DO VOTING SCHEMES MATTER?

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INTRODUCTION
It’s a cliché that when we want to make a group decision, the first option that comes to mind is “Let’s take a vote!” It makes intuitive, fair and obvious decisions difficult if there are differences of opinion about which candidate to go with or what movie to see. The fact that there are many methods of voting and not all methods produce the same result is often ignored. This is an important issue in social choice, especially corporate governance where the voting scheme used to elect a company’s board of directors influences its composition and, therefore, the company’s responsiveness to shareholder concerns.

THE EXPERIMENT
Students vote to elect 5 members of an Extra Credit Committee. All students are included on the ballot. Three votes are taken.

1. Approval Voting: voters cast one vote for up to 5 people
2. Borda Voting: voters rank all the candidates from first to last with no ties or omisions
3. Cumulative Voting: voters allocate 5 votes among their preferred candidates in any way they choose

Voting results are confidential. For the test, a question is randomly selected as the extra credit question. One point is added to the score of everyone in the class for each member of the elected committee that answers the question correctly.

After the test results are tabulated, the results of best performing committee on the extra credit because members are identified only by randomly assigned alphanumeric codes. Selected by the different voting schemes. The composition of the committees remains anonymous (usually) different committees selected by the different voting schemes as well as the (usually) question determine the extra credit for the class. The experiment is debriefed by discussing the committee that answers the question correctly.

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Pre-Test Results
After a lecture and discussion of the pros and cons of each method, students are asked which scheme will select the most effective committee. The results of this survey are below.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Approval</th>
<th>Borda</th>
<th>Cumulative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approval</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Borda</td>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Cumulative</td>
<td></td>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

RESULTS OF FIRST ELECTION

<table>
<thead>
<tr>
<th>VOTING SCHEME</th>
<th>COMMITTEE ELECTED</th>
<th>EXTRA CREDIT</th>
<th>TEST AVERAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approval</td>
<td>ABCDE (G)</td>
<td>5/5</td>
<td>85%</td>
</tr>
<tr>
<td>Borda</td>
<td>ACBDE</td>
<td>5/5</td>
<td>90%</td>
</tr>
<tr>
<td>Cumulative</td>
<td>ABCD (E)</td>
<td>5/5</td>
<td>88%</td>
</tr>
</tbody>
</table>

The schemes elect different committees and those committees perform differently. While both the APPROVAL and CUMULATIVE committees earned the maximum extra credit for the class, the BORDA committee might have been a better choice because the committee had the highest average test score and the extra credit question was randomly selected. It was also noted that if a different committee had been selected, the chances of earning points with a randomly selected question.

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MULTIPLE ITERATIONS, NO CLEAR WINNERS
This exercise was repeated twice. After the first test, most students chose Borda as their preferred method. When the results for the committee elected for the second test were similar to those of the first, students again began to move all of their preferred voting methods.

This provided a jumping off point to discuss Arrow’s Theorem and the impossibility of making a universally correct choice. Students can then discover that none of the voting schemes always produces the best result.

Approval may require more information than the voters have. Full rankings may not be possible. Cumulative voting allows people to indicate strength of preference. Approval and Cumulative voting allows voters to use incomplete information, if they don’t feel able to evaluate all candidates.

The rationality conditions are listed and discussed below.

1. UNIVERSE ADMINISTRABILITY: Every voter can have some set of rational preferences. For example, the party of each candidate can determine which is the most advantageous candidate to that voter. If the candidates are equally desirable and individual preferences are transitive.
2. UNIVERSAL ADMINISTRABILITY: Every voter can have any set of rational preferences. For example, the party of each candidate can determine which is the most advantageous candidate to that voter. If the candidates are equally desirable and individual preferences are transitive.
3. INDEPENDENT OF IRRELEVANT ALTERNATIVES: Suppose every voter prefers option A to option B and B to option C. Then Arrow’s Theorem implies that if it is preferred in B and it is preferred in C, then it is preferred to C. This means that voting produces intransitive results.

Approach: 

-transitivity implies that if A is preferred to B and B is preferred to C, then A is preferred to C.

Arrow’s Theorem
There are many ways to formulate Arrow’s Theorem. One of the most intuitive states that any voting scheme with 3 or more options that satisfies four basic rationality conditions will contain a cycle. A voting cycle is a situation where option A wins against option B and B wins against C but C wins against A. This means that voting produces intransitive results.

Transitivity implies that if A is preferred to B and B is preferred to C, then A is preferred to C. This is a natural segue to an introduction to Arrow’s Impossibility Theorem, one of the cornerstones of social choice theory.