbitrary initial wealth distribution can be complex. However, we also show that any such equilibrium path from positive initial wealths must converge to this unique steady state (Proposition 7). In this sense, we can describe fully what our equilibrium must look like, at least in the long run.

A remarkable feature of our equilibrium is that it generates (both in steady state and along the transition) an individual desire to insure against downside risk and an urge to gamble over upside risk. These are precisely the facts that drove Friedman and Savage (1948) to introduce their well-known utility function, with appropriately located concave and convex segments. But our model does not presume a utility function with different local curvatures; rather, it derives similar outcomes from a view of utility as arising from relative status.[4] Indeed, our results are fully consistent with a *strictly concave* utility function defined on consumption and relative status. With the production function concave as well, our setup is quite conventional, with the sole exception of the inclusion of relative status.

Two other distinctive features of risk-taking in our model are worth highlighting. A particular Friedman-Savage utility function might account for for the pattern of risk-taking and risk-avoidance at a particular time, but it will generally fail to work at some later date, after substantial growth in wealth. In a fully dynamic setting, the Friedman-Savage specification (with its exogenously posited curvature of utility) might generate an initial burst of gambling, with none thereafter. Our model, on the other hand, relies on endogenously generated curvature, and will exhibit risk-taking and risk-avoidance over the entire range of wealths, or across environments with different wealth levels.

Second, a central implication of our approach is that equilibrium behavior must coexist with a certain degree of consumption inequality. When that inequality is generated "naturally", as it is with an increasing-returns technology, equilibrium behavior is simple and deterministic. On the other hand, when inequality tends to go away, equilibrium behavior steps in to recreate it by endogenously generating periodic sprees of risk-taking. Paraphrasing Voltaire's famous observation about God, we might say that if inequality did not exist, it would be necessary to invent it.

At the same time, the word "necessary" is not to be confused with "desirable". It is indeed the case that risk-taking in our model is a deliberate act, in the tradition of Friedman (1953), and, more recently, Becker, Murphy and Werning (2005). When bets are taken, we think not only of individuals buying lottery tickets and investing in risky stocks, but also of individuals making risky career decisions. Friedman's view was that such risk-taking is efficient. If an individual endowed with a strictly convex utility function of own wealth

[3]In contrast, when individual production functions are convex, ongoing accumulation creates no tendency for cross-sectional convergence across individuals, and therefore no bunching of consumption. No endogenous risk-taking arises.

[4]In this context, it is of interest that Friedman and Savage also provide an informal status-based motivation for their concave-convex-concave utility function. We discuss this in more detail in the concluding section of our paper. Our model can be viewed as one possible formalization and elaboration of this interpretation of Friedman and Savage.

alone wants to gamble, that gamble must be ex-ante efficient. Friedman therefore argued that some observed inequality could be the efficient product of free choices.

In the relative status world, with consumption externalities, this is no longer true. In this model, steady states with gambling are generally Pareto-inefficient (Proposition 6).

2. Model

2.1. **Feasible Set.** There is a continuum of agents of measure one. Agent i has initial wealth $w(i)$. The cumulative distribution of initial wealth is denoted by G.

Agents consume and produce every period. Wealth at date t for agent i, $w_t(i)$, is divided between a consumption $c_t(i)$ and capital $k_t(i)$:

$$w_t(i) = c_t(i) + k_t(i). \tag{1}$$

Capital creates fresh wealth via a production function f.

$$w_{t+1}(i) = f(k_t(i)). \tag{2}$$

The same production function is available to every agent. We impose

Assumption 1 [fgen]. f is strictly increasing and differentiable, with $f(0) = 0$.

(Throughout, we place labels on assumptions as an easy mnemonic aid.)

To accommodate the possibility of endogenous risk-taking, we presume throughout that everyone has access to a "randomization technology". The technology can convert any fixed contribution into any risky gamble over outcomes, with the same expected value as the original contribution. The outcomes may be consumed or invested, or both. This technology might be a lottery as the term is commonly understood, or it could be a gamble in a more general sense, such as a risky occupation that requires upfront investment. Abstractly, one might think of competitive, risk-neutral "risk providers" standing by to deliver any fair randomization on demand.

2.2. **Status and Utility.** Let F_t be the cumulative distribution of consumption in society at date t. This will be determined "in equilibrium" but a single agent regards it as given. Define, for any $c \geq 0$,

$$\bar{F}_t(c) = \begin{cases} F_t(c) & \text{if } F_t \text{ is continuous at } c \\ \mu F_t^-(c) + (1-\mu)F_t(c) & \text{if } F_t \text{ has an atom at } c \end{cases} \tag{3}$$

where μ is some fixed number strictly between 0 and 1 (think of $\mu = 1/2$ for concreteness), and $F_t^-(c)$ is the left hand limit of F_t at c. We interpret $\bar{F}_t(c)$ as the *status* enjoyed by an individual at date t when she consumes c. It is a purely relative object, free of any absolute consumption considerations.

A general specification of individual utility is one in which both absolute consumption and relative status matter. Write $u = u(c, s)$, where $s \in [0, 1]$ stands for status.

Assumption 2 [ugen]. $u(c,s)$ is continuously differentiable, with $u(0,0) = 0$. It is increasing in s, with $u_s(c,s) > 0$ for all (c,s). It is nondecreasing and concave in c. If $u_c(c,s)$ is strictly positive, then $u_c(c,s)$ declines in c. Finally, $u(c,1)/c \to 0$ as $c \to \infty$.

Note that we do not impose any curvature restriction on utility as a function of status. However, we do require that u be concave in c, strictly so if u is sensitive to c.

A special case covered by the assumption is what we shall call the *pure status* model, in which u does not depend on c at all. In this case, with slight abuse of notation, we shall simply write utility as $u(s)$, where u is some strictly increasing and continuously differentiable function, with $u(0)$ normalized to 0.

2.3. **Optimization.** Given initial wealth w, and a sequence of consumption distributions F_t, an agent's objective is to choose a sequence of (possibly degenerate) fair randomizations of wealth, and an allocation of realized wealth to consumption and investment at every date, to maximize the present discounted value of expected payoffs over time:

$$\sum_{t=0}^{\infty} \delta^t \mathbb{E} u\left(c_t, \bar{F}_t(c_t)\right) \tag{4}$$

where $\delta \in (0,1)$ is the discount factor. The expectations are taken with respect to realizations of the endogenous randomizations at each date; there is no exogenous uncertainty. We reiterate that each agent is free to follow a fully deterministic sequence, if she so chooses. In either case, the constraints (1) and (2) must be respected at every date.

2.4. **Equilibrium.** Fix some initial distribution of wealth G. An *equilibrium* is a sequence of distribution functions $\mathbf{F} \equiv \{F_t\}$ for consumption and $\mathbf{G} \equiv \{G_t\}$ for wealth, as well as a consumption and randomization policy for every individual (possibly varying across individuals and time) such that (i) $G_0 = G$, (ii) each individual policy maximizes expected utility given $\mathbf{F}$, and (iii) at each date, given G_t, the aggregate over individual policies yields F_t and G_{t+1}.

A *steady state* is an equilibrium in which the distributions of consumption and wealth are time-stationary: there are distributions F^* and G^* such that $F_t = F^*$ and $G_t = G^*$ for all t.

3. Deterministic Equilibrium with Convex Production Functions

In this section, we develop the idea that nondecreasing returns to scale in investment, by preventing convergence, permits a remarkably simple equilibrium to exist. In such an equilibrium, individual savings policies depend only on the discount factor, and economic inequality is indefinitely perpetuated.

Consider the following two restrictions:

Assumption 3 [upure]. u has the pure status property: u depends on s alone, $u(s)$ is continuously differentiable, with $u(0) = 0$ and $u'(s) > 0$ for all $s > 0$.

Assumption 4 [fconv]. f is convex.

Observe that the convexity of the production function appears to create an individual optimization problem that is not concave. This point is apparently reinforced by the fact that we have made no assumption about the curvature of u in s. Yet, the equilibrium that we uncover will be seen to precipitate a strictly concave maximization problem for each individual.

In what follows, we also assume

Assumption 5 [init]. The initial distribution G has support $\mathbb{R}_+$, and $u(G(w))$ is continuously differentiable and strictly concave in w.

3.1. **A Simple Deterministic Equilibrium.** A deterministic equilibrium with particularly simple properties always exists, provided that the production function is convex. More precisely, we have

Proposition **1.** *Make Assumptions* 1 [fgen], 3 [upure], 4 [fconv] *and* 5 [init]*. Then there exists an equilibrium in which every individual strictly prefers a deterministic policy, and in which the equilibrium status for any individual is constant over time. In this equilibrium, every individual at every date t uses the policy function*

$$c_t = (1-\delta)w_t, \tag{5}$$

which does not depend on the utility function u nor on the initial distribution of wealth.

Two things are striking about this proposition. First, despite the presumption that f can exhibit increasing returns (to an arbitrary degree), and despite the lack of any restriction on the curvature of utility with respect to status, the equilibrium we obtain induces a strictly concave optimization problem for each individual. This is why — as the proposition observes — in equilibrium, determinism is strictly preferred by each individual to randomization. The convexity of f ensures that the wealth and consumption distributions stay suitably dispersed at every date: the marginal gain from status never becomes high enough to induce risk-taking. The reader will need to study the proof to absorb the details of the argument.

The discussion above does depend on the assumption of a pure status model. If utility were also to depend on absolute consumption, there would be an extra push towards convergence induced by the concavity of the utility function in absolute consumption. However, we conjecture that this effect would be nullified by some threshold degree of convexity in f; in the pure status model, that threshold is simply linearity.

Second, the equilibrium has an extremely simple structure. Equilibrium policy depends neither on the initial distribution of wealth, nor does it depend on the exact form of the utility function or the production function. (The equilibrium *distributions* do.) In fact, the equilibrium policy that we isolate would have been the one followed by an optimizing planner with logarithmic utility defined on absolute consumption and using a linear production technology. What accounts for this structure is the delicate balance achieved across time periods: status matters today, which increases the need for current consumption, but it matters tomorrow as well, which attenuates that need. In equilibrium

— with a convex production function — the two effects nicely cancel in a way that induces a particularly simple equilibrium structure.[5]

3.2. **Uniqueness of the Simple Equilibrium.** It may be argued that Proposition 1 only identifies a particular equilibrium. However, under some mild restrictions, it is the *only* deterministic equilibrium. To describe the restrictions, say that a deterministic equilibrium is *regular* if at each date, some individual strictly prefers her best consumption response at all but possibly a countable number of wealths. It is *smooth* if every individual employs a sequence of differentiable consumption policies $\{c_t^i\}$, with $\epsilon^i \leq c_t^{i\prime}(w) < 1$ at all wealths w and dates t, for some $\epsilon^i > 0$.

Proposition **2.** *Make Assumptions* 1 [fgen], 3 [upure] *and* 5 [init]. *Suppose that a deterministic equilibrium, described by a sequence $\{c_t^i\}$ of consumption functions, is regular and smooth. Then $c_t^i(w) = (1 - \delta)w$ for all i, all t and all w.*

The proposition states that our simple equilibrium is the only deterministic equilibrium in a broad class of policies. It is worth noticing that this proposition is independent of the curvature of f; only Assumption 1 is imposed on the production technology. This observation acts as an entry point into the concave case that we shall discuss in the next section. To see how, consider the following restriction:

Assumption 6 [fbound]. There exists $0 < \hat{k} < \infty$ with $\delta f(k) > k$ for all $k \in (0, \hat{k})$, while $\delta f(k) < k$ for all $k > \hat{k}$.

Proposition **3.** *Make Assumptions* 1 [fgen], 3 [upure], 5 [init] *and* 6 [fbound]. *Then no policy sequence of the form described in Proposition 2 can ever be an equilibrium.*

The proof of this proposition is simple. If such a policy sequence were to be an equilibrium, convergence would occur, generating large payoff gains in status from very small deviations in the proposed policy. While this proposition does not fully prove that there must be randomization in equilibrium, it comes close. For it tells us that no deterministic equilibrium exists that exhibits the mild regularity conditions imposed in Proposition 2.

4. Endogenous Risk-Taking

Proposition 3 motivates the central goal of this exercise, which is to formalize the following intuition. When the production function is concave, an equilibrium generally induces convergence across individuals. As long as relative status has some value for individuals, such convergence must be "prevented" by the endogenous creation of risky opportunities. The equilibrium must involve risk-taking, even if there is no intrinsic uncertainty to begin with.

A society that "gambles" — or more generally, engages in various risky forms of generating income — will have the property that wealth distribution remains spread out, so that high

[5]This observation may be viewed as a counterpart for rank-dependent status of the result established by Arrow and Dasgupta (2009) where status derives from the average consumption level.

status gains from relatively small deviations are eliminated. It turns out that our model not only captures this intuition, but does so in a particularly stark way. We first establish the existence of a "steady state" — a situation in which the same distribution of wealth recurs period after period — and then establish the uniqueness of that steady state. We do this in the context of an ambient model with perfect certainty. This is a deliberate choice, because we show that the randomness of incomes is generated endogenously.

We then prove that *all* equilibria must converge to this unique steady state.

We introduce the following concavity assumption on f:

Assumption 7 [fconc]. f is continuously differentiable, increasing and strictly concave on $\mathbb{R}_+$, with $\delta f'(0) > 1$, and $f(k) < k$ for all k large enough.

Under this assumption, consider the randomization policy that an individual might follow in equilibrium. Observe that there is no longer any room for *investment* (k) to be a random variable. Because f is strictly concave, any such randomization can be "dominated" by investing the expected value of the random investment, and then following up by taking a fair bet using the produced output. This domination is entirely independent of the curvature of the utility function. Without loss of any generality, then, we can work with the following equation:

$$f(k_t(i)) = b_{t+1}(i) + k_{t+1}(i),$$

where for each person i and date t, $b_t(i)$ is a "consumption budget". This budget can be used by the individual to buy any fair bet she likes at date t, entirely consuming the outcomes — call these $c_t(i)$ — which we can think of as "realized consumption". The distribution of $c_t(i)$, integrated over all individuals, then forms the society-wide distribution F_t of consumption at date t. In the remainder of the section — and, given Assumption 7, without any loss of generality — we adopt this specification.

4.1. **Steady State.** We begin with two central observations (all omitted proofs are in the Appendix).

Lemma **1.** *In any equilibrium, the function $\mu_t(c) \equiv u(c, F_t(c))$ must be concave for all t.*

This lemma has a simple intuition. No individual must want to deviate from her optimal choice by generating another fair gamble which she prefers to the one she originally chose. If the reduced-form utility function were locally strictly convex over some region, then there must exist individuals for whom any fair bet could be dominated by some other fair bet, constructed by compounding the original fair bet with additional gambling in the strictly convex segment of $u(c, F(c))$.

Lemma **2.** *Make Assumption* 2 [ugen]. *For each $b > 0$, there is a unique cdf F with the following properties:*

(i) *The mean of F equals b:*

$$\int [1 - F(c)]\, dc = b. \tag{6}$$

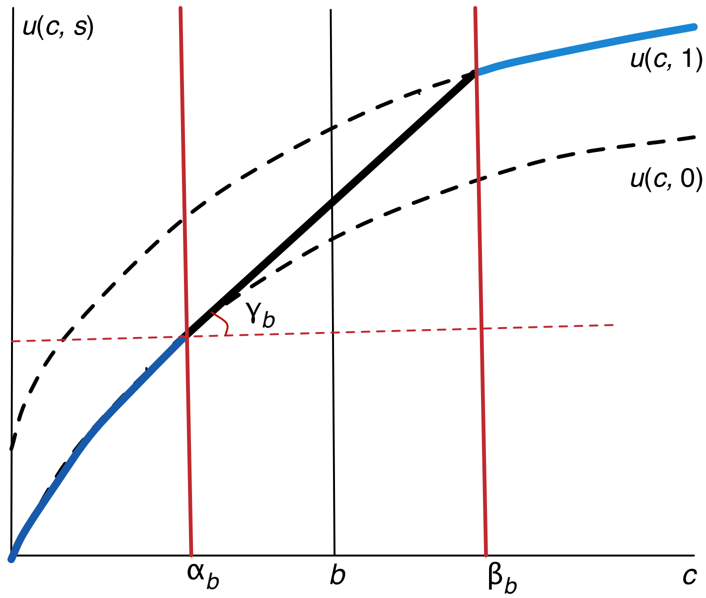

FIGURE 1. ALL ILLUSTRATION OF LEMMA 2.

(ii) *The support of F is an interval $[\alpha_b, \beta_b]$, and there exists $\gamma_b > 0$ such that*

$$\mu(c) = u(c, F(c)) = u(\alpha_b, 0) + \gamma_b(c - \alpha_b) \text{ for all } c \in [\alpha_b, \beta_b]. \tag{7}$$

(iii) *If $\alpha_b > 0$, $\gamma_b = u_c(\alpha_b, 0)$, while if $\alpha_b = 0$, $\gamma_b \geq u_c(0, 0)$.*

Moreover, the slope of the affine segment γ_b is a nonincreasing function of b.

The lemma is central, and we discuss it in some detail; refer to Figure 1 in the discussion that follows. Suppose that everyone has a common positive consumption budget of b, and suppose that it is fully consumed without randomization. With this provisional scenario in mind, look at the induced utility function for any one individual. It follows the curve $u(c, 0)$ until c increases to b, and thereafter it jumps up to follow the curve $u(c, 1)$. For this individual, the taking of a fair bet with b (and fully consuming the proceeds) is strictly profitable. It follows that the no-gambling scenario that we've assumed is invalid.

The lemma therefore describes how that common consumption budget must be spread out over a range of gambles. First, all such gambles must be fair, which explains the adding-up constraint (6). Second, the gambles are all willingly taken, which explains why the reduced-form utility function $u(c, F(c))$ needs to be weakly convex — and therefore, by Lemma 1, affine — over the overall domain of the gambles. This yields (7).

There is also the differentiable pasting carried out in part (iii), which plays an important role in pinning down F. Loosely, it is required that the slope of the reduced-form utility must equal the slope of the pure consumption utility at the point α at which the gambling realizations start. If the latter slope is higher, we cannot have concavity of the overall utility function and profitable deviations must exist. On the other hand, the latter slope cannot be lower, because the reduced-form utility must lie above the pure consumption utility everywhere.

Figure 1 depicts the final outcome, by showing the support of the gambles as well as the slope γ_b, for any budget b. (The underlying distribution F may have complicated shape, whatever it takes to linearize the reduced-form utility μ.) The figure also indicates — in the case in which $\gamma_b = u_c(\alpha, 0)$ —- why γ_b decreases as b goes up. The support of the gambles moves over to the right, and α rises. The strict concavity of u in c assures us that γ_b must therefore come down.[6]

In what follows, if a cdf F is derived from the budget b in the unique way described in Lemma 2, we will call F the *cdf associated with b*.

Proposition **4.** *Make Assumptions* 2 [ugen] *and* 7 [fconc]*. Then there exists a steady state with the following properties:*

(A) *Every individual (in a set of full measure) makes an identical investment k^*, given by the unique solution to $\delta f'(k^*) = 1$, and has equal starting wealth $w^* = f(k^*)$ at every date.*

(B) *The distribution of realized consumption is given by the unique cdf F^* associated with the budget $b^* \equiv f(k^*) - k^*$, as in Lemma 2.*

Before turning to a discussion of the steady state, we establish a converse. Observe that the steady state as described cannot be unique: for instance, the configuration in which all individuals have zero wealth, investment and consumption is (trivially) a steady state. It turns out, however, that the possibility of zero wealth is the *only* impediment to uniqueness:

Proposition **5.** *Under Assumptions* 2 [ugen] *and* 7 [fconc]*, the steady state identified in Proposition 4 is unique in the class of all steady states in which almost every individual has strictly positive wealth.*

Figure 2 provides a diagrammatic representation of the steady state. The first panel draws the cdf F^* (with compact support) that must be a feature of the steady state. This panel is deliberately drawn to suggest that F^* has no particular shape, only that it must "cancel" out all curvature in the utility function to create the linear segment (between α^* and β^*) displayed in the second panel. In addition, because consumption is generally valued for its own sake, a zone of the form $[0, \alpha^*]$ will typically be present,[7] over which no bets are taken and the utility function is strictly concave.

The two regions taken together generate the very phenomenon that Friedman and Savage (1948) sought to explain by their postulate of an (exogenous) utility function which is alternately concave and convex. In steady state, there is aversion to downside risk; no individual would ever take bets that would lead them into the consumption region $[0, \alpha^*]$. Indeed, if there were exogenous uncertainty added on to this model, they would actively seek to insure themselves against that possibility. Yet there *must* be risk-taking to the right of α^* for the reasons emphasized throughout the paper.

[6] To be sure, we obtain the same outcome when $\gamma_b > u_c(\alpha, 0)$.

[7] If u has unbounded steepness in consumption near the origin, this additional zone *must* be present.

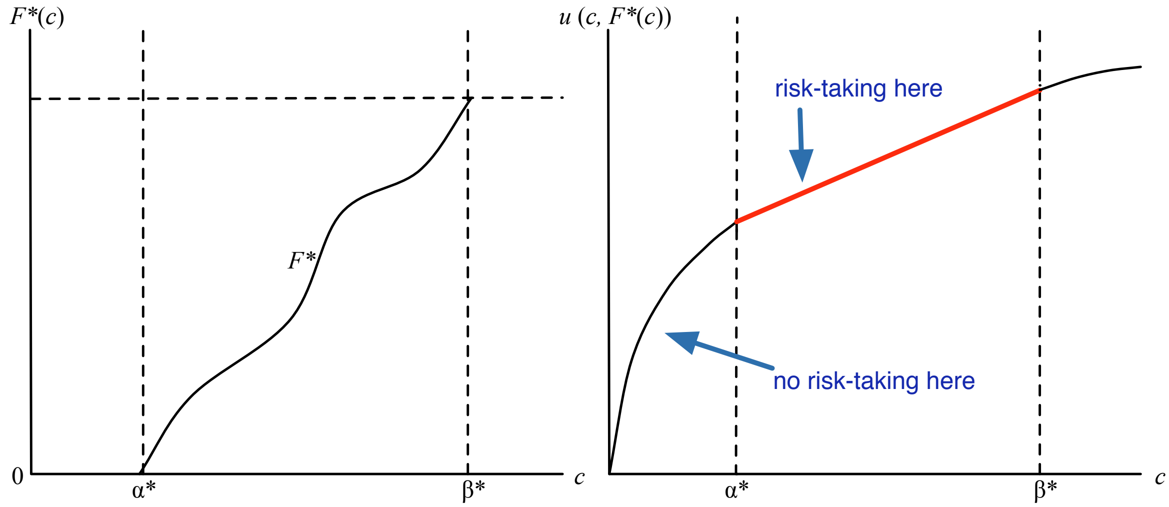

FIGURE 2. THE FRIEDMAN-SAVAGE PROPERTY

In the stark specification we study, the zone $[0, \alpha^*]$ is actually not inhabited in steady state, though such a zone will typically be inhabited in the transition dynamics that we study below. This outcome is artificial and the consequence of our assumption that there is no exogenous uncertainty in the model. That is simply a pedagogical device designed to highlight endogenous risk-taking, and it is not difficult to extend the model to include exogenous uncertainty as well. In that case both the zones will generally be actively inhabited in steady state.

It is of some interest is that this phenomenon — risk-aversion at the lowest end of the distribution coupled with risk-taking elsewhere — arises "naturally" in an environment where utility depends on status. There is no need to depend on an *ad hoc* description of preferences with varying curvature for an "explanation".[8]

Two other features of this steady state merit some emphasis, and distinguish our approach still further from Friedman and Savage. First, the use of relative status guarantees that the model is, to a large extent, scale-neutral.[9] Two insulated societies with, say, two different production technologies, will generally settle into two different steady states. *Both* the steady states will generally exhibit the Friedman-Savage property, even though they will be located at different ranges in the wealth distribution. Unless the Friedman-Savage utility function is exogenously moved around to accommodate different wealths in exactly the right *ad hoc* way, it is not possible in their approach to generate the same phenomenon at diverse aggregate wealth levels. This is another strength of the current approach.

[8]One might object that (unlike Friedman and Savage) our individuals do not *strictly* prefer to bear risk. While that is true, they *must* nevertheless bear risk for the outcome to be a bonafide steady state. Observationally, that is enough.

[9]However, the strictly concave dependence of utility on absolute consumption means that the model is not fully scale-neutral, except in the pure-status case.

Our final point, and in our opinion the most important of the three, is that gambling in the Friedman-Savage world is ex-ante efficient: there is an assumed convexity in the utility function, and this convexity is well-served by risk-taking. In the model we study, there is a pervasive externality. Equilibrium risk-taking will generally be Pareto-inefficient.

Assign a status rank of 1/2 to every individual that lives in a society of perfect equality. We may then state

Proposition **6.** *Under Assumptions 2* [ugen] *and 7* [fconc]*, and the additional restriction that u is strictly concave in (c, s), the steady state identified in Proposition 4 must be Pareto-inefficient.*

Proof. Starting from the steady state equal wealth distribution at w^*, consider an alternative arrangement in which all gambling is banned, and all individuals continue to invest k^* at every date. Their lifetime utility (suitably discount-normalized) is then given by $u(b^*, 1/2)$. In contrast, in the steady state, lifetime expected utility is given by

$$\int u(c, F^*(c))dF^*(c) < u\left(\int cdF^*(c), \int F^*(c)dF^*(c)\right) = u(b^*, 1/2),$$

where we use the strict concavity of u. □

4.2. **Nonstationary Equilibrium and Convergence to Steady State.** In the previous section, we have identified a steady state which has properties of some intrinsic interest. It involves zones over which an individual is risk averse, and other zones over which there is endogenously generated gambling. We have discussed the connections between this steady state and the set of facts that the Friedman-Savage theory purports to explain. We have also shown that our steady state is the only possible stationary outcome when all agents produce positive output.

This leaves open the question of whether any nonstationary equilibrium must converge to the steady state that we have uniquely identified. In this section, we answer that question largely in the affirmative:

Proposition **7.** *Make Assumptions 2* [ugen] *and 7* [fconc]*. Fix any initial wealth distribution with bounded support and with infimum wealth positive. Under any intertemporal equilibrium, the sequence of consumption distributions must converge over time to the steady state distribution identified in Proposition 4.*

It is will be useful to provide a rough outline of the (somewhat long) argument that leads to Proposition 7. Begin with any intertemporal equilibrium. Recall Lemma 1; we know then that the "reduced-form" utility functions $\mu_t(c) = u(c, F_t(c))$ are strictly concave at every t. This precipitates an optimal growth problem with time-varying one-period utilities. There is a "turnpike theorem" for such a problem; see Mitra and Zilcha (1981).[10] That theorem states that starting from any two (positive) initial wealths, the resulting path of capital stocks must converge to each other over time. Applying that theorem, we must conclude that all capital stock sequences bunch up very closely. When they do, the preservation of

[10]In the formal proof, we use an extension of the Mitra-Zilcha theorem due to Mitra (2009).

concavity in μ_t requires that consumption be suitably spread out using endogenous risk-taking. All consumption budgets ultimately fall into the support of these gambles, but because gambles linearize utility over their support, we must conclude that the marginal utility of consumption of all agents is fully equalized after some date. The Euler-Ramsey equation then guarantees that not only are their capital stocks are close together in all succeeding periods (as in Mitra-Zilcha), they *coincide*. The remainder of the argument consists in showing that this (common) capital stock sequence must converge. For details, consult the formal proof in Section 7.

5. Bibliographical Notes

5.1. **Status and Risk.** Criticism of the Friedman-Savage model began shortly after its publication with Markowitz (1952), who first made some of the points that motivate the present work. For example, he argues that the implied risk-taking behavior in a middle range of wealth is implausible. He suggests a modified utility of wealth with three inflection points, which is still not explicitly tied to relative wealth. Coelho and McClure (1998) sketch how defining status as wealth relative to the mean could provide an explicit tie. Although our model here is not directly inspired by Markowitz, a more important distinction is that it applies the rank order notion of status as in Robson (1992).

The present work might be seen indeed as a dynamic extension of the static model of Robson (1992), where this extension accounts for the stylized facts in a more robust and satisfactory fashion. In Robson (1992), gambling might or might not arise, apparently in a one-time fashion, depending on the exact shape of utility and the distribution of wealth. In the present model, on the other hand, persistent gambling must eventually arise, for rather general specifications of utility and the distribution of wealth. Further, Robson (1992) focussed on stable distributions of wealth in which individuals would reject all fair bets. It was shown that such distributions might be Pareto-inefficient, since fair bets might permit a Pareto improvement.

Becker, Murphy and Werning (2005) allow for equilibrium gambling in a static model with rank-dependent status. They use a rather similar construction to that here, where the gambles are such that individuals are indifferent between participating or not and so are assumed to participate. Such a result might apparently only imply a one-shot burst of fair gambles at the start in a dynamic extension of the model. As in the other static models, acceptance of fair gambles in Becker, Murphy and Werning is crucially contingent on particular properties of utility and the distribution of wealth. In the current dynamic model, however, it is shown that there must exist a unique steady state with persistent gambling and that any equilibrium trajectory must attain this steady state in finite time. Becker, Murphy and Werning show that their model could be reinterpreted as involving a market for a status good. They also stress how Pareto efficiency might then result, rather than considering the inefficiency arising here.

5.2. **Status and Saving.** Although it is not the intended focus, the present paper incorporates ancilliary results concerning how a concern with status influences savings. The most striking of these is the result for pure status utility, with a convex investment function. The savings policy function is then identical to that arising from logarithmic utility of consumption and a linear investment function. However, the more plausible case involves utility being a function of consumption per se as well as status, along with a concave investment function.

Consider, for example, the special case in which utility is additively separable. It is easily seen that a Pareto-efficient allocation would arise from each individual solving the growth problem based purely on the pure consumption component of utility. This would imply convergence of wealth and consumption levels to values that are the same for all individuals. Although these common values coincide with the long run equilibrium values obtained here, such a Pareto-efficient allocation would not be robust to allowing access to fair bets. That is, the Pareto-efficient allocation differs from the equilibrium savings allocation derived here. In particular, the equilibrium trajectories for all individuals rich or poor coalesce in finite time.

Consider then the literature that investigates how a concern with status bears on savings behavior. Perhaps the most closely related paper is Xia (2009) who also considers status from consumption to be the rank attained in the distribution of consumption. She considers a simple two period model where utility is additively separable in consumption and status, with a linear investment technology. Consistent with the results established here, the effect of the pure status term is to distort savings towards that expected with logarithmic utility of consumption per se. If the real rate of interest exceeds the pure rate of time preference, a concern with status may then then increase savings.

Hopkins and Kornienko (2006) also define status to be the rank in the distribution of consumption in a two period model. However, they assume it is only first-period consumption that is affected in this way. The effect of status is then to induce too much first period consumption and too little savings. Corneo and Jeanne (2001) consider status to be the ranking in the wealth distribution rather than the consumption distribution. They find that a more equal distribution of wealth leads to a higher growth rate of the economy. Both these contributions entail additively separable utility with logarithmic terms in consumption and status.

Finally, Arrow and Dasgupta (2009) consider an intertemporal model in which average consumption captures a concern with status. They show there may be either too much or too little savings in such a setting, and emphasize the razor's edge case in which there is no effect of status.

5.3. **Endogenous Inequality.** Our results are also related to a literature on symmetry-breaking which argues that markets must endogenously create inequality, even when we start from situations with perfect equality (Freeman (1996), Mookherjee and Ray (2003)). While the ideas are related, the mechanisms are different. In this literature, individuals must choose different occupations even when they start from identical wealth, because a variety of occupations may be needed in production. When different occupations have varying startup costs and there are credit-market imperfections, persistent economic inequality will endogenously emerge. Risk-taking plays an analogous role here, though driven by a different set of factors.

6. Conclusion

In this paper, an individual's concern for relative status is expressed in her preferences. We embed this hypothesis in an otherwise conventional model of economic growth, and examine its consequences. In our main result, obtained under entirely traditional concavity restrictions on the utility and production functions, there must be persistent and endogenously generated risk-taking in equilibrium.

We characterize the unique steady state in such a situation, and prove that any dynamic equilibrium path must converge to it.

Friedman and Savage (1948) studied the phenomenon of widespread risk-taking, often by the very same individuals who appear to be averse to downside shocks. This twin proclivity to both take on upside risk and shield against downswings led Friedman and Savage (1948) — and a large literature after them — to introduce a utility function with varying curvature. They were, of course, very aware that the shape of such a function appeared contrived, involving both concave and convex segments suitably positioned to "explain" the observations. But it is clear that they wished to take on those detractors who felt that risk-taking was better explained as systematic mistakes. In particular, Friedman (1953) makes the agenda very clear: risk-taking is deliberate and therefore (so goes the subtext) efficient.

Thus Friedman and Savage go to some pains to justify their choice of utility function:[11]

> "A possible interpretation of the utility function . . . is to regard the two [concave] segments as corresponding to qualitatively different socioeconomic levels, and the [convex] segment to the transition between the two levels. On this interpretation, increases in income that raise the relative position of the consumer unit in its own class but do not shift the unit out of its class yield diminishing marginal utility, while increases that shift it into a new class, that give it a new social and economic status, yield increasing marginal utility."

Our model can be viewed as a formalization and elaboration of this interpretation of Friedman and Savage. It substantially strengthens the purely positive implications of their theory. Yet it leads to very different welfare conclusions. Friedman (1953) forcefully made the point that if an individual wants to gamble, because that's what his attitude to risk tells him to do, that gamble must be ex-ante efficient. Some observed ex post inequality could be the efficient product of free choices. In the relative status world, with consumption externalities, this is no longer true. In stark contrast, risk-taking is generally Pareto-inefficient in the theory that we propose. Of course, this is not a surprise as models with externalities do typically display inefficient outcomes. The point is that we adopt the very model informally described by Friedman and Savage themselves.

[11]Modern defenders of the revealed preference approach, as well as Friedman himself in some of his writings, would see no urgent reason for a justification. Our suspicion is that Friedman makes some effort in this case because he does not want to abandon the welfare economics of the entire project, something that a hard-core revealed preference theorist must effectively have to do, especially in an equilibrium context where it is unclear how to equate choices with social preference.

Moving away from the Friedman-Savage results, our theory clarifies other aspects that arise in a fully dynamic setting. A key theme is that equilibrium behavior when relative status matters must coexist with a certain degree of consumption inequality. When that inequality is generated "naturally", as it is with a constant- or increasing-returns technology, behavior is simple and deterministic. On the other hand, when inequality tends to go away, equilibrium behavior steps in to recreate it by endogenously generating periodic sprees of risk-taking.

This outcome is perhaps best interpreted within our model when a "single period" is viewed as an individual lifetime, and the capital stock as an intergenerational bequest, as in the Loury (1981) reformulation of the optimal growth model. There are two implications of such an interpretation. First, as in Becker and Tomes (1981) and Loury (1981), the "production function" will include human capital accumulation, and — with imperfect capital markets for education — it will have household-specific curvature rather than being fully linear as in the case of competitive, complete markets. Second, "consumption" may be viewed as *lifetime* consumption, so that within-period risk-taking can be interpreted in a longer-term way: one might include occupational choice or entrepreneurial ventures in addition to short-term risk (such as financial market participation or property speculation). From this perspective, our model predicts the emergence of ex post lifetime inequality even when there is perfect equality from an intergenerational perspective.

7. Proofs

Proof of Proposition 1. Suppose that all individuals in a set of unit measure use the policy function (5). Let $\mathbf{G} = \{G_t\}$ be the resulting sequence of wealth distributions. It is obvious that for every date t and for every w in the support of G_t,

$$G_{t+1}\left(f(\delta w)\right) = G_t(w),$$

so that for every w in the support of G_{t+1},

$$G_{t+1}(w) = G_t\left(f^{-1}(w)/\delta)\right). \tag{8}$$

Lemma **3.** *Under Assumptions 3* [upure], *4* [fconv] *and 5* [init], $u(G_t(w))$ *is strictly concave for all dates* t.

Proof. Because f is increasing and convex, and $f(0) = 0$, $f^{-1}(w)$ is increasing and concave in w. Using (8), we therefore see that $u(G_{t+1}(w)) = u(G_t(f^{-1}(w)))$ is strictly concave provided that $u \circ G_t$ is strictly concave. Now proceed recursively from date 0, using Assumption 5. □

Fix some date t. Suppose that a particular individual employs the policy (5) for all dates $s \geq t + 1$, and that every other individual employs the policy (5) at all dates. Define $V_{t+1}(w')$ to be the discounted value to our individual under these conditions, starting from wealth w' and date $t + 1$. Then status at every $s \geq t + 1$ is simply

$$\bar{F}_s(c_s) = G_{t+1}(w'),$$

so that

$$V_{t+1}(w') = (1 - \delta)^{-1} u\left(G_{t+1}(w')\right). \tag{9}$$

Now suppose that at date t, our individual has starting wealth w, does not randomize, and chooses $k \in [0, w]$. Then her lifetime payoff at that date is given by

$$\begin{aligned} u\left(F_t(w-k)\right) + \delta V_{t+1}\left(f(k)\right) &= u\left(F_t(w-k)\right) + \delta(1-\delta)^{-1} u\left(G_{t+1}\left(f(k)\right)\right) \\ &= u\left(G_t\left([w-k]/(1-\delta)\right)\right) + \delta(1-\delta)^{-1} u\left(G_{t+1}\left(f(k)\right)\right) \\ &= u\left(G_t\left([w-k]/(1-\delta)\right)\right) + \delta(1-\delta)^{-1} u\left(G_t\left(k/\delta\right)\right), \end{aligned}$$

where the first equality uses (9), the second uses the fact that $F_t(c) = G_t\left(c/(1-\delta)\right)$ for every $c \geq 0$, and the last uses (8).

By Lemma 3, this expression is strictly concave in both w and k so if our individual maximizes her payoff at date t, she will not randomize either in w or in k. Moreover, a solution to the first-order condition

$$(10) \qquad -u'(r_t) G_t'\left([w-k]/(1-\delta)\right)(1-\delta)^{-1} + \delta(1-\delta)^{-1} u'(r_{t+1}) G_t'\left(k/\delta\right)\delta^{-1} = 0$$

(where r_s is the resulting status in date s, for $s = t, t+1$) must be the unique optimum. It is easy to see that $k = \delta w$ is indeed the solution to (10), so that optimal consumption at date t is uniquely given by (5) as well.

Because t and w are arbitrary, our proposition follows from Blackwell's unimprovability theorem.[12] □

Proof of Proposition 2. Notice that each individual is atomless and therefore has the same intertemporal utility criterion as any other. Because the equilibrium is regular, we see that at any date, the solution to the optimization problem is unique except at countably many wealth levels. But it is easy to see that such a solution cannot admit more than one differentiable selection. Therefore all individuals must use the same policy, which we denote by $\{c_t\}$.

Let **G** and **F** be the equilibrium sequences of wealth and consumption. It is clear from our assumptions that for every t, G_t and F_t have well-defined densities on the interior of their supports, that these supports are each $[0, \infty)$, and that

$$G_t(w) = F_t(c_t(w)),$$

for $w > 0$. Consequently, for such w,

$$(11) \qquad G_t'(w) = F_t'(c_t(w)) c_t'(w).$$

We also know that for every $w > 0$,

$$G_t(w) = G_{t+1}\left(f(w - c_t(w))\right),$$

so that

$$(12) \qquad G_t'(w) = G_{t+1}'(w') f'(k)[1 - c_t'(w)],$$

where $k \equiv w - c_t(w)$, and $w' \equiv f(k)$.

[12]The restriction that G has full support will be used to ensure that G_t also has full support, so that every initial w lies in the support of G_t. We can drop this restriction easily: the resulting equilibrium policy will then be flat for initial wealths that lie above the support.

Now consider any date t and any initial stock $w_t > 0$. Then, by our assumptions on equilibrium policy, equilibrium consumption c_s is strictly positive for all $s \geq t$. Write down the (necessary) Euler equation for payoff-maximization at all such dates s:

$$u'(r_s)F'_s(c_s) = \delta u'(r_{s+1})F'_{s+1}(c_{s+1})f'(k_s), \tag{13}$$

where r_s and k_s are optimally chosen ranks and capital stocks for $s \geq t$, all starting from initial wealth w_t. Because $c_s(w)$ is an equilibrium policy function with the properties mentioned in the statement of the proposition, we must have $c_s = c_s(w_s)$ and $r_s = r_{s+1}$ for all $s \geq t$. We may use this information in (13), along with (11) and (12), to conclude that

$$\begin{aligned} \frac{G'_s(w_s)}{c'_s(w_s)} &= \delta F'_{s+1}(c_{s+1})f'(k_s) \\ &= \frac{\delta f'(k_s)G'_{s+1}(w_{s+1})}{c'_{s+1}(w_{s+1})} \\ &= \frac{\delta f'(k_s)G'_s(w_s)}{f'(k_s)[1-c'_s(w_s)]c'_{s+1}(w_{s+1})} \\ &= \frac{\delta G'_s(w_s)}{[1-c'_s(w_s)]c'_{s+1}(w_{s+1})} \end{aligned} \tag{14}$$

for all $s \geq t$. Defining $a_s \equiv 1/c'_s(w_s)$ for all such s, we may conclude from (14) that

$$\delta(a_{s+1} - a_s) = (1-\delta)a_s - 1 \tag{15}$$

for all $s \geq t$.

It is easy to see from (15) that if $a_t < (1-\delta)^{-1}$, then a_s must become negative at some finite date s, which contradiction the premise that $c'_s(w) > 0$ for all s and w.

At the same time, if $a_t > (1-\delta)^{-1}$, it is easy to see that $a_s \to \infty$ as $s \to \infty$, which means that $c'_s(w_s) \to 0$. This contradicts our premise that the derivative of all consumption functions is uniformly bounded below by some $\epsilon > 0$.

So $a_t = (1-\delta)^{-1}$. Because t and w were chosen arbitrarily, the proof of the proposition is complete. □

Proof of Proposition 3. Proposition 2 tells us that if such a policy sequence exists, it must be time stationary, with

$$c_t = (1-\delta)w_t$$

at every date t and for all $w_t > 0$. It follows that for any initial value w_0, investment $\{k_t\}$ follows the difference equation

$$k_{t+1} = \delta f(k_t),$$

which means that all such sequences from any initial positive wealth must converge to $\hat{k}$, where $\hat{k}$ (strictly positive) is defined in Assumption 6, and is the unique solution to

$$k = \delta f(k)$$

This means that the distribution of wealth G_t converges weakly to the degenerate distribution with unit mass at $f(\hat{k})$, and the corresponding distribution of consumption converges weakly to unit mass on $f(\hat{k}) - \hat{k}$. It is now easy to see that some individual with status close enough to zero under the ongoing equilibrium can deviate from the proposed policy, by taking a suitable fair bet. □

Proof of Lemma 1. Suppose μ_t fails to be concave for some t. Then there exists a set I of consumptions and ϵ in $(0, 1)$ with the following properties:

(i) There is a strictly positive measure (under F_t) of consumption realizations in I, and $\inf I > \epsilon$.

(ii) For every $c \in I$,

$$(1-\epsilon)\mu_t(c+\epsilon^2) + \epsilon\mu_t(c - \epsilon[1-\epsilon]) > \mu_t(c). \tag{16}$$

For details, see this footnote.[13] Moreover, there must be some individual (indeed, a positive measure of them) who takes fair bets with positive-probability consumption realizations in I.[14] (This includes degenerate fair bets in which the consumption budget itself lies in I.) Pick any such individual, and consider the compound lottery given by her original fair bet F and another bet tagged on to each of realization c of that bet that lies in I: one that pays $c + \epsilon^2$ with probability $(1 - \epsilon)$ and $c - \epsilon(1 - \epsilon)$ with probability ϵ, where ϵ is given by (ii). Reduce this compound lottery to a single stage bet F'. Using the first part of (ii), it is trivial to check that F' is also fair. But the expected *gain* in the individual's payoff by moving to F' from F is given by

$$\int_I \left[(1-\epsilon)\mu_t\left(c+\epsilon^2\right) + \epsilon\mu_t(c-\epsilon[1-\epsilon]) - \mu_t(c)\right] dF(c)$$

which is strictly positive, by (ii). This contradicts the optimality of that individual's choice. Therefore μ_t must indeed be concave. □

Proof of Lemma 2. For every $\alpha > 0$, define a cdf F_α by

$$F_\alpha(c) = 0 \text{ for } c \in [0, \alpha], \tag{17}$$

while for $c \geq \alpha$, $F_\alpha(c)$ is the largest number in $[0, 1]$ that satisfies

$$u(c, F_\alpha(c)) \leq u(\alpha, 0) + u_c(\alpha, 0)(c - \alpha). \tag{18}$$

Because u is concave in c, we know that $u(c, 0) \leq u(\alpha, 0) + u_c(\alpha, 0)(c - \alpha)$, so that $F_\alpha(c)$ is well-defined.

We claim that F_α is a bonafide cdf with $F_\alpha(c) = 1$ at some finite c. Observe first that $F_\alpha(c)$ is nondecreasing in c. Suppose, on the contrary, that $F_\alpha(c) < F_\alpha(c')$ for some $c > c' \geq \alpha$. Then (18) holds with equality at c, so that

$$u(c, F_\alpha(c')) > u(\alpha, 0) + u_c(\alpha, 0)(c - \alpha).$$

[13] If μ_t is locally strictly convex on some interval J, take I to be some compact subinterval of J and choose ϵ small so that $c > \epsilon$ for all $c \in I$. Otherwise, F_t must possess a mass point at some consumption value $c > 0$. Take I to be the singleton set $\{c\}$ and choose $\epsilon \in (0, 1)$ smaller than c with the additional property that $(1 - \epsilon)u(c, F_t(c)) > u(c, \bar{F}_t(c))$ (by (3), this is always possible). In either case, consider a fair consumption bet which pays $c + \epsilon^2$ with probability $1 - \epsilon$ and $c - \epsilon(1 - \epsilon)$ with probability ϵ. In the former case, (16) follows right away from the strict convexity of μ_t on I. In the latter case, the expected payoff of the individual is given by $(1-\epsilon)u(c+\epsilon^2, \bar{F}_t(c+\epsilon^2)) + \epsilon u(c-\epsilon[1-\epsilon], \bar{F}_t(c-\epsilon[1-\epsilon])) \geq (1-\epsilon)u(c, F_t(c)) + \epsilon u(c-\epsilon[1-\epsilon], \bar{F}_t(c-\epsilon[1-\epsilon])) > u(c, \bar{F}_t(c))$, which yields (16) again.

[14] The overall probability of realizations in I is strictly positive, and the overall probability is a convex combination of all bets — degenerate bets included — taken by all individuals.

At the same time, because $F_\alpha(c') > 0$, we know that

$$u(\alpha, F_\alpha(c')) > u(\alpha, 0).$$

Pick $\lambda \in [0,1]$ such that $c' = \lambda c + (1-\lambda)\alpha$. Then combining the above inequalities, we must conclude that

$$\lambda u(c, F_\alpha(c')) + (1-\lambda)u(\alpha, F_\alpha(c')) > u(\alpha, 0) + u_c(\alpha, 0)(c' - \alpha) \geq u(c, F_\alpha(c')),$$

where the last inequality uses (18) at c'. But this contradicts the concavity of $u(., F_\alpha(c'))$.

Now we show that $F_\alpha(c) = 1$ for some $c < \infty$. Suppose this is false; then by (18),

$$u(\alpha, 0) + u_c(\alpha, 0)(c - \alpha) < u(c, 1) = \frac{u(c,1)}{c}c$$

for all c, but this contradicts Assumption 2.

Denote by $\mathcal{F}$ this family of all such distributions F_α.

If $u_c(0,0) < \infty$, define a second family of distributions $\mathcal{F}_0$. Members of this family will be indexed by $\gamma \geq u_c(0,0)$. For each such γ, consider the unique distribution F^γ such that for each $c \geq 0$, $F^\gamma(c)$ is the largest number in $[0,1]$ satisfying

$$u(c, F^\gamma(c)) \leq \gamma c. \tag{19}$$

An argument similar to that following (18) also shows that each F^γ is a bonafide cdf, and that $F^\gamma(c) = 1$ at some finite value of c.

Define $\mathcal{F}^* \equiv \mathcal{F}$ if $u_c(0,0) = \infty$, and $\mathcal{F}^* \equiv \mathcal{F} \cup \mathcal{F}_0$ otherwise. Note that any cdf in $\mathcal{F}^*$ satisfies properties (ii) and (iii) in the statement of the lemma, and no other cdf on c does. We claim that there is a unique $F \in \mathcal{F}^*$ such that (i) of the lemma is satisfied.

To prove this claim, suppose that $\alpha > \alpha' > 0$. Use the concavity of u in c, together with (18), to see that $F_\alpha(c) \leq F_{\alpha'}(c)$ for every c, with strict inequality over the interior of the support $[\alpha, \beta]$ of F_α. We must conclude that F_α strictly dominates $F_{\alpha'}$ in the sense of first-order stochastic dominance. It follows that the mean of F_α — call it $m(\alpha)$ — is strictly increasing in α, and it is trivial to see that $m(\alpha) \to \infty$ as $\alpha \to \infty$. We also note, using (18) and Assumption 2, that $m(\alpha)$ is continuous.

Now, whenever $u_c(0,0) = \infty$, $m(\alpha) \to 0$ as $\alpha \to 0$. Otherwise, when $u_c(0,0) < \infty$, there is a continuum of distributions in $\mathcal{F}_0$, indexed by $\gamma \geq u_c(0,0)$. Let $\hat{m}(\gamma)$ denote the mean of F^γ. An inspection of (19) immediately reveals that $\hat{m}(\gamma)$ is continuous and strictly decreasing in γ, with $\hat{m}(\gamma) \to 0$ as $\gamma \to \infty$. Finally, observe that $\hat{m}(u_c(0,0)) = \lim_{\alpha \to 0} m(\alpha)$.

These observations show that for *any* positive value of mean consumption b, there exists a unique distribution $F \in \mathcal{F}^*$ such that

$$\int [1 - F(c)] dc = b. \tag{20}$$

Let β_b for the first value of c at which $F(c) = 1$. If $F \in \mathcal{F}$, let α_b stand for the corresponding value of α, and $\gamma_b = u_c(\alpha_b, 0)$. If $F \in \mathcal{F}_0$, then let $\gamma_b \geq u_c(0,0)$ be the unique index associated with F, and set $\alpha_b = 0$.

To establish the very last part, suppose that b is increased. Then, by (20), the new fixed point F must have a higher mean. Recall that within the family $\mathcal{F}$, $m(\alpha)$ is increasing in α, while within $\mathcal{F}^0$ (if $u_c(0,0) < \infty$) $\hat{m}(\gamma)$ is decreasing in γ. By the strict concavity of u, γ_b decreases. □

Proof of Proposition 4. Define F^* as in part (B), to be interpreted as the distribution of realized consumption. Define $\mu^*(c) = u(c, F^*(c))$. If $\alpha^* > 0$, μ^* is increasing and strictly concave up to α^*, following which it is has a linear segment with slope $\gamma^* = u_c(\alpha^*, 0)$ up to β^*, the upper bound on consumption under F^*. After β^*, $\mu^*(c) = u(c, 1)$. On the other hand, if $\alpha^* = 0$, μ^* is linear with slope $\gamma^* \geq u_c(0,0)$ up to β^*, and $\mu^*(c) = u(c, 1)$ thereafter. Observe that $\beta^* > f(k^*) - k^*$.

We now describe a consumption, investment and randomization strategy for each individual. To do so, consider the following problem:

$$\max \sum_{t=0}^{\infty} \delta^t \mu^*(b_t(i))$$

subject to

$$w_t(i) = b_t(i) + k_t(i)$$

for all t, and

$$w_{t+1}(i) = f(k_t(i))$$

for all t, with $w_0(i)$ given. Because μ^* is concave and f is strictly concave, this problem has a unique optimal investment strategy associated with it, assigning an investment k for every starting wealth w. We also describe consumption strategy: if the consumption budget b at any date equals $b^* \equiv f(k^*) - k^*$, take a fair bet with cdf F^*, consuming the proceeds entirely. Otherwise consume the entire consumption budget directly. This is the candidate individual policy for the steady state.

Because μ^* *is* the utility function in the presumed steady state, the investment component of this policy must be optimal. So is the consumption component, because individual utilities are linear in realized consumption over the support of F^*.

To complete the verification that F^* is a steady state, we show that F^* is indeed replicated, period after period, by this policy, provided that all individuals start with the common wealth $w^* = f(k^*)$. All we need to show is that k^* is indeed the optimal investment at w^*.

To do this, simply choose k^* at w^*, period after period, and deploy the consumption budget as described above.[15] Then

$$\int \mu^{*\prime}(c_t) dF^*(c_t) = \mu^{*\prime}(b^*) = \delta f'(k^*) \mu^{*\prime}(b^*) = \int \mu^{*\prime}(c_{t+1}) dF^*(c_{t+1})$$

so that the Euler equation is satisfied at $k = k^*$ in every period.[16] □

[15] It does not matter whether the investment decisions are made before or after the realization of the consumption gamble.

[16] Because the program is stationary at the "modified golden rule" corresponding to f, the transversality condition is satisfied.

The remaining propositions have some lemmas in common, and we now turn to these. In what follows, we consider full intertemporal equilibria, not necessarily steady states.

Lemma **4.** *If F is a fair bet taken at some date t, then μ_t must be affine on* $[\inf F, \sup F]$.

Proof. Consider any fair bet F taken by an individual with budget b. Then

$$\int c dF(c) = b.$$

By Lemma 1, μ_t is concave. It follows that

$$\int \mu_t(c) dF(c) \leq \mu_t(b),$$

and strict inequality must hold unless μ_t is affine. However, strict inequality must contradict the fact that the agent willingly takes the bet. Therefore, μ_t must indeed be *affine* on $[\inf F, \sup F]$.[17] □

Lemma **5.** *If at any date, all agents have an identical consumption budget $b > 0$, then F_t must be the unique cdf associated with b, as in Lemma 2.*

Proof. Because μ_t is concave, while all individuals have the same consumption budget, nondegenerate fair bets must be taken at date t by a positive measure of individuals. Let F_t be the distribution of realized consumption thus induced. We must verify that parts (i)–(iii) of Lemma 2 hold for F_t.

Part (i), given by (6), is immediate from the assumption that only fair bets are available.

To verify (ii), note from Lemma 4 that $\mu_t(c)$ is affine over $[\inf F, \sup F]$ for each nondegenerate fair bet F. Define α to be the infimum value of $\inf F$ over all such bets, and similarly β to be the supremum value of $\sup F$. Because $\inf F < b < \sup F$ for all nondegenerate fair bets, we must conclude that μ_t has a *common* slope over the range of all these bets, and so μ_t is affine on $[\alpha, \beta]$.

We still need to show that the support of F_t is the interval $[\alpha, \beta]$, and that $\mu_t(\alpha) = u(\alpha, 0)$. The former follows from Assumption 2: to keep μ_t affine, F_t must be *strictly* increasing on $[\alpha, \beta]$. The latter follows right away if $\alpha > 0$, for then the concavity of μ_t assures continuity at all interior points. If $\alpha = 0$, then (7) follows from the assumption that all individuals have a strictly positive consumption budget.[18] This verifies part (ii).

Let γ be the slope of the affine segment of μ over the domain $[\alpha, \beta]$. We claim that $\gamma \geq u_c(\alpha, 0)$. Suppose not; then $u_c(\alpha, 0) > \gamma$. Because u is continuously differentiable, we can find $c > \alpha$ such that $u_c(c, 0) > \gamma$. Using the concavity of u in c, we must conclude that

$$u(c, F_t(c)) \geq u(c, 0) \geq u(\alpha, 0) + u_c(c, 0)(c - \alpha) > u(\alpha, 0) + \gamma(c - \alpha),$$

but this violates (7), a contradiction.

[17] Note that we do not need the support of F to be connected to make this argument.

[18] F_t then cannot have an atom at 0, since individuals would strictly prefer fair gambles in which 0 is not an outcome.

Finally, we claim that $\gamma = u_c(\alpha, 0)$ if $\alpha > 0$. Suppose not, then (given that $\gamma \geq u_c(\alpha, 0)$, as just proved) $\gamma > u_c(\alpha, 0)$. Using (7), it is now easy to see that μ cannot be concave, which contradicts Lemma 1. This establishes (iii). □

Proof of Proposition 5. Let F be a steady state distribution of consumption and define $\mu(b) \equiv u(b, F(b))$ for all $b \geq 0$. By Lemma 4, no individual *strictly* wishes to take a fair bet. Therefore, an individual with consumption budget b at any date gets precisely this one-period payoff. Therefore, recalling that the investment and consumption budgets of each individual are deterministic, each i must solve the optimization problem:

$$\max \sum_{t=0}^{\infty} \delta^t \mu(b_t(i))$$

subject to

$$w_t(i) = b_t(i) + k_t(i)$$

for all t, and

$$w_{t+1}(i) = f(k_t(i))$$

for all t, with $w_0(i)$ given. One can check (see, e.g., Mitra and Ray (1984)) that for each individual, k_t must converge to k^*, where $\delta f'(k^*) = 1$. In particular, in steady state, each individual must invest precisely k^*. This verifies Part A of Proposition 4.

Note that every individual has the same consumption budget $f(k^*) - k^*$. Invoke Lemma 5 to verify Part B. This completes the proof of uniqueness. □

We are left with the proof of Proposition 7. In what follows, we assume throughout that Assumptions 2 and 7 hold, and that the initial wealth distribution has bounded support with infimum wealth positive (as assumed in the statement of Proposition 7).

We quickly review the main argument to follow, as there are several details. The first critical step is Lemma 6, which is based on the Mitra-Zilcha turnpike theorem (see Mitra and Zilcha (1981) and the version we use, which is Mitra (2009)). It states that in any equilibrium, the paths followed by all agents converge *to one another*. Lemmas 7, 9 and 10 ensure that convergence occurs to some common sequence which has a strictly positive limit point (over time). The second critical step is Lemma 12, which states that when all stocks cluster sufficiently close to this common limit point, a bout of endogenous risk-taking must force all consumption budgets to lie in the same affine segment of the "reduced-form" utility function μ at that date. Lemma 13 states that all individual capital stocks must fully coincide thereafter. The remainder of the proof shows that this common path must, in turn, converge over time to k^*, with consumption distributions converging to F^*, the unique cdf associated (as in Lemma 2) with $b^* = f(k^*) - k^*$.

Lemma 6. *In any intertemporal equilibrium,* $\sup_{i,j} |k_t(i) - k_t(j)| \to 0$ *and* $\sup_{i,j} |b_t(i) - b_t(j)| \to 0$ *as* $t \to \infty$.

Proof. Pick numbers $\underline{w} > 0$ and $\bar{w} > 0$ such that $\underline{w} \leq w \leq \bar{w}$ for every initial wealth w. Denote by $\{\underline{k}_t\}$ the optimal path followed by a hypothetical individual with initial wealth $\underline{w}$, and by $\{\bar{k}_t\}$ the optimal path followed by a hypothetical individual with initial wealth $\bar{w}$. By a non-crossing argument (see Mitra (2009), Proposition 1),

$$\underline{k}_t \leq k_i(t) \leq \bar{k}_t \tag{21}$$

for all individuals i and dates t. Moreover, Lemma 1 together with our assumptions on f guarantee that all the assumptions of Theorem 2 in Mitra (2009) are satisfied, so that

$$\bar{k}_t - \underline{k}_t \to 0 \text{ as } t \to \infty. \tag{22}$$

Combining (21) and (22), we must conclude that for any two optimal paths for individuals i and j from the given set of initial wealths,

$$\sup_{i,j} |k_t(i) - k_t(j)| \to 0 \text{ as } t \to \infty. \tag{23}$$

which proves the first part of the assertion. Using the fact that for every i, $b_t(i) = f(k_t(i)) - k_{t+1}(i)$, the second part follows directly from (23). □

Lemma 7. *In any equilibrium $k_t(i) > 0$ and $b_t(i) > 0$ for all t, for all i in a set M of full measure.*

Proof. Because $f(0) = 0$, it will suffice to prove that $b_t(i) > 0$ for all t, for all i in some set M of full measure. Suppose not. Then there exists a *first* date t such that for all i in some set M' of positive measure σ, $b_t(i) = 0$. It follows from (3) that $\mu_t(b_t(i)) \leq u(0, \sigma) - d$ for all such i, for some $d > 0$.

Suppose that $t \geq 1$. Then, given how t has been chosen, $b_{t-1}(i) > 0$ for all i in a subset M'' of M' of (relatively) full measure. Pick any such i. Note that μ_{t-1} is concave and so is continuous at $b_{t-1}(i) > 0$. Therefore a small reduction in b_{t-1} will lead to a vanishingly small loss of utility at $t - 1$, but if the extra investment is fully consumed at date t it will lead to a utility jump of at least d, which contradicts the optimality of i's choices.

Otherwise $t = 0$. Because i's initial wealth is strictly positive, and μ_τ is strictly increasing at 0 for all τ, there is a first date $s > 0$ such that $b_s(i) > 0$. Once again, using the concavity of μ_s, and therefore its continuity at all positive consumptions, a small reduction in $b_s(i)$ will lead to a vanishingly small loss of utility at s. Carry out this reduction and consume the freed investment at date 0; once again we have a contradiction. □

Now we can update Lemma 1 a bit.

Lemma 8. *In any equilibrium, μ_t is continuous and concave for all t.*

Proof. Lemma 1 already establishes concavity and therefore continuity everywhere except (possibly) at zero, while Lemma 7 removes mass points at zero and therefore establishes continuity everywhere. □

Lemma 9. *Denote by b_t the average consumption among the set of individuals in the full-measure set M. Then* $\limsup_t b_t > 0$.

Proof. Suppose, on the contrary, that $b_t \to 0$ as $t \to \infty$. Then, by Lemma 6, the consumption budgets of *all* agents converge to 0 as $t \to \infty$. We can therefore also claim that $k_t(i) \to 0$ as $t \to \infty$ for every i. For if this were false for some individual j, then $\limsup k_t(j) > 0$ while

$b_t(j) \to 0$. This creates an immediate contradiction provided that u is strictly increasing in absolute consumption c. Otherwise, we are in the pure status scenario and the details of the argument are relegated to this footnote.[19]

Define $\underline{k}$ to be any stock such that $\delta f'(\underline{k}) \equiv \rho > 1$. For every $\epsilon > 0$, there is a date T such that for all dates $t \geq T$, $k_t(i) < \underline{k}$ for all individuals i in some set $M(\epsilon) \subseteq M$ of measure at least $1-\epsilon$. Fix some such (small) $\epsilon > 0$.

Choose thresholds θ_1 and θ_2 such that $\epsilon < \theta_1 < \theta_2 < 1$. For each date t, pick c_t^1 and c_t^2 by $F_t(c_t^i) \equiv \theta_i$, for $i = 1, 2$.[20] Let μ_t^+ stand for the right-hand derivative of μ_t. Then by concavity of μ_t,

$$\mu_t^+(c_t^1)[c_t^2 - c_t^1] \geq \mu_t(c_t^2) - \mu_t(c_t^1) = u(c_t^2, \theta_2) - u(c_t^1, \theta_1)$$

for every t. Therefore

$$\mu_t^+(c_t^1) \geq \frac{u(c_t^2, \theta_2) - u(c_t^1, \theta_1)}{c_t^2 - c_t^1} \to \infty \text{ as } t \to \infty \tag{24}$$

where we use the fact that $c_t^2 - c_t^1 \to 0$ as $t \to \infty$,[21] and the assumption that $u(c, s)$ is strictly increasing in s everywhere.

Now we study the consumption budget paths of individuals in the set $M(\epsilon)$. For every $i \in M(\epsilon)$ and each $t \geq T$, $b_t(i) > 0$ for all t and i, by Lemma 7. The Euler equation therefore assures us that

$$\beta_t(b_t(i)) = \delta f'(k_t(i))\beta_{t+1}(b_{t+1}(i)) \geq \rho\beta_{t+1}(b_{t+1}(i)), \tag{25}$$

where are β_t and β_{t+1} are appropriately chosen supports to the functions μ_t and μ_{t+1} respectively. Because $\rho > 1$, it follows from (25) that the right-hand derivatives of μ_t evaluated at $b_t(i)$ — $\mu_t^+(b_t(i))$ — are bounded in t (in fact, these derivatives converge to 0). Therefore, by (24),

$$\mu_t^+(c_t^1) > \mu_t^+(b_t(i)) \tag{26}$$

for all t large enough, which implies in particular that $c_t^1 < b_t(i)$ (for all t large enough). More can be said from this: it must be the case that for all *realized* consumptions $c_t(i)$,

$$c_t^1 < c_t(i) \text{ a.s.} \tag{27}$$

for all t large enough. For once (26) holds, i will never take a fair bet that yields outcomes below $c^1(t)$ with positive probability.[22]

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[19]In the pure status case, proceed as follows. If our claim is false, then it must be false for *every* i, by Lemma 6. For each $\epsilon > 0$, we can therefore find a strictly positive consumption level d and some date T such that $F_t(d) \geq 1 - \epsilon$ for all $t \geq T$, while every agent can maintain a stationary consumption of at least d starting from T. (The former is true because $b_t(i) \to 0$ for all i, and the latter is true because $k_t(i)$ is bounded away from 0 for all i along a *common* subsequence of dates and $\delta f'(0) > 1$.) But then *every* individual, by an appropriate deviation starting from T, could enjoy a status of at least $1 - \epsilon$ for every period thereafter. Such payoffs cannot happen in equilibrium for everyone, so some agents must indeed want to deviate, a contradiction.

[20]Because μ_t is concave, F_t is continuous in the interior of its support and so c_t^1 and c_t^2 can indeed be chosen as required.

[21]Because average consumption c_t equals the average consumption *budget* b_t, and because b_t converges to 0, it follows that $c_t \to 0$. The convergence of c_t^1 and c_t^2 to 0 is an immediate consequence.

[22]The utility over all fair bets must be affine in the bets, which cannot happen when (26) holds.

So we see that (27) holds for *every* individual in the set $M(\epsilon)$ which is of measure $1-\epsilon$ at least. It follows that

$$1 - F_t(c_t^1) \geq 1 - \epsilon > 1 - \theta_1,$$

because $\theta_1 > \epsilon$. But this contradicts the fact that $F_t(c_t^1) = \theta_1$ for all t. □

Lemmas 6 and 9 allow us to establish

Lemma **10.** *There exists $\sigma > 0$ so that for every $\epsilon > 0$, there is a date T*

$$b_T(i) \in [\sigma - \epsilon, \sigma + \epsilon] \tag{28}$$

for all i.

Proof. Let $\sigma \equiv \limsup_t b_t$. Then $\sigma > 0$, by Lemma 9. By Lemma 6, $b_t(i)$ must converge — uniformly in i, and along a common subsequence independent of i — to σ for all i. But this means that for every $\epsilon > 0$, there exists a date T such that $b_T(i) \in [\sigma - \epsilon, \sigma + \epsilon]$ for all i. □

Lemma **11.** *Pick any number $\sigma > 0$. Then there exists $\psi > 0$ such that for all $\epsilon < \sigma/2$,*

$$F_t(\sigma + \epsilon) - F_t(\sigma - \epsilon) \leq \psi\epsilon \tag{29}$$

independently of t.

Proof. Denote by m the smallest value of $u_s(c, s)$ for $c \in [\sigma/2, \sigma]$ and $s \in [0, 1]$. By Assumption 2, $m > 0$. Let $\psi \equiv 2\mu_t^+(\sigma/2)/m$. Then it is easy to see, using concavity, that for all $\epsilon < \sigma/2$,

$$\begin{aligned}
\psi\epsilon m = 2\epsilon\mu_t^+(\sigma/2) \geq 2\epsilon\mu_t^+(\sigma - \epsilon) \quad &\geq \quad \mu_t(\sigma + \epsilon) - \mu_t(\sigma - \epsilon) \\
&= \quad u(\sigma + \epsilon, F_t(\sigma + \epsilon)) - u(\sigma - \epsilon, F_t(\sigma - \epsilon)) \\
&\geq \quad u(\sigma - \epsilon, F_t(\sigma + \epsilon)) - u(\sigma - \epsilon, F_t(\sigma - \epsilon)) \\
&\geq \quad m[F_t(\sigma + \epsilon)) - F_t(\sigma - \epsilon)],
\end{aligned}$$

which is what needed to be proved. □

We now combine Lemmas 10 and 11 to prove

Lemma **12.** *There exists a date T such that for every i, $b_T(i)$ belongs to the interior of the same affine segment of μ_T; in particular, $\mu_T'(b_T(i))$ is a constant independent of i.*

Proof. Fix σ as given in Lemma 10, and then ψ from Lemma 11. Pick $\epsilon' \in (0, \sigma/2)$ so that $\psi\epsilon' < 1/2$. By Lemma 11,

$$F_t(\sigma - \epsilon') + [1 - F_t(\sigma + \epsilon')] > 1/2. \tag{30}$$

Choose $\epsilon < \epsilon'/7$. By Lemma 10, there is a date T so that (28) is satisfied. We claim that at that date, some individual must take a fair bet F with $\inf F < \sigma - \epsilon \leq \sigma + \epsilon < \sup F$.

Suppose the claim is false. Observe from (30) that either $F_t(\sigma - \epsilon')$ or $1 - F_t(\sigma + \epsilon')$ exceeds $1/4$. Suppose the former. Then there is *some* individual with consumption budget in $[\sigma - \epsilon, \sigma + \epsilon]$

who accepts a bet F with $F(\sigma-\epsilon') > 1/4$, so that $\inf F < \sigma - \epsilon$. The expected utility of this individual is

$$\begin{aligned}\int \mu_T(c)dF(c) &\leq (1/4)\mu_T(\sigma-\epsilon') + (3/4)\mu_T(\sigma+\epsilon)\\ &\leq \mu_T\left(\sigma + \frac{3\epsilon-\epsilon'}{4}\right)\\ &< \mu_T(\sigma-\epsilon) \qquad (31)\end{aligned}$$

where the first inequality follows from $\inf F < \sigma - \epsilon$ and our supposition that the claim is false, so that $\sigma+\epsilon \geq \sup F$, the second inequality follows from the concavity of μ_T (Lemma 8), and the last inequality from our choice of ϵ. But (31) contradicts the supposed willingness of the individual to accept such a bet.

Now suppose, on the other hand, we have $1 - F_t(\sigma+\epsilon') > 1/4$ Then there is some individual with consumption budget b in $[\sigma-\epsilon, \sigma+\epsilon]$ who accepts a fair bet F with $1-F(\sigma+\epsilon') > 1/4$, so that $\sigma+\epsilon < \sup F$. If the claim is false, then $\inf F \geq \sigma - \epsilon$ and

$$\int c dF(c) \geq (1/4)(\sigma+\epsilon') + (3/4)(\sigma-\epsilon) > \sigma+\epsilon \geq b,$$

which contradicts the fairness of the bet.

This proves the claim: at date T, some individual must take a fair bet F with $\inf F < \sigma - \epsilon \leq \sigma + \epsilon < \sup F$. By Lemma 4, μ_T must be affine on $[\inf F, \sup F]$. □

Lemma **13.** *For every date $t \geq T+1$, where T is given by Lemma 12, the wealths, investments and consumption budgets of all agents must fully coincide.*

Proof. By Lemma 12, we see that $\mu'_T(b_T(i)) = \gamma_T$ for all i, where $\gamma_T > 0$ is independent of i. Let $V_{T+1}(w)$ be the value function at date $T+1$; it is concave because μ_t is concave for all t. Therefore $V_{T+1}(f(k))$ is *strictly* concave.

Note that $b_T(i) > 0$ for all i, and that by Lemmas 6 and 7, $k_T(i) > 0$ for all i as well. So using the Bellman equation between dates T and $T+1$, and writing the first order condition, we see that

$$\gamma_T = \delta\beta_i(k_T(i))$$

for every agent i, where $\beta_i(k)$ denotes some supporting hyperplane to $V_{T+1}\circ f$ at k. Therefore $\beta_i(k_T(i)) = \beta_j(k_T(j))$ for all agents i and j. Because $V_{T+1}\circ f$ is strictly concave, this must imply that $k_T(i) = k_T(j)$. So the wealths of all agents fully coincide at date $T+1$. It is easy to see that optimal programs are unique starting from any initial wealth at any date,[23] so all wealths, investments, and consumption budgets must fully coincide from date $T+1$ on. □

In what follows, we consider only dates $t > T$. By Lemma 13, then, we may regard the equilibrium program as having common actions at all dates thereafter: (w_t, k_t, b_t), where all these values are strictly positive. By Lemma 5, the distribution F_t is also fully pinned down at all these dates. Denote by γ_t the corresponding slopes of the affine segments of μ_t, given by (7); these too are all strictly positive.

[23]The optimization problem facing each individual is strictly concave.

Lemma 14. *Suppose that for some $t \geq T+1$, we have $k_t \leq k_{t+1}$ and $\gamma_t \leq \gamma_{t+1}$. Then $k_s \leq k_{s+1}$ for all $s \geq t$.*

Proof. Suppose that for some t, we have $k_t \leq k_{t+1}$ and $\gamma_t \leq \gamma_{t+1}$. The Euler equation for utility maximization by each agent holds with equality, and this, combined with the concavity of f, tells us that

$$\frac{\gamma_t}{\gamma_{t+1}} = \delta f'(k_t) \geq \delta f'(k_{t+1}) = \frac{\gamma_{t+1}}{\gamma_{t+2}},$$

which permits us to conclude that $\gamma_{t+1} \leq \gamma_{t+2}$. Using the very last part of Lemma 2, and remembering from Lemma 5 that F_s is the unique cdf associated with b for all $s \geq T+1$, we must also conclude that $b_{t+1} \geq b_{t+2}$. Therefore

$$k_{t+1} = f(k_t) - b_{t+1} \leq f(k_{t+1}) - b_{t+2} = k_{t+2}.$$

We have therefore shown that $\gamma_{t+1} \leq \gamma_{t+2}$ and $k_{t+1} \leq k_{t+2}$. We can continue the recursive argument indefinitely to show that $k_s \leq k_{s+1}$ for all $s \geq t$. □

Lemma 15. *The common sequence of investments $\{k_t\}$, defined for $t \geq T+1$, must converge to k^*, which solves $\delta f'(k^*) = 1$.*

Proof. First we establish convergence. Suppose that for *every* date $t \geq T+1$ with $k_t \geq k_{t+1}$, we also have $k_{t+1} \geq k_{t+2}$. Then $\{k_t\}$ must converge. Otherwise, there is some date $t \geq T+1$ with $k_t \geq k_{t+1} < k_{t+2}$. Then

$$b_{t+1} = f(k_t) - k_{t+1} > f(k_{t+1}) - k_{t+2} = b_{t+2},$$

which implies (by Lemma 2) that $\gamma_{t+1} \leq \gamma_{t+2}$. But now all the conditions of Lemma 14 are satisfied, so that in this case $\{k_t\}$ must eventually be nondecreasing. By Assumption 7, k_t is bounded and so must converge. It follows that b_t and therefore γ_t also converge. Passing to the limit using the Euler equations, we must conclude that $\lim k_t = k^*$. □

Proof of Proposition 7. Lemma 12 assures us that there exists a date T at which consumption budgets $b_T(i)$ belong to the same affine segment of μ_T for every i. Lemma 13 states that for every date $t \geq T+1$, the wealths, investments and consumption budgets of all agents must fully coincide. Lemma 15 states that the common sequence of investments $\{k_t\}$, defined for $t \geq T+1$, must converge to k^*, which solves $\delta f'(k^*) = 1$.

At the same time, Lemma 5 asserts that for all $t \geq T+1$, the equilibrium distribution of consumptions must be the unique cdf associated with the common consumption budget b_t, where "association" is defined (and uniqueness established) in Lemma 2. Therefore the sequence of consumption distributions must converge to the unique cdf associated with $b^* = f(k^*) - k^*$. This is the unique steady state described in Proposition 4, and the proof is complete. □

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