Simple Analytics of the Government Expenditure Multiplier*

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Abstract

This paper explains the key factors that determine the effectiveness of government purchases as a means of increasing output and employment in New Keynesian models, through a series of simple examples that can be solved analytically. Delays in the adjustment of prices or wages can allow for larger multipliers than exist in the case of fully flexible prices and wages; in a fairly broad class of simple models, the multiplier is 1 in the case that the monetary authority maintains a constant path for real interest rates despite the increase in government spending. The multiplier can be considerably smaller, however, if the monetary authority raises real interest rates in response to increases in inflation or real activity resulting from the fiscal stimulus. A large multiplier is especially plausible when monetary policy is constrained by the zero lower bound on nominal interest rates; in this case real interest rates fall as a result of the inflationary effect of the stimulus, and a multiplier well in excess of 1 is possible. In such a case, welfare is maximized by expanding government purchases to at least partially fill the output gap that would otherwise exist owing to the central bank's inability to cut interest rates.

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[PRELIMINARY AND INCOMPLETE]

1 A Neoclassical Benchmark

I shall begin by reviewing the argument that government purchases necessarily crowd out private expenditure (at least to some extent), according to a neoclassical general-equilibrium model in which wages and prices are both assumed to be perfectly flexible. This provides a useful benchmark, relative to which I shall wish to discuss the consequences of allowing for wage or price rigidity. I shall confine my analysis here to a relatively special case of the neoclassical model, first analyzed by Barro and King (1984), though the result that the multiplier for government purchases is less than one does not require such special assumptions.[1]

1.1 A Competitive Economy

Consider an economy made up of a large number of identical, infinite-lived households, each of which seeks to maximize

$$\sum_{t=0}^{\infty} \beta^t \, [u(C_t) - v(H_t)], \tag{1.1}$$

where C_t is the quantity consumed in period t of the economy's single produced good, H_t is hours of labor supplied in period t, the period utility functions satisfy $u' > 0, u'' < 0, v' > 0, v'' > 0$, and the discount factor satisfies $0 < \beta < 1$. The good is produced using a production technology yielding output

$$Y_t = f(H_t), \tag{1.2}$$

where $f' > 0, f'' < 0$. This output is consumed either by households or by the government, so that in equilibrium

$$Y_t = C_t + G_t \tag{1.3}$$

each period. I shall begin by considering the perfect foresight equilibrium of a purely deterministic economy; the alternative fiscal policies considered will correspond to alternative deterministic sequences for the path of government purchases $\{G_t\}$. I shall also simplify by assuming that government purchases are financed through lump-sum taxation; a change in the path of government purchases is assumed to imply a change in the path of tax collections so as to maintain intertemporal government solvency.

[1] More general expositions of the neoclassical theory include Barro (1989), Aiyagari *et al.* (1992), and Baxter and King (1993).

(The exact timing of the path of tax collections is irrelevant in the case of lump-sum taxes, in accordance with the standard argument for "Ricardian equivalence.")

One of the requirements for competitive equilibrium in this model is that in any period,

$$\frac{v'(H_t)}{u'(C_t)} = \frac{W_t}{P_t}. \tag{1.4}$$

This is a requirement for optimal labor supply by the representative household, where W_t is the nominal wage in period t, and P_t is the price of the good. (That is, the real wage must equal the marginal rate of substitution between leisure and consumption.) Another requirement is that

$$f'(H_t) = \frac{W_t}{P_t}. \tag{1.5}$$

This is a requirement for profit-maximizing labor demand by the representative firm. (The real wage must also equal the marginal product of labor.) In order for these conditions to simultaneously be true, one must have $v'/u' = f'$ at each point in time. Using (1.2) to substitute for H_t and (1.3) to substitute for C_t in this relation, one obtains an equilibrium condition

$$u'(Y_t - G_t) = \tilde{v}'(Y_t) \tag{1.6}$$

in which Y_t is the only endogenous variable. Here $\tilde{v}(Y) \equiv v(f^{-1}(Y))$ is the disutility to the representative household of supplying a quantity of output Y, so that $\tilde{v}' = v'/f'$. (Note that our previous assumptions imply that $\tilde{v}' > 0, \tilde{v}'' > 0$.) This is also obviously the first-order condition for the planning problem of choosing Y_t maximize utility, given preferences, technology, and the level of government purchases; thus this equilibrium condition reflects the familiar result that competitive equilibrium maximizes the welfare of the representative household (in the case that there *is* a representative household).

Condition (1.6) can be solved for equilibrium output Y_t as a function of G_t. Note that (holding fixed both intra-temporal preferences and technology) the equilibrium level of output depends only on the current level of government purchases, so that the "multiplier" is the same regardless of whether an increase in government purchases is expected to be transitory or persistent.[2] (It is also the same regardless of whether

[2]This strong result in our simple model would not survive the introduction of endogenous capital accumulation, foreign asset accumulation, or preferences that are not time-separable.

the increased government purchases are financed by an immediate tax increase or by borrowing, owing to the "Ricardian equivalence" principle already mentioned.) Differentiation of the function implicitly defined by (1.6) yields a formula for the multiplier,

$$\frac{dY}{dG} = \frac{\eta_u}{\eta_u + \eta_v}, \tag{1.7}$$

where $\eta_u > 0$ is the negative of the elasticity of u' and $\eta_v > 0$ is the elasticity of $\tilde{v}'$ with respect to increases in Y. It follows that the multiplier is positive, but necessarily less than 1. This means that private expenditure (here, entirely modeled as non-durable consumer expenditure) is necessarily crowded out, at least partially, by government purchases. In the case that the degree of intertemporal substitutability of private expenditure is high (so that η_u is small), while the marginal cost of employing additional resources in production is sharply rising (that η_v is large), the multiplier may be only a small fraction of 1.[3]

1.2 Monopolistic Competition

The mere existence of some degree of market power in either product or labor markets does not much change this result. Suppose, for example, that instead of a single good there are a large number of differentiated goods, each with a single monopoly producer; and, as in the familiar Dixit-Stiglitz model of monopolistic competition, let us suppose that the representative household's preferences are again of the form (1.1), but that C_t is now a constant-elasticity-of-substitution aggregate of the household's purchases of each of the differentiated goods,

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \tag{1.8}$$

where $c_t(i)$ is the quantity purchased of good i, and $\theta > 1$ is the elasticity of substitution among differentiated goods. Let us suppose for simplicity that each good

[3]For example, the modal parameter estimates reported by Eggertsson (2009) imply that the elasticity of u' is -1.16, while the elasticity of v' is 1.57; these parameters would imply a multiplier of only a little over 0.4 in the case of flexible wages and prices. (Since Eggertsson's estimated model does not imply that prices are flexible, these values are perhaps not appropriate for an estimate of what an empirical flexible-price model would imply. I cite this result only for comparison with other numerical results reported below, using the same parameter values.)

is produced using a common production function of the form (1.2), with a single homogeneous labor input used in producing all goods. In this model, each producer will face a downward-sloping demand curve for its product, with elasticity θ; profit maximization will then require not production to the point where marginal cost is equal to the price for which it sells its good, but only to the point at which the price of good i is equal to μ times marginal cost, where the desired markup factor is given by

$$\mu \equiv \frac{\theta}{\theta - 1} > 1. \tag{1.9}$$

Hence condition (1.5) must be replaced by the requirement that $p_t(i) = \mu W_t / f'(h_t(i))$ for each good i.

Let us consider a monopolistically competitive equilibrium, in which each firm chooses its price optimally, taking as given the wage and the demand curve that it faces. (I continue to assume perfectly flexible prices, and a competitive labor market, or some other form of efficient labor contracting.) Since each firm faces the same wage and a demand curve of the same form, in equilibrium each firm chooses the same price, hires the same amount of labor, and produces the same quantity. It follows that we must also have

$$P_t = \mu W_t / f'(H_t), \tag{1.10}$$

where P_t is the common price of all goods (and also the price of the composite good) and H_t is the common quantity of labor hired by each firm (and also the aggregate hours worked). It also follows that aggregate output Y_t (in units of the composite good) and aggregate hours worked H_t must again satisfy (1.2). Optimal labor supply by the representative household also continues to require that (1.4) hold, where P_t is now the price of the composite good. Relations (1.2), (1.4) and (1.10) allow us to derive a simple generalization of equation (1.6),

$$u'(Y_t - G_t) = \mu \tilde{v}'(Y_t) \tag{1.11}$$

which again suffices to determine equilibrium output as a function of the current level of government purchases. While the equilibrium level of output is no longer efficient, the multiplier is still given by (1.7), regardless of the value of μ. A similar conclusion is obtained in the case of a constant markup of wages relative to households' marginal rate of substitution: aggregate output is again determined by (1.11), where μ is now an “efficiency wedge” that depends on the degree of market power in both product and labor markets, and so the multiplier calculation remains the same.[4]

A different result can be obtained, however, if the size of the efficiency wedge is endogenous. One of the most obvious sources of such endogeneity is delay in the adjustment of wages or prices to changing market conditions.[5] If prices are not immediately adjusted in full proportion to the increase in marginal cost resulting from an increase in government purchases, the right-hand side of (1.10) will increase more than does the left-hand side; as a consequence the right-hand side of (1.11) will increase more than does the left-hand side of that expression. This implies an increase in Y_t greater than the one implied by (1.11). One can similarly show that if wages are not immediately adjusted in full proportion to the increase in the marginal rate of substitution between leisure and consumption, the right-hand side of (1.11) will increase more than does the left-hand side, again implying a larger multiplier than the one given in (1.7).

As Hall (2009) emphasizes, then, the key to obtaining a larger multiplier is an endogenous decline in the markup (or more generally, the labor-efficiency wedge). However, in a model with sticky prices or wages, the degree to which the efficiency wedge changes depends on the degree to which aggregate demand differs from what it was expected to be when prices and wages were set. Equilibrium output is thus no longer determined solely by supply-side considerations; we must instead consider the effects of government purchases on aggregate demand.

2 A New Keynesian Benchmark

What is the size of the government expenditure multiplier if prices or wages are sticky — as many empirical DSGE models posit, in order to account for the observed effects of monetary policy on real activity? The answer does not depend solely on

[4]The same result is also obtained in the case of a constant rate of taxation or subsidization of labor income, firms’ payrolls, consumption spending, or firms’ revenues. The tax distortions simply change the size of the efficiency wedge μ in equation (1.11).

[5]Another possible source of endogeneity is cyclical variation in desired markups due to implicit collusion, as in the model of Rotemberg and Woodford (1992). In that model, a temporary increase in government purchases reduces the ability of oligopolistic producers to maintain collusion; the resulting decline in markups increases equilibrium output more than would occur in a perfectly competitive model.

the assumed structure of the economy. If prices or wages are sticky, monetary policy affects real activity, and so the consequences of an increase in government purchases depend on the monetary policy response. One might suppose that the question of interest should be the effects of government purchases "leaving monetary policy unchanged"; but one must take care to specify just what is assumed to be unchanged. It is not the same thing to assume that the path of the money supply is unchanged as to assume that the path of interest rates is unchanged, or that the central bank's inflation target is unchanged, or that the central bank continues to adhere to a "Taylor rule," to list only a few of the possibilities.

Here I shall consider, as a useful benchmark, a policy experiment in which it is assumed that the central bank *maintains an unchanged path for the real interest rate*, regardless of the path of government purchases. This case corresponds, essentially to the standard "multiplier" calculation in undergraduate textbooks, where the question asked is how much the "IS curve" shifts to the right — that is, how much output would be increased if the real interest rate were not to change. This is considered a useful first step, even if one recognizes that under realistic assumptions about monetary policy, the real interest rate may well change. Here I wish to consider a similar question; but in a dynamic model, it is necessary to define the hypothetical policy in terms of the entire forward *path* of the real interest rate. The answer to this question provides a useful benchmark for two reasons. The first is that it is simple to calculate; but the second is that the answer is the same under a wide range of alternative assumptions about the nature of price or wage stickiness.

2.1 The Constant-Real-Rate Multiplier

Again I consider a purely deterministic economy, and let the path of government purchases be given by a sequence $\{G_t\}$ such that $G_t \rightarrow \bar{G}$ for large t; the long-run level of government purchases $\bar{G}$ is held constant while considering alternative possible assumptions about near-term government purchases. Thus I shall consider only the consequences of *temporary* variations in the level of government purchases. I shall furthermore assume that monetary policy brings about a zero rate of inflation in the long run. (That is, the inflation rate $\{\pi_t\}$ is also a deterministic sequence, such that $\pi_t \rightarrow 0$ for large t.) Under quite weak assumptions about the nature of wage and price adjustment, these assumptions about monetary and fiscal policy in the long run imply that the economy converges asymptotically to a steady state in which government purchases equal $\bar{G}$ each period, inflation is equal to zero, and output is equal to some constant level $\bar{Y}$.[6]

Given preferences (1.1), optimization by households requires that in equilibrium,

$$\frac{u'(C_t)}{\beta u'(C_{t+1})} = 1 + r_t \tag{2.1}$$

each period, where r_t is the one-period real rate of return between t and $t+1$. It follows from (2.1) that in the long-run steady state, $r_t = \bar{r} \equiv \beta^{-1} - 1 > 0$ each period. Since I wish to consider a monetary policy that maintains a constant real rate of interest, regardless of the temporary variation in government purchases, it is necessary to assume that monetary policy maintains $r_t = \bar{r}$ for all t; this is the only constant real interest rate consistent with the assumption of asymptotic convergence to a long-run steady state. We may suppose that the central bank chooses an operating target for the nominal interest rate i_t according to a Taylor rule of the form

$$i_t = \bar{\imath}_t + \phi_\pi \pi_t + \phi_y \log(Y_t/\bar{Y}) \tag{2.2}$$

where the response coefficients ϕ_π, ϕ_y are chosen so as to imply a determinate equilibrium under this policy,[7] and where the sequence $\{\bar{\imath}_t\}$ is chosen so that $\bar{\imath}_t \to \bar{r}$ for large t (the requirement for asymptotic convergence to the zero-inflation steady state) and so that the equilibrium determined by this monetary policy involves $r_t = \bar{r}$ each period. However, there is no need to assume that the equilibrium is implemented in this way; one might alternatively assume, for example, that the central bank chooses a path for the money supply that is consistent with zero inflation in the long run and a constant real interest rate.[8] All that matters for the analysis here is that a

[6]Under many reasonable assumptions about wage and price adjustment, the steady-state level of output $\bar{Y}$ will be the same as in the model with flexible wages and prices, namely, the solution to (1.11) when $G_t = \bar{G}$.

[7]For example, in the case of flexible wages and the Calvo model of staggered price adjustment, discussed further below, a policy rule of the form (2.2) implies a determinate (locally unique) rational-expectations equilibrium as long as the coefficients satisfy $\phi_\pi, \phi_y \geq 0, \phi_\pi + (1 - \beta/\kappa)\phi_y > 1$. (See Woodford, 2003, Proposition 4.3.) In general, the precise conditions for determinacy of equilibrium will depend on the details of wage and price adjustment.

[8]In order to determine the required path for the money supply in this case, the model must be extended to include an equation for the demand for money. This can be done in a way that has no consequences for the equilibrium relations used in the discussion below, as discussed in Woodford (2003, chapter 4).

monetary policy can be specified that implements the equilibrium in the real interest rate is constant.

Let us set aside for the moment the question whether such an equilibrium exists (and what sort of monetary policy implements it), and consider what such an equilibrium must be like if it exists. If $r_t = \bar{r}$ for all t, it follows from (2.1) that $C_t = C_{t+1}$ for all t. Thus the representative household must be planning a constant level of consumption over the indefinite future, at whatever level is consistent with its intertemporal budget constraint. Convergence to the steady state referred to above implies that $C_t \to \bar{C} \equiv \bar{Y} - \bar{G}$ for large t; hence equilibrium must involve $C_t = \bar{C}$ for all t.[9] It then follows from (1.3) that

$$Y_t = \bar{C} + G_t \tag{2.3}$$

for all t. Hence in this case, we find once again that equilibrium output depends only on the level of government purchases in the current period — so that the effects of a given size increase in government purchases are the same regardless of how persistent the increase is expected to be[10] — but now the multiplier (dY_t/dG_t) is equal to 1. There is no crowding out of private expenditure by government purchases, though no stimulus of additional private expenditure, either.

An interesting feature of this simple result is that it is quite independent of any very specific assumption about the dynamics of wage and price adjustment: under the particular assumption about monetary policy made here, the effect on aggregate output depends purely on the demand side of the model. The supply side of the model matters only in solving for the implied path of inflation, wages and employment, and for the monetary policy required to achieve the hypothesized path of real interest rates. I have, however, made one crucial assumption about the supply side: I have supposed that it is *possible* for monetary policy to maintain $r_t = \bar{r}$ at all times, regardless of the chosen short-run path of government purchases. This assumption is

[9]This is the point at which it matters to the argument that I consider only paths for government purchases such that $G_t \to \bar{G}$. In the case of a change in the long-run level of government purchases, the long-run steady-state value $\bar{C}$ would also change. But the value of $\bar{C}$ depends only on $\bar{G}$, and not on the level of near-term government purchases. This conclusion also depends on the assumption of lump-sum taxation; with distorting taxes, $\bar{C}$ would only be invariant under the assumption that all contemplated fiscal policies imply the same long-run level of real public debt.

[10]This statement is subject to the proviso, of course, that the long-run level of government purchases, $\bar{G}$, is not changed.

violated by the model with fully flexible wages and prices.[11] However, under many specifications of sticky prices or wages (or both), it is possible for monetary policy to affect real interest rates, and a path for monetary policy can be chosen under which $r_t = \bar{r}$ will hold, in the case of any path for government purchases satisfying certain bounds.

2.2 Constant-Real-Rate Monetary Policy: An Example

Here, for the sake of concreteness, I shall discuss one particular example of a sticky-price model, though it should be obvious that the precise assumptions made here are stronger than are necessary in order for a monetary policy consistent with a constant real interest rate to exist. Let us assume Dixit-Stiglitz monopolistic competition, as discussed in section 1, but now let us suppose that each differentiated good i is produced using a constant-returns-to-scale technology of the form

$$y_t(i) = k_t(i) f(h_t(i)/k_t(i)), \tag{2.4}$$

where $k_t(i)$ is the quantity of capital goods used in production by firm i, $h_t(i)$ are the hours of labor hired by the firm, and $f(\cdot)$ is the same increasing, concave function as before. I shall assume for simplicity that the total supply of capital goods is exogenously given (and can be normalized to equal 1), but that capital goods are allocated to firms each period through a competitive rental market. This assumption implies that each firm will have a common marginal cost of production, a homogeneous degree 1 function of the two competitive factor prices, that is independent of the firm's chosen scale of production. Cost-minimization will imply that each firm chooses the same labor/capital ratio, regardless of its scale of production, and in equilibrium this common labor/capital ratio will equal H_t, the aggregate labor supply (recalling that aggregate capital is equal to 1). Hence the common marginal cost of production S_t in any period will equal

$$S_t = W_t/f'(H_t). \tag{2.5}$$

[11] In that case, the equilibrium path of output is determined by (1.11), regardless of monetary policy; and substitution of the implied path for consumption into (2.1) determines the equilibrium path of real interest rates, again regardless of monetary policy. Hence monetary policy can have no effect on real interest rates in that model — the "classic dichotomy" is valid.

If we assume flexible wages and a competitive labor market, (1.4) must again hold in equilibrium; substituting this for W_t in (2.5) yields

$$S_t = P_t \frac{\tilde{v}'(f(H_t))}{u'(Y_t - G_t)}. \tag{2.6}$$

Note that in the case that each firm's price is a fixed markup μ over marginal cost (as would follow from Dixit-Stiglitz monopolistic competition with flexible prices), condition (2.6) together with (1.2) would imply that output must satisfy (1.11), as concluded in the previous section.[12]

In the Calvo model of staggered price adjustment, it is assumed that fraction $1-\alpha$ of all firms reconsider their prices in any given period, while the others continue to charge the same price as in the previous period. (The probability that any firm will reconsider its price in any period is assumed to be independent of the time since it last reconsidered its price, and of how high or low its current price may be.) To a log-linear approximation,[13] the optimal price p_t^* chosen by each firm that reconsiders its price in period t will be given by[14]

$$\log p_t^* = \log \mu + \sum_{j=0}^{\infty} (1-\alpha\beta)\alpha^j \beta^j \, E_t[\log S_{t+j}]. \tag{2.7}$$

(This is just a weighted weighted geometric average of the prices $p_{t+j}^f = \mu S_{t+j}$ that a profit-maximizing flexible-price firm would choose in each of the future periods $t+j$.)

Since in each period, a fraction $(1-\alpha)\alpha^j$ of all firm chose their current price j periods earlier (for each $j \geq 0$), in a similar log-linear approximation the price index satisfies

$$\log P_t = \sum_{j=0}^{\infty} (1-\alpha)\alpha^j \, \log p_{t-j}^*,$$

[12]The derivation is more subtle here, because (2.6) has been derived without assuming that the prices of different goods are necessarily the same, as they are generally not the same in the case of staggered price adjustment.

[13]Here I log-linearize around the zero-inflation steady state, which under the assumed monetary policy is the equilibrium in the case that government purchases equal $\bar{G}$ each period; hence the approximation is valid if in all periods G_t remains close enough to $\bar{G}$. Further details of the calculation sketched here are presented in Woodford (2003, chap. 3).

[14]Here I write the condition in the more general form that applies in the case of a stochastic environment, as preparation for further applications below.

which implies that

$$\log P_t = \alpha \log P_{t-1} + (1-\alpha) \log p_t^*. \tag{2.8}$$

Condition (2.8) together with (2.7) allows one to show that

$$\log(p_t^*/P_t) = (1-\alpha\beta) \sum_{j=0}^{\infty} \beta^j \; E_t[\log \mu + \log S_{t+j} - \log P_{t+j}]. \tag{2.9}$$

Thus a firm that reconsiders its price will choose a high relative price to the extent that a weighted geometric average of the profit-maximizing relative prices $\mu S_{t+j}/P_{t+j}$ in the various future periods $t+j$ is high. In the case of fully flexible prices, P_t must equal p_t^* each period, in which case (2.9) requires that $P_t = \mu S_t$ each period, leading again to (1.11). But with sticky prices, it is possible for P_t to differ from μS_t (and hence for Y_t to violate equation (1.11)); this simply requires that firms that reconsider their prices choose a price different from the general level of prices ($p_t^* \neq P_t$), resulting in inflation or deflation ($P_t \neq P_{t-1}$) in accordance with (2.8).

A similar log-linear approximation to (2.6) takes the form[15]

$$\log(S_t/P_t) = -\log \mu + \eta_v \hat{Y}_t + \eta_u(\hat{Y}_t - \hat{G}_t), \tag{2.10}$$

where the elasticities $\eta_v, \eta_u > 0$ are defined as in (1.7), and the deviations from steady state are defined as $\hat{Y}_t \equiv \log(Y_t/\bar{Y}), \hat{G}_t \equiv (G_t - \bar{G})/\bar{Y}$.[16] Hence an increase in $\hat{Y}_t$ greater than the one implied by the flexible-price multiplier (1.7) requires that real marginal cost S_t/P_t increases. Substituting this into (2.9), we obtain

$$\log(p_t^*/P_t) = (1-\alpha\beta)(\eta_u + \eta_v) \sum_{j=0}^{\infty} \beta^j \; E_t[\hat{Y}_{t+j} - \Gamma \hat{G}_{t+j}], \tag{2.11}$$

where $\Gamma < 1$ is the flexible-price multiplier defined in (1.7). Then since (2.8) implies that the inflation rate is given by

$$\pi_t \equiv \log(P_t/P_{t-1}) = \frac{1-\alpha}{\alpha} \; \log(p_t^*/P_t), \tag{2.12}$$

[15]Note that because the steady state around which the approximation is computed involves the same level of production of each good, log-linearization of (2.4) and integration over i implies that, to this order of approximation, the aggregate quantities Y_t and H_t satisfy (1.2). This allows an expression to be derived for real marginal cost as a function of $\hat{Y}_t$ and $\hat{G}_t$ only.

[16]The latter definition is chosen so that $\hat{G}_t$ is defined even if $\bar{G} = 0$, and so that $\hat{G}_t$ and $\hat{Y}_t$ are in comparable units (i.e., percentages of steady-state output).

we obtain

$$\pi_t = \kappa \sum_{j=0}^{\infty} \beta^j \, E_t[\hat{Y}_{t+j} - \Gamma \hat{G}_{t+j}], \tag{2.13}$$

where $\kappa \equiv (1-\alpha)(1-\alpha\beta)(\eta_u + \eta_v)/\alpha > 0$.

We can now answer the question whether it is possible for monetary policy to maintain a constant real interest rate in the case of an arbitrary path $\{G_t\}$ for government purchases, at least in the case that G_t remains always close enough to $\bar{G}$ for the log-linear approximation to be accurate. For an arbitrary path $\{G_t\}$, the solution for the path of output $\{Y_t\}$ is given by (2.3). Substituting this into (2.13), one obtains a solution for the path of the inflation rate as well.[17] It is then straightforward to solve for the equilibrium path of the nominal interest rate, and for the path $\{\bar{\imath}_t\}$ of intercepts for the central-bank reaction function (2.2). One thus obtains a policy that implements the conjectured equilibrium.

It should be obvious that this last construction does not depend on the precise equations of the Calvo model of price adjustment. One might assume, for example, that the probability of a firm's reconsidering its price depends on the time since the price was adopted; Sheedy (2007) shows how a generalization of the price-adjustment dynamics presented above can be derived within a very flexible family of specifications of this kind. Again one can solve for the implied path of the inflation rate in the case of an arbitrary bounded perturbation of the path $\{Y_t\}$, so again a monetary policy exists that maintains a constant real interest rate in the case of an arbitrary bounded perturbation $\{G_t\}$. Similar calculations are possible if the assumption of a market-clearing wage is replaced by staggered wage adjustment, or by stickiness of both wages and prices, as in the model of Erceg *et al.* (2000). Alternatively, one can also derive inflation dynamics consistent with a given bounded perturbation $\{Y_t\}$ when neither wages nor prices are sticky, but prices and/or wages fail to adjust to current market conditions owing to stickiness of information, in the sense of Mankiw and Reis (2002). In any of these cases, output higher than is consistent with (1.11) is possible because some prices or wages fail to adjust to current market conditions, either because price or wage commitments were made in the past, or because price or wage offers are based on old information; it is simply necessary to solve for the degree of unanticipated price or wage increases that are required for a given degree of departure from the full-information flexible wage and price outcome. Thus in any of

[17]Note that for any bounded sequence $\{\hat{G}_t\}$, the infinite sum is well-defined.

these cases, there exists a feasible monetary policy for which the effects of government purchases are given by (2.3).

It may seem surprising that the multiplier in this baseline case is independent of the degree of flexibility of prices and wages; there thus appears to be a discontinuity in the case of *complete* flexibility (and *full* information), where the multiplier is given by (1.7). The explanation is that the derivation of (2.3) requires that it be possible for monetary policy to maintain a constant real interest rate despite an increase in government purchases; this is possible (under weak assumptions) in the case of any degree of price stickiness, but *not* when prices and wages are fully flexible. In fact, while such a policy is technically possible, according to the log-linear approximation, for any positive degree of price stickiness, as the degree of price stickiness becomes small, the required degree of inflation becomes extreme. (For example, in the case of the Calvo model, (2.12) indicates that for any given desired relative price p_t^*/P_t different from 1, the required rate of inflation or deflation becomes unboundedly large as α approaches zero.) This means that we cannot rely on the log-linear approximation to answer this particular question if the degree of price stickiness is too small;[18] but more to the point, it becomes implausible to believe that a central bank will actually maintain a constant real interest rate (even if this is feasible) if this requires extreme inflation. For this reason, the relevance of the New Keynesian benchmark does depend on the existence of a sufficient degree of stickiness of prices, wages, information (or more than one of these).

It is also noteworthy that in this benchmark case, the predicted multiplier is independent of the degree to which resource utilization is slack; in the derivation of (2.3), the costs of supplying a given level of output do not figure at all. However, supply costs do matter for the rate of inflation associated with a given size of government purchases under the assumed monetary policy; more steeply increasing marginal cost corresponds to a larger value of the factor $(\eta_u + \eta_v)$ in (2.11), which increases the elasticity of the inflation rate with respect to increases in G_t. Again, this means that it is much more plausible to imagine a central bank holding real interest rates constant in response to an increase in government purchases when there is a great deal of excess capacity (so that marginal cost increases little with increased output) than when capacity utilization is high (so that marginal cost is steeply increasing);

[18] One may doubt the continued validity of other aspects of the Calvo model, or other similar models of price adjustment, under circumstances of extreme inflation as well.

and if capacity constraints are severe enough, it may actually be infeasible to maintain a constant real interest rate under any monetary policy, because no amount of monetary stimulus can induce the increase in supply required in order for the current goods not to be expensive relative to future goods (or indexed bonds).

2.3 Extensions

While the benchmark result of a multiplier equal to 1 obtains under fairly general circumstances, it is possible under alternative assumptions about the policy experiment to obtain multipliers even larger than 1. Rather than assuming that a temporary increase in government purchases implies no change in the long-run level $\bar{G}$, one might alternatively assume that the temporary increase is offset by a *decline* in the long-run level of government purchases. For example, as proposed by Corsetti *et al.* (2009), one might suppose that the increased government purchases are at least partly financed by increased government borrowing, but that subsequently, a permanently higher level of public debt provides a reason for permanently lower government purchases than would otherwise have been affordable. In such a case (and under the assumption about monetary policy made above), the short-run increase in output in equilibrium will be equal to the short-run increase in government purchases G_t, plus Γ times the decrease in the long-run level of government purchases $\bar{G}$.[19] Hence in this case, the short-run increase in output would be greater than the short-run increase in government purchases: the observed "multiplier" would be greater than 1.[20] In addition, as Corsetti *et al.* note, such a model can explain the result of some VAR studies, according to which increases in government purchases increase consumer expenditure; and an open-economy extension of the model can explain the result of VAR studies for a number of countries, according to which increases in government purchases result in depreciation of a country's real exchange rate.

[19] The effect of the long-run level of government purchases on the level of output in the long-run steady state is the same as in the flexible-price model of section 1, in any model (such as the Calvo model of price adjustment) where the steady state with zero inflation is equivalent to the steady state of the flexible-price model. The reason for this equivalence in the case of the Calvo model is discussed in the next section.

[20] Technically, this is not a case in which there exists a purely contemporaneous "multiplier" relationship between government purchases and aggregate output, since output at a given point in time does not depend solely on the level of government purchases at that time.

The sharp result of a multiplier exactly equal to 1 in the benchmark analysis also depends on abstracting from endogenous capital accumulation; all private expenditure is treated as if it were non-durable consumer expenditure. If instead we allow for the production of new capital goods (but continue to assume a competitive rental market for the services of such goods), the desired level of capital in any period (which would be the equilibrium value, under perfect foresight and in the absence of adjustment costs) will be the value K_t^* that equates the rental rate for capital services with the user cost of capital. In the case of a Cobb-Douglas production function, cost-minimization by firms implies that the real rental rate must equal

$$\rho_t = \frac{1-\gamma}{\gamma} \frac{W_t H_t}{P_t K_t}, \tag{2.14}$$

where $0 < \gamma < 1$ is the elasticity of the function f. Under the hypothesis of a monetary policy that maintains a constant real interest rate, the real user cost will be unaffected by a change in the path of government purchases, so the desired capital stock K_t^* increases in proportion to the increase in $W_t H_t / P_t$. Since H_t must increase (even in the absence of any increase in investment demand) as a result of an increase in G_t, and W_t/P_t will increase as well in the case of flexible wages, an increase in government purchases will increase the desired capital stock. If the increase in government purchases is not purely transitory (so that at the time of the increase, government purchases are expected to remain high for some time), the increase in the desired capital stock anticipated for future periods will increase investment demand (with the precise dynamics of the adjustment depending on the magnitude of adjustment costs). Since consumption spending remains constant (as argued above), total private expenditure increases in this case, and the total (short-run) increase in output will be greater than the increase in government purchases.

Hence stickiness of prices and/or wages, under the hypothesis of an accommodative monetary policy, suffices to explain the existence of multiplier effects of government purchases of the magnitude generally found in the empirical literature. For example, Hall (2009) reviews the evidence from atheoretical regression models of various types and using data from various periods; he shows that such studies generally obtain a multiplier of 0.5 or higher, and concludes that "GDP rises by roughly the amount of an increase in government purchases" under normal circumstances,[21]

[21] Hall adds the qualification that the multiplier may be substantially larger "when monetary policy is passive because of the zero bound." This special case is discussed below in section 4.

which is to say that the multiplier is roughly 1. While this is too large an effect to be consistent with neoclassical theory, at least in standard models, it is easily consistent with a simple New Keynesian model.

3 Alternative Degrees of Monetary Accommodation

The result obtained in the previous section applies only under one specific assumption about monetary policy, namely, that the path of the real interest rate will remain fixed despite the temporary increase in government purchases. Under alternative assumptions about the degree of monetary accommodation of the fiscal stimulus, the size of the increase in output will be different. Thus while the result under the baseline analysis establishes that it is *possible* in a New Keynesian model for the multiplier to be 1 or larger, there is no necessity that this be the case; indeed, under some assumptions about monetary policy, the output response predicted by the New Keynesian model may be even smaller than in the neoclassical model. Hence an empirical finding of a multiplier less than 1, under the monetary policy that has been followed historically, does not necessarily disconfirm the validity of the New Keynesian model.

3.1 A Strict Inflation Target

As an example of another simple hypothesis about monetary policy, suppose that the central bank maintains a strict inflation target, regardless of the path of government purchases. (For conformity with the assumption made above about the long-run steady state, suppose that the inflation target is zero.) In the case of the Calvo model of price adjustment, (2.12) implies that maintaining a zero inflation rate each period requires that $p_t^* = P_t$ each period. It then follows from (2.9) that this requires that $\mu S_t = P_t$ each period.[22] If we assume flexible wages (or efficient labor contracting), (2.6) implies that this will hold if and only if Y_t satisfies (1.11) each period. Hence under this policy, aggregate output Y_t will be the same function of G_t as in the case of flexible prices, and the multiplier will be given by (1.7).

[22] One can show that this is true in the exact model, and not merely in the log-linear approximation used in (2.9).

Again, this result does not depend on the precise details of the Calvo model of price adjustment. In a wide range of specifications with sticky prices (or prices set on the basis of sticky information), a sufficient (and often necessary) condition for zero inflation each period is maintenance of aggregate conditions under which the marginal cost of production satisfies $S_t = P_{t-1}/\mu$ each period. For if this condition holds, then under the assumption that each firm that reconsiders its price at any date chooses $p_t^* = P_{t-1}$, not only will all prices remain constant over time, but each firm will find that marginal revenue equals marginal cost each period, so that no firm would expect to increase profits by deviating from this pricing strategy. But such a policy thus assures that each firm's price is equal to μS_t each period, so that the equilibrium is the same as if all prices were fully flexible and set on the basis of full information. Hence the multiplier will be given by (1.7), just as in the neoclassical model.

3.2 Monetary Accommodation under a Taylor Rule

A less extreme hypothesis would assume that policy is not tightened so much in response to a fiscal expansion as to prevent any increase in prices, but that real interest rates do rise in response to any increase in prices that occurs, rather than being held constant regardless of the consequences for inflation. For example, suppose that interest rates are set in accordance with a "Taylor rule" of the form

$$i_t = \bar{r} + \phi_\pi \pi_t + \phi_y(\hat{Y}_t - \Gamma \hat{G}_t), \tag{3.1}$$

where i_t is a short-term riskless nominal rate (the central bank's policy instrument), $\bar{r}$ is the value of this rate in a steady state with zero inflation (so that the policy rule is consistent with that steady state), and the response coefficients satisfy $\phi_\pi > 1, \phi_y > 0$, as proposed by Taylor (1993). Here $\hat{Y}_t - \Gamma \hat{G}_t$ corresponds to one interpretation of the "output gap," namely, the number of percentage points by which aggregate output exceeds the flexible-price equilibrium level.[23]

[23] Here I abstract from variations in other factors that would also cause variations in the flexible-price equilibrium level of output, such as variations in productivity. The degree to which the output-gap measure used by the central bank does or does not take into account variations in other exogenous factors of that kind has no effect on the government expenditure multiplier calculated here.

In order to determine the equilibrium implications of a policy rule of this kind, it is useful also to log-linearize equilibrium relation (2.1), yielding[24]

$$\hat{Y}_t - \hat{G}_t = E_t[\hat{Y}_{t+1} - \hat{G}_{t+1}] - \sigma(i_t - E_t\pi_{t+1} - \bar{r}), \tag{3.2}$$

where $\sigma \equiv \eta_u^{-1} > 0$ measures the intertemporal elasticity of substitution of private expenditure.[25] If we consider deterministic paths for government purchases of the simple form $\hat{G}_t = \hat{G}_0\rho^t$ for some $0 \leq \rho < 1$, then the future path of government purchases looking forward from any date t is a time-invariant function of the level of $\hat{G}_t$ at that date (the sequence $\{\hat{G}_{t+j}\}$ is always exponentially decaying at a rate ρ^j); under the Calvo model of price adjustment (in which inflation determination is purely forward-looking, as explained above), one should then expect the equilibrium values of i_t, π_t, and $\hat{Y}_t$ all to be time-invariant functions of the value of $\hat{G}_t$ at each date. Conjecturing a solution of the form

$$\hat{Y}_t = \gamma_y \hat{G}_t, \tag{3.3}$$

$$\pi_t = \gamma_\pi \hat{G}_t, \tag{3.4}$$

$$i_t = \bar{r} + \gamma_i \hat{G}_t, \tag{3.5}$$

for some coefficients $\gamma_y, \gamma_\pi, \gamma_i$, we can substitute these equations for $\hat{Y}_t, \pi_t$, and i_t in equations (2.13), (3.1) and (3.2), and solve for the values of the coefficients for which all three equilibrium conditions are satisfied each period.

There is easily seen to be a unique solution of this form, in which

$$\gamma_y = \frac{1-\rho+\psi\Gamma}{1-\rho+\psi}, \tag{3.6}$$

where

$$\psi \equiv \sigma\left[\phi_y + \frac{\kappa}{1-\beta\rho}(\phi_\pi - \rho)\right] > 0.$$

It follows from (3.3) that in this case the multiplier is simply the coefficient γ_y. One observes from (3.6) that under this policy, $\Gamma < \gamma_y < 1$. Thus the multiplier

[24] Again I write the log-linear approximation for the more general stochastic form of this equilibrium condition, as this will be used in the next section.

[25] Here i_t is a continuously compounded nominal rate — that is, $i_t \equiv -\log Q_t$, where Q_t is the nominal price of a bond that pays one unit of currency with certainty in period $t+1$ — and $\bar{r} \equiv -\log\beta$ is the corresponding continuously compounded rare of time preference. Note that this differs slightly from the definition of $\bar{r}$ in section 2.

is necessarily higher than in the flexible-price model (or under the strict inflation targeting policy), but smaller than under the constant-real-interest rate policy. It is higher than under strict inflation targeting, because under the Taylor rule, inflation is allowed to rise somewhat in response to fiscal stimulus; but lower than under the constant-real-interest rate policy, because the real interest rate is increased in response to the increases in inflation and in the output gap. In the limiting case of an extremely strong response to variations in either inflation or the output gap (so that ψ becomes very large), the multiplier is again equal to Γ, as such as policy becomes equivalent to a strict inflation target. Note also that for a given policy rule of this form, the size of the multiplier depends on the degree of stickiness of prices (through the dependence of ψ upon the value of κ); the more flexible are prices (*i.e.*, the smaller the value of α), the larger is κ and hence ψ, and the smaller is the multiplier.

A still more realistic assumption about monetary policy might be to assume a Taylor rule of the form (2.2), but with a constant intercept. (I shall assume $\bar{\imath}_t = \bar{r}$, for consistency with the zero-inflation steady state.) In this case, the central bank is assumed to respond to deviations of aggregate output from its average (or trend) level, rather than to departures from the flexible-price equilibrium level. (In fact, most central banks use measures of potential output that do not assume that potential should depend on the level of government purchases, as in the specification (3.1).) In this case, we again obtain a solution of the form (3.3)–(3.5), but with different constant coefficients; the multiplier is now given by

$$\gamma_y = \frac{1 - \rho + (\psi - \sigma\phi_y)\Gamma}{1 - \rho + \psi}. \tag{3.7}$$

The multiplier is necessarily smaller under this kind of Taylor rule, since (for any $\phi_y > 0$) the degree to which monetary policy is tightened in response to expansionary fiscal policy is necessarily greater. In fact, in the case of any large enough value of ϕ_y, the multiplier under this kind of Taylor rule is even smaller than the one predicted by the neoclassical model.[26] In such a case, price stickiness results in even less output increase than would occur with flexible prices, because the central bank's reaction function raises real interest rates more than would occur with flexible prices.

[26] This is true of the parameter values estimated by Eggertsson (2009). For those parameter values, the multiplier in the case of flexible prices would be over 0.4, as noted above; but under a Taylor rule with coefficients of $\phi_\pi = 1.5, \phi_y = 0.25$, and a persistence coefficient $\rho = 0.9$, the multiplier is only a little over 0.3, as discussed by Eggertsson.

(There is also less increase in output than would occur under a strict inflation target, because the Taylor rule implies that inflation is *reduced* to offset the increase in real activity.) Hence while larger multipliers are possible according to a New Keynesian model, they are predicted to occur only in the case of a sufficient degree of monetary accommodation of the increase in real activity; and in general, this will also require the central bank to accommodate an increase in the rate of inflation.

4 Fiscal Stimulus at the Zero Interest-Rate Lower Bound

One case in which it is especially plausible to suppose that the central bank will not tighten policy in response to an increase in government purchases is when monetary policy is constrained by the zero lower bound on the short-term nominal interest rate. In a situation where the central bank would, in the absence of the constraint of the zero lower bound, wish to push its interest-rate target below zero — and instead must settle for a target rate of zero because it has no way of driving the interest rate lower — then it is plausible that even in the event of fiscal stimulus (of a sufficiently modest magnitude), the desired interest rate will remain non-positive, so that the central bank's target will remain at zero. This is a case in which it is plausible to assume not merely that the real interest rate does not rise in response to fiscal stimulus, but that the *nominal* rate does not rise; this will actually be associated with a *decrease* in the real rate of interest, to the extent that the fiscal stimulus is associated with increased inflation expectations. Hence government purchases should have an especially strong effect on aggregate output when the central bank's policy rate is at the zero lower bound.[27] This is also a case of particular interest, since calls for fiscal stimulus become more urgent when it is no longer possible to achieve as much stimulus to aggregate demand as would be desired through interest-rate cuts alone.

It is easiest to see how the zero lower bound can pose a problem for successful

[27] In fact, it only matters that the policy rate be at a level that the central bank is unwilling to go below; this "effective lower bound" need not be zero. For example, during the current crisis, the Bank of Canada and the Swedish Riksbank have indicated that they do not intend to reduce their interest-rate targets below 25 basis points, though each bank has also indicated an expectation that the target would be kept at that level for several quarters.

stabilization through monetary policy alone if we suppose that the interest rate that is relevant in condition (2.1) for the intertemporal allocation of expenditure is not the same as the central bank's policy rate, and furthermore that the spread between the two interest rates varies over time, owing to changes in the efficiency of financial intermediation.[28] If we let i_t denote the policy rate, and $i_t + \Delta_t$ the interest rate that is relevant for the intertemporal allocation of expenditure, then (3.2) takes the more general form

$$\hat{Y}_t - \hat{G}_t = E_t[\hat{Y}_{t+1} - \hat{G}_{t+1}] - \sigma(i_t - E_t\pi_{t+1} - r_t^{net}), \tag{4.1}$$

where $r_t^{net} \equiv -\log\beta - \Delta_t$ is the real *policy* rate required to maintain a constant path for private expenditure (at the steady-state level).[29] If the spread Δ_t becomes large enough, for a period of time, as a result of a disturbance to the financial sector, then the value of r_t^{net} may temporarily be negative. In such a case the zero lower bound on i_t will make (4.1) incompatible, for example, with achievement of the steady state with zero inflation and government purchases equal to $\bar{G}$ in all periods.

4.1 A Two-State Example

As a simple example (based on Eggertsson, 2009), suppose that under normal conditions, $r_t^{net} = \bar{r} > 0$, but that as a result of a financial disturbance at date zero, credit

[28] Cúrdia and Woodford (2009a) present a complete general equilibrium model with credit frictions in which the policy rate is lower than the rate of interest that enters the equilibrium relation that generalizes (3.2), and describe a number of sources of variation in the spread between the two rates. This model depends on heterogeneity in the situations of different households, so that different interest rates are relevant for the intertemporal decisions of different economic actors; but to a log-linear approximation, aggregate expenditure continues to satisfy an equilibrium condition of the same form as (3.2), under a suitable definition of the interest rate appearing in this equation, so that the connection between the policy rate and aggregate expenditure is essentially of the kind posited in the simpler exposition given here. The consequences of the zero lower bound in the model with heterogeneity and credit frictions are discussed in Cúrdia and Woodford (2009b).

[29] Variations in credit spreads (represented here by Δ_t) are not the only possible sources of variation in r_t^{net}, and so are not the only possible reason why the zero lower bound can be a binding constraint. Variations in time preference, opportunities for private expenditure, or in expected productivity growth are other possibilities; see Christiano (2004) for quantitative analysis of the conditions under which the zero bound would be a binding constraint even in the absence of financial frictions. However, as an empirical matter, the zero bound has become a constraint on actual central-bank policies only as a result of financial crises, such as the Great Depression, the Japanese crisis of the 1990s, or the current crisis.

spreads increase, and r_t^{net} falls to a value $r_L < 0$. Suppose that each period thereafter, there is a probability $0 < \mu < 1$ that the elevated credit spreads persist in period t, and that r_t^{net} continues to equal r_L, if credit spreads were elevated in period $t-1$; but with probability $1-\mu$ credit spreads return to their normal level, and $r_t^{net} = \bar{r}$. Once credit spreads return to normal, they remain at the normal level thereafter. (This exogenous evolution of the credit spread is assumed to be unaffected by either monetary or fiscal policy choices.) Suppose furthermore that monetary policy is described by a Taylor rule, except that the interest rate target is set to zero if the linear rule would call for a negative rate; specifically, let us suppose that

$$i_t = \max\left\{\bar{r} + \phi_\pi \pi_t + \phi_y \hat{Y}_t,\ 0\right\}, \tag{4.2}$$

so that the rule would be consistent with the zero-inflation steady state, if r_t^{net} were to equal $\bar{r}$ at all times. (We shall again suppose that $\phi_\pi > 1, \phi_y > 0$, as prescribed by Taylor.) Finally, let us consider fiscal policies under which government purchases are equal to some level G_L for all $0 \leq t < T$, where T is the random date at which credit spreads return to their normal level, and equal to $\bar{G}$ for all $t \geq T$. The question we wish to consider is the effect of choosing a higher level of government purchases G_L during the crisis, taking as given the value of $\bar{G}$ (the level of government purchases during normal times) and the monetary policy rule (4.2).

Since there is no further uncertainty from date T onward, and the equilibrium conditions (2.13), (4.1) and (4.2) are all purely forward-looking, it is natural to suppose that the equilibrium from date T onward should be the zero-inflation steady state; hence the equilibrium values will be $\pi_t = \hat{Y} = 0$, $i_t = \bar{r} > 0$ for all $t \geq T$.[30] Given this solution for the equilibrium from date T onward, we wish to determine the equilibrium evolution prior to date T. Equilibrium conditions (2.13), (4.1) and (4.2 can be "solved forward" to obtain a unique bounded solution if and only if the model parameters satisfy

$$\kappa\sigma\mu < (1-\mu)(1-\beta\mu). \tag{4.3}$$

[30] One can show that this is a locally determinate rational-expectations equilibrium for dates $t \geq T$, under the policies assumed; that is, it is the only solution in which inflation and output remain within certain bounded intervals. I do not treat here the question whether other equilibria are also possible under a Taylor rule when credit spreads remain small. Additional stipulations regarding the policy regime that would exclude the possibility of either self-fulfilling inflations or deflations after date T are discussed, for example, in Woodford (2003, chap. 2, sec. 4); these additional aspects of policy have no consequences for the issues discussed here.

Note that this condition holds for all $0 \leq \mu < \bar{\mu}$, where the upper bound $\bar{\mu} < 1$ depends on the model parameters (β, κ, σ). I shall here consider only the case in which (4.3) is satisfied, which is to say, in which it is not expected that the crisis is likely to persist for too many years.[31] Then since at each date $t < T$, the probability distribution of future evolutions of fundamentals (the joint evolution of $\{r_t^{net}, \hat{G}_t\}$) is the same, the unique bounded solution obtained by "solving forward" is one in which $\pi_t = \pi_L, \hat{Y}_t = \hat{Y}_L, i_t = i_L$ for each $t < T$, for certain constant values $(\pi_L, \hat{Y}_L, i_L)$.

These constant values can be obtained by observing that (2.13) requires that

$$\pi_L = \frac{\kappa}{1 - \beta\mu}(\hat{Y}_L - \Gamma\hat{G}_L), \tag{4.4}$$

and that (4.1) requires that

$$(1 - \mu)(\hat{Y}_L - \hat{G}_L) = \sigma(-i_L + \mu\pi_L + r_L).) \tag{4.5}$$

Using (4.4) to substitute for π_L in (4.5), one obtains an equation that can be solved to yield

$$\hat{Y}_L = \vartheta_r(r_L - i_L) + \vartheta_G\hat{G}_L, \tag{4.6}$$

where

$$\vartheta_r \equiv \frac{\sigma(1 - \beta\mu)}{(1 - \mu)(1 - \beta\mu) - \kappa\sigma\mu} > 0, \qquad \vartheta_G \equiv \frac{(1 - \mu)(1 - \beta\mu) - \kappa\sigma\mu\Gamma}{(1 - \mu)(1 - \beta\mu) - \kappa\sigma\mu} > 1. \tag{4.7}$$

(Here the indicated bounds follow from (4.3) and the fact that $\Gamma < 1$.)

One can then substitute (4.6) and the associated solution for the inflation rate into (4.2) and solve the resulting equation for i_L. The solution lies on the branch of (4.2) where $i_L = 0$ for values of $\hat{G}_L$ near zero if and only if

$$\bar{r} + \left(\frac{\kappa}{1 - \beta\mu}\phi_\pi + \phi_y\right)\vartheta_r\, r_L < 0. \tag{4.8}$$

This is the case of interest here; assuming that r_L is negative enough for (4.8) to hold, the zero lower bound will bind in the case that government purchases remain at

[31] The problem of indeterminacy of equilibrium that arises in the present model if $\mu > \bar{\mu}$ occurs only because of the supposition that it is possible for the period of elevated credit spreads to last for an indefinitely long time; if there is any finite upper bound on the time that the crisis can last, no matter how long, this problem does not arise, and the equilibrium conditions can be "solved forward" for a unique bounded solution. The calculations are facilitated here by assuming a two-state Markov process for $\{r_t^{net}\}$, but the implication that there is no upper bound on the length of the crisis is not especially realistic; hence I restrict attention to the case in which (4.3) holds.

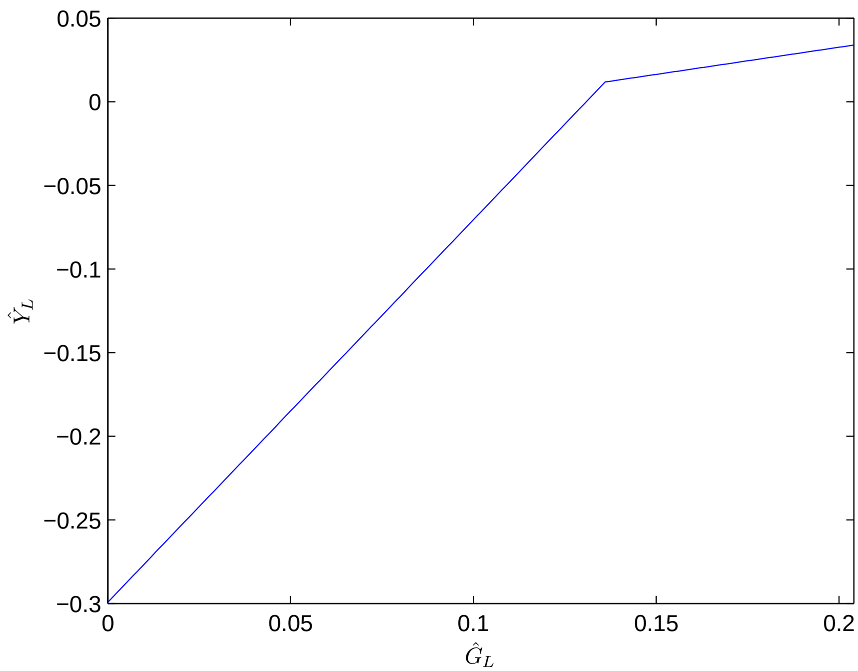

Figure 1: Output as a function of the level of government purchases during the period ($t < T$) in which credit spreads remain elevated. A "Great Depression" shock is assumed, parameterized as in Eggertsson (2009).

their normal (steady-state) level.[32] In fact, it will bind in the case of any $\hat{G}_L < \hat{G}^{crit}$, where

$$\hat{G}^{crit} \equiv \frac{\left(\frac{\kappa}{1-\beta\mu}\phi_\pi + \phi_y\right)\vartheta_r(-r_L) - \bar{r}}{\frac{\kappa}{1-\beta\mu}\phi_\pi(\vartheta_G - \Gamma) + \phi_y \vartheta_G} > 0.$$

For any level of government purchases below this critical level, equilibrium output will be given by

$$\hat{Y}_L = \vartheta_r r_L + \vartheta_G \hat{G}_L \tag{4.9}$$

for all $t < T$, and the inflation rate will equal the value π_L given by (4.4).

In this equilibrium, there will be both deflation and a negative output gap (output

[32] Note that if, as in Eggertsson and Woodford (2003), it is assumed that the central bank pursues a strict zero inflation target as long as this is consistent with the zero lower bound, then the zero lower bound necessarily binds at dates $t < T$ if $\hat{G}_L = 0$, as long as $r_L < 0$. The values computed here for the multipliers dY_L/dr_L and dY_L/dG_L are the same under that simpler hypothesis.

below its level with flexible wages and prices), for as long as credit spreads remain elevated, in the case of any level of government purchases $G_L \leq G^{crit}$.[33] The deflation and economic contraction can be quite severe, for even a modestly negative value of r_L, in the case that μ is large; in fact, ϑ_r (the multiplier dY/dr plotted in Figure 2) becomes unboundedly large as μ approaches $\bar{\mu}$. Under such circumstances, it can be highly desirable to stimulate aggregate demand by increasing the level of government purchases.

For levels of government purchases up to G^{crit}, (4.9) implies that each additional dollar spend by the government increases GDP by ϑ_G dollars. Increases in government purchases beyond that level result in even higher levels of GDP, though the increase per dollar of additional government purchases is smaller, as shown in Figure 1, owing to the central bank's increase in interest rates in accordance with the Taylor rule. (Figure 1 plots $\hat{Y}_L$ as a function of $\hat{G}_L$, for the numerical parameter values proposed by Eggertsson (2009).[34] Under these parameter values, G^{crit} is reached when government purchases exceed their steady-state value by 13.6 percent of steady-state GDP.[35]) For values $G_L > G^{crit}$, the multiplier is no longer ϑ_G, but instead the coefficient γ_y defined in (3.7), where the persistence parameter ρ is now replaced by μ.[36] It

[33] Since (4.4) implies that π_L and the output gap $\hat{Y}_L - \Gamma\hat{G}_L$ must have the same sign, it is evident that the level of government purchases that results in a solution to (4.2) of $i_L = 0$ is reached before either π_L or $\hat{Y}_L - \Gamma\hat{G}_L$ become non-negative. Hence π_L and $\hat{Y}_L - \Gamma\hat{G}_L$ are both negative for all $\hat{G}_L \leq \hat{G}^{crit}$. As illustrated in Figure 1, output may nonetheless exceed its steady-state level; for the parameter values assumed in the figure, Y_L exceeds $\bar{Y}$ ($\hat{Y}_L > 0$) for values of G_L near G^{crit}, though the output gap remains negative, because the increased government purchases increase the "natural" level of output.

[34] Eggertsson estimates parameter values which result in the best fit of the model to U.S. data during the Great Depression. According to his modal parameter estimates (for a quarterly model), $\beta = 0.997, \kappa = 0.00859, \sigma = 0.862$, and $\Gamma = 0.425$. The shock required to account for the size of the contraction during the Depression is one under which $r_L = -0.010$ (minus 4 percent per annum) and $\mu = 0.903$ (an expected mean duration a little over 10 quarters); the response coefficients for monetary policy are assumed to be $\phi_\pi = 1.5, \phi_y = 0.25$.

[35] In drawing the figure, I have also assumed that the credit spread is zero in the "normal" state, so that $\bar{r} = -\log\beta$. Allowing for a small positive credit spread in this state would raise the value of G^{crit}.

[36] Under the alternative hypothesis that the central bank implements a strict zero inflation target, unless this is prevented by the zero bound, the multiplier above the critical level of government purchases is equal to Γ. In this case, G^{crit} is the value that reduces the output gap to zero; at this point and beyond, the central bank achieves its inflation target, and (4.4) implies that $\hat{Y}_L = \Gamma\hat{G}_L$.

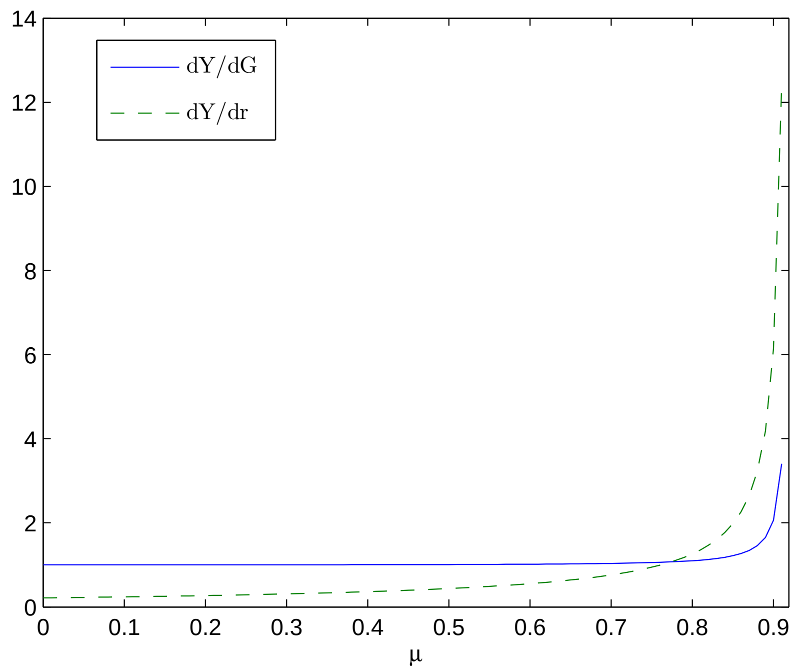

Figure 2: Derivatives of Y_L with respect to the values of r_L and G_L, for alternative assumed degrees of persistence μ of the financial disturbance. Other parameter values are taken from Eggertsson (2009).

follows from (4.7) that the multiplier $dY_L/dG_L = \vartheta_G$ for government purchases up to the level $\hat{G}^{crit}$ is necessarily greater than 1 (for any $\mu > 0$). The reason is that, given that the nominal interest rate remains at zero in periods $t < T$, an increase in G_L, which increases π_L, accordingly increases expected inflation (given some positive probability of elevated credit spreads continuing for another period), and so *lowers* the *real* rate of interest. Hence monetary policy is even more accommodative than is assumed in the benchmark analysis in section 2, and the increase in aggregate output is correspondingly higher.

The degree to which the multiplier exceeds 1 in this case can, in principle, be quite considerable. In fact, for any given values of the other parameters, the multiplier while the policy rate remains at the zero bound can be unboundedly large, for a

Similarly, if the central bank follows a Taylor rule of the form (3.1), G^{crit} is the value that reduces the output gap to zero, but the multiplier beyond this point is given by (3.6).

sufficiently value of the persistence parameter μ. Figure 2 plots the multiplier as a function of μ, holding the other model parameters fixed at the values estimated by Eggertsson (2009). The figure illustrates something that can be observed from (4.7) to hold quite generally: the multiplier is monotonically increasing in μ, and increases without bound as μ approaches $\bar{\mu}$. The figure also indicates that the multiplier is in general not too much greater than 1, except if μ is fairly large. However, it is important to note that the case in which μ is large (in particular, a large fraction of $\bar{\mu}$) is precisely the case in which the multiplier dY_L/dr_L is also large, which is to say, the case in which a moderate increase in the size of credit spreads can cause a severe output collapse.

Thus increased government purchases when interest rates are at the zero bound should be a powerful means through which to stave off economic crisis precisely in those cases in which the constraint of the zero lower bound would otherwise be most crippling — namely, those cases in which there is insufficient confidence that the disruption of credit markets will be short-lived. For example, in Eggertsson's numerical example, a contraction of the size experienced during the Great Depression occurs as a result of a disturbance with a persistence coefficient of $\mu = 0.903$; in the case of this kind of disturbance, his estimated parameter values imply a multiplier of 2.29. Christiano *et al.* (2009) similarly find that a multiplier above 2 is possible at the zero lower bound, in the context of a more complex New Keynesian model that is estimated to match a large number of features of postwar U.S. data.

Evidence on the effects of defense spending during the 1930s suggest that substantial multipliers of this kind may indeed be possible during circumstances like those of the Great Depression. For example, Almunia *et al.* (2009) estimate panel vector autoregressions using data from 27 countries for the period 1925-1939, and look at the response to innovations in defense purchases, taken to represent exogenous changes in government purchases; depending on the specification used, they find a multiplier during the year of the innovation of either 2.5 (their Figure 14) or 2.1 (their Figure 19). Gordon and Krenn (2009) similarly find a multiplier greater than 1 for the effects of innovations in government purchases on U.S. real GDP during the military buildup between 1940:Q2 and 1941:Q4. It is arguable that these relatively high multipliers for defense purchases during the Depression, relative to those found by studies of the effects of defense purchases at other times (*e.g.*, those summarized in Hall, 2009), reflect a greater degree of monetary accommodation under Depression circumstances than has been typical of other military buildups.[37]

4.2 Importance of the Duration of Fiscal Stimulus

Cogan *et al.* (2009) instead find that a leading empirical New Keynesian model of the U.S. economy predicts small multiplier effects of increased government purchases during a situation in which the zero lower bound is assumed to bind. For example, when Cogan *et al.* consider the effect of a permanent increase in government purchases of 1 percent of GDP, they find an increase in GDP of only 1.0 percent in the first quarter, which falls to only 0.6 percent by the end of the second year (the period over which they assume that the federal funds rate rate remains at zero), and to only 0.4 percent after four years. In the case of an assumed path of government purchases intended to mimic projected expenditure under the February 2009 U.S. federal stimulus package, their model implies an increase in GDP substantially smaller than the increase in government purchases in all quarters, and hence a particular modest increase in output during the first year of their simulation.

What accounts for the difference with the large multiplier obtained at the zero bound by Eggertsson (2009)? While the empirical model used by Cogan *et al.* is substantially more complex, this is probably not the most important difference in their analysis.[38] The crucial difference is that the calculations above assume an increase in government purchases that lasts precisely as long as credit spreads are elevated, and hence precisely as long as the zero lower bound is a binding constraint, following which period $G_t = \bar{G}$ again each period; Cogan *et al.* instead consider increases in government purchases that are initiated at a time when interest rates are zero, but that extend longer than the period over which the interest rate is projected to remain at zero. (In the simplest cases that they consider, the interest rate remains at zero for only one or two years, while the increase in government purchases is permanent.)

[37] In fact, the VAR results of Almunia *et al.* show central-bank discount rates being reduced, rather than increased, in response to a positive innovation in defense purchases. Other differences could also account for smaller multipliers during other military buildups. For example, Gordon and Krenn argue that the multiplier was smaller in the case of subsequent World War II increases in defense spending, owing to the wartime controls implemented after the U.S. entered the war.

[38] The empirical model considered by Christiano *et al.* (2009) has a structure very similar to the one used by Cogan *et al.*, yet Christiano *et al.* obtain multipliers well in excess of 1 for a policy experiment similar to the one analyzed above.

Also in our simple model, the increase in output is predicted to be much smaller if a substantial part of the increased government purchases are expected to occur after the zero lower bound ceases to bind. For as explained above, once interest rates are determined by a Taylor rule, a higher level of government purchases should crowd out private spending (raising the marginal utility of private expenditure), and may well cause lower inflation as well.[39] But the expectation of a higher marginal utility of expenditure and of lower inflation in the event that credit spreads normalize in the following period both act as disincentives to private expenditure while the nominal interest rate remains at zero. Hence while there is a positive effect on output during the crisis of increased government purchases at dates $t < T$, an anticipation of increased government purchases at dates $t \geq T$ has a *negative* effect on output prior to date T.

A simple calculation can illustrate this. Suppose that instead of the two-state Markov chain considered above, there are three states: after the "crisis" state (in which $r_t^{net} = r_L$ and $\hat{G}_t = \hat{G}_L$) ends, there is a probability $0 < \lambda < 1$ each period that government purchases will remain at their elevated level ($\hat{G}_t = \hat{G}_L$), even though $r_t^{net} = \bar{r}$, though with probability $1 - \lambda$ each period the economy returns to the "normal" state (in which $r_t^{net} = \bar{r}$ and $G_t = \bar{G}$) and remains there forever after. If we let $(\pi_S, \hat{Y}_S, i_S)$ be the constant values for $(\pi_t, \hat{Y}_t, i_t)$ in the transitional state (*i.e.,* for all $T \geq t < T'$, where T' is the random date at which government purchases return to their "normal" level), then the value of $E_t\hat{Y}_{t+1}$ during the "crisis" period is not $\mu\hat{Y}_L$, but $\mu\hat{Y}_L + (1-\mu)\lambda\hat{Y}_S$, and similarly for expected future government purchases and expected future inflation. We can repeat the previous derivation, obtaining instead of (4.9) the more general form

$$\hat{Y}_L = \vartheta_r r_L + \vartheta_G \hat{G}_L + \vartheta_\pi \pi_S + \vartheta_C(\hat{Y}_S - \hat{G}_L), \tag{4.10}$$

where

$$\vartheta_\pi \equiv (1-\mu)\lambda\vartheta_r > 0, \qquad \vartheta_C \equiv \sigma^{-1}\vartheta_\pi > 0.$$

The fact that $\vartheta_\pi, \vartheta_C > 0$ indicates that an expectation of either lower private expenditure or lower inflation in the transitional state will lower output during the crisis.

Using the same reasoning as in the previous section, one can show that the levels of output and inflation during the transitional state, when the interest rate is

[39] As noted above, both things occur in the case of the Eggertsson (2009) parameter values.

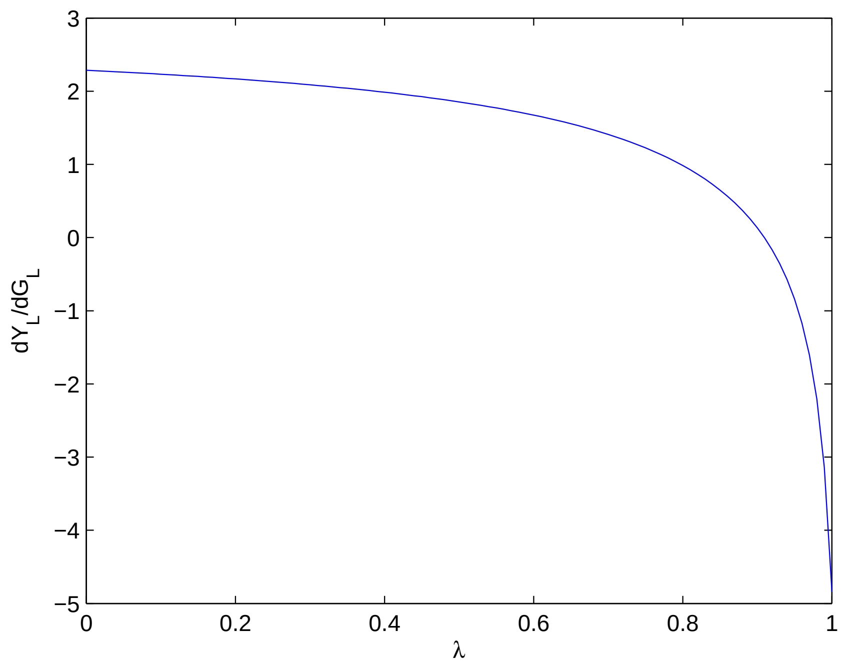

Figure 3: Derivative of Y_L with respect to G_L, for alternative degrees of persistence λ of the fiscal stimulus after the end of the financial disturbance. Other parameter values are taken from Eggertsson (2009).

determined by the Taylor rule but government purchases remain high, are given by $\hat{Y}_S = \gamma_y \hat{G}_L, \pi_S = \gamma_\pi \hat{G}_L$, where γ_y is the coefficient defined in (3.7) (but with the persistence coefficient ρ equal to λ) and γ_π is the corresponding inflation coefficient. One thus obtains a multiplier

$$\frac{dY_L}{dG_L} = \vartheta_G + \vartheta_\pi \gamma_\pi + \vartheta_C(\gamma_y - 1) \tag{4.11}$$

for government purchases below the critical level that causes the zero bound to no longer bind even in the crisis state. Since $\gamma_Y < 1$ as explained earlier, the contribution of the final term is necessarily negative. In the case that either of the response coefficients (ϕ_π, ϕ_y) is sufficiently large, the Taylor rule will not allow a large increase in inflation during the transitional phase, and one obtains a multiplier smaller than ϑ_G when $\lambda > 0$.

Figure 3 plots the value of the multiplier (4.11) as a function of λ, in the case that the other parameters take the values proposed by Eggertsson (2009). When

$\lambda = 0$, the multiplier is nearly 2.3, as reported by Eggertsson, but it steadily falls as λ is increased. For values of λ equal to 0.8 or higher (an expected duration of the fiscal stimulus for 4 quarters or more after the end of the financial disturbance), the multiplier falls below 1. For values of λ equal to 0.91 or higher (an expected duration of 10 quarters or more), the multiplier is negative. In particular, in the case of a permanent increase in the level of government purchases (the case $\lambda = 1$), as in the first case considered by Cogan *et al.,* the multiplier is strongly negative (nearly -5!). Hence a finding that a long-lasting fiscal stimulus is predicted to increase output only modestly, as in the simulations of Cogan *et al.,* does not mean that a better-targeted fiscal stimulus cannot be much more effective.

Nor is it the case that to be effective, the government spending must occur immediately. In the model considered here, an increase in government purchases during a period in which the interest rate is zero, which is expected to last for *the current quarter only,* so that there is no change in expected future government purchases, has a multiplier of exactly 1. (This is because with no change in expected future fiscal policy, there is no change in expected future output or inflation. This means no change in expected real interest rates in future periods, and, as long as the temporary increase in G_t remains within the range which implies a current nominal interest rate of zero, no change in the current real interest rate either. Hence the benchmark analysis in section 2 applies.) It follows that when Eggertsson obtains a multiplier of 2.3, 1.0 of this is due to the increase in government purchases during the current quarter, while the other 1.3 is the effect of higher anticipated government purchases in the future.

Hence even if there were no increase in government purchases in the current quarter at all, an expectation of higher government purchases in all future quarters prior to date T would increase output *immediately* by an amount that is 1.3 times as large as the promised future increase in the level of government purchases. Of course, an even longer delay would attenuate the effects on output at the time of the announcement to an even greater extent. Still, New Keynesian models certainly do not imply that a delayed fiscal stimulus will serve no purpose — as long as the eventual increase in government spending is *contingent on the continued existence of the financial disruption* that justifies the emergency measures. The kind of stimulus package that is ineffective, or even counter-productive, is one under which a large part of the increased government purchases are expected to occur in a post-crisis environment in which monetary policy is not expected to accommodate an increase in aggregate demand.[40]

5 Government Purchases and Welfare

Thus far, I have simply considered the extent to which it is possible for an increase in government spending to increase aggregate output and employment, taking it for granted (as in much popular discussion) that an increase in output would be desirable, at least under circumstances where output would otherwise be below its trend path. But it is reasonable to ask whether our models imply not only that increased government purchases will increase GDP, but that they will increase economic welfare as well. This does not follow trivially from the existence of a positive multiplier (or even a multiplier greater than 1); one must consider the value of the use to which the resources consumed by the government would otherwise be put.

5.1 Fiscal Stabilization in the Neoclassical Model

In the case of the neoclassical model, it is evident that if government purchases are of no intrinsic value ("paying people to dig holes and then fill them again"), the optimal level of government purchases must be zero, for any government purchases crowd out private expenditure *and* increase the disutility of working. But of course some kinds of government spending do benefit the public; we can represent this by making the utility of the representative household depend on G_t, the level of public goods provision. The calculations above are unaffected by this hypothesis, as long as we suppose that utility is additively separable in public goods (the tacit assumption earlier).[41] Let us suppose, then, that the utility of the representative household is given by

$$\sum_{t=0}^{\infty} \beta^t \, [u(C_t) + g(G_t) - v(H_t)], \tag{5.1}$$

[40] This is illustrated not only by the simulations of Cogan *et al.* (2009), but also by those of Erceg and Lindé (2009) for the case of a "gradual increase in government purchases" that continue beyond the point at which the zero bound ceases to bind.

[41] For extension of the neoclassical theory to the case in which public goods are at least partially substitutes for private expenditure, see, *e.g.*, Baxter and King (1993).

where $g' > 0, g'' \leq 0$. (Of course, the value of public projects does not depend solely on the amount that is spent on them. But it is an obvious principle of optimal fiscal policy that the projects financed should be those that yield the greatest additional utility per dollar spent; the function $g(G)$ accordingly indicates the utility obtained if resources G are expended on the most valuable public projects available.)

Given that for any path $\{G_t\}$ of government purchases, the competitive equilibrium will maximize the utility of the representative household, it is easily seen that the optimal path of government purchases will be the one that satisfies the first-order condition

$$g'(G_t) = u'(Y_t - G_t) = \tilde{v}'(Y_t) \tag{5.2}$$

each period. (Note that the equality of the last two expressions in equilibrium has already been assured by (1.6), so this represents only one additional condition per period, to determine the optimal level of G_t.) This condition has a simple interpretation: government purchases should be undertaken if and only if they have a marginal utility as high as that associated with additional private expenditure — *i.e.*, if they satisfy the conventional (microeconomic) cost-benefit criterion. One way of stating this criterion is to say that government purchases should be chosen so as to maximize $u(Y_t - G_t) + g(G_t)$, taking as given the quantity of aggregate expenditure Y_t. Plainly, this is not a criterion that requires one, in choosing whether to undertake a particular public project, to think about the consequences of government spending for aggregate demand.

In the case that technology, labor supply, or spending opportunities shift over time, the optimal level of government purchases is not necessarily constant. But even if it varies, the optimal variation need not be *countercyclical.* If the business cycle is due primarily to shifts in the $\tilde{v}(Y)$ function (*e.g.*, shocks to productivity, as posited by real business cycle theory), then the optimal variation in G_t will be *procyclical:* the government should cut back on its spending exactly when the private sector does, in order to maintain the equality of g' and u'.

5.2 Fiscal Stabilization When Monetary Policy is Optimal

There is greater scope for fiscal stabilization policy in the case that prices or wages are sticky (or based on older information than that available to the government). If a recession is a time when output is below the full-information flexible-wage/price level, owing to stickiness of one sort or another, this implies a misallocation of resources, and a potential justification for fiscal stimulus to "fill the output gap." If an increase in government purchases G_t is associated with an increase in output Y_t that period (abstracting, for the moment, from changes in the allocation of resources in any other periods), utility will be increased if the relative size of the two changes satisfies the condition

$$(u' - \tilde{v}')\,\frac{dY}{dG} + (g' - u') > 0. \tag{5.3}$$

In the neoclassical case, equilibrium condition (1.6) implies that the first term in (5.3) is necessarily zero, so that government purchases increase welfare only to the extent that g' exceeds u'. But if during a recession, $u' > \tilde{v}'$, the condition can be satisfied even when u' exceeds g' to some extent; this will be more likely to be true the greater the extent to which u' exceeds v' (*i.e.*, the more negative the "output gap"), and the greater the multiplier effects of government purchases.

Yet it is important to remember that in New Keynesian models, both the size of the output gap and the size of the multiplier will depend on monetary policy; and while there might well be significant opportunities for fiscal stabilization policy under the assumption that prices, wages or information are sticky *and that monetary policy is inept,* the most obvious solution in such a case is to increase the accuracy of *monetary* stabilization policy. Indeed, given that effective monetary stabilization policy should prevent there from being large variations in the ratio of u' to v' (by stabilizing the output gap), it is not obvious that the novel considerations mentioned in the previous paragraph should be of much quantitative significance when monetary policy is used optimally.

A case that is especially simple to analyze is that in which we suppose that there exists a constant employment or output subsidy, of precisely the magnitude necessary to offset the distortion owing to the market power of monopolistically competitive producers.[42] In this case, the factor $\mu > 1$ in (1.11) is canceled, and the equilibrium with (full-information) flexible prices and wages is efficient, despite the assumption of monopolistic competition. Now suppose that prices are sticky (or set on the basis of sticky information), while wages are flexible (or there is efficient contracting in the labor market). A monetary policy that maintains price stability at all times achieves the (full-information) flexible-price equilibrium allocation, regardless of the path of

[42]For example, it suffices that there be a subsidy equal to fraction τ of a firm's payroll, where $\tau = 1 - \mu^{-1} > 0$, and $\mu > 1$ is the markup factor in (1.11).

government purchases, as discussed above in section 3.1.; hence this policy maximizes expected utility, given the path of government purchases.[43] Thus one may conclude that, regardless of the path of government purchases, an optimal monetary policy achieves the allocation of resources predicted by the neoclassical model.[44] But then the condition for optimality of the level of government purchases is again simply (5.2), which is to say, only the microeconomic cost-benefit criterion is relevant.

A similar conclusion is obtained in the case that wages are sticky (or negotiated on the basis of sticky information), but prices are flexible (and based on full information). In this case the monetary policy required to achieve the competitive allocation of resources is one that completely stabilizes the aggregate level of nominal wages (as discussed in Erceg *et al.*, 2000). But again an optimal monetary policy achieves this allocation, regardless of the path of government purchases, and so again the optimal path of government purchases is the same as in the neoclassical analysis. More generally — if both wages and prices are sticky, or the subsidy assumed above to simplify the analysis does not exist — it is often not possible for even an ideal monetary policy to achieve the first-best allocation of resources. But to the extent that an optimal monetary policy can reduce the size of departures from the first-best allocation, the potential scope for fiscal stabilization policy is correspondingly reduced.

It is not simply a matter of there being two instruments which can each, in principle, address the problem of an insufficient level of aggregate nominal expenditure, given the existing level of prices or wages, so that it does not matter which instrument is used for the job. Rather, to the extent that the problem can be solved using monetary policy, it is *costless to do so,* since monetary policy has no *other* aims to fulfill; whereas, while government spending can also be used to ameliorate the problem, this has a cost, since it requires the diversion of real resources to alternative uses. Whenever government purchases are used for aggregate demand management, there is a tension between this goal and the choice of government purchases so as to maintain an optimal composition of expenditure. Since there is no equally important conflict in the case of the use of monetary policy for aggregate demand management,

[43]For a more formal presentation of this argument, see Woodford (2003, chap. 6, sec. 3.1).

[44]This result depends on an assumption that the zero lower bound on interest rates does not prevent monetary policy from achieving its inflation target at some points in time. The importance of this caveat is made clear in the following section.

monetary policy should be used to the extent possible; and this should largely allow decisions about government purchases to be made from the standpoint of the optimal composition of expenditure.

5.3 Fiscal Stabilization at the Zero Lower Bound

There is, however, one case in which a much stronger argument can be made for the usefulness of variations in government spending for stabilization purposes. This is when a financial disturbance makes it impossible for monetary policy to maintain price stability and a zero output gap at all times, as the required path for the policy rate would violate the zero lower bound. Under such circumstances, substantial distortions due to deflation and a large negative output gap can exist in equilibrium, even with a central bank that maintains a strict zero inflation target whenever this is consistent a non-negative interest rate. It can then be desirable to use government purchases to "fill the output gap," at least partially, even at the price of distorting to some extent the composition of expenditure in the economy.

As an example, let us consider the welfare effects of fiscal stimulus in the two-state example of section 4.1. Suppose that the central bank maintains a strict zero inflation target whenever this is possible, and a nominal interest rate of zero whenever deflation is unavoidable;[45] and let us consider only fiscal policies under which G_t is equal to some constant G_L for all $t < T$, equal to $\bar{G}$ for all $t \geq T$, where $\bar{G}$ is the optimal level of government purchases under "normal" conditions, that is, the value that satisfies (5.2) when $Y_t = \bar{Y}$. The analysis is simplified if we again assume the existence of a subsidy such that the flexible-price equilibrium allocation would be optimal. In this case, the steady state with $Y_t = \bar{Y}$ and $G_t = \bar{G}$ represents an optimal allocation of resources,[46] and the assumed monetary policy would be optimal in the event that credit spreads were to remain always modest in size, so that the zero bound were never a binding constraint. I wish to consider the welfare effects of increasing G_L above the normal level $\bar{G}$, and the way in which the optimal choice of G_L depends on the size and expected duration of the financial disturbance.

[45] This corresponds to a limiting case of the policy considered in section 4.1, in which ϕ_π is made unboundedly large.

[46] In this section, I also abstract from the effects of any variations in preferences or technology over time.

One can show that a quadratic approximation to the expected value of (5.1)[47] varies inversely with

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda_y (\hat{Y}_t - \Gamma \hat{G}_t)^2 + \lambda_g \hat{G}_t^2 \right], \tag{5.4}$$

where

$$\lambda_y \equiv \frac{\kappa}{\theta} > 0, \qquad \lambda_g \equiv \left[\frac{\eta_g}{\eta_u} + 1 - \Gamma \right] \Gamma \, \lambda_y > 0,$$

and $\eta_g \geq 0$ is (the negative of) the elasticity of g' with respect to G, a measure of the degree to which there are diminishing returns to additional government expenditure. Here the final two terms inside the square brackets represent a quadratic approximation to $u(Y_t - G_t) + g(G_t) - \tilde{v}(Y_t)$, which would be the period contribution to utility if the prices of all goods were the same, as would occur with flexible prices or in an environment with complete price stability; the additional π_t^2 term represents the additional welfare loss owing to an inefficient composition of the economy's aggregate product as a result of price dispersion.

If the zero bound were never a binding constraint on monetary policy, the only constraint on feasible paths for the inflation rate and the output gap $\hat{Y}_t - \Gamma \hat{G}_t$ would be (2.13), regardless of the path of $\{\hat{G}_t\}$; hence optimal monetary policy would maintain a zero inflation rate and output gap at all times, reducing each of the first two terms inside the square brackets in (5.4) to their minimum possible values each period. The optimal path of government purchases would then be chosen simply to minimize the remaining term, by setting $\hat{G}_t = 0$ each period. (This would achieve an optimal composition of expenditure, as it would result in $Y_t = \bar{Y}, G_t = \bar{G}$ each period.)

In the case considered here, however, the zero lower bound on interest rates precludes this first-best outcome. Under a policy in the family proposed above, the equilibrium is of the kind characterized in section 4.1. In any equilibrium of this kind, the objective (5.4) takes the value

$$\frac{1}{1 - \beta\mu} \left[\pi_L^2 + \lambda_y (\hat{Y}_L - \Gamma \hat{G}_L)^2 + \lambda_g \hat{G}_L^2 \right]. \tag{5.5}$$

The optimal policy within this family is therefore obtained by minimizing (5.5) with respect to $\hat{G}_L$, taking into account the dependence of $(\pi_L, \hat{Y}_L)$ on $\hat{G}_L$ implied by (4.4)

[47] See Woodford (2003, chap. 6, sec. 2) for the derivation.

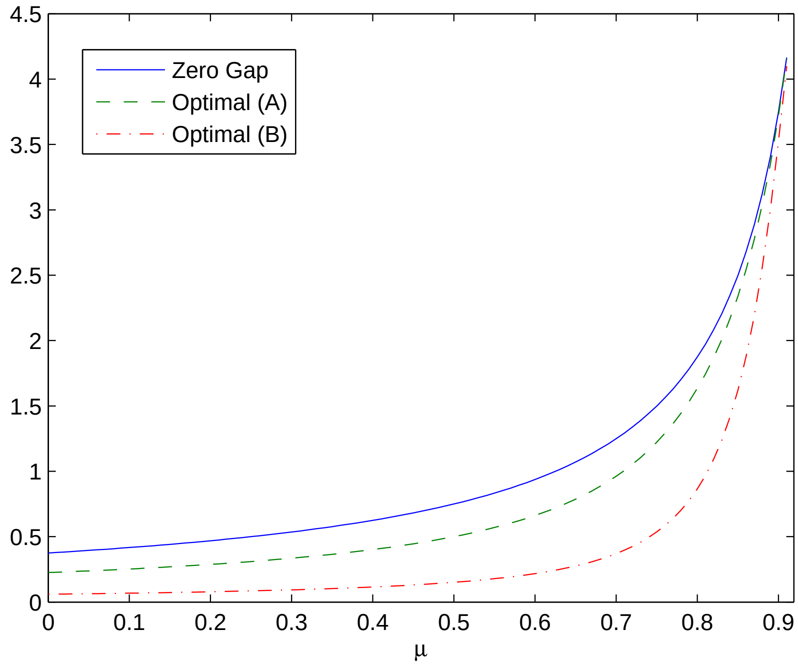

Figure 4: The optimal value of $\hat{G}_L/|r_L|$, for alternative values of μ, under two different assumptions about the size of η_g. (Case A: $\eta_g = 0$. Case B: $\eta_g = 4\eta_u$.) The solid line shows the value of $\hat{G}_L/|r_L|$ required to maintain a zero output gap.

and (4.9). The first-order conditions for the minimization of this quadratic objective subject to the two linear constraints can be uniquely solved for a linear solution,

$$\hat{G}_L = -\frac{\xi(\vartheta_G - \Gamma)\vartheta_r}{\xi(\vartheta_G - \Gamma)^2 + \lambda_g}\, r_L > 0, \tag{5.6}$$

where

$$\xi \equiv \left(\frac{\kappa}{1-\beta\mu}\right)^2 + \lambda_y > 0.$$

(This solution for the optimal value of $\hat{G}_L$ is necessarily positive, because $\vartheta_G > \Gamma$ and $r_L < 0$.)

Figure 4 plots the optimal value of $\hat{G}_L/|r_L|$ defined by (5.6), for alternative values of μ, assuming the values for the model parameters $\beta, \kappa, \sigma, \Gamma$ and θ proposed by Eggertsson (2009).[48] For a given financial disturbance parameterized by (r_L, μ), the

[48]In addition to the parameter values reported in footnote xx above, it is now also assumed that

optimal size of the increase in government purchases can be determined from the figure by observing the optimal ratio for that value of μ, and then multiplying by the value of $|r_L|$. (The value of $\hat{G}_L/|r_L|$ is reported in units of percentage points of steady-state GDP per percentage point, at an annualized rate, of reduction in the real interest rate required to maintain the steady-state level of private expenditure.) Thus a value of 2 on the vertical axis means that if r_L is equal to -4 percent per annum, it would be optimal to increase government purchases by an amount equal to 8 percent of GDP. The optimal value is plotted under two different assumptions about the degree of diminishing returns to additional government expenditure. In case A, it is assumed that utility is linear in government purchases ($\eta_g = 0$); this provides an upper bound for the degree to which it can be cost-effective to increase government purchases. In case B, it is instead assumed that $\eta_g = 4\eta_u$; this corresponds to the case in which the marginal utility of government purchases decreases at the same rate (per percentage point increase in spending) as the marginal utility of private purchases, and private expenditure is 4 times as large as government purchases in the steady state. In this case, because of the diminishing returns to additional government purchases, the optimal increase in government spending is less for any given financial disturbance. For purposes of comparison, the solid line in Figure 4 also plots the level of government purchases that would be required to fully eliminate the output gap (*i.e.,* keep output at the flexible-price equilibrium level) and prevent any decline in inflation as a result of the financial disturbance. (This line also indicates the critical level of government purchases at which the zero lower bound ceases to bind, given the central bank's assumed policy.)

The figure shows that it is optimal to use discretionary (state-dependent) government purchases to partially offset the decline in output and inflation that would otherwise occur as a result of the financial disturbance. It should be noted, however, that it is not optimal to *fully* stabilize inflation and the output gap, despite the feasibility of doing so, because of the inefficient composition of expenditure that this would involve. In the case that the financial disturbance is not too persistent ($\mu = 0.5$ or less), the optimal increase in government purchases is only a small fraction of the increase that would be required to eliminate the output gap, if we assume diminishing returns to additional public expenditure similar to those that exist for private expenditure. (The optimal fiscal stimulus would be even smaller if one were

$\theta = 12.77.$

to assume even more sharply diminishing returns to public expenditure, or if one were to take into account the distortions involved in raising government revenues.) At the same time, the optimal size of fiscal stimulus can be quite substantial, and a large fraction of the size required for full stabilization of both inflation and the output gap, in the case that μ is large. In this case — when there is believed to be a substantial probability that the financial disruption will persist for years, and when a serious depression could result in the absence of fiscal stimulus — welfare is maximized by an aggressive increase in government purchases, of nearly the size required to fully stabilize inflation and the output gap.

[ADD MORE]

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