Fuzzy Capital Requirements, Risk-Shifting and the Risk Taking Channel of Monetary Policy

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May 2010

*We thank L. Meneau and O. Prato for helpful comments and precisions on Basel regulation, and V. Acharya, G. Barlevy, J-P Benassy, C. Borio, E. Challe, S. Cecchetti, G. Demange, E. Farhi, D. Gale, E. Kharroubi, A. Kashyap, N. Kiyotaki, O. Loisel, A. Martin, H. Pages, F. Portier, J.-C. Rochet, J. Tirole, C. Upper and M. Woodford and seminar participants at the Feds of Boston, Chicago and New York, the BIS, and conference participants at the BIS, CEMFI and Tilburg for comments. All errors remain ours. The opinions expressed in this paper are the authors’ own and do not necessarily reflect the views of the Banque de France or the Eurosystem.

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**Abstract:** We set up a model where asset price bubbles due to risk shifting can be moderated by capital requirements. However, imperfect information about the ratio of required capital, or the extent of regulatory arbitrage, introduces uncertainty about the risk exposure of intermediaries. Underestimation of regulatory arbitrage may induce households to infer that higher asset prices are due to a decline of risk. This mechanism can explain the low risk premia paid by US financial intermediaries between 2000 and 2007 in spite of their increasing leverage. Moreover, the underestimation of risk is larger the lower the level of the risk free interest rate.
Many economists and institutions recognize that the wave of financial innovations that took place in the years 2000 has contributed to financial instability\(^1\). The increase of financial intermediaries’ leverage through various off-balance sheet innovations appear to have fed what will most likely enter history as another episode of asset price overpricing.

For many observers,\(^2\) the engine of this overpricing was the creativity deployed by financial intermediaries to increase their return on capital through higher leverage. In the case of financial intermediaries subject to capital requirement, this creativity is usually labelled *regulatory arbitrage*. However, this increase in leverage, which is akin to being subject to a higher risk of default, was not sanctioned by higher risk premia on intermediaries’ debts. These premia remained low and non-increasing until the summer of 2007.

The goal of this paper is to understand why financial intermediaries were able to pay non-increasing risk premia while increasing leverage and risk-taking, and to derive implications about the build-up of financial fragility. We also analyze whether the stance of monetary policy, modeled by the level of the interest rate, influences the perception of risk and the incentives of risk taking.

To do so, we build a model of asset pricing where intermediaries can default on their debt in case of bad aggregate outcomes. In this set-up we derive the interest rate margin\(^3\) paid on

\(^1\)"Those of us who have looked to the self-interest of lending institutions to protect shareholders’ equity, myself included, are in a state of shocked disbelief."

Extract of Alan Greenspan’s 23 October 2008 testimony to the House Committee on Oversight and Government Reform.

\(^2\)Brunnermeier (2008), Blanchard (2008), Greenlaw et al. (2008), Acharya et al. (2009).

\(^3\)Through out the text we will call this interest rate margin a risk premium even if, in the model, agents are risk neutral. These agents require an interest rate margin that, ex ante, covers exactly the expected cost of default. Although they do not require a premium for the risk associated to this transaction, such a premium would only strengthen our argument.
the debt of intermediaries as a function of leverage in a general equilibrium model.

Our contribution is two fold. First, the patterns of risk premia and leverage ratios observed in the US between 2000 and 2007 can be understood only if agents overestimated the constraints put on intermediaries’ capital and thus underestimated the intermediaries’ incentives to take risk. We show how rational investors may wrongly deduce from rising asset prices that the aggregate risk is decreasing, and thus charge a low risk premium on debt.

Second, we provide a theory for the risk taking channel of monetary policy. In particular, if uninformed investors underrate the extent of regulatory arbitrage, low interest rate may amplify their underestimation of risks. This is because the effect of the interest rate on asset prices is higher the higher the leverage of intermediaries and hence the lower the capital requirement. As a consequence, lower interest rates imply a larger effect of changes in capital requirements on the price of risky assets and in turn on the perception of risk by investors, who extract their estimation of risk from asset prices\(^4\).

This risk taking channel implies both mis-perception of risk by some investors and increased exposure to risks by others. It is however conditional on a lack of transparency in capital requirements, a feature of the years 2000 and, in all likelihood, of any phase of major financial innovations or deregulation.

More specifically, we set up a model à la Allen and Gale (2000), which is enriched to analyze the role of regulatory capital. In this model, investors, who do not observe the risk of the risky assets ex ante, can invest in risky assets only indirectly, through lending to financial intermediaries. Investors require a risk premium on this loan because they anticipate that financial intermediaries will default in the bad state of the world. However, the limited liabilities

\(^4\)Effectively, the model is real and the interest rate in the model is also real. We assume that monetary policy can affect, possibly only temporary, the level of the real interest rate on the storage asset. We discuss in Section 4 other factors inducing low interest rates.
of intermediaries in case of default imply that they take too much risk. A bubble results in the
sense that the price of the risky asset is higher than in the case where households can invest
directly in the risky asset. In this set-up, we introduce capital requirements as a constraint put
on intermediaries. We assume that they have to invest some of their own resources to finance
investments in the risky asset. This constraint hence moderates the degree of risk shifting by
intermediaries and the distortions induced by their limited liabilities.

We consider two assumptions on the information set of households. First, we assume that
they can observe the exact amount of capital requirement imposed on intermediaries. We prove
that they are then able to deduce the underlying risk of the risky asset from asset prices. In this
case, an (anticipated) decrease in capital requirements raises both the price of the risky asset
and the premium charged on intermediaries. This model therefore falls short of reproducing the
build up of the subprime crisis during which the increased leverage of the commercial banks did
not imply higher risk premia.

Second, we assume that households cannot observe the exact severity of the regulatory
constraint, and thus the degree of risk-shifting, and try to infer it from asset prices. One of
the reasons why the exact amount of regulatory capital ratio can be opaque is, as argued by
Acharya and Schnabl (2009), that intermediaries use off balance sheets conduit to "play" the
capital requirements. The uncertainty about the strength of the regulatory constraint implies
an uncertainty about the amount of risk associated to assets held by banks. We prove that
households underestimate the riskiness of the asset if they overestimate capital requirements
of intermediaries. The model can hence replicate one of the most puzzling stylized fact of
the banking crisis. Risk premia did not increase because the depletion of capital that financial
intermediaries effectively pledged to their riskiest investments was underestimated by uninformed
investors (be they households, pension funds, regulators or even managers of the largest banks).
We then study the effect of monetary policy on risk perception and incentives in the context of our model. We find that the level of the riskless interest rate affects the signal extraction problem of households. When there is some uncertainty about the regulatory constraint, lower real interest rates increase the scale of the underestimation of risk, which in turn amplifies the overpricing of risky assets. This sequence can account for the build up of financial fragility that occurs in times of major financial innovations, including the originate-to-distribute and securitization business model that inflated the US financial system between 2000 and 2007.

**Related literature.**

This article focuses on the link between leverage, asset prices and capital requirements. It relates to the results of Adrian and Shin (2008) and Geanakoplos (2009) who have highlighted the effects of the financial intermediaries’ leverage on asset prices. It also provides a theoretical underpinning for the empirical results of Ioanidou, Ongena and Peydro (2008), Maddaloni, Peydró and Scopel (2008), Ciccarelli, Maddaloni and Peydro (2009), Altunbas, Gambacorta and Marques-Ibanez (2009), Adrian and Shin (2008) and Shin (2009) who showed that accommodative monetary policy stance are associated with more risk taking by banks. We hence provide theory for what Borio and Zhu (2009) and Adrian and Shin (2009) call the "risk-taking" channel of monetary policy.

In the risk-shifting literature, our paper relates first to Allen and Gale (2000)’s contribution where they showed how limited liabilities of debt issuers leads to over-investment in risky assets. This is because debt issuers would then care only about the up-side of the return distribution. Barlevy (2008) proved that risk-shifting also implies bubbles in more general frameworks of financial intermediation (i.e. when the formation of financial contracts is endogenous); He also generalized risk-shifting to a continuous time dynamic framework. Challe and Ragot (2008) expand the risk-shifting model to the case where the supply of loans is endogenous\(^5\).

\(^5\)It is also important to underline the difference between the risk-shifting literature and the literature on
Our paper is also linked to the literature discussing the opacity around the real cost of risk-taking for financial intermediaries in terms of capital requirements. Asharya and Schnabl (2009) show how, in the last decade, banks have been able to under-report their "effective" leverage ratio through pseudo off-balance sheet operations. Despite the transfer of risky assets to Special Purpose Vehicle, the unwinding of the crisis demonstrates that risks were still on the book explicitly or implicitly, either because banks were tied by explicit liquidity and credit enhancement contracts or for reputational motives: ABS have frequently been brought back to intermediaries balance sheet after 2007, once in the bad state of the world. Then, the regulatory arbitrage added to the complexity of the capital requirements calculus (Rochet 2008) and blurred the information content of the capital ratio for banks’ counterparts. The contribution of our paper is to formalize the role of capital requirements over risky investments and show how opacity on the true level of capital pledged by financial intermediaries leads to endogenous uncertainty.

Our paper also shares some common feature with the recent paper by Fahri and Tirole (2009). They study the case where intermediaries do not bear the full cost of their choices because they benefit ex post from a bailout. In their model, the risk-shifting is between intermediaries and their creditors on the one hand and the tax payers on the other hand. We focus instead on the risk-shifting between intermediaries and their creditors. We believe that the US banks’ failure examples since 2008 and the dramatic jump in risk premia charged on banks’ debt at the start endogenous credit constraints. The latter analyses how asymmetric information introduces external finance premiums and collateral constraints. This literature accounts well for the financial accelerator, either in the boom phase, when the rising price of collateral releases credit constraints (Kiyotaki and Moore, 1997) or in the bust phase, when the collapse in asset prices tightens the credit constraint considerably (Holmstrom and Tirole, 1997). However, these models do not explain why there are equilibria with too much credit and overinvestment in the risky asset.

6In Washington Mutual Bank’s bankruptcy, around $13 billions of debt were left by the rescuer (JP Morgan) according to FDIC. An estimated 10,000 uninsured depositors lost over $270 million because of IndyMac failure.
of the financial crisis show that there were a significant misperception of the real risk borne by financial intermediaries bondholders. Such an increase in risk premia may actually invalidate models of the crisis whereby investors trusted that the financial system would be bailed out.

The paper proceeds as follows. Section 1 documents the stylized facts about the crisis. Section 2 presents the model. Section 3 solves the model with symmetric information. Section 4 presents the results with asymmetric information. Section 5 is the conclusion. All figures are gathered in section 6. Section 7 is the appendix.

1 Stylized facts on the pre-subprime crisis

1.1 Debt and risk premia

We dig out two major stylized facts from the literature and from our own observation of the crisis:

1. The risk premium paid by financial intermediaries on their debt has declined.

US banks benefited from extremely favorable funding conditions during this period not only because of an accommodative Fed’s monetary policy for short-term rates and the "savings glut" for long-term ones, but also because of historical low level of risk premia paid on their debt. A look at the 10 years interest rate spread between commercial paper of US banks and US government bonds\(^7\) (Fig. 1) shows for instance that the premia paid on the risk of banks’ default had been non-increasing from 2000 to mid 2007: the price of credit risk for banks has even declined markedly between 2002 and 2007.

2. The banking sector becomes more leveraged

\(^7\)A similar conclusion can be made from the observation of the spread between Libor and T-Bill rates for shorter maturities (3M, 6M).
Blundell-Wignall and Atkinson (2008) have stressed the huge increase in debt levels (in book value\(^8\)) observed in US banks’ balance sheet over the years preceding the crisis. According to Fed’s and BEA’s figures, the ratio of the debt of the US commercial banking sector over US GDP rose from 59% to 76% between 1999Q4 and 2007Q4. This higher debt level was not associated with tougher capital requirements, and the leverage ratio of the US banking sector (defined as the ratio of debt over equity, at book value) inflated during the same period, from 19 to 44 (see Flow of Funds\(^9\)).

1.2 Evolution of the Regulation

Several reasons explains the favorable norms of capital requirements in this period. For instance, Blundell-Wignall and Atkinson (2008) highlight the difficulty for outsiders to extract extensive information on risk taken by financial intermediaries, the capture of regulator in order to favour easier capital standards, or the procyclicality of the Basel capital regulation framework. Rochet (2008) focuses on the lobbying of the financial industry in the definition of Basel II. He also stresses that the Internal Rating Based (IRB) approach may have deliberately geared regulation toward complexity in the mapping from risky assets to capital requirements. Such complexity can only have favoured interpretations and implementation of capitalization that would align with the vested interest of the industry. Finally, the accounting rules concerning the consolidation of off-balance sheet entities were incriminated by the Financial Stability Forum for creating "a belief that risk did not lie with arrangers and led market participants to underestimate firms'\(^9\)

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\(^{8}\)Since we discuss the risk of bank’s default in the relation between banks’ shareholders and lenders, it is more appropriate to consider the banks’ liabilities in book value (in opposition with marked-to-market value) in our model framework.

\(^{9}\)We approximate the leverage ratio as the ratio of total financial liabilities over the difference between total financial assets and liabilities. In particular, our approximation does not take into account non-financial asset and balance-sheet elements relative to non-US area.
risk exposures" (April 2008).\textsuperscript{10}

We focus on these consensual facts because they are the most relevant to test the model’s conclusions and its ability to replicate the build up of financial fragility as we witnessed it during the last decade. We deliberately ignore the dynamics of the crisis itself, including the intertwined collapse of asset prices and flight to quality. Models of these phenomena include Holmstrom and Tirole (1997), Kiyotali and Moore (1997) and He and Krishnamurthy (2008), among others. The main goal of our paper is instead to understand how it was possible for intermediaries to increase their debt level without facing an increasing risk premium.

2 The model

2.1 Markets and assets

Timing

There are two dates $t = 1, 2$. Agents make their investment choices at date 1 and get assets returns at date 2.

Financial assets

Three financial assets are available in the economy:

1. A safe asset whose supply $X_S$ is variable, and whose return is $r_S$. The issuers of this asset get $f(X_S)$ at date 2, where $f(\cdot)$ is a continuously differentiable function, increasing in $X_S$ though with decreasing return to scale. We assume that the production function is iso-elastic $f(X_S) = X_S^{1-\eta}/(1 - \eta)$, but results are robust to the introduction of more general production function.

\textsuperscript{10}This question is actually one of the point in the agenda of the G20 and similar concerns about off-balance sheet vehicles has been brought up by academics (see Acharya and Schnabl 2009), official regulators and central bankers (see for instance speeches of C. Noyer and B. Bernanke in 2008)
2. A risky asset in fixed supply $X_R$, which return is $R^*$. $R^*$ equals $R$ with probability $\pi$ and 
0 with probability $(1 - \pi)$, which is the level of "economic risk" in the model. This asset 
is priced $P$ on the financial market at date 1. The assumption of fixed supply simplifies 
the model but it can be relaxed provided that the production function of risky assets is 
not too price sensitive.

We make moreover the following technical assumption

\[ \eta > 1 - \pi \] (1)

This assumption is fulfilled for reasonable values of the parameters, as shown below. It 
insures the uniqueness of the equilibrium.

3. A storage asset $F$ which has a constant return $\tau F$. This third asset is available in infinite 
supply.

Financial assets in this economy can be interpreted in a number of ways:

- The storage asset may for instance represent deposit facilities at the central bank or cash. 
  Indeed, it allows agents to invest without limits at a low and constant rate. In the following, 
  we will use the return on the storage asset as a proxy for the interest rate set by the 
  monetary policy authorities.

- The safe asset accounts for bonds, issued by AAA states or firms. It can be interpreted as 
  a loan to the "real" sector in order to finance investment or production.

- Finally, the risky asset encompasses all types of investments whose expected returns are 
  higher than the return on bonds. It can be either real estate mortgages, junk bonds or 
  stocks
**Agents and market segmentation**

The economy is composed of three types of agents: households, financial intermediaries and initial sellers.

1. There is a continuum of households, who are risk-neutral and who receive an endowment $W^H$ at the beginning of date 1. The households maximize their date 2 consumption.

2. There is a continuum of financial intermediaries (that we henceforth also designate as "banks"), who are risk-neutral and who receive an endowment $W^I$ at the beginning of date 1. Intermediaries maximize their consumption over the two periods. In addition, we suppose that intermediaries enjoy a private benefit $U$ from being intermediaries. This benefit will insure that these agents accept to operate as intermediaries rather than consuming all their endowment at period 1.

3. Initial sellers are agents who sell the risky assets to intermediaries at period 1, consume and leave the economy. These agents are only introduced to create a simple supply of the risky asset at the beginning of period 1.

Only financial intermediaries can invest in all the existing assets. Household can only invest in the storage asset or lend to financial intermediaries an amount $B$ at an interest rate $r$. As Allen and Gale (2000), we introduce this assumption to capture the advanced skills and accumulated rents (asset management abilities, private information and so on) needed to trade corporate bonds and sophisticated financial products. Financial intermediaries never invest in the storage asset, because they have access to the safe asset which yields a higher return.

**Financial intermediaries balance sheet and capital requirements**

Financial intermediaries’ balance sheet is composed of a risky asset $PX_R$ and a safe asset $X_S$ on the asset side, while its liabilities are either equity $K$ or debt $B$. The amount $K$ stands
for the fraction of resources invested by the intermediaries themselves in their business. As we will show in the following section, this amount is directly linked to the intermediaries incentives to take risks. The models can hence describe how bank capital requirements, for instance set by regulation policies, influences these incentives. In particular, the level of capital requirements has a direct bearing on the composition of their portfolio.

Financial regulation imposes that banks invest their own equity for at least $\Delta$ per value of unit of risky asset

$$K \geq \Delta PX_R$$

The coefficient $\Delta$ is arguably close to the original Cooke ratio of 0.08, although regulation allows for smaller value (Rochet 2008). The balance sheet is

<table>
<thead>
<tr>
<th>Balance sheet of the financial intermediaries</th>
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<tbody>
<tr>
<td>Assets</td>
</tr>
<tr>
<td>$X_S$</td>
</tr>
<tr>
<td>$PX_R$</td>
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<tr>
<td>Liabilities</td>
</tr>
<tr>
<td>$K$</td>
</tr>
<tr>
<td>$B$</td>
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</tbody>
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**Debt contract**

Following Allen and Gale (2000), we assume that households use debt contracts to finance intermediaries and are not able to observe the investment decisions of financial intermediaries. This asymmetric information structure leads to moral hazard because banks do not bear the full costs of the risk incurred by the risky asset. In the bad state of the world, intermediaries file for bankruptcy because the residual value of their portfolio is inferior to their debt. Hence, banks have incentives to take more risks than what would be optimal from the households’ standpoint. Households, who anticipate that the possibility to default induces risk shifting, will lend to intermediaries as long as the expected return on this loan remains superior or equal to the return on the storage asset.
Households cannot discriminate between banks because these are identical \textit{ex post}. Hence, households will demand the same interest rate $r$ whatever the size of the loan they grant to the financial intermediary. They supply loanable funds inelastically and the interest rate clears the market.

\textbf{Information structure.}

All agents observe asset prices and the interest rates: the safe interest rate $\tau$, the interest rate $r$ paid by intermediaries and the period-1 price of the risky asset $P$ are known. Intermediaries know the amount of risky asset in the economy $X_R$ and the production function $f(.)$ which produces the safe asset, but households do not observe $X_R$ nor the production function $f(.)$. The basic justification for this assumption is that households can easily observe the liability side of banks, but it is much more difficult to obtain information about the risk at the asset side. Many observers now recognize that it was difficult even for the rating agencies and financial analysts to assess the real risk born by some restructured assets. This assumption seems thus a natural benchmark for the analysis of risk-shifting in this framework.

We then solve the model with two assumptions concerning the regulatory constraint that intermediaries face. In section 3.2 we assume that households observe the exact value $\Delta$. In section 4, we assume that households do not observe $\Delta$. As discussed in introduction and in section 1.2, this last assumption is based partly on the unknown extent of regulatory arbitrage.

\section{2.2 Agents}

We now describe the problem of each agent.
2.2.1 Households

Households choose the composition of their financial portfolio in order to maximize their final consumption\(^{11}\), taking market prices, their expectations of aggregate risk and the capital ratio of banks as given:

\[
\max_{c,F,B} E[c^H]
\]

where \(E[\cdot]\) is the expectation operator. The expectations are made conditional on two different information structures, which are specified in section 3.2 and 4. The households’ income constraints are:

\[
F + B \leq W^H \text{ (at date 1)}
\]

\[
c^H \leq \rho B + \tau F \text{ (at date 2)}
\]

\(W^H\) is the households wealth at the first period. \(F\) is the amount invested in the storage asset, and \(B\) is the amount lent to intermediaries. For households, the stochastic interest rate \(\rho\) that they receive \textit{ex-post} on their loans to financial intermediaries is the only source of uncertainty. If the intermediaries do not default, they get the return defined \textit{ex ante} \(r\). In case of default, households get the residual value of the banks’ portfolio \(r_S X_S\) so that the \textit{ex post} return per unit of loan is \(\frac{r_S X_S}{B}\). Since the loans to financial intermediaries are risky, we define the risk premia paid by intermediaries as the difference between their cost of borrowing \(r\) and the riskless rate for household \(\tau\).

As the return of the risky asset is 0 in the bad state of the world, financial intermediaries

\(^{11}\)We could relax this assumption and make households maximise their aggregate consumption across periods 1 and 2, at the cost of much more algebra.
will always default in the bad state of the world. Hence, the return on $B$ is

$$\rho = r \text{ with probability } \pi$$

$$\rho = \frac{r_S X_S}{B} \text{ with probability } 1 - \pi$$

### 2.2.2 Financial intermediaries’ problem

Financial intermediaries seek to maximize their aggregate consumption at date 1 and 2 with a discount factor $\beta$. They obtain private returns $U$ from being active financial intermediaries, which insure their participation to the economy. We assume that

$$\beta < 1/\tau$$

This assumption implies the intermediaries are relatively impatient. They choose their debt level $B$, and the composition of their portfolio $(X_S, X_R)$ taking market prices and the regulation as given. Their program is

$$\max_{K, B, X_R, X_S} c_1^f + \beta E \left[ c_2^f \right] + U \quad \text{s.t.} \quad c_1^f \leq W^f - K$$

$$c_2^f \leq \max \{ R^* X_R + r_S X_S - rB, 0 \}$$

$$PX_R + X_S = B + K$$

$$K \geq \Delta PX_R$$

In the constraint (3), the max operator indicates the intermediaries’ option to default. This option will be considered depending on the value of the stochastic return $R^*$. 


2.2.3 Initial sellers

Initial sellers have no choice and simply consume in period 1 the amount obtained from the sale of the risky asset:

\[ c^i = PX_R \]

3 Full information resolution of the model

3.1 Pareto Efficient Equilibria

Before solving the model for each of the two information structures above mentioned, we derive the set of Pareto efficient allocations. To do this we maximize the average expected welfare of financial intermediaries for a given expected welfare of households and initial sellers.

This maximization is

\[ \max_{F, X_s} c_1^f + \beta E c_2^f + U \]

\[ Ec^H = \bar{c} \]

\[ c^i = \bar{d} \]

\[ W^f + W^h = c_1^f + F + X_S + c^i \]

\[ \tau F + f(X_S) + RX_R = c^H + c_2^f \]

As \( \tau < 1/\beta \), the solution to this maximization is simply

\[ F = 0 \]

\[ f'(X_S) = \frac{1}{\beta} \]

If there were no market segmentation, each allocation of this set could be reached thanks to appropriate first period lump-sum transfers. In these equilibria, the interest rate on the safe
asset would be $1/\beta$ and the price of the risky asset would be equal to the fundamental price:

$$P^* = \beta \pi R \quad (5)$$

### 3.2 Market equilibrium with known regulation

In this section, we assume that households can observe $\Delta$. Fig. 2 summarizes the structure of the model. The choices of households and banks are presented. In this setup, the uncertainty is only on the payoff of the risky asset in period 2.

#### 3.2.1 Financial intermediaries

Intermediaries default in the bad state of the world because their overall return would be negative if they repay their debt (because $B > X_S$).

Their program can be written as

$$\max_{K,B,X_R,X_S} W^f - K + \beta (RX_R + r_SX_S - rB) + (1 - \pi) \times 0 + U$$

$$PX_R + X_S = B + K$$

$$K \geq \Delta PX_R$$

We solve the model under the conjecture that the constraint $K \geq \Delta PX_R$ is binding. This case is of course the relevant one for this model. The binding constraint is equivalent to the following inequality

$$\pi r < 1/\beta \quad (6)$$

This inequality stipulates that expected cost of the debt $\pi r$ (because debt is repaid only in case of the high return which occurs with probability $\pi$) must not be too high. If the expected cost of the debt is too high, intermediaries could want to invest all their wealth to decrease their
expected debt burden. Hence, the regulatory constraint would not bind. As $r$ is determined in equilibrium, we show below that the condition (6) is fulfilled for a wide range of parameter values.

The program yields the only asset prices for which the asset market clears.

$$P = \frac{\beta \pi R}{\Delta + \beta \pi r (1 - \Delta)}$$

(7)

This asset price equilibrium is the main equation of the model. First note that when there is no capital requirement ($\Delta = 0$), the price is simply $P = R/r$ which is the case studied by Allen and Gale (2000).\footnote{In their model, Allen and Gale show how incomplete debt contracts limit debtors losses in the bad state of the world (losses fall on lenders). In other words, debt contracts act as call options for borrowers. This implies that borrowers only focus on the good state of the world when deciding the composition of their portfolio: the share of the portfolio at risk is higher and the price of risky assets is inflated above its level in a world without segmentation or complete contracts.} As intermediaries default in case of a bad aggregate shock, their demand for the risky asset is always higher than under the first best equilibrium. Indeed, as $\pi r < 1/\beta$, one finds $P > P^*$. Asset prices are thus too high. Second, when capital requirements increase, the price of the risky asset decreases. Taking $r$ as given, increasing $\Delta$ implies a cost in the form of additional foregone consumption in period 1, an effect that dominates the reduction in size of the loan that needs to be repaid with probability $\pi$.

Thus, in partial equilibrium, the price of the risky asset can increase for two reasons: either because $\pi$ increases, which means that the expected return of the risky asset is higher or because $\Delta$ decreases (the amount of \textit{ex ante} risk shifting increases). Finally, the demand for the safe asset yields

$$f'(X_S) = r_S \implies X_S = [f'(r_S)]^{-1}$$

(8)
3.2.2 Households

It has been assumed that the households do not observe the amount of risky asset \( X_R \) and the risk of this asset, \( \pi \), but they observe the regulatory requirement \( \Delta \). One can first show that households can deduce both \( X_R \) and \( \pi \) from asset prices and from the value of \( \Delta \). Indeed, they can deduce the amount of risk \( \pi \) from the price level (7). Then, they can deduce the amount of aggregate exposure to risk from the amount of regulatory capital and with their knowledge of the coefficient \( \Delta, K = \Delta P_X R \). Finally, they can infer the amount of safe asset \( X_S \) from the balance sheet constraint of the intermediary. To summarize, households can deduce the structure of the asset side of intermediaries and the aggregate risk from the regulatory constraint and from the structure of financial intermediaries’ liabilities.

Households anticipate rightly that, in the bad state, the intermediaries’ default implies that they get the residual value of the bank’s portfolio. With probability \( \pi \) their return per unit invested is \( r \) and with probability \( (1 - \pi) \) it is \( r_S X_S/B \). The no-arbitrage condition can be written as

\[
\pi r + (1 - \pi) \frac{r_X X_S}{B} = \tau
\]  

(9)

3.2.3 Market clearing

First, competition in the financial sector yields \( r = r_S \), i.e. the funding cost of the financial intermediaries is equal to the return on the safe asset. This is necessary and sufficient to avoid infinite riskless profit opportunities by the financial intermediaries. In addition, since \( \pi \in [0, 1[ \) and \( X_S < B \), the no-arbitrage condition (9) implies \( r > \tau \). The return on the safe asset \( X_S \) for the intermediaries is then strictly higher than the return on the storage asset, and intermediaries never invest in storage.
Equality (9) can be written as

\[ B(r) = \frac{(1 - \pi) r X_S}{\tau - \pi r} \]  

(10)

We can substitute \( K, X_S \) and \( P \) by their equilibrium value given by equations (7) and (8) to find an expression \( B(r) \).

\[ B(r) = \frac{(1 - \Delta) R}{\Delta + r (1 - \Delta)} X_R + f^{\prime -1}(r) \]

Using this expression in the equality (9) together with the expression of \( X_S \) given by (8), one finds an equation where \( r \) is the sole unknown. The solution to this equation gives the equilibrium level of the interest rate.

In this economy, changes in \( \Delta \) have two effects:

1. a direct effect through the intermediaries incentives to take risk,

2. an indirect one through the evolution of the interest rate \( r \), as households require a higher return when \( \Delta \) declines.

The increase in the risky asset price will be moderated because \( r \) increases. The next section summarizes the effect of a change in \( \Delta \).

### 3.3 Risk-shifting and debt

We perform some comparative statics to analyze how allocation and asset prices change after in the regulatory constraint.

**Proposition 1** Both debt level \( B \) and the risk premia \( r - \tau \) increase when the capital requirement decrease (i.e. \( \Delta \) decreases inducing more risk-shifting)

\[ \frac{\partial B}{\partial \Delta} < 0 \quad \text{and} \quad \frac{\partial (r - \tau)}{\partial \Delta} < 0 \]
The debt level of the financial intermediaries $B$ increases when $\Delta$ decreases. Reducing $\Delta$ has two effects: the demand for $B$ increases and the household supply of $B$ decreases (they require a larger risk premium $r - \tau$). However the overall general equilibrium effect is an increase in the intermediaries debt level since the direct negative effect of $\Delta$ on capital requirement dominates other general equilibrium effects through the changing level of $B$. (see appendix A.2).

However, it should be stressed that when households observe changes in $\Delta$, the effects of risk-shifting on asset prices is somewhat moderated by the response of the risk premium $r - \tau$. Investors realize that the intermediaries take more risks, they require to be compensated. This version of the model is therefore incompatible with the stylized facts of the sub-prime cycle. As showed by figure 1, banks have been able to borrow at lower risk premium during the five years that preceded the crisis, in spite of increasing their leverage and their exposure to US housing assets.

To summarize, if an increase in the effective leverage of financial intermediaries, which could for instance be due to a larger scale of off balance sheet activities, can account for an increase in their debt level, it cannot explain the path of the risk premia between 2000 and 2007. We therefore assert that risk-shifting per se is not enough to replicate the stylized fact of the subprime crisis. Banks and financial intermediaries have effectively benefited from extremely favorable funding conditions before the crisis, a feature that cannot arise in the context of a known regulatory constraint.

4 Uncertainty on the capital requirements

We now assume that the regulatory constraint is not observed by the households, especially as financial intermediaries manage to increase leverage through regulatory arbitrage. To model this information structure, we introduce another level of uncertainty, in addition to the shock at
date 2 on the return on the risky asset. This uncertainty affects the value at date 1 of the capital requirement. For simplicity, we assume that the regulatory capital requirement $\Delta$ can take two values: $\Delta^{up}$ with a probability $p^{up}$, and $\Delta^{low} < \Delta^{up}$ with a probability $p^{low}$, with $p^{low} + p^{up} = 1$. Nature will eventually set either $\Delta = \Delta^{up}$ or $\Delta = \Delta^{low}$. Only financial intermediaries then learn in period 1 the true value of $\Delta$.

Considering the possibilities that capital requirement is either high or low, with the relevant probabilities, households will infer from asset prices their best estimate of the aggregate risk $\pi$, and charge a risk premium accordingly. We characterize the inference by households for each state. We use the upper script $up$ for values conditional on the fact the regulatory constraint is high $\Delta = \Delta^{up}$, and we use the upper script $low$ for values conditional on the light regulatory constraint, $\Delta = \Delta^{low}$.

Fig. 3 summarizes the structure of the model. Both the payoff of the risky asset in period 2 and the level of capital requirement in period 1 are now uncertain for the households.

First, the price of the risky asset is still given by equation (7) because it results from a no-arbitrage condition for intermediaries, who know the real value $\pi$ and the real $\Delta$. Households deduct $\pi^{up}$ and $\pi^{low}$ from their observation of the price $P$ the real interest rate $r$, and their belief that be either $\Delta^{up}$ or $\Delta^{low}$ The following equation must hold:

$$\frac{\beta \pi R}{\Delta + r \beta \pi (1 - \Delta)} = \frac{\beta \pi^{up} R}{\Delta^{up} + r \beta \pi^{up} (1 - \Delta^{up})} = \frac{\beta \pi^{low} R}{\Delta^{low} + r \beta \pi^{low} (1 - \Delta^{low})}$$  \hspace{1cm} (11)

Note that we denote $\Delta$ (without upper script) is the level of capital requirement effectively imposed on financial intermediaries. We deduce the following inference for households

$$\pi^{up} = \frac{\pi}{\Delta^{up} (1 - r \beta \pi) + r \beta \pi}$$  \hspace{1cm} (12)

$$\pi^{low} = \frac{\pi}{\Delta^{low} (1 - r \beta \pi) + r \beta \pi}$$  \hspace{1cm} (13)
These equations give the inference about the level of aggregate risk consistent with anticipated regulatory constraint and from asset prices. In case nature chooses $\Delta = \Delta^{\text{low}}$, $\pi^{\text{low}} = \pi$ and, given that $r\beta \pi < 1$ in the equilibrium under consideration, $\pi^{\text{up}} > \pi$. The household inference on $\pi$ is biased upward when households overestimate the regulatory constraint: the higher the ratio $\Delta^{\text{up}}/\Delta$, the higher $\pi^{\text{up}}$, the belief about the probability of the good state of nature for the risky asset. Symmetrically, in case nature chooses $\Delta = \Delta^{\text{up}}$, $\pi^{\text{low}} < \pi$ for most parameter values.

Second, the households form their inference of the residual value of their portfolio $X_{S}^{r}$ in the following way. First, from the observation of the regulatory capital of the banks $K$ and from their belief over $\Delta$, households assume that the level of investment in the risky asset is:

$$
X_{R}^{\text{up}} = K/(\Delta^{\text{up}} P) \text{ and } X_{R}^{\text{low}} = K/(\Delta^{\text{low}} P)
$$

From the balance sheet constraint of banks given by equality (4), households form the following expectation of the amount of safe asset

$$
X_{S}^{\text{up}} = B + K - P \frac{\Delta}{\Delta^{\text{up}}} X_{R} \text{ and } X_{S}^{\text{low}} = B + K - P \frac{\Delta}{\Delta^{\text{low}}} X_{R}
$$

(14)

Note the in the previous equalities $X_{R}$ denotes the true level of the risky asset in the balance sheet of financial intermediaries.

Third, the no-arbitrage for households must now be written according to the expectations of households. As they anticipate that the regulatory constraint is high with a probability $p^{\text{up}}$ and low with a probability $p^{\text{low}}$, they adjust their portfolio such that:

$$
p^{\text{up}} \left( \pi^{\text{up}} r + (1 - \pi^{\text{up}}) \frac{r X_{S}^{\text{up}}}{B} \right) + p^{\text{low}} \left( \pi^{\text{low}} r + (1 - \pi^{\text{low}}) \frac{r X_{S}^{\text{low}}}{B} \right) = \tau
$$

(15)

Using the equations (12 and 13) to substitute for $\pi^{\text{up}}$ and $\pi^{\text{low}}$, the expression of $X_{S}^{\text{up}}$ and $X_{S}^{\text{low}}$ given by (8) and finally the value of $B$ implied by the balance sheet constraint of the intermediary
and the fact that \( r_S = r \), one finds one equation in the equilibrium interest rate \( r \) which depends only on known parameters and functional forms.

To focus on the interesting case, we assume that the binding regulatory constraint (chosen by nature in this framework) is \( \Delta^{low} \). This may for instance illustrate the possibility of increasing effective leverage through securitization.

In this case, when \( p^{low} = 1 \) one finds the outcome of the previous section, when agents know the true regulatory constraint. We study below the cases where \( p^{low} < 1 \).

4.1 An extreme case

To get insights on the economic mechanisms in the model, we consider the simplest case where households associate a probability \( p^{up} = 1 \) to a capital requirement regime that amount to a form of narrow banking. All the investment in the risky asset is funded by the own capital of financial intermediaries, hence \( \Delta^{up} = 1 \). This case is only used here to exhibit the main effects at stake in the model, which is solved in the next section in a more general, though numerical, case. Households anticipate in this case that \( X_S^{up} = B \) from the budget constraints of banks. As a consequence, they anticipate that the residual value of banks portfolio fully covers the debt of the banks. Using (15) one finds that the equilibrium interest rate is

\[
r = \tau
\]

As households expect that all the risk is born by banks, the return on the portfolio liquidation is the same as the return on the safe asset. Hence households charge no premium on banks. The risk perception error \( (E(\pi) - \pi) \) is:

\[
E(\pi) = p^{up} \pi^{up} + p^{low} \pi^{low}
\]

\[
E(\pi) = \pi^{up} = \frac{\pi}{\Delta + (1 - \Delta) \beta \pi > \pi}
\]
The price of the risky asset can written simply as

\[ P = \frac{\beta \pi R}{\Delta + (1 - \Delta) \beta \tau \pi} \]

This extreme example also illustrate the role of the level of interest rates. Lower interest rates \((\downarrow \tau)\) leads to a rise in \(P\) the price of the risky asset and a larger risk perception error \((E(\pi) - \pi)\). Households infer from the rise in \(P\) that the aggregate risk has declined (i.e. \(E(\pi)\)). Indeed, households underestimate the effect of a decrease in the riskless rate \(\tau\) on the price of the risky asset because they underestimate the incentives to take risk. Households do not require premia to compensate for the default risk and the price of the risky asset increases. In what follows, we study the more general case.

4.2 Model simulations for the general case

This section studies the general case where households form expectations about the level of capital requirement. Unfortunately, it is not insightful to derive an analytical expression of the endogenous variables. We thus illustrate the main conclusions of the model with a simple calibration exercise. We show how the perception of risk \(E(\pi)\) and the risk premium paid by intermediaries \(r - \tau\) are affected by the riskless interest rate \(\tau\).

We take the following numerical values \(\beta = 0.96\), \(R = 1\), \(\eta = 0.9\) and \(X_R = 1\). We also set \(\pi = 0.8\) and \(\Delta^{low} = \Delta = 0.04\) and \(\Delta^{up} = 0.07\). With these parameter values, the three conditions on the parameters (1), (2) and (6) are always fulfilled for the range of parameters considered.

To observe the effect of the riskless interest rate, we construct the average (unconditional) expectation about the average risk

\[ E(\pi) = p^{up} \pi^{up} + p^{low} \pi^{low} \]
$E(\pi)$ is the expected probability that the risky investment succeeds and that the financial intermediary does not default. As we assume that nature draws $\Delta = \Delta^{low}$, we have $\pi^{low} = \pi$. Hence, the difference between $E(\pi)$ and $\pi$ stems from the deviation of $\pi^{up}$ from $\pi$.

Fig. 4 presents the value of $E(\pi)$ as a function of $\tau$ for $p^{up} = 0.4$; $p^{up} = 0.5$ and $p^{up} = 0.6$.\textsuperscript{13} Recall that the true value of the risk is $\pi = 0.8$. Households underestimate the amount of aggregate risk ($E(\pi) > 0.8$), because putting a positive probability on having a high capital requirement implies that higher asset prices can be due to a higher probability of success of the risky asset.

In fact, higher asset prices are due to an increased leveraged of financial intermediaries. Ex ante, households assign a positive probability to the state of nature where $\Delta = \Delta^{up}$. The expectation of such a state lead them to require a risk premium on $B$ which turns out insufficient when nature chooses $\Delta = \Delta^{low}$. As shown in Fig. 4, the higher the household’s prior that capital requirements are high, the higher the error in the perception of risk.

This mechanism whereby households are misled because there is uncertainty on the level of capital pledged by financial intermediaries, and therefore on their incentives to shift risk, may have played an important role in the period preceding the 2007-2009 crisis. First, as explained in section 2 and in Asharya and Schnabl (2009), there has been a considerable increase in regulatory arbitrage to increase leverage in the financial industry. Second, the exposure to US mortgages risks, through CDOs, ABS, various repackaging of tranches along credit risk seniority, had become increasingly complex. Altogether, the assumption of the model that risk incentives of financial intermediaries are not easily observable seems highly plausible for the last 10 years. The model shows how this uncertainty may lead investors to believe that the level of financial risk in the economy has declined, and therefore to lend to financial intermediaries at relatively

\textsuperscript{13}In the trivial case where $p^{up}=0$, note that $E(\pi) = \pi$ whatever the level of $\tau$. 

27
small risk premia (see Fig. 1). However, unlike the model of Shleifer and Vishny (2010) where market sentiment evolves exogenously, our model obtains an endogenous form of optimism in financial markets.

An alternative explanation of the low level of risk premia is that investors expected to be bailed out collectively by government and central banks. Farhi and Tirole (2008) propose a model whereby financial institutions coordinate their exposure to risks in order to increase systemic risks and the chances that public authorities would bail out all financial intermediaries. While their model hints at risk shifting from investors to tax payers, we focus instead on the risk shifted from banks to bond holders. We see our model as complementary to theirs. It is important to stress though that the increase in credit spreads of US commercial banks in 2007 (see Fig. 1) indicates that at least some investors feared that the US commercial banks may not be bailed out. The capital loss on a portfolio of US commercial banks bonds associated to these fall in spreads is all but negligible.

4.3 A model of the risk-taking channel of monetary policy

The second important result of the model is that the underestimation of credit risk, $E(\pi) - \pi$ decreases and the risk premia, $r - \tau$, increases with $\tau$, the level of the risk free interest rate. This is showed, for our calibrated example, in Fig. 4 and Fig. 5. The intuition for these effects has been identified in the previous section: households do not fully assess the marginal effects of changes of $\tau$ (which is the floor for $r$) on the incentive to take risk and thus on the asset price $P$. In our framework, the effects of interest rates on asset prices is magnified by risk shifting. The capital requirements curb this effect, but their strength are overestimated, leading households to wrongly infer that a part of the increase in asset prices is due to a reduction in aggregate risk. This overestimation is however larger when interest rates are low because the valuation of our
assets are non-linear. A wrong prior on $\Delta$ has a larger effect on the price of the risky asset $P$ at lower levels of interest rates. The higher response of $P$ translates, through 11, into a higher risk perception error.

The model therefore provides an explanation of two complementary aspects of "the risk taking channel of monetary policy" (Borio and Zhu, 2008): "the impact of changes in policy rates on either risk perceptions or risk-tolerance and hence on the degree of risk in the portfolios, on the pricing of assets, and on the price and non-price terms of the extension of funding".14

The predictions of the model on risk perception are actually consistent with the empirical results of Altumbas, Gambacorta and Marquez (2009). They find that the Expected Default Frequencies, and other market based measures of bank’s risks as perceived by financial market participants react positively to changes in interest rates. A lower interest rate lead investor to perceive that banks are less risky 15. Turning to bank’s risk taking, which may be seen as banks exploiting their ability to borrow cheaply from financial markets, a number of recent studies, including Jimenez et al, (2008), Ioannidou, Ongena and Peydro (2008) Ciccarelli, Maddaloni and Peydro (2009) show that credit standards are correlated to the level of interest rates. Lower interest rates imply lower credit standards including to customers that are perceived as presenting a higher credit risk.

It is important to stress however that the model bears results on the effects of the level of interest rates on risk perception and risk taking irrespective of the source of variation of interest

14 Adrian and Shin (2008) have a model where leverage increases with the liquidity of the repo market. However their model applies more to primary dealer than to commercial banks more generally. Also, their model is based on the management by banks of their value at risk, where, arguably, the default probability of banks is constant. We focus instead on the response of asset prices to the risk free rate in a context where the default probability of intermediaries is not known by households.

15 Like in our model, this measure of the perception of risk need not be accurate. For instance, the average Expected Default Frequencies for US banks reaches its minimum in 2007!
rates. The interest rates in the model are real and therefore can be influenced by several factors. In the context of the decade preceding the crisis, several explanations have been put forth to explain the low level of nominal and real interest rates. According to Taylor and Williams (2007), US monetary policy has been overly accommodative. However, Bernanke (2010) stresses instead that China’s excess savings have had a major role keeping the long end of the US yield curve at low levels. Either of these factors may in turn have been amplified by "search for yields", as emphasized by Rajan (2006). We don’t take a stand on these alternative drivers of the level of interest rate and stress that the endogenous mechanism described in the model would hold for either of them.

4.4 Relevance for economic policies

Uncertainty about the level of regulatory constraint can thus yield to greater mispricing when the level of regulatory constraint happens to be lower than what was expected. This effect is magnified when the real interest rate is low. These results carry important intuitions for the conduct of monetary and supervision policies. In an uncertain world, these policies are likely to be interdependent.

In principle, the third pillar of Basel II, which requires more transparency on risk management should limit the relevance of the model. If the risk exposure of banks and their capital positions are well understood by market participants, riskier positions will be sanctioned by higher costs of funding for financial intermediaries. However, recent history recalls that regulatory arbitrage can take a very large scale. And regulators may, like households in the model, be behind the curve, or, even if they identify risks, be delayed because international coordination to impose a new regulatory framework can be slow.

Our model suggests that it may be desirable to change the level of interest rates to fulfil a
financial stability objective in the circumstances when capital requirements appear deficient and cannot be adjusted directly. Such a situation can arise for instance when capital requirements cannot be adjusted unilaterally in one country because they are typically negotiated between countries over several years. However, even in such circumstances, the monetary authorities should act only if reasonably convinced either that the level of capital requirements is not well perceived by investors or that a class of investors underestimate the true level of risk.

Finally the key mechanism underlying the misperception of risk is macroeconomic in essence. It is through rising asset prices that optimism and insufficient interest rate margins occur. Hence the need to monitor asset prices, leverage and risk premia jointly at the aggregate level. A simultaneous increase in leverage and asset prices while risk premia decline may reflect an endogenous optimism that is not borne by better fundamentals. The assessment of such mis-pricing by financial markets, which, as the 2007-2009 crisis showed can have dire consequences, may belong to macroprudential policies.

5 Concluding Remarks : Can the model explain the build up of financial fragility?

In this paper, we showed first that the combination of risk-shifting and fuzzy capital requirement can explain one of the most puzzling stylized fact of the sub-prime crisis i.e. that banks could ever increase their exposure to risk without having to pay higher risk premia on their debt.

In a situation of uncertainty with respect to regulatory constraints, the increase in the observed asset prices can be interpreted as a lower aggregate risk in the economy while, effectively, asset prices were driven by greater risk-taking by financial intermediaries. We also showed that this model give rise to a risk-taking channel of monetary policy: the influence of the level of interest rate on risk perception by some agents and exposure to risks by others.
Our result extends the popular intuition that financial markets participants can form wrong inference on risks. In particular, the signal extracted from market price is polluted by noise coming from excessive risk-taking behavior when the effectiveness of capital requirement is not observable by agents. In our model, market forces are not able to lead by themselves the economy to the optimum allocation of capital because risk incentives are not correctly understood.

We see two obvious extensions to our model. First, it is possible to endogenize the expectations of households in a dynamic setting where households learn about the relevant parameters. Although, the results of our models would hold if the prior of the households are far enough from the true parameters, the resulting dynamics may lead to interesting patterns. Second, it would be interesting to study the political economy associated with the assessment of risk within the economy. Sellers of the assets have incentives to underestimate the level of economic risk or to generate complexity to increase the cost of signal extraction. This should be anticipated by households who then would look for other sources of information. For instance, we understand the current discussion about rating agencies as a part of the political economy debate on the management of risk expectations in economies where intermediaries play an important role.

References


Fig 1: Spread between 10Y US T-Bonds and 10Y Bonds of US AAA Financial Companies

Source: Bloomberg
Fig 2: Structure of the model with full information about capital requirement ($\Delta$)
Fig 3: Structure of the model with uncertain capital requirement ($\Delta$)
Fig. 4: Expected probability of success $E(\pi)$ (y-axis) as a function of $\tau$ (x-axis)
Fig 5: Interest rate margin \( r - \tau \) (y-axis) as a function of \( \tau \) (x-axis)
7 Appendix

A Proof of $\frac{\partial (r-\tau)}{\partial \Delta} < 0$

Recall that:

$$B = (1 - \Delta)PX_R + X_S = \frac{(1 - \Delta)R}{\beta \pi} + r(1 - \Delta)X_R + f^{r-1}(r)$$ (16)

and

$$\tau = \pi r + \frac{1 - \pi}{B} rf^{r-1}(r)$$ (17)

Let us define: $\Theta = \frac{\Delta}{1 - \Delta}$. Then from (16)

$$B = \frac{R}{\beta \pi + r}X_R + f^{r-1}(r)$$

and (17)

$$\tau = r \left( \pi + (1 - \pi) \frac{f^{r-1}(r)}{\beta \pi} X_R + f^{r-1}(r) \right)$$

$$\Rightarrow \Theta = \beta \pi \left( \frac{R(\tau - \pi r)}{1 - \frac{r}{\tau}} r^{\frac{1}{\eta}-1} X_R - r \right)$$

The last equality defines a function $\Theta (r)$, which gives the value of $\Theta$ (and hence $\Delta$) necessary to obtain an equilibrium interest rate $r$. We prove that $\Theta (r)$ is decreasing, and hence that the function $r (\Theta)$ is decreasing. Differentiating $\Theta (r)$, a sufficient condition to obtain $\Theta' (r) < 0$ is

$$\left( \frac{1}{\eta} - 1 \right) \left( \frac{\tau}{r} - \pi \right) < \pi + \frac{\pi \tau/r - \pi}{r 1 - \tau/r}$$

Define $x \equiv \tau/r$. Along the equilibrium under consideration $\pi < x < 1$. After some algebra, one finds that a sufficient condition is

$$\eta > 1 - \pi$$

The condition is satisfied for instance for $\eta > 1/2$ and $\pi > 1/2$. In this case, $r$ is decreasing with $\Delta$. CQFD
B  Proof of $\frac{\partial B}{\partial \Delta} < 0$

From equality (9), one finds

$$B = \frac{1 - \pi}{\tau - \pi r}^{1 - \frac{1}{\eta}}$$

After some algebra, a sufficient condition for $B$ to increase with $r$ is

$$\eta > 1 - \pi$$

If the previous condition is fulfilled one finds $\frac{\partial B}{\partial r} > 0$ and $\frac{\partial (r - \tau)}{\partial \Delta} < 0$. As a consequence, $\frac{\partial B}{\partial \Delta} < 0$. 