Decoupling and Contagion

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The financial crisis that has engulfed the world over the past three years started out in a relatively small set of sectors in a select number of countries, with a particularly important contribution from the real estate sector in the United States. At first (February 2007 to May 2008), financial problems seemed to stay confined to their sectors and countries of origin, with little repercussion on other sectors in the United States or on emerging countries. In fact, policymakers in the developing world would brag about this “decoupling” from the United States as a sign of the economic maturity reached by their domestic economies.1 Starting around May 2008, however – and particularly after the collapse of Lehman (September 15, 2008) – the financial crisis began to spread like wildfire, affecting countries all around the world, with asset and stock prices collapsing in unison.2

This paper develops a stylized model of decoupling and recoupling that captures these phenomena in an environment where heterogeneous entrepreneurial sectors face financial constraints in their relationship with a common set of lenders. In response to adverse shocks, a financially constrained sector must reduce its borrowing and cut down on production. In particular, as the constrained sector can absorb less and less capital, the interest rate in the economy declines. Other sectors that compete for the same inputs (including capital) experience positive terms of trade effects that lower their costs and boost their output and profits, reflecting the phenomenon of “decoupling.” As long as the shock is sufficiently small in magnitude, the entrepreneurial sector repays what is owed and the capital position of lenders is unaffected. If the adverse shock passes a certain threshold, the constrained sector is no longer able to honor its debts in full and lenders experience losses that erode their capital base. This induces them to cut their supply of credit to the rest of the economy, which reduces output and profit for all other entrepreneurial sectors, capturing the phenomenon of “contagion.”3

1 In mid-September 2008, Brazil’s president, Lula da Silva, was quoted as saying “What crisis? Go ask Bush.” A few weeks later, Brazil’s stock market and currency plummeted by 20 and 13 percent, respectively (Bloomberg.com, December 3, 2008, “Lula, Like Bush, Gives Bad Shopping Advice”).

2 This “decoupling-recoupling” sequence is well documented in Michael P. Dooley and Michael M. Hutchinson (2009) for emerging markets as a whole and in Alejandro Izquierdo and Ernesto Talvi (2009) for Latin America.

3 For empirical documentation that episodes of contagion typically involve common lenders see e.g. Graciela L. Kaminsky, Carmen M. Reinhart and Carlos A. Vegh (2003).
leading to the first-order condition

\[ R_2 = \frac{u'(c_1)}{u'(c_2)} \]

From these equilibrium conditions we find that \( \partial R_2 / \partial d_1 < 0 \) as outlined in the appendix, i.e. the richer households are in period 1, the lower the interest rate that they charge. Furthermore \( \partial R_2 / \partial d_2 > 0 \), i.e. the supply of loans to entrepreneurs in period 1 is increasing in the interest rate. As long as households/bankers are net lenders, their utility is an increasing function in the amount of loans intermediated, \( \partial U / \partial d_2 > 0 \).

### B. Entrepreneurial Sector

We assume that each entrepreneurial sector consists of a continuum of identical entrepreneurs of mass 1 that are risk-neutral and value their profits \( \pi^i \), which they consume at the end of period 2, according to the linear utility function \( U^i = \pi^i \). A representative entrepreneur in sector \( i \) enters period 1 with a predetermined debt obligation of \( R_1^i d_1^i \) that is due in period 1 and with productive output of \( \bar{A}_1^i F \left( k_1^i \right) \), where \( \bar{A}_1^i \) is a random variable with support \([0, \bar{A}]\), \( k_1^i \) is a predetermined level of capital and \( F(\cdot) \) is a decreasing returns-to-scale production function that fully depreciates the capital employed. If his production is insufficient to cover his debt, the entrepreneur goes bankrupt and lenders obtain his entire output. In summary, the lender receives

\[ \bar{d}_1^i = \min \left\{ d_1^i, \bar{A}_1^i F \left( k_1^i \right) / R_1^i \right\} \]

and the entrepreneur’s net worth at the beginning of period 1 is

\[ n_1^i = \max \left\{ \bar{A}_1^i F \left( k_1^i \right) / R_1^i d_1^i, 0 \right\} \]

After these variables are realized, the entrepreneur decides how much debt \( d_2^i \) to issue at a gross interest rate of \( R_2 \) and how much to invest in next-period production. His total period 2 investment into capital is financed from his net worth and the proceeds of his borrowing,

\[ k_2^i = n_1^i + d_2^i \]

This capital investment produces period 2 output of \( \bar{A}_2^i F \left( k_2^i \right) \), where we set for simplicity \( \bar{A}_2^i = A \forall i \). This implies that there are no bad shocks and we can rule out bankruptcy due to low productivity shocks in period 2.

However, we assume that there is a moral hazard problem in period 1, which imposes a credit limit on \( d_2^i \). After having borrowed in period 1, a producer has an opportunity to move his project into a scam that hides his income in period 2. Creditors can challenge this in court but can recover at most a fraction \( \frac{1}{1 + \alpha} \) of his total assets because of imperfect enforcement. To avoid losses from potential fraud, creditors limit the amount of bonds that producers can sell to

\[ d_2^i \leq \frac{\alpha}{1 + \alpha} k_2^i \text{ or } d_2^i \leq \alpha n_1^i \]

The optimization problem of a representative entrepreneur in sector \( i \) is described by the Lagrangian

\[ L^i = AF \left( n_1^i + d_2^i \right) - R_2 d_2^i - \lambda^i \left[ d_2^i - \alpha n_1^i \right] \]

where \( \lambda^i \) is the shadow price on the borrowing constraint. The problem results in the first-order condition

\[ AF' \left( k_2^i \right) = R_2 + \lambda^i \]

If the constraint is loose, this reduces to the standard neoclassical condition. Entrepreneurs invest and borrow optimally,

\[ k_2^*(R_2) = F^{-1}(R_2/A) \]

\[ d_2^i = k_2^*(R_2) - n_1^i \]

The optimal capital stock is independent of individual-specific variables and only depends on the cost of capital \( R_2 \) in the economy. This yields a level of period 2 profits

\[ \pi^i_{unc} = AF \left( k_2^*(R_2) \right) - R_2 \left[ k_2^*(R_2) - n_1^i \right] \]

where it is straightforward to show that \( \partial \pi^i / \partial n_1^i > 0 \) and that \( \partial \pi^i / \partial R_2 < 0 \) as long as the entrepreneur is a net borrower, i.e. \( n_1^i < k_2^*(R_2) \).

If the constraint is binding, a wedge opens between the entrepreneur’s cost of funds and his mar-
original product, and the level of borrowing and of investment is determined by the constraint
\[ d_2^i = \alpha n_1^i, \]
\[ k_2^i = (1 + \alpha) n_1^i. \]

Now the capital stock is independent of the interest rate in the economy and only depends on entrepreneurial net worth. This results in a level of period 2 profits
\[ \pi_i^2 = AF \left( (1 + \alpha) n_1^i \right) - \alpha R_2 n_1^i \]
which also satisfies \( \hat{c} \pi^i R_2 < 0 \). Note that if \( n_1^i = 0 \) because of bankruptcy in period 1, the entrepreneur cannot borrow and invest due to the constraint, and he produces and consumes zero in period 2.

II. Equilibrium

For given initial conditions, a decentralized equilibrium in the economy consists of a bundle \( \left\{ d_2^i, k_2^i, R_2 \right\} \) that is a solution to the maximization problems of the household/banking and the entrepreneurial sectors and satisfies the market clearing condition for debt
\[ d_2 = \Sigma d_2^i. \]

Our characterization of the economy’s equilibrium allows us to study the dynamics of decoupling and contagion. For simplicity we assume there are two productive sectors labeled by \( i = X, Z \), of which sector \( Z \) always experiences a constant period 1 productivity shock \( A_1^Z = A \) that is sufficiently high so the sector is always unconstrained during the ensuing experiment. We study the response of the economy as we vary the productivity of sector \( X \) over the admissible interval \( [0, \bar{A}] \) for given initial capital and debt positions. To capture the traditional role of entrepreneurs as net demanders of finance, we assume the initial capital and debt levels of both entrepreneurial sectors are such that they remain net borrowers in period 1.

A. Unconstrained Economy

If period 1 productivity in sector \( X \) is sufficiently high \( A_1^X \geq A_{unc} \), the sector is unconstrained and the economy follows standard neoclassical rules. The threshold is determined by the productivity level \( A_1^X \) that leads to a sectoral net worth \( n_1^X \) such that
\[ (1 + \alpha) n_1^X = k_2^X \]
Households receive the promised amount \( R_1 d_1 = R_1^X d_1^X + R_1^Z d_1^Z \) in period 1 and supply loans according to (1), and both entrepreneurial sectors demand loans according to their optimality condition (3). Within this region, greater productivity means higher entrepreneurial net worth \( n_1^X \) and therefore a lower demand for loans \( d_2^X \) \( (R_2) \). As a result, the interest rate \( R_2 \) declines, and the optimum amount of investment as well as profits in both sectors increase. A positive shock in sector \( X \) therefore spills over positively to sector \( Z \).

B. Decoupling

If the productivity of sector \( X \) drops below \( A_{unc} \), the sector becomes constrained. As long as net worth \( n_1^X \) is positive, the sector can honor its repayments and households receive the promised amount \( R_1 d_1 \) in period 1. This is the case as long as
\[ A_1^X \geq R_1^X d_1^X / F \left( k_1^X \right) \equiv A_{fail} \]
Within this region, the loan demand of sector \( X \) is constrained, reducing aggregate loan demand and therefore the interest rate. Specifically, lower period 1 productivity for a constrained entrepreneur tightens the constraint, leads to lower loan demand and a lower interest rate \( R_2 \). Sector \( Z \) reacts by increasing investment and profits, i.e. a negative shock in sector \( X \) spills over positively to sector \( Z \). The worse the productivity shock for sector \( X \), the better off sector \( Z \) – there is decoupling.\(^5\)

C. Contagion

For \( A < A_{fail} \), sector \( X \) defaults and households receive a total repayment of \( R_1 d_1 = R_1^X d_1^X + R_1^Z d_1^Z \) +

\(^5\)Note that there are two effects on the welfare of sector \( X \): on the one hand, it is hurt by the binding constraint, but on the other hand it benefits from the lower interest rate. The net effect of the two can initially be positive. However, as productivity declines further and the sector approaches the bankruptcy threshold, sectoral welfare will unambiguously decline until it reaches zero at the threshold.
A lower productivity shock reduces period 1 wealth of households/bankers, which makes them less willing to lend and increases the interest rate \( R_2 \) at which they are willing to provide loans. As a result sector \( Z \) invests less and obtains lower profits. Within this region, negative shocks to sector \( X \) spill over to sector \( Z \) – there is contagion.

III. Illustration

In figure 1 we illustrate the three regions through which the economy passes as we vary the productivity shock \( \tilde{A}_X^1 \) between 0 and \( \bar{A} \) on the horizontal axis (from right to left): coupling, decoupling, and contagion, each separated by a dotted vertical line. For the utility function of households and the production function we used \( u(c) = \log c \) and \( F(k) = \sqrt{k} \), and the parameter values chosen are \( \alpha = .5, \epsilon = 0 \), \( A = 1.2, A_i^C = 1 \), and \( k_i^1 = 1 \) and \( R_i^1d_i^1 = .5 \) in each sector \( i \).

For high realizations of the productivity shock \( \tilde{A}_X^1 \), i.e. to the right of the figure, both sectors are unconstrained and higher productivity in sector \( X \) raises welfare in both sectors as the cost of capital declines. In the center of the figure there is decoupling: since the demand for loans of sector \( X \) is progressively constrained, the interest rate declines and sector \( Z \) is better off. In the left region of the figure, sector \( X \) goes bankrupt and the supply of capital to the economy is reduced, pushing up the interest rate \( R_2 \). This hurts sector \( Z \), i.e. there is contagion.

IV. Extensions

There are several dimensions in which our benchmark model can be extended to provide further insights:

Time Structure In the recent financial crisis decoupling and contagion occurred consecutively, whereas the two are mutually exclusive outcomes in our stylized analysis. In a multi-period version of our model, contagion could occur after an episode of decoupling, if a series of adverse shocks progressively depletes the net worth of a constrained sector to the point where it is pushed into bankruptcy.

Factor Prices In our benchmark model, the only factor of production was capital. More generally, other factors such as labor or commodities are complements to capital in standard production functions. The less capital is employed in the economy, the lower demand for other factors. In labor markets with rigid wages, this may lead to unemployment; in commodity markets to price declines.

Bankruptcy Costs If we augment our model to incorporate bankruptcy costs that reduce the receipts of banks by a factor \( (1 - \delta) \) in case of bankruptcy, then there would be a discontinuity at \( A_{\text{fail}} \) that would cause the interest rate to jump up and welfare to jump down, magnifying the impact of defaults.

Leveraged Banks Modifying our benchmark model by separating households and the banking sector and introducing leverage in the latter can amplify the propagation of defaults. Leverage implies that the impact of a shock on the net worth of banks is magnified. If banks experience financial constraints in their relationship with households, our contagion result are strengthened.

We explore these extensions in more detail in our companion paper (Anton Korinek, Agustín Roitman and Carlos A. Végh, 2010). with the behavior of actual lending rates during the recent crisis, though not with the Fed Funds rate: after the demise of Lehman, lending rates (and, since there was widespread rationing, shadow lending rates) shot up sharply.

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\( \bar{A}_i^X \) with sector \( X \) being wiped out, period 1 productivity in this region affects directly the capital position of households/bankers. A lower productivity shock reduces period 1 wealth of households/bankers, which makes them less willing to lend and increases the interest rate \( R_2 \) at which they are willing to provide loans. As a result sector \( Z \) invests less and obtains lower profits. Within this region, negative shocks to sector \( X \) spill over to sector \( Z \) – there is contagion.

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V. Conclusions

We have presented a stylized model that captures the decoupling-recoupling phenomenon observed after the subprime crisis erupted in the United States in February 2007. There are two “sectors” in our model that experience first decoupling and then recoupling as productivity falls in one of them. These two sectors could be given a literal interpretation (i.e., the real estate and manufacturing sectors within a country being financed by the financial sector) or a broader interpretation in terms of different countries (i.e., United States and Brazil being financed by international capital markets). In our companion paper, we embed this mechanism in a model with leverage and show how this decoupling-recoupling cycle is further amplified.

REFERENCES


