Intrepreting Labor Supply Regressions in a Model of Full and Part-Time Work*

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Abstract

We construct a family model of labor supply that features adjustment along both the intensive and extensive margin. Intensive margin adjsutment is restricted to two values: full time work and part-time work. Using simulated data from the steady state of the calibrated model, we examine whether standard labor supply regressions can uncover the true value of the intertemporal elasticity of labor supply parameter. We find positive estimated elasticities that are larger for women and that are highly significant, but they bear virtually no relationship to the underlying preference parameters.

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Consider an individual with time separable preferences and period utility function of the form

$$u(c_t) + B\frac{(1-h_t)^{1-1/\gamma}}{1-1/\gamma}.$$

The value of γ is critical for many economic analyses. One common strategy for uncovering the value of γ is to carry out structural estimation on micro panel data. One general issue in structural estimation exercises using micro data is that misspecification of the constraints that individuals face is likely to influence inference about preference parameters. In one popular specification of the constraints faced by individuals–the canonical model of life cycle labor supply–individuals can vary their hours continuously, face a wage per unit of time that is independent of hours worked and are subject to a present value budget constraint. In this setting, Thomas E. MaCurdy (1981) and Joseph G. Altonji (1986) show how simple linear regressions can be used to estimate the value of γ. At the other extreme, many aggregate models (e.g., Gary D. Hansen (1985) and Richard Rogerson (1988)) assume that workers face a binary choice with regard to working: they can either work a fixed amount that corresponds to full time work, or they can not work. In this setting, the key regression coefficient from either the MaCurdy or Altonji specifications will always be zero but does not provide any information about the value of γ.[1]

While both of these two extreme cases capture some elements of reality in terms of the choices that individuals face, it seems reasonable to think that reality probably lies in between the two extreme cases.[2] In this paper we consider one empirically appealing intermediate case in which individuals who decide to participate in the labor market can choose between full-time work and part-time work. We build and calibrate such a model and use it to generate micro panel data. We then ask whether standard labor supply regressions typically justified by the assumption of a continuous hours choice and a linear earnings schedule will uncover the true value of γ. As this model allows for some choice along the intensive margin, it is not clear a priori what to expect.[3]

[1] Alternative specifications that are of particular interest include allowing for human capital accumulation (Susumu Imai and Michael P. Keane (2004)), borrowing constraints (David Domeij and Martin Floden (2006)), non-linear wage schedules (Eric French (2005) and Rogerson and Johanna Wallenius (2009)), or optimization frictions (Raj Chetty (2010)).

[2] See Rogerson (forthcoming) for a review of the literature on hours constraints.

[3] One may ask whether it is of interest to estimate the parameter γ in a model that features discrete choice. As argued in both Rogerson (forthcoming) and Chetty et al. (forthcoming), if the discrete choices are dictated by the need to coordinate, then the values of these choices will respond to changes in the aggregate economic environment, and in this case the curvature parameter will play a key role.

Because the prevalence of part-time work in reality varies significantly across males and females, and since the results may be sensitive to the frequency of part-time work, we consider a model that features both male and female labor supply. Additionally, since most labor supply takes place in multi-member households, we model households as consisting of two members and consider the joint labor supply problem of the household in a dynamic setting. The resulting model is essentially the family labor supply model of Yongsung Chang and Sun-Bin Kim (2006), extended to allow for part-time work.

In contrast to what one would find if full time work and no work were the only options, we find that standard labor supply regressions on individual panel data do generate non-zero estimates for labor supply elasticities. However, the estimated values are virtually unrelated to the underlying preference parameter that the methods are designed to uncover. Moreover, the estimates are higher for women than for men, even if the underlying preference parameters are identical. We conclude that standard methods do not correctly identify individual preference parameters in our model featuring a mix of adjustment along the intensive and extensive margins. While we focus on one specific economy, our results point to the importance of properly modeling the choices of hours and earning schedules that individuals actually face and using methods that can uncover the underlying preference parameters given the nature of these choice sets.

Several papers in the literature have discussed the issue of estimating individual preference parameters if workers are not free to choose hours, or more generally can choose among hours and wage bundles. Jeff E. Biddle (1988) argues for using standard labor supply methods but on a sample that is restricted to self-employed workers that are free to adjust hours. In addition to possible selection biases, a shortcoming of this approach is that self-employed workers are likely to face stochastic employment opportunities as opposed to being able to choose hours at a fixed wage. Altonji and Christina H. Paxson (1988, 1992), French (2005) and Rogerson and Wallenius (2009) consider the case in which workers face a continuous hours choice but face a wage that is contingent on hours. Chetty et al. (forthcoming) consider a model in which each firm offers a fixed workweek and search frictions prevent workers from being able to instantaneously adjust hours of work by switching to another firm.[4] Our paper is the first to explore the performance of standard labor supply methods in a context that features multiple but discrete choices for hours of work and tied hours-wage bundles.

[4] See the discussion in Rogerson (forthcoming) for additional references to the literature.

I. The Model

There is a continuum of measure one of ex-ante identical households, each consisting of one male and one female member. We assume a unitary household that has preferences over streams of (household) consumption and individual leisure given by:

$$\max_{\{c_t,h_{mt},h_{ft}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ 2\log(0.5c_t) + B_m \frac{(1-h_{mt})^{1-1/\gamma}}{1-1/\gamma} + B_f \frac{(1-h_{ft})^{1-1/\gamma}}{1-1/\gamma} \right\} \tag{1}$$

where c_t is household consumption, h_{mt} is hours of market work by the male member and h_{ft} is hours of work by the female member. There are only three possible choices for hours of market work by an individual: full time work, denoted by h^F, part time work, denoted by h^P, or zero. Individuals are subject to idiosyncratic shocks that affect their productivity in market work. Letting x_{it}, $i = m, f$ denote the idiosyncratic productivity for male and female household members respectively, we assume:

$$\log x'_{it+1} = (1-\rho_i)\log \bar{x}_i + \rho_i \log x_{it} + \epsilon_{it}, \qquad \epsilon_{it} \sim N(0, \sigma_i^2) \tag{2}$$

The innovations for these markov processes are i.i.d. across individuals, households, and time.

We assume that there is a wage penalty associated with part-time work. Additionally, to capture the fact that men and women have differential labor supply across occupations and that this penalty may differ across occupations, we allow the penalty to be gender specific. In particular, if w_t is the wage rate per unit of labor services in period t, and an individual of gender i has idiosyncratic productivity x_{it}, then he or she receives labor earnings of $x_{it}w_t h^F$ if working full time and $(1-\lambda_i)x_{it}w_t h^P$ if working part time. It follows that the wage rate *per hour* of market work that an individual of gender i faces is a function of both his or her own idiosyncratic productivity shock and choice of work hours. We denote this by $\tilde{w}_{it}(x_{it}, h_{it})$. There are no markets for insurance against idiosyncratic productivity shocks, and the only asset available to households is claims to physical capital. Letting a_t denote the capital that the household carries into period t, and r_t the rate of return on capital net of depreciation, the one period budget constraint faced by the household is given by:

$$a_{t+1} = \tilde{w}_{mt}(x_{mt}, h_{mt})h_{mt} + \tilde{w}_{ft}(x_{ft}, h_{ft})h_{ft} + (1+r_t)a_t - c_t,$$

We assume that capital holdings cannot go below zero, i.e., $a_t \geq 0$ for all t.

A representative firm produces output according to a constant-returns-to-scale Cobb-Douglas technology in capital (K_t) and efficiency units of labor (L_t): $Y_t = K_t^{\alpha} L_t^{1-\alpha}$.. Capital depreciates at rate δ.

We focus on a steady state equilibrium in which the aggregates K_t and L_t as well as the two prices r_t and w_t are constant. The household's optimization problem can be conveniently reformulated in the recursive form. The individual state variable for a household will be the triplet (a, x_m, x_f). Let $V_{jk}(a, x_m, x_f)$ denote the value to a household when the male member's labor market state is j and the female member's is k. Due to indivisibility of labor as assumed above, j, $k \in \{F, P, N\}$ where F stands for full-time work, P part-time work and N not-working. Hence a household has nine types of value functions associated with its two members' labor market states. For example, the value to a household in which both male and female members work full time is defined by the following Bellman equation:

$$V_{FF}(a, x_m, x_f) = \max_{a' \in \mathcal{A}} \left\{ u\big(c, h^F, h^F\big) + \beta E\Big[V(a', x_m', x_f') \big| x_m, x_f\Big] \right\} \tag{3}$$

$$\text{s.t. } c = w_m^F x_m^F h^F + w_f^F x_f^F h^F + (1+r)a - a', \text{ and } a' \geq 0.$$

Other value functions, V_{FP}, V_{FN}, V_{PF}, V_{PP}, V_{PN}, V_{NF}, V_{NP}, V_{NN}, are defined in a similar way. Dropping the arguments of the value functions for notational convenience, a household's joint decision for labor supply can be characterized as:

$$V(a, x_m, x_f; \mu) = \max \Big\{ V_{FF}, V_{FP}, V_{FN}, V_{PF}, V_{PP}, V_{PN}, V_{NF}, V_{NP}, V_{NN} \Big\}. \tag{4}$$

II. Quantitative Analysis

We set a time period equal to one year. As shown in Chang and Kim (2006), some interesting implications arise when allowing for a shorter time period and then time aggregating to generate panel data at annual frequency. Because we want to focus on different issues, we set a time period equal to one year and thereby abstract from these time aggregation effects.

Our main goal is to assess how the relationship between the preference parameters γ_m and γ_f and the estimates from standard labor supply regressions is affected by the presence of restricted hours choices. For this reason we will consider five different economies that differ in their values for these two labor preference parameters. For simplicity we will consider the symmetric case in which the two parameters are the same, and denote the common value by γ. The five economies that we consider correspond to $\gamma = .2$, $.4$, $.6$, $.8$, and 1.0. Given these assumed values for γ, the rest of the model's parameters will be calibrated as follows.

There are many papers in the literature that estimate idiosyncratic wage and/or earnings shocks, though almost exclusively for prime aged males that are employed full time. The consensus from this literature is that wage shocks are very persistent and large. For our benchmark analysis we again impose symmetry and use the same values for both male and female workers: $\rho_i = .92$ and $\sigma_i = .21$ for $i = m, f$. This particular choice corresponds to the estimates of Floden and Jesper Linde (2001) for prime aged males. The value of $\bar{x}_m$ is set to zero as a normalization, while $\bar{x}_f$ is set so that the unconditional average productivity for females is 35% less than that of males.[5]

The labor share, α, is set to .64, and the annual depreciation rate, δ, is set to .08. The discount factor β is chosen so that in steady state we have $r = .05$. At work, individuals supply $h = 0.4$ and $h = 0.2$, as a full-time and part-time worker, respectively.

This leaves four parameters: the two utility parameters B_m and B_f, and the two part-time wage penalty parameters, λ_m and λ_f. For each value of γ we choose the values of these parameters so as to match four employment targets: the fraction of the male and female population that are employed full time and part time. This requires taking a stand on how to map hours of work in the data into the categories of non-employment, part-time and full-time. For our benchmark results we use the following mapping between annual hours of work and employment status: annual hours greater than 1800 corresponds to full time work, annual hours between 400 and 1800 corresponds to part time work and annual hours less than 400 corresponds to zero work. Our results are not much affected by small changes in the values of the cutoffs. The hours worked distribution from the data is clearly much more spread out than the three point distribution implied by our model. Adding measurement error to hours in the model could produce a distribution in the model that would better resemble the one from the data. To the extent that measurement error in hours generates results that are well-understood, we choose to abstract from this feature. More generally, whereas we assume that the full and part-time options are the same for all individuals, it would be reasonable to assume that there is heterogeneity in these

[5] We incorporate this feature so as to generate a gender wage gap in equilibrium, though this feature plays no role in our results. Because the preference parameters B_i are allowed to be gender specific, the value of B_f plays a similar role as $\bar{x}_f$ in terms of influencing labor supply. Despite this, it is interesting to note that the model has some difficulties in matching the gender wage gap found in the data. For example, in the equilibrium with $\gamma = .40$, the gender wage gap is 25%, but for full-time workers the wage gap is only 7.7%, reflecting the presence of strong selection effects in labor-market participation for females.

values associated with, for example, occupational differences.

Based on data from the PSID for the years 1968-2002 for married couples with the age of the household head between 25 and 55, the target values for the fraction of individuals employed full time is .891 and .402 for males and females respectively, while the corresponding values for part time employment are .057 and .226 with the above values of cutoffs for employment status.[6]

The required penalty is decreasing in γ, and ranges from zero to as much as 50%. While several issues make it difficult to measure this penalty in the data, there is a small literature that provides some estimates this penalty, including the early contribution of Robert A. Moffitt (1984) and more recent papers by Keane and Kenneth I. Wolpin (2001) and Daniel Aaronson and French (2004). A reasonable upper bound for the size of the penalty seems to be around 20%. All of our specifications with the exception of $\gamma = .2$ are within the reasonable range. While this might suggest that the $\gamma = .2$ specification is not reasonable, but we will continue to consider it below. The model is calibrated so as to exactly match the distribution of both males and females across the three different levels of work. However, the model also does a good job of capturing the joint distribution of male and female employment status across households, as well as the year-to-year transition rates for both men and women across the three employment states. In the interest of space we do not report these results. In summary, our model does a reasonable job of matching both the distribution of males and females across the three employment states as well as the transitions among the three states.

Our main goal is to assess how standard methods for estimating labor supply elasticities based on the assumptions of a continuous choice of working hours and a linear earnings schedule (i.e., earnings are linear in time devoted to work at a particular point in time) fare in our setting. Assuming a continuous choice and a linear earnings schedule, our period utility function implies that the following condition for male and female labor supply must hold in any period in which hours are positive:

$$\log(1 - h_{it}) = A - \gamma \log w_{it} + \gamma \log c_t$$

where A is a constant and c_t is household consumption in period t.

To begin our analysis we use the steady state equilibrium decision rules to generate a panel data set consisting of 5000 households with 100 years of observations and then run the above regression using all observations with positive hours.[7] Later on we will consider how various

[6] Our results are not much affected by small changes in the values of the cutoffs.

[7] For the results we report we imposed that the coefficients on log wages and log consumption are the same, though the results are effectively the same if this is not imposed.

selection criterion influence the estimates. The results are in Table 1. In running this regression we impose that the coefficients on $\log w_{it}$ and $\log c_t$ are equal in absolute value and of opposite sign, though the estimates are effectively unchanged even if we do not impose this condition. We do not report standard errors in the table, but note that they are all very close to zero.

Table 1: Estimates of γ

	$\gamma = .2$	$\gamma = .4$	$\gamma = .6$	$\gamma = .8$	$\gamma = 1.0$
$\hat{\gamma}_m$	.10	.09	.09	.09	.09
$\hat{\gamma}_f$	.25	.34	.37	.37	.38

Several interesting results emerge. First, whereas in a pure indivisible labor model in which full time work is the sole option for working, one would necessarily find a zero estimate for the labor supply elasticity parameter (recall that we abstract from time aggregation effects), here we obtain positive values for both males and females in all cases. Second, the estimated preference parameter is always larger for women than for men. Third, the estimated values $\hat{\gamma}_i$ bear essentially no relationship to the true preference parameters. For men we find that the estimated value of $\hat{\gamma}_m$ is effectively equal to .10 independently of the true value of γ. A similar result arises for the estimated preference parameter for women. Although when $\gamma = .20$ the estimated value $\hat{\gamma}_f$ is substantially lower than for the other cases, for γ between .40 and 1.0 we see that there is relatively little change in $\hat{\gamma}_f$ despite a more than doubling of γ.

While space limitation prevents us from reporting results from many sensitivity studies, we briefly mention three. First, we have examined how sample selection rules influence the results. Many studies use selection rules that oversample highly attached workers. Our main results also apply in this case. While we do find higher estimated elasticities for low attachment samples, it remains true that the estimates are virtually unrelated to the true value of γ. Second, while the above results assumed a very long panel, in practice researchers often have a much shorter panel. We have also run the same exercises with a sample panel that lasts for ten years, and obtained similar results. Third, the above regressions have used the log of leisure as the left hand side variable, since this is consistent with our specification of preferences. We have also run the same regressions using log of hours worked as the left hand side variable. The results are effectively identical, though as expected the estimated values of $\hat{\gamma}_i$ are now larger since the labor supply elasticity of labor supply for hours worked is greater than the labor supply elasticity for leisure. Specifically, based on the 100 year sample, the estimates are roughly .21 and .88 for men and women respectively.

Lastly, we report one exercise that captures the properties of aggregate labor supply in our setting. Specifically, motivated by the exercise of Edward C. Prescott (2004), we ask what happens to aggregate hours worked in steady state if a proportional tax on labor income is introduced and the proceeds are used to fund a lump-sum transfer. We find that moving from a tax rate of zero to 20% results in a decrease in aggregate hours worked of roughly 13% in all five economies. Interpreted from the perspective of a stand-in household model with period household utility function $\log c - B\frac{h^{1+1/\gamma}}{1+1/\gamma}$, this change is consistent with a value of γ of roughly 1.6. This is quite similar to the implied value that Chang and Kim (2006, 2007) have found for models that do not allow for part time work.

III. Conclusion

We built a model of family labor supply in which individuals choose between full-time work, part-time work and non-employment. The model is calibrated so as to replicate the movements of both male and female workers among these states. Although the individual labor supply problem is a discrete choice problem, individuals are able to adjust hours along the intensive margin by moving between part-time and full-time work. Intuitively, adjustment along the intensive margin potentially allows one to estimate the true value of the underlying curvature parameter describing the utility from leisure. We explore the extent to which standard labor supply methods can achieve this in our setting. Although these methods deliver precise estimates that are significantly different from zero, the estimates are effectively unrelated to the true underlying values. These methods also deliver elasticity estimates for women even when the underlying preference parameters are the same for men and women.

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