The Education Risk Premium

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Abstract

College graduates earn substantially higher lifetime income than do workers who do not graduate college. This skill premium is both persistently high and has increased over time. Nonetheless, the skill premium has not been accompanied by exceedingly high college enrollment rates: close to one-third of all high-school graduates currently do not enroll in any form of college.

In this paper, we reconcile observed college enrollment with a high skill premium. We show that when households face empirically observed failure and earnings risk, even skill premia in excess of current levels should not be associated, ceteris paribus, with far higher enrollment rates than seen now. We also show that subsidies to college likely play a very important role in the size of the response of enrollment to skill premia.

Our findings help explain what Altonji, Baradwaj, and Lange (2008b) term the “anemic” response of enrollment to changes in the skill premium, and arise from the following simple and intuitive mechanism. The presence of failure risk generates asymmetric changes in the net return to college investment: those with low failure risk see a large increase in expected returns, but are inframarginal because they will enroll under most circumstances. Those with high failure risk see a much smaller increase in expected returns, and hence remain largely inframarginal. Lastly, despite this dampening effect of risk on enrollment, we also show that education subsidies of various forms are sufficiently effective to be positive NPV projects from the point of view of a tax-paying public.

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1 Introduction

The skill premium is one of the most robust empirical facts in economics. An individual who has completed college can expect to earn over her lifetime between 1.5 and 1.7 times as much in present value terms as her non-college-educated counterpart. Human capital therefore appears to generate an enormous premium, and one that far exceeds that historically available on financial market (traded) equity. At its peak, the financial market equity premium identified by Mehra and Prescott (1985) averaged 6%. By nearly all estimates, the college premium has consistently averaged approximately twice that much: between 10 and 15%, depending on the measure used (see, e.g. Goldin and Katz (2000)). A second striking observation concerns the magnitude of the response of enrollment to changes in the college premium. When measured by the ratio of hourly wages of skilled to unskilled workers, the college premium increased by nearly 20% between 1980 and 1996 (see Autor, Katz, and Krueger (1998)). However, enrollment did not respond substantially. Over the period 1979-2005, even though the fraction of young adults (29 years and younger) with a college degree rose by 9 percentage points (23% to 32%), the increase in male enrollment accounted only for one percentage point (Bailey and Dynarski (2009)).

The broad trends in college enrollment and the skill premium are shown in Figure 1. At a glance, one sees that even though enrollment is currently at its historically highest level, approximately one-third of all high-school completers still do not immediately proceed to any form of college.

Figure 1: Recent Trends in Enrollment Rates and Skill Premia

1See e.g., Restuccia and Urrutia (2004), or Heckman (2007).
Averett and Burton (1996) argue further that changes in wage premia by themselves had little effect on female enrollment rates growing over this period, perhaps reflecting other, more transitional phenomena arising from broader social changes. Bound, Loevenheim, and Turner (2007) report similar results, showing not only that enrollment rates failed to rise substantially, but that they even fell for some groups. Moreover, as documented by these authors, the completion rate for current cohorts has fallen, and those who do complete appear to take longer to do so. As a result, the overall response of enrollment—and subsequent skill formation—to changes in the skill premium itself is typically described as “anemic,” as argued by Altonji, Bharadwaj, and Lange (2008).

How should one interpret these observations? Should one take the observed premium to college completers and the apparent insensitivity of enrollment to increases in skill premia as reflective of important constraints facing households considering investments in human capital? Or, instead, do these features reflect compensation for, and responses to, an investment opportunity that is lumpy, irreversible, and most crucially, risky? The goal of this paper is to address the preceding by posing and answering two more specific questions. First, what does theory predict that enrollment rates across various ex-ante heterogeneous groups should be under a given skill premium? Second, given the underlying joint distribution of wealth and collegiate-preparedness, how large should one expect the response of enrollment of a representative cohort to be in the face of changes in the college premium?

Our findings are as follows. First, we will demonstrate that a fairly simple model of college enrollment that is quantitatively accurate in its representation of the lumpiness, irreversibility, and risk inherent in the college entrance decision can reconcile the high rates on return available to those who succeed with observed rates of enrollment, as well as the observation that changes in skill premia have not been met with by “large” changes in enrollment. In particular, we will show that an investment in human capital is unlikely to be a good deal for significant portions of the US population, even at rates of return that appear, a priori, to be extremely high, and even when no one is constrained with respect to financing college. Second, we will show that the underlying heterogeneity in failure risk and wealth is such that a large share of potential college enrollees are inframarginal with respect to the skill premia. In other words, our benchmark model suggests that those who did not enroll in college have been, by and large, those who still would not enroll even when the skill premium increases. Moreover, even to the extent that increases in skill premia increase enrollment rates, the incremental populations will be increasingly less well-prepared, and will therefore fail at higher rates than the cohorts who enrolled in the pre-increase period. As a result, the effective increase in the stock of skilled labor associated with from an increase in the skill premium will be reduced by this composition effect. Third, our model suggests that current higher education policy, particularly the large direct subsidies which reduce the out-of-pocket costs for all enrollees irrespective of need or preparedness, are playing an
important role. The model shows that as a quantitative matter, when faced with observed skill premia, but subsidy rates that are much lower than current levels, *far fewer* would enroll than currently do.

Our emphasis above on the role of uninsurable risks associated with the collegiate investment decision is motivated by four related pieces of evidence. First, there is abundant evidence for “completion risk,” measured by the probability that a student will fail to complete college. Failure rates at public 4-year colleges, which account for the majority of undergraduate enrollment, are approximately 50% (see e.g. Bound, Loevenheim, and Turner (2007), and NCES (2001)). Second, the uncertainty over eventual completion is not quickly resolved: it takes, at the median, two years of foregone earnings, and explicit costs of tuition to realize an earnings stream that may deliver a near zero return. Third, from all available evidence, the return to partial completion of college is low (i.e. attending but not obtaining a diploma); early documentation includes Psacharapoulous and Layard (1974), and more recently Hungerford and Solon (1987). Lange and Topel (2006) argue forcefully that the most reasonable interpretation of this is that students learn about their future productivity. These authors also take the data as suggesting that the bulk of learning takes place in the latter parts of college. The lumpiness of initial investment along with the poor returns to non-completers render failure risk potentially very important to would-be enrollees. Fourth, even upon completing college, a vast literature, starting perhaps most famously with Lillard and Willis (1978) and continuing to the present (e.g. Heathcote, Storesletten, and Violante (2009)) has documented the presence of significant uninsurable idiosyncratic risk (in addition to aggregate risk) in the returns to human capital. Even the successful college completer is not guaranteed anything. In particular, even college educated households face earnings processes with substantial persistent (and by several accounts, e.g. Hryshko (2010), nearly unit root) uninsurable shocks. It is therefore entirely possible for relatively young college graduates to receive earnings shocks that immediately, and substantially, lower the expected present value of remaining lifetime income. Finally, the persistence of these shocks also makes them inherently difficult to self-insure as well, making the absence of market-based insurance more problematic.

One key aspect of our analysis is that we do not impose frictions on credit markets. In part this is because of the existence of significant policy interventions aimed at ameliorating credit-related impediments to college financing. In particular, the statutory availability of federally subsidized student loans in amounts capable of covering the entire cost of most four-year degree-granting institutions (Stafford loans, plus the PLUS loan program), and the most detailed measurement of borrowing constraints for college-bound households available to date, that of Carneiro and Heckman (2002), both cast serious doubt on the strength of borrowing constraints. The latter in particular finds that few households are meaningfully “borrowing constrained” at the time they decide on collegiate enrollment. Thus, the most commonly cited constraint, that of limits on the ability of enrollees to borrow, seems unlikely to be a quantitatively important barrier to investment.
While the high observed rates of return to investment in human capital cannot easily be ascribed directly to credit market frictions, credit will in general interact with the frictions we emphasize. Specifically, leverage magnifies the impact of uninsurable risks. For a household with currently low wealth and non-trivial failure risk, for example, financing education with a fundamentally non-contingent instrument, such as debt, magnifies the risk of failure. Were default possible, this is precisely the type of event in which the bankruptcy option would be most beneficial to households. It is therefore critical that U.S. government-guaranteed student loans are explicitly non-dischargeable in bankruptcy. As a result, an enrollee who experiences failure must lower long-run consumption even more than they otherwise might have to, while also smoothing the transition. Ex-ante, the lottery over future consumption (especially in the near-term) induced by debt-financed college enrollment, ceteris paribus, makes college less attractive. We will show that even without direct credit constraints, students do not always go to college even when the financial returns appear to be positive.

These results explain why college enrollment is not universal, even when the rate of return appears to be high, and why enrollment appears insensitive to further increases in the skill premium. Students expect to receive the skill premium upon college completion. By contrast, education subsidies and financial aid are conditional only on enrollment, not on completion. Hence, the dampening role of failure risk is reduced, because students receive the aid regardless of graduation or not. We show that indeed such policies are somewhat effective in promoting enrollment. Importantly, even accounting for failure risk, we show that these policies are often positive NPV investment projects from the point of view of the tax-paying public. The policies counter failure-risk-aversion and encourage a sufficient amount of enrollment and completion to be self-financing out of subsequent tax revenue. Of course, a fundamental friction in the model is failure risk itself, so we conclude by examining the benefits of reducing failure probabilities (rather than policies to counter a given distribution of failure probabilities).

The vast literature on human capital acquisition has long emphasized its importance (see e.g. Altonji et al. (2008), Goldin and Katz (2008)). Leveling access to education has been viewed as among the least distortionary ways in which to encourage greater equality within society. To the extent that unequal access to human capital acquisition is to blame for subsequent inequality in earnings and wealth, expanding educational opportunities directly limits the growing dispersion in income and wealth that now occurs dramatically over the life-cycle (see e.g. Storesletten, Telmer, and Yaron (2004)). Of course, education has also long been viewed as an engine of growth, both through direct effects on the accumulation of a factor of production, but also through indirect “spillover” effects which hold the promise of increasing returns and thereby efficiency.

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2 Recent legislation has allowed for more income-based repayment options to make student loans more equity-like. However, these options are available only under limited circumstances. In practice, the Department of Education does enforce the no-bankruptcy rule, making it the largest U.S. garnisher of wages behind the IRS.
gains. Our model addresses the first issue, but abstracts from any growth externalities.

Our work is related most closely to early work of Altonji (1993), Chen (2001), and more recently to a series of quantitative general equilibrium models of higher education. The latter include important papers of Heckman, Lochner and Taber (1998a,b), and Restuccia and Urrutia (2004). Most recently, related work includes He (2005), Akyol and Athreya (2005), Garriga and Keightley (2007), Gallipoli, Meghir, and Violante (2010), Ionescu (2009), Schiopu (2009), and Castex (2009, 2010). Several of the preceding papers study higher education decisions in settings where enrollees may fail. Aside from our work, fewer papers, however, feature both failure risk and rate of return risk, with some examples including Chen (2001), Restuccia and Urrutia (2004) and Akyol and Athreya (2005). A paper that is highly complementary to ours is that of Ionescu and Chatterjee (2010) who study the problem of how to insure against college failure risk, and in turn, show that an insurance program can increase enrollment rates substantially—suggesting that risk is indeed a relevant consideration in enrollment decisions.

The main distinctions between our paper and existing work are twofold. First, while our model structure shares features in common with existing work, we employ the model rather differently. The previously cited general equilibrium work first specifies policies, and then aims at understanding their (long-run) implications for prices (and allocations). By contrast, our approach is to first specify prices—and all other objects that are parametric to the individual agent—and then analyze the individual-level enrollment decision and ask what, when aggregated, such decisions should lead one to expect vis-a-vis enrollment and failure.

The approach taken here allows us to address a question of central interest to us: to what extent is a given skill premium by itself responsible for, or capable of, explaining observed enrollment rates? Relatedly, we ask to what extent such rates are dependent in important ways on other aspects of the household’s environment, such as subsidies or need-based aid. Our approach also will help shed light on why certain constellations of skill premia and policy may not be sustainable as long-run outcomes, as they might be associated with extremely high or low enrollment rates. A main finding of the model suggests that the current skill premium is not even close to sufficient for generating observed enrollment. Instead, it is only when current skill premia are combined with observed rates of subsidies and need-based aid that one generates reasonable enrollment rates. The insufficiency of the skill premium to spur enrollment is initially surprising, but we will show that it follows fairly naturally from the presence of risk and heterogeneity in household wealth and preparedness.

Our second innovation is in the modeling structure, which employs the richest model of both failure- and rate-of-return risk of which we are aware. We are motivated in particular by estimates of Chen (2001) showing that both transitory and persistent risk components are important in accounting for the rate of return to college, and our model accommodates both forms of risk.\footnote{A third, and more minor, distinction between our approach and the preceding literature is that we choose parameters directly to match their empirically observable counterparts, rather than calibrating parameters such that the model generates...}
2 Model

We study the decision problem of a household in an environment in which college investment carries the three classes of risk discussed above. First, students must decide whether or not to enroll in college, given failure risk. Second, subsequent to completion of college investment, and regardless of its success, households will choose consumption and savings, given earnings risk. Third, all potential enrollees are restricted to the use of pure non-defaultable debt if their personal resources at the time of enrollment are insufficient to finance college investment, exposing them to leverage risk.

2.1 Preferences

Households go through three phases in life: they are born Young at which point they make human capital investment decisions, they work as Adults, and then they are Retirees. Households are Young for $K$ model periods, to reflect the period between high school and successful college completion. Households then become workers for $J$ periods, which will be set to reflect the length of time between the age at college completion and retirement age. Young and Adult households order stochastic processes over consumption using a standard time-separable CRRA utility function. As Retirees, households value resources taken according to a “retirement felicity function”, $\phi$, that is defined on wealth $x_R$ taken into retirement. All households have a common discount factor $\beta$ and discount exponentially.

The general problem for the Young household is to choose consumption $\{c_k\}_{k=1}^K$ and make risky human capital investment (enrollment) decisions. Their enrollment decisions will leave them with a human capital level $h \in \{HS, SC, C\}$ corresponding either to high school ($HS$), some college ($SC$), or college ($C$) attainment which, to avoid clutter, we will suppress in the notation below wherever it is obvious. The realized human capital attainment conditional on the enrolling will depend on the realization of uncertainty over college completion. When Adults, households then choose consumption $\{c_j\}_{j=1}^J$, and then wealth $x_R$ with which to enter retirement.

Denote by $\Theta(\Psi)$ the set of feasible combinations $(\{c_k\}, \{c_j\}, x_R)$, given initial state $\Psi$. The household’s optimization problem is then:

$$\sup_{((c_k), (c_j), x_R) \in \Theta(\Psi)} E_0 \sum_{j=1}^J \beta^j \frac{c_j^{1-\sigma}}{1-\sigma} + \phi(x_R)$$

(1)

matching moments. We will show that nonetheless, this parsimonious structure accounts well for the behavior of enrollment in college and its response to changes in the skill premium. Some details: Ionescu (2009) abstracts from both failure risk and subsequent riskiness of returns to human capital, which are the risks of central interest in this paper. Ionescu and Chatterjee (2010), by contrast, allow for failure and moral hazard in effort while enrolled. The model is stylized, and like Ionescu (2009), abstracts from earnings risk. In addition, for policy analysis, the reader is directed to the particularly rich models of Garriga and Keigthley (2007) and Gallipoli et al. (2010) who begin, unlike us, by calibrating their model to match enrollment behavior.
Retirement felicity as a function of retirement wealth takes the same form as preferences over consumption in working life, but also includes a weighting factor $\nu$, which will be calibrated.

$$\phi(x_R) = \nu x_R^{1-\sigma} \frac{1}{1-\sigma}$$ (2)

This approach is taken in Athreya (2008) and Akyol and Athreya (2010), and offers a convenient way of generating consumption and wealth accumulation during working life that generates the appropriate valuations of the college investment given a young agent’s state. It is particularly useful given our focus on the early-life decision problem of households who face a given skill premium and earnings and failure risk, as such decisions will remain insensitive to the temporally distant events of retirement.

2.2 Endowments

All agents are endowed with one unit of time, which they supply inelastically.

2.2.1 Labor Income

Young and Adult households face stochastic productivity shocks to their labor supply. Because households do not value leisure, they are modeled as simply receiving stochastic endowments of the single consumption good in each period. The income process faced by households in the model is intended to represent precisely those risks which remain, net of (i) all private insurance mechanisms and (ii) all non-means-tested public insurance programs, such as the US unemployment insurance system.

A key aspect of our approach is to specify an empirically accurate description of the risk to income, subsequent to the enrollment decision. The work of Chen (2001) in particular is suggestive in its assessment of the role played especially by persistent income risk in creating a premium, and thereby being fundamentally in the nature of a compensating differential, for investment in college. We disaggregate log endowments into three components: an age-specific mean of log income $\mu_j$, persistent shocks, $z_j$, and transitory shocks, $u_j$.4

All components of income will depend on the eventual education attainment of an agent, $h$. Our specification follows Hubbard et al. (1994), and specifies log income for a household with human capital $h$ to evolves as:

$$\ln y^h_j = \mu^h_j + z^h_j + u^h_j$$ (3)

where

$$z^h_j = \rho^h z^h_{j-1} + \eta^h_j, \ \rho^h \leq 1, \ j \geq 2$$ (4)

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4Standard specifications of this, are, e.g. Hubbard et al. (1995), Storesletten et al. (2004), Huggett and Ventura (2000).
\[
\ln u^h_j \sim \text{i.i.d } N(0, \sigma^2_{u,h}), \quad \ln \eta^h_j \sim \text{i.i.d. } N(0, \sigma^2_{\eta,h}), \quad u^h_j, \eta^h_j \text{ independent} \quad (5)
\]

In addition, all household begin life as unskilled households, \( h = HS \), and receive their initial realization of the persistent shock, \( z_1 \), from a distribution with different variance than at all other ages. That is,

\[
z_1 = \xi \quad (6)
\]

where

\[
\ln \xi^h \sim N(0, \sigma^2_{\xi}) \quad (7)
\]

The income process can be interpreted as follows. To reflect heterogeneity prior to any direct exposure to labor market risk, households first draw a realization of the persistent shock \( z_1 \) from the random variable \( \xi \) with distribution \( N(0, \sigma^2_{\xi}) \). In subsequent periods, household non-asset income is determined as the sum of the the unconditional mean of log income \( \mu_j \), the innovation to the persistent shock \( \eta_j \) and the transitory shock, \( u_j \). The shocks to labor earnings during working age will depend on the human capital level of agents, to reflect the fact that the risk-characteristics of labor earnings appear to differ systematically by human capital level (e.g. Chen (2001), Hubbard et. al. (1994, 1995), and Storesletten, Telmer, Yaron (2004)).

### 2.2.2 Means-Tested Transfer Income

Our model also allows for means-tested transfers, \( \tau(\cdot) \), represented as a function of current age \( j \), net assets \( x_j \), and income level \( y_j = \exp(\mu_j + z_j + u_j) \). Including this is potentially relevant as it is a source of wealth to households that may be large enough to alter the decisions of related to college investment. In the benchmark model, transfers will not depend explicitly on age. Transfers are specified as in the seminal work of Hubbard et al. (1995):

\[
\tau(j, x_j, y_j) = \max\{0, \tau - (\max(0, x_j) + y_j)\} \quad (8)
\]

and what follows is detailed in Athreya (2008).

### 2.2.3 Retirement Income

As we stated earlier, households select retirement savings according to the function \( \phi(x_R) \). Following Athreya (2008), a household’s wealth level at retirement is then the sum of the household’s personal savings \( x_{J+1} \) and the baseline retirement benefit \( x_\tau \)

\[
x_R = x_{J+1} + x_\tau \quad (9)
\]
This amount $x_\tau$ is the wealth level that, when annuitized at the discount rate $R^f$, and adjusted for the probability of survival for $k$ periods, $\pi_k$, yields a flow of income each period equal to the societal minimum consumption floor $\tau$. That is, minimal retirement wealth $x_\tau$ solves:

$$\sum_{k=1}^{K} \frac{\pi_k x_\tau}{(R^f)^k} = \tau$$

(10)

### 2.3 Young Households and The College Investment Decision

As described above, Young households make decisions for $K$ periods. In the first period, households first draw income shocks which inform them of their potential income if they decide not to enroll in college. If they enroll in college, they cannot work. If their private wealth is insufficient to fund college investment, they must borrow by issuing non-defaultable personal debt. Given the knowledge of both the explicit costs of college, as well as the level of earnings that will be foregone, households make the decision to enroll or not in college. If they enroll, they must attend college for $\tau_1 < K$ periods before they learn whether or not they will succeed. If they are informed that they will succeed, they have the option to invest the remainder of their time $\tau_2 \equiv K - \tau_1$ in college, after which they will enter working life as a Skilled agent. Not all households who are informed of success may choose to continue; they receive earnings shocks in each period, and a sufficiently high and persistent realization of this random variable may make the expected payoff from “dropping out” greater than that of continuing. Simultaneously with college investment, Young households choose consumption and savings. Those choosing to drop out and those failing to succeed then draw income from the shock process applicable to those with “some college.” Thus, the investment is lumpy, as it does not offer partial returns for partial investment. Lastly, after $\tau_2$ periods, Young households transition to being Adults, after which they solve a life-cycle consumption savings problem in the face of stochastic, education-dependent earnings. The preceding structure therefore captures the central features which make human capital investment risky: leverage, completion risk, lumpiness, irreversibility, and risky returns given completion.

#### 2.3.1 Recursive Formulation

Given the timing described above, the recursive formulation is straightforward. The state of the household can be expressed as follows. First, let $k$ and $a$ denote age and household resources at the beginning of the period. For Young agents the wealth level $a$ should be thought of as the transfer that college-bound children are expected to receive from their parents. Next, let $z_{\tau_1}$ and $u_{\tau_2}$ represent the persistent and transitory shocks to earnings received by households, respectively. Lastly, $h$ and $\pi$ denote human capital and failure risk.

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5See again, Athreya (2008) for details.
respectively. The state of a household is summarized by the vector \( x = (k, a, z_{\tau_1}, u_{\tau_1}, h, \pi) \), and to avoid clutter, in what follows we will refer to the household state by \( x \).

In the first period of being Young, households make the decision to enroll in post-secondary education by comparing the value of enrollment \( V^E(x) \) with the value of not enrolling \( V^{NE}(x) \). Therefore, we denote the maximal utility attainable by a young agent in the first period of youth as:

\[
V^{Y_1}(x) = \max(V^E(x), V^{NE}(x))
\]

The value of enrolling is itself the solution to the following problem:

\[
V^E(x) = \max \left[ \frac{c^{1-\alpha} - 1}{1 - \alpha} + \beta^{\tau_1} \left( \pi E_{z,u} V^F(x') + (1 - \pi) E_{z,u} V^S(x') \right) \right]
\]

subject to the budget constraint if they enroll:

\[
c + qa' + \tau_{1,1} \Phi_{1}^{\text{pr}} [1 - \gamma^{\text{need}}(x) - \gamma^{\text{merit}}(x) - \gamma^{\text{direct}}] \leq a_0
\]

\[
a' > a
\]

In the preceding, \( \pi \) denotes the probability of failure faced by the enrollee, and \( V^F(\cdot) \), and \( V^S(\cdot) \) are the values of failure and success, respectively, in the first phase of college. The first phase of college takes \( \tau_1 \) units of time, and therefore future payoffs are discounted accordingly via the term \( \beta^{\tau_1} \). The term \( \Phi_{1}^{\text{pr}} > 0 \) denotes the cost of college, prior to all subsidies directly received by educational institutions from state, local, and Federal sources. Direct subsidies are denoted \( \gamma^{\text{direct}} \), and apply to all enrollees. The terms \( \gamma^{\text{need}}_1 \) and \( \gamma^{\text{merit}}_1 \) denote further proportional reductions in the private cost of college arising from need- and merit-based aid, respectively. Lastly, \( a_0 \) denotes the wealth or resources available to an enrollee (in general, much of this will represent parental resources), and in the event that an agent does not enroll, they can earn \( y(x) \) as labor earnings. Insurance markets against income risk are also incomplete, and all agents are endowed with the ability to save their risk-free assets in a form which earns them return \( 1/q \).

All enrolling students will learn after a period \( \tau_1 \) whether they are performing well enough to successfully complete college, or must leave. If they remain in college, there is no more uncertainty, and the enrollee graduates in \( \tau_2 \) units of time. However, since the agent receives a productivity draw in each period, even those who succeed in the first part of college may choose to drop out. The state vector of a household in the second phase of Young life is given by: \( x = (k, a_0, z_{\tau_1}, u_{\tau_1}, h) \)

These options result in the following value functions. First, the maximal value attainable after a failure from college, \( V^F(\cdot) \), satisfies the following recursion:
\[ V^F(x) = \max \left[ \frac{c^{1-\alpha}}{1-\alpha} + \beta^{\tau z} E_{z,u} V^A(x') \right] \]

and the flow constraint faced by the household if they fail is:

\[ c + qa' \leq a + \tau_2 y^{SC}(x) \]

\[ a' > a \]

Next, the value of success in the first phase of college is given by:

\[ V^S(x) = \max(V^C(x), V^D(x)) \]

with \( V^C(\cdot) \) and \( V^D(\cdot) \) denoting the values of choosing to continue in college, or drop out, respectively. Let \( V^A(\cdot) \) denote the value function applicable to Adults. These value functions are given as follows.

\[ V^C(x) = \max \left[ \frac{c^{1-\alpha}}{1-\alpha} + \beta^{\tau z} E_{z,u} V^A(x') \right] \]

subject to the budget constraint that applies to continuing students:

\[ c + qa' + \tau_2 \Phi_2^{\text{net}} [1 - \gamma_2^{\text{need}}(x) - \gamma_2^{\text{merit}}(x) - \gamma^{\text{direct}}] \leq a \]

\[ a' > a \]

For those choosing to drop out, the value function is:

\[ V^D(x) = \max \left[ \frac{c^{1-\alpha}}{1-\alpha} + \beta^{\tau z} E_{z,u} V^A(x') \right] \]

and the flow constraint they face if they choose to dropout is:

\[ c + qa' \leq a + \tau_2 y^{SC}(x) \]

\[ a' > a \]

If an agent chooses not to enroll, their decision problem collapses to a standard consumption-savings problem. In the first period of being Young, they attain a value function that satisfies:

\[ V^{NE}(x) = \max \left[ \frac{c^{1-\alpha}}{1-\alpha} + \beta E_{z,u} V^Y_2(x') \right] \]
where \( V^{Y_2}(\cdot) \) denotes the maximal value attainable as an agent in the second period of being Young. The constraint households face in the first period of being Young, if they choose not to enroll is:

\[
c + qa' \leq a + \tau_1 y^{HS}(x)
\]

\( a' > a \)

In the second period of being Young, given the continuation value \( V^A(\cdot) \), optimal decisions imply that \( V^{Y_2}(\cdot) \) satisfies:

\[
V^{Y_2}(x) = \max \left[ \frac{c^{1-\alpha}}{1-\alpha} + \beta E_{\zeta,u}V^A(x') \right]
\]

subject to the associated constraints:

\[
c + qa' \leq a + \tau_2 y^{HS}(x)
\]

\( a' > a \)

### 2.4 Adults

Once agents are Adults, they face a finite horizon consumption savings problem, given an income process which fluctuates about a deterministically evolving mean that reflects the accumulation of experience and human capital over the life-cycle. Both the shock processes and the evolution of the mean earnings process will reflect educational attainment. Therefore, optimal decision making of adults will satisfy the Bellman equation:

\[
V^A(x) = \max \left[ \frac{c^{1-\alpha}}{1-\alpha} + \beta E_{\zeta,u}V^A(x') \right]
\]

subject to the flow budget constraint

\[
c + qa' \leq a + y^h(x)
\]

\( a' > a \)

Lastly, in the period immediately prior to retirement, households’ optimal decisions satisfy:

\[
V^A(x) = \max \left[ \frac{c^{1-\alpha}}{1-\alpha} + \beta \phi(x_R) \right]
\]
subject to the flow budget constraint

\[ c + x_R \leq a + y^h(x) \]

\[ x_R > 0 \]

2.5 Aggregating Individual Decisions to Enrollment and Failure Rates

As clarified at the outset, our primary focus will be on understanding the investment decision of a cohort of young enrollees. To do this, we solve for the flow of new enrollees predicted by theory, under a given expected skill premium, educational policy, and the joint density of failure risk and available resources for college. We will show that despite the seeming attractiveness of college from the perspective of risk-neutrality, once risk-aversion and empirically reasonable measures of failure are taken into account, significant portions of the population will elect not to enroll in college at current skill-premia.

Given any constellation of behavioral parameters, income process parameters, and educational policy parameters, household optimization will generate decision rules governing college enrollment. These decision rules will of course be functions of the household’s state vector. Therefore, the aggregate enrollment flow of any new cohort of Young agents will depend on the joint distribution describing how Young households are distributed over the values of these state variables. Letting \( \Gamma(x) \) denote the (cumulative) joint distribution of Young households over the state vector, and \( I(\cdot) \) an indicator function over enrollment in college, the aggregate enrollment rate, \( \Phi \), is given by:

\[ \Phi \equiv \int I(V^E(x) > V^NE(x))d\Gamma \]

Similarly, the aggregate failure rate is given by:

\[ \Pi \equiv \int f(\pi|x)d\Gamma \]

2.6 Parameters

There are three classes of parameters in the model: those related to preferences, those related to education, and those related to income and its risks. There are only two preference parameters, the annual discount factor \( \beta \), which is set to 0.96, and risk-aversion \( \sigma \), which is set to 3, as is standard. We turn next to education-related parameters.
2.6.1 Education Related Parameters

To calibrate the investment in education, we choose parameters governing college completion, the cost of attendance, and student preparedness and resources. We emphasize that we do not set any of the following parameters to help the model match observables. They are all assigned values based on direct measures from data. The first parameter is given by the data on the difference in average after-tax lifetime earnings, which is based on findings of Restuccia and Urrutia (2004) and Chatterjee and Ionescu (2010), is set to 1.5.

The next two parameters are those governing median time-to-failure and time to subsequent completion, \( \tau_1 \) and \( \tau_2 \), respectively. The next two parameters specify the average subsidy \( \gamma^{direct} \) that is received by public higher education institutions and the private cost of college \( \Phi_1^{pvt} \). Turning next to the distribution of wealth available to potential enrollees, the fifth and sixth parameters specify the mean \( \mu_{a_0} \) and median \( median_{a_0} \) of the distribution of initial wealth for Youths, which is assumed to be log normal, in line with SCF data. Similarly, as an empirical matter, the marginal distribution of standardized test scores, e.g. the SAT, is given by a normal distribution, whose mean and standard deviation we denote by \( \mu^{SAT} \) and \( \sigma^{SAT} \).

This distribution is then translated into a risk of failure that is set to match observed failure rates across institutions with varying mean SAT scores. We also do not a priori restrict the distributions of standardized test performance and household resources to be independent. To allow for dependence in a tractable manner, we assume that these two objects are jointly lognormal, and therefore specify the covariance between test scores and family resources, which we denote by \( cov(SAT, a_0) \). The final parameter we choose is the (common) borrowing limit available to households, denoted by \( a \).

Starting with the parameters governing college completion, the median time to failure is parameterized according to Fang (2006), as is identical to Akyol and Athreya (2005) (who use NCES data), and is set to \( \tau_1 = 2 \). Following ([10], Table D-1), college completion time is set at five years, which implies \( \tau_2 = 3 \). That is, college takes five years to complete, and failure, if it occurs, does so at the end of the second year. In the section on Robustness, we will allow for alternative failure times, and also for a more gradual resolution of failure uncertainty. The real resource cost of college, \( \Phi_1^{pvt} \), is set to match the mean “sticker price” of college. The upper and lower bounds for this come from adding, or leaving out, room and board. If the former, the annual cost is approximately $30,393 at 4-year private colleges; if the latter, the cost falls to $21,588. The cost of college to an enrollee is denoted \( \Psi_1^{pvt} = \theta \mu^{coll} \) where \( \theta \) is a fraction of mean annual income of college-educated households. The latter is $75,000 (Census (2007)). Using the midpoint for college costs, we set \( \theta = 0.35 \). The level of direct subsidization, prior to any need- or merit-based aid available to those enrolling, is set to match that prevailing at public four year institutions. It is denoted by \( \gamma_{benchmark}^{direct} \), and is measured

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6 The Department of Education compiles a set of studies each estimating the distribution of completion times, and nearly all of them place the median completion time at 5 years. See http://www2.ed.gov/pubs/CollegeForAll/completion.html

by Caucutt and Kumar (2006) at 0.425. Chatterjee and Ionescu (2010) measure the cost of public 4 year college to be closer to $8000, implying a similar, but slightly higher subsidy rate closer to 0.5. However, some other measures suggest higher rates of subsidization. In particular, if we measure subsidization rates as the ratio of the costs of tuition and fees at public four-year colleges relative to that at private four-year colleges, the rate is 0.72 ($5,950/$21,588, NCES (2008)). The out of pocket cost for tuition and fees in the model is given by $\theta_{\text{coll}}^{\text{coll}} \gamma_{\text{direct}}^{\text{benchmark}}$. Under our benchmark parameterization, this yields an average cost (tuition plus fees) of approximately $11,100 annually. To parameterize need-based aid, we follow Clayton and Dynarski (2007), and employ a simple linear function with two parameters governed by (i) the maximal Pell grant of $4,000, and (ii) the constant reduction in Pell grants as a linear function of family resources, $a_0$, set such that it completely disqualifies households with income greater than $50,000.

To set the limit on borrowing by enrollees, as mentioned at the outset, we are guided by the work of Carneiro and Heckman (2002) who argue that widespread borrowing constraints for education are implausible. Moreover, there exists at present an explicit set of guaranteed loan programs to finance any amount in excess of the so-called “Expected Family Contribution.” These are the PLUS loan programs. We therefore will set the debt limit to always allow a household to finance the entire cost of college, given the set of subsidies that are in place. Specifically, given the costs of college inclusive of all subsidies, we set $a$, the borrowing limit that enrolling households face at $a = -\sum_{i=1}^{2} \tau_i \Psi_i^{\text{pt}, \gamma_{\text{direct}}}^{\text{benchmark}}$.

Since the joint distribution of initial enrollee wealth and enrollee failure risk is specified as bivariate log-normal, it is governed by five parameters. The first two, which give the distribution of SAT scores is taken from the College Board, and yields value for the mean and standard deviation of total (critical writing plus mathematics) SAT scores as $\mu_{\text{SAT}} = 1000$ and $\sigma_{\text{SAT}} = 200$, respectively. The mean score is also close (by construction) to the median score.

The available wealth of enrollees will reflect not only their own private resources, if any, but also parental transfers. The latter, however, are not obviously proxied for by parental wealth, since the willingness of parents to make such transfers is not directly observable. For the same reason, the level and covariance of familial resources available to potential enrollees (not just those who ultimately enroll) with test scores is not well-measured in the data. We use the Survey of Consumer Finances (SCF) from 2004 to compute the moments of the wealth distribution of households whose head is the median age of the parents of college-bound students. This yields a lognormal distribution of enrollee initial familial resources with mean $\mu_{a_0} = $40,000 and a median $median_{a_0} = $20,000. Our parameterization specifies that the median transfer to an enrollee from within the family is on the order of twice median annual household income, and so is not likely

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8 See http://www.fafsa.ed.gov/what010.htm
9 These numbers are the mean and standard deviation of the total score on the Critical Reading and Mathematics sections of the SAT. The raw data are here: http://professionals.collegeboard.com/profdownload/sat-percentile-ranks-composite-cr-m-2010.pdf
to understate the actual transfer. As a result, our parameterization is not likely to overstate the benefits of cost-reductions for college simply by making households counterfactually wealth-poor. This choice is disciplined by the proportion who receive Pell grants, and the conditional mean of grants among this group.

With respect to the covariance of resources and test scores, it is plausible that while not perfectly positive, wealth and parental education are strongly correlated and that the latter is in turn correlated with failure risk (see e.g. Athreya and Akyol (2006) and the references therein). Letting $\nu^{SAT}$ denote SAT score, we set $\text{cov}(\nu^{SAT}, a_0) = 0.3$ in our benchmark, which is the midpoint measured of the range estimated by Castex (2009) for the correlation between “Family Income” and “Ability” (as measured by AFQT) in the NLSY79 and 97. These values are also consistent with students’ self-reported wealth in the demographic section of the SAT data reported by the College Board. However, because family income and resources available to enrolling students may not be the same thing, we will examine the effects of alternatives in the Robustness section. Lastly, to map SAT scored into failure risk, we use observed data on institution-level failure rates by SAT score to estimate a linear map that takes the percentile of the test scores $g(\nu^{SAT})$ into a probability of failure as follows: $\pi = 1 - \lambda_{grad} g(\nu^{SAT})$; where $\lambda_{grad}=0.9$. We specify $g(\cdot)$ such that the top percentile of SAT scores will fail with probability $(1-\lambda_{grad})$, while the first percentile will fail with probability $\lambda_{grad}$.

Table 11 displays the benchmark specification of all education-related parameters in the model.

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10 While in principle there is an issue of selection here, in particular coming from the possibility that those who take the SAT are disproportionately well-prepared, this is not likely to alter our conclusions, for two reasons. First, there are several states in the US in which SAT participation rates are very high, and in Maine for instance, it is 100%. The moments of the score distributions from these states are very similar to that seen amongst only the enrollees at large state universities. Second, if the bias were to be important, the risk facing most students is actually even greater, making non-enrollment rates even easier to account for.
Main Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skill Premium</td>
<td>$1.5$ (Restuccia and Urrutia (2004))</td>
</tr>
<tr>
<td>$\tau_1$</td>
<td>$2$ (Fang(2006))</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>$3$ (Bound, Loevenheim, Turner (2007))</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$0.35$ (NCES(2008), Authors’ Calculation)</td>
</tr>
<tr>
<td>$\gamma_{\text{direct}}$</td>
<td>$0.425$ ((Caucutt and Kumar (2005), NCES (2008))</td>
</tr>
<tr>
<td>$\gamma_{\text{benchmark}}$</td>
<td>$0.425$ ((Caucutt and Kumar (2005), NCES (2008))</td>
</tr>
<tr>
<td>$\gamma_{\text{need}}$</td>
<td>$4,800-0.4a_0$ (Clayton and Dynarski (2007))</td>
</tr>
<tr>
<td>$\mu^{\text{SAT}}$</td>
<td>$1000$ (College Board (2010))</td>
</tr>
<tr>
<td>$\sigma^{\text{SAT}}$</td>
<td>$200$ (College Board (2010))</td>
</tr>
<tr>
<td>median$_{a_0}$</td>
<td>$20,000$ (SCF 2004)</td>
</tr>
<tr>
<td>$\mu_{a_0}$</td>
<td>$40,000$ (SCF 2004)</td>
</tr>
<tr>
<td>$\text{cov}(SAT, a_0)$</td>
<td>$0.3$ (Castex (2009))</td>
</tr>
<tr>
<td>$\lambda_{\text{grad}}$</td>
<td>$0.9$ (Authors’ Calculation from College Board)</td>
</tr>
<tr>
<td>$a$</td>
<td>$-\sum_{i=1}^{2} \tau_i \Psi_{1}^{\text{pvt}} \gamma_{\text{direct}}$</td>
</tr>
</tbody>
</table>

2.6.2 Income Parameters

Income risk is assigned entirely standard values employed in the literature. We follow Hubbard, Skinner, and Zeldes (1994), Table A.2, who express age-specific means of after-tax labor income for the three education groups are given by simple polynomials that are cubic in age. Throughout the paper, we maintain the assumption that subsidies represent a negligible portion of total government expenditures, and therefore have negligible tax consequences. The parameters for the stochastic process for shocks to earnings are described in equations 3 through 7, and summarized below.

Income Risk Parameters

<table>
<thead>
<tr>
<th>Parameter $\backslash$ Education Level</th>
<th>HS</th>
<th>Some Coll</th>
<th>Coll</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_u$</td>
<td>$0.15$</td>
<td>$0.15$</td>
<td>$0.15$</td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>$0.16$</td>
<td>$0.16$</td>
<td>$0.11$</td>
</tr>
<tr>
<td>$\sigma_\zeta$</td>
<td>$0.5$</td>
<td>$0.5$</td>
<td>$0.5$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$0.95$</td>
<td>$0.95$</td>
<td>$0.95$</td>
</tr>
</tbody>
</table>

3 Results: Enrollment and Risk

First, we provide simple measures of internal rates of return to college. These calculations are sufficient for a risk-neutral decision maker to choose whether or not to attend college. We then study the implications of our benchmark model, in which decision makers are risk-averse and prefer intertemporally smooth consumption. The main focus of our analysis will be to examine the effects of risk on the decision making process of
individual agents, and in turn, to show that aggregate implications of our model for college investment, especially for overall enrollment and failure rates, are very close to the data. This is striking, as no parameters in the model were set to help match these facts. This gives us confidence that our model indeed captures the salient forces, especially those related to failure risk. Given this, our final section develops predictions for the likely consequences of various policies aimed at encouraging collegiate enrollment.

3.1 Risk-Neutrality

We first abstract from risk, and present and compute measures of the “internal rate of return” (IRR). Such measures will, given the high average skill premium, provide an indication of the “attractiveness” of college. Indeed, such measures lie behind the intuitive view that college enrollment should be nearly universal and that failure to enroll is a symptom of resource misallocation.

<table>
<thead>
<tr>
<th>failure risk</th>
<th>π = 0.03</th>
<th>π = 0.26</th>
<th>π = 0.50</th>
<th>π = 0.74</th>
<th>π = 0.98</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRR</td>
<td>0.0648</td>
<td>0.0585</td>
<td>0.0486</td>
<td>0.0301</td>
<td>-0.0426</td>
</tr>
</tbody>
</table>

Table 13 reports the discount rate at which a risk-neutral agent with given failure probability π and direct tuition subsidy rate of 42.5 percent would be just indifferent between enrolling in college or not. When failure risk is low, say 3 percent, a risk-neutral agent would invest in college for any discount rate below 6.48 percent. The results indicate that failure risk meaningfully alters the expected benefits from enrollment. Nonetheless, it is true that for all but the least likely to succeed, a risk-neutral investor would choose to attend. Even at failure risks of nearly 75%, the internal rate of return exceeds the risk-free rate. In this sense, the risk-neutral perspective does indeed generate the conclusion that college makes financial sense for the vast majority of students. If this is so, and if borrowing constraints are so unimportant, why is the observed enrollment rate fully 30 percentage points short of universal attendance? Our contention is that the answer lies in the interaction of modest levels of risk-aversion with the lumpy and risky investment characterizing college, which we now examine.

3.2 Risk-Aversion, Failure-Risk, and the College Investment Decision: The Benchmark Model

While risk-neutrality is perhaps appropriate as a guide for firms evaluating investment projects, or for households considering “local” gambles, investment in college is neither conducted by firms, nor is it local in size. Moreover, as we noted at the outset, many households are not wealthy at the time of the enrollment decision. These households will have to borrow (being unable to issue equity claims on the fruits of their
human capital). Given the size of debts needed for many to finance college, leverage in the relevant amounts carries a potentially serious risk. For all these reasons, it is a priori likely that risk-aversion plays a significant role in governing decisions. We therefore turn now to the role played by failure risk by using the model of enrollment decision-making.

We begin by studying the performance of the benchmark model. This holds fixed all education policies such as subsidies, need-based, and merit-based aid at currently observed values. We will show that the model does a good job at describing the decisions of an entering cohort. Specifically, we compare the model’s performance along two dimensions: enrollment and failure rates. Turning first to the unconditional moments, we see that the model does quite well, as seen next in Tables 14 and 15.

**Unconditional Enrollment and Failure Rates**

<table>
<thead>
<tr>
<th></th>
<th>Enroll Rate (Φ)</th>
<th>Failure Rate (Π)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>0.67</td>
<td>0.45</td>
</tr>
<tr>
<td>Data (Bailey/Dynarski (2009))</td>
<td>0.74</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Turning next to the more disaggregated relationship between preparedness (as measured by the SAT) and enrollment rates, we find that the model again does well at generating aggregate enrollment with the “right” enrollees. The data are taken from Chatterjee and Ionescu (2010).

**Enrollment Rates and Test Scores**

<table>
<thead>
<tr>
<th>SAT</th>
<th>&lt;900</th>
<th>901-1100</th>
<th>1101-1250</th>
<th>≥ 1251</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>33%</td>
<td>86%</td>
<td>97%</td>
<td>97%</td>
</tr>
<tr>
<td>Data(Chatterjee/Ionescu (2010))</td>
<td>—</td>
<td>89%</td>
<td>95%</td>
<td>96%</td>
</tr>
</tbody>
</table>

The key finding from the preceding two tables is that the benchmark model successfully approximates both aggregate enrollment and failure rates, without being targeted to do so. We now use the model to obtain predictions for the main question of interest: what should one expect for the enrollment response to a change in skill premia?

### 3.3 Failure Risk and The Response of Enrollment to Changes in the Skill Premium

Having described the performance of the benchmark model, and some of its properties, we now turn to one of the main questions posed at the outset: how should a given cohort’s enrollment behavior change in response to an increase in the skill premium? The assessment of many (see e.g., Altonji et al. (2008)) is that the

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11See Bailey and Dynarski (2009), Table 1, for enrollment data, and Table 4 for completion rates. We use the rates reported for the 1988 birth cohort for enrollment, and the 1982 birth cohort for completion (to allow for suitable length of time to determine completion). Similar estimates are found in Bound, Loevenheim, and Turner (2007). For example, for completion rates, see their Table 1 with the 8-year completion rate (proportion of enrollees with bachelor’s degree within 8 years of graduating high school) among the 1988 NELS cohort being 45.3% and that for the NLS72 group being higher at 51.1%.
enrollment response to the steady increase in the skill premium over the 1970s and 1980s has been “anemic.” Such a view has motivated a variety of policy responses, most notably a substantial increase in Pell Grant generosity. We use our quantitative model to provide an answer to just how large such a response should have been expected. Each row gives a value of the skill premium, measured by the ratio of skilled worker income to unskilled income. Each column gives the enrollment rate in college for a given value of failure probability. The final column integrates over the distribution of types (in terms of their failure probabilities) to give the aggregate enrollment rate for a given skill premium.

<table>
<thead>
<tr>
<th>Lifetime Prem/π</th>
<th>0.03</th>
<th>0.18</th>
<th>0.34</th>
<th>0.50</th>
<th>0.66</th>
<th>0.82</th>
<th>0.98</th>
<th>Φ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>0.88</td>
<td>0.94</td>
<td>0.93</td>
<td>0.76</td>
<td>0.16</td>
<td>0.00</td>
<td>0.00</td>
<td>0.56</td>
</tr>
<tr>
<td>1.5 (benchmark)</td>
<td>1.00</td>
<td>0.97</td>
<td>0.96</td>
<td>0.89</td>
<td>0.33</td>
<td>0.00</td>
<td>0.00</td>
<td>0.67</td>
</tr>
<tr>
<td>1.6</td>
<td>1.00</td>
<td>0.98</td>
<td>0.98</td>
<td>0.94</td>
<td>0.47</td>
<td>0.01</td>
<td>0.00</td>
<td>0.74</td>
</tr>
<tr>
<td>1.7</td>
<td>1.00</td>
<td>0.99</td>
<td>0.99</td>
<td>0.96</td>
<td>0.56</td>
<td>0.01</td>
<td>0.00</td>
<td>0.77</td>
</tr>
<tr>
<td>1.8</td>
<td>1.00</td>
<td>1.00</td>
<td>0.99</td>
<td>0.98</td>
<td>0.70</td>
<td>0.02</td>
<td>0.00</td>
<td>0.82</td>
</tr>
</tbody>
</table>

The final column shows that increases in the skill premium of the magnitude observed, say from 1.5 to 1.7, should not be expected to generate an enormous increase in enrollments; our model predicts that, ceteris paribus, enrollment should be ten percentage points higher, going from 0.67 to 0.77. As the skill premium rises further, the response of aggregate enrollment is even smaller: increases in the skill premium from 1.6 to 1.7 to 1.8, raise enrollments by only three to five percentage points for each increment. These modest increases in aggregate enrollments mask some dramatic responses for particular groups, however. For example, taking again the group with failure probability of 66%, the model predicts that enrollment under the higher skill-premium of 1.7 will be 0.56, compared with the enrollment rate of 0.33 predicted to follow from a skill-premium of 1.5.

3.4 A Punchline: Most US Households are Inframarginal To College Investment

Taken as a whole, our findings help explain what Altonji, Baradwaj, and Lange (2008b) term the “anemic” response of enrollment to changes in the skill premium, and arise from the following simple and intuitive mechanism. The presence of failure risk generates asymmetric changes in the net return to college investment: those with low failure risk see a large increase in expected returns, but are inframarginal because they will enroll under most circumstances. Those with high failure risk see a much smaller increase in expected returns, and hence remain largely inframarginal.

The overall importance of failure risk is seen again in the fact that while a substantial proportion of households are inframarginal with respect to the return to college, those who are marginal are overwhelmingly those for whom college investment is genuinely risky—in the sense of carrying a high variance of future utility. Households who are quite sure to fail or succeed do not face meaningful risk: the variance of future utility.
induced by an investment in college is low. And as such, the mean return, roughly speaking, will dominate decision making. In turn, movements in the skill premium are not likely to matter. Specifically, well-prepared enrollees face low failure risk and so already receive a high rate of return from college under any skill premium in the approximate vicinity of the current one. Similarly, the ability of the skill premium to meaningfully alter mean returns for the ill-prepared is minimal. The only remaining question is then: how large is the set of marginal households? The answer provided by the model is: not very big.

4 Why are So Many Inframarginal? Higher Education Policy and the Enrollment Decision

We have shown above that incorporating quantitatively reasonable wealth and preparedness heterogeneity into a model of college investment can account reasonably well for both the level of college enrollment (far less than universal, even with apparently extremely high returns) and its failure to increase even when the skill premium rises further. We now ask about the role being played by policy in driving such a large fraction of each cohort of potential enrollees to be inframarginal with respect to changes in the return to college investment. Our answer here is that this role is large.

Before addressing the effect of changes in skill premia across different subsidy rates, we first evaluate the power of direct subsidies to alter decision making under a given skill premium (in this case, the benchmark lifetime earnings premium of 1.5). Given our primary focus on understanding the decision-making process leading to college enrollment, we now display enrollment rates and the enrollment decision rule as a function of both subsidies and preparedness. The Figure 2 documents enrollment rates across subsidy rates and SAT scores (as a proxy for preparedness levels). This figure shows that under current skill-premia, college is not financially attractive at low levels of subsidies for even for very well-prepared students, as even top SAT-scroers enroll at rates below 20% when college is unsubsidized. However, as subsidies rise, the well-prepared enroll at high rates, while those with middling scores enroll at lower rates. When subsidies reach 50%, in the broad range of empirically-observed rates for public colleges, students with median scores enroll at very high rates while less-well-prepared students hardly enroll at all. In essence, Figure 2 shows that the drop-off of enrollment for less-well-prepared students is very steep, because even subsidized education is not desirable if the failure probability is too high. The “steepness” of the enrollment surface reflects the fundamental message of the paper: most are inframarginal. In other words, unless a large mass of household types are located at a “ridge” where the slope of the surface in any direction is steep (particularly in the direction of subsidies) one should not expect changes in the underlying environment to yield large changes in enrollment.
Enrollment Rates by SAT score and subsidy rate

Figure 2: Subsidy Policy, Failure Risk, and Enrollment Rates
4.1 Wealth and Enrollment: Seeing that Risk Matters

Having described the predictions of our model for the aggregate enrollment behavior of a cohort across subsidies and failure risk, we turn now to understanding the decisions that underlie the aggregates seen in Figure 2. In particular, we show “iso-enrollment” contours across three dimension: failure risk, subsidies, and wealth. Figure 3 is perhaps the most important figure in this paper. It shows the “critical” levels of personal wealth at which investment in college becomes desirable as a function of the failure risk of an enrollee, for a variety of subsidy levels. The lowest line is associated with the highest subsidy rate, and the relationship is monotone. We emphasize three results from this figure.

First, college is not a “no-brainer.” There are many households for whom an unsubsidized investment in college is simply not financially worthwhile. As can be seen from the figure, if one fixes a wealth level, then looking across failure risk, we see that for enrollees with a success probability of less than 30%, it takes a subsidy in excess of 50% before low-wealth households (e.g. those with assets less than $50,000) find it worthwhile to invest in college.

Second, wealth and preparedness are very clearly substitutes in influencing the decision to enroll. This is not what would obtain under risk-neutrality: recall that the model does not impose any borrowing constraints. Therefore, personal wealth would, under risk neutrality play no role in the investment decision. The fact that it seems, on the contrary, to play a substantial role in the enrollment decisions of poorly-prepared households, holding the subsidy fixed, is consistent with risk playing an important role. Moreover, as a quantitative matter, the relationship is highly nonlinear. Starting in the neighborhood of the unconditional mean of failure risk in the economy (approximately 54%), we see that under benchmark subsidy rates (the red line) that an enrollee with no internal wealth would be just indifferent to enrolling or not. But, a less well-prepared student, with a success probability of 0.25, will only enroll if he has in excess of $10,000 in personal wealth. As failure probabilities rise, the requisite internal wealth rises rapidly. Subsidies do not change the qualitative nature of decisions, but alter the threshold level of internal wealth significantly. Under the lowest subsidy rates, an average would-be enrollee requires approximately $30,000 more internal wealth than she would under the most generous subsidy regime. This gap persists for even very well-prepared students, but shrinks for those least-prepared. Intuitively, as the likelihood of failure grows, the subsidy acts as a form of wealth, as all households wish to enroll, while the subsidy can do little to alter the expected net benefit for those likely to fail. Moreover, those with a low likelihood of success will face heavy debts from enrolling unless they have significant internal funds; hence, the vertical distance between critical wealth levels grows with failure risk. For the highest failure risk however, the subsidy has little influence as the expected return is deeply negative.

Third, and perhaps most suggestive of the role of risk is that the gap between critical wealth levels shrinks
in either direction of failure risk: as failure becomes either highly probable or improbable. For intermediate ranges, households become sensitive to subsidies. Interestingly, on this point notice that in contrast to what occurs at very high and low success rates, households are sensitive to subsidies in a nonlinear manner, with initial increases doing much more to drive down the critical wealth levels that make enrollment acceptable than they do for those with high success likelihoods. In other words, when success is very likely or unlikely, increases in subsidies will operate to make enrollment more attractive via only one channel: a pure “price effect.” That is, whenever the outcome of the investment is more or less guaranteed, subsidies cannot by definition have any insurance effects. At intermediate failure likelihoods, however, this is no longer true. Now, in addition to the price effect, we have an insurance effect whereby the subsidy makes a given investment more tolerable. A sense of the size of this insurance effect is captured by the difference in reductions in critical wealth among those sure to succeed, relative to those seen at intermediate values of failure risk.

Since wealth can so strongly influence the enrollment decision, one would expect the financial resources of enrolled students to be very different. We now examine the implications for the mapping from household resources available to a student (proxied by parents’ earnings or wealth) and their enrollment decisions. Figure 4 and Figure 5 clarify this relationship.

The model predicts that the most poorly prepared households enroll only when they are very wealthy.
The Mean Wealth of Enrollees By Preparedness

Figure 4: The Mean Wealth of Enrollees
Figure 5: The Mean Wealth of Non-Enrollees

This reflects the “cutoff” levels of wealth seen in Figure 3. In equilibrium, the richest enrollees in college will be those with the highest failure risk. Moreover, as a quantitative matter, the wealth of enrollees depends strongly on their failure risk: households in the lowest quartile of SAT scores (below 700), have a wealth level under the no-subsidy regime that is nearly five times as high as those in the top quartile of SAT (above 1200). However, subsidies have the power to flatten this relationship. Under near-benchmark subsidy rates (42.5%), the model predicts that the observed relationship between failure risk and wealth should nearly disappear. Of course, this is achieved, as seen earlier, not by discouraging the poorly prepared, but rather by encouraging them, and thereby driving down the mean wealth required of enrollees. Taken as a whole, this again suggests that the risk of failure deters many, and induces a relationship between initial wealth and enrollment that is not consistent with either risk-neutrality or full insurability of college failure risk.

Next, we see that for failure-risk groups who essentially never enroll in college, mean wealth is by definition close to the unconditional mean. Amongst failure risk groups that do enroll at non-trivial rates, we see that mean wealth falls. The reason is that the set of non-enrollees among these groups will in general be disproportionately poor. In Figure 3 notice that those with intermediate and low levels of failure risk have far lower critical wealth levels needed to induce enrollment compared to those for whom failure is relatively high.
While we do not present it here, at the margin, such households are likely to generate substantial increases in human capital as subsidies rise, even though their enrollment rates remain low even at high subsidy rates. We revisit this feature below in the context of the fiscal implications of human capital subsidies. The converse occurs at high wealth levels, whereby enrollment is consistently high, even at low subsidy rates, while failure rates are high. For example, under the highest subsidy level, the children of the richest households enroll at rates exceeding 70%, but those who enroll actually complete at only a 50% rate.

4.2 Enrollment Responses to Changes in Skill Premia: Policy Matters

The results thus far are suggestive of current policy playing a strong role in driving enrollment, and that successive increases in subsidies will be met with by shrinking, though strictly positive, changes in enrollment rates. We now turn to the issue of the extent to which enrollment responses to skill premium are altered by the subsidy policy in place. The results are striking, and are presented below in Table 16:

<table>
<thead>
<tr>
<th>Skill Premium</th>
<th>$\gamma^{direct} = 0.0$</th>
<th>$\gamma^{direct} = 0.425$</th>
<th>$\gamma^{direct} = 0.75$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>0.10</td>
<td>0.57</td>
<td>0.74</td>
</tr>
<tr>
<td>1.5</td>
<td>0.18</td>
<td>0.67</td>
<td>0.83</td>
</tr>
<tr>
<td>1.6</td>
<td>0.22</td>
<td>0.74</td>
<td>0.86</td>
</tr>
<tr>
<td>1.7</td>
<td>0.26</td>
<td>0.77</td>
<td>0.88</td>
</tr>
<tr>
<td>1.8</td>
<td>0.30</td>
<td>0.82</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Two points follow from the results in Table 16. First, skill premia do not appear to be capable of explaining enrollment by themselves. The unsubsidized cost of college, along with reasonable measures of the distributions of failure risk and parental resources available to potential enrollees, leaves most households ambivalent about college—*even at high skill premia*. Second, when direct subsidies rise from low levels to higher ones ($\gamma^{direct} = 0$ to 0.425) subsidies to higher education matter a great deal for household decisions. However, at subsidies and skill premia near current levels, additional subsidization (from $\gamma^{direct} = 0.425$ to 0.75) does not meaningfully change the response to further increases in the skill premium. This is seen clearly by comparing the bottom two rows of the Table 16. The message, once again, is that a skill premium of near 1.5 or greater means that most households are inframarginal with respect to further increases in expected wages. In other words, both “tails” of the failure risk distribution are inframarginal: those with high failure risk find that an increase in the skill premium only slightly increases the expected return to college enrollment.

One way of summarizing the information above is that subsidies matter far more than skill premia for enrollment. The unsubsidized enrollment rate under the highest skill premium we study (1.8) is far lower than the enrollment obtaining under the lowest skill premium (1.4) with a 75% subsidy. Relatedly,
changes in enrollment only when overall education subsidy policy is very stringent. Under current or higher subsidization rates, however, changes in skill premia should not be expected to bring forth important changes in enrollment: the marginal households have been exhausted. One implication of this finding is that if one views a substantial component of increases in skill-premia arising from shifts in the “demand” for skilled labor inputs, then it is likely that the net result, barring an increase in subsidization, will be only modest increases in enrollment. Moreover, given the increasing marginality of the additional enrollee, the resulting skill formation will likely be even smaller. As a result, the model suggests that increases in skill premia followed by stable enrollment rates may well be a possibility.

4.3 Subsidies, Grants and Aggregate Enrollment and Failure Rates

Historically, the most important policy aimed at encouraging college enrollment has been the creation of public college and universities that feature heavily subsidized tuition, and living expenses. In fall 2010, 74% of college students are enrolled at public colleges and universities (NCES 2010). Having shown above that subsidies governed the size of the response to skill premia by governing the size of the marginal population, we now use our model to briefly assess the likely effect of subsidy policies on aggregate enrollment and completion rates under the benchmark skill premium. Recall that in our model, direct subsidies will not only reduce the cost of college, but will also reduce the risk faced by students. Subsidies reduce the amount of wealth devoted, or the amount of borrowing required, to finance college. Given precautionary motives to avoid low-wealth states, this alters the risk associated with college attendance. Both effects will be larger for students who are wealth-poor, less-well-prepared, or both.

The enrollment rates as a function of wealth and preparedness, along with the underlying behavior of the joint distribution of SATs and initial wealth imply the following for aggregate enrollment rates and failure rates, as a function of subsidies.

<table>
<thead>
<tr>
<th>Subsidy Rate ($γ^{direct}$)</th>
<th>Enroll Rate ($Φ$)</th>
<th>Failure Rate (II)</th>
<th>Mean Failure Prob ($μ_{II}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.18</td>
<td>0.34</td>
<td>0.54</td>
</tr>
<tr>
<td>0.25</td>
<td>0.53</td>
<td>0.42</td>
<td>0.54</td>
</tr>
<tr>
<td>0.425</td>
<td>0.67</td>
<td>0.45</td>
<td>0.54</td>
</tr>
<tr>
<td>0.5</td>
<td>0.75</td>
<td>0.47</td>
<td>0.54</td>
</tr>
<tr>
<td>0.75</td>
<td>0.83</td>
<td>0.48</td>
<td>0.54</td>
</tr>
</tbody>
</table>

This table integrates enrollment decisions over the underlying joint distribution of SAT scores and initial wealth available to households of typical enrollment age. As the subsidy rate rises, enrollment responds strongly. This subsidy is universally available to all students, and (unlike the skill premium) is received

\[12\] In the longer run, the net effect of the preceding is likely to lead to greater inequality.
regardless of whether or not the student graduates. As more students attend, the selection becomes less favorable, so the aggregate failure rate rises also. This suggests that while enrollees may be willing to enroll, that taxpayers as a whole may lose, a question we now turn to.

4.4 Need-Based Aid

The direct subsidies we have studied so far are, by construction, received by all enrollees, and so are a blunt policy instrument. It is of interest to examine the effectiveness of need-based aid to alter decisions. Our decision model allows predictions about the long-run effects of changes in such aid. As mentioned in the Calibration section, we employ a simple representation, based on Dynarski and Clayton (2008) that provides a good approximation of need-based aid. Specifically, all households face a maximum amount of need-based aid (what they would get if their familial resources were zero) of $4000. Given this maximum, the Pell program essentially deducts two-thirds of any additional resources from the maximum amount. As a result, Pell benefits reach zero at roughly $40000 of household resources. In 2010, the maximal benefit was increased to approximately $5550, with the reduction for additional resources remaining unchanged. We therefore look at the effects arising from three levels of maximal Pell grant, $4,100 (‘Pell Level 1’), $4,800 (‘Pell Level 2’—the benchmark), and $5550 (‘Pell Level 3’). To parallel the earlier discussion, we first show that the Pell program, which essentially augments household wealth—is likely to have meaningful effects on the critical wealth levels of households that render them indifferent to enrolling or not. In Figure 6 we see two things. First, that under current skill premia, for any given subsidy rate, the higher the Pell grant, the lower the critical wealth level that makes college worthwhile. Second, the higher the subsidy rate, the less that the Pell grant matters, as is natural.

The overall impact of the Pell program depends not just on the wealth thresholds described above, but also on the characteristics of the joint distribution of wealth and standardized test scores. The following Table shows the results for these values of the Pell program for all the agent types in the economy. Given the positive correlation between wealth and collegiate preparedness, the recipients of need-based aid will disproportionately be drawn from a relatively less well-prepared population. The following Table shows the model’s predictions for the response of enrollment by failure-risk type to systematic increases in the generosity of the Pell Grant program. Each row of the table gives the enrollment rate for a given maximum Pell grant, varying across failure probabilities in each column. The final column integrates over the distribution of failure probabilities to give the aggregate enrollment rate.

| Enrollment rate by Failure Risk and Pell Grant Maximum |
The results in the final column suggest that under the benchmark parameterization, modest increases in need-based aid of the magnitude we have observed can be expected to generate modest increases in enrollment. The issue here is that the increase in need-based aid is small compared to the subsidy already available (57% in the benchmark). Once again, these results highlight the inframarginality to college investment of most households. Students with low wealth and a good chance of success are already enrolled (see the first four columns above), while those with weak preparedness tend not to enroll (see the last two columns with $\pi = 0.82$ and higher). A small grant does not, for most potential enrollees, overcome the cost of school which must be paid even if graduation is not attained. Nonetheless, we see that a substantial change for enrollees with a failure risk of $\pi = 0.66$. 

<table>
<thead>
<tr>
<th>Pell Max $\backslash \pi$</th>
<th>0.03</th>
<th>0.18</th>
<th>0.34</th>
<th>0.50</th>
<th>0.66</th>
<th>0.82</th>
<th>0.98</th>
<th>$\Phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4,100$</td>
<td>1.00</td>
<td>0.97</td>
<td>0.94</td>
<td>0.86</td>
<td>0.25</td>
<td>0.00</td>
<td>0.00</td>
<td>0.65</td>
</tr>
<tr>
<td>$4,800$ (benchmark)</td>
<td>1.00</td>
<td>0.97</td>
<td>0.96</td>
<td>0.89</td>
<td>0.33</td>
<td>0.00</td>
<td>0.00</td>
<td>0.67</td>
</tr>
<tr>
<td>$5,550$</td>
<td>1.00</td>
<td>0.97</td>
<td>0.98</td>
<td>0.90</td>
<td>0.40</td>
<td>0.00</td>
<td>0.00</td>
<td>0.71</td>
</tr>
</tbody>
</table>
5 The Fiscal Implications of College Financial-Aid: Are College Subsidies a Good Investment for Taxpayers?

In this section, we evaluate the value of subsidies from a more narrow fiscal perspective by measuring the return the government (and hence, taxpayers) receive for investments in the human capital of young cohorts that arise from our model of enrollment decisions. We will hold the skill premium fixed throughout, something that allows for comparability across subsidy rates, but also provides a good approximation given that the flow of new enrollees, in whom we are interested, will constitute only a small proportion of the overall stock of college-educated workers.

Specifically, we calculate the net present value (in per student terms) arising from the initial investment in educational subsidies, followed by increased income tax receipts for the remainder of working life, collected from those who succeed. At low levels of subsidies, the NPV of college subsidies may increase as subsidies rise. This is because, as we showed earlier (see, e.g. Figure 3), low wealth can present a serious barrier to even well-prepared students without borrowing constraints. As a result, if potential enrollees respond strongly to subsidies, the net payoff to taxpayers can be positive: many marginal enrollees will be well-prepared, will therefore succeed frequently at college, and given the high skill premium and non-trivial average tax rate that applies, will repay the taxpayers in excess of the initial investment. Of course, not all will succeed, and in fact, as subsidies get larger, enrollees will, as seen earlier, be progressively less well-prepared. As a result, even abstracting from the potential for lower ex-post returns to college among those with higher ex-ante failure probabilities, a government will eventually be financing more poorly prepared students. Still, the NPV may still remain positive even at very high subsidy rates, something which crucially depends on the size of the skill premium. Eventually the marginal payoff from increasing the subsidy or need-based aid is likely to begin to decline.

The specific calculations we make are the following. For a given value of need-based aid, $\gamma_{\text{need}}$, and given a subsidy rate $\gamma_{\text{direct}}$, the population (measure) of young agents $\mu_{\pi}^Y$ of failure-type $\pi$, and the full annual unsubsidized cost of college, $\Phi_{pvt}^1$, we first solve the model to obtain enrollment rates $\Phi_{\pi}$ for each failure-risk group. Given this, we then calculate the total present value of public investment in a college-bound student as follows:

$$I_{\text{public}}(\gamma_{\text{direct}}, \pi|\gamma_{\text{need}}) = \mu_{\pi}^Y \Phi_{\pi} [\gamma_{\text{direct}} \Psi_{pvt}^1 \tau_1 + (1 - \pi) \gamma_{\text{direct}} \Psi_{pvt}^2 \tau_2]$$

That is, the total cost to the fiscal authority is the per-enrollee expenditure in the first part of college, $\gamma_{\text{direct}} \Psi_{pvt}^1 \tau_1$, and if the student is successful, which occurs with probability $\pi$, again in the second part of college, $\gamma_{\text{direct}} \Psi_{pvt}^2 \tau_2$ times the total number (measure) of enrollees of failure-type $\pi$, $\mu_{\pi}^Y \Phi_{\pi}$.

The revenue generated by this public-sector investment is the flow of additional tax revenues received by
the fiscal authority in taxes implied by the skill-premium multiplied by the tax rate. Denote by \( \bar{\omega}_j^{C_{pre-tax}} \) and \( \bar{\omega}_j^{HS_{pre-tax}} \) the pre-tax age-specific mean levels of earnings for college- and high-school-educated households, respectively. Assuming that the taxes \( T \in [0, 1] \) are levied as flat proportions on labor earnings alone, and given a public-sector discount factor \( Q \), we have the present value of flows arising from a subsidy level \( \gamma \) to a failure-risk-type \( \pi \) as \( \Omega^{public} \):

\[
\Omega^{public}(\gamma, \pi) = \sum_{j=1}^{J} Q^j (\bar{\omega}_j^{C_{pre-tax}} - \bar{\omega}_j^{HS_{pre-tax}}) \mu^Y \Phi_\pi (1 - \pi) T
\]

The net present value to the public of a subsidy program of generosity \( \gamma \) targeted at failure-risk-group \( \pi \) is then simply:

\[
NPV(\gamma, \pi) = -I^{public}(\gamma^{direct}, \pi|\gamma^{need}) + \Omega^{public}(\gamma, \pi)
\]

As stated earlier, we assume that the incremental increase to taxes arising from an expansion of the subsidy program are negligible. Therefore, we present calculations under the presumption of an overall flat tax rate \( T \) of 25% that remains constant across subsidy levels. We set \( Q = 0.96 \) to reflect a 4% discount rate by the government.\(^\text{13}\)

5.1 Direct Subsidies and Merit Aid

Since the most common way to reduce the cost of college is to provide subsidized tuition, we first examine the net present value implications of providing subsidies, and collect the results in the Table 18. Each row of the table gives the NPV as calculated above for a given subsidy rate. Each column gives the NPV for a given failure probability cohort. By looking at a column alone, we can also measure the NPV of giving aid to a particular preparedness cohort, which is a version of merit-based aid. In other words, fix a class of agent in terms of failure risk, and fix a subsidy regime. Then ask: “given that the set of agents with such failure risk, facing such a subsidy, currently enroll at rate \( \Phi_{ij} \), what is the net present value to the government of giving this class of youth this subsidy?”\(^\text{14}\) Of course, the issue for the government-as-investor is whether the incremental enrollees (those enrolling in response to the change in subsidy) subsequently earn enough to offset the public’s investment in their education. This is because under any given subsidy regime, many will be inframarginal. Therefore, we will present the changes in \( NPV(\gamma, \pi) \) arising from systematic increases in subsidy levels. The results are given below.

\(^{13}\)This is equivalent to the discount rate of the agent for simplicity, and also to reflect the notion that much of the failure risk faced by agents is diversifiable from the point of view of the government. This should not suggest, however, that the government’s investment is risk-free. The stream of tax revenue has systematic risk, which the government bears and should be priced. We include higher discount rates in our robustness calculations to allow for this risk.

\(^{14}\)Of course, under a subsidy rate of zero, the net present value is positive, and simply represents the present value of tax collections from a given cohort (of a given failure-risk type) over their lifetime. Once the subsidy is strictly positive, these benefits get partially offset by the costs of providing the subsidy.
Simply put, the results suggest that under current skill premia, the taxpayer is likely to be gaining from current rates of college subsidies, and would be unlikely to lose at even higher rates of subsidization of higher education. For low subsidy rates, even amongst groups with very high rates of failure, we find that as long as these groups enroll in positive proportions, the gains over a lifetime from success in college make the investment a positive net present value one. Interestingly, though they are overwhelmingly positive for all failure risk groups at low subsidy rates, NPVs fall monotonically with subsidies. This is for two reasons. First, the direct effect is a trivial one: higher subsidies are more expensive, by definition. Therefore, unless the enrollment rates of relatively well-prepared groups rise rapidly, the gains will recede, and eventually become negative. However, the inframarginality of well-prepared households—who enroll at very rates at even low subsidies—prevents this from occurring: only the moderately prepared and high-risk potential enrollees are available to change their enrollment rates. But, as emphasized repeatedly, and shown earlier in Figure 2 those who face low “risk” because they are very poorly-prepared also do not respond much to subsidies. As a quantitative matter, the “intermediate” failure groups that remain (e.g. \( \pi = 0.50 \) and \( \pi = 0.82 \)) do respond, but fail at such high rates that as subsidies rise, the average payoff per enrollee to the taxpayers declines steadily. For those who do respond however, failure rates are non-trivial, and this lowers the average gains that arise from successively higher subsidy rates, even while preserving the positivity of the gains themselves.

In a clear sense then, at low to moderate subsidy rates, the traditional view of college as a “no-brainer” appears to be validated at the societal level. Nonetheless, we showed earlier that no such feature need be true at the household level, where risk looms large. In a sense, a highly subsidized higher education system allows society to take a equity-like position in the human capital of households, whereby those who are ex-post successful pay back more than enough that ex-ante, tax payers are willing to fund the project. Our results should be interpreted correctly, though: they refer to the gains from investing in a single cohort—and if one views general equilibrium effects are vital, then such measures may overstate the gains of a sustained effort at subsidizing many successive cohorts.\(^{15}\)

\(^{15}\)“May” is the operative word. Since at least Acemoglu (1994), there has been evidence for capital-skill complementarity that it may be endogenous. As such, higher subsidization rates may well be met with by increasing demand for skill premia—fortifying the gains presented here.
5.2 Universal Tuition Subsidies

The preceding results are important to policy decisions in cases where the subsidy can be targeted by failure risk. In essence, limiting subsidies to particular subgroups on the basis of the NPV calculations would function as merit-based aid. Now, we consider the case in which the government has no such ability, and instead must offer a subsidy rate that is accessible to all enrollees. Naturally, this will shrink the overall NPV of such a change, as the enrollment responses of the poorly-prepared will factor in along with the increased enrollment, at least at low subsidy levels, of the well-prepared. Specifically, the calculation we present is as follows. Let \( P(\pi|\text{enroll}) \) denote the probability of a household being of failure-type \( k \), given that it has enrolled. This is an input that allows then to compute the overall measure (size) of each failure group in the pool of enrollees. Given this, the unconditional expected NPV, \( E(NPV) \), to government just takes the NPVs for each failure risk group (holding fixed the subsidy), and then weights each group by the preceding probability distribution. That is, we define \( E(NPV) = \sum_{\pi} NPV(\gamma, \pi)P(\pi|\text{enroll}) \). We now assess the payoff to the public’s investment in a universal subsidization scheme for higher education. The results are reported in following Table.

<table>
<thead>
<tr>
<th>( \gamma_{direct} )</th>
<th>( E(NPV) )</th>
<th>( \Phi )</th>
<th>( II )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>$44,400</td>
<td>0.18</td>
<td>0.34</td>
</tr>
<tr>
<td>0.25</td>
<td>$23,500</td>
<td>0.53</td>
<td>0.42</td>
</tr>
<tr>
<td>0.425</td>
<td>$8,800</td>
<td>0.68</td>
<td>0.45</td>
</tr>
<tr>
<td>0.50</td>
<td>$1,300</td>
<td>0.75</td>
<td>0.46</td>
</tr>
</tbody>
</table>

The main finding here is that this reflects, in yet another way, the importance of the size of the inframarginal population. Namely, as the subsidy goes up, this number stays essentially flat, mainly because the well-prepared students enroll at extremely high rates, and the higher the subsidy, the more badly-prepared enrollees there are (as seen by the failure rates in the third column). Nonetheless, current subsidy rates appear to be substantially positive NPV.

However, on balance, the most noticeable feature is the fact that an unrestricted subsidy program produces substantial gains, even at high subsidy rates, and even when the aggregate failure rate is high. The results are driven by the fact that the high skill premium, even when college success is remote ex-ante, trump the costs of failure—especially when these costs are automatically truncated by the fact that failure occurs relatively early in the college experience. Hence, subsidies encourage more experimentation, and even when few succeed, the payoff to success is high.
6 How much are programs that alter failure risks likely to be worth?

The results presented thus far have all been aimed at understanding the role played by failure risk in a setting in which the underlying distribution of preparation has been taken as given. However, the efficacy of the policy changes we have considered have all depended importantly on their interaction with the distribution of college success. Given the centrality of failure risk, it may be most important to instead better understand the extent to which altering the distribution of collegiate-preparedness may in fact be the relevant policy instrument. Such programs might involve more rigorous high-school curricula, as well as more active intervention into the pre-collegiate experiences of potential entrants. Our model allows features three sources of risk: initial wealth, the randomly determined ability to complete college, and the waiting time for the uncertainty over collegiate success to resolve itself. We will focus on understanding the extent to which the latter two would be worth pursuing.

Specifically, we conduct two classes of experiments. First, we consider the gains available from putting in place measures that will detect an inability to complete college at an earlier date. That is, we consider the effect on enrollment and NPVs from reductions in $\tau_1$, the date at which students learn whether they will succeed or fail in college. We then allow for changes in the overall preparedness of high-school completers. Specifically, we study allocations obtained from a setting in which a given SAT score is associated with a $\kappa$–percent reduction in the likelihood of failure. Here again, we will ask the extent to which there are gains available, and compare them to the gains from increased subsidies.

[TO BE COMPLETED]

6.1 Implications of Fiscal Federalism

We noted earlier that one of the most important college subsidy programs is the existence of modestly priced state colleges and universities. The fiscal implications of this subsidy require a different parameterization of our fiscal model, since most states have lower tax rates than the 25% we use above. These states ultimately recoup their investment under the presumption that those who are subsidized remain in state and pay taxes. To examine this hypothesis, we consider outcomes from a variety of settings for the two parameters that govern the NPV reaped (or paid) by a given state. These are, respectively, the state’s share of subsidies $\psi_s$, and the state’s tax rate relative to overall tax rates, $\tau_s$. The results are given for the NPV arising from a regime in which the benchmark subsidy of 42.5% is in place.

[TO BE COMPLETED]
7 Conclusions

We have shown that a high and increasing college skill premium can be reconciled with the observed level and response of college enrollment, which has been termed “anemic” by higher education researchers. Empirically reasonable measures of college failure risk and income uncertainty deter enrollment when potential students are uninsured and especially when they are also leveraged. The model replicates both the observed enrollment rate of college graduates, their subsequent failure experience once in college, and the manner in which the enrollment rates responds to increases in the skill premium.

Our findings, especially those that examine household decision-making, emphasize that while the return to higher education is high, it is not a safe investment for many households. The deterrent effect of risk survives the ability of agents to feasibly debt finance college with student loans. The main force here is that debt makes higher education riskier for student borrowers, suggesting that alternatives that are mindful of risk may be more effective. In turn, the focus of the policy experiments we consider is on direct subsidies and need-based aid, both of which help lower the risk over future utility induced by college.

Ideally, as was noted many years ago by Milton Friedman, equity financing is one way to reduce failure risk, because the equity “investor” shares some of the risk with the student. While we have not explicitly modeled equity-financing of higher education, its risk-sharing properties are implicit in both the direct subsidy programs and need-based grant aid we consider. In particular, this is effectively what the government does when it subsidizes education and then recoups its investment through income taxation that is either neutral (flat-rate) or progressive. Under a flat-rate tax system of empirically reasonable magnitudes, we have shown that these investments tend to be positive NPV to the government: the present value of a share of the skill premium exceeds the cost of the subsidy provided.

While our results emphasize the impact of failure risk, as a given, on the enrollment decision, we are currently examining the effect of reducing failure risk, by increasing student preparedness directly. We expect that the impact of improving student preparedness would be large in our model.

Two further caveats are in order. First, we have assumed that the skill premium, and indeed all of the parameters in the model, are fixed and known to potential students in advance. Uncertainty about the skill premium, and in particular the possibility that it may decline, may further deter enrollment. The financial aid policy parameters are also fixed. However, subsidy parameters are only relevant for the period while in college, and so this imposes a weaker assumption than the lifetime skill premium we have assumed. However, we do assume that loans continue to be non-dischargeable and that other long run features of student lending do not change.

Second, we have also abstracted from substantial and interesting heterogeneity among schools. We have parameterized the enrollment decision to a public four-year institution. Public higher education enrolls the
majority of college students (74% of all college students), so we view this parameterization as capturing
the cost structure facing the marginal student deciding whether or not to enroll in college. Those who
enroll in more expensive schools face higher costs, so we assume that they would surely have enrolled in the
"cheaper" school that we model and hence this variation does not change our calculations of the enrollment
rate. Nonetheless, the structure we employ could be used to model the distribution of students across schools,
where the returns to attending various schools could vary along with their costs. This would be an interesting
extension and application of the framework we develop here.
References

[1] [To be completed!]


8 Appendix: Robustness [to be completed]

8.1 Sensitivity to Risk and The Roles of Risk-Aversion and Discounting

8.1.1 Risk-Aversion

8.1.2 Discounting

8.2 The Importance of Household Wealth on the Fiscal Consequences of Public Support for College

All of these effects are enhanced in settings where the wealth of enrolling households is relatively lower, and stunted in the opposite case.

8.3 The Interaction of Failure Risk and Low Familial Resources

In this section we demonstrate that initial wealth has important effects on enrollment decisions. We study enrollment in the case where the distribution of parental resources available to enrollees at the time of enrollment is halved from that used in the benchmark. Specifically, median parental transfers are lowered to $5000, while the mean is set to $10,000. The most prominent form way to deal with the presence of households with low availability of resources has been to ensure that guaranteed loan programs exist to finance college, and the need-based aid for college is that provided by the Federal Pell Grant program. We first assess the likely impact of direct subsidies..