On the Unstable Relationship between Exchange Rates and Macroeconomic Fundamentals[1]

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Abstract

It is well known from anecdotal, survey and econometric evidence that the relationship between the exchange rate and macro fundamentals is highly unstable. This could be explained when structural parameters are known and very volatile, neither of which seems plausible. Instead we argue that large and frequent variations in the relationship between the exchange rate and macro fundamentals naturally develop when structural parameters in the economy are unknown and change very slowly. We show that the reduced form relationship between exchange rates and fundamentals is driven not by the structural parameters themselves, but rather by expectations of these parameters. These expectations can be highly unstable as a result of perfectly rational "scapegoat" effects. This happens when parameters can potentially change much more in the long run than the short run. This generates substantial uncertainty about the *level* of parameters, even though monthly or annual *changes* are small. This mechanism can also be relevant in other contexts of forward looking variables and could explain the widespread evidence of parameter instability found in macroeconomic and financial data.

1 Introduction

"*The dollar's resilience in the wake of recent dire US economic data has raised the prospect that the currency market may be experiencing one of its periodic changes in focus*" (Financial Times, February 11, 2008)

"*The dollar's latest stumble ... came despite optimistic economic data from the US. But analysts said the movement of the US currency was no longer driven by growth fundamentals. All the focus is on the deficit now...*" (Financial Times, February 11, 2003)

As reflected in these quotes, foreign exchange traders regularly change the weight they attach to different macro indicators. Cheung and Chinn (2001) have documented these changes through a survey of U.S. foreign exchange traders. Frequent changes in focus lead to an unstable relationship between exchange rates and macro fundamentals. Such parameter instability is confirmed in formal econometric evidence. Rossi (2005) conducts a battery of parameter instability tests and finds "overwhelming evidence of parameter instability". Sarno and Valente (2009) find that "(exchange rate) models that optimally use the information in the fundamentals change often and this implies frequent shifts in the parameters".

One way to explain the highly unstable relationship between exchange rates and macro fundamentals is to assume large and frequent changes in structural parameters that are known to all agents. This does not appear very plausible though as these parameters are not directly observed and hard to estimate. Moreover, many structural changes in the economy, such as those associated with technological and financial innovation and institutional reform, are gradual.

The main goal of this paper is to show that large and frequent variations in the relationship between the exchange rate and macro fundamentals can occur quite naturally even when structural parameters in the economy are unknown and change very slowly. We show that the relationship between a forward looking variable like the exchange rate and macro fundamentals is determined not by the structural parameters themselves, but rather by the expectations of these structural parameters. We show that these expectations can vary significantly over time, giving rise to a highly unstable reduced form relationship between exchange rates and fundamentals. This happens even though agents are perfectly rational Bayesian learners and changes in structural parameters are small and gradual.

While the focus of this paper is on exchange rates, our explanation for the unstable reduced-form relationship could apply similarly to other forward looking financial or macroeconomic variables. As first shown by Stock and Watson (1996), and since then by many others, the phenomenon of parameter instability in macroeconomic data is widespread.[1] The same is the case for financial data. In a survey, Pastor and Veronesi (2009) point out that "parameter uncertainty is ubiquitous in finance" and "many facts that appear baffling at first sight seem less puzzling once we recognize that parameters are uncertain and subject to learning".[2]

The estimation mistakes that agents make when continuously updating their views on structural parameters are to a large extent a result of what we refer to as "scapegoat" effects. Some information about the nature of structural parameters can be derived by analyzing macroeconomic data and exchange rates. But these data are also driven by shocks to *unobserved* fundamentals. Such unobserved fundamentals can generate considerable confusion in the short to medium run. When the exchange rate fluctuates as a result of an unobserved macroeconomic shock, it can be optimal for agents to blame this on an observed macro fundamental by giving it more weight and therefore making it a "scapegoat".[3] For example, when the dollar depreciates it is natural to attribute it to a large current account deficit. This

[1]Recent contributions include Boivin (2006), Canova (2005), Clarida, Gali and Gertler (2000), Cogley and Sargent (2005), Del Negro and Otrok (2007), Inoue and Rossi (2007), Primiceri (2005), Sims and Zha (2006) and Fernandez-Villaverde and Rubio-Ramirez (2007). There has also been great interest in the impact of parameter or model uncertainty on optimal monetary policy. See for example contributions by Hansen and Sargent (2008), Onatski and Williams (2003) or Levin et al. (2006).

[2]For example, Cogley (2005) and Piazzesi and Schneider (2007) introduce uncertainty about time-varying parameters to explain the term spread.

[3]In a previous short paper, Bacchetta and van Wincoop (2004), we developed the idea of such a scapegoat effect in the context of a simple static noisy rational expectations model in which some parameters are unknown. We showed that excessive weight could be given to a variable depending on the correlation between the noise shock and the fundamental shock. However, since that model is static it could not be used to address the unstable dynamic relationship between exchange rates and fundamentals and its implications. Apart from the dynamic setup, the model in this paper also differs in that there is no private information as in Bacchetta and van Wincoop (2004). Scapegoat effects naturally develop as long as there is incomplete information about parameters; the information does not need to be private.

happens even when the depreciation is unrelated to the current account deficit.

There is significant potential for such scapegoat effects when the uncertainty about structural parameters is large. Two factors contribute to this. First, parameters can potentially change much more in the long-run than the short-run. This generates substantial uncertainty about the *level* of parameters, even though monthly or annual *changes* are small. Second, agents cannot observe these structural parameters and obtain only very indirect information about their level through inference from the data.

In illustrating the importance of such scapegoat effects and their role in the unstable reduced form relationship between exchange rates and fundamentals, we slightly generalize the "canonical" exchange rate model. This is actually a broad class of exchange rate models that can be reduced to a single stochastic difference equation, which is derived from two equations: an interest rate parity equation and an equation that relates the interest differential to observed fundamentals. The latter can be obtained either from monetary policy specifications or money market equilibrium in a standard monetary model (see Engel and West, 2005, for examples). We generalize this by introducing time variation in the interest rate differential equation. While we illustrate the source of this time variation in the context of the standard monetary model, in general it can have many possible sources. Examples are changes in monetary policy parameters, changes in money demand parameters, or changes in the relationship between policy targets and observed fundamentals.

We calibrate the model to data for 5 industrialized countries, matching moments related to interest rates and exchange rates and the explanatory power of observed fundamentals. We consider a particular process for time-varying structural parameters that satisfies two features. First, changes in these parameters are small over short horizons of a month or a year. Second, changes in structural parameters gradually build over time, so that they can change substantially over long periods. These features are plausible when we think of long-term technological, institutional or cultural changes. Such a process generates large scapegoat effects as there is substantial uncertainty about the level of parameters even when month-to-month changes are small.

We do not estimate the process of structural parameters. That would be nearly impossible to do. First, the data can tell us very little about the exact nature of the process of time-varying parameters, even if there is clear evidence of parameter instability (e.g., see Elliott and Timmermann, 2008, for a discussion). Second, even when a particular process is assumed, its parameters are notoriously hard to estimate with any precision. While we focus on a specific process in the benchmark analysis, we examine the robustness of our results to a wide range of alternative processes.

The next section presents the model. It also discusses the signal extraction method used to solve the model and the implications for the relationship between exchange rates and fundamentals. Section 3 calibrates the model based on data on interest rates and exchange rates and presents numerical results for the relationship between exchange rates and fundamentals based on simulations. Section 4 concludes.

2 A Model with Unknown Parameters

We first describe the model when parameters are constant and known. Then we introduce unknown and time-varying coefficients and examine how the impact of fundamentals on the exchange rate is affected. For that purpose, we need to derive how expectations about parameters are formed. We show that this process leads to an unstable relationship between fundamentals and exchange rates. The final subsection provides intuition on the mechanism leading to this instability.

2.1 Basic Framework with Constant Parameters

We consider the class of fundamental-based exchange rate models that can be reduced to a single stochastic difference equation. The equilibrium value of the exchange rate in these models depends on the present value of expected future fundamentals. We start with the usual case of constant and known parameters. We follow Engel and West (2005) and slightly rewrite their equation (1):

$$s_t = (1-\lambda)\left[F_t + b_t + \sum_{j=1}^{\infty} \lambda^j E_t\left(F_{t+j} + b_{t+j}\right)\right] - \lambda\left[\phi_t + \sum_{j=1}^{\infty} \lambda^j E_t \phi_{t+j}\right] \qquad (1)$$

where s_t is the log nominal exchange rate (domestic per foreign currency), E_t is the expectation of the representative investor, ϕ_t is the risk premium and $0 < \lambda < 1$.

We denote by F_t a linear combination of observed macro fundamentals: $F_t = \mathbf{f}_t'\boldsymbol{\beta}$ where $\mathbf{f}_t = (f_{1t}, f_{2t}, ..., f_{Nt})'$ is the vector of N observed macroeconomic fundamentals and $\boldsymbol{\beta} = (\beta_1, \beta_2, ..., \beta_N)'$ is the vector of associated parameters. Finally, b_t represents unobserved macro fundamentals.

Engel and West (2005) present several models that lead to this equation.[4] For illustrative purposes we focus on the familiar flexible-price monetary model. A two-country model can be described by the standard four equations:

$$E_t s_{t+1} - s_t = i_t - i_t^* + \phi_t \tag{2}$$

$$s_t = p_t - p_t^* \tag{3}$$

$$\mu m_t = p_t - \alpha i_t + \boldsymbol{\gamma}'\mathbf{z}_t + \nu_t \tag{4}$$

$$\mu m_t^* = p_t^* - \alpha i_t^* + \boldsymbol{\gamma}'\mathbf{z}_t^* + \nu_t^* \tag{5}$$

As usual, i_t and i_t^* represent the domestic and foreign nominal one-period interest rates, p_t and p_t^* are the domestic and foreign log prices, and m_t and m_t^* are the log nominal money supplies. We denote by $\mathbf{z}_t$ and $\mathbf{z}_t^*$ the vectors of other observed fundamentals affecting money demand. Unobserved velocity shocks are denoted ν_t and ν_t^*. The parameter μ is usually set at 1, but does not need to be 1 when the vector $\mathbf{z}_t$ includes nominal variables as well.[5]

By combining equations (3), (4), and (5), we find:

$$i_t - i_t^* = \frac{1}{\alpha}s_t - \frac{1}{\alpha}\left[\mu(m_t - m_t^*) - \boldsymbol{\gamma}'(\mathbf{z}_t - \mathbf{z}_t^*)\right] + \frac{1}{\alpha}(\nu_t - \nu_t^*) \tag{6}$$

This equation can be rewritten in a more compact form as:

$$i_t - i_t^* = \frac{1}{\alpha}s_t - \frac{1}{\alpha}(F_t + b_t) \tag{7}$$

where $b_t = -(\nu_t - \nu_t^*)$ is an unobserved fundamental and $F_t = \mathbf{f}_t'\boldsymbol{\beta} = \mu(m_t - m_t^*) - \boldsymbol{\gamma}'(\mathbf{z}_t - \mathbf{z}_t^*)$ is a linear combination of observed fundamentals. Combining equations (2) and (7), integrating forward and assuming no bubble gives equation (1), where $\lambda = \alpha/(1+\alpha)$.

[4] See also Nason and Rogers (2008) who derive this equation from a DSGE model.

[5] Examples of nominal variables in $\mathbf{z}_t$ include lagged money demand, lagged prices or nominal financial wealth. Introducing μ gives us a parameter multiplying the money supply fundamental, just like $\boldsymbol{\gamma}$ is a vector of parameters multiplying the other observed fundamentals $\mathbf{z}_t$. But it is not critical to the analysis in any substantial way.

Since s_t and fundamentals are typically non-stationary in the data, it is usual to consider first differences. As an illustration, consider the special case without a risk premium and where b_t and $\Delta\mathbf{f}_t$ are iid. More precisely, assume that: i) $\phi_t = 0$, $\forall t$; ii) $b_t = \varepsilon_t^b$ with $\varepsilon_t^b \sim N(0, \sigma_b^2)$; iii) $\Delta f_{nt} = \varepsilon_{nt}^f$ with $\varepsilon_{nt}^f \sim N(0, \sigma_f^2)$. In this case, we have:

$$\Delta s_t = \Delta\mathbf{f}_t'\boldsymbol{\beta} + (1-\lambda)\Delta b_t \tag{8}$$

The impact of a change in fundamental f_{nt} on the exchange rate is simply given by β_n: $\partial\Delta s_t/\partial\Delta f_{nt} = \beta_n$.

2.2 Time-varying and Unknown Parameters

We now depart from the standard model by assuming that parameters can vary over time. We introduce parameter instability to the first difference of the money demand equation (4).[6] Adding a time subscript to the parameters μ and $\boldsymbol{\gamma}$ after taking the first difference of (4), we get

$$\mu_t\Delta m_t = \Delta p_t - \alpha\Delta i_t + \boldsymbol{\gamma}_t'\Delta\mathbf{z}_t + \Delta\nu_t \tag{9}$$

In level terms we can write this specification as

$$\begin{aligned}
\tilde{m}_t &= p_t - \alpha i_t + Z_t + \nu_t && (10)\\
Z_t &= Z_{t-1} + \boldsymbol{\gamma}_t'\Delta\mathbf{z}_t && (11)\\
\tilde{m}_t &= \tilde{m}_{t-1} + \mu_t\Delta m_t && (12)
\end{aligned}$$

Together with an analogous specification for money demand in the other country, and defining $F_t = (\tilde{m}_t - \tilde{m}_t^*) - (Z_t - Z_t^*)$, the solution for the interest differential remains the same as in (7). This again yields the present value equation (1) when combined with (2). All we have really done is to replace $\Delta F_t = \Delta\mathbf{f}_t'\boldsymbol{\beta}$ with

$$\Delta F_t = \Delta\mathbf{f}_t'\boldsymbol{\beta}_t \tag{13}$$

[6]It is easy to show that introducing time-varying parameters in the levels equation is inconsistent with the stationarity of Δm_t, Δp_t and Δi_t in the data. When introducing parameter instability to the levels equation, and then taking first differences, there are terms that involve the product of fundamentals and the change in parameters. Such terms are non stationary to the extent that there are non-stationary fundamentals.

where

$$\mathbf{f}_t = \begin{pmatrix} m_t \\ \mathbf{z}_t \end{pmatrix} \quad ; \quad \boldsymbol{\beta}_t = \begin{pmatrix} \mu_t \\ \gamma_t \end{pmatrix} \tag{14}$$

With a total of N fundamentals we will also write $\boldsymbol{\beta}_t = (\beta_{1t}, \beta_{2t}, ..., \beta_{Nt})'$.

We therefore replace the specification $\Delta F_t = \Delta \mathbf{f}_t' \boldsymbol{\beta}$ for constant parameters with the specification $\Delta F_t = \Delta \mathbf{f}_t' \boldsymbol{\beta}_t$ for time-varying parameters. While for illustrative purposes we have motivated this in the context of the familiar flexible price monetary model, it can also be obtained from other models that lead to the present value equation (1). One example is to replace the money market equilibrium by an interest rate rule that depends on a number of observed fundamentals. Time-varying parameters are then associated with time variation in the monetary policy parameters. Another possibility is that these monetary policy parameters are constant but the (possibly unknown) policy targets have a time-varying relationship to the observed fundamentals. The exact source of the time-varying parameters is not critical to the qualitative findings of the paper.

The major difference with the standard framework is that F_t is not directly observable. Investors need to estimate current and future $\boldsymbol{\beta}_t$. They have two sources of information regarding $\boldsymbol{\beta}_t$. First, they know the process of $\boldsymbol{\beta}_t$, which we will specify below. Second, by observing the exchange rate and the interest rate differential, they know $F_t + b_t$ from (7). We describe below how investors combine optimally these two sources of information to form expectations about $\boldsymbol{\beta}_t$.

The signal $F_t + b_t$ provides information about the parameters, but is also a source of estimation errors. Consider for example the expectation of parameter β_{nt} for fundamental n. While β_{nt} affects $F_t + b_t$, the latter is also affected by b_t, all current and past fundamentals and all current and past parameters. Therefore, to the extent that $F_t + b_t$ is used as a source of information about β_{nt}, its expectation can change without any change in β_{nt} itself. We will see that it is this rational confusion that is the key driver behind the unstable relationship between exchange rates and observed fundamentals.

2.3 Exchange Rates and Fundamentals

For convenience, in the remainder of this section we consider the special case without a risk premium and where b_t and $\Delta \mathbf{f}_t$ are iid, as described above. A more general specification will be considered in the numerical analysis in the next section. We maintain the assumption throughout the paper that shocks to f_{nt}, b_t and parameters are uncorrelated with each other.

Under these assumptions, $(1-\lambda)\sum_{j=1}^{\infty}\lambda^j E_t F_{t+i} = \lambda E_t F_t$ because $E_t F_{t+i} = E_t F_t$. The first difference of the present value equation (1) then becomes:

$$\Delta s_t = (1-\lambda)\Delta F_t + (1-\lambda)\Delta b_t + \lambda(E_t F_t - E_{t-1}F_{t-1}) \tag{15}$$

If the parameters $\boldsymbol{\beta}_t$ were known, then F_t is known as well at time t and (15) becomes

$$\Delta s_t = \Delta \mathbf{f}_t' \boldsymbol{\beta}_t + (1-\lambda)\Delta b_t \tag{16}$$

This generalizes (8) by replacing the constant vector of parameters $\boldsymbol{\beta}$ that multiplies the fundamentals $\Delta \mathbf{f}_t'$ by the vector of time-varying parameters $\boldsymbol{\beta}_t$. When the parameters β_{nt} are not only known, but also very volatile, it could explain the unstable relationship between exchange rates and fundamentals.

However, in reality the time-varying parameters are unknown. In that case the last term $E_t F_t - E_{t-1}F_{t-1}$ in (15) is a complex expression that depends on expectations of parameters. In order to avoid the technical problem of computing expectations of parameter innovations going back to the infinite past, we assume that parameters are known after T periods. Therefore the total number of unknown parameter innovations is NT, which is finite. In practice we will set T very large. In that case, we can write (15) as:

$$\Delta s_t = \Delta \mathbf{f}_t' \left((1-\lambda)\boldsymbol{\beta}_t + \lambda E_t \boldsymbol{\beta}_t\right) + (1-\lambda)\Delta b_t + \lambda \sum_{i=1}^{T} \Delta \mathbf{f}_{t-i}' (E_t \boldsymbol{\beta}_{t-i} - E_{t-1}\boldsymbol{\beta}_{t-i}) \tag{17}$$

As can be seen from the first term in (17), $\Delta \mathbf{f}_t$ is now multiplied by a weighted average of actual and expected parameter values. Since the discount rate λ tends to be close to 1 (see Engel and West, 2005), almost all of the weight is on the expected value of parameters rather than the actual level of parameters. The reason is that the exchange rate is forward looking and depends on expectations of future fundamentals. In this particular example, where fundamentals follow a random walk, expected future levels of F are equal to the expected level of F today, which depends on the expectation of the current set of parameters $\boldsymbol{\beta}_t$. More generally, if changes in fundamentals are not iid, Δs_t also depends on expectations about future values of the parameters. The general setup is discussed in Appendix A.

In the last term of (17) we see that Δs_t also depends on the change in expectations $E_t\boldsymbol{\beta}_{t-i} - E_{t-1}\boldsymbol{\beta}_{t-i}$ of past parameters, interacted with changes in past fundamentals. Intuitively, since $F_t = \sum_{i=0}^{\infty} \Delta\mathbf{f}'_{t-i}\boldsymbol{\beta}_{t-i}$, changes in the expectation of past parameters lead to a change in the expectation of F_t and therefore the exchange rate. We will show that changes in current fundamentals lead to changes in the expectation of both current and past parameters. This is therefore an additional channel through which changes in current fundamentals affect the exchange rate.

To examine the impact of fundamentals on the exchange rate, we simply consider the derivative of the exchange rate with respect to current fundamentals:

$$\frac{\partial \Delta s_t}{\partial \Delta f_{nt}} = (1-\lambda)\beta_{nt} + \lambda E_t \beta_{nt} + \lambda \sum_{i=0}^{T} \Delta\mathbf{f}'_{t-i} \frac{\partial E_t \boldsymbol{\beta}_{t-i}}{\partial \Delta f_{nt}} \tag{18}$$

The rest of this section analyzes in more detail the last two elements on the right-hand side of (18).

2.4 Expectation of Parameters

In order to determine the impact of fundamentals on the exchange rate, we need to determine the expectation of current and past parameters. We do this by first assuming a process for structural parameters and then solving a signal extraction problem.

We consider the case where a structural parameter β_{nt} depends on a finite number T of past innovations:

$$\beta_{nt} = \beta_n + \sum_{i=1}^{T} \theta_{ni}\varepsilon_{n,t-i+1} \tag{19}$$

where $\varepsilon_{nt} \sim N(0, \sigma_\beta^2)$. In this section we consider a rather general process characterized by the parameters θ_{ni}. In the next section we will pick a particular process for the numerical analysis that satisfies the criteria discussed in the Introduction.

As discussed in section 2.3, we assume that parameter innovations at dates $t-T$ and earlier are known at date t in order avoid an infinite number of unknown parameter innovations about which expectations need to be formed. In addition (19) assumes that parameter innovations at $t-T$ and earlier do not affect parameters at time t. This is a different assumption, which we make to assure stationarity of the structural parameters. In practice the impact of these assumptions is minimized by setting T very high in the numerical analysis. In addition we will consider an alternative process in section 3.3 where parameters depend on an infinite number of lagged innovations.

In vector notation (19) can be written as

$$\boldsymbol{\beta}_t = \boldsymbol{\beta} + \Theta \boldsymbol{\xi}_t \tag{20}$$

where $\boldsymbol{\beta}=(\beta_1, \beta_2, ..., \beta_N)'$ is a N-vector of constants; $\boldsymbol{\xi}_t$ is a NT vector that stacks all the vectors $\boldsymbol{\xi}_{nt} = (\epsilon_{nt}, ..., \epsilon_{n,t-T+1})'$; and Θ is a $N \times NT$ matrix with $\Theta[n, T(n-1)+1 : Tn] = \boldsymbol{\theta}'_n = (\theta_{n1}, \theta_{n2}, ..., \theta_{nT})$ and zeros otherwise.

In order to form expectations about current and past values of $\boldsymbol{\beta}_t$ we need to compute expectations about the vector $\boldsymbol{\xi}_t$ of current and past parameter innovations. Since the problem is linear and all the shocks are normal, we can use standard signal extraction techniques. Leaving some of the details to Appendix B, we sketch how this is done. We start from the knowledge that the unconditional distribution of $\boldsymbol{\xi}_t$ is normal with mean zero and variance $\sigma^2_\beta \mathbf{I}_{NT}$, where $\mathbf{I}_{NT}$ is an identity matrix of size NT. We combine this with knowledge of $d_t = F_t + b_t$ over the past T periods. Defining $\mathbf{Y}_t = \left(d^*_t, .., d^*_{t-T+1}\right)'$, where d^*_t subtracts the known components from d_t, we have

$$\mathbf{Y}_t = \mathbf{H}'_t \boldsymbol{\omega}_t \tag{21}$$

where $\boldsymbol{\omega}'_t = (\boldsymbol{\xi}'_t, \varepsilon^b_t, \varepsilon^b_{t-1}, ..., \varepsilon^b_{t-T+1})$ and $\mathbf{H}_t$ is a matrix that depends on current and lagged changes in observed fundamentals: $\Delta \mathbf{f}_{t-i}$ for $0 \leq i \leq T$. The precise form of $\mathbf{H}_t$ can be found in Appendix B.

The unconditional distribution of $\boldsymbol{\omega}_t$ is normal with mean zero and variance

$$\tilde{\mathbf{P}} = \begin{pmatrix} \sigma^2_\beta \mathbf{I}_{NT} & \mathbf{0} \\ \mathbf{0} & \sigma^2_b \mathbf{I}_T \end{pmatrix} \tag{22}$$

Combining this with (21), standard signal extraction[7] implies that the conditional distribution of $\boldsymbol{\omega}_t$ is normal with mean

$$\begin{aligned} E_t \boldsymbol{\omega}_t &= \mathbf{M}_t \mathbf{Y}_t \\ \mathbf{M}_t &= \tilde{\mathbf{P}} \mathbf{H}_t \left[\mathbf{H}'_t \tilde{\mathbf{P}} \mathbf{H}_t\right]^{-1} \end{aligned} \tag{23}$$

[7] See for example Townsend(1983, p.556).

and variance

$$\mathbf{P}_t = \tilde{\mathbf{P}} - \mathbf{M}_t \mathbf{H}_t' \tilde{\mathbf{P}}$$

Therefore

$$E_t \boldsymbol{\omega}_t = \mathbf{C}_t \boldsymbol{\omega}_t \tag{24}$$

where $\mathbf{C}_t = \mathbf{M}_t \mathbf{H}_t'$. Together with knowledge of parameter innovations of at least T periods ago, (24) gives expressions for $E_t \boldsymbol{\xi}_{t-i}$, for $i = 0, 1, ..., T-1$. We use this to compute $E_t \boldsymbol{\beta}_{t-i}$ from (20).

We then have

$$E_t \boldsymbol{\beta}_{t-i} = \hat{\boldsymbol{\beta}}_{t-i} + \boldsymbol{\Omega}_{ti} \boldsymbol{\omega}_t \tag{25}$$

Here $\hat{\boldsymbol{\beta}}_{t-i}$ is equal to $\boldsymbol{\beta}$ plus (for $i > 0$) a vector that depends on parameter innovations of at least T periods ago that are known at time t. The matrix $\boldsymbol{\Omega}_{ti}$ is equal to $\Theta \widetilde{\mathbf{I}}_i \mathbf{C}_t$, where $\widetilde{\mathbf{I}}_i$ is a matrix of zeros and ones that maps $\boldsymbol{\omega}_t$ into the unknown elements of $\boldsymbol{\xi}_{t-i}$.

There are two important features to notice from (25). First, $E_t \boldsymbol{\beta}_{t-i}$ is determined by a combination of shocks contained in $\boldsymbol{\omega}_t$. Thus, the expectation of a specific parameter β_{nt-i} depends on its own innovations, but also on current and past innovations to the noise vector $\mathbf{b}_t$ and to all other parameters. Second, $\boldsymbol{\Omega}_{ti}$ depends on current and past $\Delta \mathbf{f}_t$ so that shocks to fundamentals affect parameter expectations.[8]

As we will see, the expectation of β_{nt} can change significantly over a relatively short period even when the actual structural parameters change very slowly. What matters is not the monthly (or even annual) fluctuations in structural parameters but rather their potential to fluctuate over a very long period of time (decades or longer). The unconditional standard deviation of the parameters then becomes large even though changes from period to period are small. A large unconditional standard deviation of structural parameters, together with the difficulty in learning about their level, may imply large and frequent changes in expectations about these parameters. This allows expectations to become significantly disconnected from the true value of the parameters.

[8]Current and past $\Delta \mathbf{f}_t$ enter $\mathbf{H}_t$, which affects $\mathbf{M}_t$, $\mathbf{C}_t$, and therefore $\boldsymbol{\Omega}_{ti}$.

2.5 Derivative of Exchange Rate with respect to Fundamentals: Intuition

After substituting the solution for the expected parameters into (18), we have an expression for the derivative of the exchange rate with respect to fundamentals as a function of all the underlying shocks in the model: shocks to fundamentals, ε^f_{nt}, shocks to parameters, ε_{nt}, and shocks to unobservables, ε^b_t. We can solve the model numerically to show how the derivative evolves over time. However, it is hard to get much intuition out of the algebraic expression. It is highly non-linear in the shocks, which enter through large matrices and their inverse. To provide some intuition, especially regarding the scapegoat effect, in this section we decompose the derivative into components of different orders.[9] We also consider a simple example that in many ways captures the essence of the more general case. We summarize the findings of this order decomposition analysis in the form of five intuitive Results that connect to the outcome of the numerical analysis in the next section.

For convenience we repeat expression (18) for the derivative of the exchange rate with respect to fundamentals:

$$\frac{\partial \Delta s_t}{\partial \Delta f_{nt}} = (1-\lambda)\beta_{nt} + \lambda E_t \beta_{nt} + \lambda \sum_{i=0}^{T} \Delta \mathbf{f}'_{t-i} \frac{\partial E_t \boldsymbol{\beta}_{t-i}}{\partial \Delta f_{nt}} \tag{26}$$

A Technical Appendix that is available on request computes the various order components of this expression. But before we turn to that it is useful to first consider a simple example.

Assume that $T = N = 1$. In this case, only the most recent parameter innovation ε_t is unknown. Apart from knowing the unconditional distribution of ε_t, we have one other signal: $F_t + b_t$, for which the only unknown component in this case

[9] Any variable can be written as the sum of its components of all orders. For example, for a variable x_t we have $x_t = x(0) + x_t(1) + x_t(2) + ...$ The zero-order component of a variable, $x(0)$, is its value when the standard deviation of shocks in the model approaches zero. The first-order component, $x_t(1)$, is proportional to the shocks. The second-order component, $x_t(2)$, is proportional to the product of two shocks (or the same shock squared). Notice that we only compute these order components for the purpose of intuition. The simulations reported below are based on the exact expressions.

is simply $\varepsilon_t \Delta f_t + \varepsilon_t^b$. The expectation of the parameter innovation is then

$$E_t \varepsilon_t = \frac{\sigma_\beta^2}{(\Delta f_t)^2 \sigma_\beta^2 + \sigma_b^2} \left((\Delta f_t)^2 \varepsilon_t + \Delta f_t \varepsilon_t^b \right) \tag{27}$$

It depends both on the parameter innovation itself and on the innovation ε_t^b in the unobserved fundamental. Two points stand out, which can be summarized in the following two Results:

Result 1 *The expectation of structural parameters is affected by unobserved fundamental shocks that are entirely unrelated to the structural parameters. This leads to the 'scapegoat effect'.*

Result 2 *Parameter innovations themselves have an impact on the expectations of parameters that are of third order and generally small.*

In order to understand Result 1 and the scapegoat effect, consider again the signal $\varepsilon_t \Delta f_t + \varepsilon_t^b$. Assume that Δf_t and ε_t^b are both positive, but there is no actual parameter innovation: $\varepsilon_t = 0$. Agents do not know ε_t, while they can see the signal and Δf_t. From the signal they know that $\varepsilon_t \Delta f_t + \varepsilon_t^b$ is positive. Since $\Delta f_t > 0$, agents naturally increase their expectation of ε_t. We refer to this as a scapegoat effect as the fundamental f_t becomes the scapegoat for the positive signal even if in reality the positive signal is due to the noise shock ε_t^b. Notice that there is a significant scapegoat effect only if both Δf_t and ε_t^b are large: from (27) we see that the expectation depends on the product of Δf_t and ε_t^b.

Result 2 says that parameter innovations themselves have only a small effect on the expectation of parameters. This can be seen from (27), which shows that the expectation of ε_t depends on the product of ε_t and $(\Delta f_t)^2$. The term $(\Delta f_t)^2$ is very small since Δf_t is small.[10] In more technical terms, only the third-order component of $E_t \varepsilon_t$, $(\sigma_\beta^2/\sigma_b^2)(\Delta f_t)^2 \varepsilon_t$, depends on ε_t. The impact of parameter innovations is also an order of magnitude smaller than the impact of the noise ε_t^b, which multiplies Δf_t rather than $(\Delta f_t)^2$ in the expectation of ε_t. The small effect of parameter changes on the expectations of parameters is caused by the fact that it is hard to learn about them through the signal as parameter innovations are multiplied by fundamental innovations that are on average zero.

[10] Since f_t is in logs, Δf_t is the percentage change in money supply, output or the interest rate.

To gain further insight, we need to consider the order decomposition. Setting $\theta_1 = 1$, so that $\beta_t = \varepsilon_t$, the sum of the zero, first and second-order component of the derivative of the exchange rate with respect to fundamentals is equal to

$$\frac{\partial \Delta s_t}{\partial \Delta f_{nt}} = \beta + (1-\lambda)(\beta_t - \beta) + 2\lambda(\sigma_\beta^2/\sigma_b^2)\Delta f_t \varepsilon_t^b \tag{28}$$

This needs to be compared to the case where time-varying parameters are known and the derivative is equal to β_t. Three factors contribute to a divergence between the derivative of exchange rate with respect to fundamentals and the structural parameter β_t. First, as emphasized by Engel and West (2005), the discount factor λ is close to 1. This implies that the expectation of the parameter β_t gets much more weight than the actual parameter in the first two terms of the derivative in (26): $(1-\lambda)\beta_t + \lambda E_t \beta_t$. Second, as captured in Result 1, the expectation of parameters is affected by unobserved fundamental shocks. Third, as captured in Result 2, structural parameters themselves have only a third-order effect on the expectation of parameters. We can summarize this as follows:

Result 3 *The derivative of the exchange rate with respect to fundamentals depends mainly on the expectation of structural parameters as opposed to their actual value. Together with Results 1 and 2 this implies substantial volatility in the derivative of the exchange rate with respect to fundamentals that is unrelated to structural parameter changes themselves.*

In the remainder of this section we consider the more general process for parameters described in (19). We will assume that all N parameters are drawn from the same process, so that $\beta_n = \beta$ and $\theta_{ni} = \theta_i$ for all n. The sum of the zero and first-order components of the derivative is $\beta + (1-\lambda)(\beta_{nt} - \beta)$. This is again not much affected by the impact of the actual parameter innovations as only a small weight $1-\lambda$ in the derivative is on the actual structural parameters rather than their expectation. In the remainder we focus on the second-order component of the derivative (26), which is

$$\frac{\partial \Delta s_t}{\partial \Delta f_{nt}}(2) = \frac{\sigma_\beta^2}{\sigma_b^2}\lambda \sum_{i=0}^{T-1} \zeta_{t,i}\varepsilon_{t-i}^b + \frac{\sigma_\beta^2}{\sigma_b^2}\lambda \vartheta_t \varepsilon_t^b \tag{29}$$

where:

$$\zeta_{t,i} = \sum_{k=i}^{T-1}\sum_{j=1}^{T-k} \theta_j \theta_{j+k} \delta_{ik} \Delta f_{n,t-k}$$

$$\vartheta_t = \sum_{i=0}^{T} \left(\sum_{j=1}^{T-i} \theta_j \theta_{j+i} \right) \Delta f_{n,t-i}$$

and where δ_{ik} is 1 for $k = i$ and $1 - \rho_b$ for $k > i$. The two terms on the right hand side correspond to the second-order components of respectively the second and third terms of the derivative (26).

These terms again involve the product of innovations in the unobserved fundamentals and observed fundamentals, reflecting scapegoat effects. The first conclusion that can be drawn from this expression is summarized as follows:

Result 4 *Scapegoat effects have a bigger impact on the derivative of the exchange rate with respect to fundamentals when parameter innovations have long-lasting effects.*

This can be seen by noting that $\zeta_{t,i}$ and ϑ_t depend on the products $\theta_j \theta_{j+k}$ of coefficients of the process of structural parameters. When parameter innovations have a long-lasting effect on the level β_{nt} of structural parameters, coefficients θ_j for $j > 1$ are positive. Clearly, the more persistent the effect, the larger $\zeta_{t,i}$ and ϑ_t and therefore the bigger the scapegoat effects. This reflects the fact that when parameter innovations have long-lasting effects, there is significant uncertainty about the level of the structural parameters that enter the change in the observed signal: $\sum_{n=1}^{N} \beta_{nt} \Delta f_{nt} + \Delta b_t$. This leaves plenty of room for the scapegoat mechanism to operate. This is especially the case when θ_j rises with j, which implies a gradual change in parameters in response to an innovation, so that parameters can change much more in the long run than the short run.

A second conclusion that can be drawn from (29) is:

Result 5 *The impact of scapegoat effects on the derivative of the exchange rate with respect to fundamentals has both a persistent and transitory component.*

The two components are readily seen on the right hand side of (29). The first term on the right hand side depends on innovations in the unobserved fundamental over the past T periods and therefore has significant persistence. These past T noise innovations all affect the expectation of β_{nt} as agents learn about β_{nt} from all T past signals $F_{t-i} + b_{t-i}$ $(i = 0, .., T-1)$.

The second term on the right hand side of (29) is entirely transitory as it is only the current noise innovation ε_t^b that enters. This component leads to very high frequency fluctuations in the derivative. It is associated with the last term in the derivative (26). The impact of a fundamental innovation Δf_{nt} on the exchange rate depends not only on the expectation of β_{nt} that multiplies the fundamental innovation. It also depends on how the change in the fundamental leads to changes in the expectation of current and past parameters, as seen in the last term of (26). A change in the current fundamental affects current and past parameter expectations only to the extent that it becomes a scapegoat in the face of a current noise shock ϵ_t^b.[11]

Finally, while we do not report the third-order component, it is worth pointing out that it captures another type of rational confusion. Instead of confusing unobserved parameter shocks with unobserved fundamental shocks, agents may also confuse the unobserved innovations in one parameter with unobserved innovations in another parameter. This is reflected in the third-order component, which is a complicated expression that multiplies current and past parameter innovations (including those associated with other parameters) with the product of fundamental innovations (current and past).

In order to illustrate these findings and show the magnitude of the scapegoat effect, we now turn to a calibration of the model that is grounded in monthly data of exchange rates and interest rates.

3 Numerical Analysis

3.1 Calibration

We calibrate the model to data for exchange rates, interest rates and observed fundamentals. A description of the data can be found in Appendix C. In the previous section, we considered a special case with no risk-premium shocks and where both b_t and $\Delta \mathbf{f}_{nt}$ are iid. For calibration purposes we now turn to a somewhat more general form of the model.

[11] To see this point, one can alternatively write the signals $F_{t-i} + b_{t-i}$ for $i = 0, .., T-1$ as $(1-\rho_b L)(F_{t-i} + b_{t-i}) = F_{t-i} - \rho_b F_{t-i-1} + \varepsilon_t^b$. The current fundamental innovation Δf_{nt} only enters in the most recent signal ($i = 0$), in which only the most recent noise innovation enters.

First, we assume that b_t and Δf_{nt} follow AR(1) processes:

$$\begin{aligned}\Delta f_{nt} &= \rho_f \Delta f_{n,t-1} + \varepsilon_t^f \\ b_t &= \rho_b b_{t-1} + \varepsilon_t^b\end{aligned}$$

Second, in order to match observed exchange rate volatility we allow for a time-varying risk premium. Let v_t be the present discounted value of the risk premium:

$$v_t = \sum_{k=0}^{\infty} \lambda^k E_t \phi_{t+k}$$

To match the observed volatility and autocorrelation of Δs_t, we assume that v_t follows the process

$$v_{t+1} - v_t = \psi_1(v_t - v_{t-1}) - \psi_2 v_t + \varepsilon_{t+1}^v \tag{30}$$

where $\varepsilon_{t+1}^v \sim N(0, \sigma_v^2)$.[12]

The process for the structural parameters is determined by the values of the parameters θ_{in} in equation (19). We assume that the parameters associated with all observed fundamentals are the same, so that $\beta_n = \beta$ and $\theta_{in} = \theta_i$ for all n. As discussed in the introduction, we consider structural parameters that exhibit two features that would appear plausible in terms of gradual changes in the structure of the economy, for example associated with technological and financial innovation, or cultural and institutional changes. First, structural parameter changes are small over short horizons of a month or a year. Second, changes in structural parameters gradually build over time and can be significant over long horizons of many years or decades. Parameters therefore can change much more in the long run than the short run, generating substantial uncertainty about the *level* of parameters, even though monthly or annual *changes* are small.

In order to get these features, we set $\theta_1 = 1$ and then choose the other parameters θ_i $(i = 2, .., T)$ such that we maximize the ratio of the unconditional standard deviation of β_t relative to the standard deviation of monthly changes in β_{nt}. In other words we maximize

$$\frac{\sigma_{\beta_{nt}}}{\sigma_{\Delta\beta_{nt}}}.$$

[12]These risk-premia shocks are assumed to be uncorrelated with the observed fundamentals Δf_{nt}, which exogenously generates a disconnect between Δs_t and the observed fundamentals. For a more endogenous explanation of the disconnect between exchange rates and observed fundamentals, related to private information, see Bacchetta and van Wincoop (2006).

The resulting process implies that an innovation impacts the parameter β_{nt} slowly over time in the form of a hump shape. It builds up to a maximum impact after $T/2$ periods and then gradually declines. We will examine other processes in section 3.3.

Table 1 reports the parameters adopted for the benchmark parameterization. The first four parameters relate to the processes for β_{nt}. We set $T = 1000$. Since we assume that one period is one month, this implies that the current level of structural parameters is determined by parameters innovations over the last 1000 months or 83 years. We set $N = 5$, so that the total number of structural parameters (and fundamentals) is 5. Therefore the total number of unknown structural parameter innovations that agents need to learn about is 5000. We normalize by setting the mean value of the parameters at $\beta = 1$. We set $\sigma_\beta = 0.000165$. As reported in the last row of Table 2, this implies a monthly standard deviation of the change in β_{nt} of 0.3% of the mean value of parameters, which is small. But there is considerable uncertainty about the level of parameters as their unconditional standard deviation is 1.2, or 120% of their steady state level. This is because parameter changes build gradually over time.

The next five parameters are associated with the process for b_t and v_t. These are set to closely match four moments related to exchange rates and interest rates: the standard deviation of Δs_t, the standard deviation of $i_t - i_t^*$, the first-order autocorrelation of Δs_t and the first-order autocorrelation of $i_t - i_t^*$. In doing so, we use monthly data from 1975(9) to 2008(9) for exchange rates and interest differentials of 5 countries relative to the United States. The countries are Canada, Germany, Japan, Switzerland, and United Kingdom. These moments are reported in the first column of Table 2 (first 4 rows). We match these moments in the model for the case of constant parameters ($\sigma_\beta = 0$). The moments for constant parameters are reported in the second column of Table 2. But the moments are virtually identical under the benchmark assumption about time-varying parameters, as shown in column 3.[13]

As a by-product the model also generates a significant negative correlation

[13] Both under constant and time-varying parameters the moments are computed based on a simulation over 1300 months (108 years). So they can reasonably be considered population moments. Prior to the 1300 months over which we compute the moments we first simulate the model for $T = 1000$ months (83 years) in order to obtain a history.

between the change in the exchange rate and lagged interest differential. The Fama regression coefficient, reported in the fifth row of Table 2, is even slightly more negative than in the data. We emphasize that this is not intended as an explanation for the forward discount puzzle as it is due to entirely exogenous risk-premium shocks (see Bacchetta and van Wincoop, 2010, for a more plausible explanation for the forward discount puzzle). It does imply though that the model is well grounded in the data as it conforms to the basic statistical properties of exchange rates and interest rates.

The next two parameters relate to the process of the observed fundamentals. We set the number of fundamentals at $N = 5$. We do not take a strong stand on exactly which observed fundamentals affect exchange rates. This is not necessary as the finding that observed fundamentals have limited explanatory power for exchange rates is well known and applies broadly across fundamentals. But for concreteness in terms of the calibration, we use some representative results from Bacchetta, van Wincoop and Beutler (2010). For the same 5 currencies and sample period used to calibrate exchange rate and interest rate moments, they regress Δs_t on changes in 5 fundamentals (Δf_{nt} in our model): changes in money supply, industrial production, unemployment rate, and oil price and the level of lagged interest rates. They obtain an average R^2 of 0.023.

We set the standard deviation σ_f of fundamental innovations in the model equal to 0.125% in order to match the average R^2 in the data when computed over a sample of 397 months (33 years) that corresponds to the sample in the data. As shown in Table 2, we match this for both constant parameters and the benchmark assumption of time-varying parameters. We set the persistence ρ_f of the process for fundamentals equal to 0 under the benchmark parameterization. This is also closely consistent with the specific fundamentals listed above.[14] We will also consider positive persistence in sensitivity analysis.

Finally, we set $\alpha = 100/3$, implying a discount rate λ in the present value equation for the exchange rate of 0.97. This is consistent with evidence by Engel and West (2005) that the discount rate is close to 1.

[14] The change in money supply, industrial production, unemployment rate, and the oil price all have low persistence, with first-order autocorrelations averaging to 0.02. Only the lagged interest rate differential has a high persistence of 0.94.

3.2 Results

We simulate the model over 2300 months. All moments reported drop the first 1000 months in order to generate a prior history of shocks. Unless otherwise indicated, the results reported are based on the subsequent 1300 months.

Derivative of Exchange Rate with Respect to Fundamentals

Figures 1 and 2 show $\partial\Delta s_t/\partial\Delta f_{nt}$ for each of the five fundamentals. From now on we simply refer to this as the derivative of the exchange rate with respect to fundamentals. Figure 1 does so for a 10-year period (observations 1540-1659 in the simulation), while Figure 2 does so for a 100-year period (observations 1001-2200 in the simulation).[15] Both Figures also show β_{nt}, which would be the derivative of the exchange rate with respect to fundamentals if parameters were known.

It is evident from Figure 1 that the derivative of the exchange rate with respect to fundamentals is far more volatile than the underlying structural parameters. As reported in Table 2, the average standard deviation of monthly changes in the derivative is 25.9% of the mean value of the derivative. By contrast, the standard deviation of monthly changes in the underlying structural parameters is only 0.3%. We will call the ratio between these two standard deviations the "scapegoat ratio" as scapegoat effects are responsible for the increased instability in the relationship between the exchange rate and fundamentals. In the benchmark case, this ratio is equal to 85.1.

This disconnect between structural parameters and the derivative of the exchange rate with respect to fundamentals illustrates Result 3 in section 2.5. We have seen that several factors are behind this. First, the derivative is mostly driven by expectations of structural parameters rather than structural parameters themselves. Second, structural parameters have very little impact on the expectation of structural parameters (Result 2). Third, scapegoat effects lead to an impact of noise innovations ε_t^b on the expectation of parameters (Result 1). In addition we saw that scapegoat effects are bigger the more persistent the process for parameters (Result 4). We found that this is especially the case when coefficients θ_i increase with i as is the case for our assumed process (for $i < 0.5T$).

While Figure 1 would suggest that the derivative of exchange rates with respect

[15]Figure 1 corresponds to the middle observations of Figure 2.

to fundamentals is entirely disconnected from the true underlying structural parameters, Figure 2 shows that this is not the case when we take a longer 100-year view. There are large changes in parameters over long cycles of several decades, while the derivative of the exchange rate with respect to the fundamentals broadly catches up with these long term swings. This implies that when there are persistent changes in parameters, agents do eventually learn about them when they are consistently reflected in the data $F_t + b_t$ for several decades.

But, as illustrated in both Figures 1 and 2, short to medium-term fluctuations around such long-term cycles can be large and even dominate the trend itself. It is precisely the possibility that parameters can change a lot in the long run that creates significant uncertainty about their level and gives rise to scapegoat effects that lead to large changes in the derivatives over the short to medium run.

Expectation of Parameters

It is useful to recall equation (18) of the derivative of the exchange rate with respect to fundamentals, which is displayed here again for convenience:

$$\frac{\partial \Delta s_t}{\partial \Delta f_{nt}} = (1-\lambda)\beta_{nt} + \lambda E_t \beta_{nt} + \lambda \sum_{i=0}^{T} \Delta \mathbf{f}'_{t-i} \frac{\partial E_t \boldsymbol{\beta}_{t-i}}{\partial \Delta f_{nt}} \tag{31}$$

Since λ is close to 1, the derivative of the exchange rate with respect to fundamentals is primarily driven by the last two terms. We have seen that the impact of scapegoat effects on the derivative of the exchange rate with respect to fundamentals has both a persistent and transitory component (Result 5). These are associated with respectively the second and third term in (31).

The persistent scapegoat effects enter through the expectation $E_t \beta_{nt}$ of structural parameter n. The persistence results from the fact that all unobserved fundamental innovations over the past $T = 1000$ months generate scapegoat effects. Focusing on variable 1, Figure 3 compares the evolution of β_{1t} with $E_t \beta_{1t}$ over the samples of 10 and 100 years used in Figures 1 and 2. The top panels illustrate that $E_t \beta_{1t}$ is more volatile than the underlying parameter β_{1t} and that fluctuations have significant persistence at various frequencies.

But a comparison with Figures 1 and 2 also shows that the overall derivative $\partial \Delta s_t / \partial \Delta f_{nt}$ has even much larger fluctuations at high frequencies. This is the result of the transitory scapegoat effects that are associated with the last term in (31). As explained in section 2.5, the last term in (31) has a second-order component that is proportional to ε_t^b (zero and first-order components are zero). Therefore, this term has no persistence and gives rise to very high frequency fluctuations. It is illustrated in the bottom panels of Figure 3, which show β_{1t}, $E_t\beta_{1t}$ as well as $\partial\Delta s_t/\partial\Delta f_{1t}$.

To summarize, very gradual changes in structural parameters can lead to a highly unstable relationship between exchange rates and observed fundamentals. This is the result of both persistent and transitory scapegoat effects. We have seen in section 2.5 that these scapegoat effects (both persistent and transitory components) are largest when the process of structural parameters is highly persistent, leading to large long-run uncertainty about the level of structural parameters. To illustrate this point, we now turn to a discussion of sensitivity analysis with respect to the nature of the process for structural parameters.

3.3 Sensitivity to Process of Structural Parameters

Perhaps most relevant when conducting sensitivity analysis with respect to our findings is to consider how the results depend on the process of parameters. This is the only aspect of the model that we could not calibrate to the data. There are good reasons for this. It is impossible to know what exactly the process of structural parameters is. As emphasized in the Introduction, econometric analysis cannot distinguish between lots of different processes. Nonetheless it is important to consider alternative processes. We will do so in order to make a general point, which is key to our results. There is significant reduced-form parameter instability relative to structural parameter instability when structural parameters can potentially change much more in the long-run than the short-run. This implies significant uncertainty about the *level* of parameters relative to monthly *changes* in parameters. Or in more technical terms, for any process where $\sigma_{\beta_{nt}}/\sigma_{\Delta\beta_{nt}}$ is high, there will be a high scapegoat ratio.

In order to illustrate this point, we consider four alternative processes. These are all special cases of the process

$$\beta_{n,t+1} - \beta_{nt} = \delta_1(\beta_{nt} - \beta_{n,t-1}) - \delta_2(\beta_{nt} - \beta) + \epsilon_{nt} \tag{32}$$

with different values for δ_1 and δ_2. In terms of an MA process, (32) can be written as

$$\beta_{nt} = \beta_n + \sum_{i=1}^{\infty} \theta_i \varepsilon_{n,t-i+1}. \tag{33}$$

with $\theta_1 = 1$, $\theta_2 = 1 + \delta_1 - \delta_2$ and

$$\theta_{i+1} - \theta_i = \delta_1(\theta_i - \theta_{i-1}) - \delta_2 \theta_i \tag{34}$$

for $i \geq 2$.

The benchmark process is a special case of this process as well, with $\delta_1 = 1$ and $\delta_2 = 0.00000985$. The benchmark process truncates the MA process to an $MA(T)$ by setting $\theta_i = 0$ for $i > T$. The alternative Process 1 is different in that we do not truncate. As is the case for the benchmark process, the parameters θ_i are chosen to maximize the standard deviation of $\tilde{\beta}_{nt}$ relative to $\Delta\beta_{nt}$, where $\tilde{\beta}_{nt} = \sum_{i=1}^{T} \theta_i \epsilon_{n,t-i+1}$ captures the component of β_{nt} that is unknown at time t (most recent T innovations). But the coefficients θ_i are not restricted to be zero for $i > T$. In the alternative Processes 2 through 4 we truncate $\theta_i = 0$ for $i > T$ as for the benchmark process.

For all 4 alternative processes, Figure 4 shows the impulse response functions of the structural parameters after a one standard deviation parameter innovation. For comparison each chart also shows the impulse response function for the benchmark parameterization. The top of each chart shows the parameters δ_1 and δ_2 for each of the alternative processes. In each case the standard deviation of $\tilde{\beta}_{nt}$ is kept the same as under the benchmark parameterization.

In Process 1 the structural parameter rises gradually in response to an innovation, until it peaks at a new level where it will remain. From a theoretical standpoint this process has the unattractive feature that the structural parameters are non-stationary and therefore unbounded. But in practice we only simulate the model over a finite 2300 months (192 years) and the uncertainty of β_{nt} due to parameter innovations over the past $T = 1000$ months (83 years) is kept identical across all parameterizations. This process captures the idea that certain structural changes, such as technological and financial innovation, are indeed permanent. It also connects well to a lot of the econometrics literature that tests for structural breaks in parameters. This amounts to testing for permanent changes in parameters. In a way Process 1 captures even better than the benchmark parameterization what we have in mind with gradual and long-lasting changes in parameters. We only chose to truncate the benchmark process after T innovations in order to assure stationarity for theoretical reasons.[16]

Process 2 is a truncated AR(1) process with AR coefficient of 0.99. Process 3 is a truncated random walk process. In Process 4 the structural parameter gradually rises over time in response to an innovation and peaks a bit earlier than under the benchmark. In these three cases the response is truncated to zero after T periods.

Table 3 shows the scapegoat ratio for each of the processes, as well as $\sigma_{\tilde{\beta}_{nt}}/\sigma_{\Delta\beta_{nt}}$. Since $\sigma_{\tilde{\beta}_{nt}}$ is kept the same across all processes, a higher ratio means a smaller standard deviation $\sigma_{\Delta\beta_{nt}}$ of monthly changes in structural parameters. Table 3 clearly shows that the higher the long-run uncertainty about the level of the structural parameters relative to monthly changes in structural parameters, the bigger the scapegoat ratio. For Process 1, where $\sigma_{\tilde{\beta}_{nt}}/\sigma_{\Delta\beta_{nt}}$ is about double that under the benchmark process, the scapegoat ratio is now an amazing 267.7. This is more than three times that under the benchmark. Figure 5 illustrates the scapegoat effect in this case. A major difference is that structural parameters are more stable, even at very low frequency. Both the expectation of β_{nt} and the reduced form parameters $\partial\Delta s_t/\partial\Delta f_{nt}$ remain highly unstable and are now even more disconnected from the smooth structural parameters.

By construction $\sigma_{\tilde{\beta}_{nt}}/\sigma_{\Delta\beta_{nt}}$ is less for the other processes than under the benchmark parameterization and therefore the scapegoat ratio is lower as well.[17] Beyond that, two points are worth making. First, even when $\sigma_{\tilde{\beta}_{nt}}/\sigma_{\Delta\beta_{nt}}$ is much lower than under the benchmark parameterization, there can still be a substantial scapegoat ratio. For example, for Process 4, where $\sigma_{\tilde{\beta}_{nt}}/\sigma_{\Delta\beta_{nt}}$ is less than one sixth that under the benchmark, the reduced form monthly parameter instability is still more than 10 times the structural parameter instability. Second, even when the scapegoat ratio is close to 1, as it is for Process 2, this does not mean that that reduced form parameters are similar to structural parameters. Indeed, even for Process 2, the correlation between monthly changes in structural parameters and reduced form parameters is only 0.33 (it is 0.02 in the benchmark case).

[16]Of course one could truncate process 1 after $\bar{T}$ innovations, with $\bar{T}$ much larger than T. The results will then be very similar to what we report for Process 1 even though technically the process will then be stationary.

[17]Recall that the benchmark process is chosen to maximize $\sigma_{\tilde{\beta}_{nt}}/\sigma_{\Delta\beta_{nt}}$ for processes that are truncated after T periods.

We should finally point out that we have restricted ourselves to processes with normally distributed innovations. It is possible that some parameter changes are big and infrequent. One can imagine a process where there is a big change in parameters with some very small probability $p > 0$. In that case parameters are perfectly constant almost all of the time. But even when structural parameters do not change at all, reduced-form parameters will be very volatile as the infrequent large parameter changes contribute to significant uncertainty about the level of parameters.[18]

3.4 Other Sensitivity Analysis

We now return to the benchmark process and examine the extent to which the results are sensitive to changes in various parameters. We consider four types of parameters: the standard deviation of structural parameter innovations; the variability and persistence of fundamentals; the horizon T after which parameters are known; and the volatility of the unobserved fundamentals.

3.4.1 Sensitivity Moments to Parameter Instability

When considering alternative processes for the parameters in the previous subsection, we held constant the overall parameter instability as measured by the standard deviation of $\tilde{\beta}_t$. We now consider the impact of a change in the standard deviation σ_β of parameter innovations for the benchmark process.

Table 2 reports moments for three values of σ_β. In addition to the constant parameter and the benchmark time-varying parameter cases, the fourth column shows the case where the standard deviation of parameter innovations is twice that under the benchmark ($\sigma_\beta = 0.00033$). In the latter case the standard deviation of monthly changes in the derivative of the exchange rate with respect to fundamentals is 45%, while the same moment is only 0.6% for the structural parameters. This implies a scapegoat ratio of 73.8. While this remains very high, it is slightly lower than under the benchmark parameterization. The reason for this

[18]For example, when the structural parameter follows a Markov process with two states $1 + a$ and $1 - a$ and the probability of changing from one state to another is a small p, then $\sigma_{\tilde{\beta}_{nt}} / \sigma_{\Delta\beta_{nt}}$ is equal to $1/(4p)$. This can get very large for small p. Our results from Table 3 suggest that this will again generate a very large scapegoat ratio.

is that when structural parameters become sufficiently volatile, it becomes easier to learn about them through data on $F_t + b_t$. This reduces the rational confusion and associated scapegoat effects, although numerically the difference is small.

Even though we have seen that gradual changes in structural parameters lead to a highly unstable relationship between exchange rates and fundamentals, some basic moments involving exchange rates and interest rates are remarkably insensitive to the degree of parameter instability. Exchange rate volatility rises only slightly. The standard deviation of exchange rate changes rises from 2.90% to 3.04%, from the case of constant parameters to the extreme case where parameter volatility is twice that under the benchmark. The standard deviation of the interest rate differential, as well as the autocorrelation of monthly exchange rate change and the interest differential, are all virtually unaffected by parameter volatility. The same is the case for the monthly Fama regression coefficient of Δs_{t+1} on $i_t - i_t^*$. The reason for these results is that most exchange rate volatility is unrelated to changes in fundamentals. For the benchmark parameterization the R^2 is 0.023, as in the data.

3.4.2 Sensitivity to Process Fundamentals

We first examine the impact of the fundamentals process on the link between exchange rates and these fundamentals. We consider a higher standard deviation of the innovations of the fundamentals and positive persistence of changes in the fundamentals. We find that the volatility of $\frac{\partial \Delta s_t}{\partial \Delta f_{nt}}$ decreases with σ_f. When we set the standard deviation of innovations four times as large as under the benchmark ($\sigma_f = 0.005$), the scapegoat ratio declines from 85.1 to 53.7.

The explanation for these results is that when σ_f is larger, the signal $F_t + b_t$ becomes more informative about structural parameters as they are multiplied by fundamentals that fluctuate more. Consequently, there is less confusion. Scapegoat effects are smaller and therefore the derivative $\frac{\partial \Delta s_t}{\partial \Delta f_{nt}}$ is somewhat less volatile. We should not overstate this though as monthly changes in this derivative remain 54 times more volatile than monthly change in the structural parameter β_{nt}. Moreover, a standard deviation of $\sigma_f = 0.005$ is implausibly high as it leads to an R^2 of 0.15. This is well above representative results for a sample of at least 3 decades.

We also consider raising the persistence ρ_f of Δf_{nt} from 0 to 0.2. As shown in

Appendix A, the derivative of exchange rates with respect to fundamentals is then also affected by expectations of future levels of the structural parameters. But the overall impact on the unstable relationship between exchange rates and observed fundamentals is small. The scapegoat ratio increases slightly from 85.1 to 96.7.

3.4.3 Sensitivity to the horizon T

A smaller T implies that there are fewer parameter innovations to learn about. This reduces rational confusion and scapegoat effects. This is illustrated by comparing the case of $T = 1000$ to the case of $T = 300$. For $T = 300$ we find a scapegoat ratio of 9.3. While this still reflects significant scapegoat effects, it is much smaller than scapegoat ratio of 85.1 found in the benchmark of $T = 1000$. Conversely, the scapegoat ratio would rise as we make T even bigger than 1000. However, this would take an excessive amount of computer time. With $T = 1000$, 5 fundamentals and a simulation over 2300 months we already need to solve 2300 signal extraction problems that each involve 5000 unknown parameter innovations.[19]

3.4.4 Sensitivity to σ_b (volatility of unobserved fundamentals)

Shocks to unobserved fundamentals play a crucial role in generating scapegoat effects. However, there is a non-linear relationship between the volatility of unobserved fundamentals and the magnitude of the scapegoat effect as measured by the scapegoat ratio. This is illustrated in Figure 6, which plots the scapegoat ratio as a function of σ_b. As the standard deviation σ_b of the unobserved fundamentals rises, the scapegoat ratio first increases and then eventually starts to fall. This non-linear relationship can be explained by the inference process. At low values, an increase in σ_b generates more rational confusion as $F + b$ becomes more volatile. But when b becomes too volatile, $F + b$ is a less valuable source of information for investors. They will then attach less weight to it when forming expectations about parameters, which reduces scapegoat effects.

[19]With our current technology, this takes about 40 hours of computer time.

4 Conclusion

Anecdotal, survey and econometric evidence all suggest that the relationship between the exchange rate and macro fundamentals is highly unstable. One possible way to explain this is by assuming large and frequent known changes in the structural parameters. But this does not seem very plausible as structural parameters are hard to observe and estimate and many changes in the structure of the economy are gradual as a result of technological and financial innovation and institutional changes. We have therefore developed a model where structural parameters are not observed and changes in these structural parameters are very gradual. We have shown that the relationship between a forward looking variable like the exchange rate and macro fundamentals is determined not by the structural parameters themselves, but rather by the expectations of these structural parameters.

We have also shown that expectations of these parameters can change significantly and frequently, even when changes in structural parameters are small and gradual. This is a result of scapegoat effects, where changes in the exchange rate, or other macro data, are attributed to certain observed fundamentals even when they are driven by unobserved fundamental shocks. Such scapegoat effects occur in an environment where agents are rational Bayesian learners that incorporate all available information to revise their view on the parameters. When structural parameters can potentially change significantly over long horizons of several decades, there is substantial room for scapegoat effects as agents are trying to learn about the level of the parameters.

While our focus has been on the exchange rate, an analogous explanation could also account for the extensive evidence of parameter instability seen in other forward looking macroeconomic and financial data. Two key ingredients, which are not limited in any way to exchange rate models, drive our unstable reduced form results. First, there must be unobserved fundamental shocks. This applies surely to other asset prices as well and more generally to other macroeconomic data as factors driving business cycles and long term growth rates are not perfectly understood. Second, structural parameters must have the potential to change significantly over long horizons. This would be hard to dispute as well, especially in the context of major technological, financial and institutional changes over the past two centuries.

Appendix

A Solving the General Model

In this Appendix we describe the model's solution in the more general case, where the processes for Δf_{nt}, b_t, and v_t are as specified in Section 3. A Technical Appendix provides further details towards the implementation of the simulations with Gauss. We start from the present value equation (1) of the exchange rate. We need to express it in way we can easily substitute the expectation terms. This equation can be rewritten as:

$$s_t = (1-\lambda)F_t + (1-\lambda)b_t - \lambda v_t + (1-\lambda)\sum_{k=1}^{\infty}\lambda^k E_t\left(F_{t+k} + b_{t+k}\right) \tag{35}$$

First, consider the term involving the present discounted value of F. Use that

$$F_{t+k} = F_t + \sum_{n=1}^{N}\sum_{i=1}^{k}\beta_{n,t+i}\left(f_{n,t+i} - f_{n,t+i-1}\right) \tag{36}$$

Therefore

$$\sum_{k=1}^{\infty}\lambda^k F_{t+k} = \frac{\lambda}{1-\lambda}F_t + \frac{1}{1-\lambda}\sum_{n=1}^{N}\sum_{i=1}^{\infty}\lambda^i\beta_{n,t+i}\left(f_{n,t+i} - f_{n,t+i-1}\right) \tag{37}$$

The present value of b can be written as $\tilde{b}E_t b_t$, where

$$\tilde{b} = (1-\lambda)\frac{\rho_b\lambda}{1-\rho_b\lambda} \tag{38}$$

Using this, (35) becomes

$$\begin{aligned} s_t &= (1-\lambda)F_t + \lambda E_t F_t + (1-\lambda)b_t - \lambda\nu_t \\ &\sum_{n=1}^{N}\sum_{i=1}^{\infty}(\rho\lambda)^i E_t\beta_{n,t+i}\left(f_{n,t} - f_{n,t-1}\right) + \tilde{b}E_t b_t \end{aligned} \tag{39}$$

Therefore

$$\begin{aligned} s_t - s_{t-1} &= (1-\lambda)\sum_{n=1}^{N}\beta_{nt}\left(f_{nt} - f_{n,t-1}\right) + \lambda\left[E_tF_t - E_{t-1}F_{t-1}\right] + \\ &\sum_{n=1}^{N}E_t\tilde{\beta}_{nt}\left(f_{n,t} - f_{n,t-1}\right) - \sum_{n=1}^{N}E_{t-1}\tilde{\beta}_{n,t-1}\left(f_{n,t-1} - f_{n,t-2}\right) + \\ &(1-\lambda)(b_t - b_{t-1}) + \tilde{b}\left(E_tb_t - E_{t-1}b_{t-1}\right) - \lambda\left(v_t - v_{t-1}\right) \end{aligned} \tag{40}$$

where

$$\tilde{\beta}_{nt} = \sum_{i=1}^{\infty} (\rho\lambda)^i \beta_{n,t+i} \tag{41}$$

Finally, we can write

$$\begin{aligned} &E_t F_t - E_{t-1}F_{t-1} = E_t(F_t - F_{t-1}) + [E_t F_{t-1} - E_{t-1}F_{t-1}] = \\ &\sum_{n=1}^{N} E_t\beta_{nt} (f_{nt} - f_{n,t-1}) + \sum_{n=1}^{N}\sum_{i=1}^{T} (f_{n,t-i} - f_{n,t-i-1}) [E_t\beta_{n,t-i} - E_{t-1}\beta_{n,t-i}] \end{aligned} \tag{42}$$

Using (42) and collecting terms multiplying $f_{nt} - f_{n,t-1}$, (40) becomes

$$\begin{aligned} s_t - s_{t-1} = &\sum_{n=1}^{N} \Big((1-\lambda)\beta_{nt} + \lambda E_t\beta_{nt} + E_t\tilde{\beta}_{nt}\Big) (f_{nt} - f_{n,t-1}) + \\ &- \sum_{n=1}^{N} E_{t-1}\tilde{\beta}_{n,t-1} (f_{n,t-1} - f_{n,t-2}) + \\ &\lambda \sum_{n=1}^{N}\sum_{i=1}^{T} (f_{n,t-i} - f_{n,t-i-1}) [E_t\beta_{n,t-i} - E_{t-1}\beta_{n,t-i}] + \\ &(1-\lambda)(b_t - b_{t-1}) + \tilde{b} (E_t b_t - E_{t-1}b_{t-1}) - \lambda (v_t - v_{t-1}) \end{aligned} \tag{43}$$

Given the processes of $\boldsymbol{\beta}_t$ and b_t, the terms including expectations can be written as:

$$\begin{aligned} &E_t\beta_{nt} - \beta = \hat{\omega} E_t \boldsymbol{\xi}_{nt} \\ &E_t\tilde{\beta}_{nt} - \frac{\rho\lambda}{1-\rho\lambda}\beta = \hat{\theta} E_t \boldsymbol{\xi}_{nt} \\ &E_t b_t = \hat{b} E_t \mathbf{b}_t + \rho_b^T b_{t-T} \\ &\sum_{i=1}^{T} (f_{n,t-i} - f_{n,t-i-1}) [E_t\beta_{n,t-i} - E_{t-1}\beta_{n,t-i}] = \\ &\qquad \sum_{i=1}^{T} (f_{n,t-i} - f_{n,t-i-1}) \theta_{T-i+1}\epsilon_{n,t-T} + \hat{h}_{nt} E_t \boldsymbol{\xi}_{nt} - \hat{f}_{n,t-1} E_{t-1} \boldsymbol{\xi}_{n,t-1} \end{aligned}$$

where $\bar{\theta}$, $\hat{\theta}$, $\hat{b}$, $\hat{h}$ and $\bar{h}$ are 1 by T vectors with

$$\hat{\omega}(j) = \theta_j \tag{44}$$

$$\hat{\theta}(j) = \sum_{i=1}^{T-j} \theta_{j+i} (\rho\lambda)^i \tag{45}$$

$$\hat{b}(j) = \rho_b^{j-1} \tag{46}$$

$$\hat{h}_{nt}(j) = \sum_{i=1}^{j-1}(f_{n,t-i} - f_{n,t-i-1})\theta_{j-i} \tag{47}$$

$$\hat{f}_{n,t-1}(j) = \sum_{i=1}^{j}(f_{n,t-i} - f_{n,t-i-1})\theta_{j-i+1} \tag{48}$$

and $\hat{h}_{nt}(1) = 0$.

Substituting these results into (43) gives

$$\begin{aligned} s_t - s_{t-1} = & \sum_{n=1}^{N}\left(\frac{\beta}{1-\rho\lambda} + (1-\lambda)(\beta_{nt} - \beta) + \left[\lambda\hat{\omega} + \hat{\theta}\right] E_t\boldsymbol{\xi}_{nt}\right)(f_{nt} - f_{n,t-1}) + \\ & - \sum_{n=1}^{N}\left(\frac{\rho\lambda\beta}{1-\rho\lambda} + \hat{\theta}E_{t-1}\boldsymbol{\xi}_{n,t-1}\right)(f_{n,t-1} - f_{n,t-2}) + \\ & \lambda\sum_{n=1}^{N}\sum_{i=1}^{T}(f_{n,t-i} - f_{n,t-i-1})\,\theta_{T-i+1}\epsilon_{n,t-T} + \lambda\sum_{n=1}^{N}\left(\hat{h}_{nt}E_t\boldsymbol{\xi}_{nt} - \hat{f}_{n,t-1}E_{t-1}\boldsymbol{\xi}_{n,t-1}\right) + \\ & (1-\lambda)(b_t - b_{t-1}) - \lambda\,(v_t - v_{t-1}) + \tilde{b}\left(\hat{b}(E_t\mathbf{b}_t - E_{t-1}\mathbf{b}_{t-1}) + \rho_b^T(b_{t-T} - b_{t-T-1})\right) \end{aligned} \tag{49}$$

The expectation terms can be derived from the signal extraction problem, where $E_t\boldsymbol{\omega}_t = \mathbf{C}_t\boldsymbol{\omega}_t$. This gives:

$$\begin{aligned} s_t - s_{t-1} = & \sum_{n=1}^{N}\left(\frac{\beta}{1-\rho\lambda} + (1-\lambda)(\beta_{nt} - \beta) + \left[\lambda\bar{\omega}^n + \bar{\theta}^n\right]\mathbf{C}_t\boldsymbol{\omega}_t\right)(f_{nt} - f_{n,t-1}) + \\ & - \sum_{n=1}^{N}\left(\frac{\rho\lambda\beta}{1-\rho\lambda} + \bar{\theta}^n\mathbf{C}_{t-1}\boldsymbol{\omega}_{t-1}\right)(f_{n,t-1} - f_{n,t-2}) + \\ & \lambda\sum_{n=1}^{N}\sum_{i=1}^{T}(f_{n,t-i} - f_{n,t-i-1})\,\theta_{T-i+1}\epsilon_{n,t-T} + \lambda\sum_{n=1}^{N}\left(\bar{h}_t^n\mathbf{C}_t\boldsymbol{\omega}_t - \bar{f}_{t-1}^n\mathbf{C}_{t-1}\boldsymbol{\omega}_{t-1}\right) + \\ & (1-\lambda)(b_t - b_{t-1}) - \lambda\,(v_t - v_{t-1}) + \tilde{b}\left(\bar{b}(\mathbf{C}_t\boldsymbol{\omega}_t - \mathbf{C}_{t-1}\boldsymbol{\omega}_{t-1}) + \rho_b^T(b_{t-T} - b_{t-T-1})\right) \end{aligned} \tag{50}$$

Here $\bar{\theta}^n$ is a 1 by $(N+1)T$ vector with $\hat{\theta}$ in elements $T(n-1)+1$ through Tn and zeros otherwise. The vectors $\bar{\omega}^n$, $\bar{h}_t^n$ and $\bar{f}_{t-1}^n$ are defined analogously. $\bar{b}$ is a 1 by $(N+1)T$ vector with $\hat{b}$ in elements $NT+1$ through $NT+T$ and zeros otherwise.

Collecting terms in $\mathbf{C}_t\boldsymbol{\omega}_t$ and $\mathbf{C}_{t-1}\boldsymbol{\omega}_{t-1}$, we can rewrite this as

$$\begin{aligned} s_t - s_{t-1} = & \sum_{n=1}^{N}\left(\frac{\beta}{1-\rho\lambda} + (1-\lambda)(\beta_{nt} - \beta)\right)(f_{nt} - f_{n,t-1}) + \\ & \left(\sum_{n=1}^{N}\left[\lambda\bar{\omega}^n + \bar{\theta}^n\right](f_{nt} - f_{n,t-1}) + \lambda\sum_{n=1}^{N}\bar{h}_t^n + \tilde{b}\bar{b}\right)\mathbf{C}_t\boldsymbol{\omega}_t - \end{aligned} \tag{51}$$

$$\left(\sum_{n=1}^{N} \bar{\theta}^n \left(f_{n,t-1} - f_{n,t-2}\right) + \lambda \sum_{n=1}^{N} \bar{f}_{t-1}^n + \tilde{b}\bar{b}\right) \mathbf{C}_{t-1}\boldsymbol{\omega}_{t-1} +$$

$$-\sum_{n=1}^{N} \frac{\rho\lambda\beta}{1-\rho\lambda} \left(f_{n,t-1} - f_{n,t-2}\right) + \lambda \sum_{n=1}^{N}\sum_{i=1}^{T} \left(f_{n,t-i} - f_{n,t-i-1}\right) \theta_{T-i+1}\epsilon_{n,t-T} +$$

$$(1-\lambda)(b_t - b_{t-1}) + \tilde{b}\rho_b^T(b_{t-T} - b_{t-T-1}) - \lambda\left(v_t - v_{t-1}\right)$$

The derivative with respect to the current fundamental is:

$$\partial\Delta s_t/\partial\Delta f_{nt} = \left(\frac{\beta}{1-\rho\lambda} + (1-\lambda)(\beta_{nt} - \beta)\right) + \tag{52}$$

$$\frac{\partial\left(\sum_{n=1}^{N}\left[\lambda\bar{\omega}^n + \bar{\theta}^n\right]\left(f_{nt} - f_{n,t-1}\right) + \lambda\sum_{n=1}^{N}\bar{h}_t^n + \tilde{b}\bar{b}\right)\mathbf{C}_t\boldsymbol{\omega}_t}{\partial\Delta f_{nt}}$$

B Signal Extraction

The signal extraction problem is described in Section 2.3. The matrix $\mathbf{H}_t$ is defined as:

$$\mathbf{H}_t' = [\mathbf{A}_{1t}, ..., \mathbf{A}_{Nt}, \mathbf{B}] \tag{53}$$

with

$$\mathbf{A}_{nt} = \begin{bmatrix} \hat{f}_{nt}(1) & \hat{f}_{nt}(2) & ... & \hat{f}_{nt}(T) \\ 0 & \hat{f}_{n,t-1}(1) & ... & \hat{f}_{n,t-1}(T-1) \\ & ... & ... & ... \\ 0 & 0 & ... & \hat{f}_{n,t-T+1}(1) \end{bmatrix}$$

and

$$\mathbf{B} = \begin{bmatrix} 1 & \rho_b & ... & \rho_b^{T-1} \\ 0 & 1 & ... & \rho_b^{T-2} \\ & ... & ... & ... \\ 0 & 0 & ... & 1 \end{bmatrix}$$

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Table 1 Benchmark Parameter Assumptions*

T	1000
N	5
β	1
σ_β	0.0165
σ_b	2.2
ρ_b	0.96
σ_υ	2.7
ϕ_1	0.06
ϕ_2	0.1
σ_f	0.125
ρ_f	0
α	33.3

*** Standard deviations are given in %.**

Table 2 Moments: Data and Model*

	Data	σ_β=0	Benchmark σ_β=0.0165	σ_β=0.033
Standard Deviation Δs_t	2.91	2.90	2.99	3.04
Corr($\Delta s_t, \Delta s_{t-1}$)	0.04	0.04	0.04	0.04
Standard Deviation $i_t - i_t^*$	0.22	0.23	0.23	0.23
Corr($i_t - i_t^*, i_{t-1} - i_{t-1}^*$)	0.92	0.92	0.93	0.93
cov($\Delta s_t, i_{t-1} - i_{t-1}^*$) / var($i_{t-1} - i_{t-1}^*$)	-1.25	-1.82	-1.86	-1.83
R^2 monthly	0.023	0.022	0.022	0.031
s.d. Monthly Change $\partial \Delta s_t / \partial \Delta f_{nt}$	-	0	25.9	45.0
s.d. Monthly Change $\Delta \beta_{nt}$	-	0	0.30	0.61

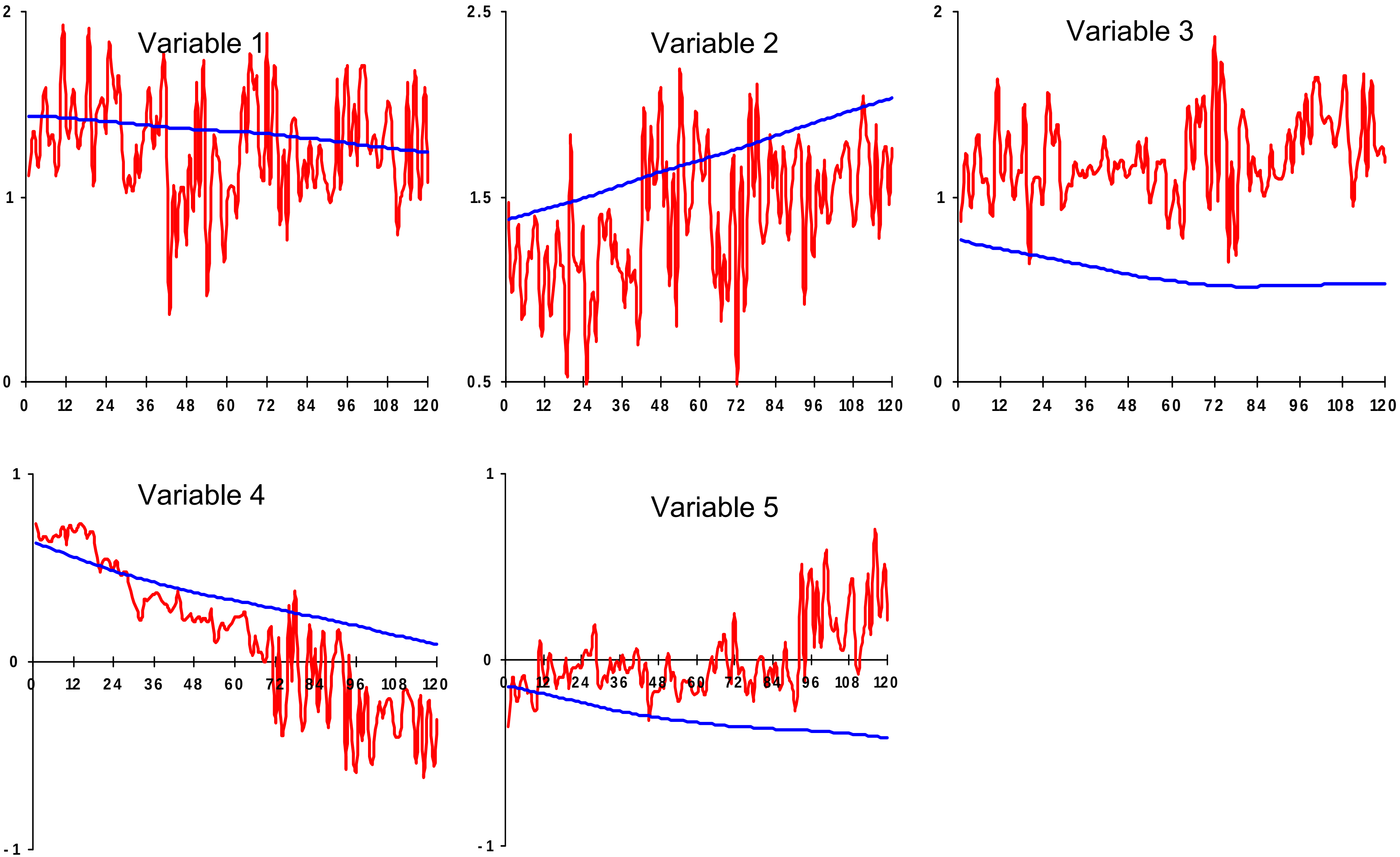
*** Standard deviations are given in %.**

Table 3   Scapegoat Ratio*

	$\frac{s.d.(\tilde{\beta}_{nt})}{s.d.(\Delta\beta_{nt})}$	Scapegoat ratio
Benchmark Process	319	85.1
Process 1	637	267.7
Process 2	7	1.6
Process 3	22	4.4
Process 4	49	10.1

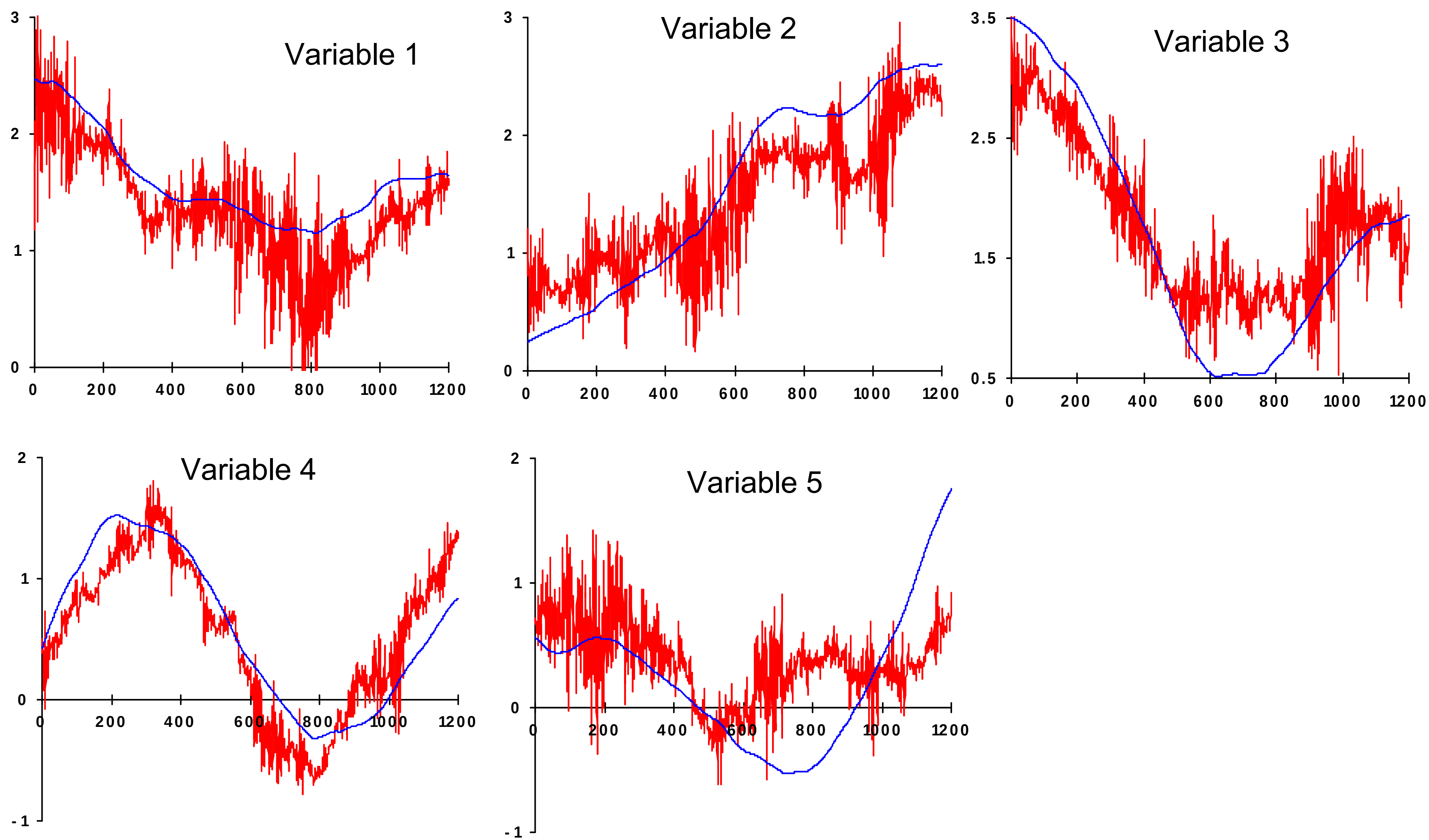
*** The scapegoat ratio is the standard deviation of monthly changes in the reduced form derivative $\partial\Delta s_t/\partial\Delta f_{nt}$ of the exchange rate with respect to fundamentals relative to the standard deviation of monthly changes in structural parameters.**

Figure 1 Derivative Δs_t with respect to Δf_{nt} (10 years)*



* The smooth line is β_{nt}, while the volatile line represents the derivative of Δs_t with respect to Δf_{nt}.

Figure 2 Derivative Δs_t with respect to Δf_{nt} (100 years)*



* The smooth line is β_{nt}, while the volatile line represents the derivative of Δs_t with respect to Δf_{nt}.

Figure 3 Expectations β_{nt} (variable 1)

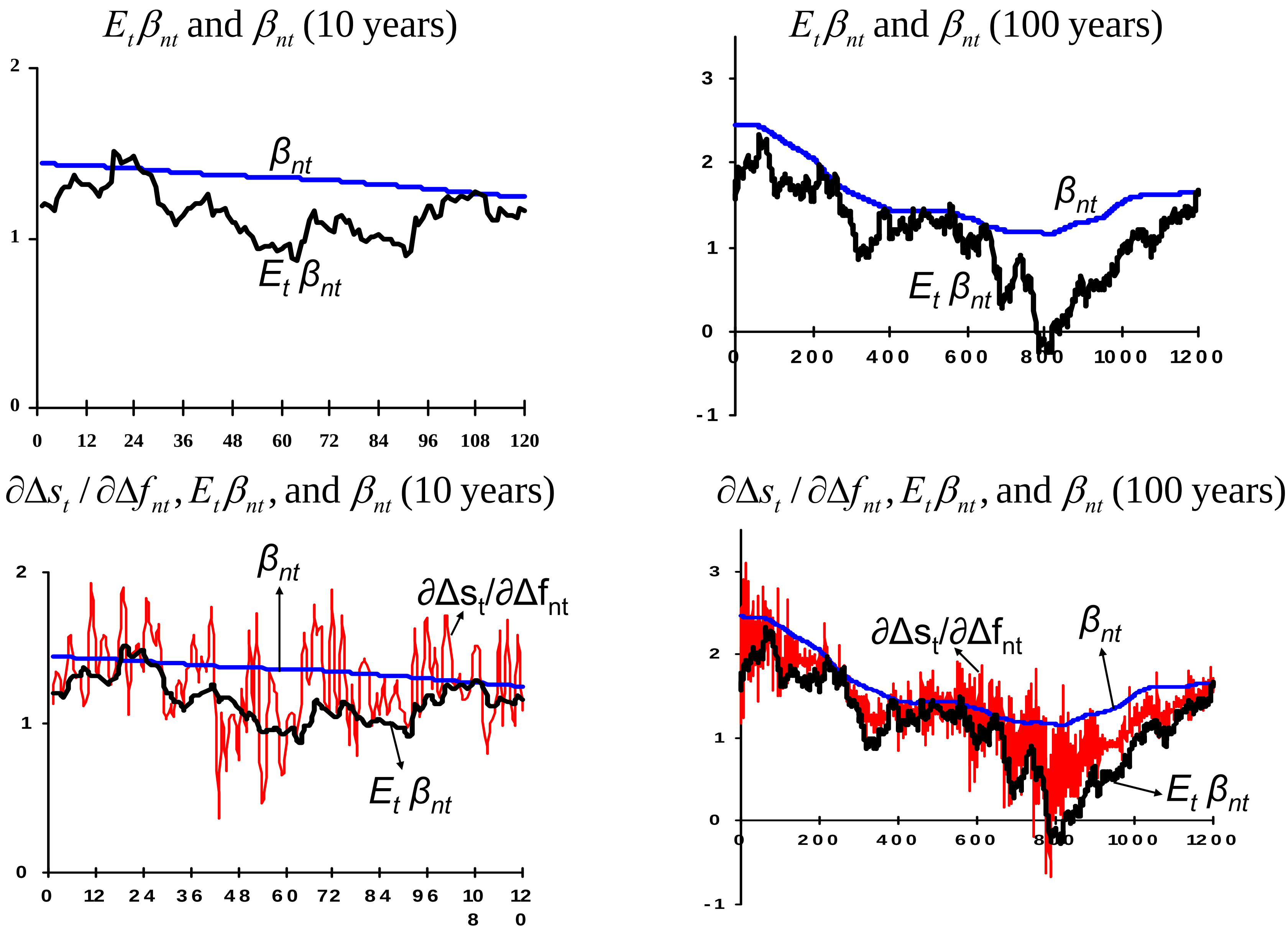
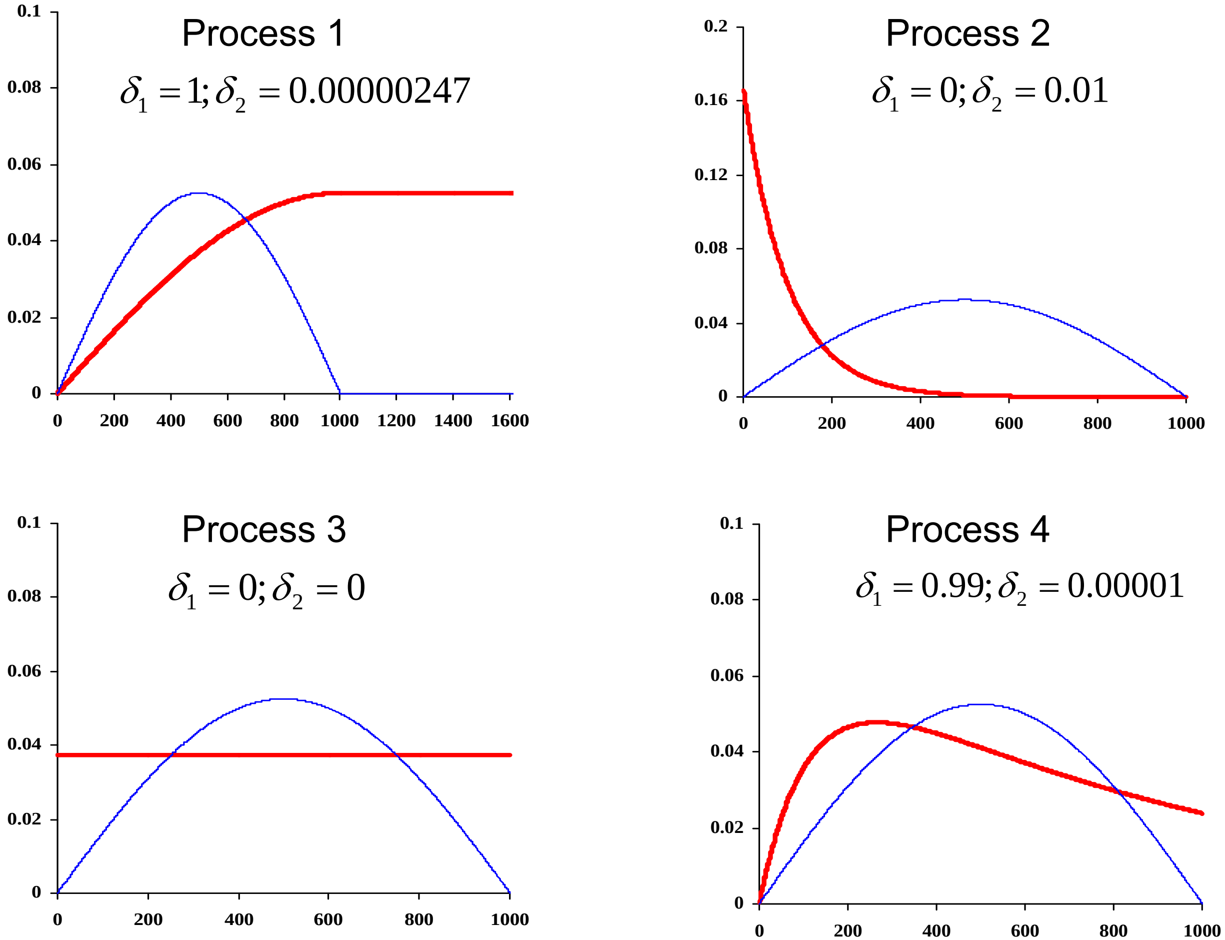
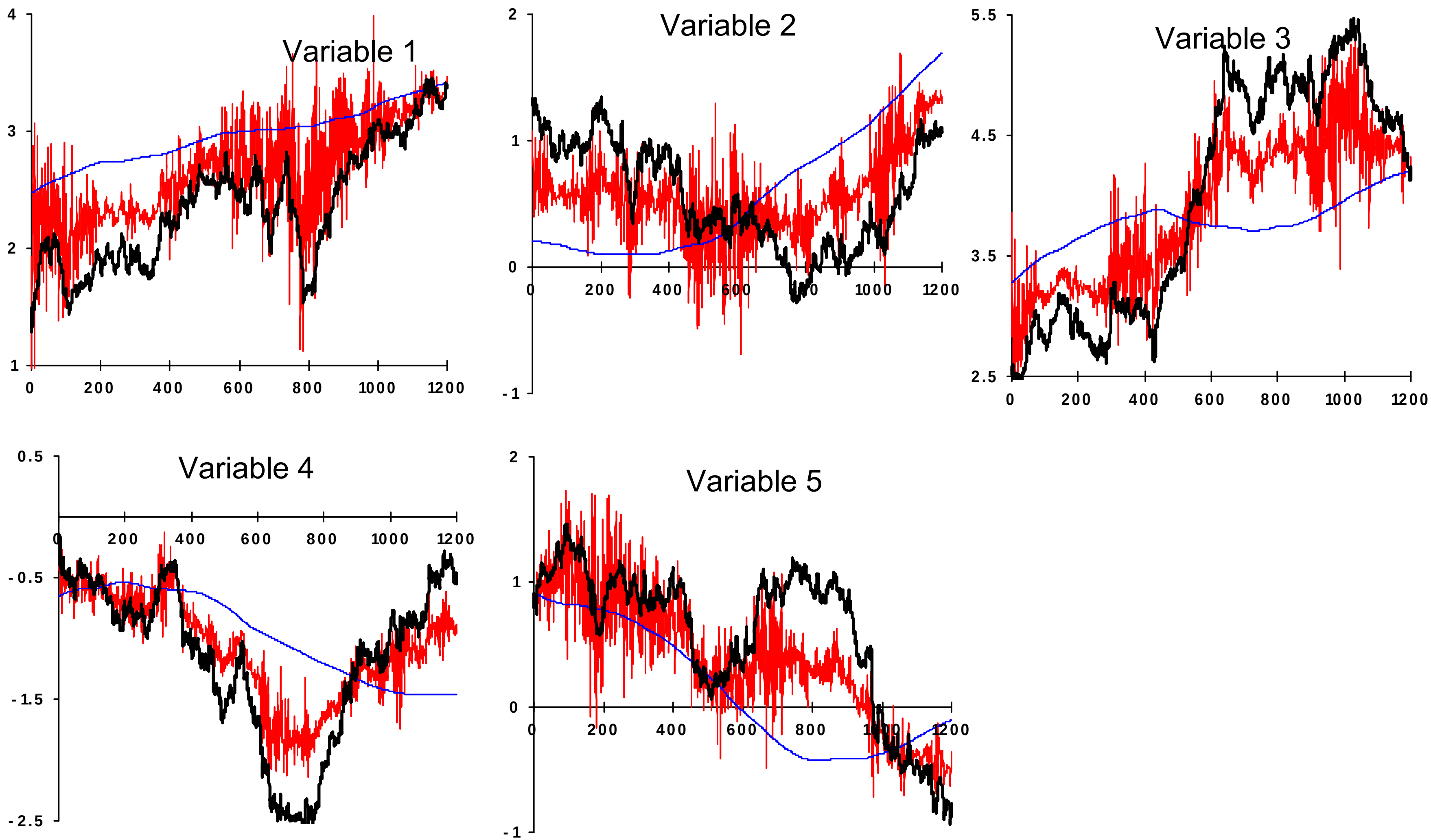


Figure 4 Impulse Response Functions for Alternative Processes for Structural Parameters*



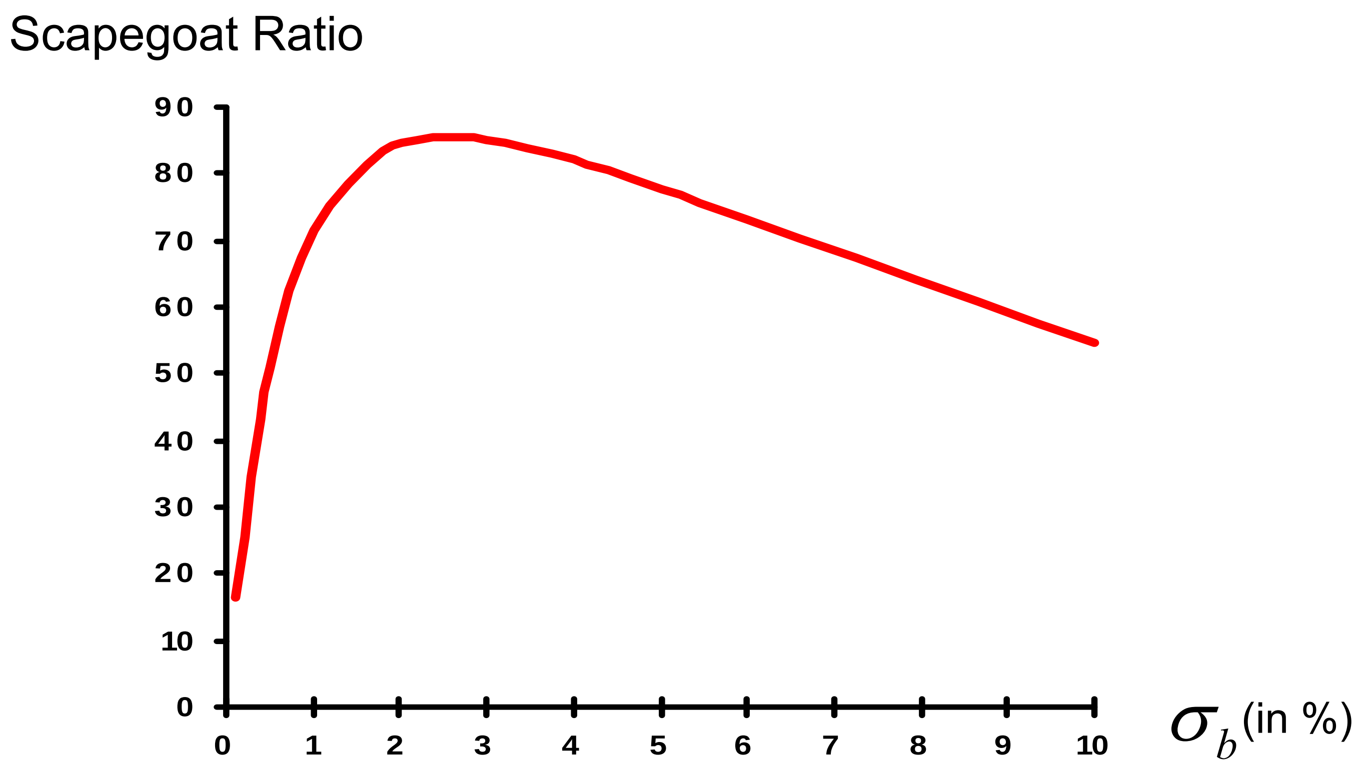
* Each graph shows the impulse response functions of the structural parameters in response to one standard deviation parameter innovations. The different charts correspond to different values for δ_1 and δ_2 for the process described in the paper. For comparison each graph also shows the impulse response function for the benchmark process (thinner hump-shaped line).

Figure 5 Derivative Δs_t with respect to Δf_{nt} (100 years)* Process 1



* The smooth thin line is β_{nt}, the thick line is $E_t\beta_{nt}$, while the most volatile line represents the derivative of Δs_t with respect to Δf_{nt}.

Figure 6 Unobservable Shocks and Scapegoat Ratio*



*Scapegoat ratio=standard deviation monthly changes in $\partial\Delta s_t/\partial\Delta f_{nt}$ relative to standard deviation monthly changes in β_{nt}.