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ABSTRACT. We extend the existing literature on group lending contracts by analyzing the underinvestment problem of Stiglitz and Weiss (1981) in a context where client's project payoffs are correlated. We show that while in an independent project model there are no contracts that achieve the full information welfare allocation, this outcome can still be achieved provided that there are differences in project correlation across client types. Subsequently, we compare the performance of these contracts to that of guarantor contracts, where clients with a relatively high project payoff pledge to repay the loans of clients with lower payoffs in case of default.

#### 1. INTRODUCTION

Since the late 1970s the poor in developing economies have increasingly gained access to small loans with the help of microfinance programs. Especially during the past ten years, these programs have been introduced in many developing economies. Well-known examples are the Grameen Bank in Bangladesh, Banco Sol in Bolivia and Bank Rakyat in Indonesia. Stimulated by the success of the microfinance programs, the academic world has shown increased interest in this field. The literature focuses on explaining how and why microfinance works from a theoretical perspective. Group lending contracts based on joint liability lending, which are used in many programs, have received special attention.With joint liability lending the group of borrowers is made responsible for the repayment of the loan, i.e. all group members are jointly liable. Many models focus on the advantages of such schemes as compared to individual contracts in settings where providers of loans cannot distinguish safe from risky borrowers due to asymmetric inormation (see e.g. Banerjee et al., 1994; Ghatak, and Guinnane, 1999; Ghatak, 2000, Laffont, and Rey, 2003;Gangopadhyay, Ghatak, and Lensink, 2005).

With some notable exceptions (e.g. Ahlin and Townsend, 2007) existing joint liability lending models assume that project returns of group members are independent. In general, however, people who participate in microfinance projects live quite close to each other and are exposed to similar risks. Therefore it seems highly important to analyze joint liability lending in a setting where project returns are correlated. This is the first objective of our paper. More specifically, we focus on two issues. First, we show that, whereas in an underinvestment model with uncorrelated projects (Ghatak,2000) group lending schemes cannot yield the full information solution, this actually can be the case in a model with correlated projects, even if the correlation between some projects is highly positive, as is likely to be the case in practice. This may help explain the success of many group lending schemes based on joint liability contracts.

However, we also show that joint liability lending schemes may have considerable drawbacks. Specifically, the client's project payoff necessary to make such contracts work may be prohibitively high for small entrpreneurs. Therefore we compare the effectiveness of joint liability contracts, which have received most attention in the literature and guarantor contracts, similar to those proposed by Gangopadhyay and Lensink (2005). In case of a guarantor contract, one client receives a contract without joint liability, while the other client pledges to repay his peer's loan, should his project fail, in exchange for an interest rate discount on his own loan. We find that in certain settings such contracts can significantly reduce the threshold value of project return required to ensure the participation of clients in a microfinance scheme.

### 2. The Model

**2.1.** Agents. There is a population of potential clients, normalized to unity and consisting of two client types, safe (S) and risky (R). These types occur in proportions q and 1-q respectively, with  $q \in (0, 1)$ . Clients are endowed with one unit of labor, but have no capital. At time T = 0 they can either borrow a unit of capital and embark on a project with random payoff at T = 1 or carry out a project which requires no capital and yields certain payoff u at T = 1. The random payoff is given by  $X_i R_i, i \in \{S, R\}$ , where  $X_i$  is a Bernoulli random variable with parameter  $p_i$ , and  $p_S > p_R$ . To avoid technicalities, we will assume that  $p_R > \frac{1}{2}$ .<sup>1</sup> Project payoffs upon success,  $R_S$  and  $R_R$ , are known constants. All clients are risk-neutral and their utility at T = 1 equals their project payoff less loan repayments. At T = 0 they have no assets that can serve as collateral. It is assumed that all clients know each other's types.

Loans are provided by a risk-neutral microfinance institution (MFI), which requires a repayment of  $\gamma > 1$  per unit of capital to break even. The MFI acts as a benevolent social planner and has the objective of maximizing the clients' total welfare. It is fully informed about the market structure described above, except for the fact that it cannot a priori distinguish risky clients from safe clients.

It is assumed that

$$p_R R_R \ge p_S R_S > \gamma + u \tag{1}$$

which implies that both projects are socially efficient, and would be financed by the MFI in the full information case. Notice that we allow the risky project to have a higher expected payoff than the safe project. Furthermore the outside utility u of the safe project satisfies

$$u > p_S \left( R_S - \frac{\gamma}{\overline{p}} \right) \tag{2}$$

where  $\overline{p} = qp_S + (1-q)p_R$ . This implies that the safe project payoff is too low to make borrowing at the individual break-even pooling rate  $\overline{p}$  profitable for safe clients. Individual lending under asymmetric information thus leads to underinvestment; while the safe types have a socially efficient project, the presence of risky types drives the individual interest rate up to a level at which it is unprofitable for safe clients to carry it out (Stiglitz and Weiss (1981)).

**2.2.** Loan types. The MFI can offer a combination of individual loans, symmetric group loans and guarantor group loans. Individual loans specify only a repayment amount r at T = 1. Symmetric group loans are defined as in Ghatak (2000). For each of two clients who decide to form a group together, they consist of an identical repayment amount r, which has to be repaid whenever the client's project succeeds, and of an identical joint liability amount c, which has to be paid in addition to r whenever the client's success coincides with his peer's failure. Guarantor loans take the following form: one of the two clients in a group gets a loan which specifies a repayment component r only. His peer gets a loan which specifies a repayment r' and, additionally, a joint liability

<sup>&</sup>lt;sup>1</sup>This is a common assumption that is also made in, for example, Ghatak (2000).

component c = r. As the proceeds from his projects are the only assets available to a client at T = 1, his payments to the MFI upon project failure equal 0 for both symmetric and asymmetric contracts.

These contracts must satisfy three conditions. First, all repayment amounts and joint liability amounts must be positive. Second, for any contract that will be chosen with nonzero probability by client type i, the total payments should be lower than the project return  $R_i$ , as at T = 1 the project return is the client's only collateral. Also, symmetric group lending loans must satisfy the ex post incentive compatibility condition  $c \leq r$ (Gangopadhyay, Ghatak and Lensink (2005)). Were this not the case, a client would prefer to donate r to his failing peer, so that he could feign success and repay his loan, rather than paying the higher amount c to the MFI himself. This condition is satisfied by definition for a guarantor contract.

We will denote the set of all feasible contracts, that is, of all individual contracts and all group lending contracts that satisfy the three conditions above, by  $\mathcal{F}$ . Upon observing a subset of contracts  $F \in \mathcal{F}$  offered by the MFI, clients form groups with the objective of maximizing their expected utility. The group formation process is assumed to be free of costs or frictions for the clients. During this process, a type *i* client can offer a side contract, consisting of a claim of size  $b < R_i$  contingent upon the success of his project to potential partners. The proceeds from such side contracts can be claimed as collateral by the MFI at T = 1.

2.3. The first-best benchmark. In order to compare the effectiveness of symmetric contracts and guarantor contracts in different market settings, we will make use of a first-best full information benchmark. In this full information benchmark the expected contract payments of both client types equal the MFI's break-even value  $\gamma$  and side contracts are unnecessary. From a theoretical perspective, this benchmark is suitable for comparison with the analysis of, for example, Ghatak (2000). It is also relevant from a practical perspective, as MFI's often have to compete with local profit-maximizing moneylenders, who possess more information about client types. Whenever the first-best benchmark can be attained, both client types will prefer to borrow from the MFI rather than from fully informed moneylenders, provided that the cost of capital of the moneylenders is higher than the cost of capital of the MFI. Moreover, this benchmark rules out cross-subsidization of one client type by the other type, which is undesirable from the perspective of the social planner.

In our analysis we will derive the full set of market parameters q,  $p_i$  and  $R_i$  for which the first-best benchmark can be attained using the contracts defined in the previous section, and the contract sets  $F \in \mathcal{F}$  necessary to do so. In section 3 we first focus on symmetric contracts. Subsection 3.1 briefly introduces the Ghatak (2000) model based on independent projects, as extended by Gangopadhyay, Ghatak and Lensink (2005). Subsequently we extend this model to a general project correlation pattern in subsections 3.2 and 3.3. In section 4 we analyze the properties of asymmetric contracts within this general setting and compare the performance of both contract types in a setting with correlated projects.

### 3. Symmetric contracts

**3.1.** Review of the analysis for independent projects. In the context of the underinvestment problem, the challenge is to design a group lending contract which yields

the first-best for the safe clients, as the risky clients can always be offered an individual lending contract at a rate  $\frac{\gamma}{p_R}$ . As shown in Ghatak (1999,2000), when projects are independent any symmetric contract C = (r, c) will induce *positive assortative matching*. The optimal choice of safe clients upon observing a symmetric contract will be to form a group with safe peers, even if risky clients offer them side contract payments. Consequently, if a risky client wants to take a symmetric contract, he will have to do so with a risky peer.

Let us denote the expected contract payments of a client of type i who forms a group with a client of type j and signs contract C by  $P_{ij}^e(C)$ . By definition, a contract Cthat yields the first-best must satisfy  $P_{SS}^e(C) = \gamma$ . A further necessary condition is  $P_{RR}^e(C) \geq \gamma$ . If this condition is not satisfied, a pair of risky clients will find it profitable to take contract C together. The expected payments on this contract will be less than the MFI's break-even rate, thus making contract C infeasible for the MFI.

The expected payments of a client of type i are given by

$$P_{ii}^e = p_i r + p_i (1 - p_i) c.$$

In Figure 1 we display the sets of contracts such that  $P_{SS}^e(C) = \gamma$  and  $P_{RR}^e(C) = \gamma$ . The set of contracts satisfying the two necessary conditions for the first-best is given by all combinations of r and c on the line  $P_{SS}^e(C) = \gamma$  with  $r < \tilde{r}$  and  $c > \tilde{c}$ , where  $(\tilde{r}, \tilde{c})$  is the intersection point of the solid lines. The coordinates of this point are given by

$$(\widetilde{r},\widetilde{c}) = \left(\frac{p_S + p_R - 1}{p_S p_R}, \frac{1}{p_S p_R}\right).$$

The intuition behind this result is as follows: the probability that a safe client has to repay for a safe peer is lower than the probability that a risky client has to repay for a risky peer. Thus, a safe client who is in a group with a safe peer (which is guaranteed by the positive assortative matching result) will be more willing to accept a contract with a high joint liability component c than a risky client who is in a group with a risky peer. If we want to prevent that risky clients take the contract designed for safe clients, leading to an expected loss for the MFI, we must set a joint liability component that is high enough to discourage them from doing so.

## (Figure 1 about here)

Unfortunately, however, there are no *feasible* contracts satisfying  $P_{SS}^e(C) = \gamma$  and  $P_{RR}^e(C) \geq \gamma$ . As pointed out by Gangopadhyay, Ghatak and Lensink (2005) the intersection point has  $\tilde{r} < \tilde{c}$ . Therefore, the expost incentive compatibility condition is violated for all contracts on the line segment that could yield the first-best. This is bad news for symmetric group lending schemes. What is the second-best alternative for the safe clients? Dropping the first-best restriction that *both* clients should have expected payments equal to  $\gamma$ , we turn to contracts that yield the *aggregate* welfare-maximizing solution. Any feasible symmetric contract C must satisfy  $qP_{SS}^e(C) + (1-q)P_{RR}^e(C) \geq \gamma$ , as both client types will be able to sign it, if they so desire. The set of contracts that satisfies this condition with equality has

$$\overline{p}r + (\overline{p} - \overline{p^2})c = \gamma \tag{3}$$

where we define  $\overline{p^2} = qp_S^2 + (1-q)p_R^2$ . Linear optimization shows that, in this set, the contract that yields the lowest expected payments to safe clients has the maximum feasible

value of the joint liability component. This optimal contract will depend on the return of the safe client's project  $R_S$ . Straightforward algebra yields that the intersection of (3) with boundaries of the feasible region in the positive quadrant is given by

$$(r^*, c^*) = \begin{cases} \left(\frac{\gamma}{2\overline{p} - \overline{p^2}}, \frac{\gamma}{2\overline{p} - \overline{p^2}}\right) & \text{for } R_S > \frac{2\gamma}{2\overline{p} - \overline{p^2}}\\ \left(\frac{\gamma - (\overline{p} - p^2)R_S}{\overline{p^2}}, \frac{\overline{p}R_S - \gamma}{\overline{p^2}}\right) & \text{else.} \end{cases}$$

If the return of the safe client's project is high enough, the expost incentive compatibility condition will be binding, while the limited liability condition will bind in all other cases. The corresponding expected payments of a safe client group equal

$$P_{SS}^{e}(r^*, c^*) = \begin{cases} \frac{2p_S - p_S^2}{2\overline{p} - \overline{p}^2} \gamma & \text{for } R_S > \frac{2\gamma}{2\overline{p} - \overline{p}^2} \\ \frac{p_S^2}{\overline{p}^2} \left(\gamma - \frac{(1-q)p_R(p_S - p_R)}{p_S \overline{p}^2} R_S\right) & \text{else.} \end{cases}$$

Of course this group lending contract can still result in a Pareto improvement for all clients as compared to the individual lending case, provided that  $P_{SS}^e(r^*, c^*) < p_S R_S - u$ . Instead of opting out, the safe clients will now participate. The risky clients will also be better off as they will be effectively cross-subsidized by their safe peers. The contract is not robust to competition, however. The MFI's outreach to safe clients can be hampered by fully informed competitors, such as moneylenders. If these moneylenders have breakeven cost of capital  $\gamma'$  such that  $\gamma' < P_{SS}^e$ , they can, for example, offer an individual contract with interest rate  $r^m = (P_{SS}^e - \varepsilon)/p_S$  and capture the whole market of safe clients, causing an expected welfare loss of  $r^m - \gamma$  on each client as compared to the first-best benchmark. Contract  $(r^*, c^*)$  will be infeasible in that case, as it will only be chosen by risky clients, resulting in an expected loss for the MFI.

These problems can be expected to occur in markets where the difference in success probabilities between the two client types is large, the project return of the safe client is relatively low, or the proportion of safe clients is relatively small, as we find

$$\frac{\partial P_{SS}^{e*}}{\partial p_R}, \frac{\partial P_{SS}^{e*}}{\partial q}, \frac{\partial P_{SS}^{e*}}{\partial R_S} < 0.$$

However, projects are very unlikely to be independent in the typical context of a microfinance program. Clients embarking on similar project types will be affected by the same shocks, in terms of, for example, weather conditions or aggregate market demand. This induces positive correlation between project success between clients of the same type. One can also think of market settings where the correlation between projects of the same type is negative due to, for example, limited total demand for project output in a local market. If one of the clients manages to market his crops successfully, the probability that his peer will do so too may decrease.

In the following sections, we will show that for certain correlation patterns, the firstbest benchmark can be attained using symmetric contracts. In these settings traditional symmetric microfinance contracts can be expected to perform particularly well in terms of outreach. Interestingly, strong positive correlation between safe projects turns out to facilitate this. This may seem somewhat counterintuitive, as positive project correlation generally has a negative effect on repayment, as the joint liability component is paid less often when projects tend to succeed or fail together (Ghatak (2000), p.625). However, if this effect is larger for safe clients than it is for risky clients, it can help to achieve a

first-best separating equilibrium. It is not the magnitude of correlations that matters for screening purposes, but the relative values of the correlations between two safe projects and two risky projects.

**3.2.** Modeling correlations and expected contract payments. Let us first model the relationship between project correlations and expected payments on a group lending contract. A group consists of two clients of types i and j,  $i, j \in \{R, S\}$ . Recall that client i's project success corresponds to the random variable  $X_i$  taking the value 1 and failure corresponds to this random variable taking the value 0. By definition, the correlation between the projects equals

$$\rho_{ij} = \frac{E(X_i X_j) - p_i p_j}{\sqrt{p_i (1 - p_i)} \sqrt{p_j (1 - p_j)}}.$$
(4)

where we use  $E(X_i) = p_i$  and  $Var(X_i) = p_i(1 - p_i)$ .

Let us denote the probabilities of the four possible combinations of the realizations of  $X_i$  and  $X_j$  by

$$\mathbf{p}_{ij} = \begin{pmatrix} p_{ij}^s \\ p_{ij}^c \\ p_{ji}^c \\ p_{ij}^f \\ p_{ij}^f \end{pmatrix} = \begin{pmatrix} P\{X_i = 1, X_j = 1\} \\ P\{X_i = 1, X_j = 0\} \\ P\{X_i = 0, X_j = 1\} \\ P\{X_i = 0, X_j = 0\} \end{pmatrix}$$
(5)

where the superscript s stands for success of both projects, the superscript f stands for failure of both projects and the superscript c stands for the situations in which joint liability needs to be paid.

By definition we must have  $p_{ij}^s + p_{ij}^c = p_i$  and  $p_{ij}^s + p_{ji}^c = p_j$ . Also,  $E(X_iX_j)$  in (4) equals  $p_{ij}^s$ . Using these relationships we find that, for identical projects (i = j), the probabilities corresponding to the minimal and the maximal correlation,  $\rho_{l,ij}$  and  $\rho_{h,ij}$ are given by<sup>2</sup>

$$\mathbf{p}_{l,ij} = \begin{pmatrix} 2p_i - 1\\ 1 - p_i\\ 1 - p_i,\\ 0 \end{pmatrix}, \mathbf{p}_{h,ij} = \begin{pmatrix} p_i\\ 0\\ 0\\ 1 - p_i \end{pmatrix}.$$

From substituting in equation (4) we find that the minimal correlation between identical projects equals  $\rho_l = \frac{p_i - 1}{p_i}$  while the maximal correlation equals  $\rho_h = 1$ . Likewise, for different project types  $(i \neq j)$ , the probabilities for minimal and maximal correlation correspond to

$$\mathbf{p}_{l,ij} = \begin{pmatrix} p_S + p_R - 1\\ 1 - p_R\\ 1 - p_S\\ 0 \end{pmatrix}, \mathbf{p}_{h,ij} = \begin{pmatrix} p_R\\ p_S - p_R\\ 0\\ 1 - p_S \end{pmatrix}.$$

with minimal and maximal correlations

$$\rho_{l,ij} = \frac{-\sqrt{(1-p_S)(1-p_R)}}{\sqrt{p_S p_R}}, \rho_{h,ij} = \frac{\sqrt{p_R(1-p_S)}}{\sqrt{p_S(1-p_R)}}$$

<sup>&</sup>lt;sup>2</sup>Maximizing/minimizing the correlation amounts to maximizing/minimizing  $p_{ij}^s$ . By definition the maximal value of  $p_{ij}^s$  equals  $p_i$ . Using the restrictions  $p_{ij}^s + 2p_{ij}^c + p_{ij}^f = 1$  and  $p_{ij}^s + p_{ij}^c = p_i$ , we obtain the minimal value of  $p_{ij}^s$  by setting  $p_{ij}^f = 0$ .

respectively. For any correlation  $\rho_{ij} \in [\rho_{l,ij}, \rho_{h,ij}]$ , we can use equation (4) to find the corresponding probability  $p_{ij}^s$  and the remaining probabilities follow immediately:

$$\mathbf{p}_{ij} = \begin{pmatrix} p_{ij}^s \\ p_{ij}^c \\ p_{ji}^c \\ p_{jj}^f \\ p_{ij}^f \end{pmatrix} = \begin{pmatrix} p_i p_j + \widetilde{\rho}_{ij} \\ p_i (1 - p_j) - \widetilde{\rho}_{ij} \\ p_j (1 - p_i) - \widetilde{\rho}_{ij} \\ (1 - p_i)(1 - p_j) + \widetilde{\rho}_{ij} \end{pmatrix}$$
(6)

where  $\tilde{\rho}_{ij} = \rho_{ij} \sqrt{p_i p_j (1 - p_i)(1 - p_j)}$ . In our subsequent analysis, we will assume that  $\rho_{RR}$  can take any value between  $\frac{p_i - 1}{p_i}$  and 1. As for safe project correlations, we will assume that they are positive. Also, we normalize the correlation between safe and risky projects  $\rho_{SR}$  at  $0.^3$ 

**3.3.** The full information benchmark with correlated projects. Introducing correlations to the independent project model induces two major changes as compared to the analysis of section 3.1. First, the set of contracts  $(r_S, c_S)$  that yield the first-best outcome for a safe client who matches with a risky peer now satisfies

$$p_S r_S + p_{SS}^c c_S = \gamma. \tag{7}$$

As compared to the case of independent projects, these contracts are on a line that is pivoted clockwise around the point  $\left(\frac{1}{p_i}, 0\right)$  for positive values of  $\rho_{SS}$ . This situation is shown by in Figure 2, where we have introduced positive correlations  $\rho_{SS} = 0.8$  and  $\rho_{RR} = 0.2$  for both projects. The intuition is that if projects are positively correlated, clients are more likely to either succeed or fail together and the probability of having to pay the joint liability payment decreases. Therefore, clients are willing to accept a higher value of c for a given decrease in r as compared to the case of independent projects.

The intersection with the line  $p_R r_R + p_{RR}^c c_R = \gamma$  now occurs in the point

$$(r^*, c^*) = \left(\frac{(p_{RR}^c - p_{SS}^c)\gamma}{p_S p_{RR}^c - p_R p_{SS}^c}, \frac{(p_S - p_R)\gamma}{p_S p_{RR}^c - p_R p_{SS}^c}\right).$$

This means that the problem with the expost incentive compatibility condition we encountered in the independent project model is solved whenever

$$(p_S - p_R) \le p_{RR}^c - p_{SS}^c.$$
 (8)

which can be written as

$$\rho_{SS} \ge 1 - \frac{p_R(2 - p_R) - p_S}{p_S(1 - p_S)} + \frac{p_R(1 - p_R)}{p_S(1 - p_S)}\rho_{RR}.$$
(9)

in terms of our original model parameters.<sup>4</sup>

## (Figure 2 about here)

 $<sup>^{3}</sup>$ Both the assumptions of positive safe project correlation and zero correlation between safe and risky projects can be relaxed at the expense of extra notation. In our opinion, however, this is not conducive to our analysis.

<sup>&</sup>lt;sup>4</sup>Note that this relationship implies  $\rho_{RR} < 1 - \frac{p_S - p_R}{p_R(1 - p_R)}$  as  $\rho_{SS}$  must be smaller than one.

Assuming that condition (9) is satisfied, the MFI can offer any symmetric contract (r, c) satisfying equation (7) with  $c^* \leq c \leq r$ . We only need to ensure that homogeneous groups will be formed if it does so, which we have assumed this far. This is immediate whenever  $\rho_{SS} \geq 0$  and  $\rho_{RR} \geq 0$ . The positive assortative matching result will hold a fortiori as homogeneous groups will be more attractive for both safe types and risky types as compared to the independent case. If the correlation between risky projects is negative, however, risky clients will be willing to offer a larger side contract to the safe clients than they would do if projects were independent. The positive assortative matching result does not hold for all combinations of  $\rho_{SS}$  and  $\rho_{RR}$ . This is the second change induced by introducing correlations.

We therefore need to verify which correlation patterns result in the formation of homogenous groups.<sup>5</sup> The expected payments of both client types, as compared to their benchmark payments of  $\gamma$  are given by

$$P_{S,SR}^{e}(C) - \gamma = p_{S}(p_{S} - p_{R} + \rho_{SS}(1 - p_{S}))c$$

$$P_{R,SR}^{e}(C) - \gamma = \frac{p_{R} - p_{S}}{p_{S}}\gamma + \rho_{SS}p_{R}(1 - p_{S})c$$
(10)

The expected payments of both clients are increasing in c. The safe client is always worse off matching with a risky client as (10) is always positive. However, the risky client may be able to bribe the safe client into matching with him. The minimal side contract payment  $b^S$  that the safe client will demand as compensation and the maximal payment  $b_R$  the risky client will be willing to offer satisfy, respectively<sup>6</sup>,

$$p_{S}p_{R}b^{S} = p_{S}(p_{S} - p_{R} + \rho_{SS}(1 - p_{S}))c$$
  
$$p_{R}b_{R} = \frac{p_{S} - p_{R}}{p_{S}}\gamma - \rho_{SS}p_{R}(1 - p_{S})c.$$

Straightforward algebra shows that the threshold value of c such that  $b_R = b^S$  is given by

$$c^{t} = \frac{(p_{S} - p_{R})\gamma}{p_{S}(p_{S} - p_{R} + \rho_{SS}(1 + p_{R})(1 - p_{S}))}$$

Whenever this value is below  $c^*$  homogeneous matching is guaranteed for all feasible contracts satisfying (7). This amounts to the correlation between safe client's projects being sufficiently positive, hence reducing the expected payments of a homogeneous safe group, so that safe clients will refuse the risky client's side contract offer. In terms of model parameters, this amounts to

$$\rho_{SS} > -\frac{1-p_R}{1-p_S}(p_S - p_R) - p_R \frac{1-p_R}{1-p_S} \rho_{RR}.$$
(11)

<sup>&</sup>lt;sup>5</sup>Remark that if a symmetric contract is to yield the first-best benchmark, it must induce homogenous group formation. If two different client types take a symmetric contract, then equal expected payments on this contract imply  $p_S r + p_S (1 - p_R)c = p_R r + p_R (1 - p_S)c$ , that is c = -r, which is not feasible.

<sup>&</sup>lt;sup>6</sup>Recall that the risky client pays the side contract whenever he succeeds; the safe client will only derive utility from this payment if he succeeds as well; otherwise the side payment is taken as (partial) collateral for his non-repaid loan by the MFI.

Given that  $\rho_{SS}$  is positive, this condition is automatically satisfied for  $\rho_{RR} > 1 - \frac{p_S}{p_R}$ . In that case, contract  $(r^*, c^*)$  solves the underinvestment problem whenever condition (9) holds. We require condition (11) for  $(r^*, c^*)$  to be optimal only for negative risky project correlations, that is, for  $1 - \frac{1}{p_R} < \rho_{RR} < 1 - \frac{p_S}{p_R}$ . If  $\rho_{SS}$  is too low to meet this condition, the first-best may still be feasible, though. A contract with  $c > c^t$  needs to be set to prevent heterogeneous group formation. Recall that the highest value of c the MFI can set, due to the restriction r > c, equals  $c^{\max} = \frac{\gamma}{p_S + p_{SS}^c}$ . We find that  $c^t < c^{\max}$  amounts to  $\rho_{SS} > \frac{p_S - p_R}{1 + p_S}$ . Therefore, even if risky project correlations are negative and (11) does not hold,  $\rho_{SS} > \frac{p_S - p_R}{1 + p_S}$  together with (9) suffices for a first-best solution to exist. We conclude that, as opposed to the case of independent projects, symmetric con-

We conclude that, as opposed to the case of independent projects, symmetric contracts can be expected to work well in practice across a wide range of relevant, positive correlation values. There still is an important problem, however. The contract  $(r^*, c^*)$ (and, to a larger extent, contract  $(r^t, c^t)$ ) requires the return on the safe project to be of a considerable magnitude for the limited liability condition to be satisfied. In order for the contract  $(r^*, c^*)$  to satisfy the limited liability requirement, the corresponding safe project return must satisfy

$$R_{S} \ge \frac{((p_{S} - p_{R}) + (p_{RR}^{c} - p_{SS}^{c}))\gamma}{p_{S}p_{RR}^{c} - p_{R}p_{SS}^{c}}$$

We have drawn these threshold values for  $R_S$  in Figure 3 for the range of safe project correlations under which the contract  $(r^*, c^*)$  is feasible, fixing the correlation between risky projects at 0 and normalizing  $\gamma$  at 1. The green line in this Figure represents a market where the clients have success probabilities  $p_S = 0.95$  and  $p_R = 0.9$ , respectively. It shows that, in this case, the safe project payoff should exceed the MFI's cost-of capital by 95 percent if the correlation between safe projects is at the lowest feasible value (about 0.2) and by 70 percent as the correlation coefficient approaches unity. The red line depicts a scenario where the difference between success probabilities is considerable high ( $p_S = 0.95$ and  $p_R = 0.8$ ). This is very disadvantageous for safe clients. As we saw in the independent project case, symmetric joint liability contracts are hampered by such differences. There is only a small range of correlations for which ( $r^*, c^*$ ) is feasible and the required payoff in excess of the MFI's cost of capital is over 100 percent for safe clients, even if a payoff of a mere 6 percent could have made the project interesting, had the MFI known the client's types.<sup>7</sup>

### (Figure 3 about here)

Of course these figures are not to be interpreted literally, as in particular the assumptions that clients have no collateral at all and that their projects yield zero payoff upon failure are obviously stylized. We will use them primarily for a relative comparison with guarantor contracts, which will be introduced in the next section. Nevertheless, a peculiar feature of the screening mechanism underlying symmetric contracts is that it requires exactly those clients who have a relatively low project payoff to produce joint liability payments. This is somewhat unsatisfactory, as in the context of the underinvestment problem it's exactly due to a relative low project payoff that safe clients are driven out of the market. In microfinance it is of the essence to reach clients whose fledgling small-scale

<sup>&</sup>lt;sup>7</sup>Remark that if the correlation between risky projects becomes positive, this threshold value will increase ceteris paribus, as the risky client's first-best line will pivot outward.

enterprises do not yet yield high payoffs. This is an important reason for turning to alternative group lending solutions, like guarantor contracts.

## 4. GUARANTOR CONTRACTS

In this section we will focus on group contracts which do *not* require safe clients to produce high joint liability payments, should their peer fail. If we can design a guarantor contract which will induce risky clients to co-sign for their safe peers, we will be able to make use of the risky clients' high project returns, which are relatively high by assumption, for joint liability payments.

Consider the guarantor contract  $G = \{(r_S, 0), (r_R, r_S)\}$ . This contract consists of a subcontract  $(r_S, 0)$  destined for the safe client, which does not specify a joint liability component, and a part destined for the risky client, which specifies a joint liability component equal to the repayment component of the safe client  $r_S$ , next to an individual repayment component  $r_R$ . The risky client acts as a guarantor for the safe client, pledging to repay his peer's debt should he fail. Alternatively, the contract can be interpreted as microfinance scheme with the risky client acting as a group leader and taking responsibility for the other client's repayment, in return for a discount on his own loan repayment.

To achieve the first-best solution for the safe clients we must set  $r_S = \frac{\gamma}{p_S}$ . The firstbest solution for the risky clients implies  $r_R = \gamma (1 + \frac{1}{p_R} - \frac{1}{p_S})$ , which is a discount of  $\frac{1=p_S}{p_S}\gamma$  as compared to their individual lending rate. Limited liability then implies that the risky client's return satisfies  $R_R \ge (1 + \frac{1}{p_R})\gamma$ .

Of course this scheme yields the first-best for all safe clients only if  $q \leq \frac{1}{2}$ , as each safe client should be able to find a guarantor. If  $q > \frac{1}{2}$ , at least part of the safe clients will achieve the first-best. The remaining risky clients can obtain an individual contract with interest rate  $\frac{\gamma}{p_R}$ . We assume that, faced with the choice between two contracts between which they are indifferent, risky clients will choose the socially optimal one, that is, they will rather act as a guarantor for a safe peer than take an individual contract provided their expected payoffs from both contracts are equal.

For the guarantor contract to yield the first-best outcome in this setup, we only require that it is indeed optimal for both client types to form heterogeneous groups, with the risky client playing the role of a guarantor. There are three possible deviations from this outcome: it can be chosen by a risky pair, a safe pair, or by a safe-risky pair with the safe client taking the role of the guarantor. In the subsequent paragraphs, we will define market types in which these deviations are precluded and guarantor contracts will yield the first-best outcome.

Let us first consider the optimal decision of two risky clients who observe that the MFI offers a guarantor contract. If both of them take the guarantor part of contract G together with a safe peer, by construction they will have expected contract payments equal to the first-best value  $\gamma$ . If instead they take the guarantor contract together, the client who gets the 'safe' part will clearly be better off, expecting to pay only  $\frac{\mathcal{P}R}{\mathcal{P}_S}\gamma$ . If the correlation between risky projects is high enough, the client who plays the guarantor role will actually also be better off, as his expected joint liability payoffs will be low. Straightforward calculations show that this happens whenever  $\rho_{RR} > \frac{\mathcal{P}S - \mathcal{P}R}{1-\mathcal{P}_R}$ . Even if this is not the case, two risky clients may still come to an agreement using side contracts. The risky client who gets the safe part is willing to promise a maximal amount of  $b_R =$ 

 $\frac{(p_S - p_R)\gamma}{p_S p_R}$  at T = 1. The expected payments of his risky peer amount to

$$P_{RR}^{e}(G) = p_{R}r_{R} + p_{CR}^{c}r_{S} - p_{RR}^{s}b = \left(\frac{p_{S}p_{R} + p_{S} - p_{R}}{p_{S}} + \frac{p_{CR}^{c}}{p_{S}}\right)\gamma - (p_{R} - p_{RR}^{c})b.$$

The value of b for which a risky client is just indifferent between taking the guarantor role with a risky peer or a safe peer follows from solving  $P_{R\mathbf{R}}^e(G) = \gamma$ . By comparing this value to  $b_R$  it can be verified that the condition  $\rho_{RR} \leq 0$  is required to prevent the formation of homogeneous risky groups, which would imply that safe clients are forced out of the market once again. Thus, in a single-period model, guarantor contracts will only work in markets where circumstances like competition induce negative correlation between risky clients' project success.<sup>8</sup>

It can also be profitable for a pair of safe clients to take the guarantor contract. The client who ends up with the contract designed for the safe client will just have expected payments equal to  $\gamma$ . The other client may be even better off, provided that  $r_R < r_S$  and his probability of having to pay joint liability for the safe peer is low. This situation is precluded if the probability of success of the safe client is high enough; this means that the guarantor's interest discount will be low, as he faces little risk of his peer's failure, and consequently  $r_R$  will exceed  $r_S$ . In terms of model parameters this implies

$$p_S > \frac{2p_R}{1+p_R}.\tag{12}$$

For lower values of  $p_S$  we must distinguish two cases. First, if the safe project return is too low to pay the joint liability component, that is,  $R_S < 1 + \frac{1}{p_R}$  the safe client will be able to pay only  $R_S$  instead of  $1 + \frac{1}{p_R}$  if his peer fails. His expected utility from the guarantor contract equals

$$P_{SS}^{e}(G) = p_{SS}^{s}(R_{S} - r_{R}) = (p_{S} - p_{SS}^{c})(R_{S} - r_{R})$$

as he only gets a positive payoff in the event that both the and his peer succeed. Comparing this to the payoff  $p_S R_S - \gamma$  in the first-best case, we find that a safe client will prefer to take the guarantor contract whenever  $(p_S - p_{SS}^c)r_R + p_{SS}^cR_S \leq \gamma$ . This implies that, if the difference in success probabilities between safe and risky clients is not large enough to satisfy equation (12), the safe project return must exceed a threshold value for first-best guarantor contracts to exist. This value is given by the expression

$$R_{S}^{t} = \left(\frac{1 - p_{SS}^{s}}{p_{SS}^{c}} - \frac{(p_{S} - p_{R})p_{SS}^{s}}{p_{S}p_{R}p_{SS}^{c}}\right)\gamma$$

If the payoff of the safe client does exceeds  $(1 + \frac{1}{p_R})\gamma$  and  $p_S$  is low enough for  $r_R$  to be lower than  $r_S$ , we just require  $p_S r_R + p_{SS}^c c_R \ge \gamma$ . In terms of correlations, this means

<sup>&</sup>lt;sup>8</sup> The requirement that risky projects are negatively correlated is induced by the normalization  $\rho_{RS} = 0$ . If we allow risky projects to be correlated with safe projects, this requirement can be relaxed to allow for positive correlations between risky projects such that  $\rho_{RR} < \rho_{RS}$ . A natural interpretation for this correlation pattern would be that the outputs of risky clients are each other's substitutes and compete in the marketplace, while the outputs of safe clients are complements for the outputs of risky clients.

$$\rho_{SS} < \frac{p_S - p_R}{p_R (1 - p_S)}.$$

So if safe clients' project payoff is high, a relatively low correlation between safe project is sufficient to prevent them from taking the guarantor contract together.

Let us finally consider the situation in which the risky client and the safe client swap positions. Suppose first that the safe client's project return is low, so that he will not be able to pay the joint liability component of this contract, should this necessity arise. Recall that the risky client is willing to offer a side contract with a payment up to  $b_R = \frac{(p_S - p_R)\gamma}{p_S p_R}$ to be allowed to take the 'safe' part of a guarantor contract. Given this side contract payment, the expected payoffs of a safe client who takes the guarantor contract will equal

$$P_{SR}^{e}(G) = p_{S}p_{R}(R_{S} - r_{R} + b_{R})$$
$$= p_{S}p_{R}(R_{S} - \gamma)$$

Comparing to the first-best payoff  $p_S R_S - \gamma$  we find the condition

$$R_S > \frac{1 - p_S p_R}{p_S - p_S p_R}.\tag{13}$$

Likewise, if the safe client's project payoff is high enough, it can be verified in a similar way that this matching pattern is never optimal for the safe client.

#### (Figure 4 about here)

Figure 4 displays the minimal values of safe project payoffs that make guarantor contracts feasible, for the same combinations of success probabilities as in the previous section. Note that condition (12) holds for these probabilities, so guarantor contracts are feasible for all possible safe project correlations. We need only consider the threshold value on safe project payoffs given in (13). The required excess returns amount to 53 percent for the case  $p_S = 0.95$ ,  $p_R = 0.9$  and to only 26 percent for the case  $p_S = 0.95$ ,  $p_R = 0.8$ ! The large difference in success probabilities actually turns into an advantage if we use guarantor contracts, as it makes the 'swapping' strategy less attractive for the safe client.

## 5. DISCUSSION

We conclude that the performance of symmetric joint liability contracts and guarantor contracts depends crucially on the structure of the market in which they are employed. Symmetric joint liability contracts can be expected to perform well in markets where the differences in project riskiness are limited, the correlation between safe projects is high as compared to the correlation of risky projects, and the payoffs of safe clients are high. Especially this last requirement may pose a problem if the objective of the MFI is to reach out to starting small-scale entrepreneurs. In this case, guarantor contracts may provide a solution, however. Such contracts can turn risky clients from effectively causing a welfare loss by driving their safe counterparts out of the individual lending market, to actually ensuring that these very same clients obtain access to credit.

Our results suggest that there is no unique matching pattern or contract type which performs well across a wide range of markets. In recent theoretical work, a similar finding was obtained by Chowdhury (2007) in a framework employing social capital. An advantage of the approach we employ in this paper is that the model parameters can be estimated from data on group lending projects. Ahlin and Townsend (2007) and Ahlin (2009) provide guidance about obtaining proxies of success probabilities and project correlations from survey data. Empirical scrutiny of the main predictions of our paper will therefore be our next objective.

#### 6. LITERATURE

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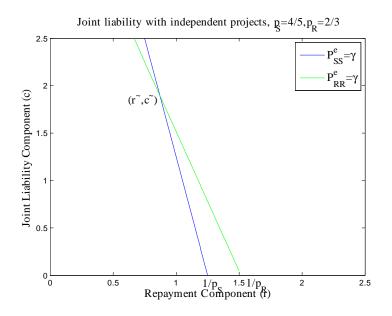


Figure 1: The screening mechanism behind symmetric group lending contracts.

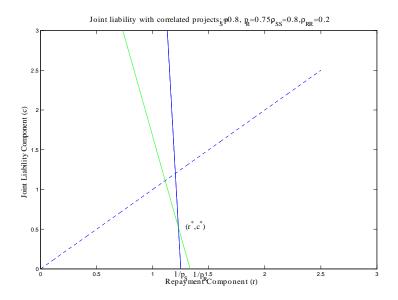


Figure 2: Achieving the first-best outcome when projects are correlated.

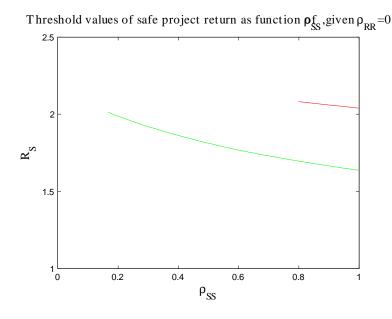


Figure 3: Required safe project payoffs using symmetric contracts.

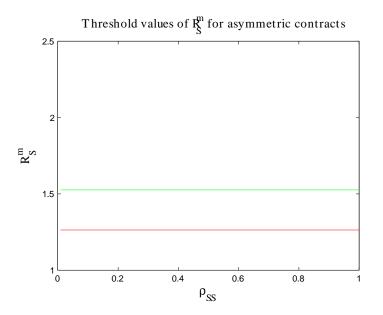


Figure 4: Required safe project payoffs using guarantor contracts.