Selection, Growth and Learning

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January 3, 2009
Firm behavior crucially depends on age (e.g. Evans ’87)
  - Young firms grow faster, more likely to exit (even conditional on size)

Large part of young firm growth due to demand (vs productivity)
  - Haltiwanger et al ’09: evidence from homogenous good industries
  - EEKT, Albornoz et al ’09: large growth for new exporters into individual destinations
What about idiosyncratic productivity shocks?

- Stochastic productivity essential modeling component
  - Arkolakis ’08 extending Luttmer ’07, Hopenhayn ’92
    - Explain US cohort turnover & growth
    - Explains dependence of growth and turnover on firm size

- *Cannot* explain dependence of firm growth & turnover on *age*
  - Reason: one state Markov structure
This Paper: Quantitative Framework of Firm Demand-Learning

- Revisit findings of Evans ’87 using Colombian plant data

- Develop a benchmark framework of firm learning and productivity
  - Learning generates age dependent turnover & growth (Jovanovic ’82)
  - Approach related to Ruhl & Willis ’09. Also to EEKKT ’09
    - Simpler framework, going further in characterizing model implications
Agenda: is learning the missing link?

- Learning has a number of advantages vs e.g. financial constraints
  - Tractability
  - Results largely independent of productivity shock structure (Cooley & Quadrini ’01)
  - Demand explanation: useful to model growth in individual markets

- Develop a benchmark framework of firm learning & productivity
  - SR: Estimate importance of firm learning vs productivity
  - MR: Perform counterfactual policy experiments
  - LR: Understand how learning affects trade
The data
Data

- Colombian data (DANE survey)
  - Dataset covers all plants with 10+ employees

- Look real production 83-91, treat each plant-year as an observation
  - Yearly turnover and growth
Evidence from Colombian Data

Hazard Rates by Age

Age Interval

0-2  3-4  5-8  9-16  17-32  >32
Evidence from Colombian Data

Hazard Rates by Size

Size Percentile
Evidence from Colombian Data

Hazard Rates by Size Percentile

- 0-10 perc
- 10-20
- 90-100th

Age Interval:
- 0-2
- 3-4
- 5-8
- 9-16
- 17-32
- >32
Evidence from Colombian Data

Log Growth Rates by Age

Age Interval

0-2  3-4  5-8  9-16  17-32  >32
Evidence from Colombian Data

Log Growth Rates by Size

Size Percentile

10th 20th 30th 40th 50th 60th 70th 80th 90th 100th
Evidence from Colombia Data

Log Growth Rates by Size Percentile

Age Interval

0-2  3-4  5-8  9-16  17-32  >32

10-20 perc

0-10 perc

90-100th perc
The model
Consumer Preferences

- Unit mass of consumers with preferences over a composite good, $C_t$:

$$E_t \left( \sum_{t=0}^{+\infty} \beta C_t^\gamma \, dt \right)^{\frac{\gamma}{\gamma-1}}$$

where

$$(C_t)^\rho = \int_{\omega \in \Omega} \left[ e^{a_t(\omega)} \right]^{1-\rho} q_t(\omega)^\rho \, d\omega$$

- $e^{a_t(\omega)}$: good $\omega$ idiosyncratic demand component
- $q_t(\omega)$: quantity consumed from good $\omega$
Consumer Demand

- Modeling of representative consumer is parsimonious

- Implies demand for good $\omega$

$$q_t(\omega) = e^{a_t(\omega)} \frac{p_t(\omega)^{-\sigma}}{P_t^{1-\sigma}}$$

where $w_t$ is worker wage, $P_t$ is the CES price index, $\sigma = \frac{1}{1-\rho} > 1$ is the elasticity of substitution.

- Each firm is a monopolist of one good. Takes demand as given
Information Frictions

- The demand realization for the good of a firm $\omega$ is given by:

$$a_t(\omega) = \theta(\omega) + \epsilon_t(\omega), \quad \epsilon_t(\omega) \sim N(0, \sigma^2) \text{ i.i.d}$$

- Permanent demand realization $\theta(\omega)$ unobserved by the firm
  - Drawn from normal with known mean & variance.
  - Firm observes $a_t(\omega)$, updates beliefs for $\theta(\omega)$ in Bayesian fashion.
Firm Production and Equilibrium Conditions

- Firms use a CRS production function, productivity $z$

- We assume free entry condition to close the model.
  - Firms enter with a productivity drawn from $g_e(z)$

- Labor market clears
Timing of Firm Actions

- **Timing**

<table>
<thead>
<tr>
<th>Period $t$ begins.</th>
<th>Firms die with prob. $\delta$, new productivity is realized</th>
<th>Firm makes quantity decisions, Pays fixed cost</th>
<th>Demand uncertainty is realized, production takes place</th>
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</thead>
<tbody>
<tr>
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<td>Updating of belief takes place. Firm decides whether to produce next period or endogenously exit.</td>
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<td>Period $t+1$ begins. Firms die with prob. $\delta$, new productivity is realized</td>
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- Firm updates beliefs (learns) even if there is very little production
  - Firm optimization wrt to quantities is in fact static
  - But beliefs do affect quantity and entry-exit decisions
Firm Optimization

- Firm chooses quantity, $q_t$ to maximize expected profits:

$$
\pi_t (z, \overline{a}_t, n) = \max \int q_t \left[ p_t q_t - q_t \frac{w_t}{z} \right] g_a (d a_t | \overline{a}_n, n) - w_t f
$$

subject to:

$$
q_t = e^{a_t} \frac{p_t^{1-\sigma}}{p_t^{1-\sigma}}
$$

where $g_a (\cdot | \overline{a}_n, n)$ is the pdf of the firm beliefs at $t$ regarding the realization $a_t$, conditional on having $n$ signals with mean $\overline{a}_n$. 
Characterization of learning

- \((\overline{a_n}, n)\) is a sufficient statistic for firm beliefs at \(t\) regarding \(a_t\).

- Define firm expected demand,
  
  \[ b_t = E_t [e^{a_t}] = \int \left( e^{a_t} \right)^{\frac{1}{\sigma}} g_a \left( da_t | \overline{a_n}, n \right) \]

  - Turns out that also \((b_t, n)\) is a sufficient statistic for firm learning.
  - Firm state is \((z, b_t, n)\).
Characterization of a Stationary Equilibrium
Optimal choice of quantity for a firm \((z, b)\)

\[
q_t(z, b) = \frac{\left(\frac{\sigma}{\sigma - 1} \frac{w}{z}\right)^{-\sigma}}{(P^{\sigma - 1}Lw)^{-1}} (b)^{\sigma}
\]

Market clearing price:

\[
p(z, b) = \frac{\sigma}{\sigma - 1} \frac{w}{z} \frac{(e^a)^{\frac{1}{\sigma}}}{b}
\]
Firm Growth

- **Proposition**: The growth rate of the sales is higher for Young firms \((n < +\infty)\) versus Old firms \((n \to \infty)\) (assuming there is no exit).

- Intuition of the result: Jensen’s inequality
  - Young firms: Chance to be superstar, production expected to increase
  - Old firms: no uncertainty of true \(\theta (\omega)\), production roughly constant
  - Result does not depend on normality of \(\theta (\omega)\)
Firm Growth

- **Proposition**: The growth rate of the sales is higher for Young firms \((n < +\infty)\) versus Old firms \((n \rightarrow \infty)\) (assuming there is no exit).

- Furthermore, proposition is true for any prior distribution of \(\theta(\omega)\).
Firm Entry-Exit

• Each period the firm can either stay in the market or exit.
  • Its value function is given by:
    \[ V(z, b, n) = \pi(z, b) + \beta(1 - \delta) \int \max [V(z, b', n), 0] g_b(db'|b, n) \]
    where \( g_b \) distrib. of next period \( b \).

• Proposition:
  • Value function is unique.
  • Value function is increasing in \( z \) and \( b \).
    • Thus, given \( n, z, \exists b^*(z, n) \) s.th. \( \forall b \geq b^*(z, n) \) firms operate
Numerical Simulations

- A stationary equilibrium exists
  - Belief process is positive recurrent

- Some quantitative preliminary results with homogeneous $z$
  - Model can deliver both age and size dependent growth
    - Consumer Parameters: $\sigma = 6$, $\beta = 0.99$
    - demand shock true mean: $\sigma_\theta = 1$. Noise st.dev: $\sigma_\varepsilon = 0.5$
    - Exogenous death: $\delta = .03$
Model Simulation

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Model Simulation

Log Growth Rates by Size

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Model Simulation

Hazard Rates by Size Percentile

Log Growth Rates by Size Percentile
Summary

- Model of learning and productivity heterogeneity
  - Tractable framework, easy to extend to productivity dynamics

- Tractable framework.
  - Continuous time version would allow more tractability
  - Some positive preliminary results.

- Working on finding better data and on estimation
  - Trade extension (similar to Ruhl & Willis ’09)